Light-quark mass dependence of QCD: *Myths and Facts*

INT 2012
Seattle, WA, USA

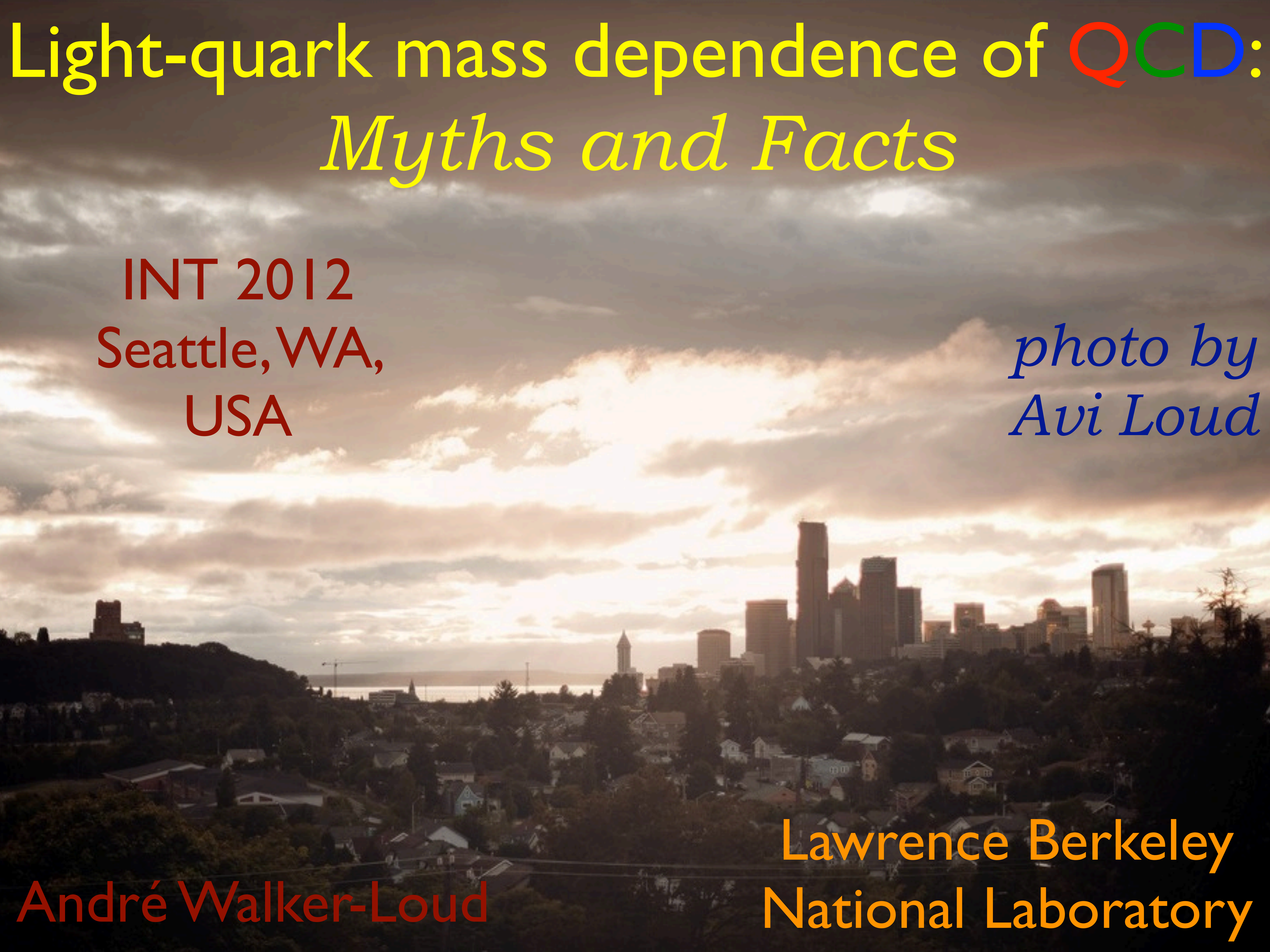

photo by Avi Loud

André Walker-Loud

Lawrence Berkeley National Laboratory

Light-quark mass dependence of QCD:

- Myths: Effective Field Theory
  Chiral Perturbation Theory

- Facts: Numerical Lattice QCD results

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Caveat: I am a born and raised Effective Field Theorist

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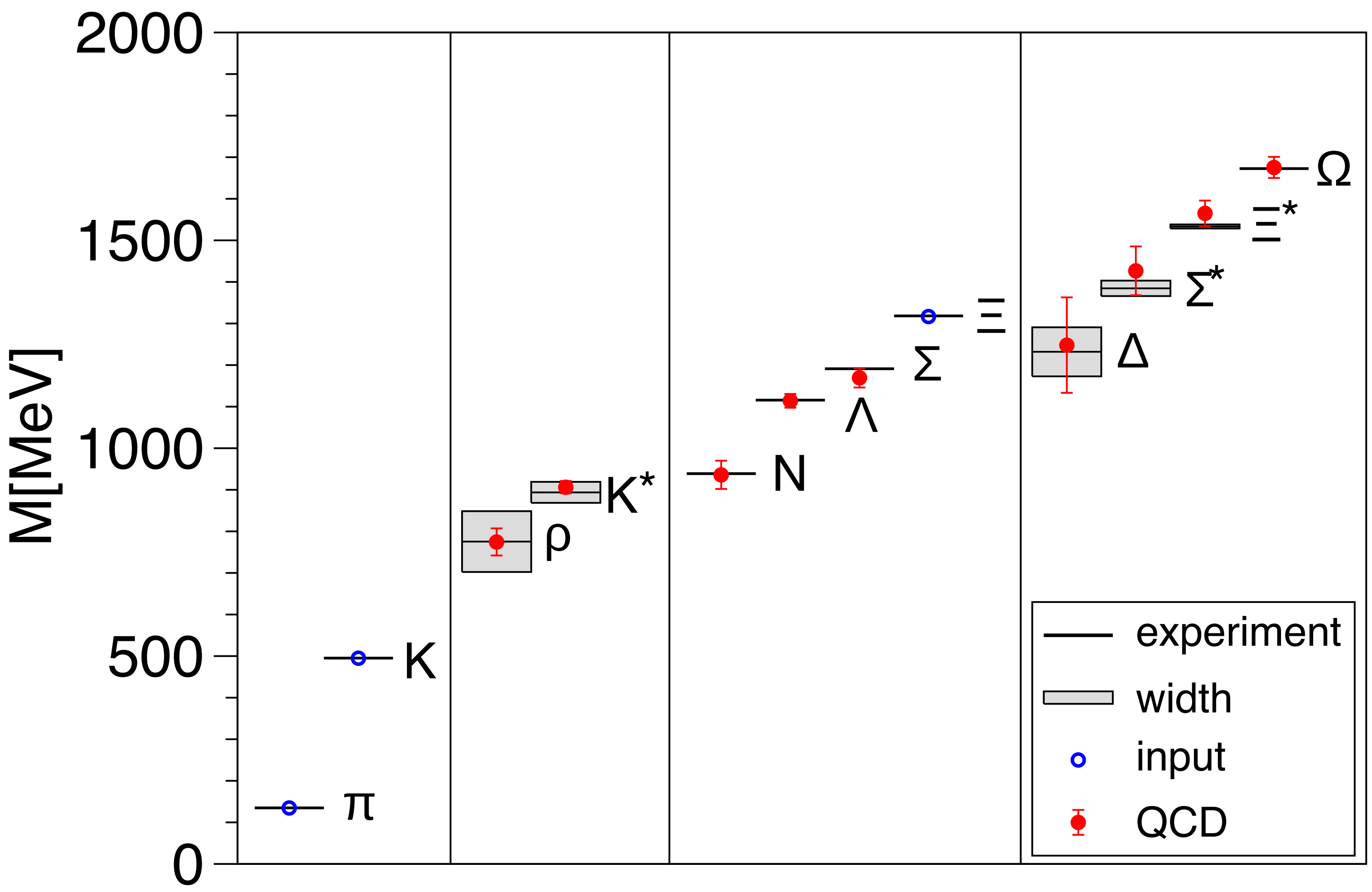
I am having fun with my own *faith*, while making a serious point

Light-quark mass dependence of QCD:

- Light quark mass dependence of the baryon spectrum

- Hadron Electromagnetic Polarizabilities from Lattice QCD

Light quark mass dependence of M_B



BMWc
Science 21 Nov 2008
Vol. 322 no. 5905 pp. 1224

$$m_\pi \to m_l$$

$$m_K \to m_s$$

$$m_\Xi \to \text{scale}$$

This heralded the paradigm change in the relation between lattice QCD and effective field theory at least for simple quantities

Light quark mass dependence of M_B

- Chiral perturbation theory ($\chi\mathrm{PT}$) provides a complete (but non predictive) description of low-energy QCD
- The chiral logarithms (non-analytic dependence upon the light quark masses) are the "predictions" of $\chi\mathrm{PT}$ as they encode long-range IR physics not contained in local operators
- For some (small) values of m_q, $\chi\mathrm{PT}$ should provide a precise and accurate description of low energy hadronic phenomena
- confidence in our understanding requires evidence of the chiral logarithms from lattice QCD

Light quark mass dependence of M_B

Heavy Baryon Chiral Perturbation Theory ($\mathrm{HB}\chi\mathrm{PT}$)

E. Jenkins and A. Manohar PLB 255 (1991)

Expand about the static heavy baryon limit

$$\mathcal{L} = \bar{N} i v \cdot \partial N + 2\alpha_M \bar{N} N \mathrm{tr}(\mathcal{M}_+) - \bar{T}^\mu \left[i v \cdot \partial - \Delta_0 \right] T_\mu - 2\bar{\gamma}_M \bar{T}^\mu T_\mu \mathrm{tr}(\mathcal{M}_+)$$

$$+ 2 g_A \bar{N} S \cdot A N + 2 g_{\Delta\Delta} \bar{T}^\mu S \cdot A T_\mu + g_{\Delta N} \left(\bar{T}^\mu A_\mu N + \bar{N} A^\mu T_\mu \right)$$

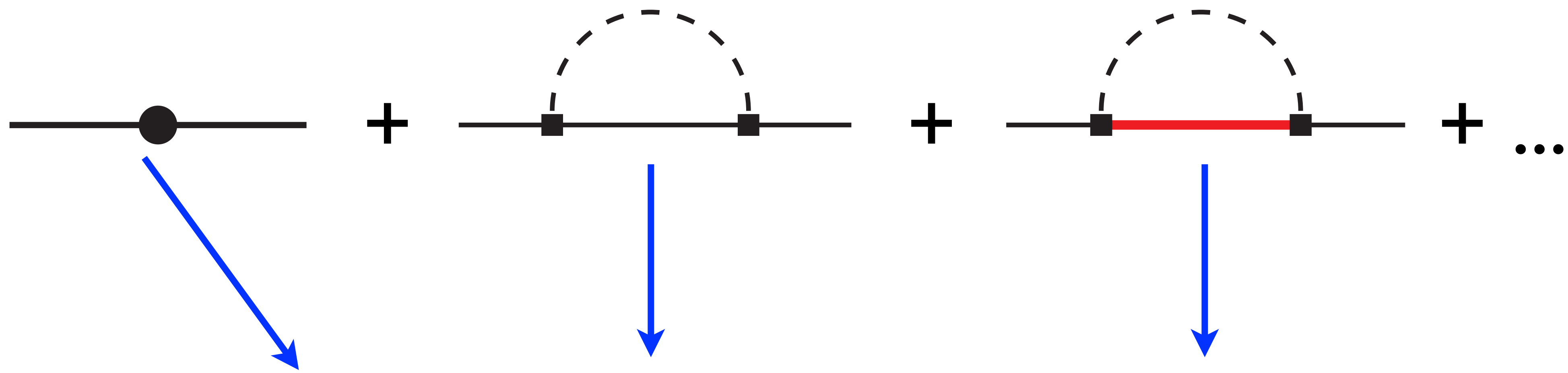
$$\Delta_0 = M_\Delta - M_N \Big|_{m_q=0}$$

phenomenologically $\Delta \simeq 290 \text{ MeV}$

Light quark mass dependence of M_B

nucleon mass to nlo

$-i\Sigma =$



$$M_N = M_0 - 2\alpha_M(\mu)\, m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu)\,,$$

$$\mathcal{F}(m,\Delta,\mu) = (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln\left(\frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}\right) - \frac{3}{2}\Delta\, m^2 \ln\left(\frac{m^2}{\mu^2}\right) - \Delta^3 \ln\left(\frac{4\Delta^2}{m^2}\right)$$

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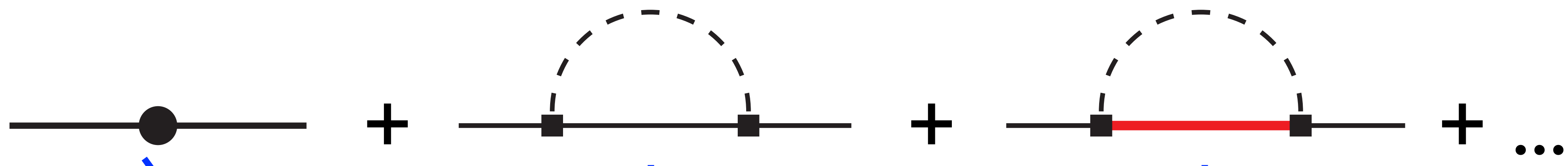

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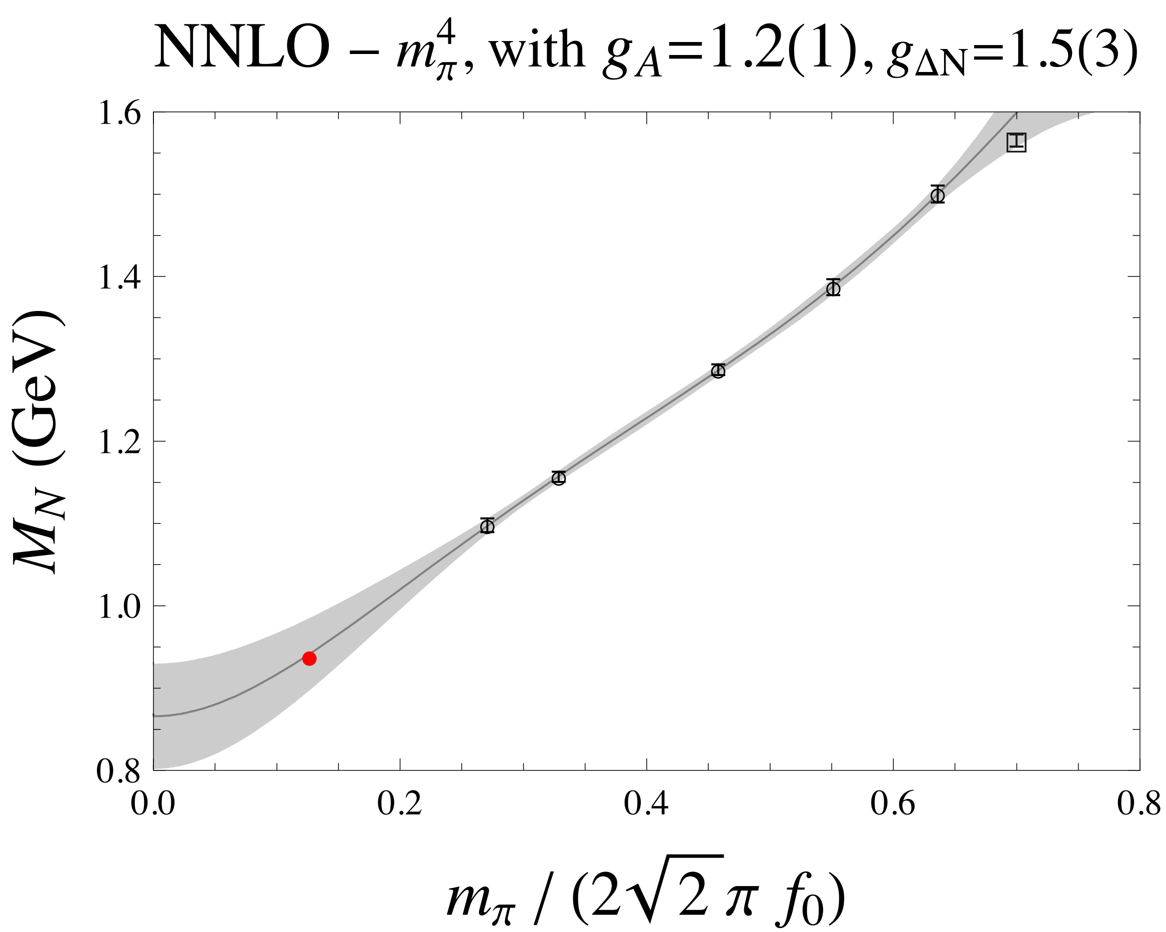
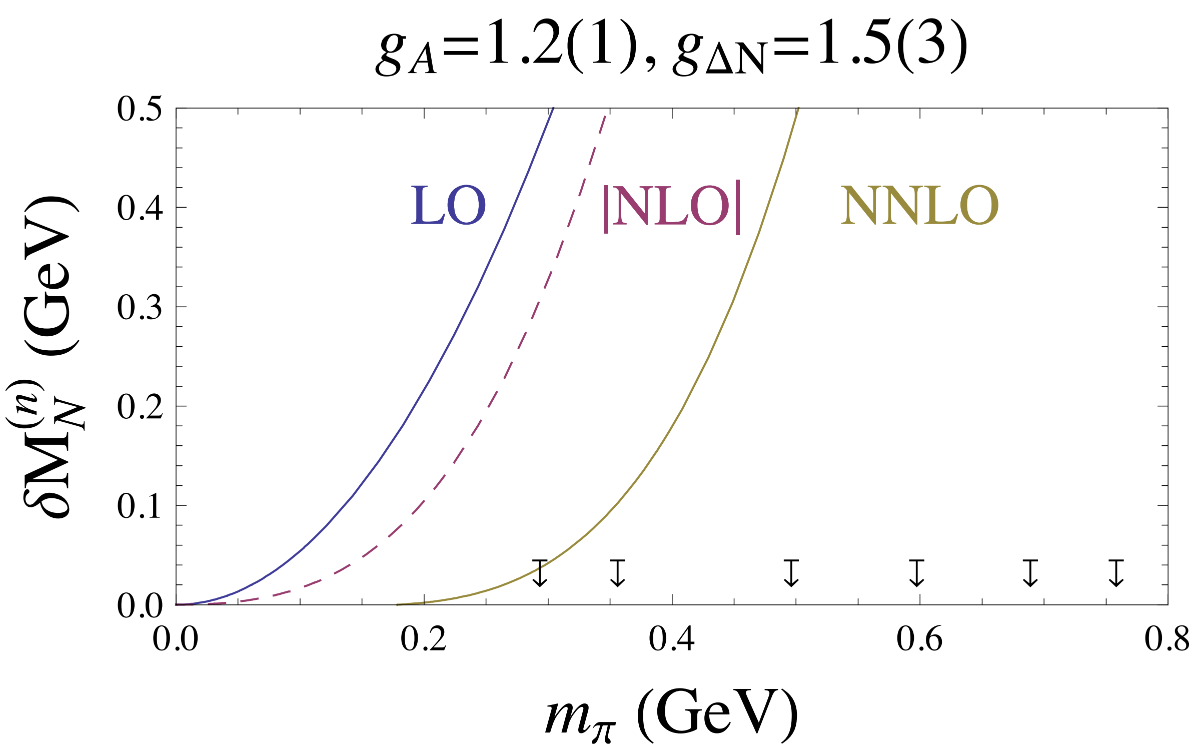
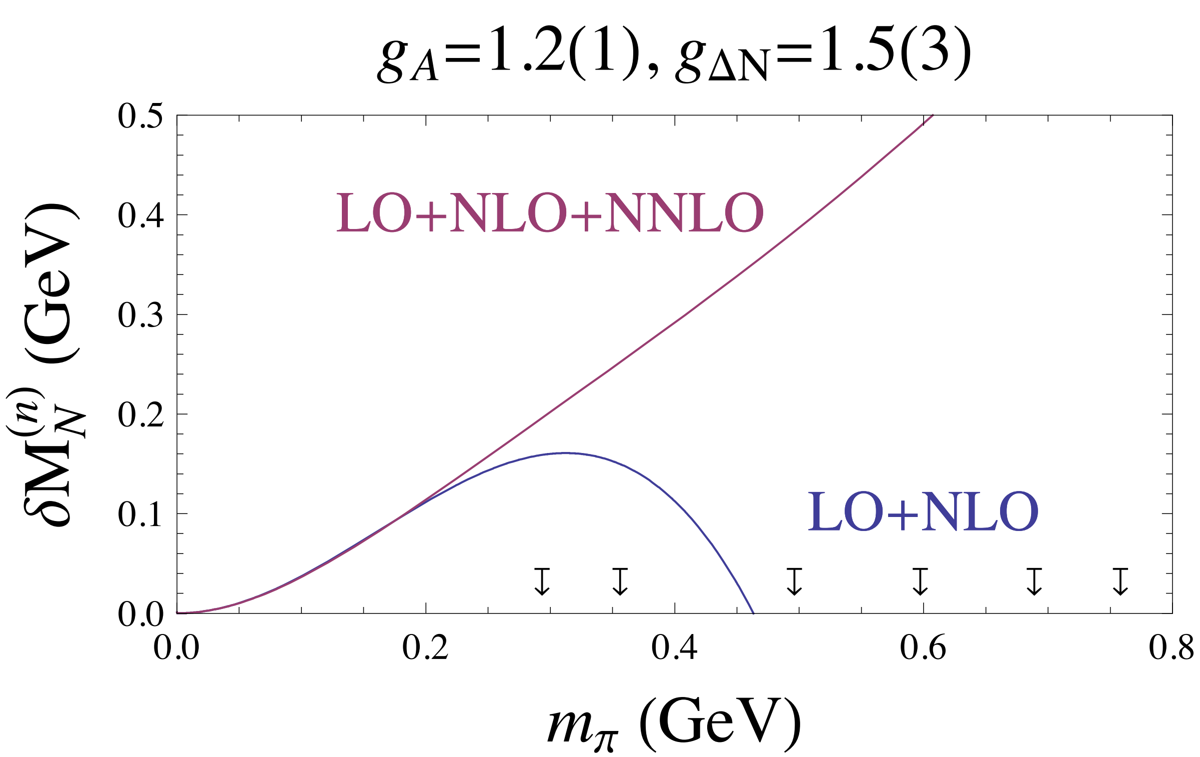
$m_\pi^3 \sim m_q^{3/2}$ leading non-analytic chiral behavior

renders the chiral expansion less convergent (than for mesons)

Light quark mass dependence of M_B







NNLO Heavy Baryon Fit

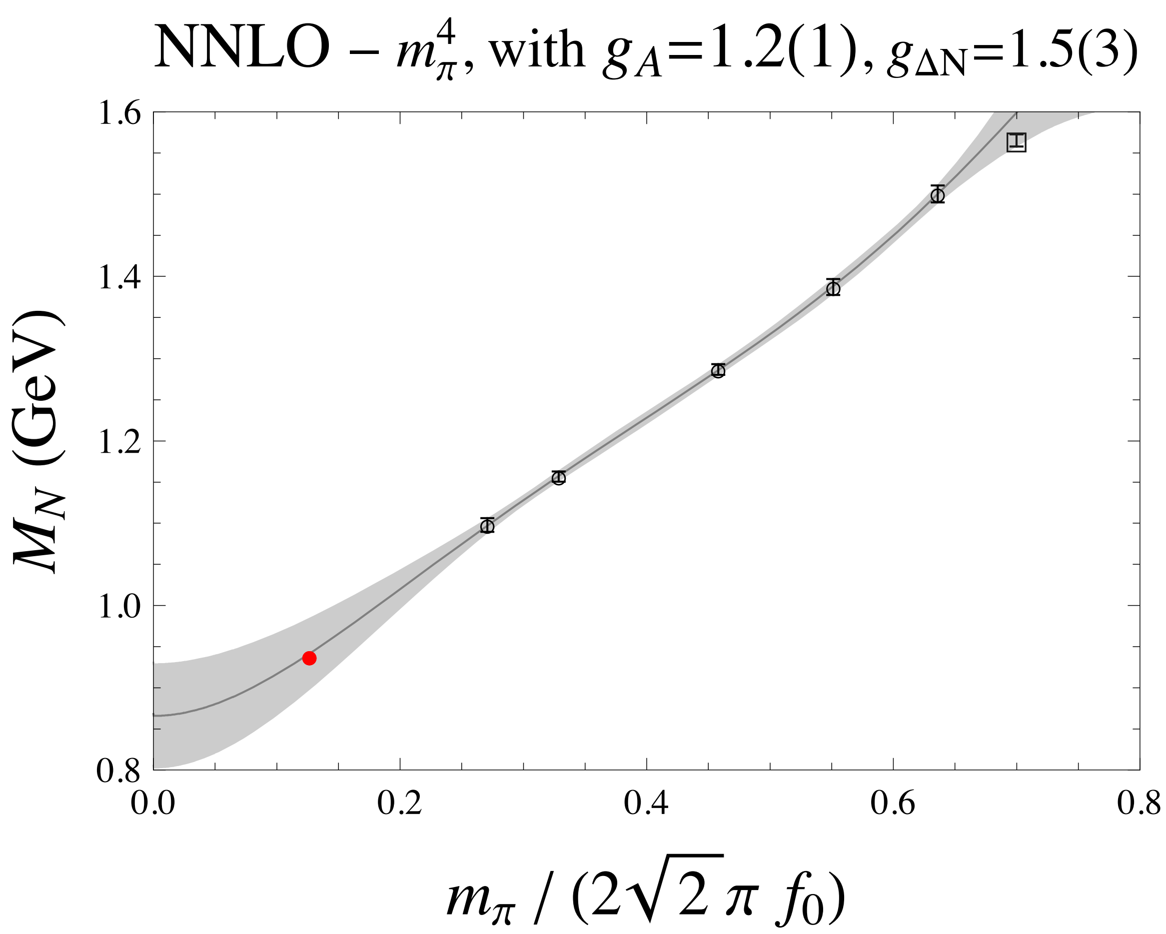
$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

