Anatomy of Hadronic Parity Violation on the Lattice

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NSF

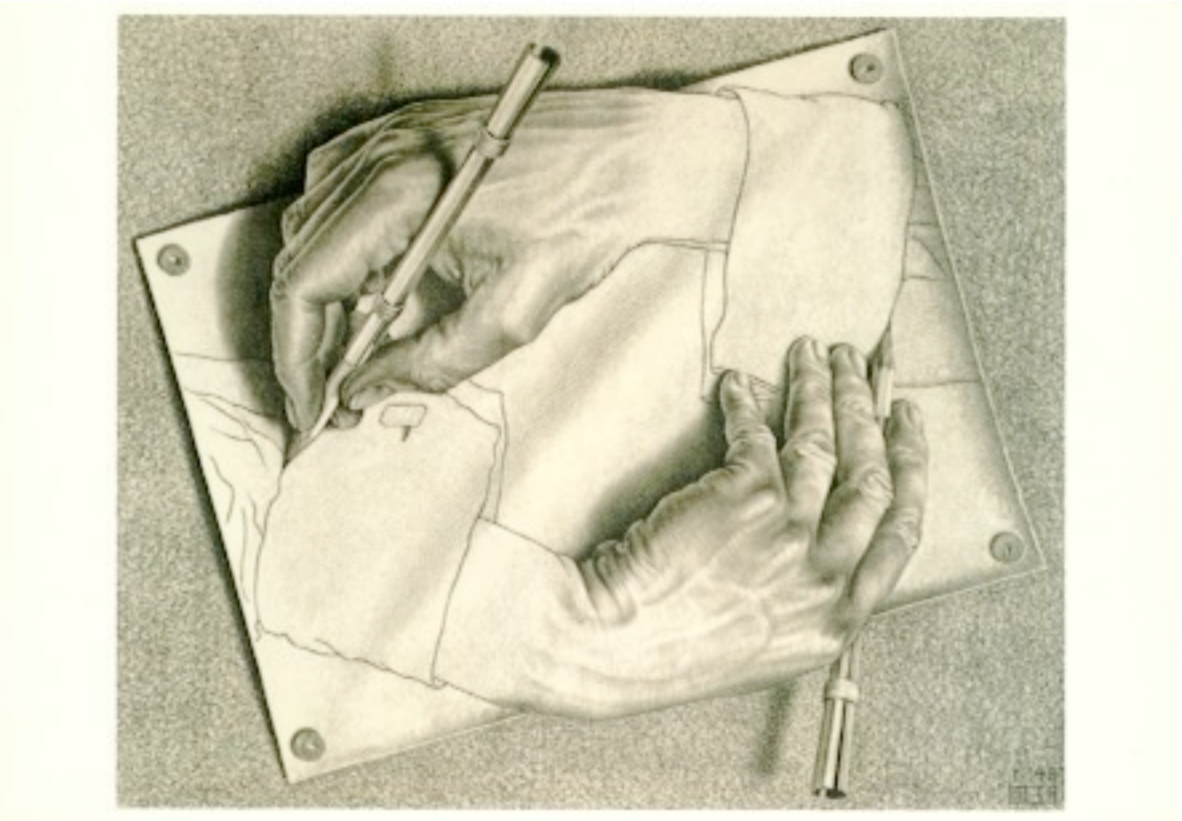
the City College of New York

CUNY

RIKEN BNL Research Center

The Anatomy of Hadronic Parity Violation

- **Parity Violation, Nuclear Parity Violation, Hadronic Parity Violation**
  Weak interactions between quarks



- **New Experiments, New Motivation**
  Fundamental neutron physics beamline

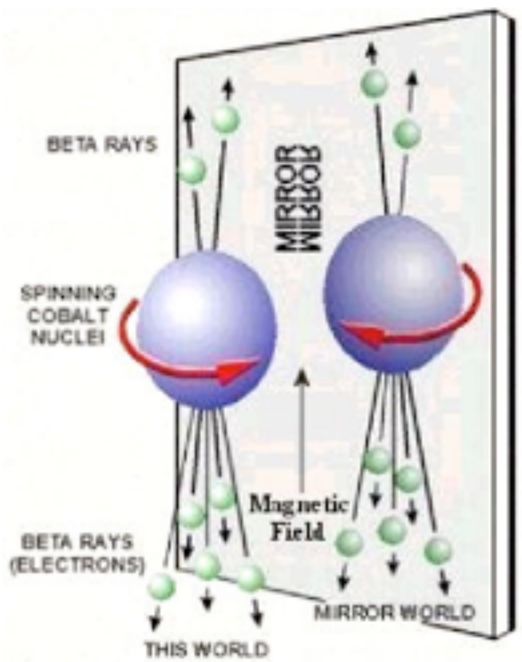
- **From Quarks to Nuclei**
  ~~Impossibility~~ in non-perturbative QCD
  Opportunity

