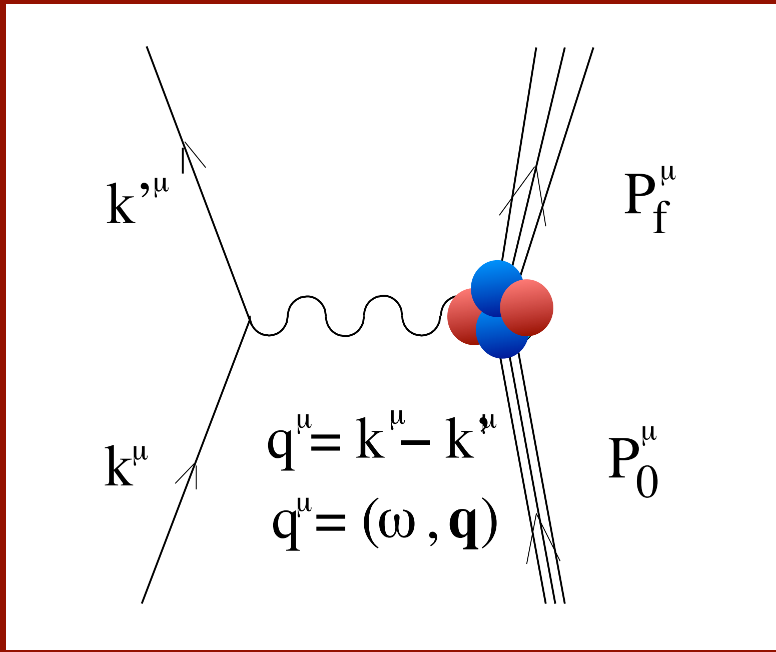
Three-nucleon Force Effects in Electron Scattering Observables

Sonia Bacca | Theory Group | TRIUMF

In collaboration with:
Nir Barnea, Winfried Leidemann, Giuseppina Orlandini



Electron Scattering Reaction

Outline:

- Motivation
- Theoretical tools
- Results on ^4He

Workshop on "Electroweak properties of light nuclei"
INT Seattle, Nov. 5-9 2012

Motivation

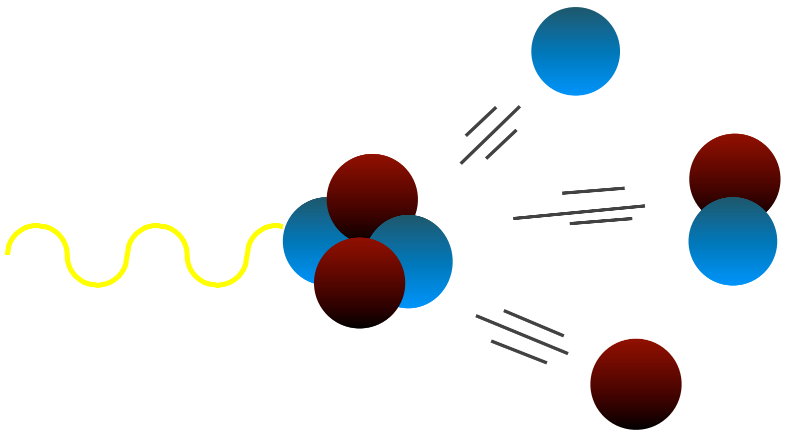
- The coupling constant << 1

"*With the electro-magnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"

[De Forest-Walecka, Ann. Phys. 1966]

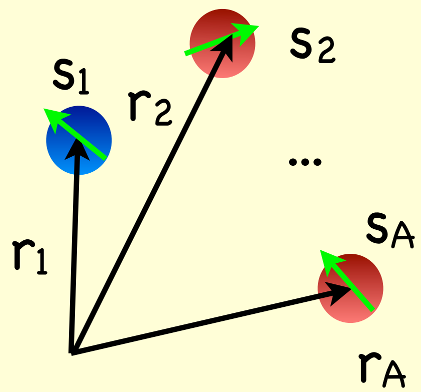
$$\sigma \propto \left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

- In few-body physics one can perform exact calculations both for bound and scattering states ➡ test the nuclear theory on light nuclei



- Provide useful numbers for astrophysics:
  - radiative capture reactions
  - interaction of photons with nucleonic matter ...

Electromagnetic Reactions: Ingredients



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \dots$$

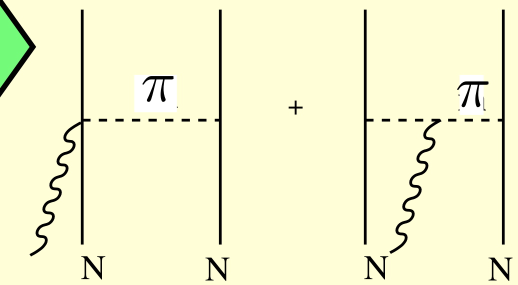
High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

Conventional Nuclear Physics
AV18+UIX, ..., J_2

Effective Field Theory
N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$



exchange currents

subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$\sigma \propto \left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

Exact Initial state &
Final state in the continuum at different energies and for different A

Objective

★ Compare with experiment to test nuclear theories and learn about its predictive power

★ Learn about the role of many-nucleon forces and currents by switching them on/off

Final State Interaction

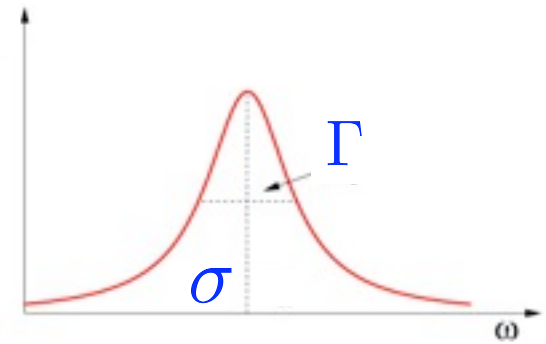
Exact evaluation of the final state in the continuum is limited in energy and A

Solution: The Lorentz Integral Transform Method

Efros, Leidemann, Orlandini, PLB **338** (1994) 130
Efros, Leidemann, Orlandini, Barnea, JPG.: Nucl.Part.Phys. **34** (2007) R459

$$R(\omega,\mathbf{q}) = \sum_f |\langle\psi_f|J^\mu(\mathbf{q})|\psi_0\rangle|^2\,\delta(E_f - E_0 - \omega)$$

$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega,\mathbf{q})}{(\omega-\sigma)^2+\Gamma^2} = \left\langle\tilde{\psi}|\tilde{\psi}\right\rangle$$



where $\left|\tilde{\psi}\right\rangle$ is obtained solving

$$(H - E_0 - \sigma + i\Gamma)|\tilde{\psi}\rangle = J^\mu(\mathbf{q})|\psi_0\rangle$$

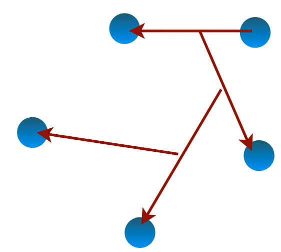
- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- If the r.h.s. is finite $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

$$L(\sigma,\Gamma) \xleftrightarrow{\text{inversion}} R(\omega,\mathbf{q})$$ The exact final state interaction is included

Hyper-spherical Harmonics Expansion

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\dots,\vec{\eta}_{A-1})\rangle$$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM}$ $\vec{\eta}_1, \dots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

ρ, Ω $$\rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H_0(\rho,\Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right)[\mathcal{Y}_{[K]}^{\mu}(\Omega)\chi_{ST}^{\bar{\mu}}]_{JT}^a$$

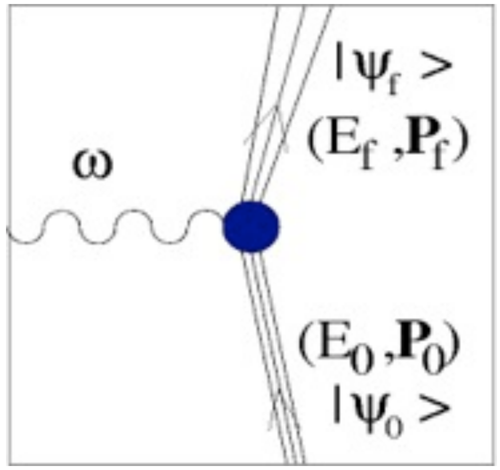
Asymptotic $e^{-a\rho}$ $\rho \to \infty$

Model space truncation $K \leq K_{max}$, Matrix Diagonalization $$\langle\psi|H_{(2)}|\psi\rangle = \frac{A(A-1)}{2}\langle\psi|H_{(A,A-1)}|\psi\rangle$$

Antisymmetrization algorithm Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

Introduce an effective interaction a la Lee-Suzuki Barnea, Leidemann, Orlandini PRC **61** (2000) 054001

Photo-disintegration Reaction



Real Photon

$$\omega = \mathbf{q} = E_f - E_0$$

Inclusive cross section γ + A $\longrightarrow$ X

$$\sigma_\gamma \propto \sum_f \left| \left\langle \Psi_f | J_T | \Psi_0 \right\rangle \right|^2$$

$$J_T = (J_1^c + J_1^s + J_2)_T$$

explicit degrees of freedom: p and n

implicit degrees of freedom $\nabla \cdot J_2 = -i[V, \rho]$

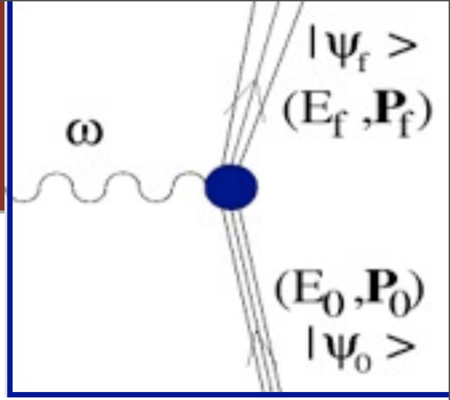
Lucky case: Photoabsorption cross section at low energy!
Via “Siegert theorem” MEC are implicitly included in the dipole response

