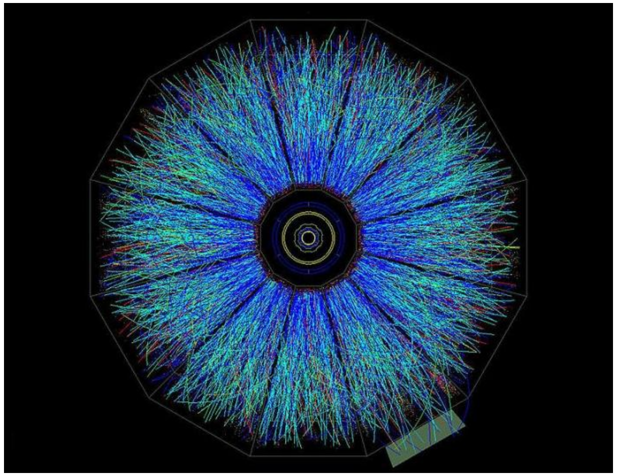
Joseph Wasem, Michael Buchoff, Tom Luu
Lawrence Livermore National Laboratory

Omega Baryon Interactions with Lattice QCD

LLNL-PRES-533073

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Omega Physics



- Experiments beginning to probe hyperon physics
- Omega physics least understood
- Model calcs disagree:

$$\Delta E_{\Omega\Omega} = 43 \pm 18\,\mathrm{MeV} \quad \text{(Quark Disloc./Color Screening Model)}$$

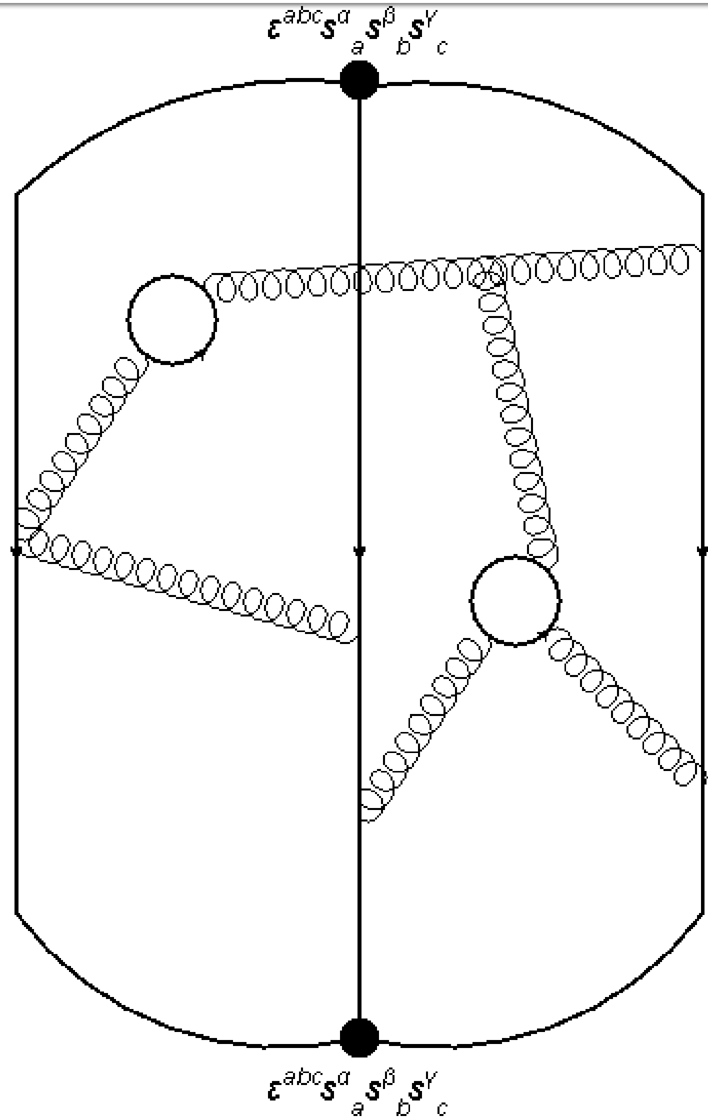
F. Wang, J.-l. Ping, G.-h. Wu, L.-j. Teng, and J. T. Goldman, Phys. Rev. **C51**, 3411 (1995), nucl-th/9512014.

$$\Delta E_{\Omega\Omega} = -116\,\mathrm{MeV} \quad \text{(SU(3) Chiral Quark Model)}$$

Z. Y. Zhang, Y. W. Yu, C. R. Ching, T. H. Ho, and Z.-D. Lu, Phys. Rev. **C61**, 065204 (2000).

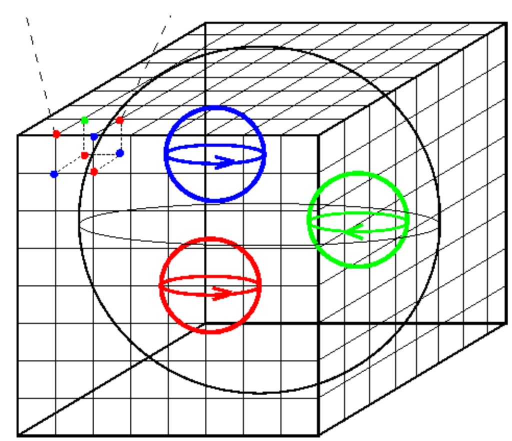
- Lattice QCD can provide model-independent resolution to this question!

Steps to a Lattice Omega



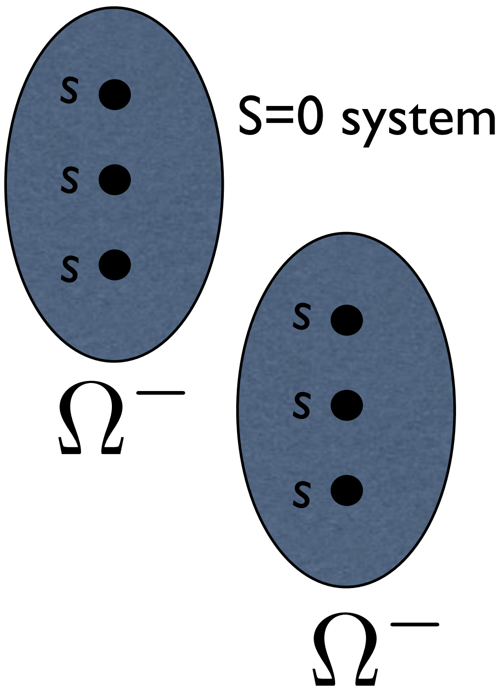
1. Gauge Configurations
2. Propagator Generation
3. Quark Contractions

$$\Rightarrow \int [d\phi] \rightarrow \prod_n \int_{-\infty}^{\infty} d\phi_n$$



Strange Quarks in Lattice QCD

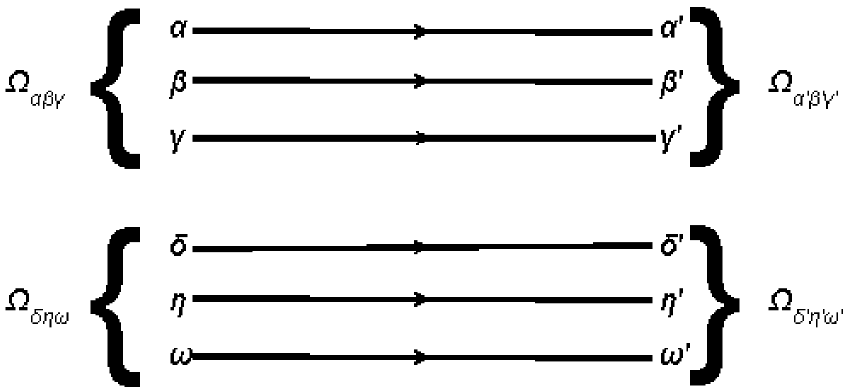
- Omega are strange.....
- Purely strange particles easier to deal with...
  - Heavier mass => Faster inversions
  - More measurements
- ...leading to cleaner signals (at a given level of computing power).
  - Cost is scalable to physical point