... especially for those interested in multi-hadrons

Disclaimer: P odd and CP even processes

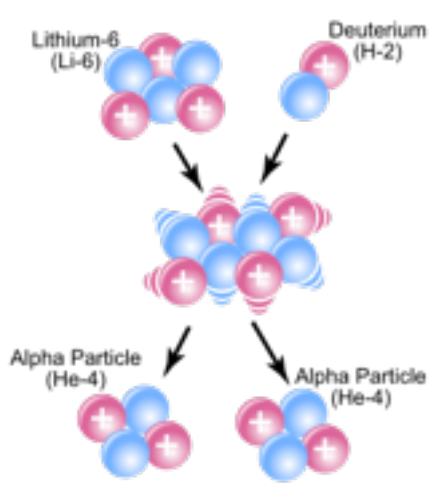
Historical Introduction

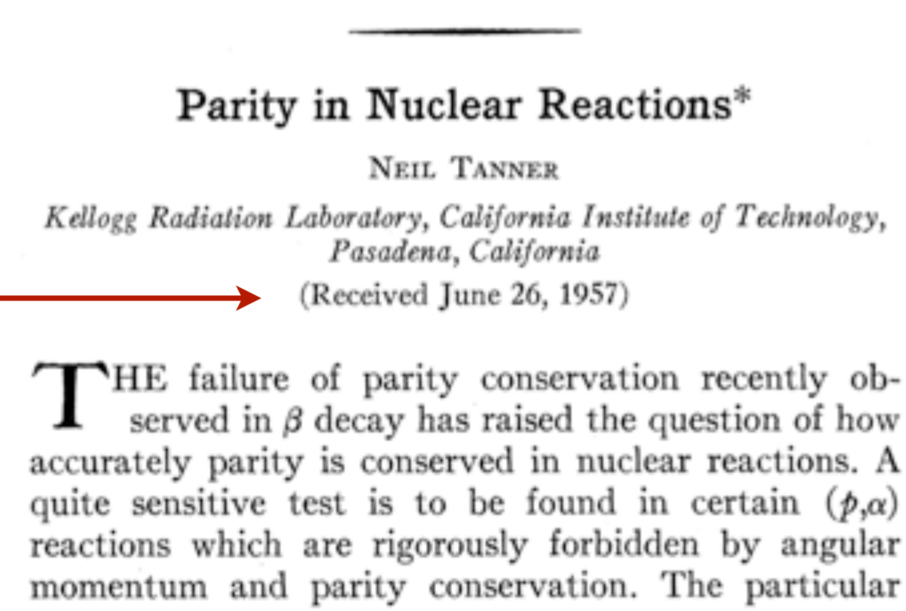
- **Parity Violation in the Weak Interaction ca. 1956**



Maximal violation 100%

- **Parity Violation in Nuclear Interactions**





Parity in Nuclear Reactions*

NEIL TANNER

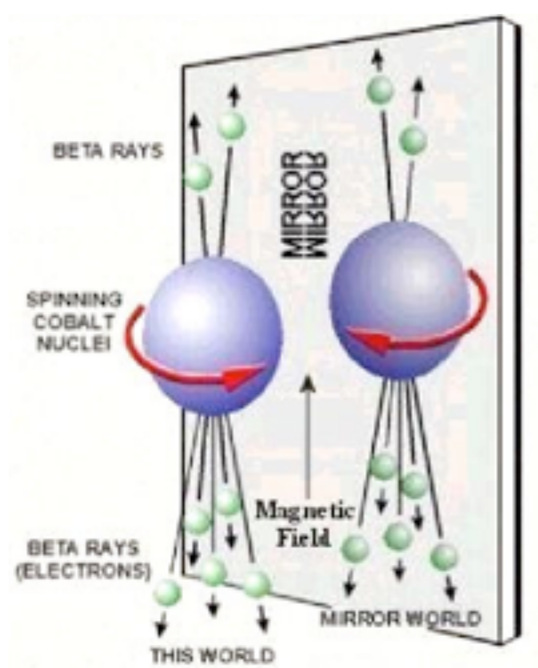
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California

(Received June 26, 1957)

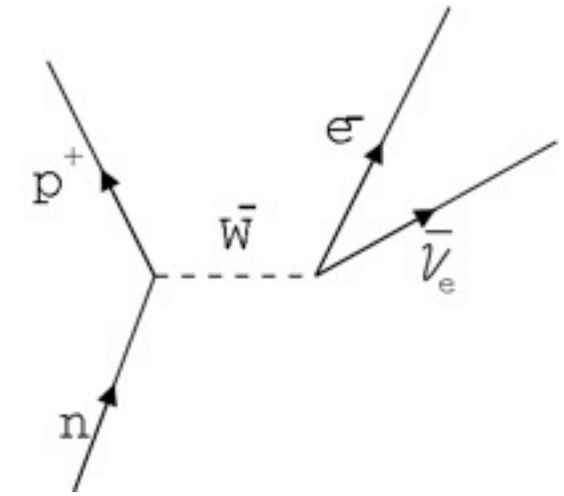
THE failure of parity conservation recently observed in β decay has raised the question of how accurately parity is conserved in nuclear reactions. A quite sensitive test is to be found in certain (p,α) reactions which are rigorously forbidden by angular momentum and parity conservation. The particular