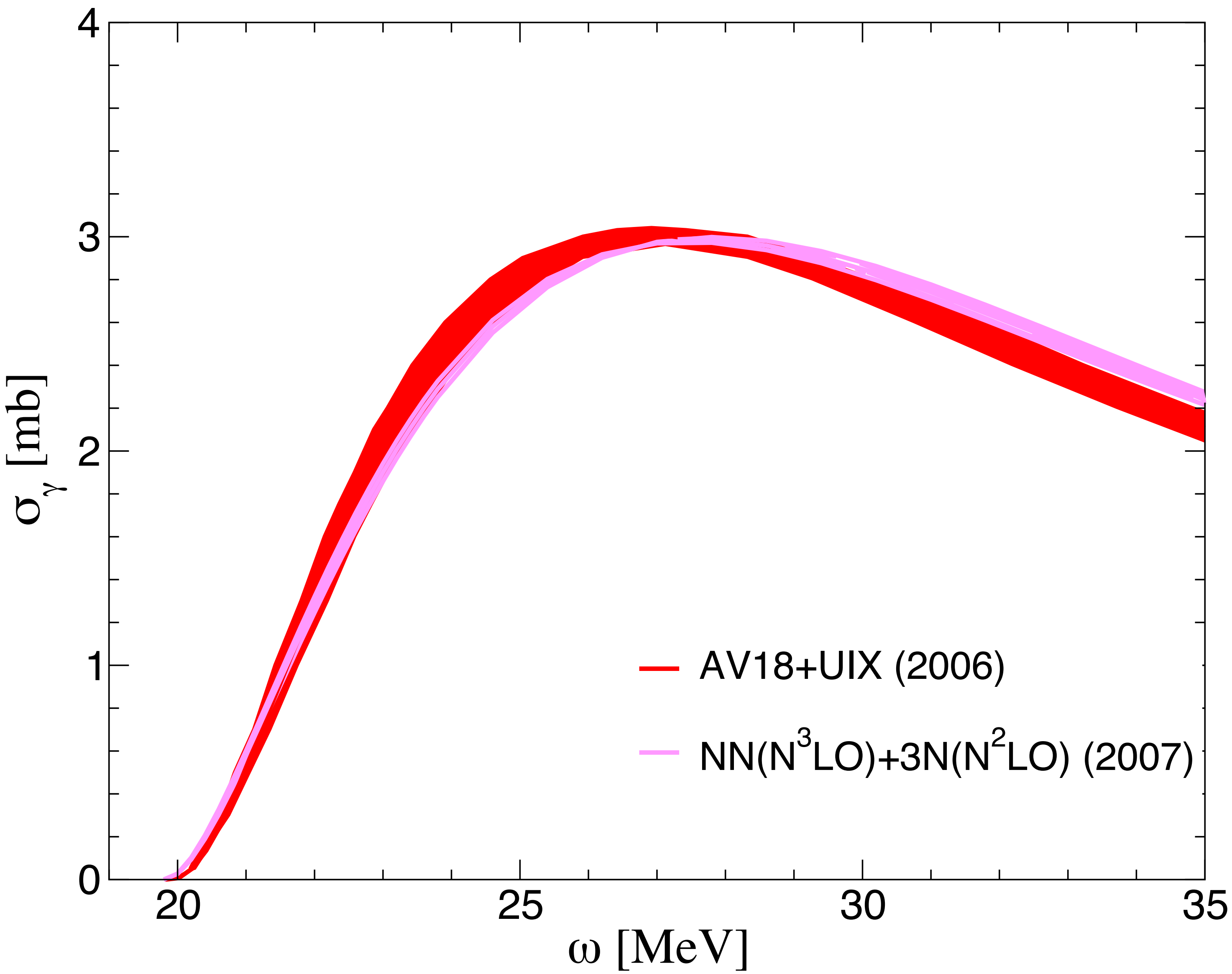
$$\sigma_\gamma = \frac{4\pi^2\alpha}{3} \omega R^{E1}(\omega) \qquad R^{E1}(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \Psi_f | E1 | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

Unretarded dipole approximation: d → 87% of MEC considered up to 100 MeV!

Theory

$$\gamma + {}^4\mathrm{He} \longrightarrow X$$





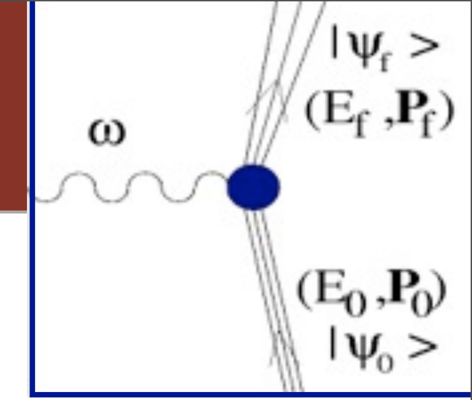
Past

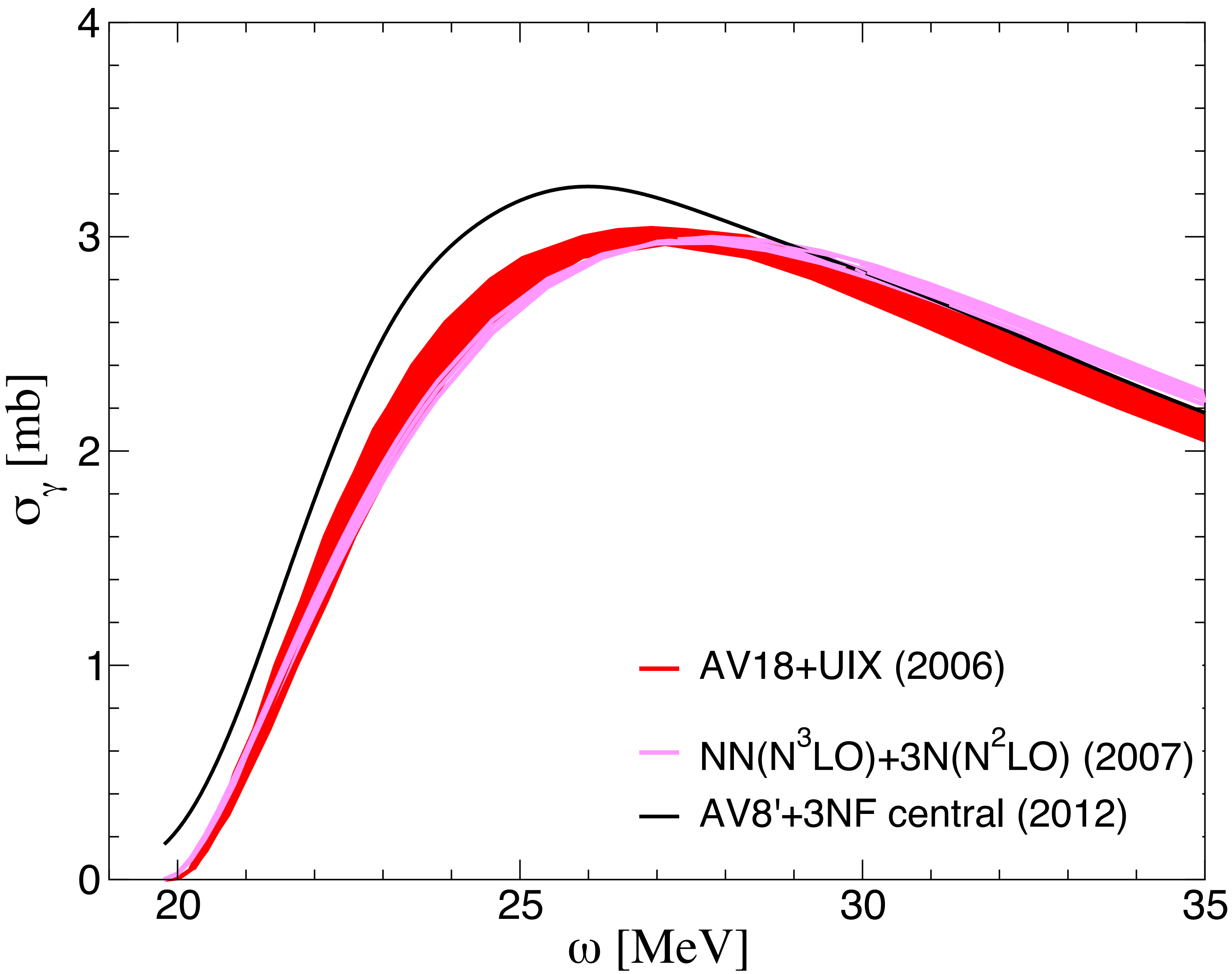
Conventional Hamiltonian
D.Gazit, S.B. *et al.* PRL **96** 112301 (2006)

EFT Hamiltonian
S.Quaglioni and P.Navratil PLB **652** (2007)

NN(N^3LO) Entem-Machleidt PRC68, 041001(R) (2003)
3N(N^3LO) local version from Navratil with
C_D=1 C_E=-0.029

Theory $\gamma + {}^4\mathrm{He} \longrightarrow X$





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New

Phenomenological Hamiltonian
W.Horiuchi *et al.* PRC **85** 054002 (2012)

⇨ Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak

Theory/Experiment $\gamma + {}^4\mathrm{He} \longrightarrow X$

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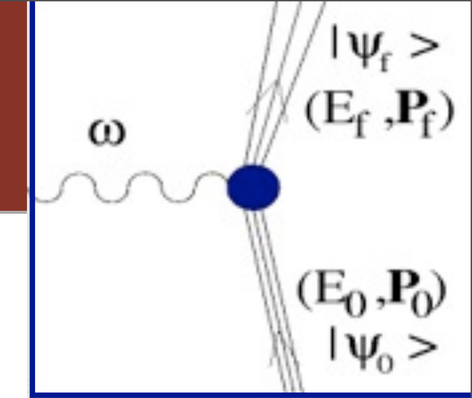