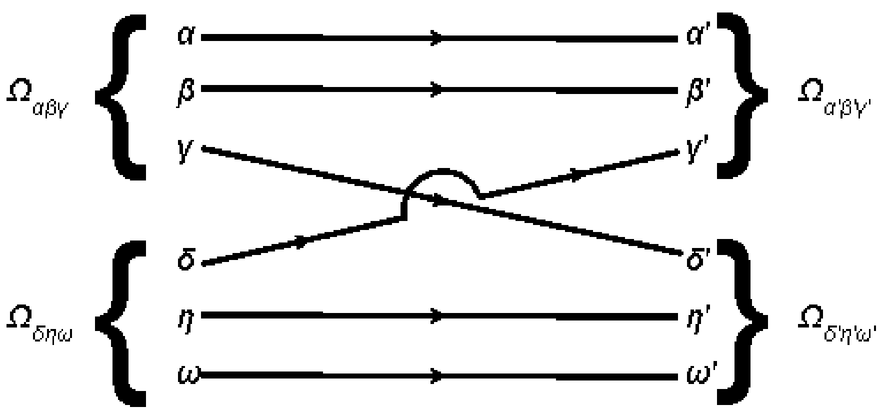


Lattice Details

- Gauge Configurations
  - Anisotropic, Jlab parameters
  - $20^3 \times 256$ [$(2.5\ \mathrm{fm})^3 \times 9.2\ \mathrm{fm}$]
  - $32^3 \times 256$ [$(3.9\ \mathrm{fm})^3 \times 9.2\ \mathrm{fm}$]
  - $m_\pi \sim 390$ MeV
- Propagator Generation/Contractions
  - GPUs, Thanks to BU Group & Balint Joo
  - 55k on 20^3
  - 12k on 32^3

Omega Interpolators





1. Single Omega
   - Choice of Spin Indices
   - Lattice Symmetries…
2. Two Omega
   - Allowed combinations of single omegas
   - S=0 & S=2
   - Excited States
   - Combine carefully…

Lattice Symmetry

- Discretization breaks O(3) symmetry to octahedral subgroup

$\Gamma\ (L \neq \infty)$	$J\ (L = \infty)$
A_1^+	0
T_1^-	1
E^+	2
T_2^+	2
G_1^+	$\frac{1}{2}$
H^+	$\frac{3}{2}$

Lattice Symmetry

- Discretization breaks O(3) symmetry to octahedral subgroup

$\Gamma\ (L \neq \infty)$	$J\ (L = \infty)$
A_1^+	0
T_1^-	1
E^+	2
T_2^+	2
G_1^+	$\frac{1}{2}$
H^+	$\frac{3}{2}$

ΩΩ Ground

ΩΩ Excited

Ω

- Different linear combinations of $\Omega_{\alpha\beta\gamma}$ are in different irreps/embeddings/rows
  - S. Basak et al., Phys. Rev. D72, 074501.

Lattice Symmetry

$\overline{\Psi}^{\Lambda,k}_{S,S_z}$	$\overline{\Delta}_{\mu_1\mu_2\mu_3}$
$\overline{\Psi}^{G_{1g},1}_{\frac{1}{2},\frac{1}{2}}$	$\overline{\Delta}_{134} - \overline{\Delta}_{233}$
$\overline{\Psi}^{G_{1g},1}_{\frac{1}{2},-\frac{1}{2}}$	$\overline{\Delta}_{144} - \overline{\Delta}_{234}$
$\overline{\Psi}^{H_g,1}_{\frac{3}{2},\frac{3}{2}}$	$\overline{\Delta}_{111}$
$\overline{\Psi}^{H_g,1}_{\frac{3}{2},\frac{1}{2}}$	$\sqrt{3}\ \overline{\Delta}_{112}$
$\overline{\Psi}^{H_g,1}_{\frac{3}{2},-\frac{1}{2}}$	$\sqrt{3}\ \overline{\Delta}_{122}$
$\overline{\Psi}^{H_g,1}_{\frac{3}{2},-\frac{3}{2}}$	$\overline{\Delta}_{222}$
$\overline{\Psi}^{H_g,2}_{\frac{3}{2},\frac{3}{2}}$	$\sqrt{3}\ \overline{\Delta}_{133}$
$\overline{\Psi}^{H_g,2}_{\frac{3}{2},\frac{1}{2}}$	$2\overline{\Delta}_{134} + \overline{\Delta}_{233}$
$\overline{\Psi}^{H_g,2}_{\frac{3}{2},-\frac{1}{2}}$	$\overline{\Delta}_{144} + 2\overline{\Delta}_{234}$
$\overline{\Psi}^{H_g,2}_{\frac{3}{2},-\frac{3}{2}}$	$\sqrt{3}\ \overline{\Delta}_{244}$

- Different linear combinations of $\Omega_{\alpha\beta\gamma}$ are in different irreps/embeddings/rows
  - S. Basak et al., Phys. Rev. D72, 074501.

$\Gamma\ (L \neq \infty)$	$J\ (L = \infty)$	
A_1^+	0	ΩΩ Ground
T_1^-	1	
E^+	2	ΩΩ Excited
T_2^+	2	ΩΩ Excited
G_1^+	$\frac{1}{2}$	
H^+	$\frac{3}{2}$	Ω

Correlation Functions

- After spatial sum, and irrep choice:

$$C_{\Omega}(t) = \sum_{x} \left\langle \Omega_{\alpha\beta\gamma}(x,t) \middle| \Omega_{\alpha\beta\gamma}(0,0) \right\rangle = \sum_{n} \left| \left\langle n \middle| \Omega_{\alpha\beta\gamma} \middle| 0 \right\rangle \right|^2 e^{-E_{\Omega} t}$$

$$\xrightarrow[t\to\infty]{} \left| \left\langle \Omega \middle| \Omega_{\alpha\beta\gamma} \middle| 0 \right\rangle \right|^2 e^{-E_{\Omega} t}$$