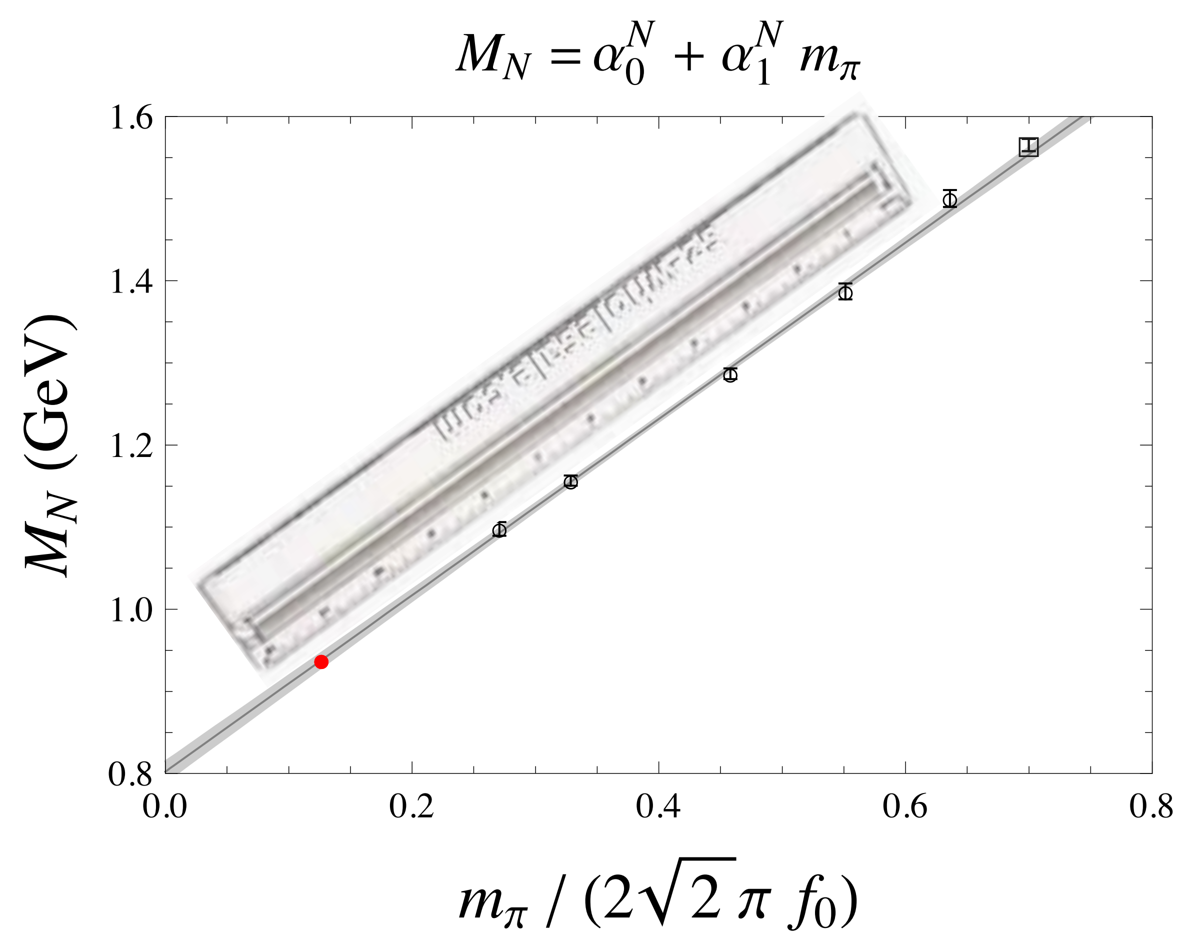
statistical

varying inputs

LHP Collaboration arXiv:0806.4549

Light quark mass dependence of M_B





NNLO Heavy Baryon Fit

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Ruler Approximation

$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$

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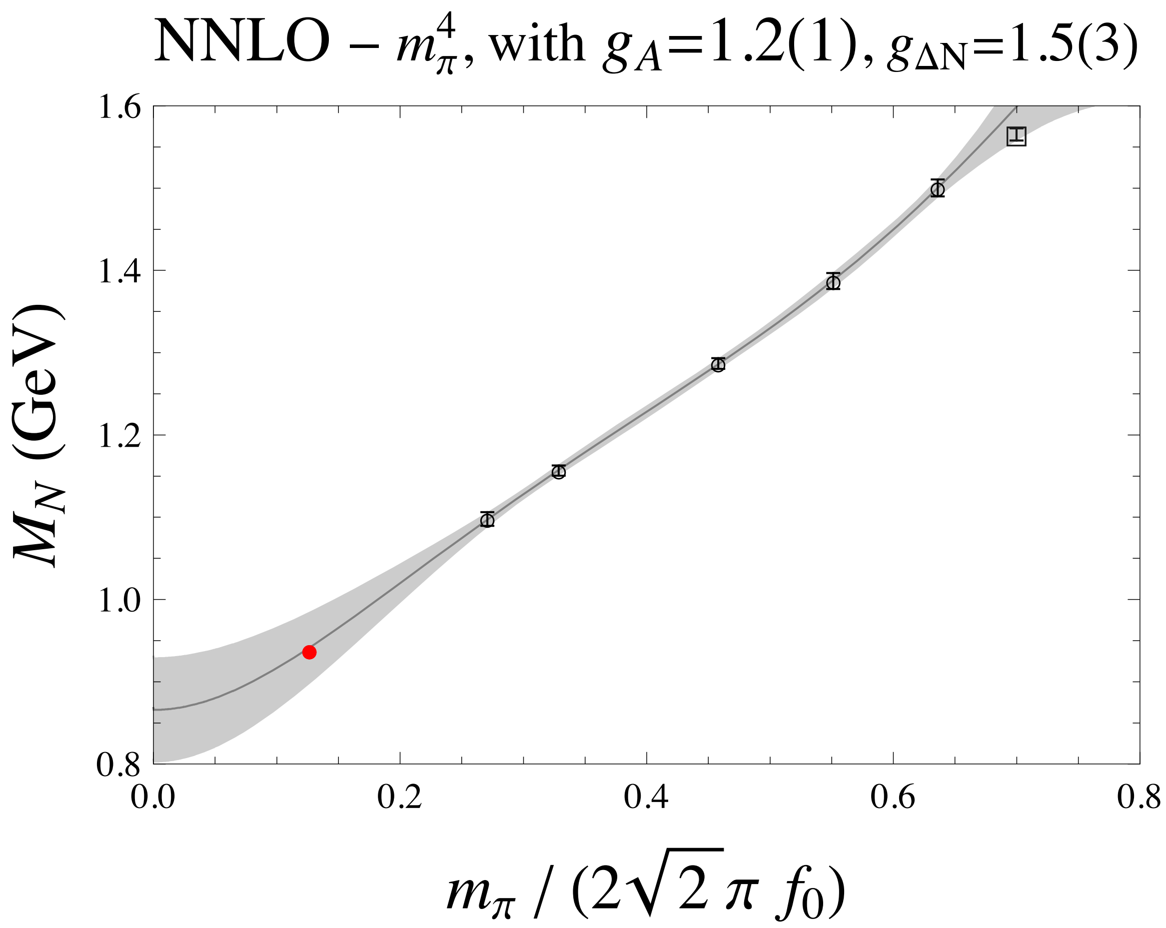

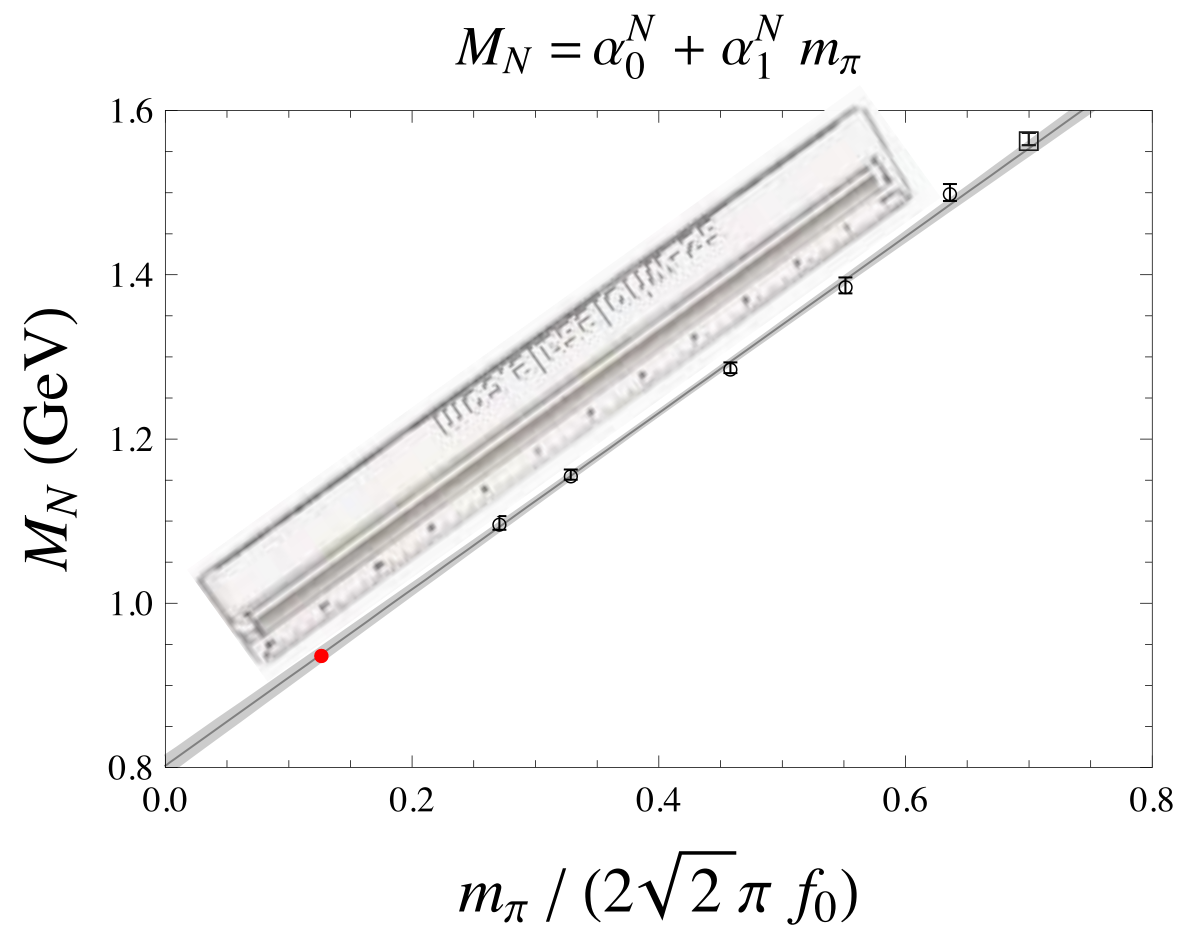

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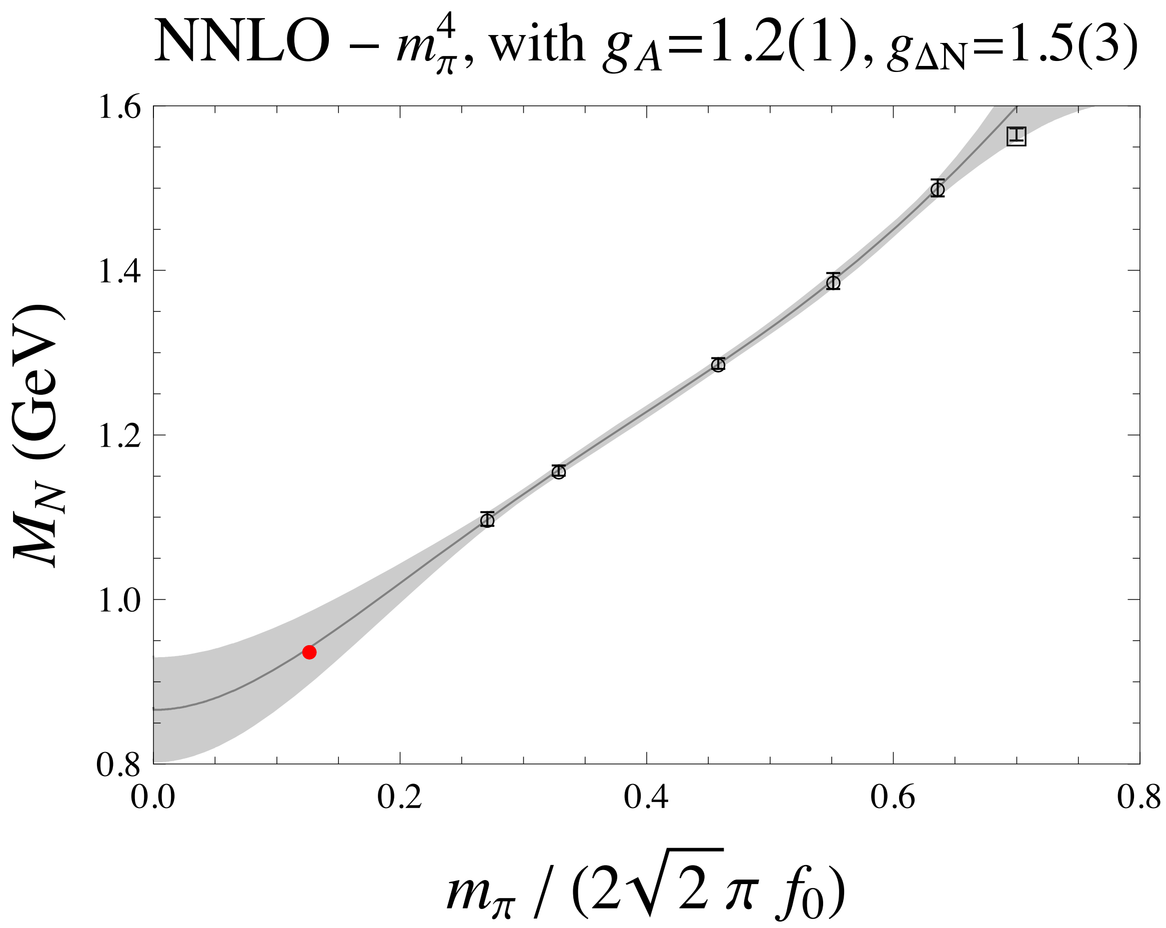

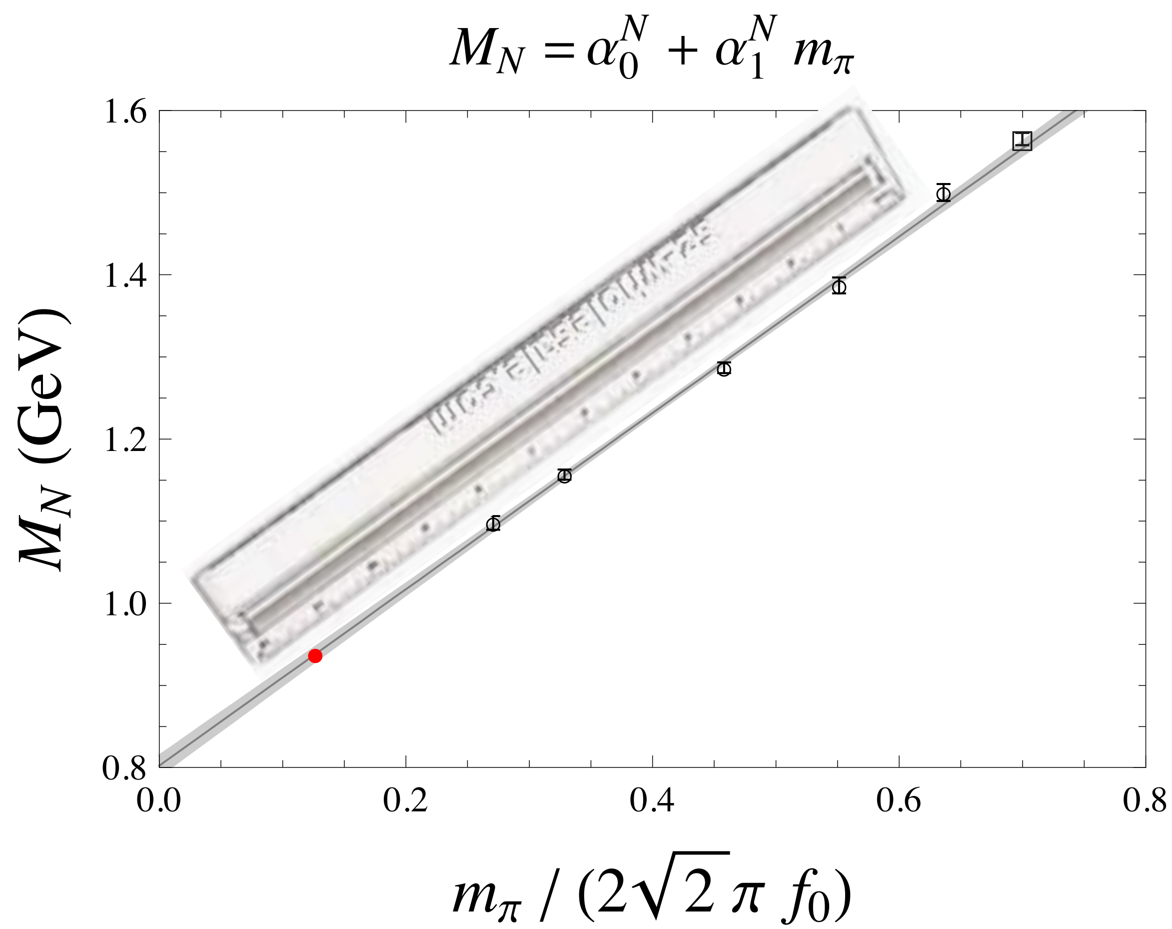

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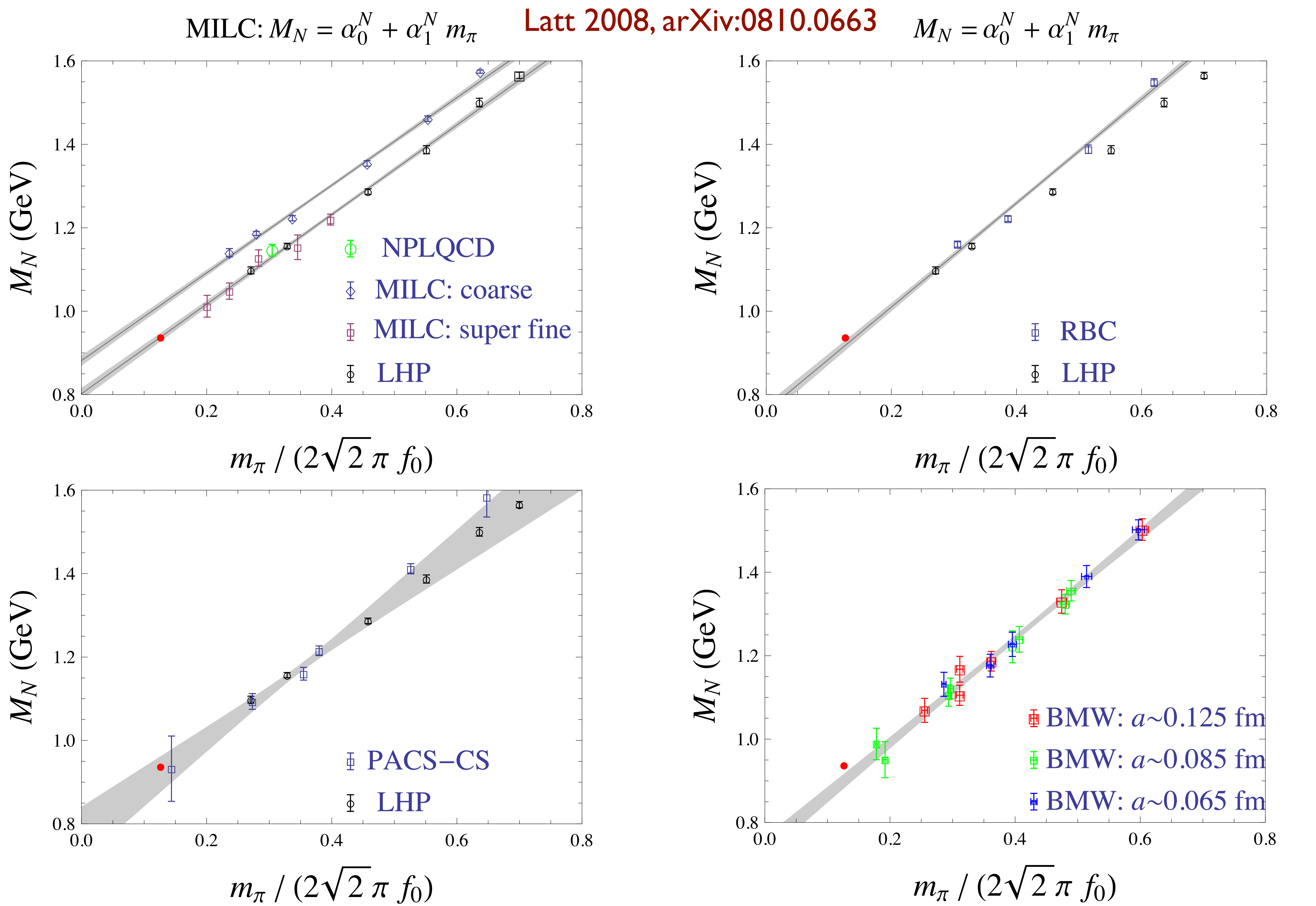

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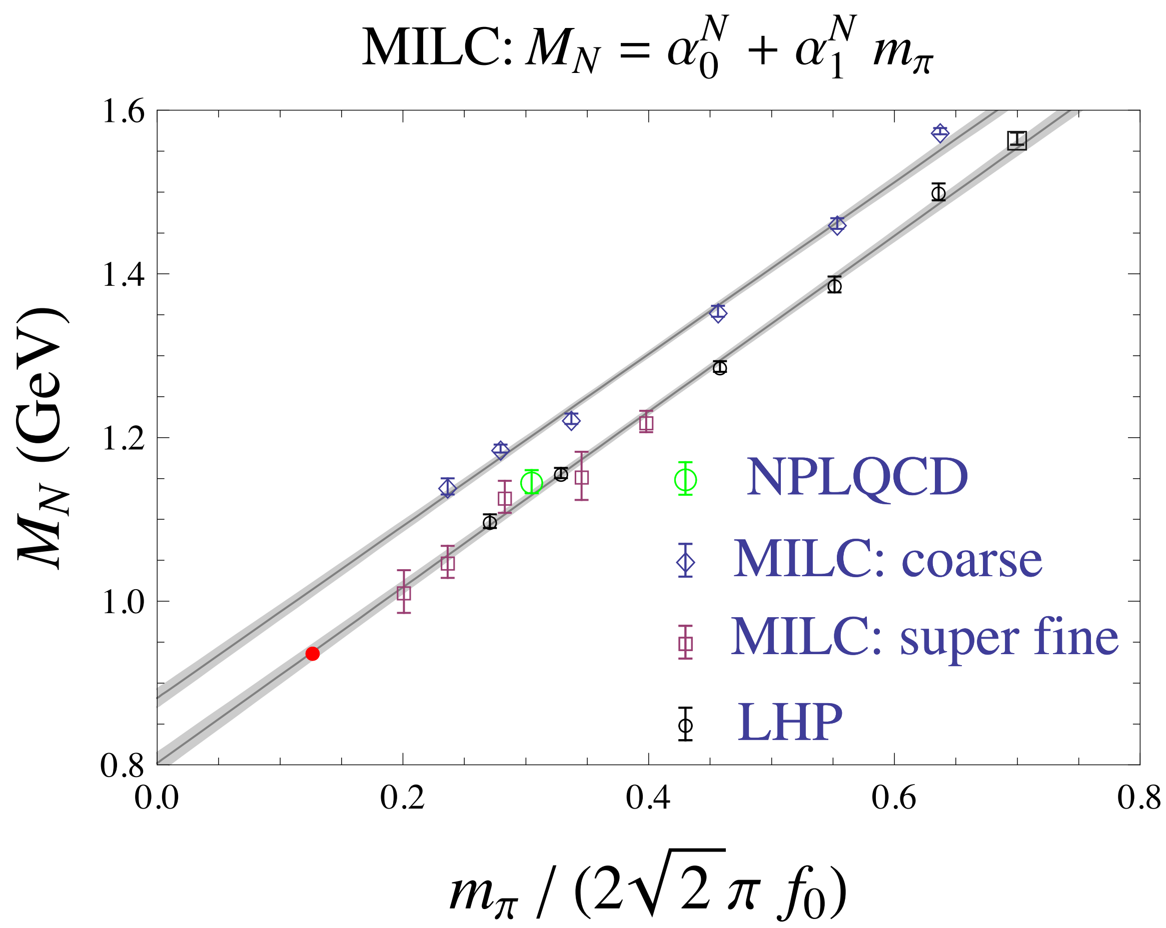


What does this teach us?

