Electro-Magnetic Reactions in Few-Body Systems From Nuclei to Cold-Atoms

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The Racah institute for Physics
The Hebrew University, Jerusalem, Israel

INT Program
Light Nuclei From First Principles
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האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

Collaboration

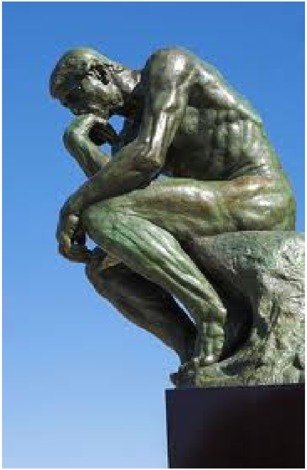
Jerusalem, Israel
B. Bazak*, D. Gazit, E. Liverts, N. Nevo*

Trento, Italy
W. Leidemann, G. Orlandini

Moscow, Russia
V. Efros

TRIUMF, Canada
S. Bacca

What can we learn from photo reactions?



1. Understanding of the systems at hand.
2. A test of the Hamiltonian at regimes not accessible by elastic reactions.
3. Reaction rates as input for experiments or applications (e.g. astrophysics).
4. Underlying degrees of freedom.
5. The transition from single particle to collective behavior.

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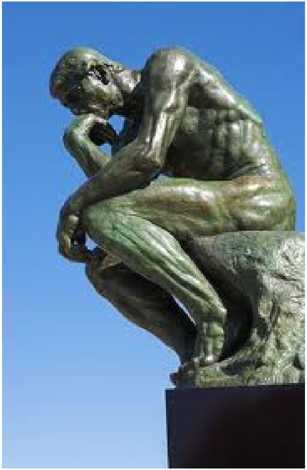

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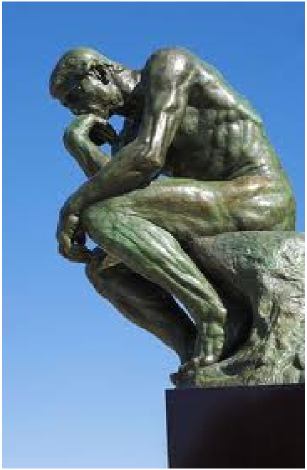

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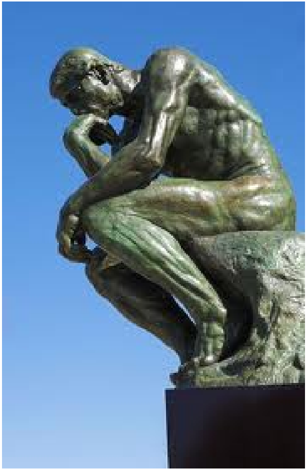

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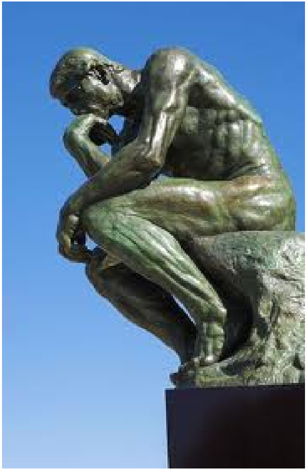

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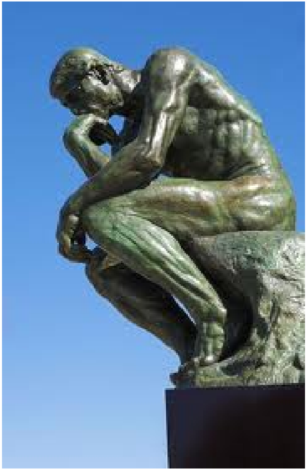
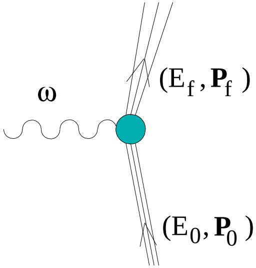