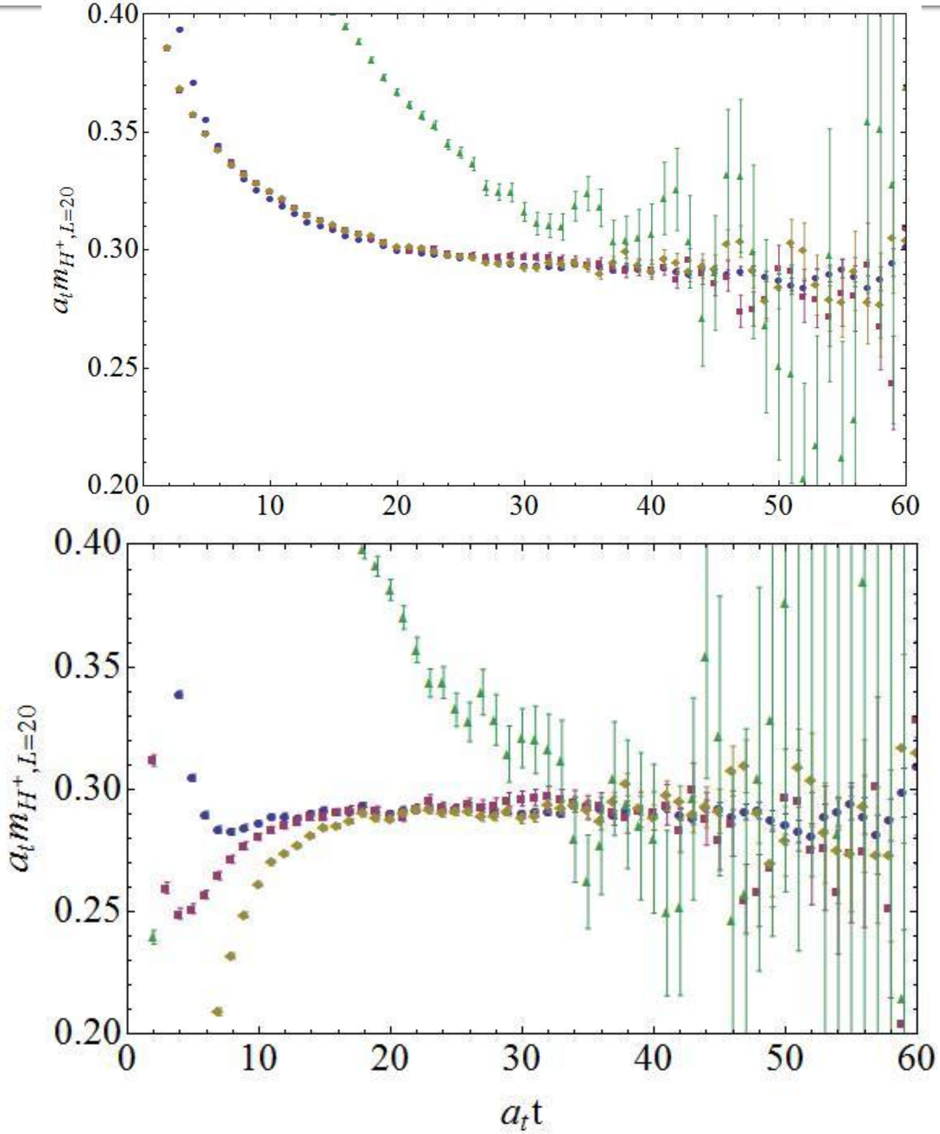
$$C_{\Omega\Omega}(t) = \sum_{x} \left\langle \Omega_{\alpha\beta\gamma}(x,t)\Omega_{\delta\eta\lambda}(x,t) \middle| \Omega_{\alpha\beta\gamma}(0,0)\Omega_{\delta\eta\lambda}(0,0) \right\rangle = \sum_{n} \left| \left\langle n \middle| \Omega_{\alpha\beta\gamma}\Omega_{\delta\eta\lambda} \middle| 0 \right\rangle \right|^2 e^{-E_{\Omega\Omega} t}$$

$$\xrightarrow[t\to\infty]{} \left| \left\langle \Omega\Omega \middle| \Omega_{\alpha\beta\gamma}\Omega_{\delta\eta\lambda} \middle| 0 \right\rangle \right|^2 e^{-E_{\Omega\Omega} t}$$

- Energy levels extracted using

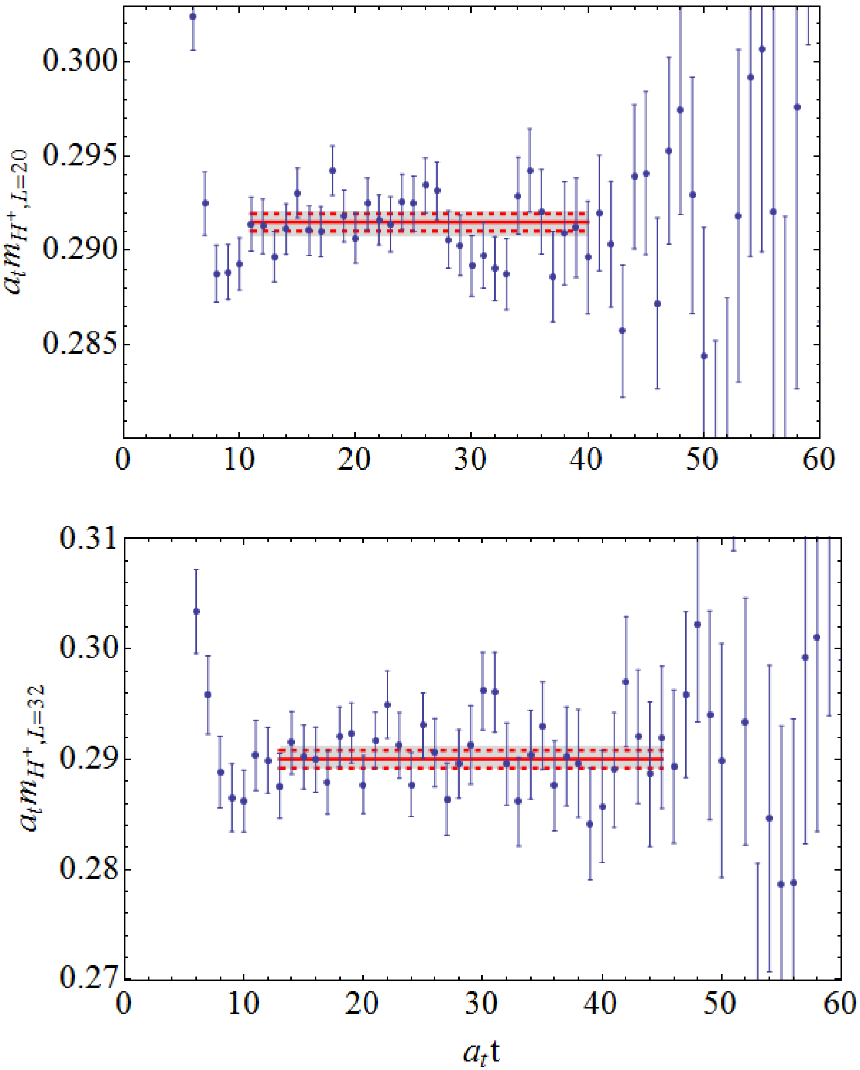
$$E = \log\left[\frac{C(t)}{C(t+1)}\right]\Bigg|_{t\to\infty}$$