For these pion masses, there is a strong cancelation between LO, NLO and NNLO χPT contributions

perhaps should have been expected given poor convergence (but just not a straight line!!!)

Light quark mass dependence of M_B

What if we consider the octet and decuplet in the three flavor theory?

$$M_N = M_0 + \alpha_N^\pi m_\pi^2 + \alpha_N^K m_K^2$$
$$- \frac{1}{16\pi^2 f^2} \left[3\pi(D+F)^2 m_\pi^3 + \frac{\pi}{3}(D-3F)^2 m_\eta^3 \right.$$
$$+ \frac{2\pi}{3}(5D^2 - 6DF + 9F^2) m_K^3$$
$$\left. + \frac{8}{3}\mathcal{F}(m_\pi, \Delta, \mu) + \frac{2}{3}\mathcal{F}(m_K, \Delta, \mu) \right]$$

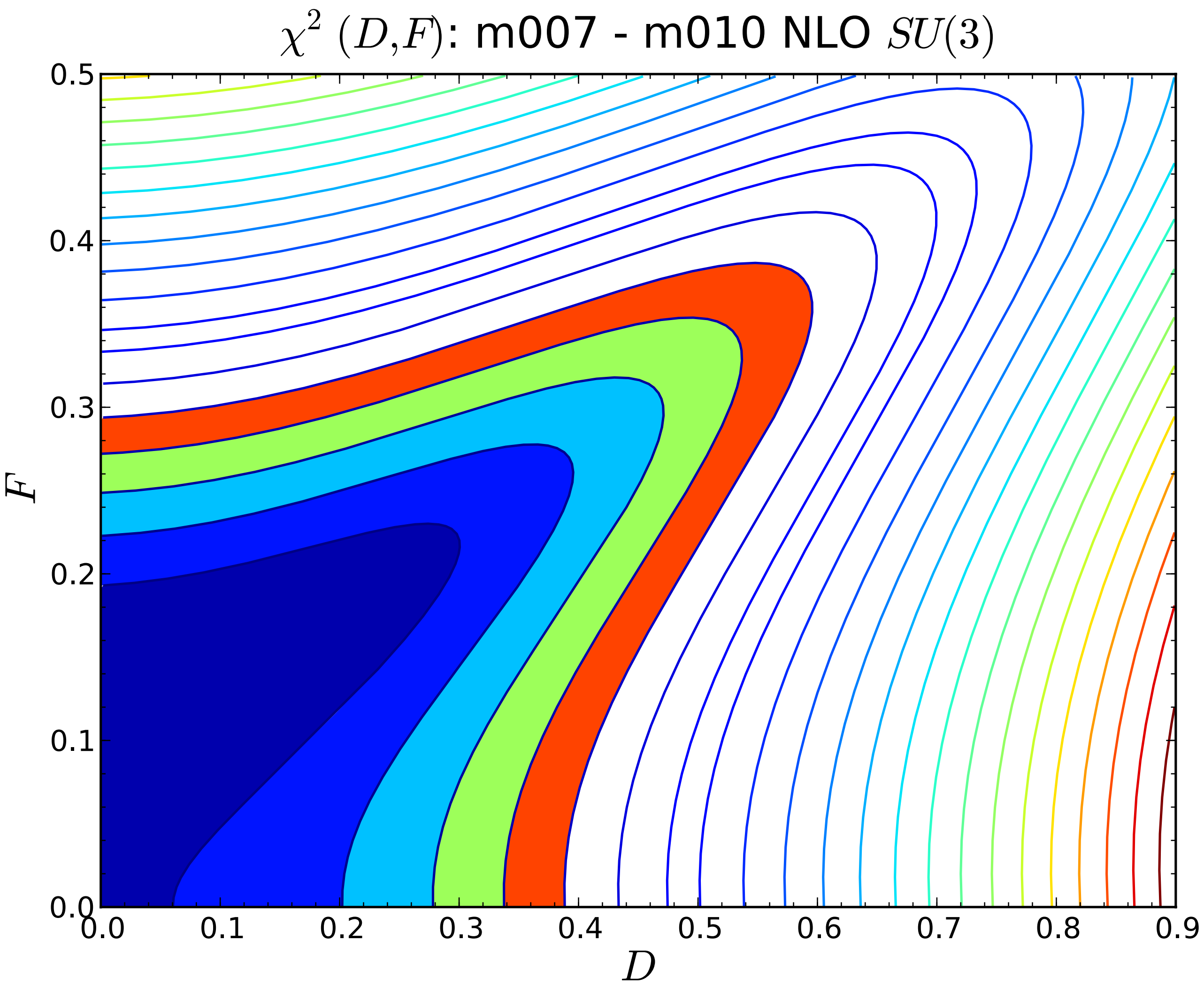
Possible convergence is significantly challenged (fails) by kaon and eta loops

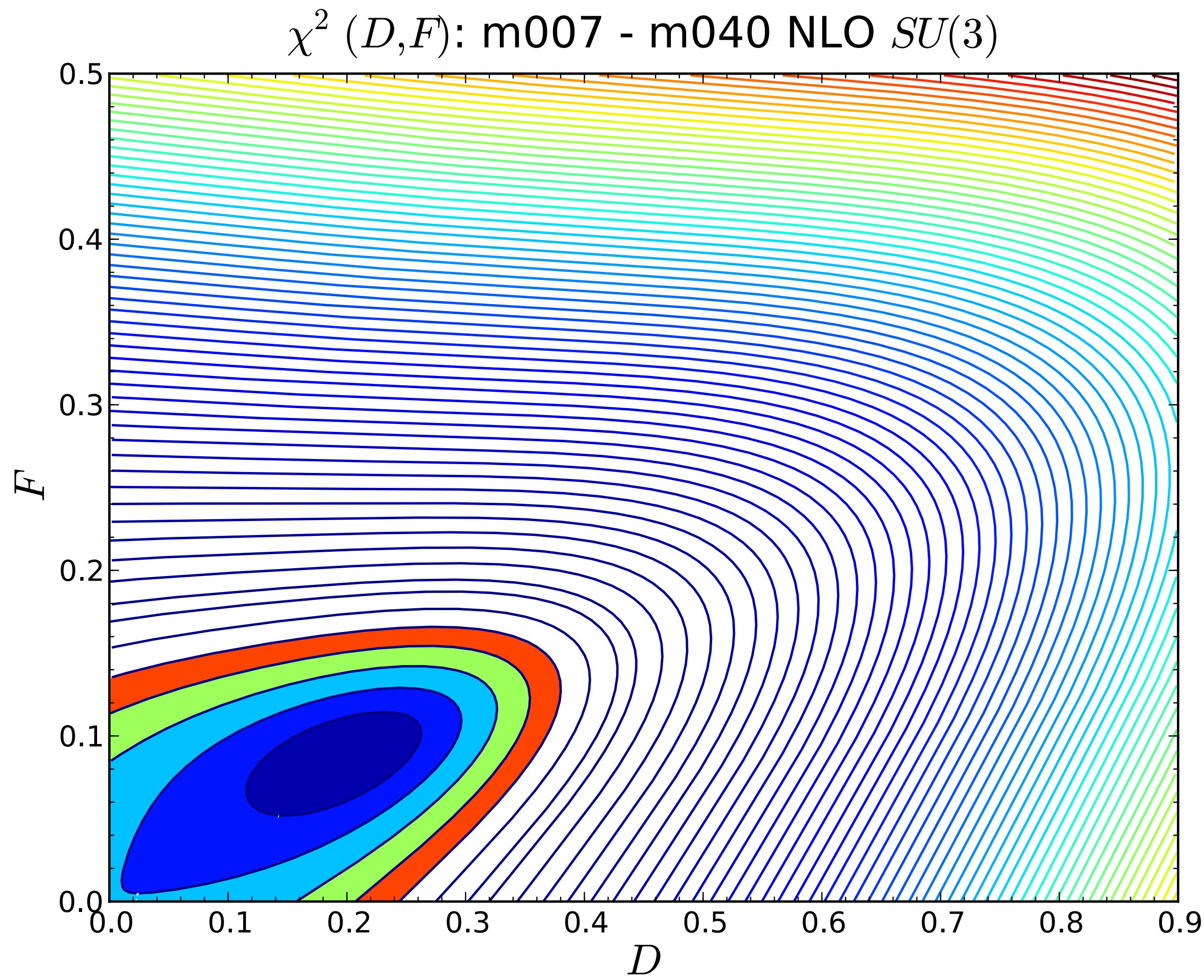
LHP Collaboration arXiv:0806.4549

PACS-CS Collaboration arXiv:0905.0962

Light quark mass dependence of M_B

figures: Jenkins, Manohar, Negele and AWL arXiv:0907.0529



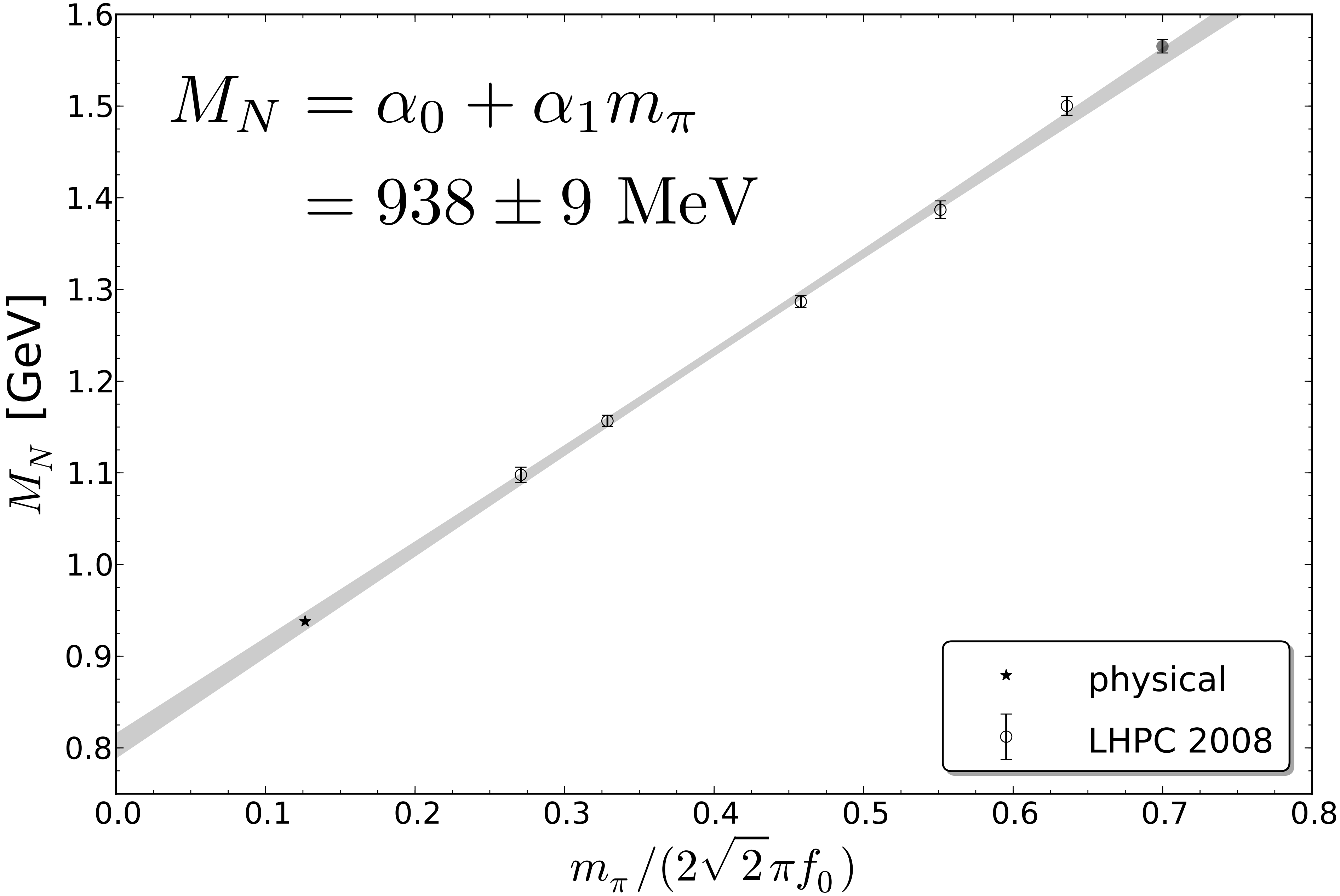


NLO SU(3) chiral fits to spectrum are not consistent with phenomenological values of D, F

$$D \sim 0.75, \quad F \sim 0.50$$

Light quark mass dependence of M_B

What is the status now (2012)?



Physical point NOT included in fit

Light quark mass dependence of M_B

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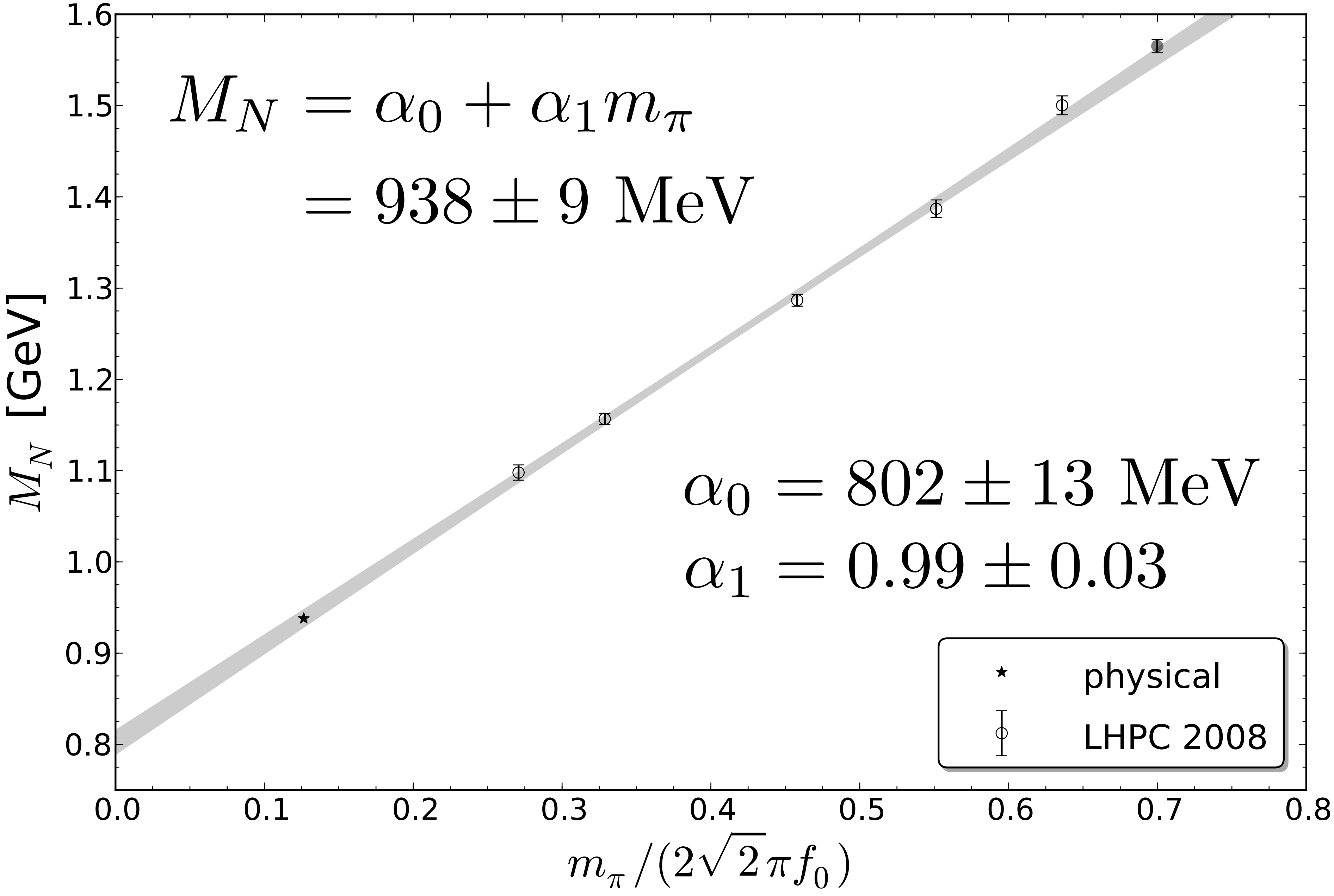

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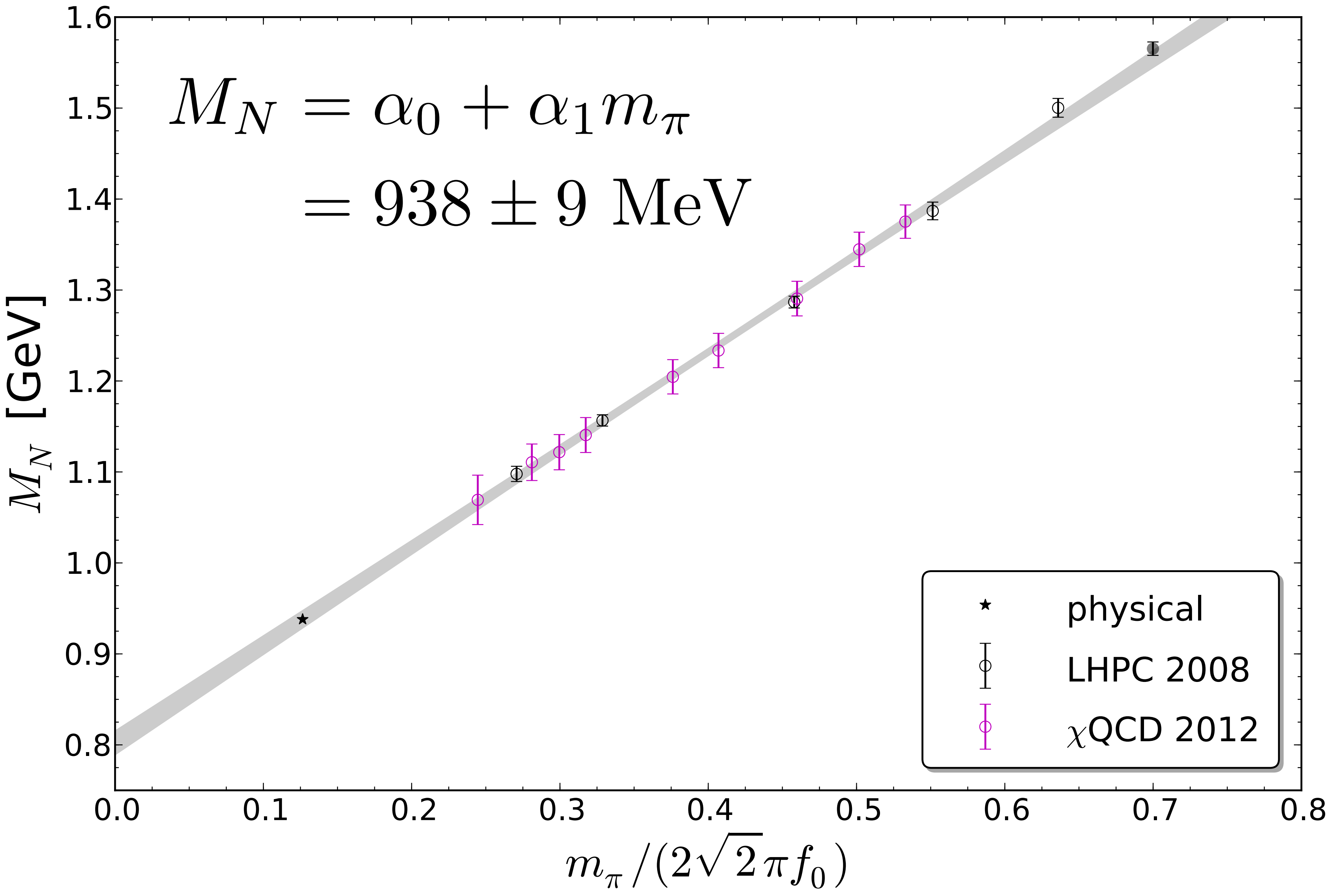

χQCD Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions

Light quark mass dependence of M_B

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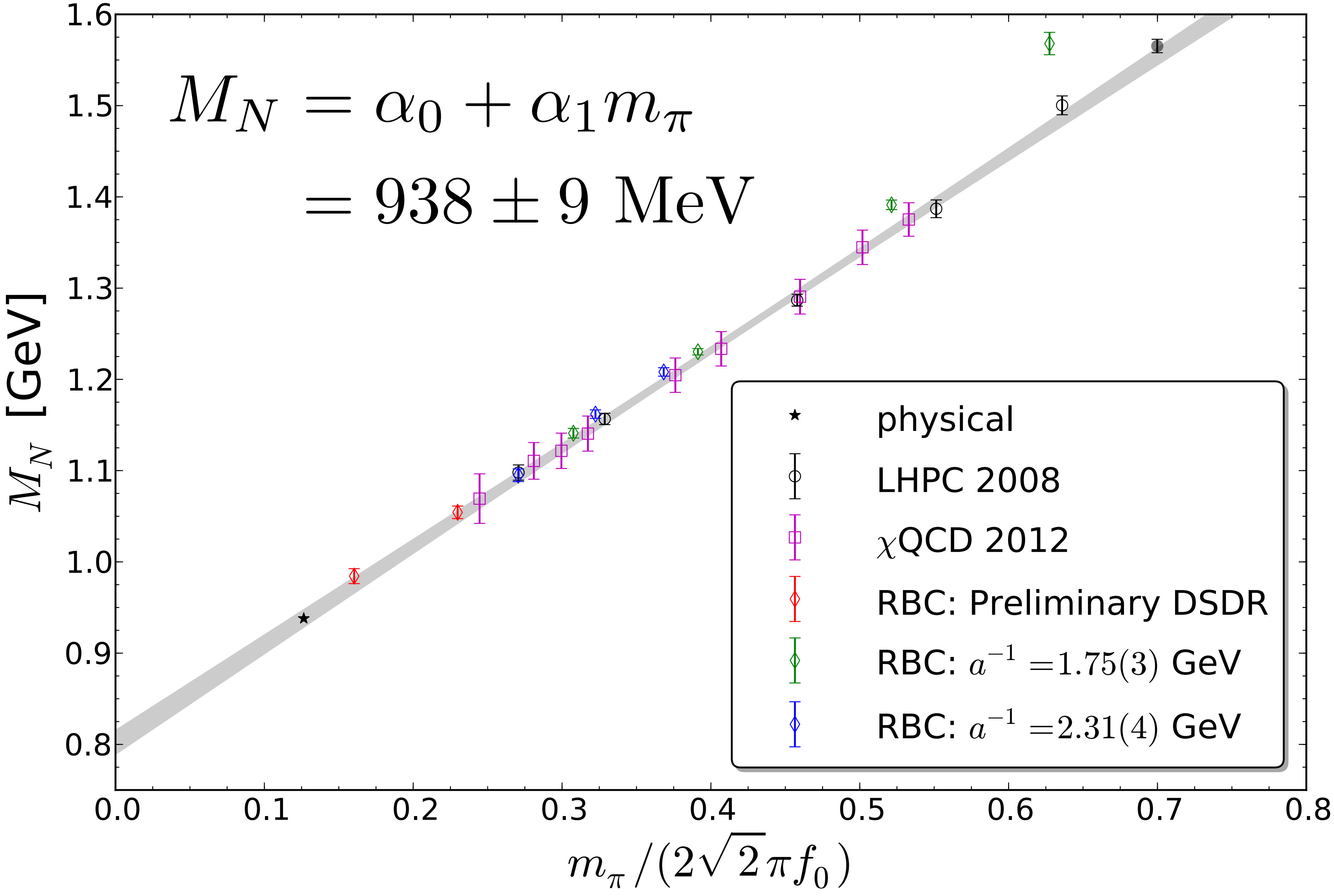

RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

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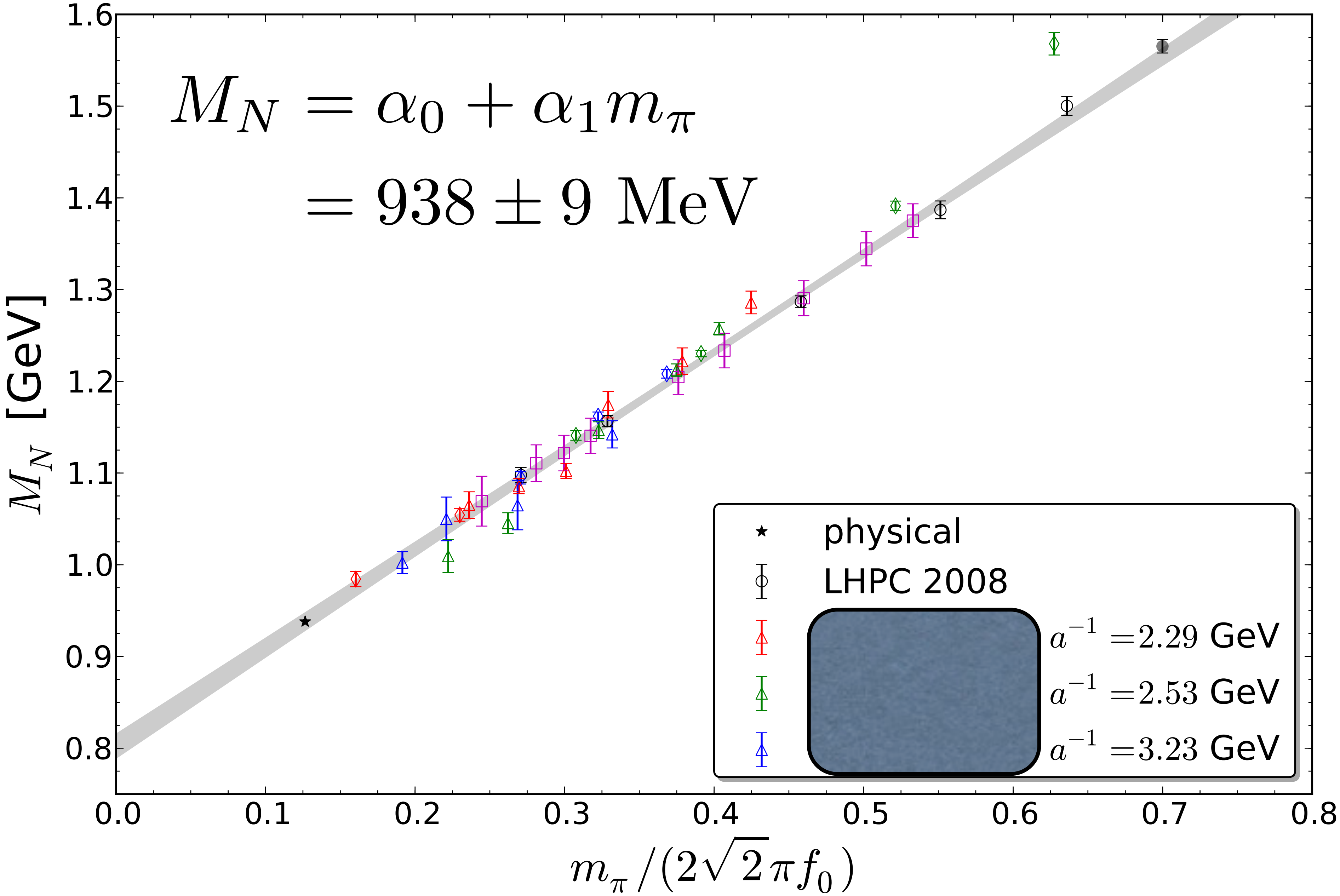

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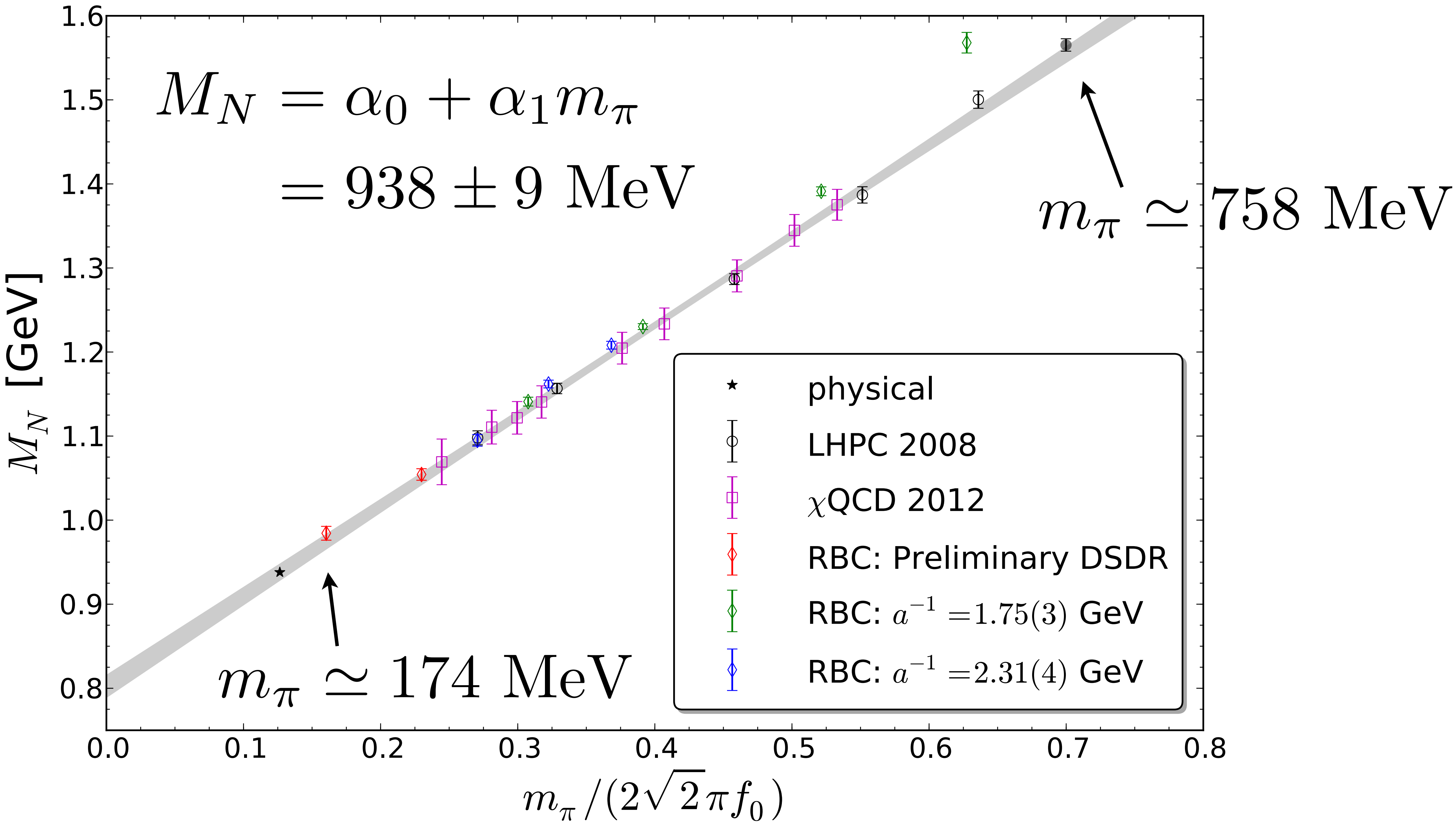

Taking this seriously yields

$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

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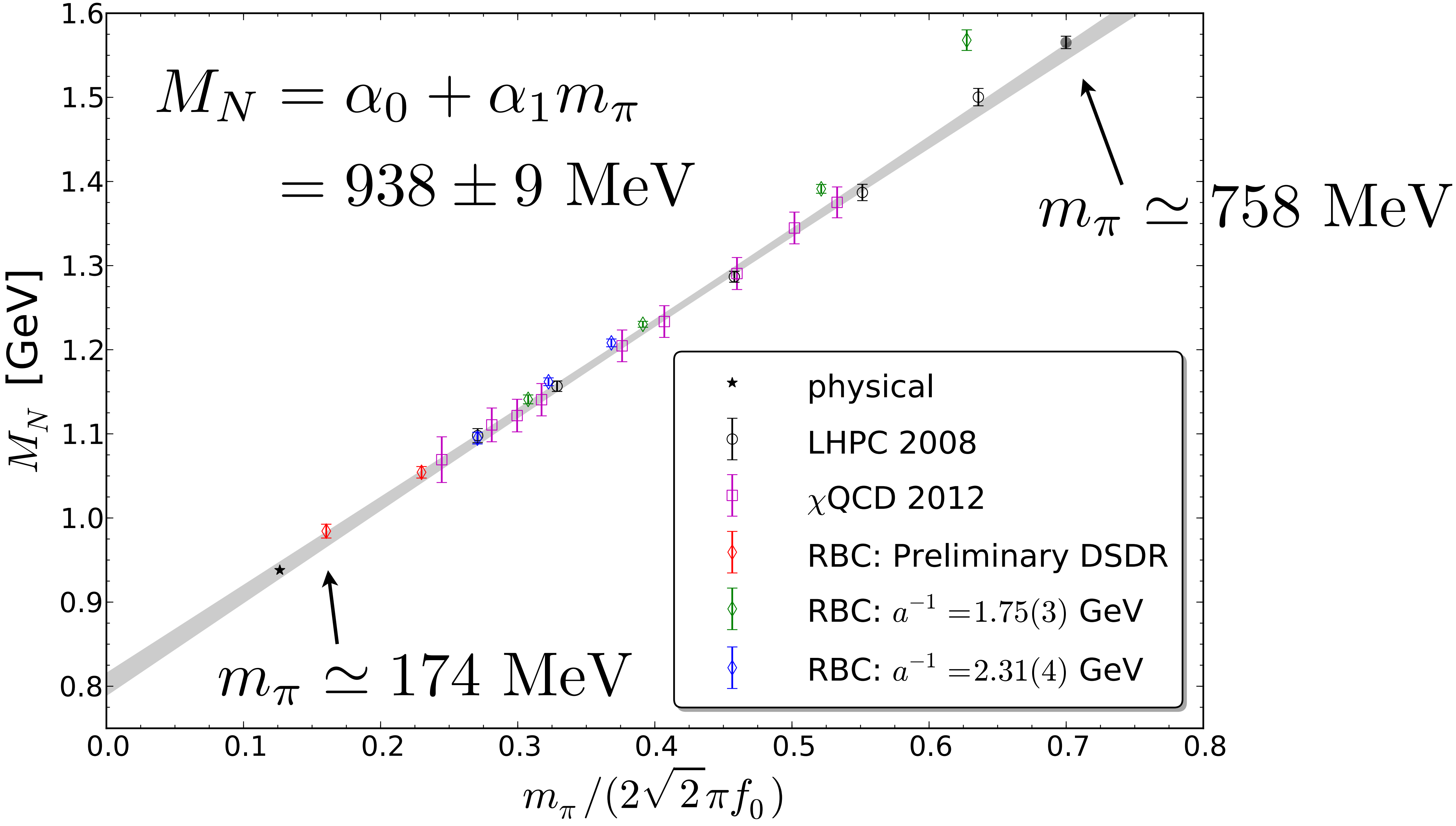

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Light quark mass dependence of M_B M. Alberg, G. Miller PRL 108 (2012)

$$\Sigma_\pi(N) = \frac{3g_{\pi N}^2}{M}\int\frac{d^4kF^2(k^2)}{i(2\pi)^4}$$
$$\times\frac{k\cdot p}{(k^2-\mu^2+i\epsilon)[(p-k)^2-M^2+i\epsilon]}.$$

To evaluate integral, used light cone coordinates

Light quark mass dependence of M_B M. Alberg, G. Miller PRL 108 (2012)

$$\Sigma_\pi(N) = \frac{3g_{\pi N}^2}{M}\int dk^+ d^2k_\perp J$$

$$J = \frac{1}{i(2\pi)^4}\frac{1}{2}\int dk^- F^2(k^2)$$
$$\times \frac{k\cdot p}{k^+(p-k)^+(k^- - \frac{k_\perp^2+\mu^2-i\epsilon}{k^+})[(p-k)^- - \frac{k_\perp^2+M^2-i\epsilon}{p^+-k^+}]}.$$

This allows one to pick the contour such that the intermediate nucleon (delta) is on shell - simplifying the numerator structure

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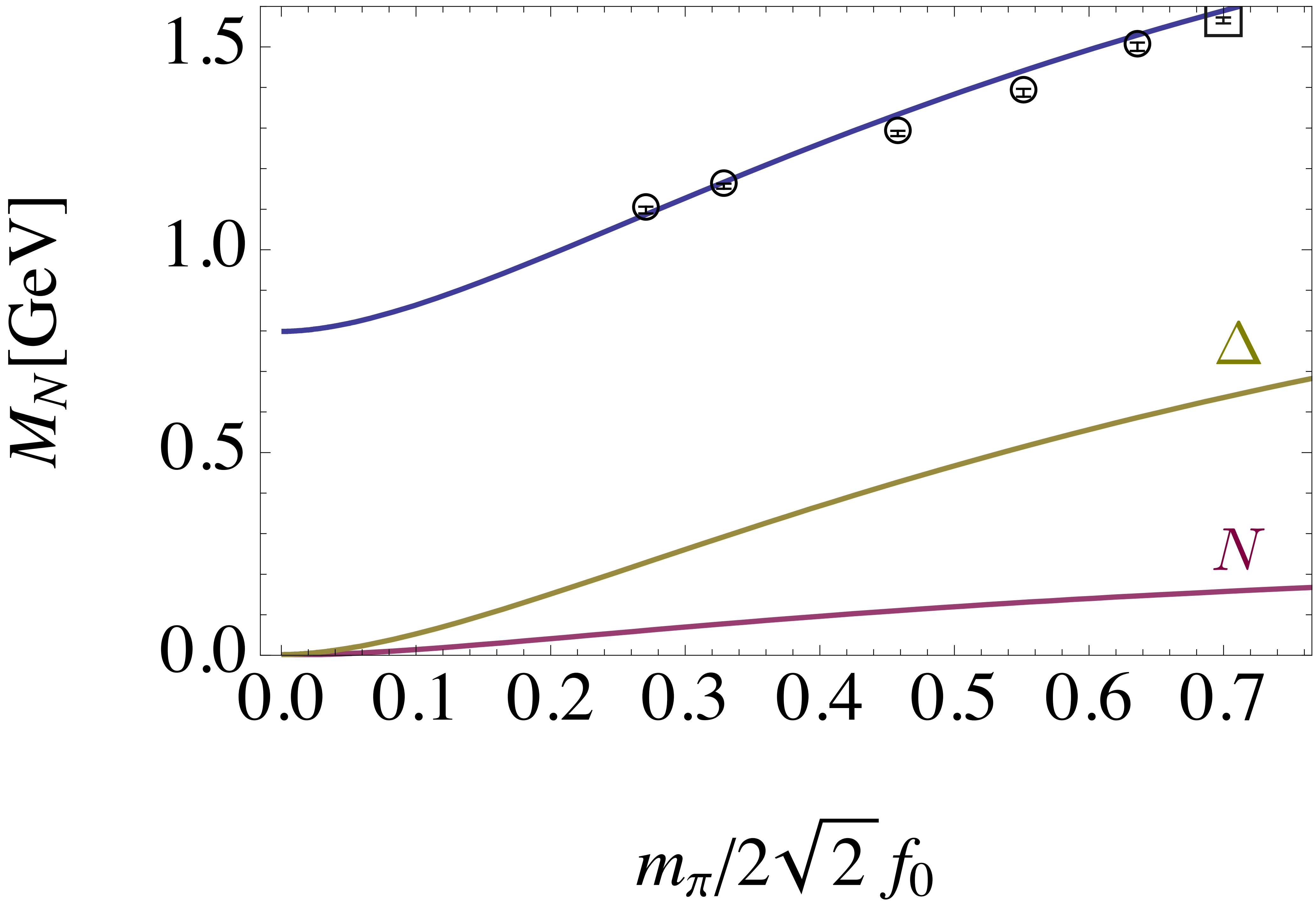
expanding for small pion mass (μ) one recovers the HBChiPT expression

Light quark mass dependence of M_B M. Alberg, G. Miller PRL 108 (2012)



Nucleon and Delta loop contributions set to zero at origin

Large N_c and SU(3) Chiral Perturbation Theory

What can we do?

- Consider 2-flavor expansion for hyperons

Beane, Bedaque, Parreno and Savage nucl-th/0311027

Tiburzi and AWL arXiv:0808.0482

Jiang and Tiburzi arXiv:0905.0857

Mai, Bruns, Kubis and Meissner arXiv:0905.2810

Jiang, Tiburzi and AWL arXiv:0911.4721

Jiang and Tiburzi arXiv:0912.2077

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- Read the literature and apply an old idea to our new problem

  combine the constraints of large N_c and SU(3) symmetries

Large N_c and SU(3) Chiral Perturbation Theory

Combined large N_c and SU(3) symmetries

't Hooft 1974
Witten 1979
Coleman 1979
Dashen, Jenkins, Manohar 1993

...

Large N_c and SU(3) Chiral Perturbation Theory

- theory is placed on solid theoretical foundation

$$\lim_{N_c \to \infty} M_B = \infty$$

controlled expansion in $1/N_c$ (at least formally)

- inclusion of spin 3/2 dof well defined field theoretically

$$M_\Delta - M_N \propto \frac{1}{N_c}$$

- naturally explains smallness of baryon octet GMO relation

$$N_c m_s^{3/2} \propto \text{flavor-}\mathbf{1}$$

$$m_s^{3/2} \propto \text{flavor-}\mathbf{8}$$

$$m_s^{3/2}/N_c \propto \text{flavor-}\mathbf{27}$$ leading correction to GMO

Large N_c and SU(3) Chiral Perturbation Theory

- gives you "smarter" observables to measure/calculate

eg: Spectrum $M = M^{1,0} + M^{8,0} + M^{27,0} + M^{64,0}$

$$M^{1,0} = c^{1,0}_{(0)} N_c \mathbf{1} + c^{1,0}_{(2)} \frac{1}{N_c} J^2$$

$$M^{8,0} = c^{8,0}_{(1)} T^8 + c^{8,0}_{(2)} \frac{1}{N_c} \{J^i, G^{i8}\} + c^{8,0}_{(3)} \frac{1}{N_c^2} \{J^2, T^8\}$$

$$M^{27,0} = c^{27,0}_{(2)} \frac{1}{N_c} \{T^8, T^8\} + c^{27,0}_{(3)} \frac{1}{N_c^2} \{T^8, \{J^i, G^{i8}\}\}$$

$$M^{64,0} = c^{64,0}_{(3)} \frac{1}{N_c^2} \{T^8, \{T^8, T^8\}\}$$

$J^i = q^\dagger (J^i \otimes \mathbf{1}) q$ one-body spin operator

$T^a = q^\dagger (\mathbf{1} \otimes T^a) q$ one-body flavor operator

$G^{ia} = q^\dagger (J^i \otimes T^a) q$ one-body spin-flavor operator

Large N_c and SU(3) Chiral Perturbation Theory

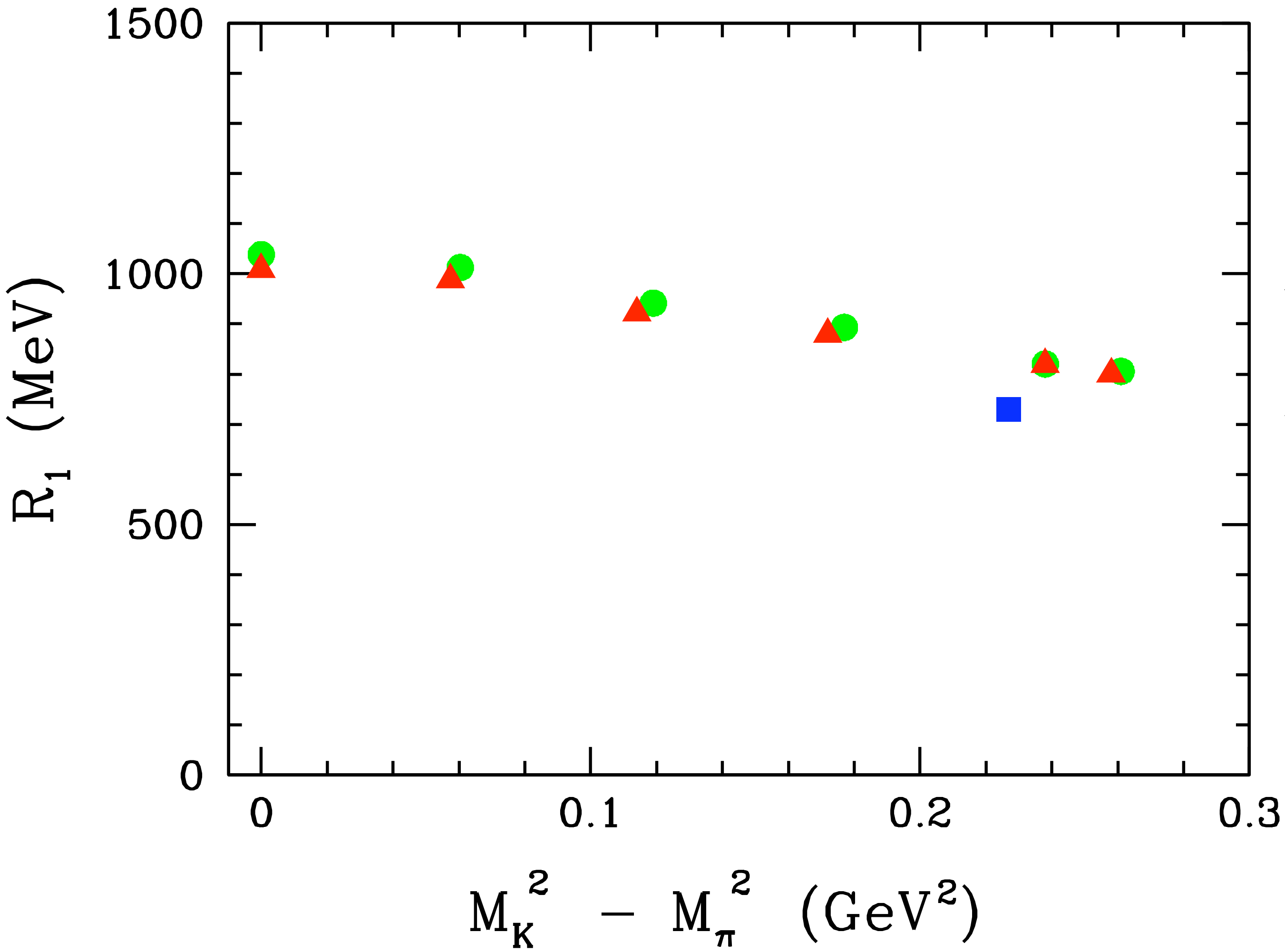
Jenkins and Lebed hep-ph/9502227

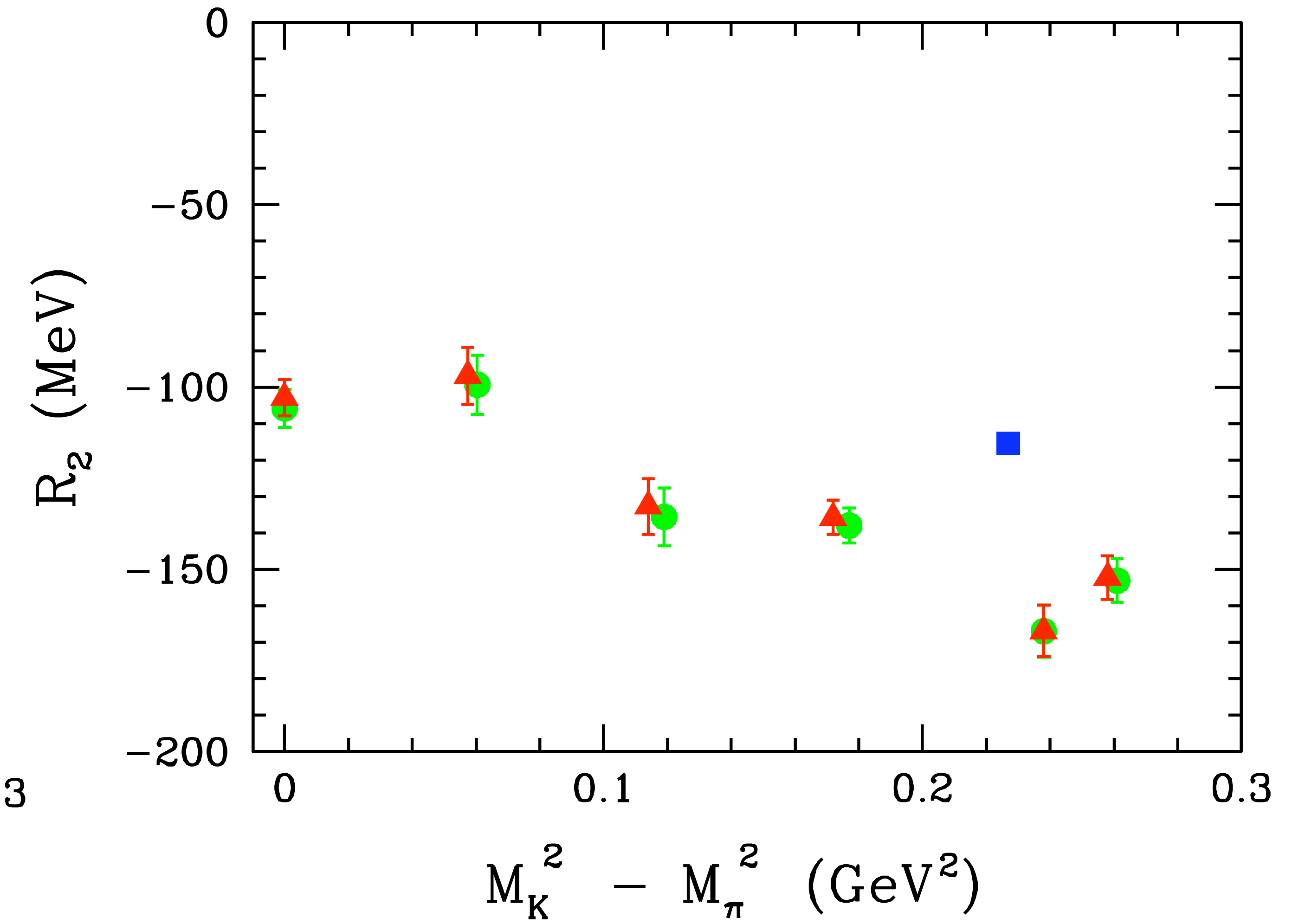
Label	Operator	Coefficient	Mass Combination	$1/N_c$	$SU(3)$
M_1	$\mathbb{1}$	$160\, N_c\, c^{1,0}_{(0)}$	$25(2N+\Lambda+3\Sigma+2\Xi)-4(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	N_c	1
M_2	J^2	$-120\, \frac{1}{N_c}\, c^{1,0}_{(2)}$	$5(2N+\Lambda+3\Sigma+2\Xi)-4(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$	$1/N_c$	1
M_3	T^8	$20\sqrt{3}\,\epsilon\, c^{8,0}_{(1)}$	$5(6N+\Lambda-3\Sigma-4\Xi)-2(2\Delta-\Xi^*-\Omega)$	1	ϵ
M_4	$\{J^i, G^{i8}\}$	$-5\sqrt{3}\,\frac{1}{N_c}\,\epsilon\, c^{8,0}_{(2)}$	$N+\Lambda-3\Sigma+\Xi$	$1/N_c$	ϵ
M_5	$\{J^2, T^8\}$	$30\sqrt{3}\,\frac{1}{N_c^2}\,\epsilon\, c^{8,0}_{(3)}$	$(-2N+3\Lambda-9\Sigma+8\Xi)+2(2\Delta-\Xi^*-\Omega)$	$1/N_c^2$	ϵ
M_6	$\{T^8, T^8\}$	$126\,\frac{1}{N_c}\,\epsilon^2\, c^{27,0}_{(2)}$	$35(2N-3\Lambda-\Sigma+2\Xi)-4(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c$	ϵ^2
M_7	$\{T^8, J^iG^{i8}\}$	$-63\,\frac{1}{N_c^2}\,\epsilon^2\, c^{27,0}_{(3)}$	$7(2N-3\Lambda-\Sigma+2\Xi)-2(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$	$1/N_c^2$	ϵ^2
M_8	$\{T^8,\{T^8,T^8\}\}$	$9\sqrt{3}\,\frac{1}{N_c^2}\,\epsilon^3\, c^{64,0}_{(3)}$	$\Delta-3\Sigma^*+3\Xi^*-\Omega$	$1/N_c^2$	ϵ^3
M_A			$(\Sigma^*-\Sigma)-(\Xi^*-\Xi)$	$1/N_c^2$	–
M_B			$\frac{1}{3}(\Sigma+2\Sigma^*)-\Lambda-\frac{2}{3}(\Delta-N)$	$1/N_c^2$	–
M_C			$-\frac{1}{4}(2N-3\Lambda-\Sigma+2\Xi)+\frac{1}{4}(\Delta-\Sigma^*-\Xi^*+\Omega)$	$1/N_c^2$	–
M_D			$-\frac{1}{2}(\Delta-3\Sigma^*+3\Xi^*-\Omega)$	$1/N_c^2$	–