Photo Reactions

The Interaction Hamiltonian between the photon field $\boldsymbol{A}(\boldsymbol{x})$ and the atomic/nuclear system

$$H_I = -\frac{e}{c}\int d\boldsymbol{x}\boldsymbol{A}(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x})$$



The current is a sum of convection and spin currents

$$\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{J}_c(\boldsymbol{x}) + \boldsymbol{\nabla}\times\boldsymbol{\mu}(\boldsymbol{x})$$

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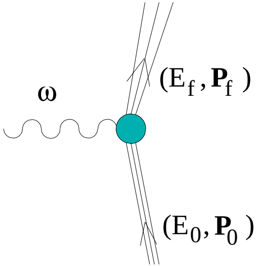

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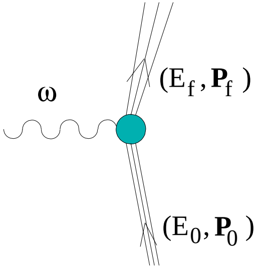

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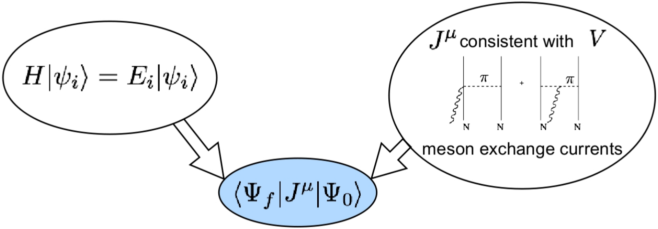

The Wave Functions

- We solve the A-body non-realtivistic Schroedinger equation.
- The Hamiltonian

$$H = T + \sum_{ij} V_{ij}^{(2)} + \sum_{ijk} V_{ijk}^{(3)} + \dots$$

High precision two-nucleon potentials, well constraint by NN phaseshifts
Less established 3NF

- EFT provides a solid theoretical framework for construction of the potentials.
- Phenomenological potential models are not that bad either.

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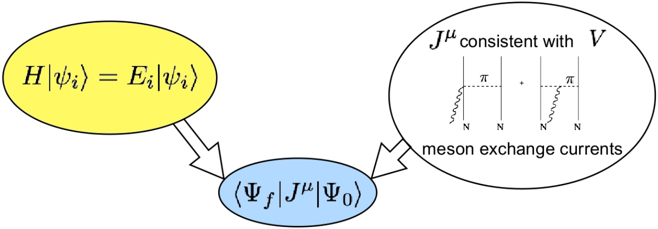

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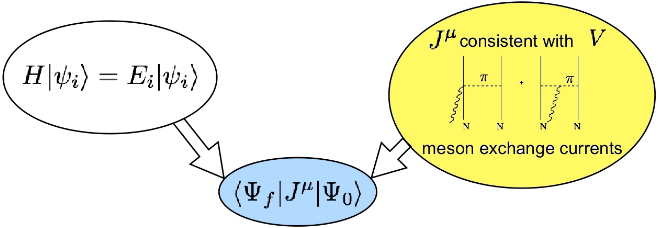

The Electro-Magnetic Current

- The EM current is a sum of convection and spin currents

$$\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{J}_c(\boldsymbol{x}) + \boldsymbol{J}_s(\boldsymbol{x}) = \boldsymbol{J}_c(\boldsymbol{x}) + \nabla \times \boldsymbol{\mu}(\boldsymbol{x})$$

- Classicaly, the convection current $\boldsymbol{J}_c = \sum_i Z_i \boldsymbol{v}_i$ is the flow of the charged particles.
- In nuclei $\boldsymbol{J}_c(\boldsymbol{x})$ is mainly due to proton movement.
- Meson exchange between nucleons leads to $2, 3, \ldots$-body currents $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2 + \ldots$
- Cold atoms are neutral $\boldsymbol{J}_c(\boldsymbol{x}) = 0$ and the current $\boldsymbol{\mu}(\boldsymbol{x})$ is dominated by the electronic spins.