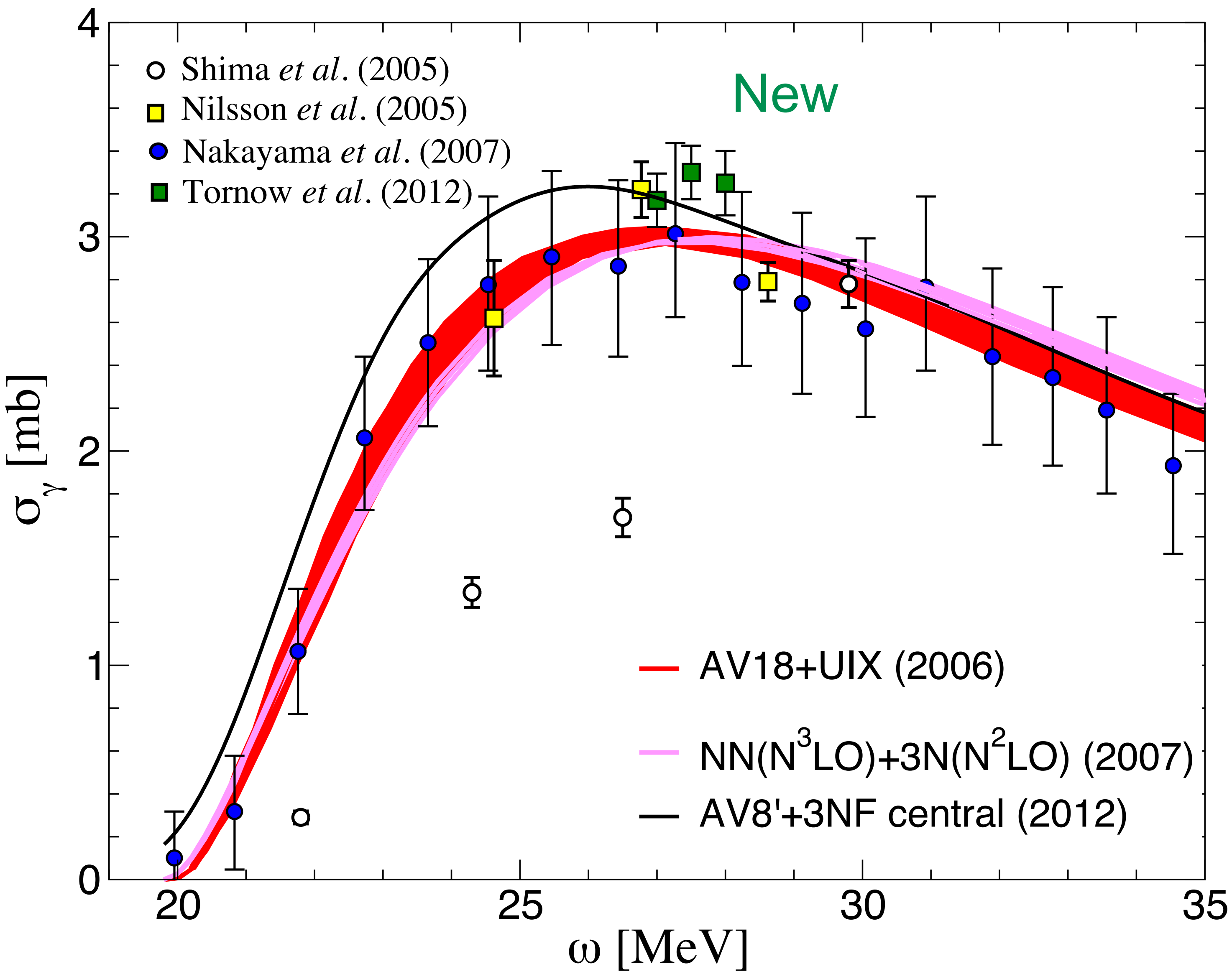

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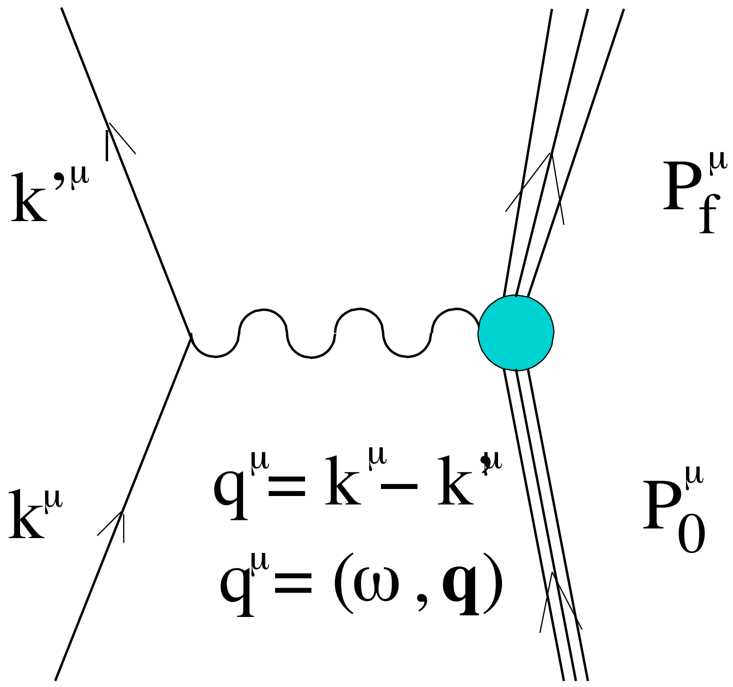
New

Phenomenological Hamiltonian
W.Horiuchi *et al.* PRC **85** 054002 (2012)

- Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak
- More recent experimental activity seems to confirm higher data with peak around 27 MeV

Electron Scattering Reaction



Virtual Photon

$$(\omega, \mathbf{q})$$

can vary independently

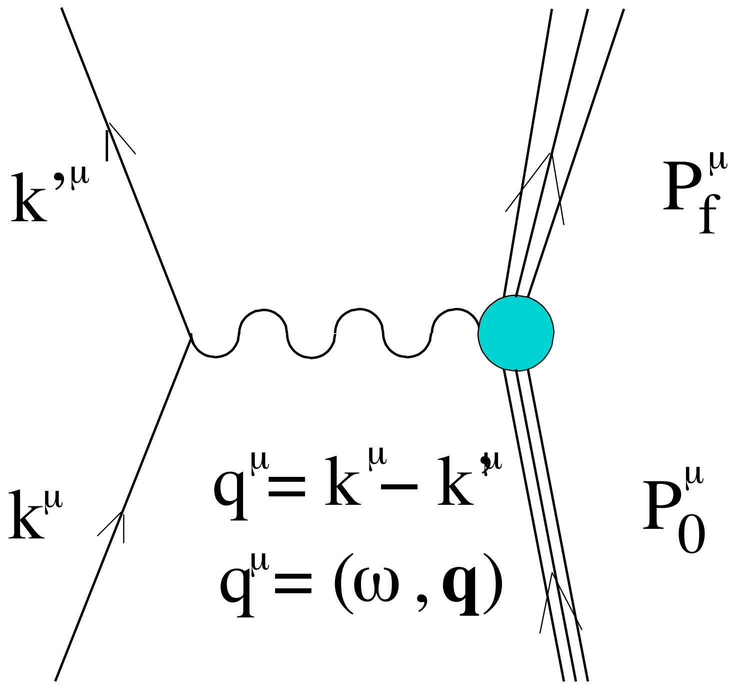
Inclusive cross section A(e,e')X

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

with $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ scattering angle

and σ_M Mott cross section

Electron Scattering Reaction



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Inclusive cross section A(e,e')X

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega,\mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega,\mathbf{q}) \right]$$

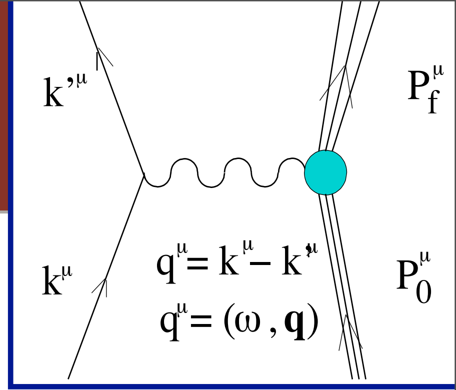
$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | \, \rho(\mathbf{q}) \, | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$ ← charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | \, J_T(\mathbf{q}) \, | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$ ← current operator

Electron Scattering Reaction

^{4}He → First step:
Study $R_L(\omega, \mathbf{q})$ (many-body operators negligible)
to investigate the effect of 3NF

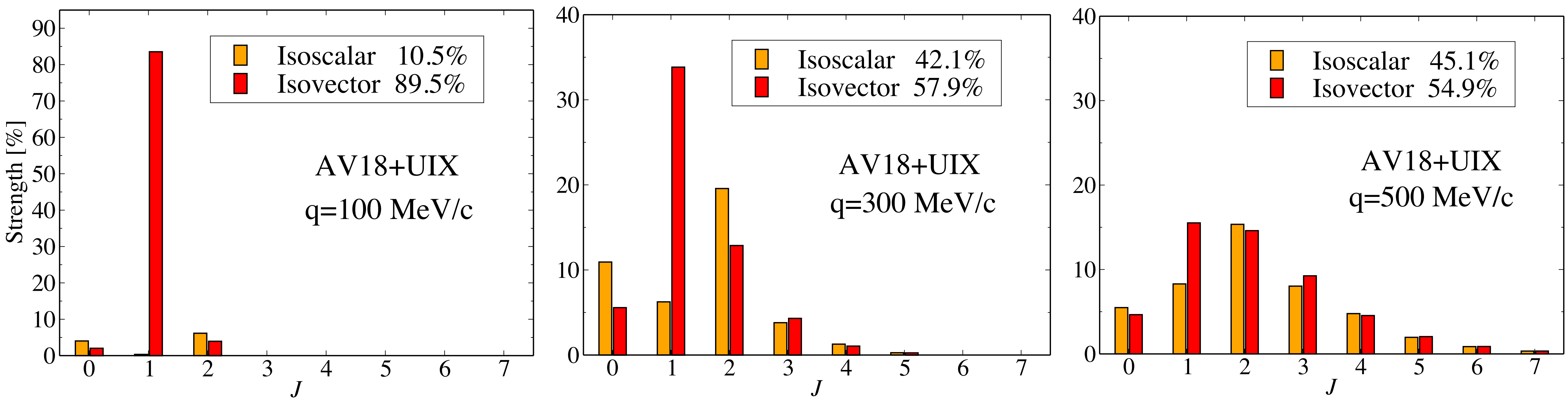
Electron Scattering Reaction



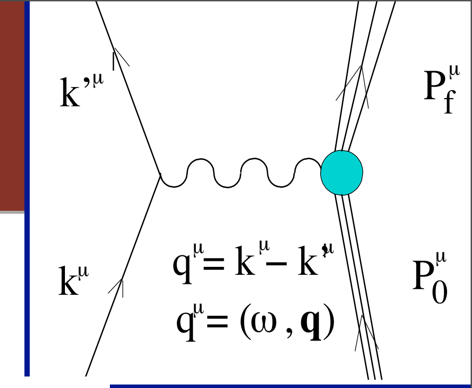
$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle|^2 \, \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

$$\rho(\mathbf{q}) = \sum_k^A e^{i\mathbf{q}\cdot\mathbf{r}'_k} \frac{1+\tau_k^3}{2} = \sum_J^\infty C_J^S(\mathbf{q}) + C_J^V(\mathbf{q})$$