$$R \equiv \frac{\sum_i c_i M_i}{\sum_i |c_i|}$$

$$\epsilon \propto m_s - m_l$$

Large N_c and SU(3) Chiral Perturbation Theory





$$R_1 \sim \mathcal{O}(N_c) \times \mathcal{O}(\epsilon^0)$$

$$R_2 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^0)$$

Jenkins, Manohar, Negele + AWL arXiv:0907.0529

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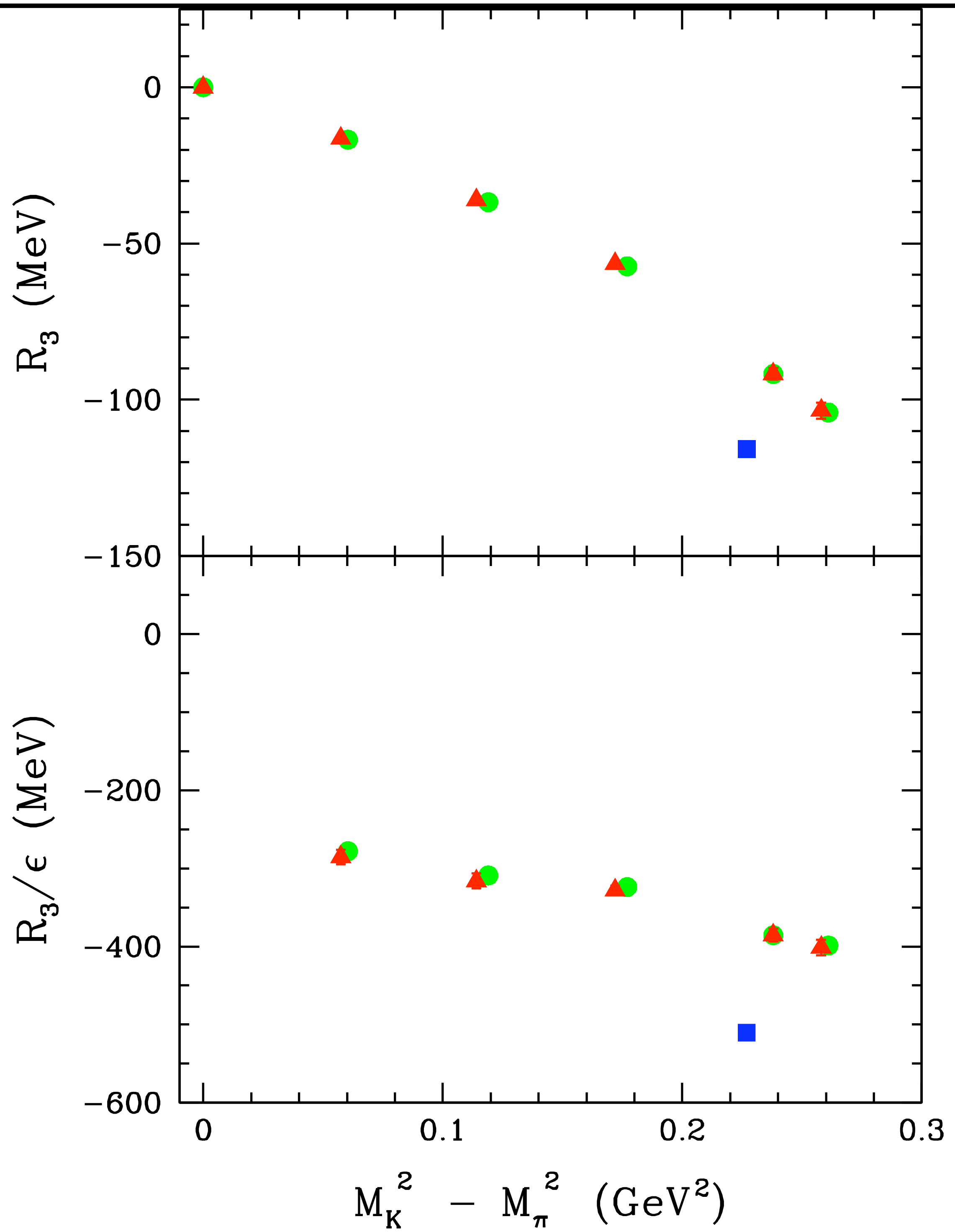

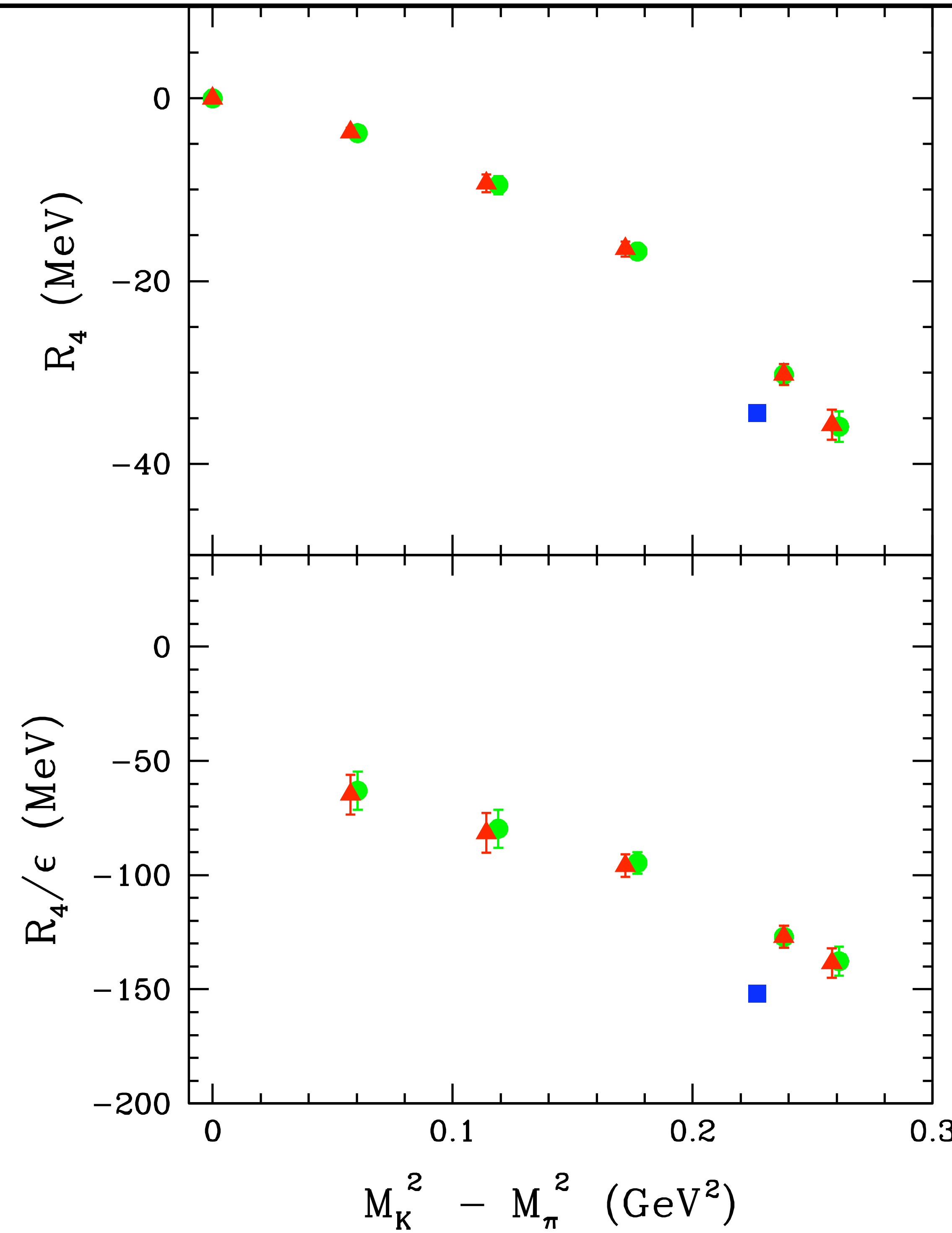

$$R_3 \sim \mathcal{O}(N_c^0) \times \mathcal{O}(\epsilon)$$

$$R_4 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon)$$

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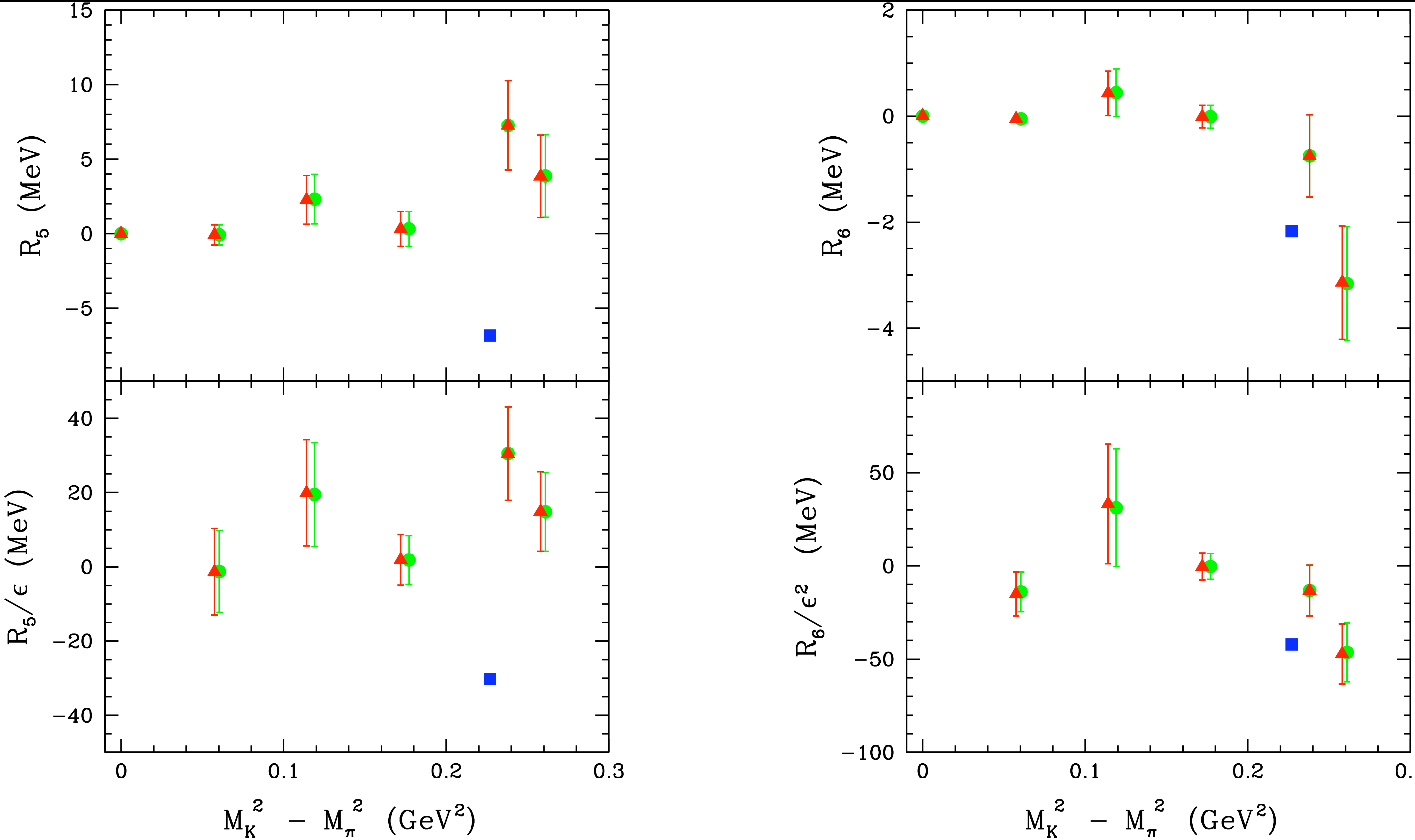


$$R_5 \sim \mathcal{O}(1/N_c^2) \times \mathcal{O}(\epsilon)$$

$$R_6 \sim \mathcal{O}(1/N_c) \times \mathcal{O}(\epsilon^2)$$

Jenkins, Manohar, Negele + AWL arXiv:0907.0529

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

$$\mathcal{L} = i\ \mathrm{Tr}\ \bar{B}_v\,(v\cdot\mathcal{D})\,B_v - i\ \bar{T}^{\mu}_v\,(v\cdot\mathcal{D})\,T_{v\,\mu} - \frac{1}{4}\Delta_0\ Tr\ \bar{B}_v B_v + \frac{5}{4}\Delta_0\ \bar{T}^{\mu}_v T_{v\,\mu}$$
$$+ 2D\,\mathrm{Tr}\left(\bar{B}_v S^{\mu}_v\left\{\mathcal{A}_\mu, B_v\right\}\right) + 2F\,\mathrm{Tr}\left(\bar{B}_v S^{\mu}_v\left[\mathcal{A}_\mu, B_v\right]\right)$$
$$+ \mathcal{C}\ \left(\bar{T}^{\mu}_v\mathcal{A}_\mu B_v + \bar{B}_v\mathcal{A}_\mu T^{\mu}_v\right) + 2\mathcal{H}\,\bar{T}^{\mu}_v S^{\nu}_v\mathcal{A}_\nu T_{v\,\mu}$$
$$+ 2\sigma_B\ \mathrm{Tr}\left(\bar{B}_v B_v\right)\mathrm{Tr}\mathcal{M}_+ - 2\sigma_T\ \bar{T}^{\mu}_v T_{v\,\mu}\mathrm{Tr}\mathcal{M}_+$$
$$+ 2b_D\mathrm{Tr}\left(\bar{B}_v\left\{\mathcal{M}_+, B_v\right\}\right) + 2b_F\mathrm{Tr}\left(\bar{B}_v\left[\mathcal{M}_+, B_v\right]\right) + 2b_T\,\bar{T}^{\mu}_v\mathcal{M}_+ T_{v\,\mu}$$

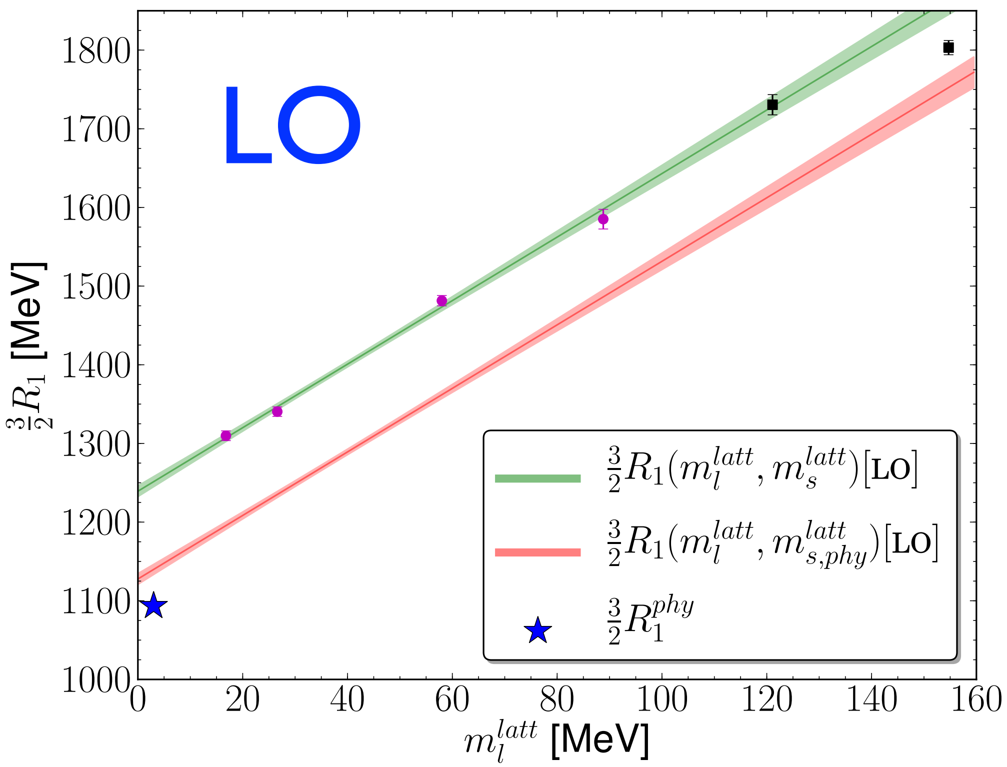
Large N_c expansion simplifies operators: Jenkins hep-ph/9509433

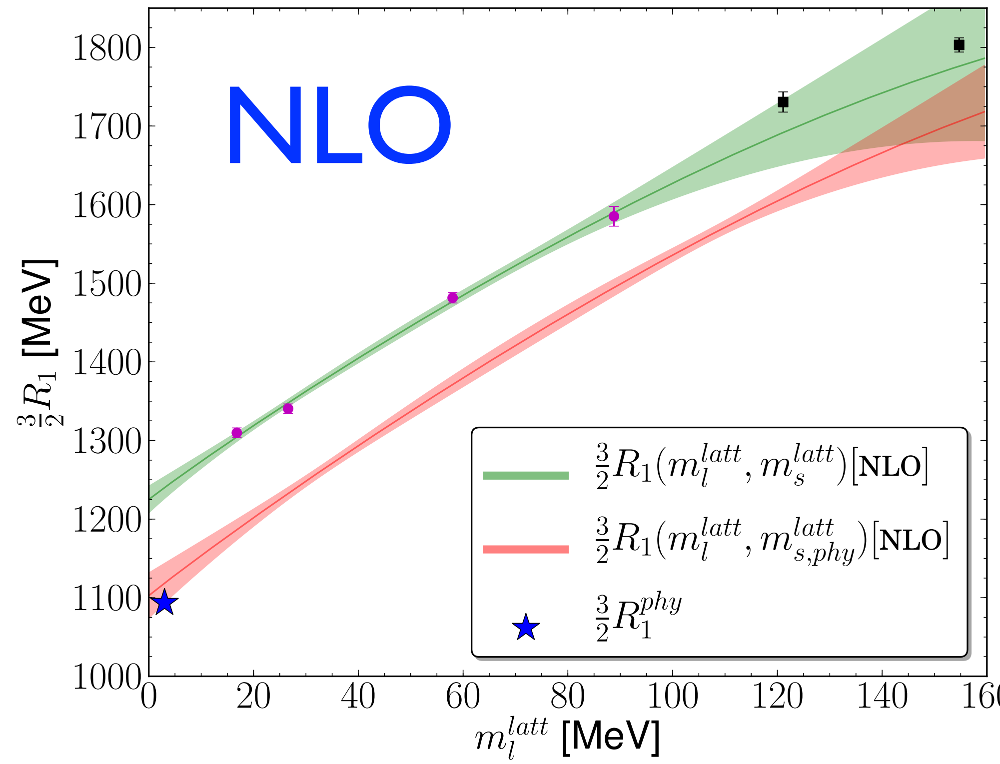
$$b_D = \frac{1}{4}b_{(2)}\,, \qquad b_F = \frac{1}{2}b_{(1)} + \frac{1}{6}b_{(2)}\,, \qquad b_T = -\frac{3}{2}b_{(1)} - \frac{5}{4}b_{(2)}$$
$$\sigma_B = \frac{1}{2}b_{(1)} + \frac{1}{12}b_{(2)}\,, \qquad \sigma_T = \frac{1}{2}b_{(1)} + \frac{5}{12}b_{(2)}\,.$$

$$D = \frac{1}{2}a_{(1)}\,, \qquad F = \frac{1}{3}a_{(1)} + \frac{1}{6}a_{(2)}\,,$$
$$\mathcal{C} = -a_{(1)}\,, \qquad \mathcal{H} = -\frac{3}{2}a_{(1)} - \frac{3}{2}a_{(2)}$$

$$\mathcal{C} = -2D,$$
$$\mathcal{H} = 3D - F.$$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

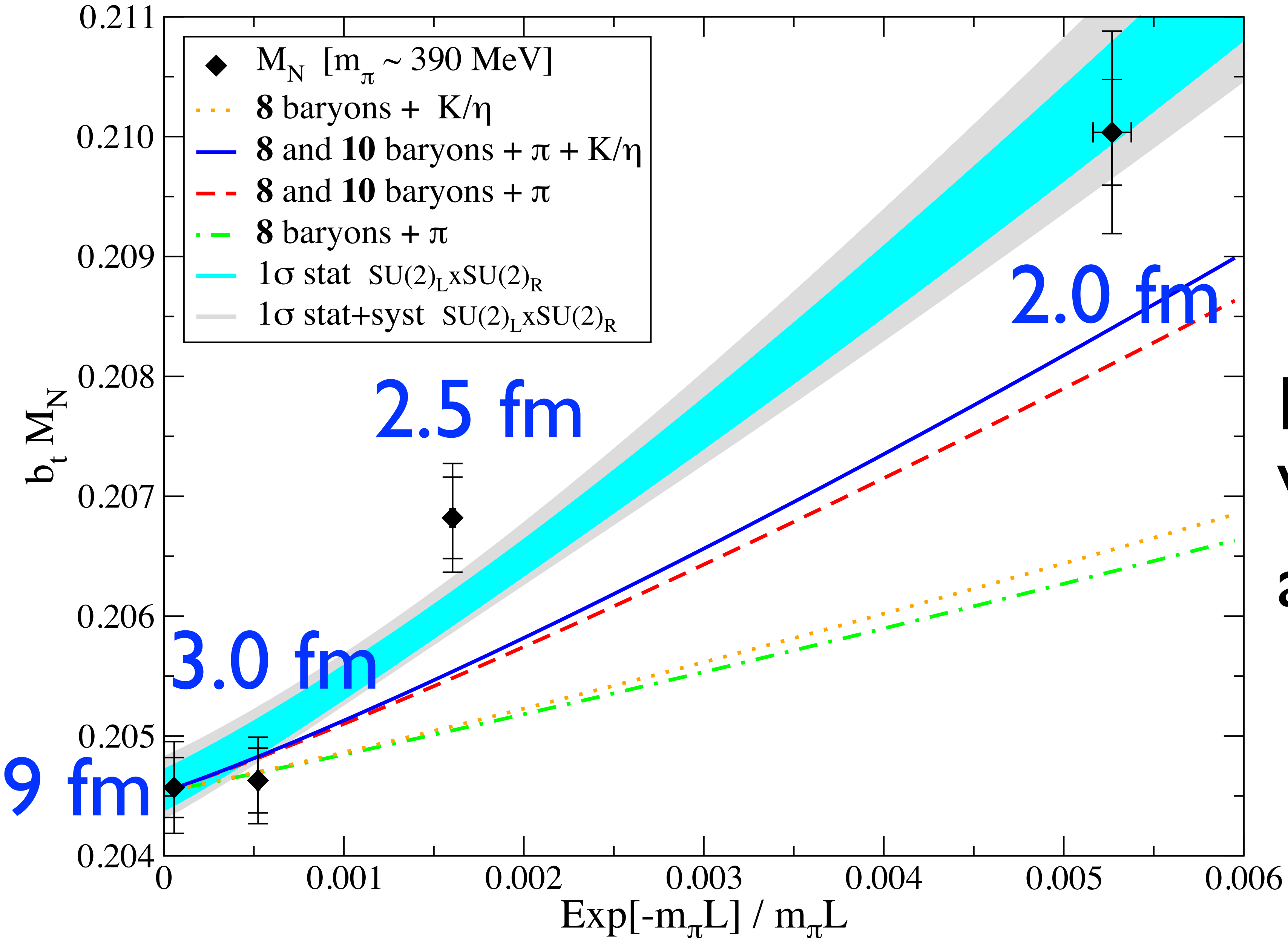




$$
\begin{aligned}
\frac{3}{2}\ R_1(m_l, m_s) = M_0 &- \left(\frac{3}{4}b_{(1)} + \frac{5}{24}b_{(2)}\right)(2m_l + m_s) \\
&- \frac{1}{12}\left(35a_{(1)}^2 - 5a_{(2)}^2\right)\left(\frac{3\mathcal{F}(m_\pi, 0, \mu) + 4\mathcal{F}(m_K, 0, \mu) + \mathcal{F}(m_\eta, 0, \mu)}{8(4\pi f)^2}\right) \\
&- \frac{1}{12}a_{(1)}^2\left[50\left(\frac{3\mathcal{F}(m_\pi, \Delta, \mu) + 4\mathcal{F}(m_K, \Delta, \mu) + \mathcal{F}(m_\eta, \Delta, \mu)}{8(4\pi f)^2}\right)\right. \\
&\qquad \left. - 4\left(\frac{3\mathcal{F}(m_\pi, -\Delta, \mu) + 4\mathcal{F}(m_K, -\Delta, \mu) + \mathcal{F}(m_\eta, -\Delta, \mu)}{8(4\pi f)^2}\right)\right]
\end{aligned}
$$