Nuclear Physics - A tale of two potentials

- The nuclear Hamiltonian

$$H = -\sum_i \frac{\hbar^2}{2m_N}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- The Potential is composed of EM and NUCLEAR terms.
- The NUCLEAR force cannot be derived from QCD and must be modeled.

The AV18 NN-Force

The JISP16 Potential

- A Kernel expansion of the potential

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- v_{ij}^{π} - A Yukawa type interaction $e^{-\mu r}/r$, $v_{ij}^{2\pi} \propto e^{-2\mu r}/r$.
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- NNN force must be supplemented to reproduce 3,4-body binding-energies.

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- A formal expansion of the potential

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- JISP16 J-matrix Inverse Scattering Potential, Shirokov *et* al.

Binding Energies

	AV18+UBIX	JISP16	Nature
D	2.24	2.24	2.24
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^{3}He	7.74	7.65	7.72
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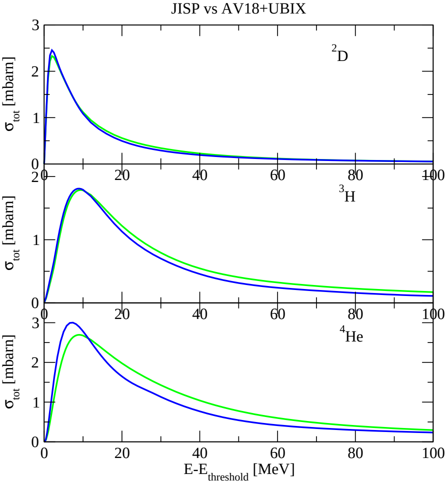
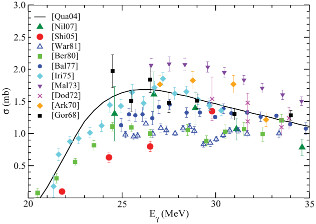
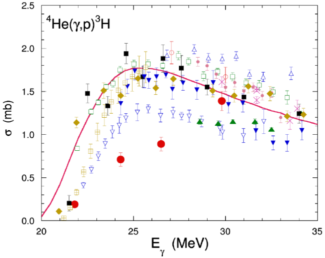
Photodisintegration cross-section for A=2,3,4



The Experimental Verdict !





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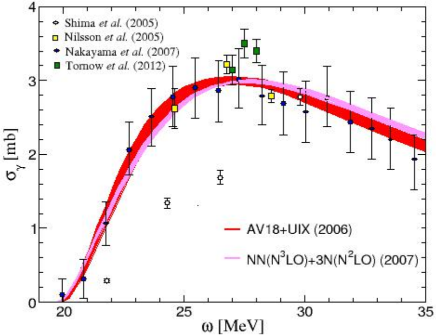

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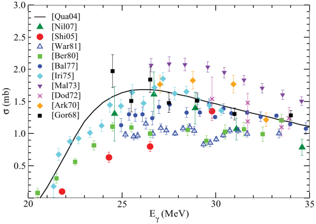

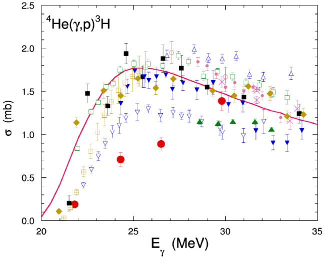

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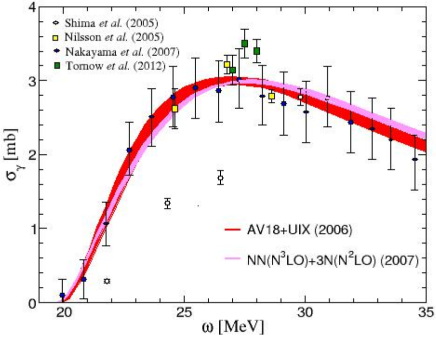
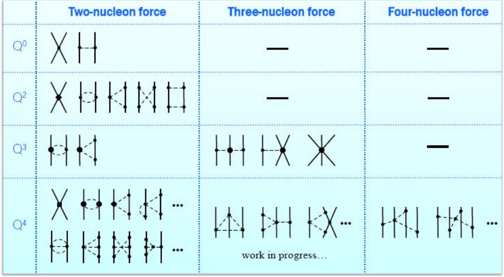