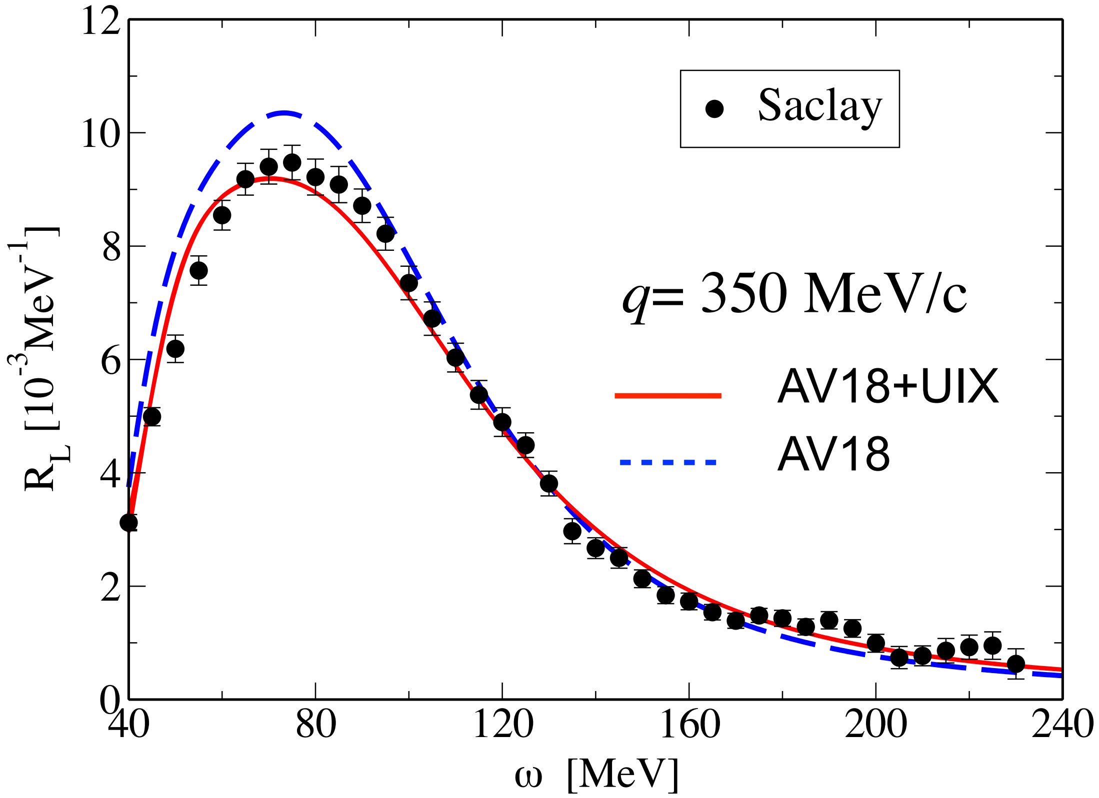
Distribution of the total inelastic strength among multipoles: first sum rule



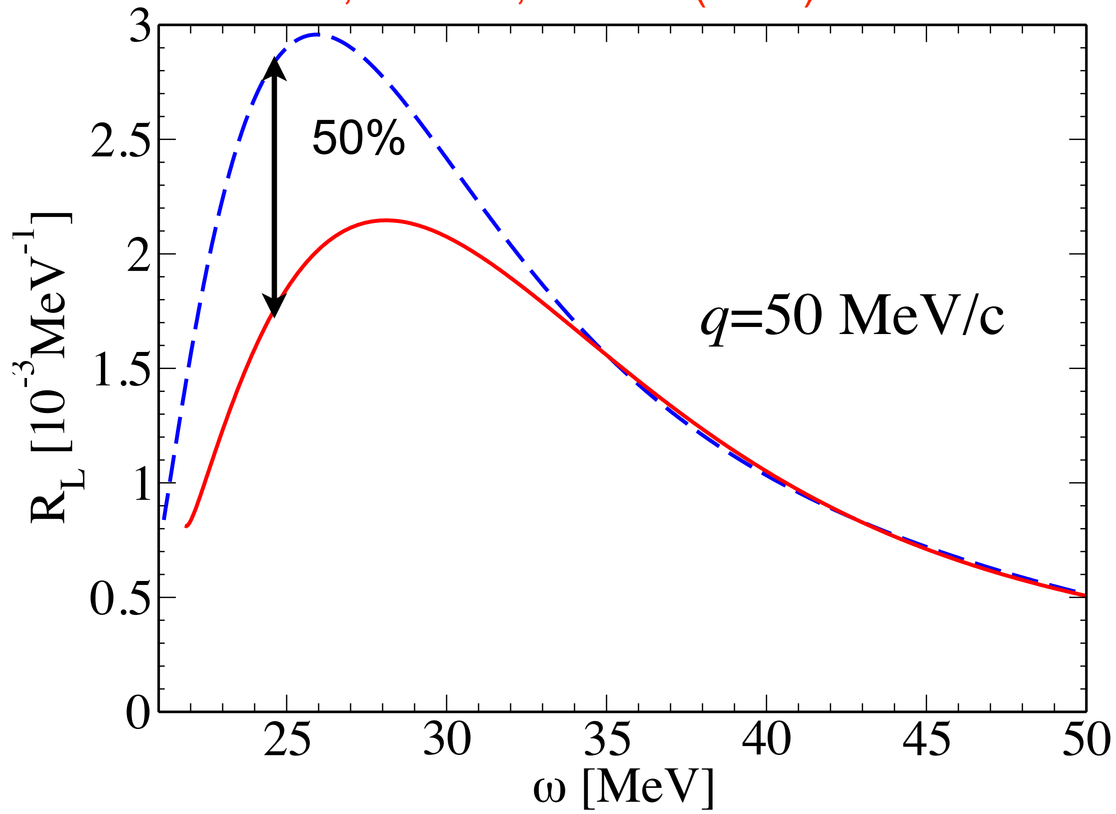
Electron Scattering Reaction



S.B. *et al.*, PRL **102**, 162501 (2009)



S.B. *et al.*, PRC **80**, 064001 (2009)

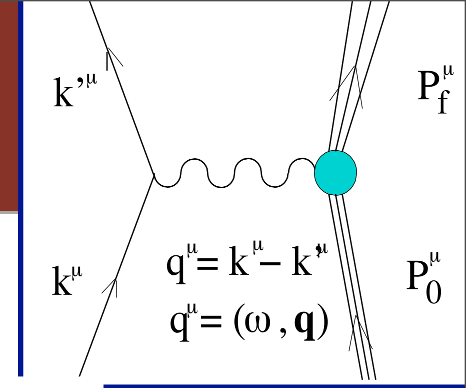


Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%

Note:

Structure of the response close to threshold is not considered here.
This is the response 1-2 MeV above threshold.

Electron Scattering Reaction



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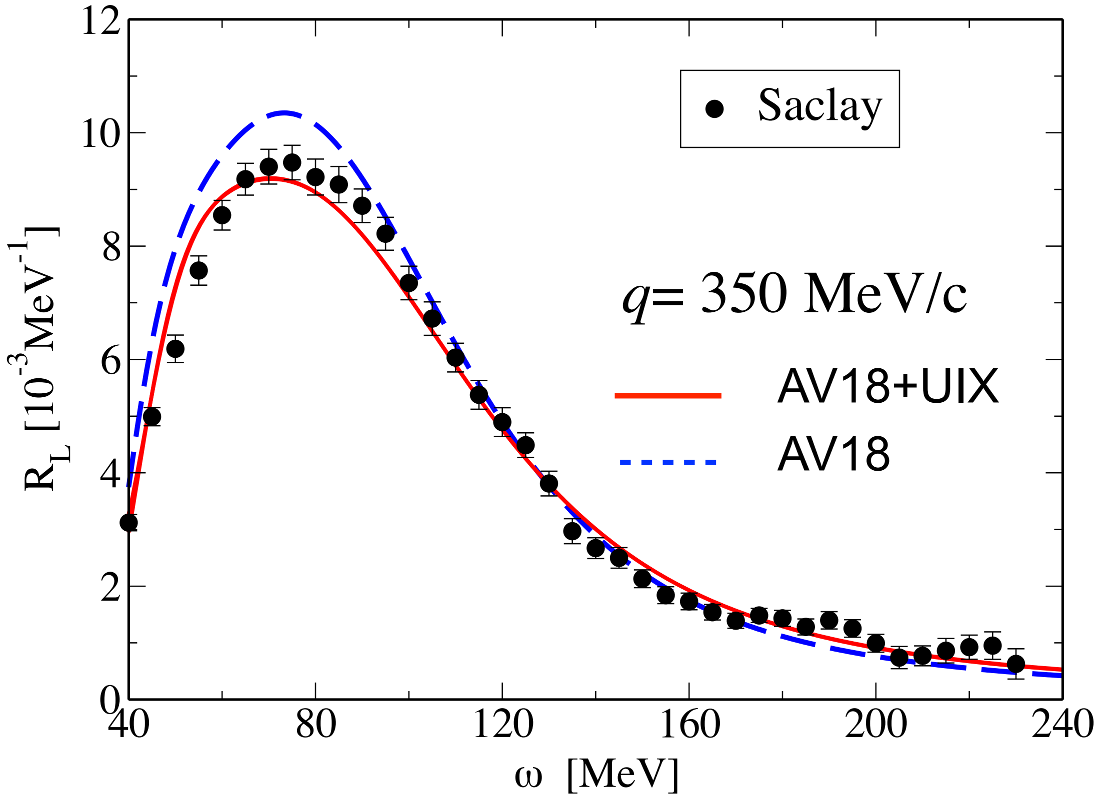