H⁺ Embeddings (Single Ω)



- Choice of embedding combo
  - E1-e1, e1-e2, e2-e1, e2-e2
- E2-e2 is very different
  - Pure Shell-Shell shows significant difference
  - Matrix-Prony on SS/SP also
- Can significantly affect plateau extraction

H⁺ Lattice Data (Single Ω)

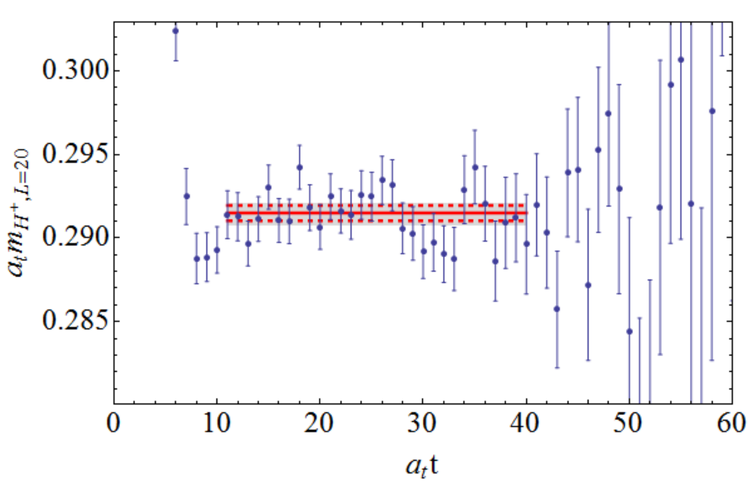


- Sub-1% extraction
- Sub-1% Volume effects
- Long plateaus
- χ^2 of 1.003 & 0.85, respectively

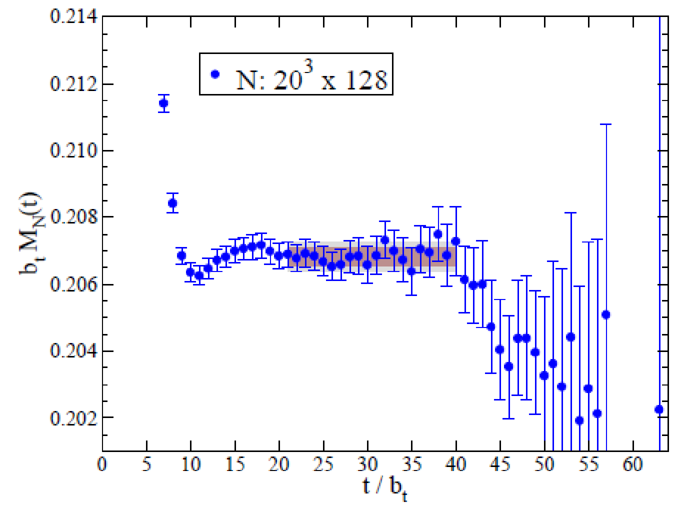
H⁺ Lattice Data (Single Ω)