55 Years Later: Standard Model

- Parity Violation in the Weak Interaction



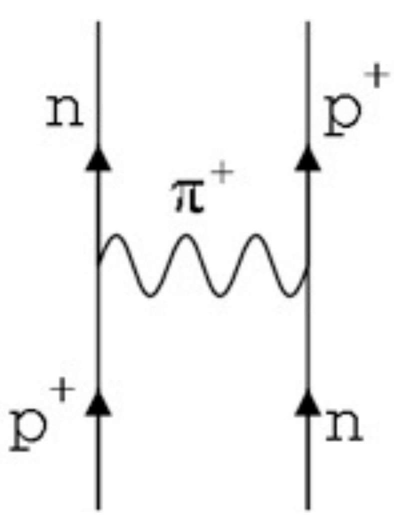
$$G_F = \frac{\sqrt{2}g^2}{8M_W^2} = 10^{-5}/\mathtt{GeV}^2$$



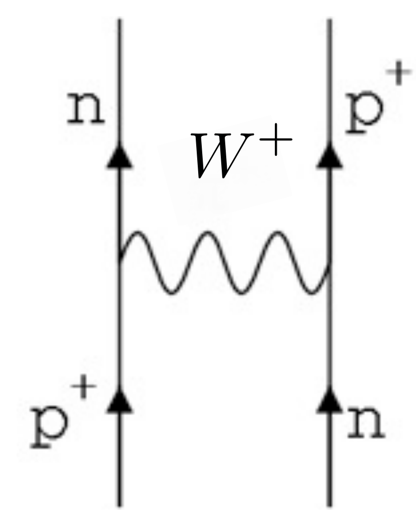
$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left(\overline{u}_L \gamma_\mu d_L\right) \left(\overline{\nu}_L \gamma^\mu e_L\right)$$

$$\langle p|V|n\rangle \sim g_V \qquad \langle p|A|n\rangle \sim g_A$$

- Long-Range Nuclear Force from Strong Interactions



$$\sim \left(\frac{g_A}{f_\pi}\right)^2 \frac{q\cdot\sigma_1\, q\cdot\sigma_2}{q^2+m_\pi^2} \qquad G_F$$