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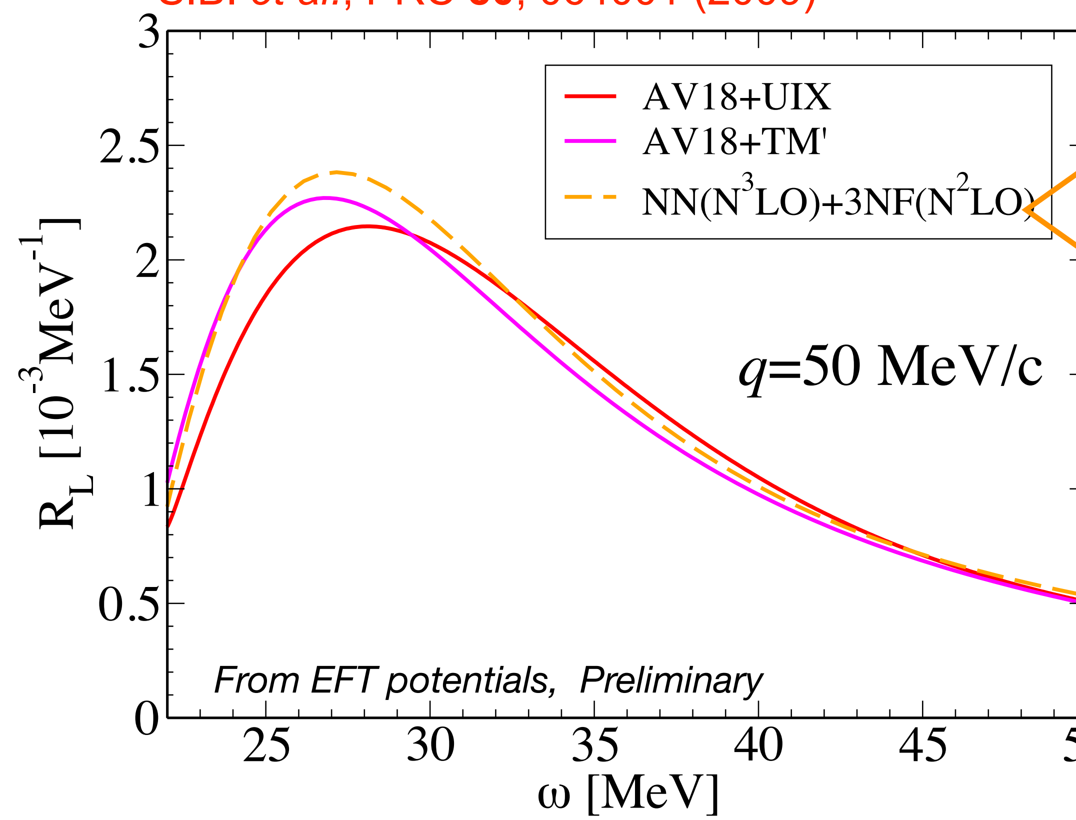
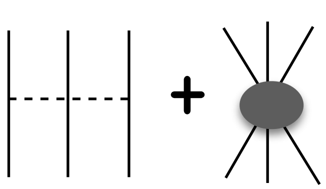
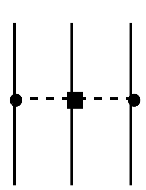


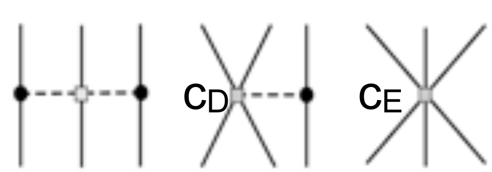
$C_D=1$
$C_E=-0.029$

- Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%
- It is not a simple binding effect!

	AV18+UIX	AV18+TM'	NN(N³LO)+3NF(N²LO)
B.E./MeV	28.40	28.46	28.357(7)



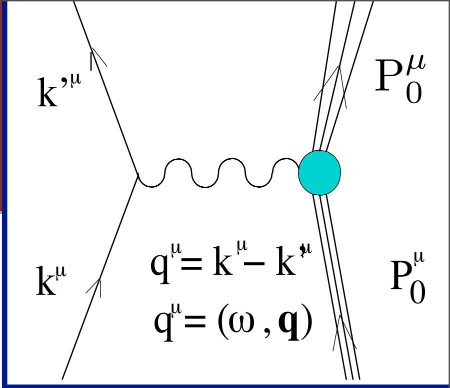




In agreement with:
28.34(2) NCSM FBS **41** (2007)
28.37 Pisa JPG **35** 063101 (2008)

- Stimulating new experiments: MAMI taken data q>150 MeV/c; S-Dalinac will maybe take data at lower q

Elastic Electron Scattering

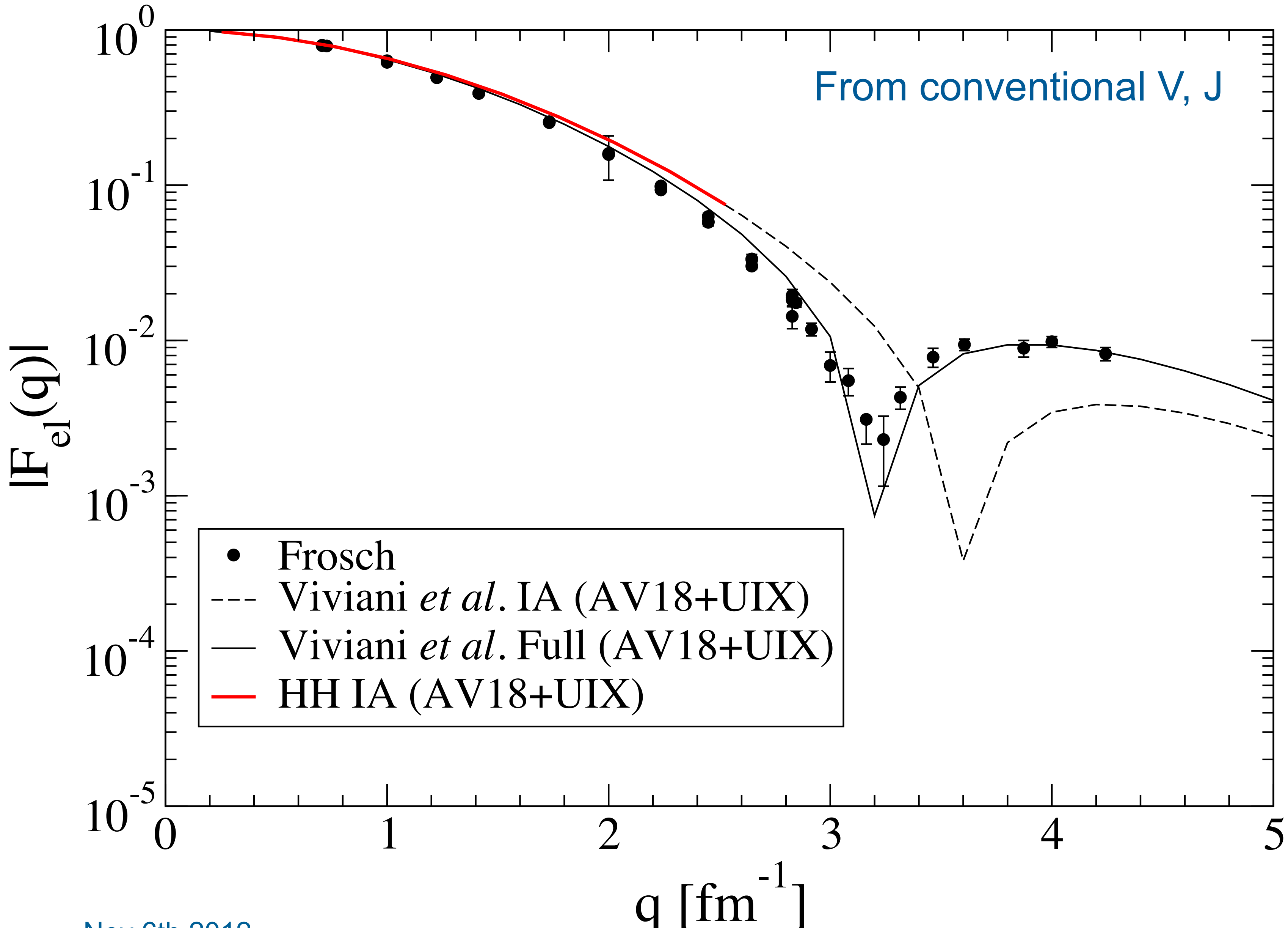


Elastic Form Factor $^4He(e,e')^4He$

$$F_{\rm el}(q) = \frac{1}{Z}\langle\Psi_0\,|\rho(q)|\,\Psi_0\rangle$$

Because J_0=0 only isoscalar monopole contribute

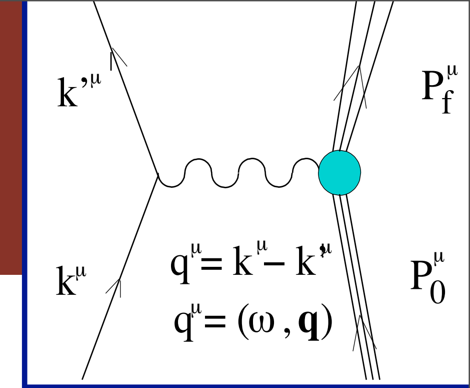
$$\rho(q) \rightarrow \mathcal{M}(q) = \frac{G_E^s(q)}{2}\sum_k^A j_0(qr_k)$$



Impulse approximation (IA) valid below 2 fm^{-1}
Viviani *et al.*, PRL **99** (2007) 112002