Effective Field Theory potentials

Effective Field Theory

- Expansion in small momentum Q.
- Contains all terms compatible with QCD up to a given order.
- NNN and NNNN forces come in naturally at orders N2LO and N3LO.



2 nucleon force ≫ 3 nucleon force ≫ 4 nucleon force ...

χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^{3}LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

$$
\begin{aligned}
V &= -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{\boldsymbol{q}^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
&+ C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
&+ V_{NLO} + V_{N2LO} + \dots
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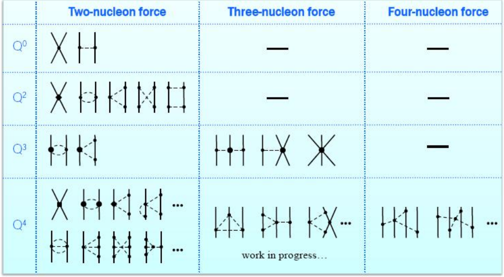

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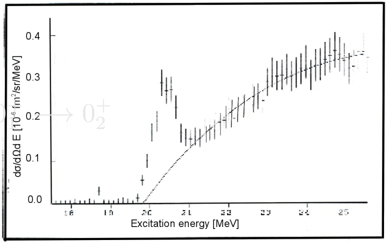
^{4}He*	7.3(1)	7.1(2)	8.21

The transition form factor $0_1^+ \longrightarrow 0_2^+$

Electron scattering on ^{4}He, the 0_2^+ resonance

G. Koebschall et al./ Quasi bound state in ^{4}He - Nucl. Phys. A405, 648 (1983)



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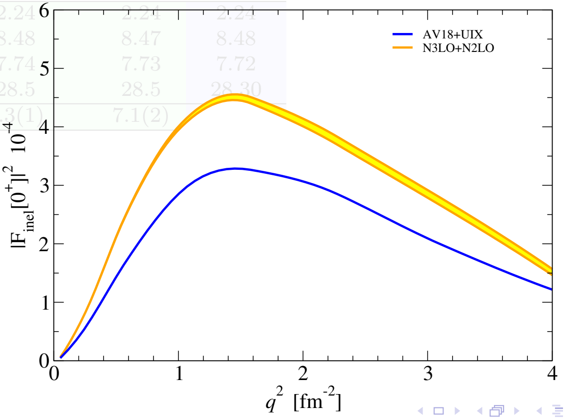
D	2.24	2.24	2.24
^{3}H	8.48	8.47	8.48
^{3}He	7.74	7.73	7.72
^{4}He	28.5	28.5	28.30
^{4}He*	7.3(1)	7.1(2)	