Nucleon-Nucleon Weak Interactions

$$G_F f_\pi^2 \sim 10^{-7}$$

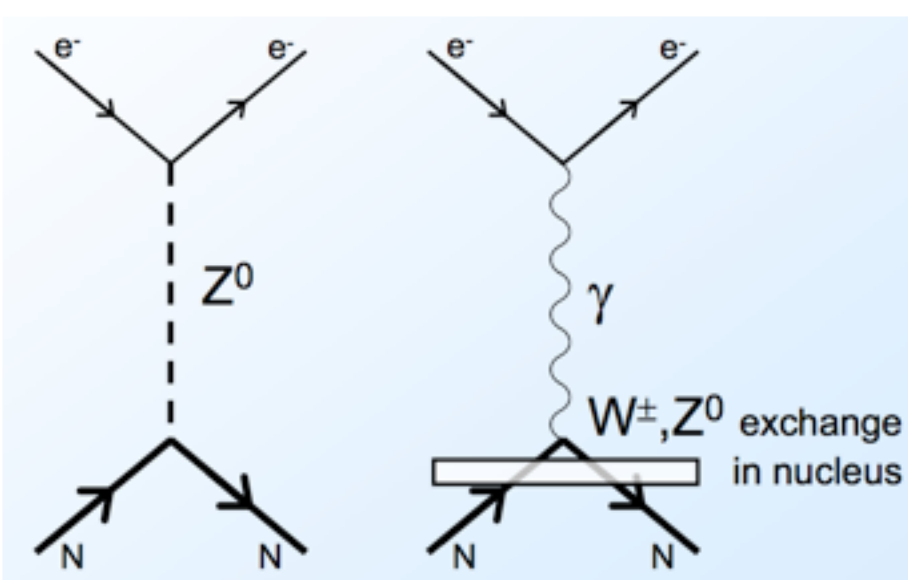
Nuclear Parity Violation

Violate strong interaction symmetries to expose weak nuclear force

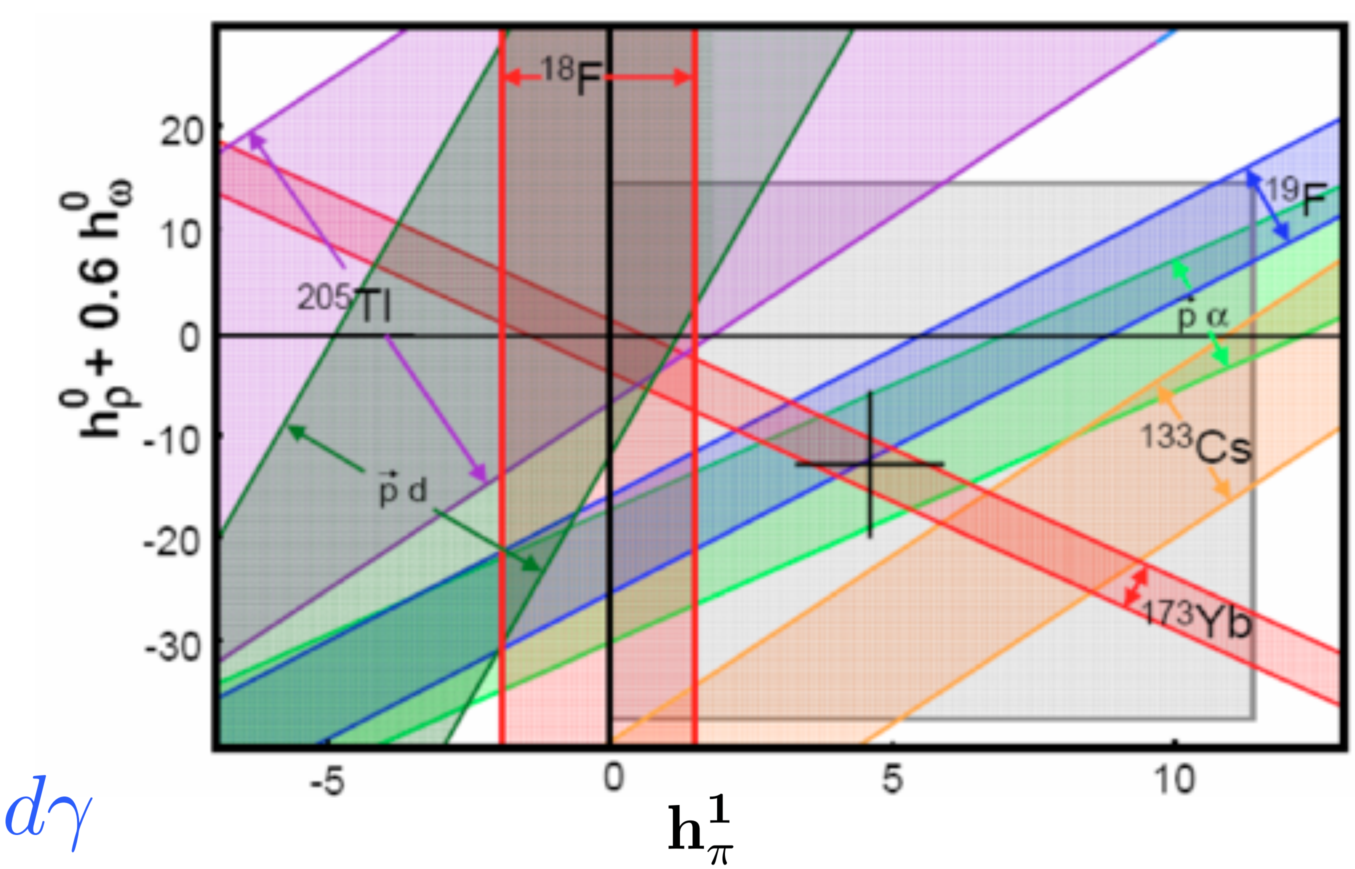
- **(Many) Parity Violating Nuclear reactions have been seen starting in 1967**
- **1989** From one in ten million to one in ten... $^{139}\texttt{La}$ $|P_+\rangle \longrightarrow |P_+\rangle + \epsilon|P_-\rangle$

$$\epsilon = \langle P_+|\frac{1}{E_+ - E_-}\mathcal{H}_{PV}|P_-\rangle \qquad \Delta E/E \sim 10^{-6} \quad \text{for} \quad \Delta E \sim 0.7\,\texttt{keV}$$

- Same ideas are being applied in **Atomic Parity Violation** expts. atoms, molecules, solids



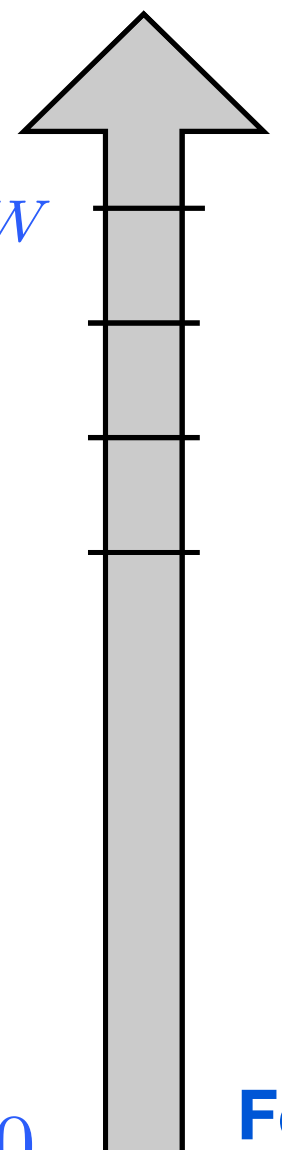
Parity Violating Nuclear Force



- **Forthcoming:** $n + {}^4\text{He}$ $\vec{n}p \to d\gamma$

Panoply of Parity Violation

- PV nuclear transitions, PV photo-nuclear transitions (anapole moment), PV nucleon-nucleon interaction, PV nucleon-meson couplings, ... , ...
- **Organize with Effective Theory mindset**



μ			
Weak Scale PV	M_Z, M_W		boson exchange between quarks
Fermi Theory PV			four quark operators
	m_Q		below heavy quark thresholds
QCD Scale PV	Λ_{QCD}		non-perturbative
	0	**Few Body**	