- NPLQCD, **Phys.Rev. D84 (2011) 014507**

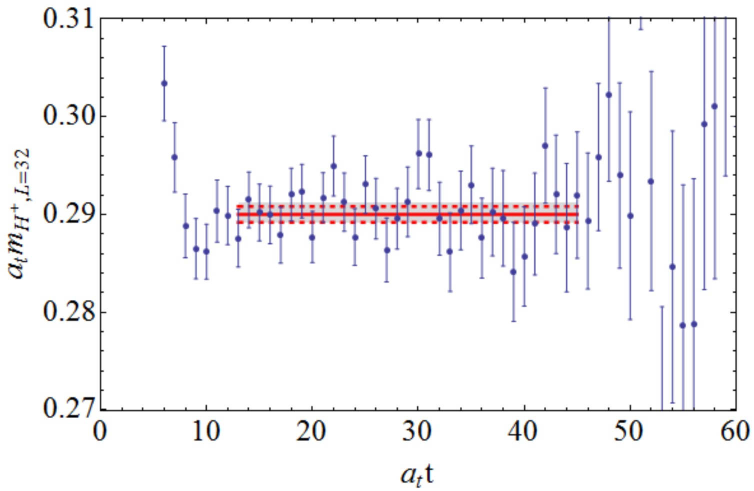
55k
~7.5k GPU-hr



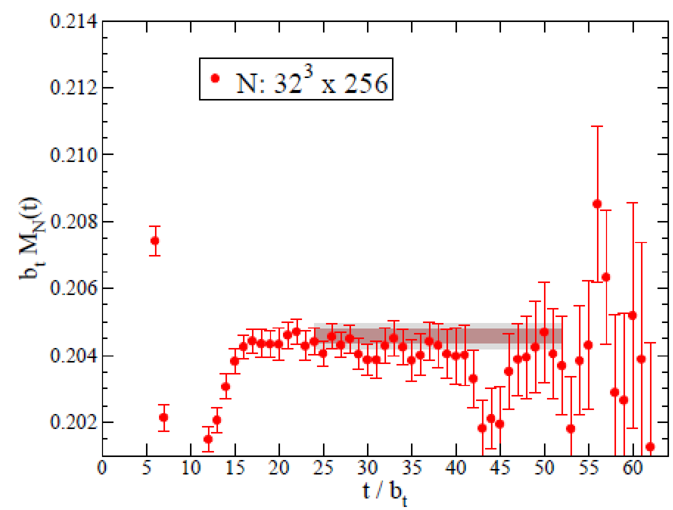
403k
~102k GPU-hr



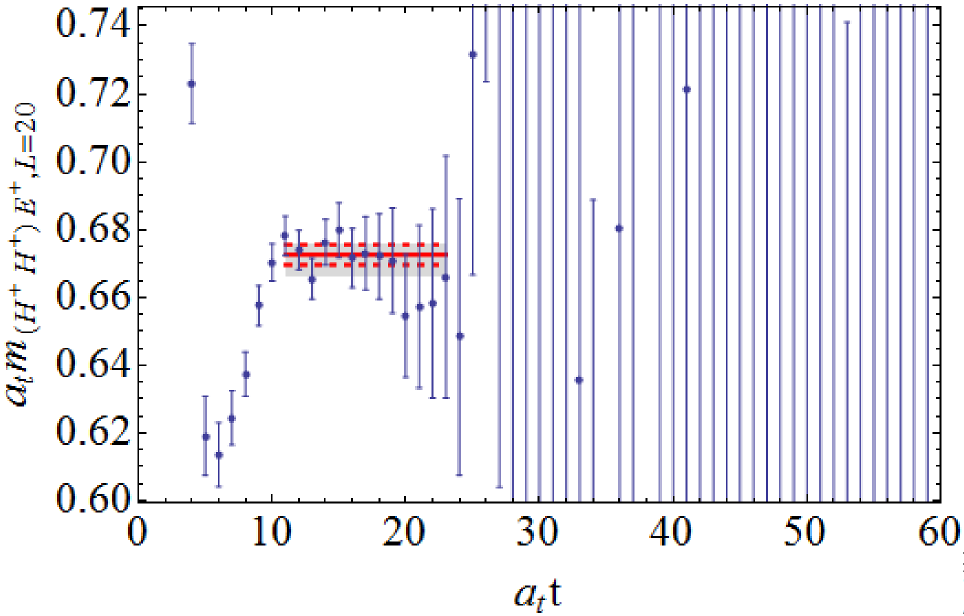
12k
~10k GPU-hr



130k
~200k GPU-hr

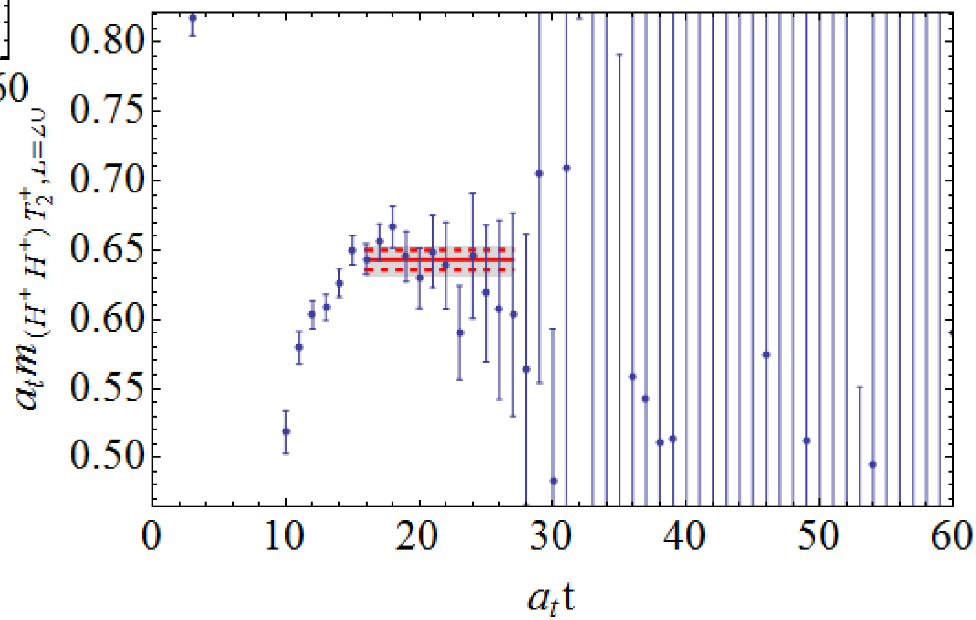


E^+ & T_2^+ Lattice Data (Two Ω)