We reproduce perfectly Viviani's results in impulse approximation

Monopole Resonance ^{4}He(e,e')0$^+$

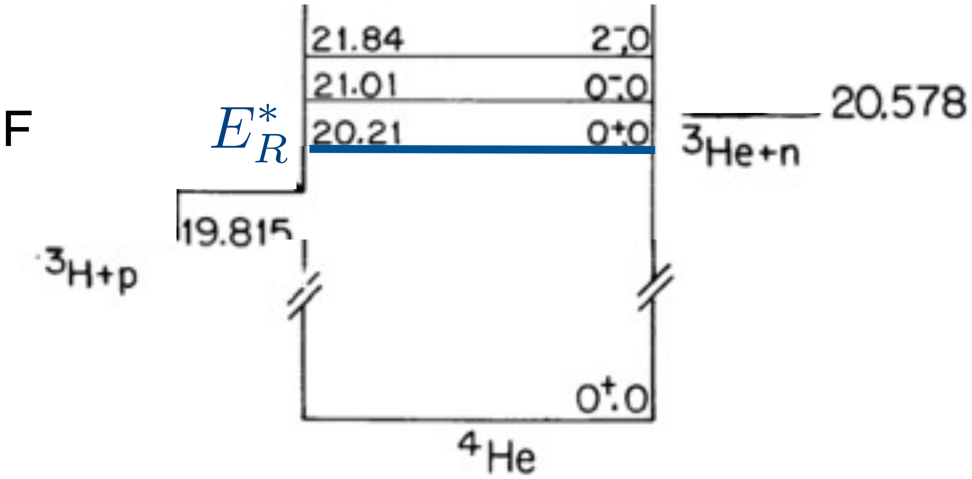


Resonant Transition Form Factor

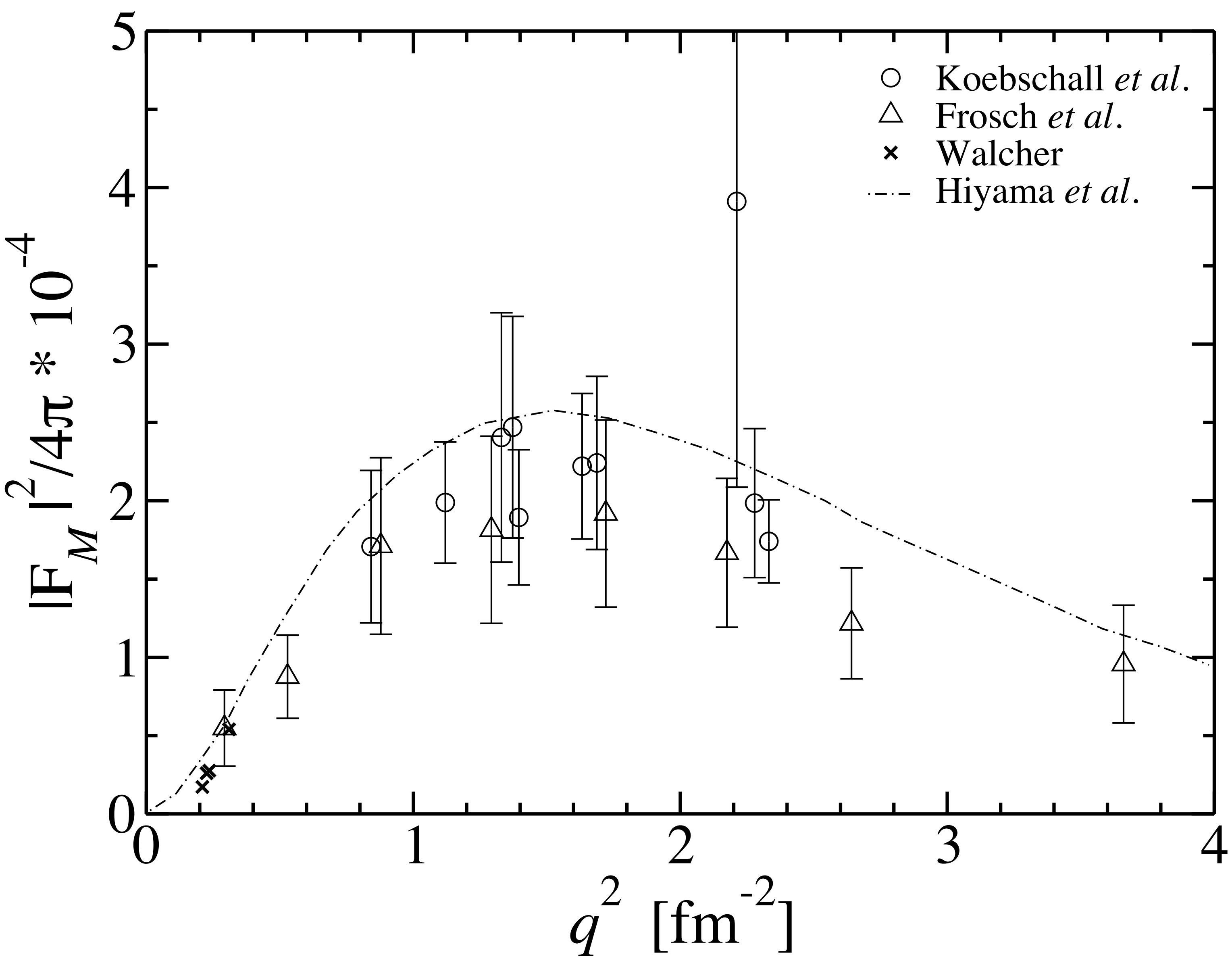
$0_1^+ \longrightarrow 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2} \int d\omega R^{\rm res}_{\mathcal{M}}(q,\omega)$$

First ab-initio calculation: Hiyama *et al.*, PRC **70** 031001 (2004)
obtained good description of data with phenomenological central 3NF



$\Gamma_R = 270 \pm 70$ keV

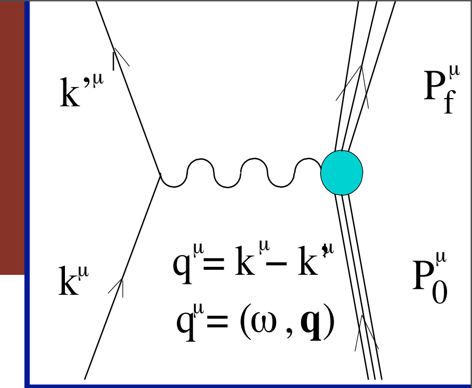


AV8' + central 3NF

E_0 = -28.44 MeV

E_0^{exp}= -28.30 MeV

Monopole Resonance $^4He(e,e')0^+$



Resonant Transition Form Factor

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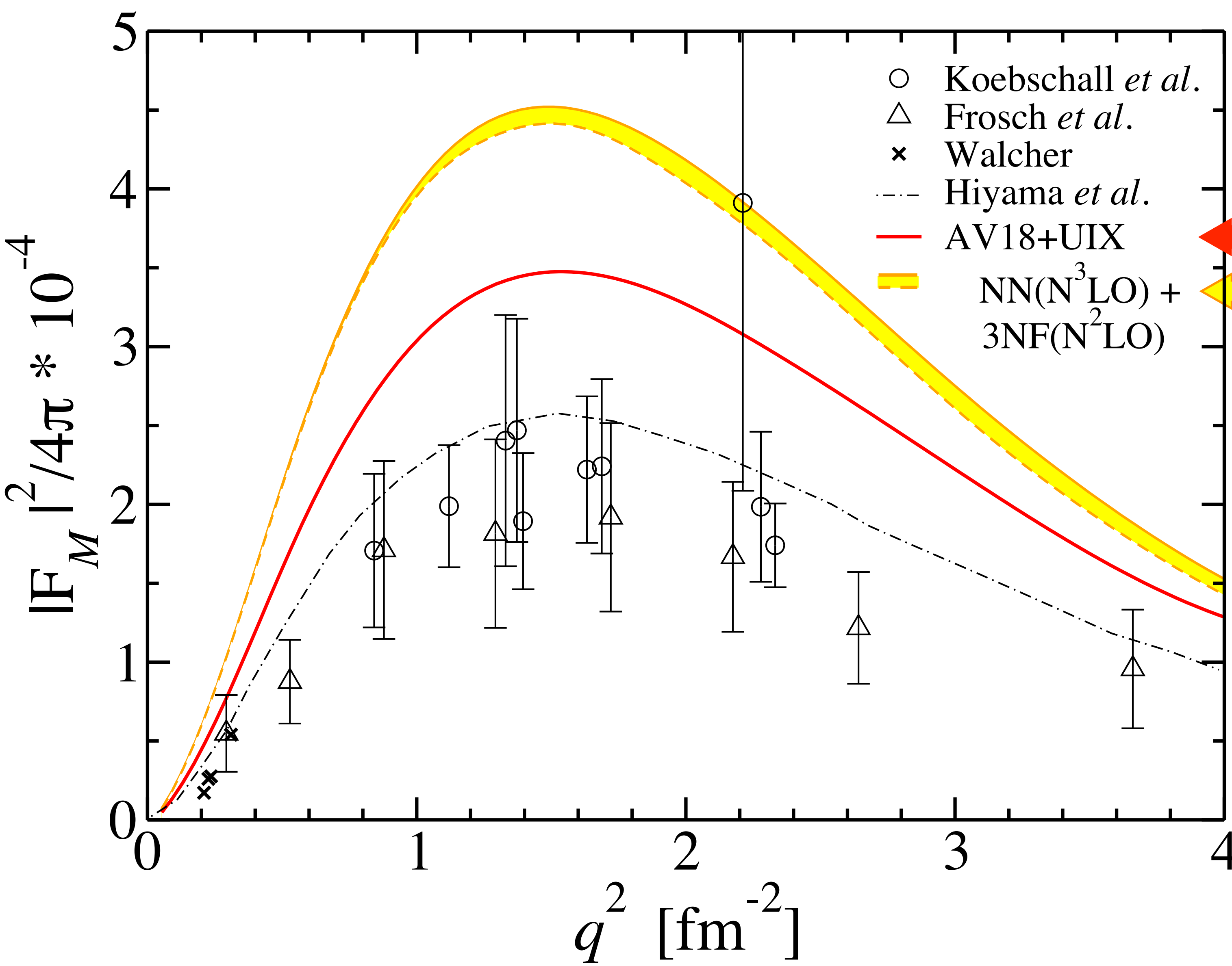
$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2} \int d\omega R_{\mathcal{M}}^{\rm res}(q,\omega)$$

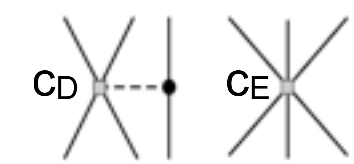
First ab-initio calculation with realistic three-nucleon forces and with the Lorentz Integral Transform method

S.B. *et al.*, **arXiv:1210.7255**



conventional forces

EFT forces



Two-parameterization of the 3NF from EFT:

C_D=1 C_E=-0.029 ^3H g.s. energy
Navratil, FBS **41** (2007) 117-140

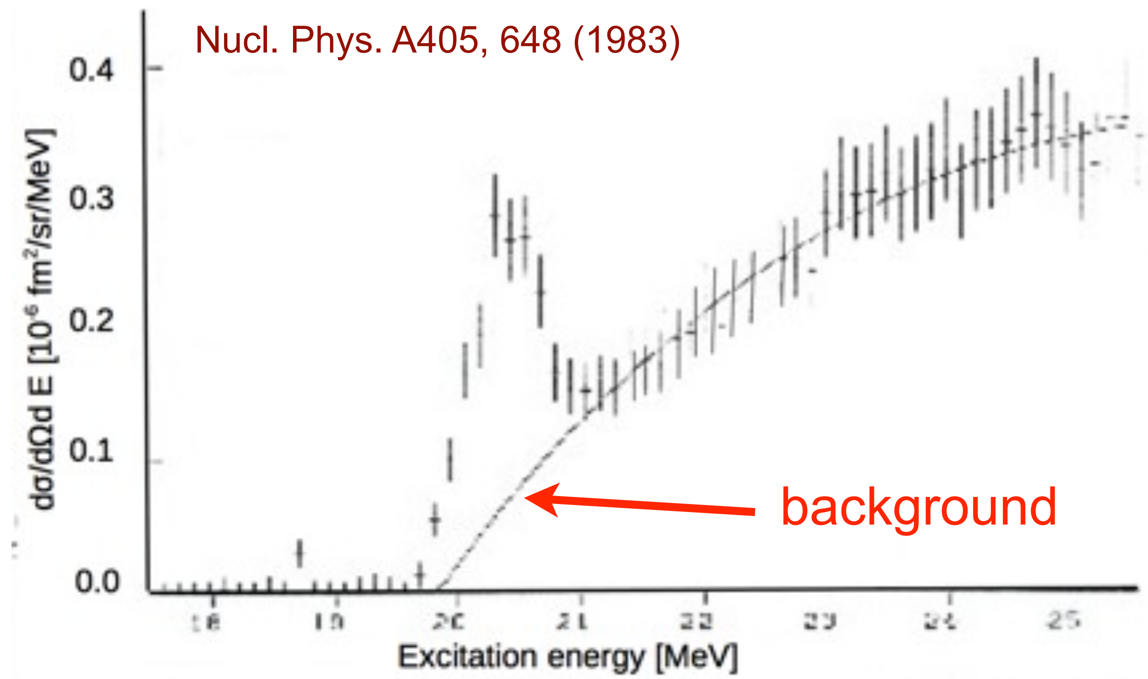
C_D=-0.2 C_E=-0.205 ^3H g.s.energy and beta-decay
Gazit *et al.*, PRL **103** (2009) 102502