Panoply of Parity Violation

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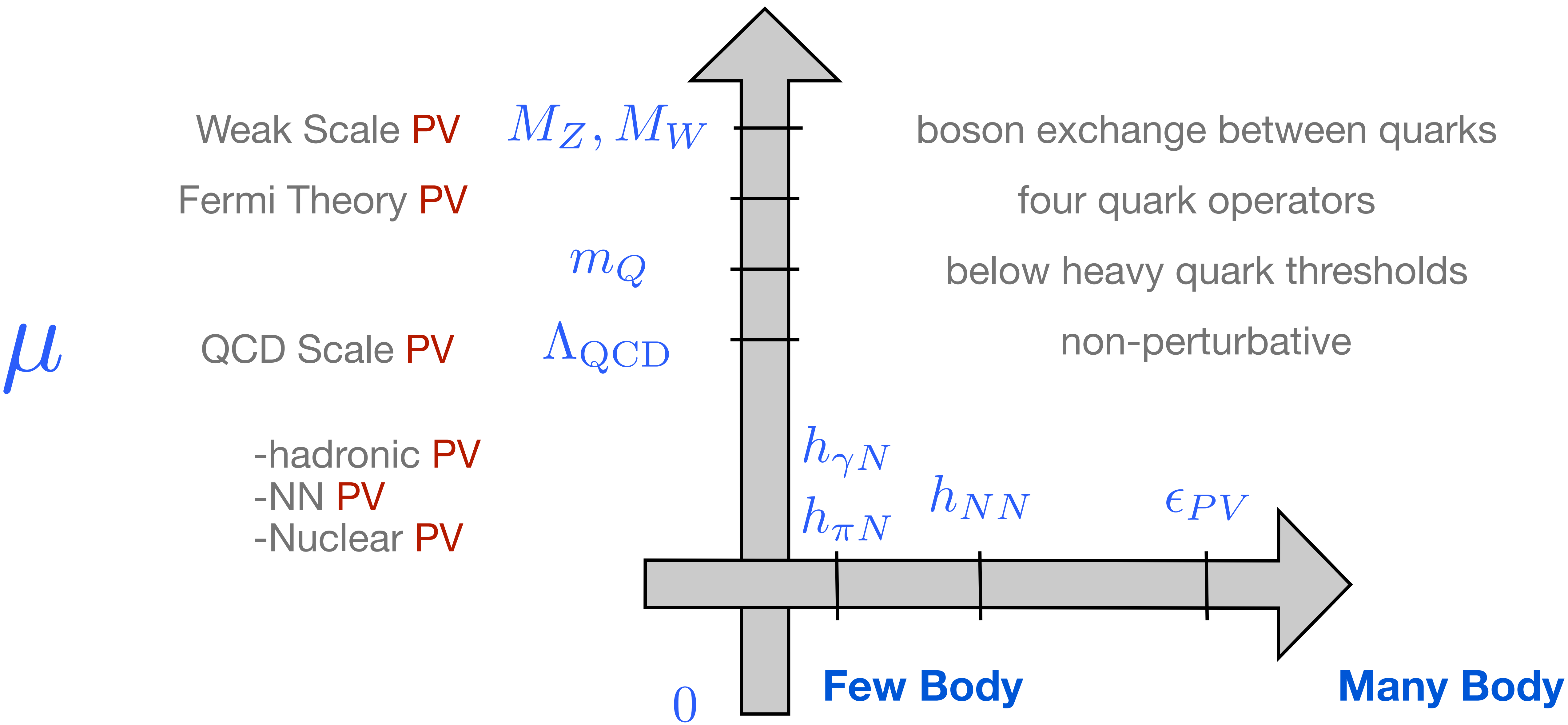