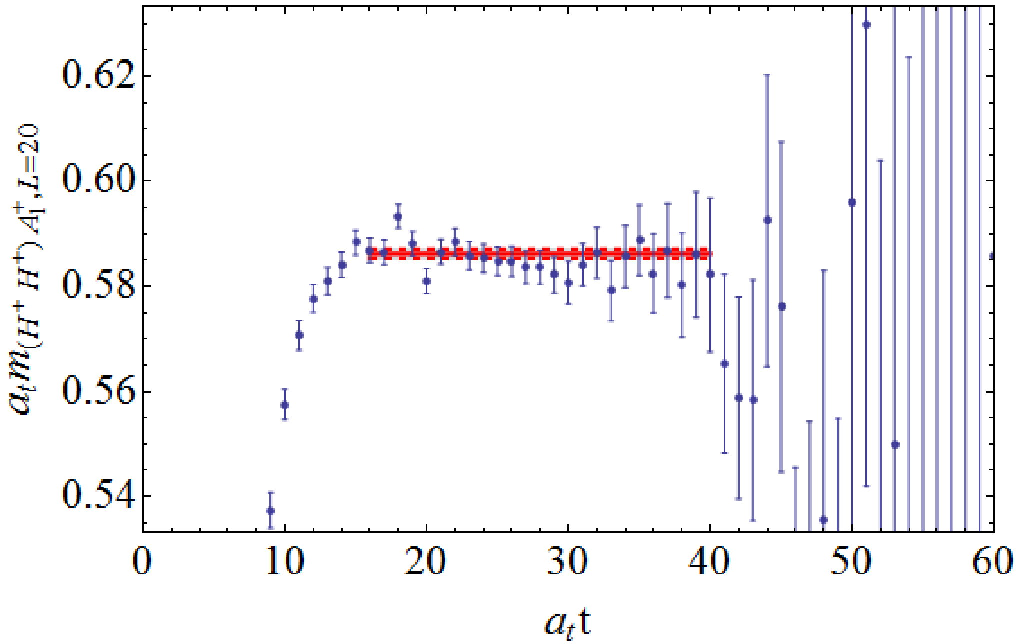


- Excited states
- χ^2 of 0.925 & 0.5, respectively

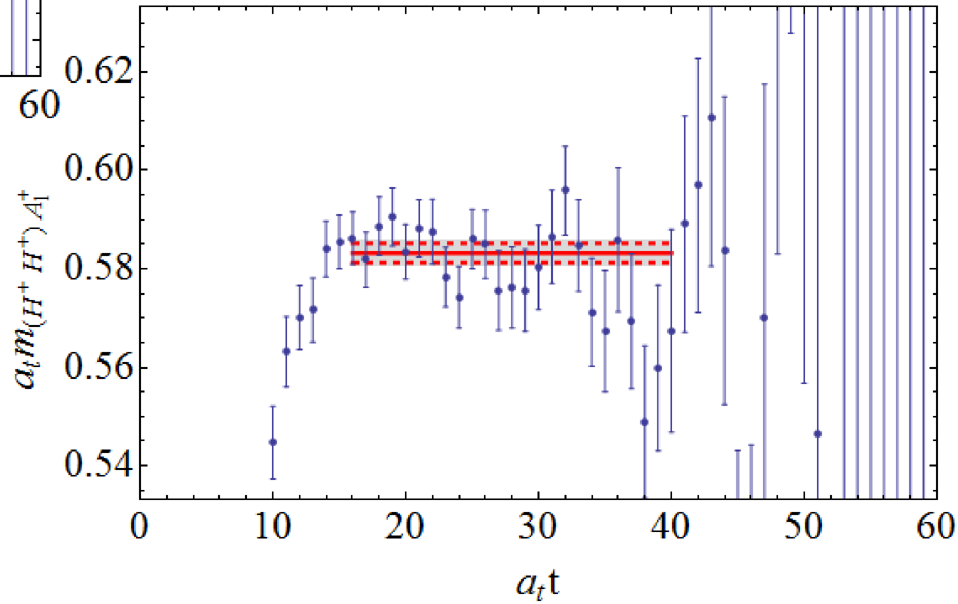
- Much noisier
- Large ΔE



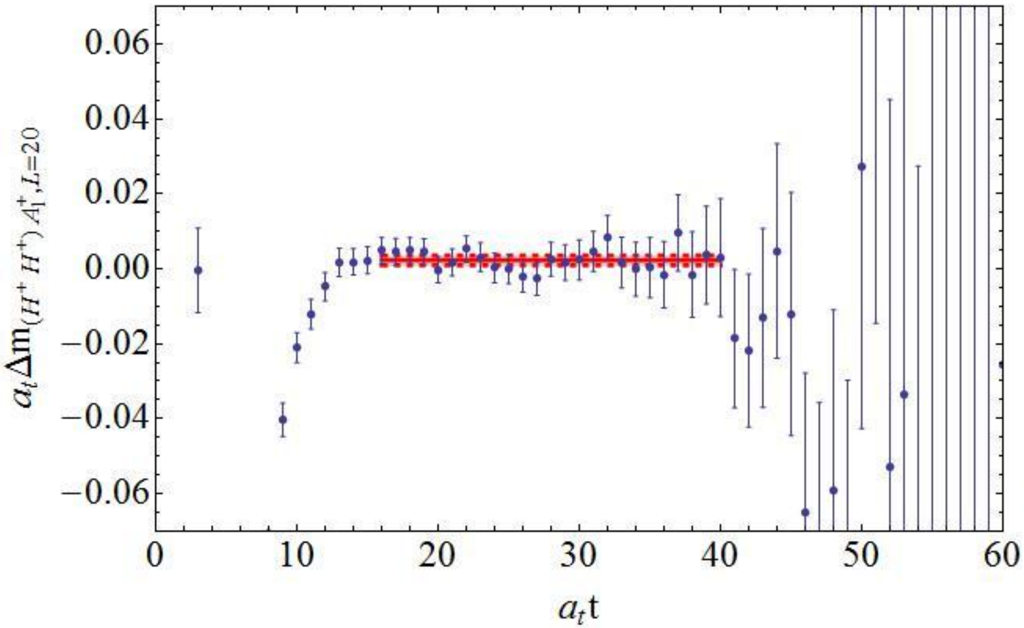
A_1^+ Lattice Data (Two Ω)



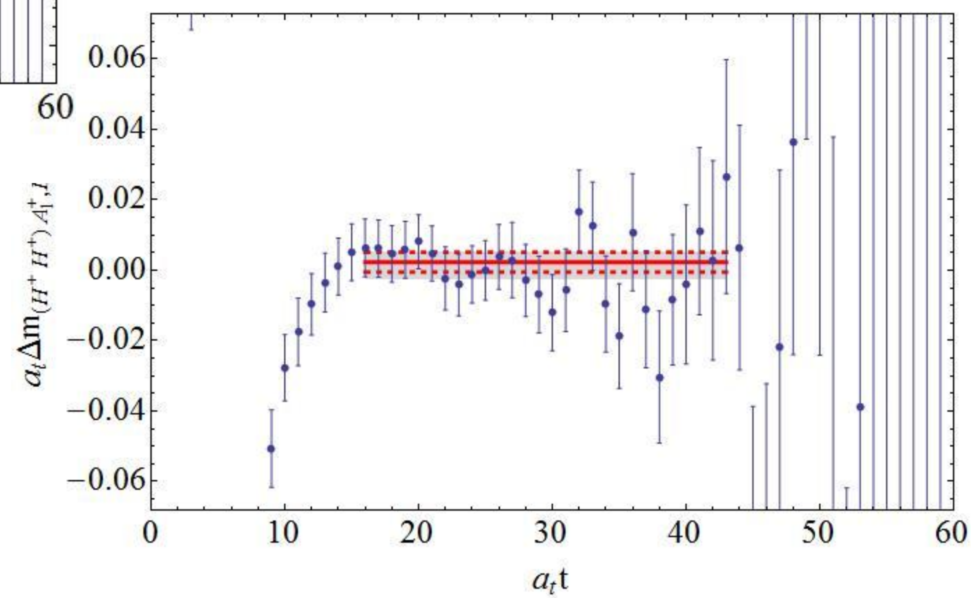
- Relatively clean
- χ^2 of 1.105 & 1.086, respectively



A_1^+ Lattice Data (Two Ω)



- Energy Shift
- Near zero
- Positive
- Greater than 1σ



- Ratio of MP'ed H+ & A1+
  - NOT MP of ratio
- Plateau within previous

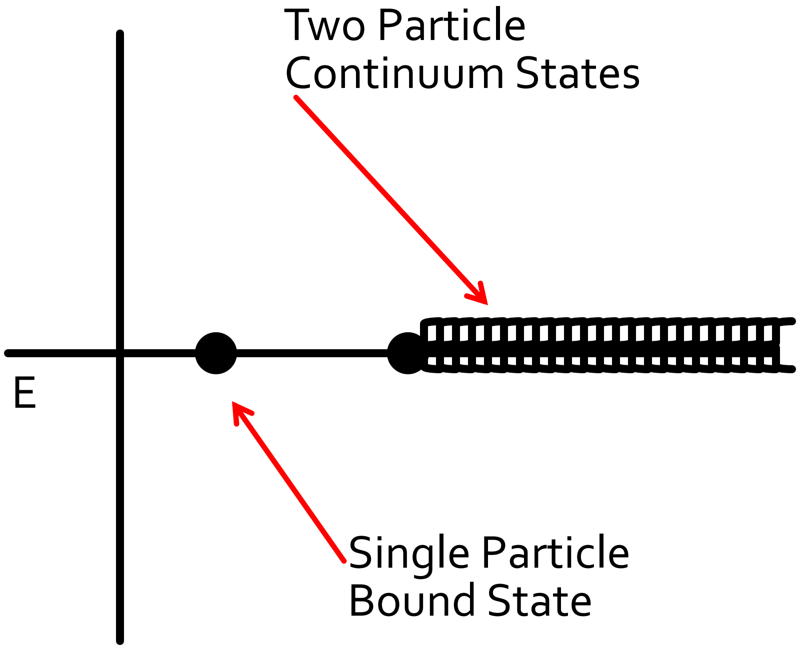
Lattice Data

Irrep	Lattice Size	$a_t E$	$\sigma_{E,stat.}$	$\sigma_{E,sys.}$	χ^2/dof	Q	$a_t \Delta E$	$\sigma_{\Delta E,stat.}$
H^+	$20^3 \times 256$	0.291501	0.000457	$^{+0.000099}_{-0.000268}$	1.003	0.460		
	$32^3 \times 256$	0.290001	0.000804	$^{+0.000418}_{-0.000001}$	0.850	0.708		
A_1^+	$20^3 \times 256$	0.586235	0.000843	$^{+0.000091}_{-0.000348}$	1.105	0.327	0.00323	0.00124
	$32^3 \times 256$	0.583224	0.002002	$^{+0.000577}_{-0.000680}$	1.086	0.350	0.00322	0.00257
T_2^+	$20^3 \times 256$	0.642961	0.007136	$^{+0.002502}_{-0.005120}$	0.925	0.514	0.05996	0.00719
E^+	$20^3 \times 256$	0.67256	0.00293	$^{+0.00013}_{-0.00329}$	0.500	0.916	0.08956	0.00307