The transition form factor $0_1^+ \longrightarrow 0_2^+$



A tale of two potentials II

AV18+UIX ⇔ EFT

Electron scattering on ^{4}He, the 0_2^+ resonance

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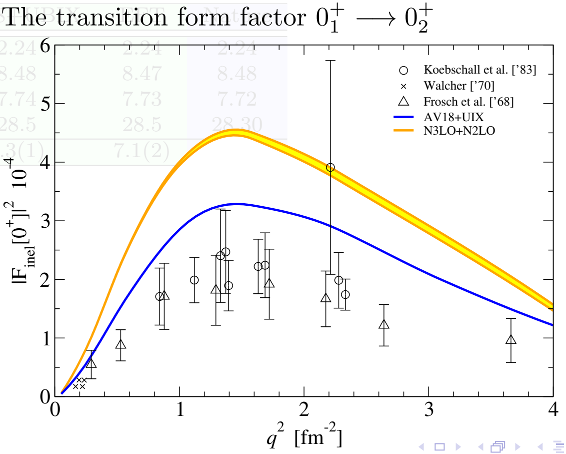
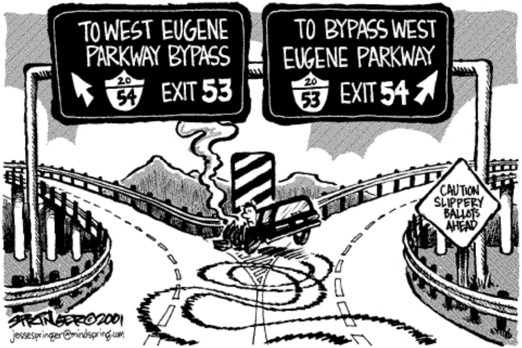


The ^{4}He 0_2^+ state - A short summary



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Bose systems, short range force, energy scale 10^{-9}eV

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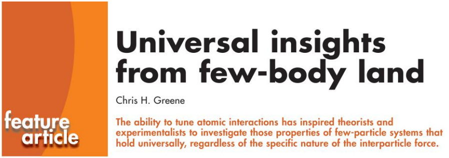
$$A + A + A \longrightarrow A_2 + A$$



feature article

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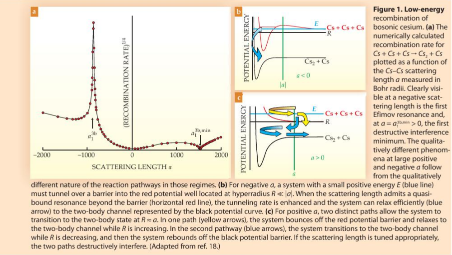
Physics Today, March 2010



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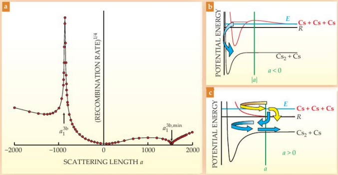

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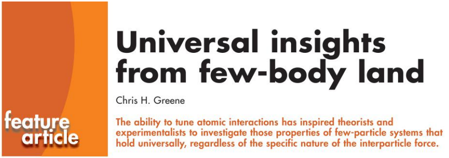
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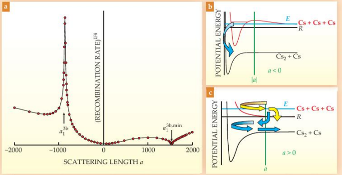

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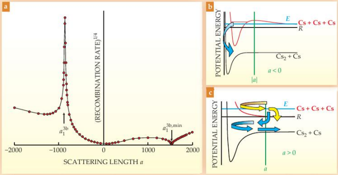

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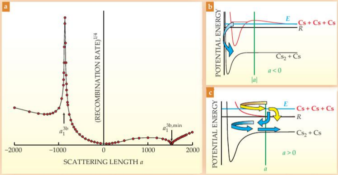

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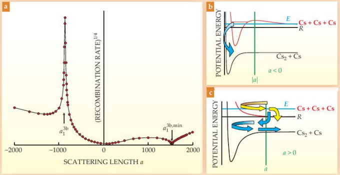