$a_{(1)} = 0.2(5)$ → $D = 0.10(25)$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658



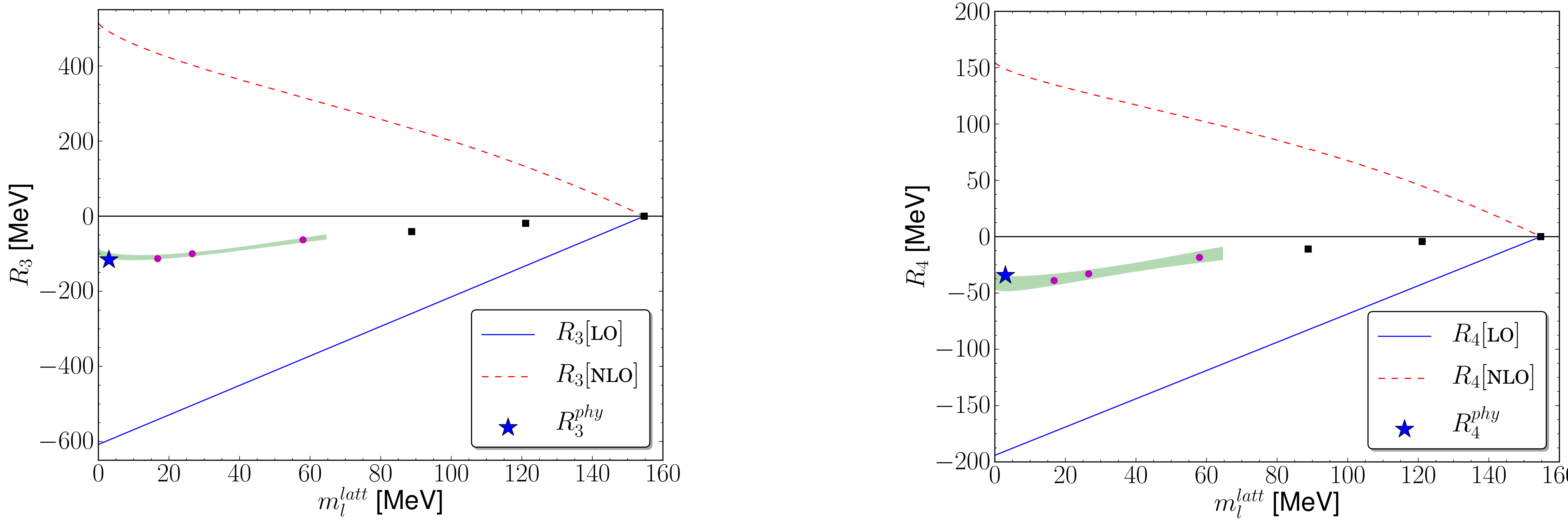
$m_\pi L =$ 7.7,5.8, 4.8 3.9

High statistics (IV):
Volume Dependence
arXiv:1104.4101

While SU(3) HBChiPT fails to converge with acceptable values of D, F, H, C, provides a quantitatively accurate description of finite volume corrections (with acceptable D, F, H, C)

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

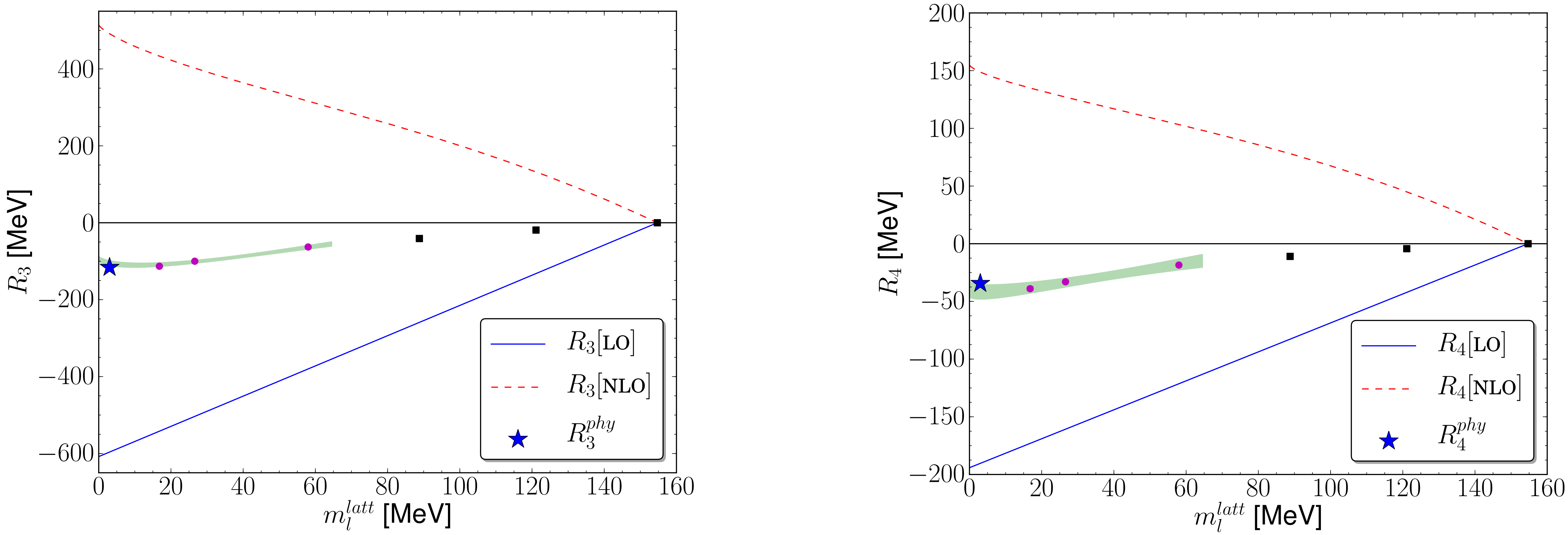
$$R_3 \propto m_s - m_l \qquad\qquad R_4 \propto (m_s - m_l)/N_c$$



$$R_3(m_l, m_s) = \frac{20}{39}\, b_1\,(m_s - m_l) - \frac{20a_1^2 - 5a_2^2}{117}\,\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{a_1^2}{117}\left[35\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2} - \frac{3\mathcal{F}_\pi^{-\Delta} - 2\mathcal{F}_K^{-\Delta} - \mathcal{F}_\eta^{-\Delta}}{(4\pi f)^2}\right],$$

$$R_4(m_l, m_s) = -\frac{5}{18}\, b_2\,(m_s - m_l) + \frac{a_1^2 + 4a_1a_2 + a_2^2}{36}\,\frac{3\mathcal{F}_\pi^0 - 2\mathcal{F}_K^0 - \mathcal{F}_\eta^0}{(4\pi f)^2} - \frac{2a_1^2}{9}\,\frac{3\mathcal{F}_\pi^\Delta - 2\mathcal{F}_K^\Delta - \mathcal{F}_\eta^\Delta}{(4\pi f)^2}$$

Evidence for non-analytic light quark mass dependence arXiv:1112.2658



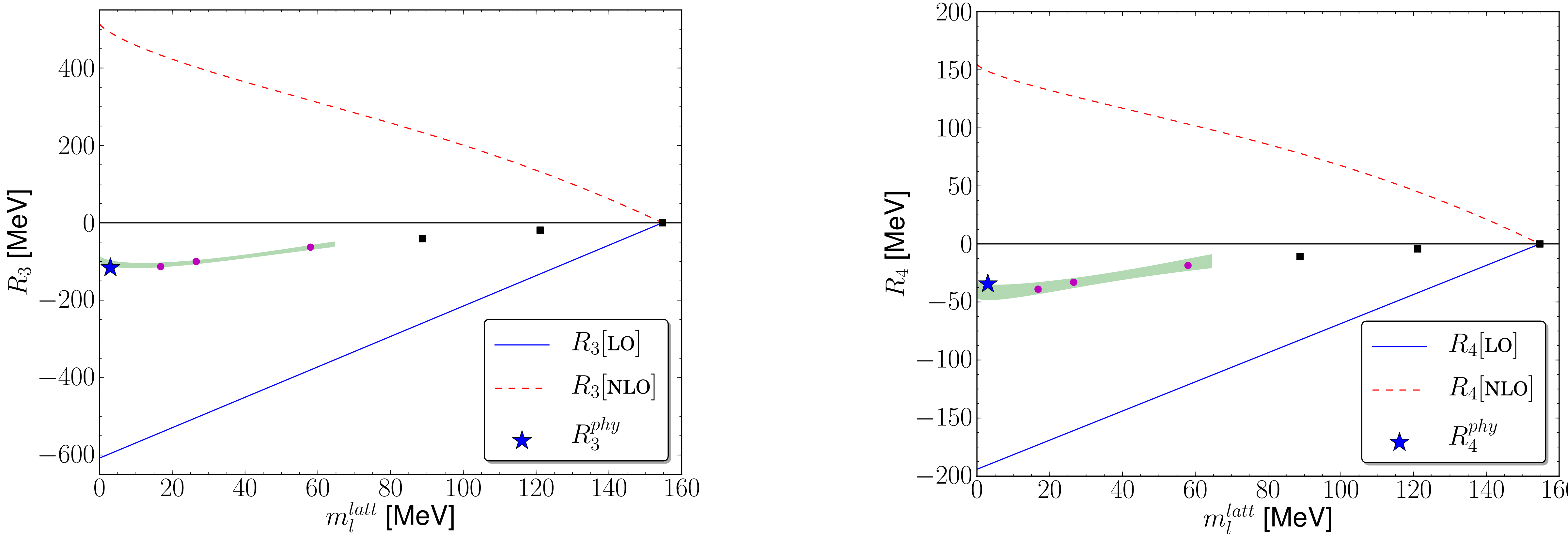
Fit yields

$$b_1[\mathrm{NLO}] = -6.6(5)\,, \qquad b_2[\mathrm{NLO}] = 4.3(4)\,, \qquad a_1[\mathrm{NLO}] = 1.4(1)\,.$$

$$\Rightarrow \quad D = 0.70(5)\,, \qquad F = 0.47(3)\,, \qquad C = -1.4(1)\,, \qquad H = -2.1(2)$$

First time axial couplings left as free parameters and: values consistent with phenomenological determinations

Evidence for non-analytic light quark mass dependence arXiv:1112.2658



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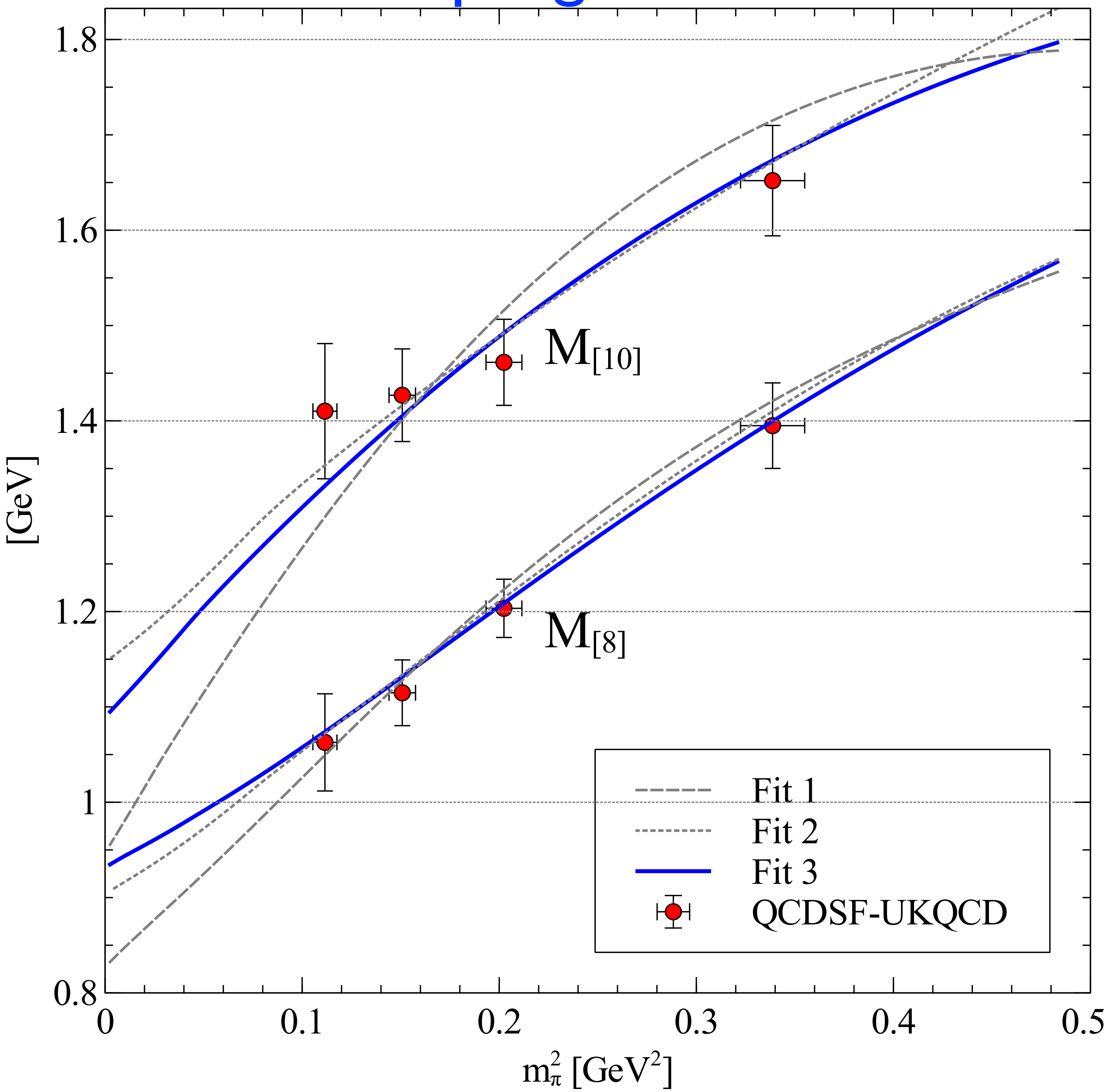
$$\Rightarrow \quad D = 0.70(5)\,, \qquad F = 0.47(3)\,, \qquad C = -1.4(1)\,, \qquad H = -2.1(2)$$

but still observe large cancellations between LO and NLO

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Work of Mathias Lutz and Alexandre Semke who fit the masses (not mass splittings) of 4 different lattice QCD groups, and obtained similar axial couplings



Fit i:

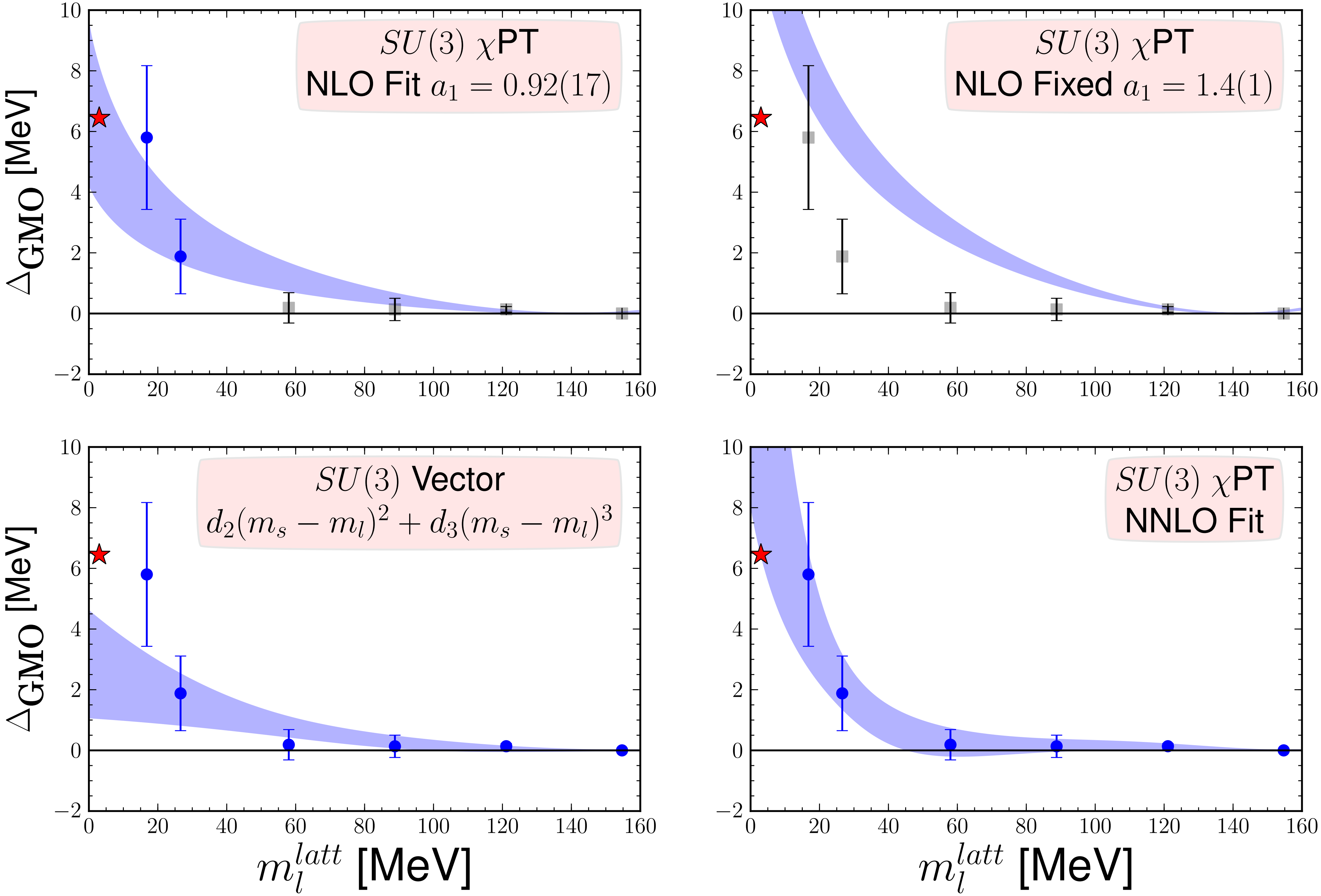
each fit is to set of BMW, LHPC, PACS-CS

none of the fits include QCDSF-UKQCD, who computed masses in SU(3) limit as well as SU(3)-broken (with similar agreement)

I do not understand - but this agreement is remarkable

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

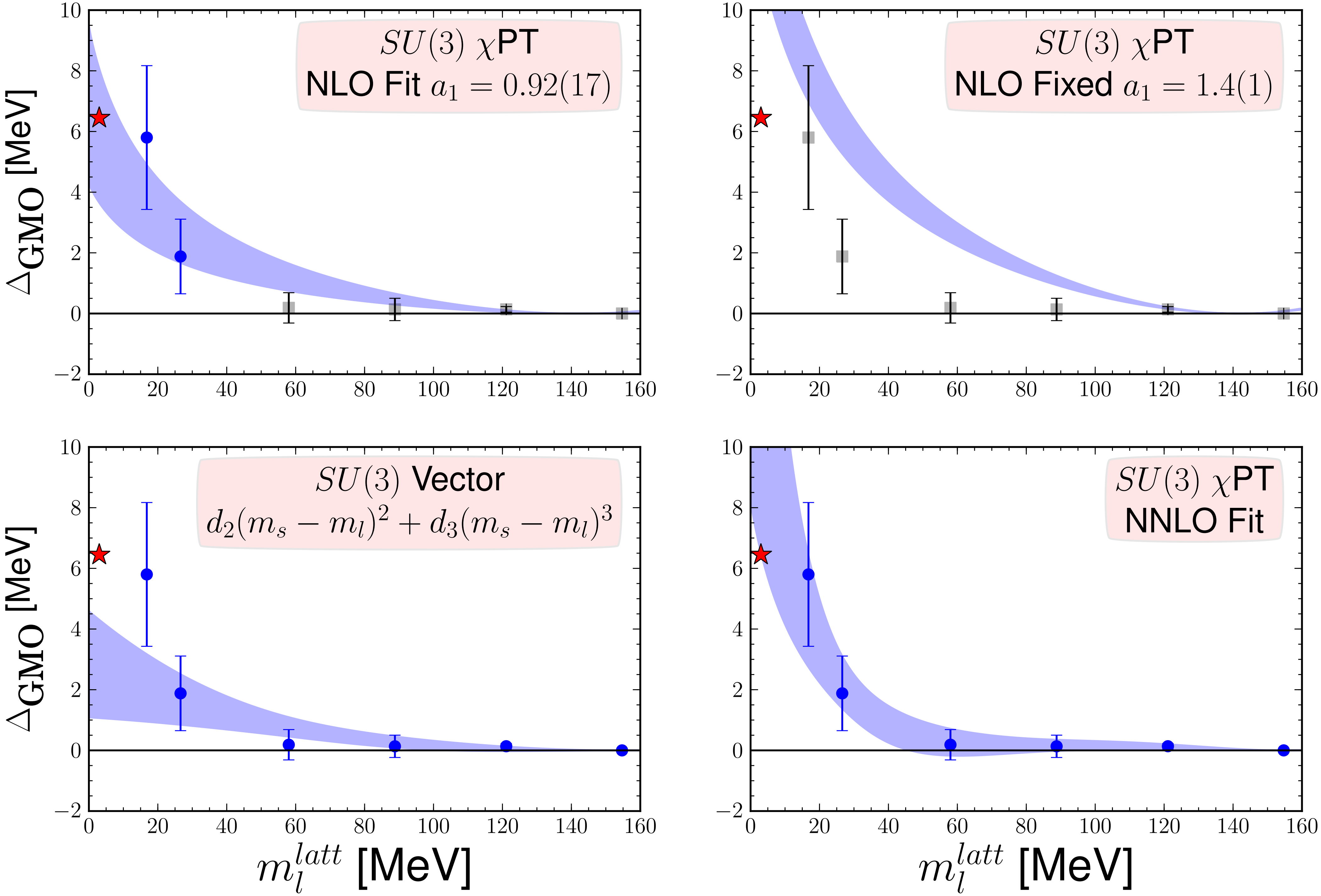
Gell-Mann--Okubo Relation



Only NNLO SU(3) naturally supports strong light quark mass dependence

Evidence for non-analytic light quark mass dependence arXiv:1112.2658

Gell-Mann--Okubo Relation



Combined with R3 and R4 - provides first compelling evidence of non-analytic light quark mass dependence in the baryon spectrum

Light quark mass dependence of M_B

- the more I study baryons, the more confused I get
  - there now seems to be un-ignorable evidence for entirely unexpected light quark mass dependence in the nucleon (baryon) spectrum, basically down to the physical pion mass

$$M_N = \alpha_0 + \alpha_1 m_\pi$$

  - combining large N_c with SU(2) and SU(3) flavor symmetry is showing promise - at least qualitatively
  - what is clearly (still) needed is high statistics study of baryons with (with the aim of understanding chiral perturbation theory)

$$120 \leq m_\pi \leq 400 \text{ MeV}$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

- electric polarizabilites and magnetic moments of the nucleon from lattice QCD

electromagnetic collaboration:
Will Detmold, Brian Tiburzi, AWL

Hadron Electromagnetic Polarizabilities from Lattice QCD

- Compass at CERN will measure pion and kaon polarizabilites through Primakoff process
- Compton MAX-lab (Lund) will extract neutron $\mathcal{E}M$ polarizabilities from Compton scattering on deuterium
- HIγS TUNL will make high precision measurements of proton and neutron electromagnetic and spin polarizabilites

Hadron Electromagnetic Polarizabilities from Lattice QCD

comparison of experiment and phenomenological prediction

pion

two-loop ChPT prediction

U.Burgi; NPB 479(1996), PLB 377(1996)
J. Gasser et.al.; NPB 745 (2006)

$$\alpha_E^\pi = 2.4 \pm 0.5$$

$$\beta_E^\pi = -2.1 \pm 0.5$$

experimental determination

Y.M.Antipov et.al.; PLB 121(1983), Z.Phys. C 26 (1985)

$$\alpha_E^\pi = -\beta_M^\pi = 6.8 \pm 1.4 \pm 1.2$$

assumed $(\alpha_E^\pi = -\beta_M^\pi)$

nucleon

Polarizability	Proton	Neutron
α $[10^{-4}\,\mathrm{fm}^3]$	11.9 ± 1.4	12.5 ± 1.7
β $[10^{-4}\,\mathrm{fm}^3]$	1.2 ± 0.9	2.7 ± 1.8
γ_1 $[10^{-4}\,\mathrm{fm}^4]$	*1.1±0.25*	*3.7±0.4*
γ_2 $[10^{-4}\,\mathrm{fm}^4]$	*-1.5±0.36*	*-0.1±0.5*
γ_3 $[10^{-4}\,\mathrm{fm}^4]$	*0.2±0.24*	*0.4±0.5*
γ_4 $[10^{-4}\,\mathrm{fm}^4]$	*3.3±0.11*	*2.3±0.35*
γ_π $[10^{-4}\,\mathrm{fm}^4]$	-38.7 ± 1.8	58.6 ± 4.0

measured

expected

(theoretical disagreements)

Hadron Electromagnetic Polarizabilities from Lattice QCD

Prediction from Chiral Perturbation Theory (χPT):
Non-analytic dependence on the light quark masses

$$m_\pi^2 = 2Bm_q \left[1 + \frac{2Bm_q}{(4\pi f)^2} \ln\left(\frac{2Bm_q}{\mu^2}\right) + 4\frac{2Bm_q}{f^2} l_3^r(\mu)\right] + \ldots$$

Polarizabilites:

$$\alpha_E^{\pi^\pm} = \frac{8\alpha_{f.s.}}{f_\pi^2}\frac{L_9 + L_{10}}{m_\pi} \qquad \text{LO } \chi\text{PT}$$

$$\alpha_E^N = \frac{5\alpha_{f.s.}\, g_A^2}{192\pi f_\pi^2}\frac{1}{m_\pi} + \Delta\text{-contributions} \qquad \text{NLO } \chi\text{PT (leading loop)}$$

$$\beta_B^N = \frac{\alpha_{f.s.}\, g_A^2}{384\pi f_\pi^2}\frac{1}{m_\pi} + \Delta\text{-contributions} \qquad \text{NLO } \chi\text{PT (leading loop)}$$

$$\gamma_{E_1E_1}^N = -\frac{5\alpha_{f.s.}\, g_A^2}{192\pi^2 f_\pi^2}\frac{1}{m_\pi^2} + \Delta\text{-contributions} \qquad \text{NLO } \chi\text{PT (leading loop)}$$

Evidence for this non-analytic light quark mass dependence is smoking gun for being in the chiral regime.