- Ω mass ~1640 MeV
- Ω Ω energy > 2x Ω mass
  - Scattering state, not bound

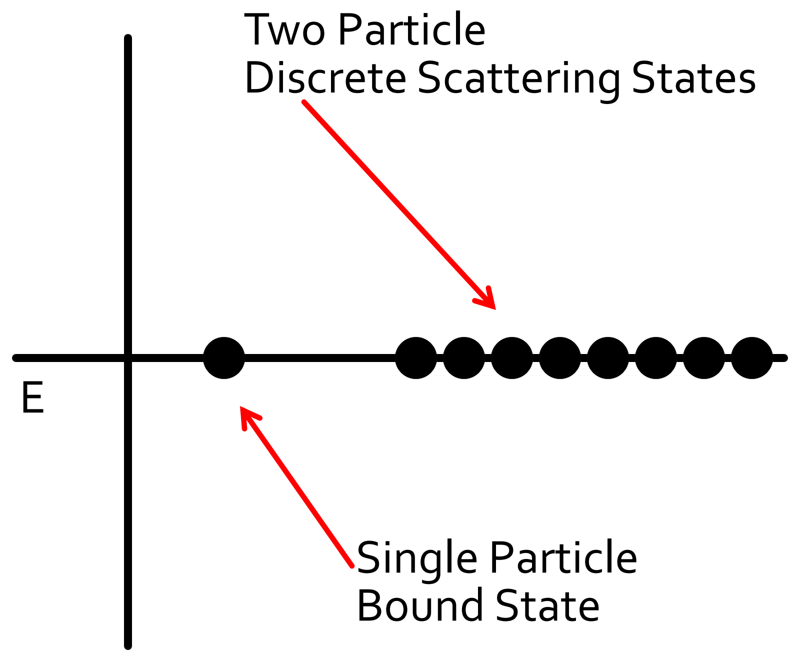
Scattering on the Lattice

- In Euclidean space, LSZ holds only at kinematic thresholds
- Solution lies at finite volume....

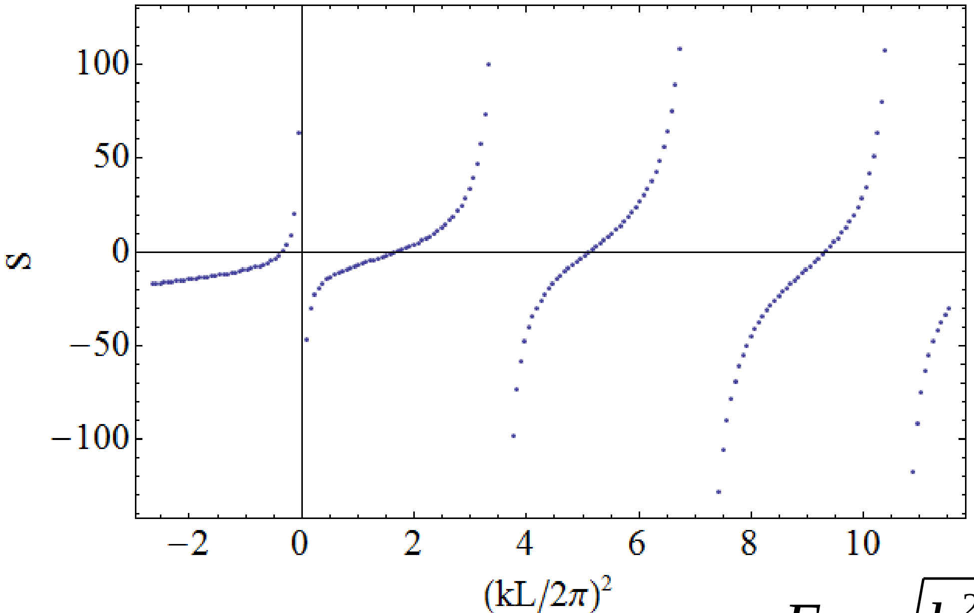


Scattering on the Lattice

- In Euclidean space, LSZ holds only at kinematic thresholds
- Solution lies at finite volume....
- .....where continuum scattering states become discrete energy levels.
- How these levels shift with volume reveals scattering information.



Scattering at Finite Volume



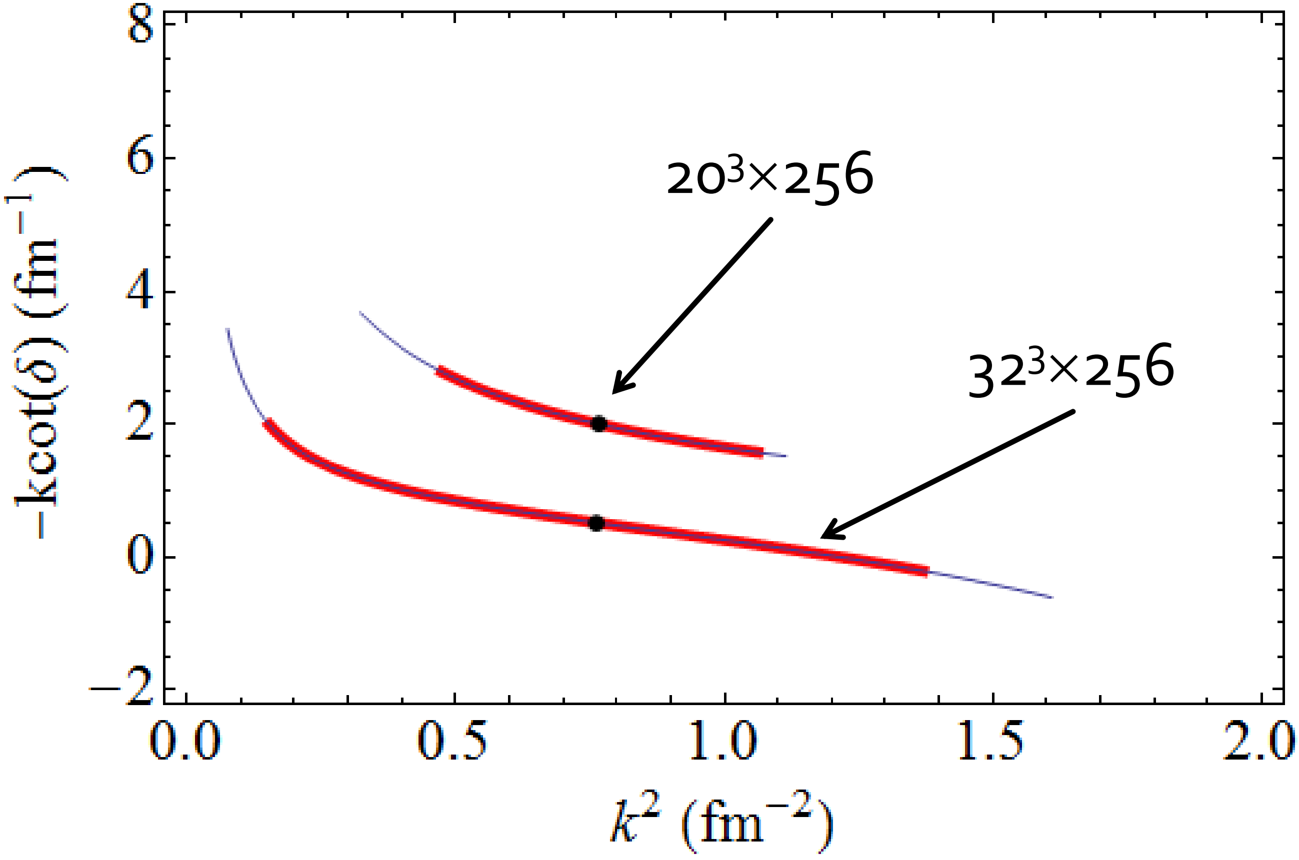
- kcotδ at finite volume is related to the interaction energy