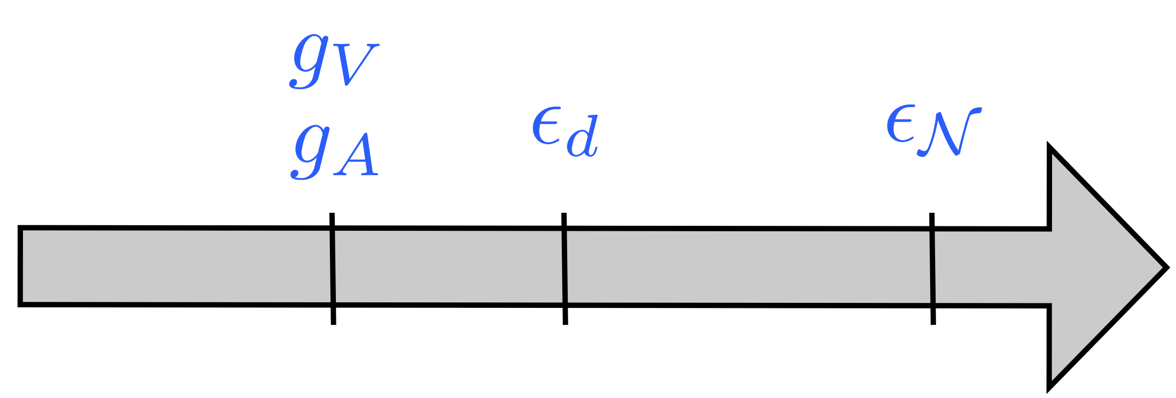
Parallel to Parity Violation

Lattice QCD: Connect Quarks to Hadrons, Few Body

Quantum Many Body: Connect Few Body to Nuclei

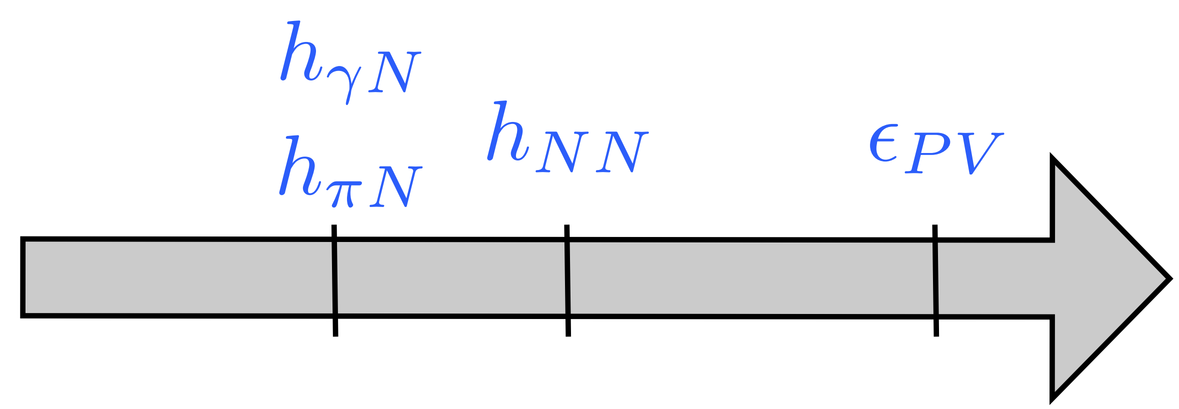
- **Organize with Effective Theory mindset**