Pronounced sensitivity to the input Hamiltonian

The LIT for the resonant transition

In proximity of the resonance
both in theory and experiment

$$R_{\mathcal{M}}(q,\omega) = R^{\rm res}_{\mathcal{M}}(q,\omega) + R^{\rm bg}_{\mathcal{M}}(q,\omega) \quad (\star)$$



We use a square integrable basis (HH) to
calculate the LIT, not the response
rigorous because of finite Γ

$$\mathcal{L}_{\mathcal{M}}(q,\sigma,\Gamma) = \frac{\Gamma}{\pi}\sum_{\nu=1}^{N}\frac{|\langle\Psi_\nu|\mathcal{M}(q)|\Psi_0\rangle|^2}{(\sigma - e_\nu + E_0)^2 + \Gamma^2}$$

where Ψ_ν, e_ν are eigenstate and eigenvalues of H on our basis

We see one very pronounced strength $|\langle\Psi_{\nu_R}|\mathcal{M}(q)|\Psi_0\rangle|^2$ located at the energy

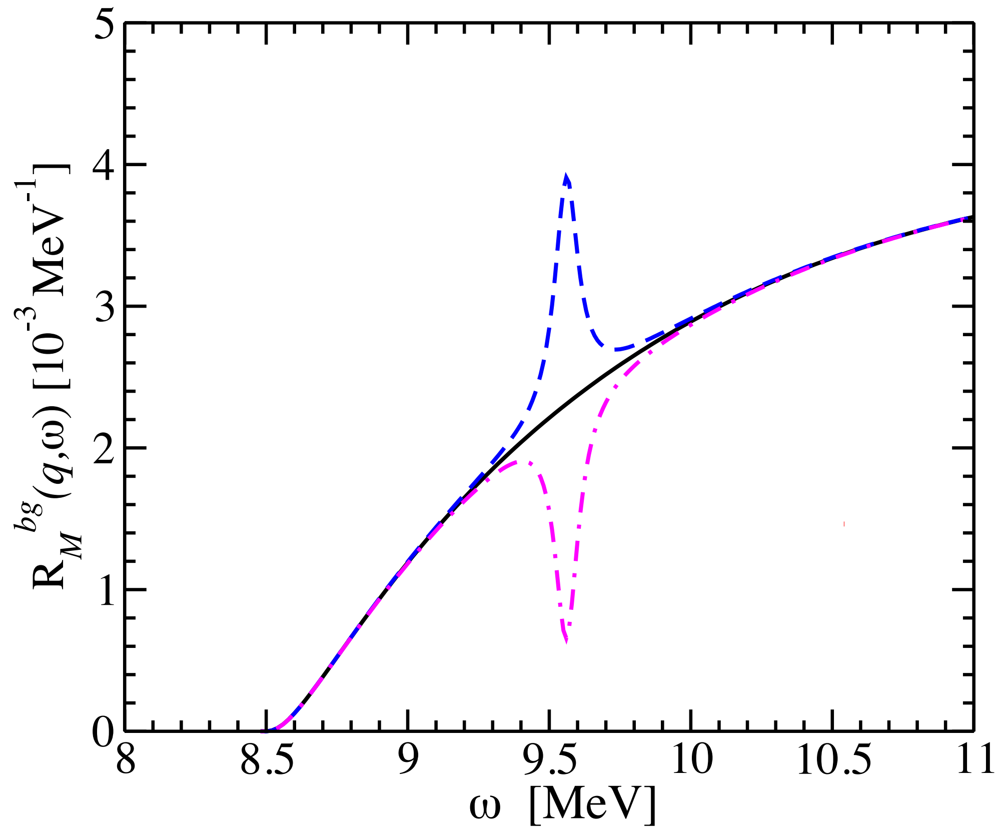
$$e_\nu - E_0 = E^*_R$$

Exploit the power of the LIT method (calculate the far continuum) to subtract the background

The LIT for the resonant transition

In proximity of the resonance both in theory and experiment

$$R_{\mathcal{M}}(q,\omega) = R^{\text{res}}_{\mathcal{M}}(q,\omega) + R^{\text{bg}}_{\mathcal{M}}(q,\omega) \quad (\star)$$



Inversion of the LIT

ansatz

$$\mathcal{R}_{\mathcal{M}}(q,\omega) = \sum_i c_i \chi_i(\omega,\alpha)$$

$$\mathcal{L}_{\mathcal{M}}(\sigma,\Gamma) = \sum_i c_i \mathcal{L}[\chi_i(\omega,\alpha)]$$

least square fit of c_i ↘ 0

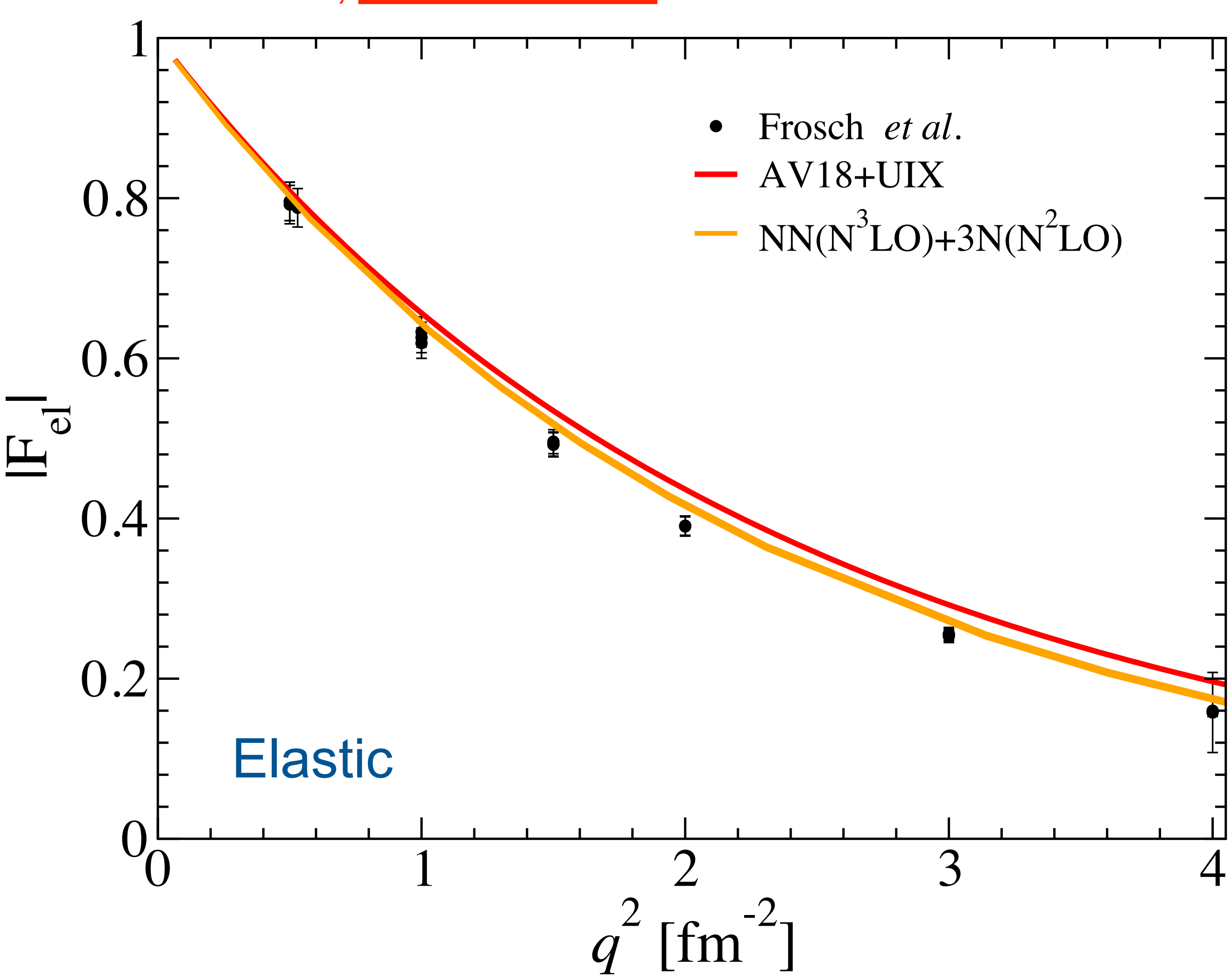
$$f_R(q)\frac{\Gamma}{\pi}\frac{1}{(\sigma - E_R + E_0)^2 + \Gamma^2}$$