$$E = \sqrt{k^2 + m_A^2} + \sqrt{k^2 + m_B^2} = \Delta E + m_A + m_B$$

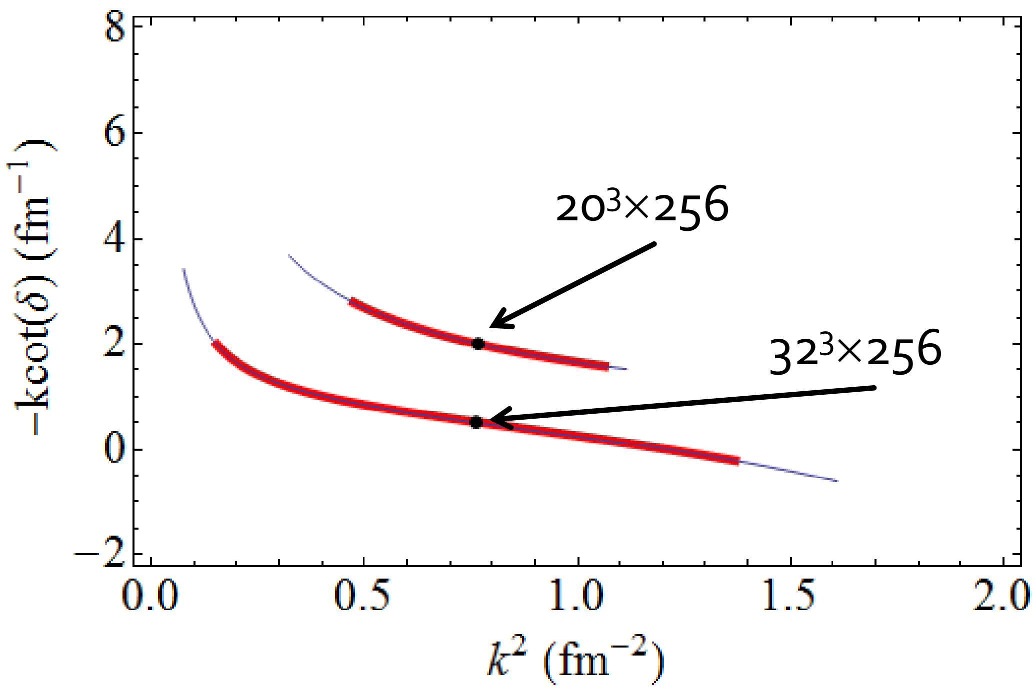
$$k \cdot \cot\delta(k) = \frac{1}{\pi L} S\left(\frac{k^2 L^2}{4\pi^2}\right) = -\frac{1}{a} + \tfrac{1}{2} r k^2 + \ldots.$$

$$S(\eta) = \sum_{\vec{j}\neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$$

A_1^+ Scattering

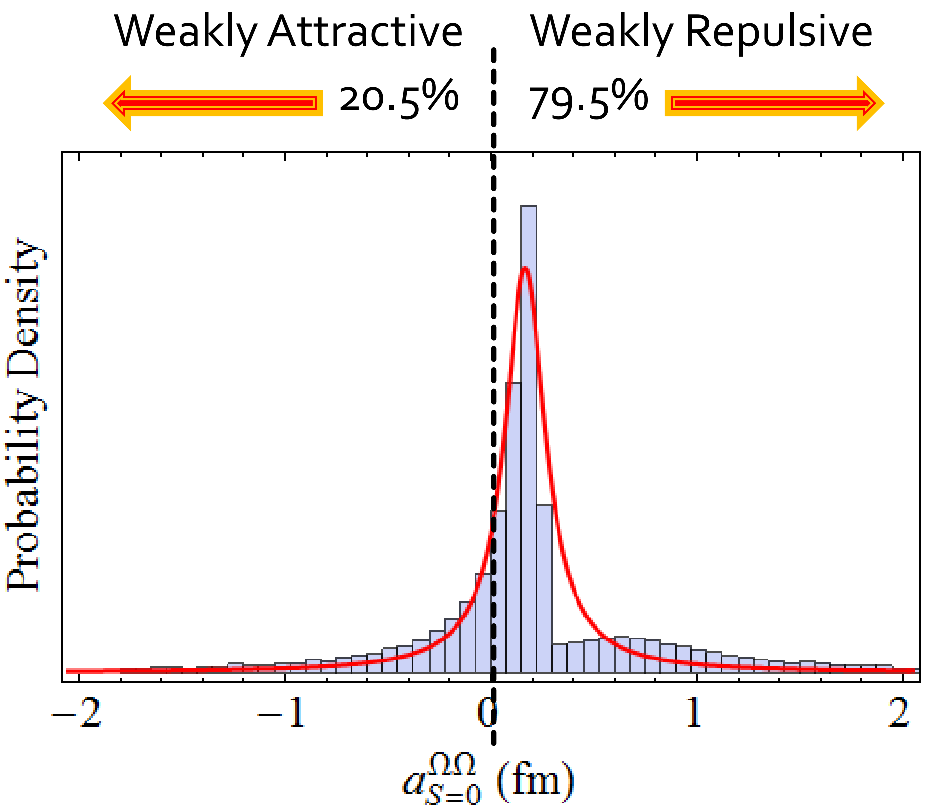


A_1^+ Scattering



- k^2 is only quantity that is Gaussian
- Pick 10k random pairs from k^2 distributions
- Determine kcotδ
- Fit to effective range expansion

A_1^+ Scattering



$$k \cdot \cot\delta(k) = -\frac{1}{a} + \tfrac{1}{2} r k^2 + \ldots$$

- *r* distribution absorbs higher orders
- *a* distribution is Lorentzian

$$a_{S=0}^{\Omega\Omega} = 0.16 \pm 0.22\ \text{fm}$$

Toward the Physical Point

- Light quark dependence expected to be small
- Currently taking measurements at $m_\pi \sim 230$ MeV
- Sequoia is generating $64^3 \times 128$ isotropic physical point lattices

Conclusions

- Results indicate a weakly repulsive system.
- $a^{\Omega\Omega}_{S=0} = 0.16 \pm 0.22\ \mathrm{fm}$ at m_π~390 MeV
- Light quark dependence (small)
  - Running now with m_π~230 MeV
  - Physical point lattices running now on Sequoia
- Contrast with other lattice hyperon results that are bound states.
  - May just reflect smaller influence of light quark dynamics
- Phys.Rev. D85 (2012) 094511
- arXiv:1201.3596 [hep-lat]