-hadronic PC
-NN PC
-Nuclear PC



μ QCD Scale PV $\Lambda_{\rm QCD}$ non-perturbative

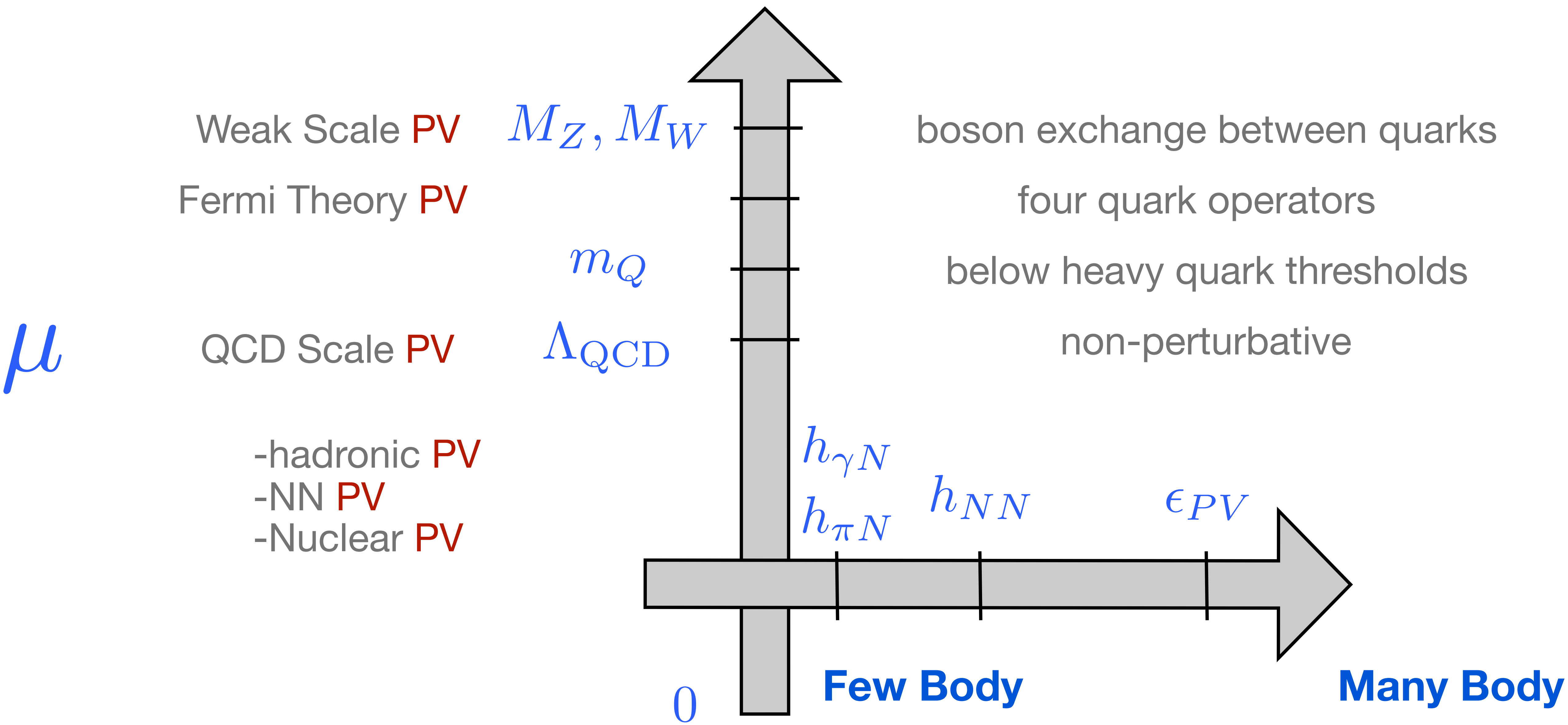
-hadronic PV
-NN PV
-Nuclear PV



Few Body **Many Body**

Parallels to Parity Violation

- **Organize with Effective Theory mindset**



Parallels to Parity Violation

Perturbative QCD: Connect Standard Model to QCD scale

Lattice QCD: Connect Four Quark Ops. to Observables

- **Organize with Effective Theory mindset**

$\Delta F = 1, 2$

μ

Weak Scale PV M_Z, M_W

Fermi Theory PV

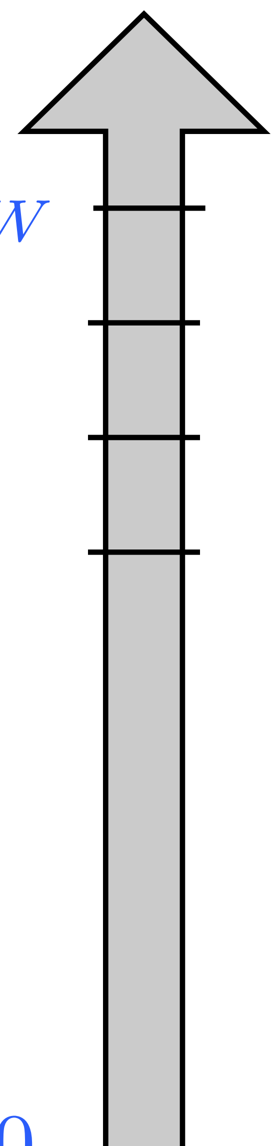
m_Q

QCD Scale PV Λ_{QCD}

Four-Quark Correlations deep inside hadrons

$\sim 1/100\,\mathtt{fm}$

0



Weak Scale FC

Fermi Theory FC

QCD Scale FC

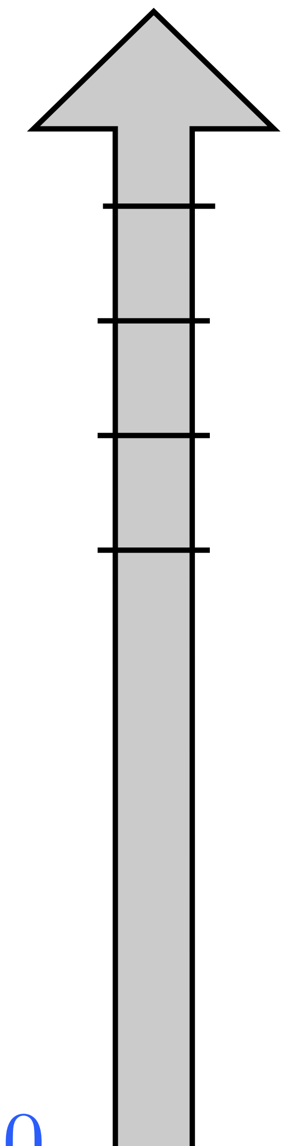
e.g.

$K \to \pi\pi$

$\Delta I = 1/2$

$\cdots$

0



Hadronic Parity Violation in QCD

Perturbative QCD: Connect Standard Model to QCD scale
Lattice QCD: Connect Four Quark Ops. to Observables

- **QCD renormalization of PV**

$$\mathcal{L}_{\rm PV}^{I=1} = \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

B Tiburzi, PRD **85** 054020 (2012)

- **(First) Lattice QCD calculation of PV**

$$\langle p | \mathcal{L}_{\rm PV}^{I=1} | \pi n \rangle = h_\pi^1$$

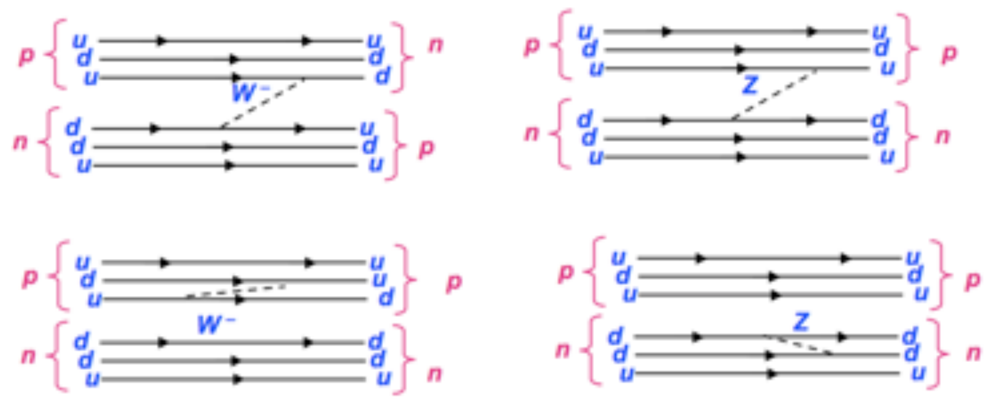
J Wasem, PRC **85** 022501(R) (2012)

- **In tandem: program to remove model dependence in NN, NNN, ...**

Zhu Maekawa Holstein Ramsey-Musolf van Kolck, Phillips Schindler Springer Grießhammer, Shin Ando Hyun, Vanasse, . . .