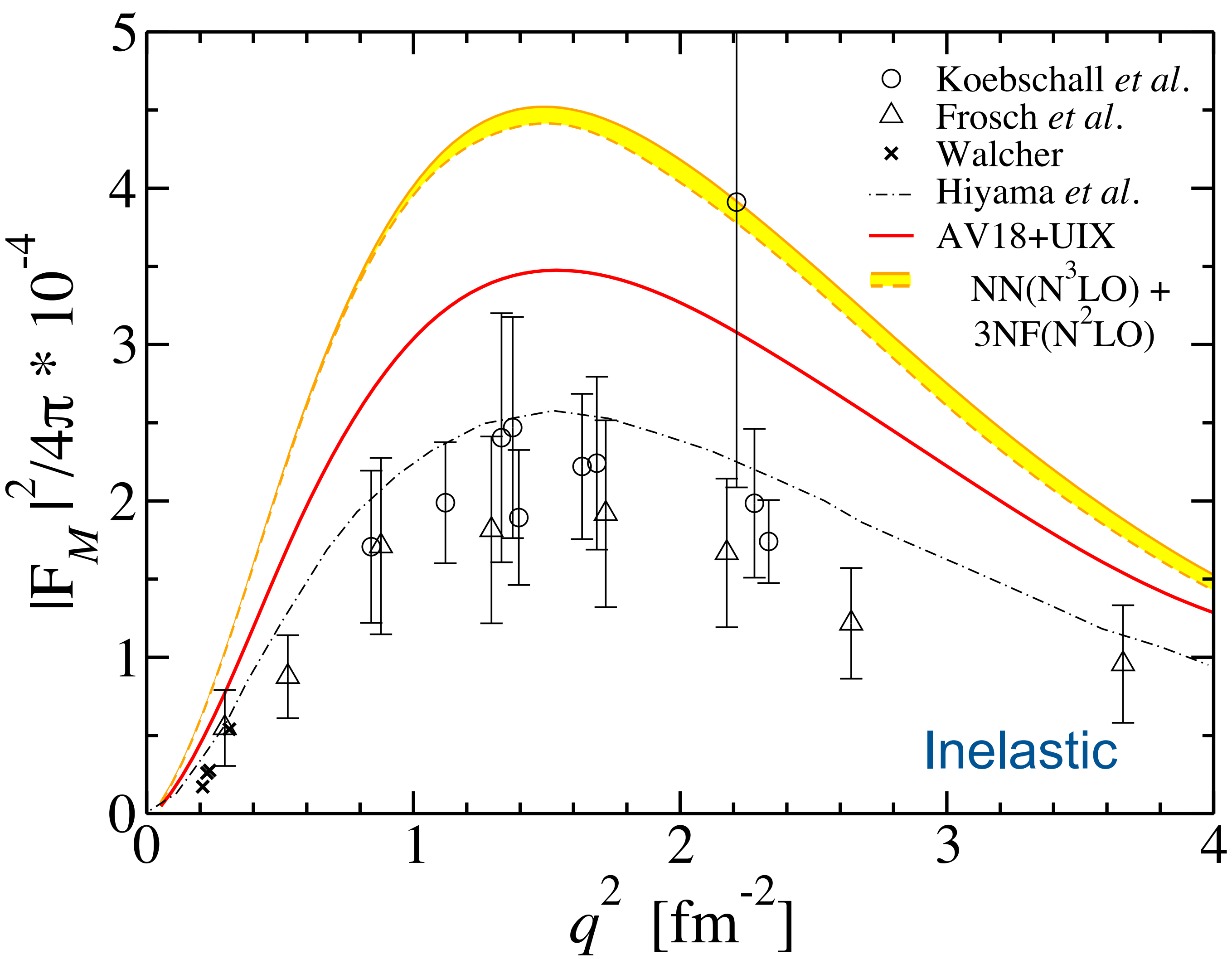
LIT of a delta by numerically choosing $\gamma \ll \Gamma$

Fit $f_R(q)$ to obtain a smooth background $\rightarrow f_R(q)$ is related to the resonant form factor

Sensitivity to Nuclear Hamiltonians

S.B. *et al.*, **arXiv:1210.7255**



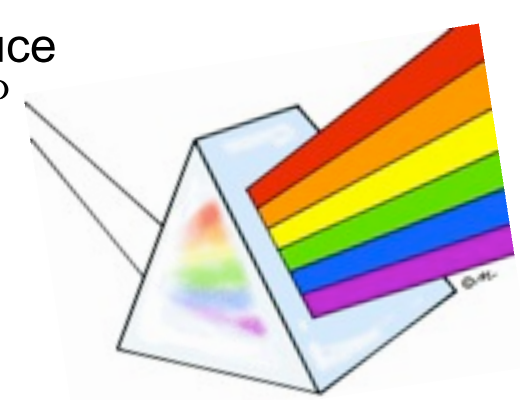


➡ The inelastic monopole resonance acts as a prism to nuclear Hamiltonians.

AV8' + central 3NF	E_0	= -28.44 MeV
AV18+UIX	E_0	= -28.40 MeV
NN(N^{3}LO)+3NF(N^{2}LO)	E_0	= -28.357 MeV

E_0^{exp}= -28.30 MeV

H reproduce E_0^{exp}



spectrum in $|F_{\mathcal{M}}|^2$

TRIUMF

Analysis of this result

Realistic three-nucleon forces do not reproduce the data for $|F_{\mathcal{M}}|^2$
Particularly large difference are found with chiral EFT potentials.
This is unexpected! What can be the source of this behaviour?

- Numerics? Our calculations are well converged (few % level) in the HH basis

$K_{\rm max}$	12	14	16	18
$10^4 \lvert F_{\mathcal{M}} \rvert^2$	4.59	4.75	4.85	4.87

- Many-body charge operators?

Conventional Nuclear Physics

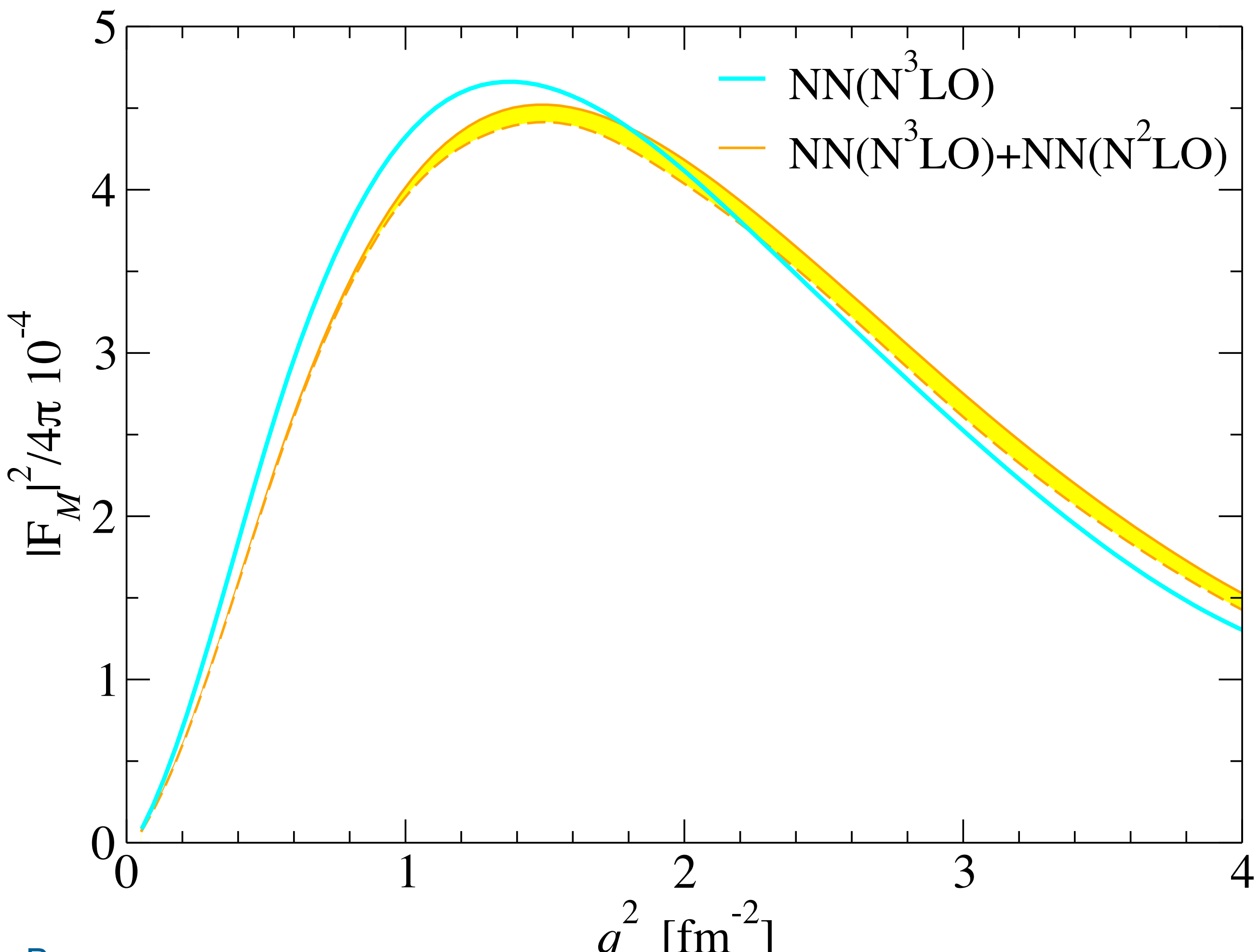
Impulse approximation valid for elastic form factor below 2 fm^{-1}
Viviani *et al.*, PRL **99** (2007) 112002

EFT approach

work done by Park *et al.*, Epelbaum, Koelling *et al.*, Pastore *et al.*, many-body operators appear at high oder in EFT

- Higher order 3NF (N^3LO)?

Unlikely...

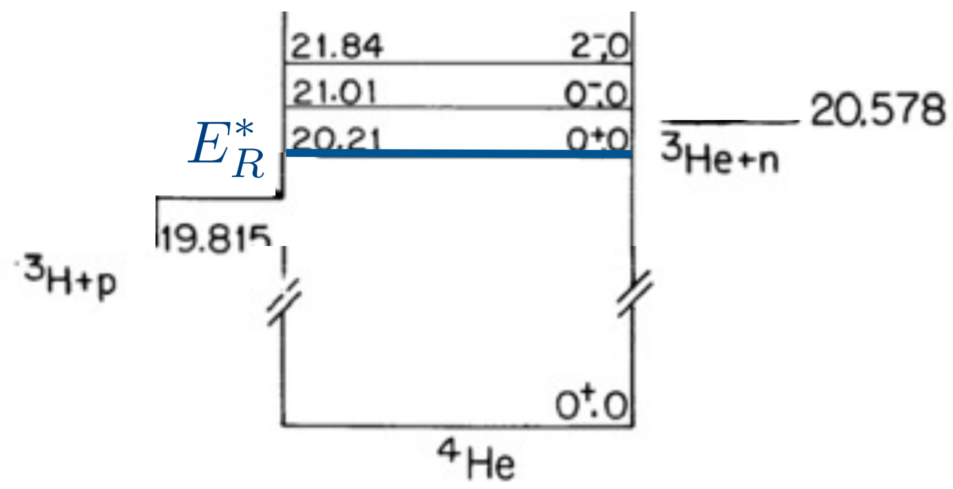


Analysis of this result

- Location of the resonance?

AV8' + central 3NF $E_R^* = 20.25$ MeV
AV18+UIX $E_R^* = 21.00(20)$ MeV
NN(N^{3}LO)+3NF(N^{2}LO) $E_R^* = 21.01(30)$ MeV

$E_R^* = 20.21$ MeV



The "realistic Hamiltonians" fail to reproduce the correct position of the 0^+_2 resonance

More theoretical work needed to understand this.

- Can this be measured again?

Outlook

- The LIT is a very powerful method to an exact study of electron scattering observables
  - ★ Showed results obtained in conjunction with the HH for A=4
- The investigation of electromagnetic observables allows to
  - ★ Shed more light on role of 3NF → Electromagnetic reactions are sensitive to 3NF and to different nuclear Hamiltonians
  - ★ Study the effect of exchange currents

Future

- ★ Use forces and currents from EFT
- ★ Extend these studies to heavier nuclei