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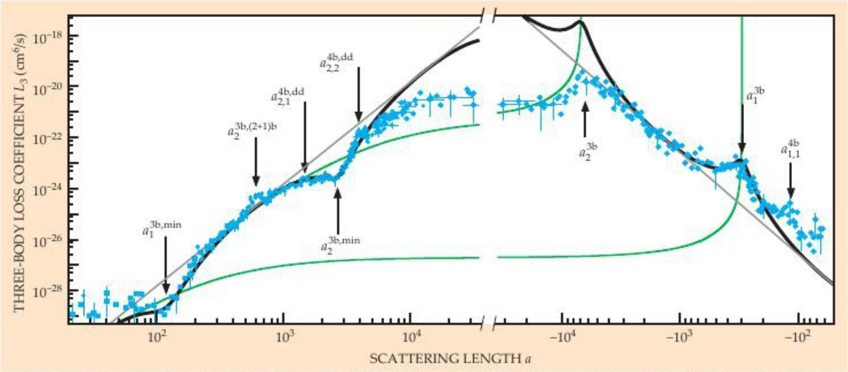
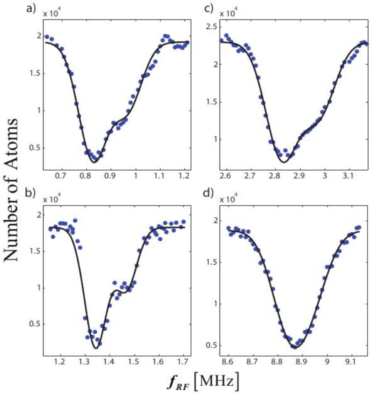
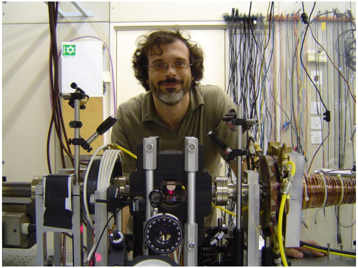


Photoassociation of Atomic Molecules

The quest for the Efimov Effect

RF-induce atom loss resonaces for different values of bias magnetic fields.





O. Machtey, Z. Shotan, N. Gross and L. Khaykovich
PRL **108**, 210406 (2012)

The Static Response - Inelastic Reactions

- The response of an A-particle system is closely related to the static moments of the charge density

$$\rho(\boldsymbol{x}) = \sum_{i}^{A} Z_i \delta(\boldsymbol{x} - \boldsymbol{r}_i)$$

- The Fourier Transform

$$\rho(\boldsymbol{q}) = \int d\boldsymbol{x} \rho(\boldsymbol{x}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} = \sum_{i}^{A} Z_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

- In the long wavelength limit $q \longrightarrow 0$

- For a system of **identical** particles

- **Conclusion A:** In general the Dipole is the leading term.
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- In the final state the photon can either change one of the spins or leave them untouched.
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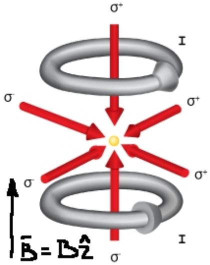

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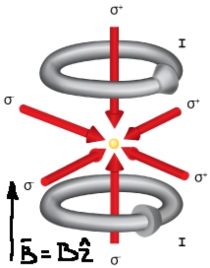

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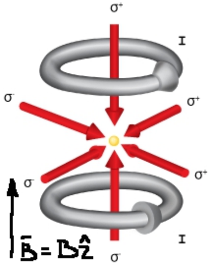

Photo Reactions with Cold-Atoms

- For RF photons in the MHz region the wave length is meters so $qR \ll 1$.
- The Atoms reside in a strong magnetic field, thus spins are "frozen"

$$|\Psi_0\rangle = \Phi_0(\boldsymbol{r}_i)|m_F^1 m_F^2 \ldots m_F^A\rangle$$

- In the final state the photon can either change one of the spins or leave them untouched.
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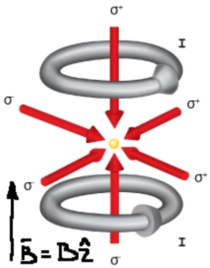

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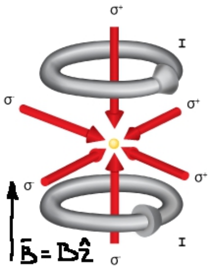