Hadron Electromagnetic Polarizabilities from Lattice QCD

For sufficiently low energy ($\omega << m_\pi$), a spin $1/2$ baryon has the effective Hamiltonian

$$H_{eff} = \frac{(\vec{p} - Q\vec{A})^2}{2M} + Q\phi - \frac{1}{2}4\pi\Big(\alpha\vec{\mathcal{E}}^2 + \beta\vec{\mathcal{B}}^2$$

$$\gamma_{E_1E_1}\vec{\sigma}\cdot\vec{\mathcal{E}}\times\dot{\vec{\mathcal{E}}} + \gamma_{M_1M_1}\vec{\sigma}\cdot\vec{\mathcal{B}}\times\dot{\vec{\mathcal{B}}} + \gamma_{M_1E_2}\sigma_i\mathcal{E}_{ij}\mathcal{B}_j + \gamma_{E_1M_2}\sigma_i\mathcal{B}_{ij}\mathcal{E}_j\Big)$$

where

$$\mathcal{E}_{ij} = \frac{1}{2}(\nabla_i\mathcal{E}_j + \nabla_j\mathcal{E}_i) \qquad \mathcal{B}_{ij} = \frac{1}{2}(\nabla_i\mathcal{B}_j + \nabla_j\mathcal{B}_i)$$

$$\gamma_{E_1E_1} = -\gamma_1 - \gamma_3 \qquad \gamma_{M_1M_1} = \gamma_4$$

$$\gamma_{E_1M_2} = \gamma_3 \qquad \gamma_{M_1E_2} = \gamma_2 + \gamma_4$$

For specific choices of A_μ, one can isolate the various (spin) polarizabilites W. Detmold, B.C. Tiburzi, AWL PRD 73 (2006).

Hadron Electromagnetic Polarizabilities from Lattice QCD

For our calculation, we want Euclidean action which respects periodic boundary conditions (hyper-torus)

$$e^{-i\int d^4x_M \frac{1}{4}F_{\mu\nu}F^{\mu\nu}} = e^{i\int d^4x_M \frac{1}{2}\left(\mathcal{E}_M^2 - \mathcal{B}_M^2\right)}$$

$$\longrightarrow e^{-\int d^4x_E \frac{1}{4}F_{\mu\nu}F_{\mu\nu}} = e^{-\int d^4x_E \frac{1}{2}\left(\mathcal{E}_E^2 + \mathcal{B}_E^2\right)}$$

In this way, the $U(1)$ gauge links are given by a phase

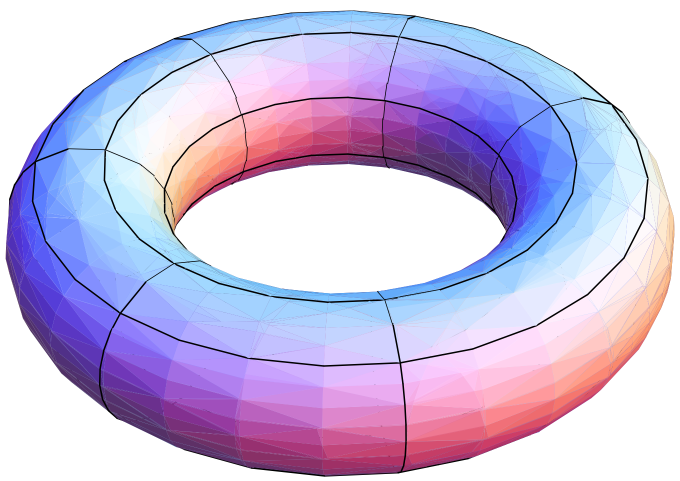
$$U_\mu(x) = e^{iaqA_\mu(x)}$$

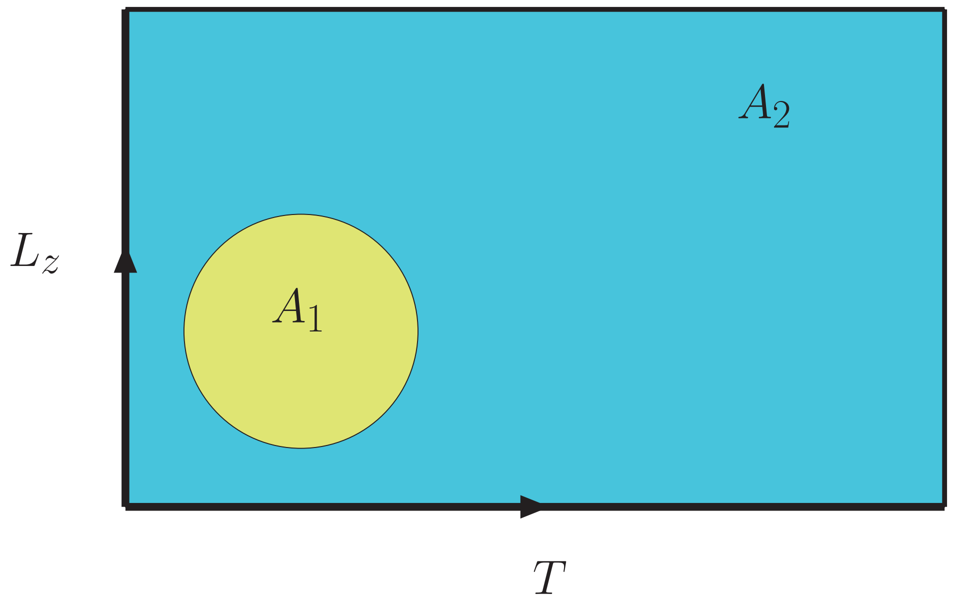
Consequences:

$$M(\mathcal{E}_M) = M_0 - 2\pi\alpha\mathcal{E}_M^2 + \ldots \longrightarrow M(\mathcal{E}_E) = M_0 + 2\pi\alpha\mathcal{E}_E^2 + \ldots$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

On a compact torus, not all values of the field strength are allowed:
G. 't Hooft NPB 153 (1979)





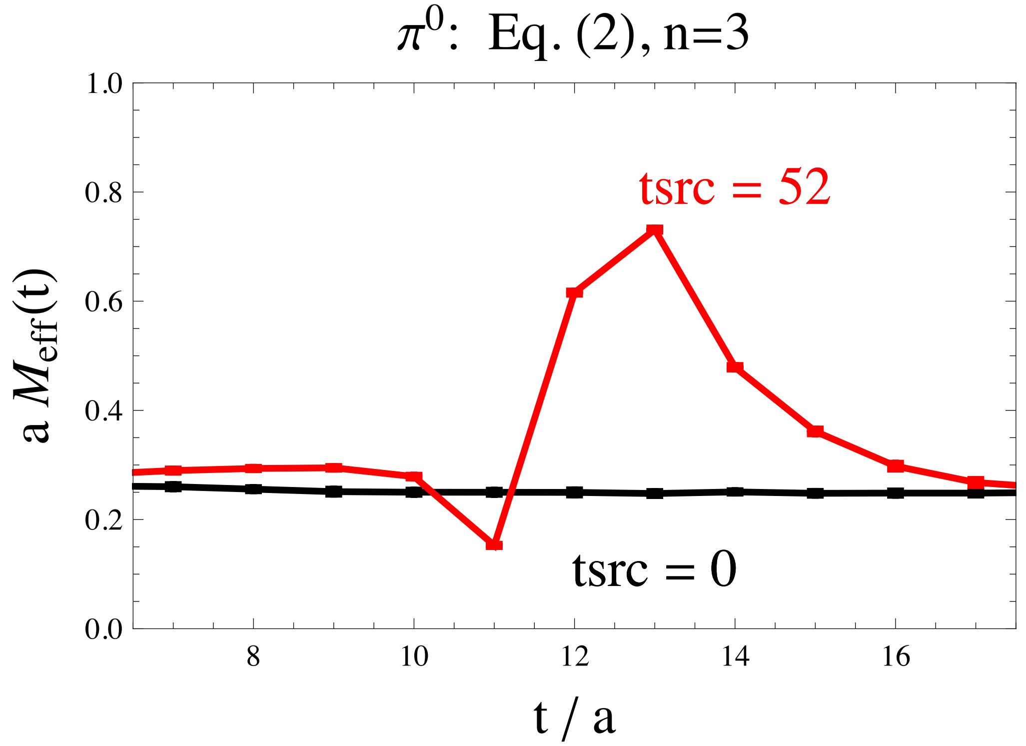
$$0 = \Phi = \Phi_1 + \Phi_2 \qquad\qquad A_1 = TL_z - A_2$$

$$\longrightarrow \exp\{iq\mathcal{E}A_1\} = \exp\{iq\mathcal{E}\,(TL_z - A_2)\} \qquad \longrightarrow 1 = \exp\{iq\mathcal{E}\,TL_z\}$$

$$q\mathcal{E} = \frac{2\pi}{TL_z}n \qquad\qquad \text{for } n = 1, 2, \ldots$$

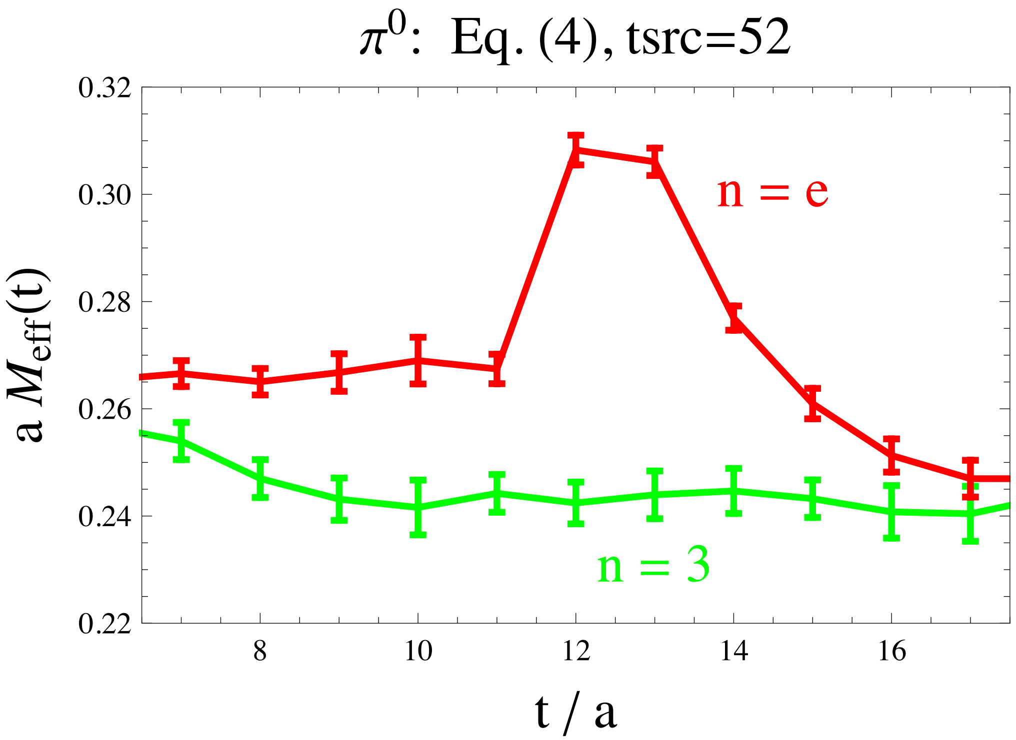
Hadron Electromagnetic Polarizabilities from Lattice QCD

Non-Quantized



- $n = 3,\ t_{src} = 0$
- $n = 3,\ t_{src} = 52$

Quantized



- $n = 3,\ t_{src} = 52$
- $n = e,\ t_{src} = 52$

$$aM_{eff}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

In a background field, what do we expect the correlation functions to look like?

$$J = 0, Q = 0; \qquad C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) e^{-E_n(\mathcal{E})t}$$

$$J = 1/2, Q = 0; \qquad C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}, \mu_n) e^{-E_n(\mathcal{E}, \mu_n)t}$$

$$J = 0, Q = 1; \qquad C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(E_n, \mathcal{E}, t)$$

$$J = 1/2, Q = 1; \qquad C(t, \mathcal{E}) = \sum_n Z_n(\mathcal{E}, \mu_n) G(E_n, \mathcal{E}, \mu_n, t)$$

Consider spin-less, relativistic particle of unit charge coupled to an electric field

$$\mathcal{L} = D_\mu \pi^\dagger D_\mu \pi + m_{eff}^2 \pi^\dagger \pi, \qquad D_\mu = \partial_\mu + iA_\mu, \qquad A_\mu = (0, 0, -\mathcal{E}t, 0)$$

integrating by parts and changing variables

$$D^{-1} = p_\tau^2 + \mathcal{E}^2\tau^2 + E_{k_\perp}^2 \equiv 2\left(\mathcal{H} + \frac{1}{2}E_{k_\perp}^2\right),$$

$$\tau = t - \frac{k_z}{\mathcal{E}}, \qquad E_{k_\perp}^2 = E_k^2 - k_z^2$$

solution B.C. Tiburzi Nucl.Phys. A 814 (2008)

$$D(\tau', \tau) = \frac{1}{2}\int_0^\infty ds \langle \tau', s|\tau, 0\rangle e^{-sE_{k_\perp}^2/2}$$

$$\langle \tau', s|\tau, 0\rangle = \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E}s}} \exp\left\{-\frac{\mathcal{E}}{2\sinh \mathcal{E}s}\left[(\tau'^2 + \tau^2)\cosh \mathcal{E}s - 2\tau'\tau\right]\right\}$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

Take $\tau = 0$, $\vec{k} = 0$:

$$C(\tau, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau, \mathcal{E})$$

$$G(\tau, \mathcal{E}) = \frac{1}{2} \int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E} s}} \exp\left\{ -\frac{1}{2} \left(\mathcal{E}\tau^2 \coth \mathcal{E} s + s\, m_{eff}^2 \right) \right\}$$

in the weak field limit

$$C(\tau, \mathcal{E}) = Z(\mathcal{E}) \exp\left\{ -M(\mathcal{E})\tau - \frac{\mathcal{E}^2}{M(\mathcal{E})^4} \left(\frac{1}{6}(M(\mathcal{E})\tau)^3 + \frac{1}{4}(M(\mathcal{E})\tau)^2 + \frac{1}{4}(M(\mathcal{E})\tau) \right) \right\}$$

$$M(\mathcal{E}) = M_0 + 2\pi\alpha\mathcal{E}^2 + \mathcal{O}(\mathcal{E}^4)$$

computing hadron deformations in background $\mathcal{E}M$ fields amounts to spectroscopy

Hadron Electromagnetic Polarizabilities from Lattice QCD

neutron in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

$$S = \int d^4x\, \overline{\psi}(x) \left[\partial\!\!\!/ + E(\mathcal{E}) - \frac{\mu(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x)\,,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$

$$\sigma_{\mu\nu} F_{\mu\nu} = 2\vec{K} \cdot \mathcal{E}\,, \qquad \text{for background } \mathcal{E}\text{-field and } \vec{K} = i\vec{\gamma}\gamma_4$$

$$\mu(\mathcal{E}) = \mu + \mu'' \mathcal{E}^2 + \ldots \qquad \text{anomalous magnetic coupling}$$

motion of the quarks in the $\mathcal{E}$-field gives rise to the magnetic coupling

with $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$, construct

$$G_\pm(t, \mathcal{E}) \equiv \mathrm{tr}[\mathcal{P}_\pm G(t, \mathcal{E})] = Z(\mathcal{E}) \left(1 \pm \frac{\mathcal{E}\mu(\mathcal{E})}{2M^2} \right) \exp\left[-t\, E_{eff}(\mathcal{E})\right]\,,$$

$$\mathcal{P}_\pm = \frac{1}{2}\left[1 \pm K_3\right] \qquad E_{eff} = E(\mathcal{E}) - \frac{\mu(\mathcal{E})^2 \mathcal{E}^2}{8M^3}$$

$$= M + \frac{1}{2}\mathcal{E}^2 \left(4\pi\alpha_E - \frac{\mu^2}{4M^3} \right) + \ldots$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

proton in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

$$S = \int d^4x \, \overline{\psi}(x) \left[\not{D} + E(\mathcal{E}) - \frac{\tilde{\mu}(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x) \,,$$

$$D_\mu = \partial_\mu + iQA_\mu \qquad\qquad \mu = Q + \tilde{\mu}(0)$$

boost projected correlation functions

$$G_\pm(t, \mathcal{E}) = Z(\mathcal{E}) \left(1 \pm \frac{\tilde{\mu}\mathcal{E}}{2M^2} \right) D\left(t, E_{eff}(\mathcal{E})^2 \mp Q\mathcal{E}, \mathcal{E} \right)$$

$$D(t, E^2, \mathcal{E}) = \int_0^\infty ds \sqrt{\frac{Q\mathcal{E}}{2\pi \sinh(Q\mathcal{E}s)}} \exp\left[-\frac{Q\mathcal{E}t^2}{2} \coth(Q\mathcal{E}s) - \frac{E^2 s}{2} \right]$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

Results I am going to present are from

- mesons: W. Detmold, B.C. Tiburzi, AWL PRD 79 (2009)
- proton and neutron: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

To date, we have set $q_{sea} = 0$ (Quenched $\mathcal{E}M$)

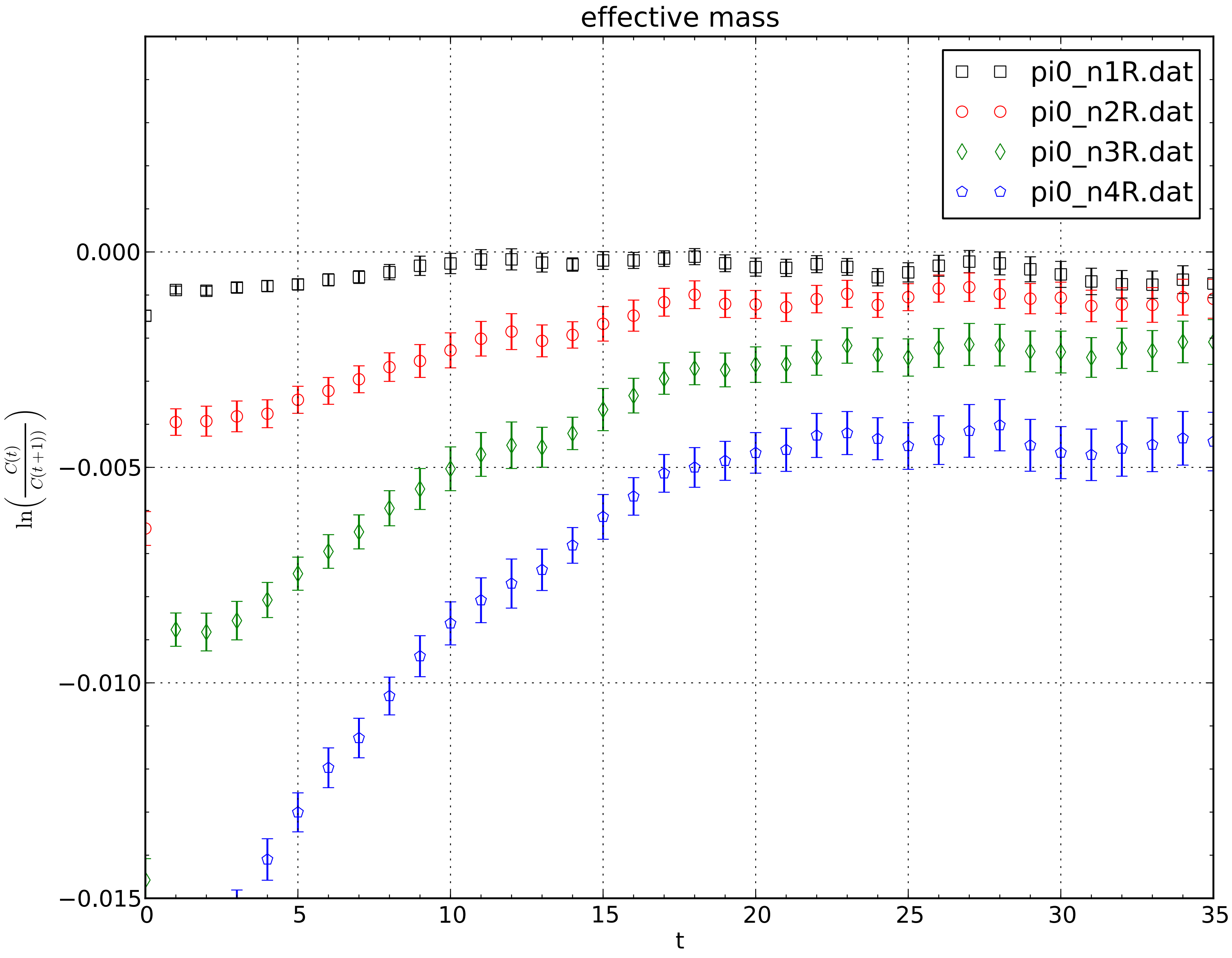
$m_\pi \sim 390$ MeV $\qquad L = 2.5$ fm

TABLE I: Propagators generated to date with our 2008-09 and 2009-10 USQCD allocations.