Photo Reactions with Cold-Atoms

- For Spin-flip reactions we get the "Fermi" operator

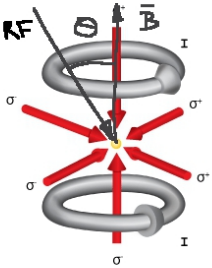
$$R(\omega) = Ck \sum_{f,\lambda} \left| \langle \Phi_f | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

- For Frozen-Spin reactions we get a sum of the monopole operator $\hat{M} = R^2 = \sum r_i^2$ and the Quadrupole operator $\hat{Q} = \sum r_i^2 Y_2(\hat{r}_i)$

$$\boldsymbol{O} = \alpha \hat{M} + \beta \hat{Q}$$

- The response is given by

$$R(\omega) = k^5 \sum_{f,\lambda} \left| \langle \Phi_f | \boldsymbol{O} | \Phi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$



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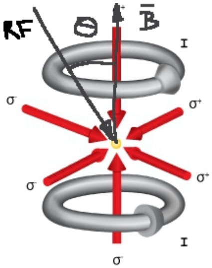

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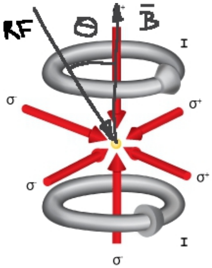

Photoassociation of The Atomic Dimer

- For the dimer case the response function can be written as

$$R(\omega) = C\omega^5 \left[\frac{1}{6^2} |\langle \varphi_0(q) \| \hat{M} \| \psi_0 \rangle|^2 + \frac{1}{5 \cdot 15^2} |\langle \varphi_2(q) \| \hat{Q} \| \psi_0 \rangle|^2 \right]$$

- Where the G.S. wave function is given by

$$\psi_0 = Y_0 \sqrt{2\kappa} e^{-\kappa r}/r \quad ; \quad \kappa \approx 1/a_s$$

- The continuum state is given by $\varphi_\ell(q) = Y_\ell(\hat{r})\chi_\ell(r)/r$

$$\chi_\ell(r) = 2qr[\cos\delta_\ell j_\ell(qr) - \sin\delta_\ell n_\ell(qr)]$$

- The $\ell = 0$ matrix element

$$|\langle \varphi_0(q) \| \hat{M} \| \psi_0 \rangle|^2 = \frac{1}{4\pi} \left(\frac{4q\sqrt{2\kappa}}{(q^2+\kappa^2)^3} \right)^2 \left[\cos\delta_0(3\kappa^2 - q^2) - \sin\delta_0 \frac{\kappa}{q}(3q^2 - \kappa^2) \right]^2$$

- The $\ell = 2$ matrix element, assuming $\delta_2 = 0$

$$|\langle \varphi_2(q) \| \hat{Q} \| \psi_0 \rangle|^2 = \frac{5}{4\pi} \left[\frac{16q^3\sqrt{2\kappa}}{(q^2+\kappa^2)^3} \right]^2$$

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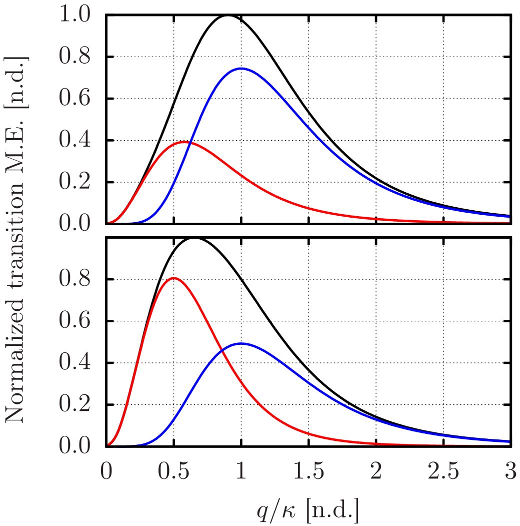
Photoassociation of The Atomic Dimer