V	a_s [fm]	a_s/a_t	$a_t m_u^0$	$a_t m_s^0$	m_π [MeV]	m_K [MeV]	Field Strength	$N_{src} \times N_{cfg}$	total # of props(u,d,s)
$20^3 \times 128$	0.123	3.5	-0.0840	-0.0743	390	546	0	15×200	6,000
							± 1	15×200	9,000
							± 2	10×200	6,000
							± 3	10×200	6,000
							± 4	10×200	6,000
$24^3 \times 128$	0.123	3.5	-0.0840	-0.0743	390	546	0	10×195	3,900
							± 1	10×195	5,850
							± 2	10×195	5,850
							± 3	10×195	5,850
							± 4	10×195	5,850
$32^3 \times 256$	0.123	3.5	-0.0860	-0.0743	225	467	0	7×106	2,226

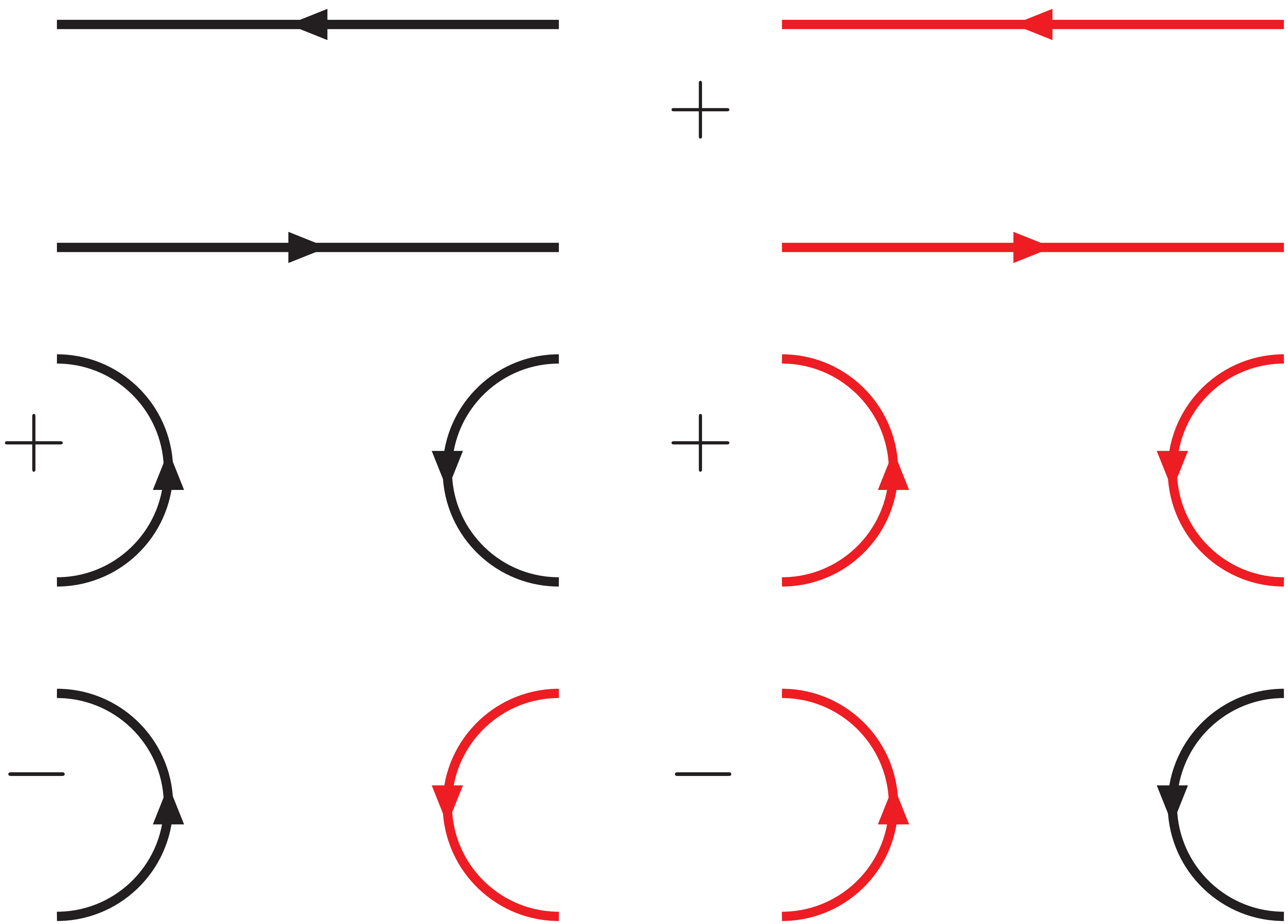
Hadron Electromagnetic Polarizabilities from Lattice QCD

π^0 Mass Shift:



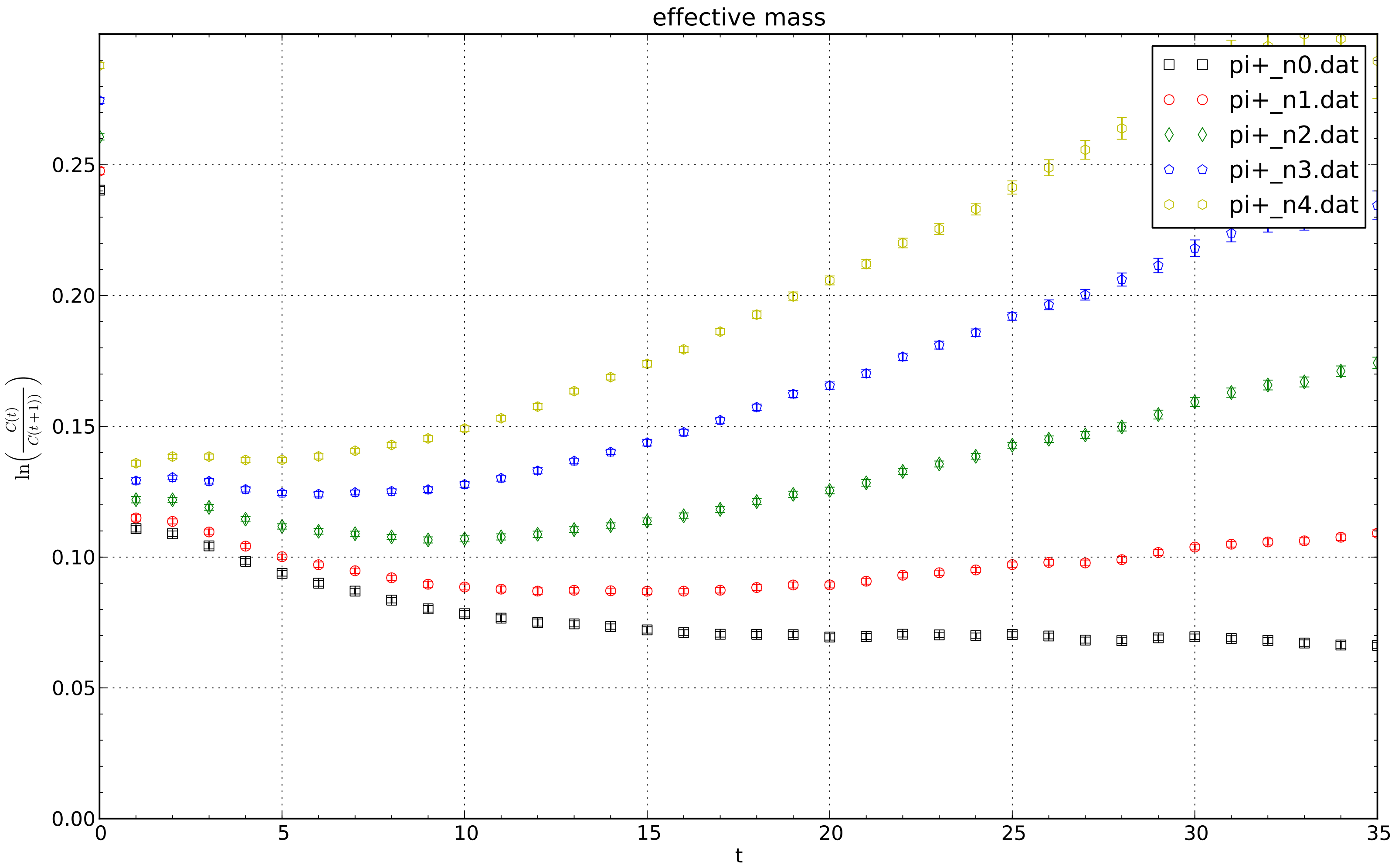
Hadron Electromagnetic Polarizabilities from Lattice QCD

π^0



Hadron Electromagnetic Polarizabilities from Lattice QCD

π^+ Effective Mass



$$C(\tau,\mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau,\mathcal{E})$$

$$G(\tau,\mathcal{E}) = \frac{1}{2}\int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E}s}} \exp\left\{-\frac{1}{2}\left(\mathcal{E}\tau^2 \coth \mathcal{E}s + s\, m_{eff}^2\right)\right\}$$

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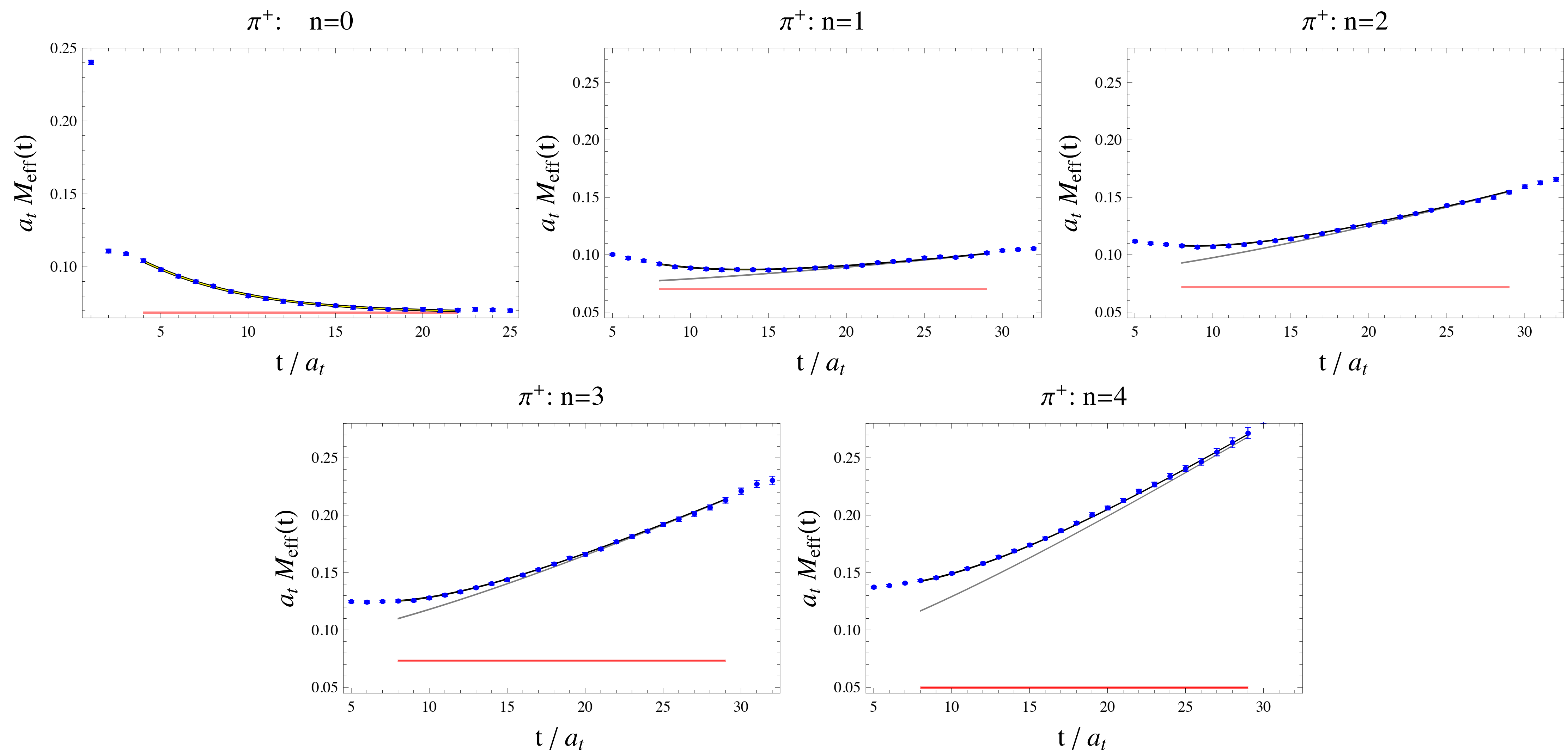
π^+



	n				
	0	1	2	3	4
$m(\mathcal{E})$	0.0691(4)	0.0702(6)	0.0718(8)	0.0733(16)	0.0497(129)

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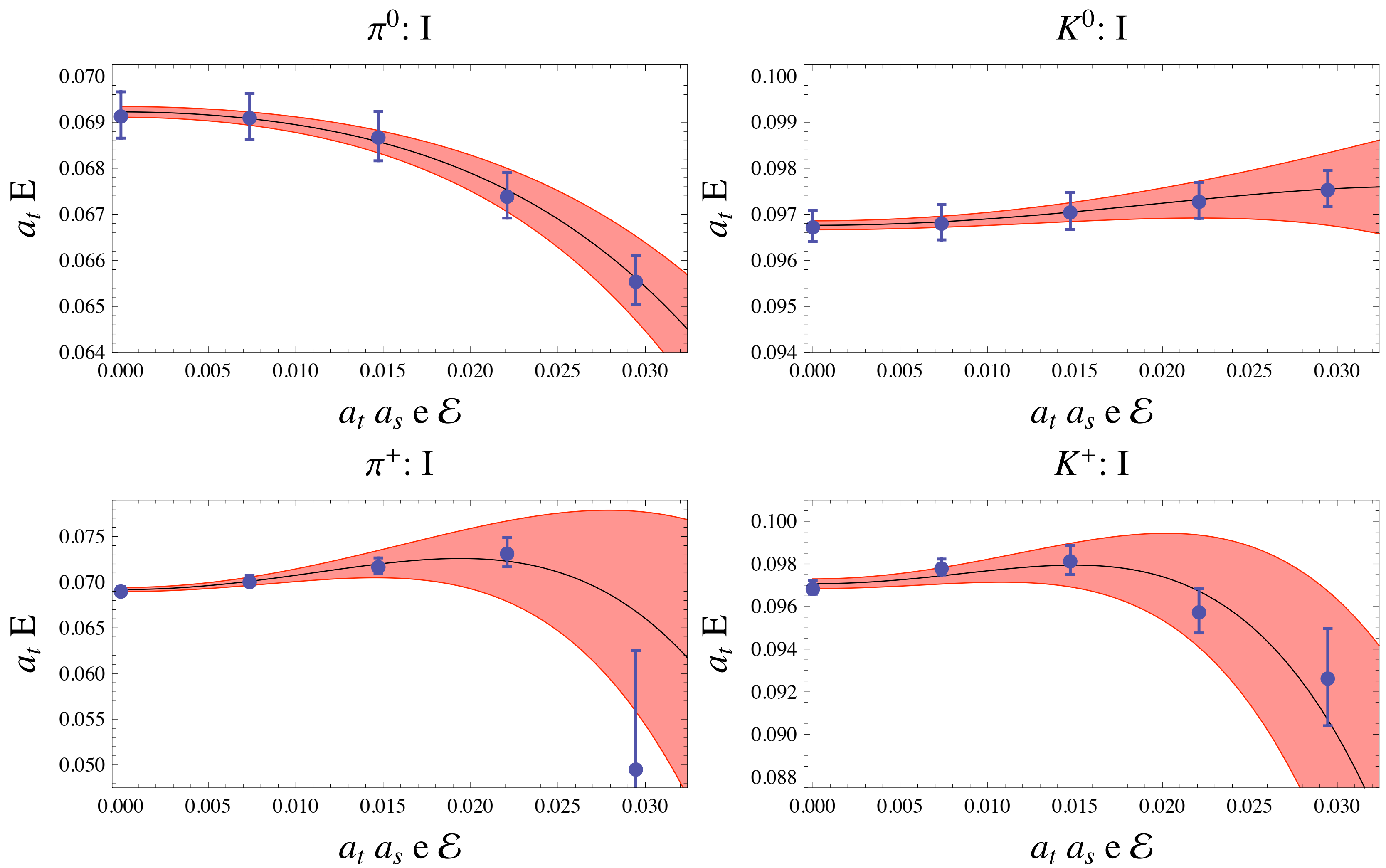

$$m(\mathcal{E}) = m_0 + \alpha_E^{latt}\mathcal{E}^2 + \bar{\alpha}_{EEE}^{latt}\mathcal{E}^4$$

	π^0	π^+	K^0	K^+
α_E^{latt}	-2.6(5)(9)	18(4)(6)	1.5(4)(7)	8(3)(1)
$\bar{\alpha}_E^{latt}$	1.8(5)	24(10)	0.6(5)	17(5)

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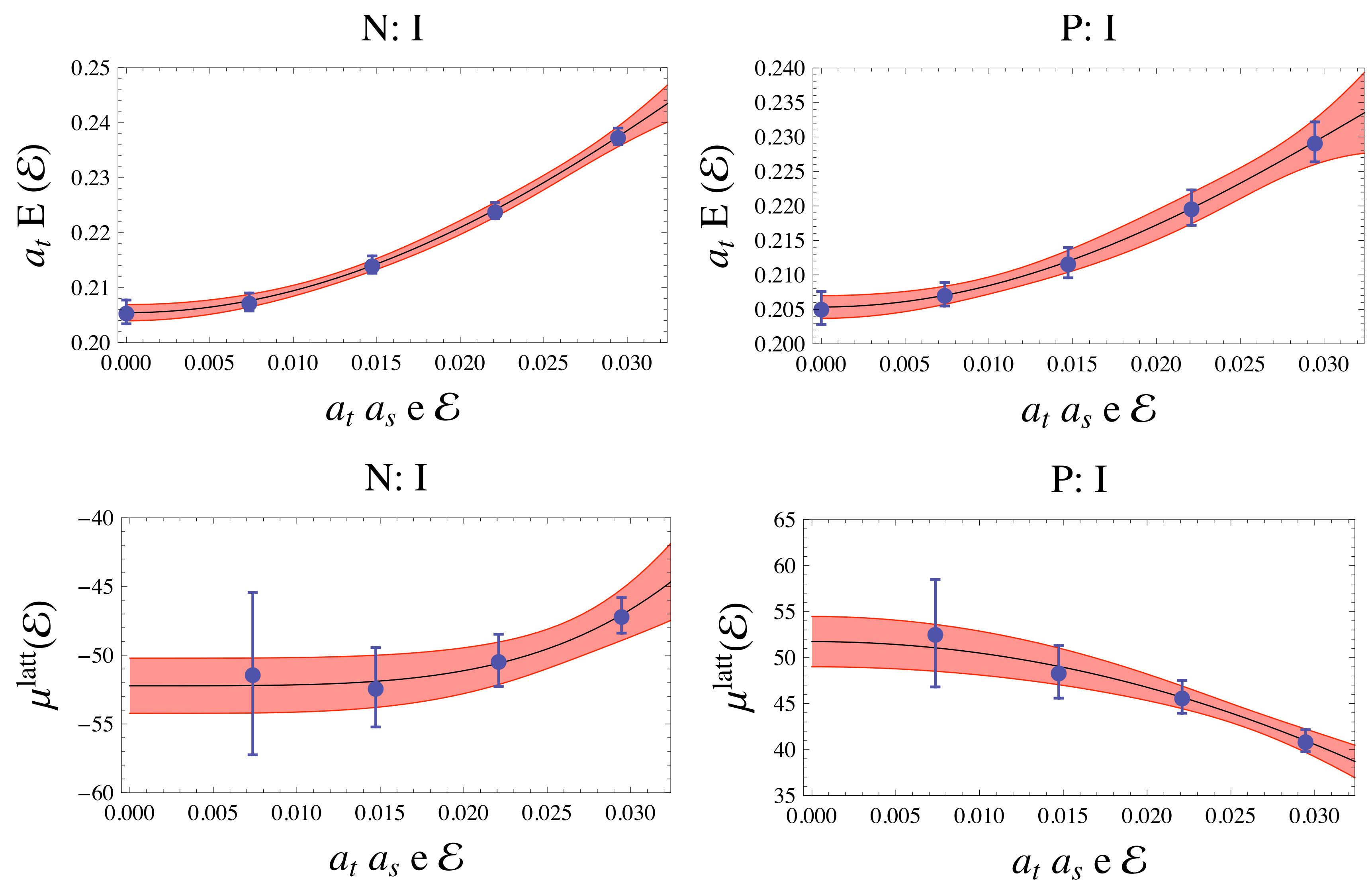

$$G_{\pm}(t,\mathcal{E}) = Z(\mathcal{E})\left(1 \pm \frac{\tilde{\mu}\mathcal{E}}{2M^2}\right) D\left(t, E_{eff}(\mathcal{E})^2 \mp Q\mathcal{E}, \mathcal{E}\right)$$

$$D(t,E^2,\mathcal{E}) = \int_0^\infty ds \sqrt{\frac{Q\mathcal{E}}{2\pi \sinh(Q\mathcal{E}s)}} \exp\left[-\frac{Q\mathcal{E}t^2}{2}\coth(Q\mathcal{E}s) - \frac{E^2 s}{2}\right]$$

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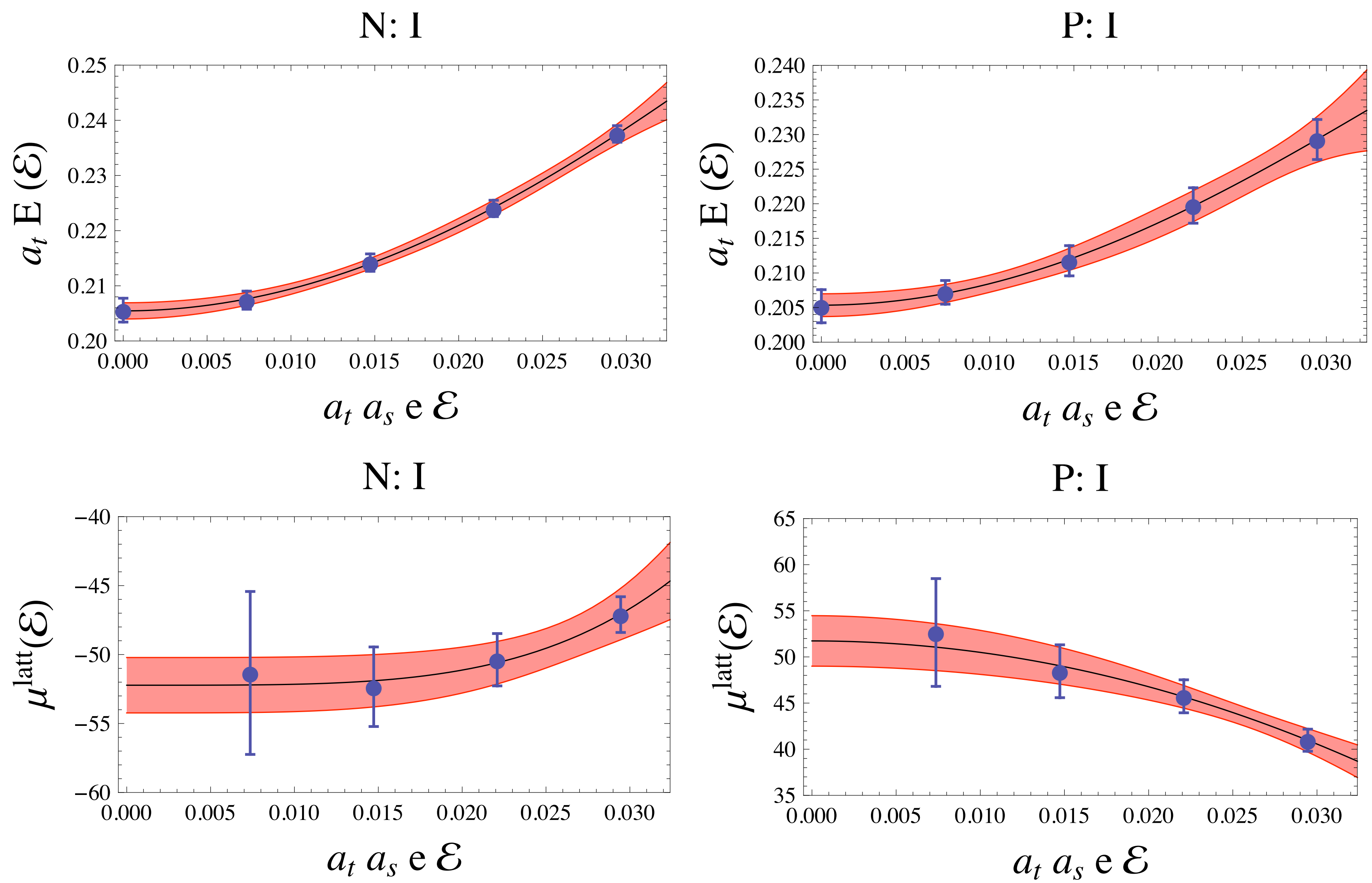

N	α_E^{latt}	$\tilde{\mu}^{latt}$	μ^{latt}
neutron	40(9)(2)	-52(2)(1)	-52(2)(1)
proton	32(13)(1)	52(3)(1)	83.9(3)(1)

$$\alpha_E^V(m_\pi = 390 \text{ MeV}) = -0.9(2.5)(.3)(.4) \times 10^{-4} \text{ fm}^3 \qquad \mu^V(m_\pi = 390 \text{ MeV}) = 4.3(.2)(.1)(.1)[\mu_N]$$

Hadron Electromagnetic Polarizabilities from Lattice QCD

- over the last few years, we have established a program to compute polarizabilities of hadrons as well as magnetic moments, utilizing background electromagnetic fields

- we now have to address several systematics (which need more computing time)
  - sea quark electric charges need to be "turned on"
  - light quark mass extrapolation - do we see $1/m_\pi$ behavior?
  - nucleon spin polarizabilities (need field gradients - more difficult quantization condition if any)
  - explicit magnetic background fields

Light-quark mass dependence of QCD:

- the era of physical quark mass lattice QCD calculations is just around the corner - exciting time
- care must be taken to understand the light quark mass dependence of observables - unique predictions from effective field theory - are these predictions verified in the numerical simulations?
- effective field theory provides us with a deeper understanding of the underlying physics

(I realize here I am preaching to the choir)

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