The s-wave and d-wave components in the response function

- upper panel $a/r_{eff} = 2$
- lower pannel $a/r_{eff} = 200$
- red - r^2 monopole
- blue - quadrupole



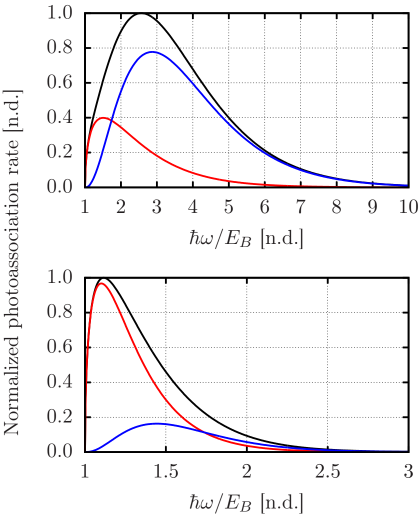
Photoassociation rates

Photoassociation of ^{7}Li atoms

$a_s = 1000 a_0$
$T = 5\mu$K (lower panel), $T = 25\mu$K (upper panel)

red - r^2 monopole, blue - quadrupole

The relative contribution to the peak



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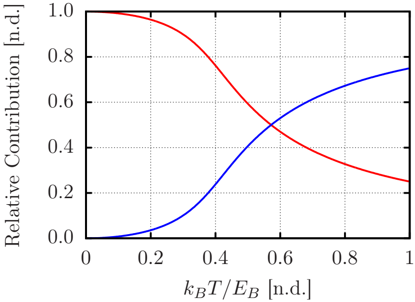
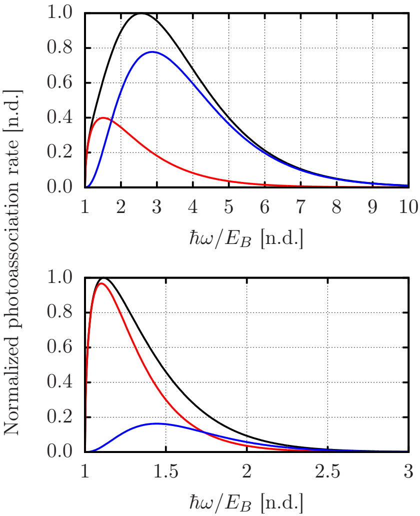
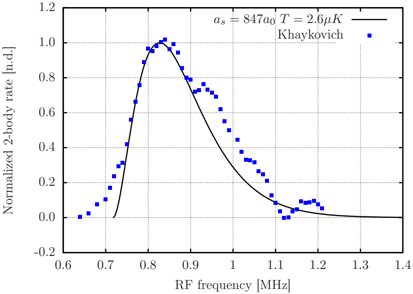
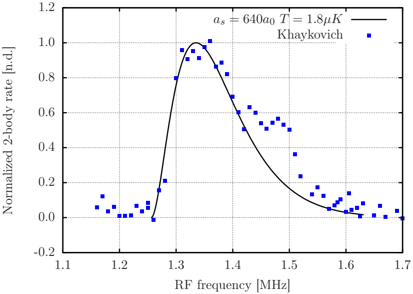




Photoassociation of The Atomic Dimer

Comparison to the Khaykovich group data





- The fitted values of a_s and T are in reasonable agreement with the estimates of the experimental group.
- Effect of RF field on dimers not included.
- Finite time effect
- Disagreement are due to 3-body (4-body?) association.
- Effects of $\delta_2 \neq 0$ are negligible.

Summary and Conclusions

1. EM reaction provides a prism of the nuclear Hamiltonian.
2. The 0_2^+ poses a problem to our contemporary understanding.
3. Relation to the A_y problem?
4. The new RF experiments in Cold-Atoms systems carry much in common with photo-reactions and charged current reactions in nuclei.
5. For spin-flip reaction a Fermi type operator is the leading contribution to the cross-section, and $R(\omega) \propto \omega$.
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Few fish (but from first principles)...
Russian river, Alaska