Isovector Parity Violation in QCD

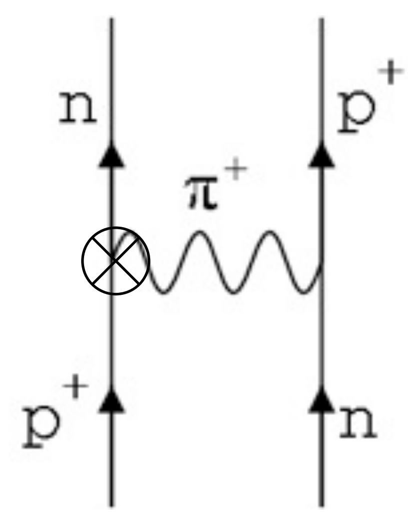
Why Isovector?



- **QCD renormalization of PV**
- **(First) Lattice QCD calculation of PV**

$$\mathcal{L}_{\rm PV}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$$\langle p|\mathcal{L}_{\rm PV}^{I=1}|\pi n\rangle = h_\pi^1$$

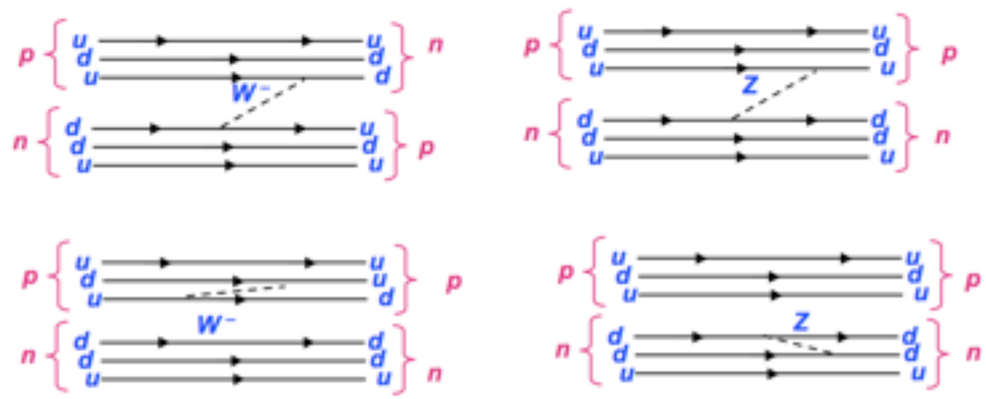


$$\sim \frac{h_\pi^1 g_A}{f_\pi^2}\frac{\sigma\cdot q}{q^2+m_\pi^2}$$

Alleged: Longest range piece of PV NN interaction

Isovector Parity Violation in QCD

Why Isovector?



- **QCD renormalization of PV**

$$\mathcal{L}_{\rm PV}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$$W^{\pm} : \ \Delta I = 0, 2 \propto |V_{ud}|^2$$

$$\Delta I = 1 \quad \propto |V_{us}|^2$$

$$Z^0 : \ \Delta I = 0, 1, 2$$

- **(First) Lattice QCD calculation of PV**

$$\langle p | \mathcal{L}_{\rm PV}^{I=1} | \pi n \rangle = h_\pi^1$$

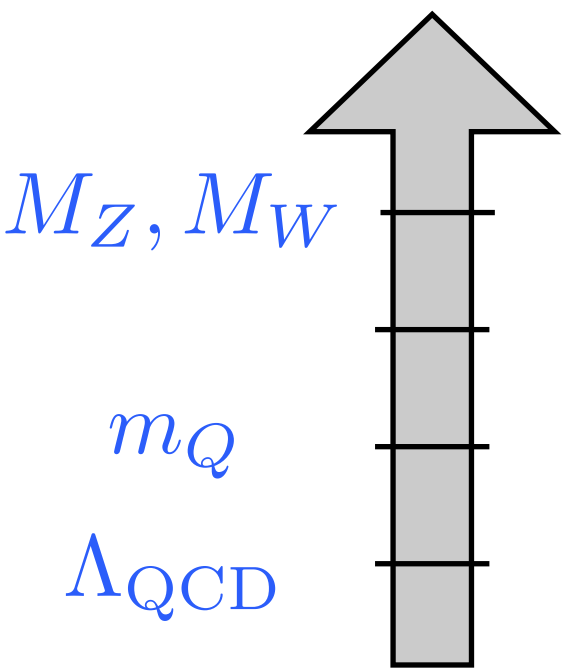
$$J_\mu^{W^-} = \overline{U}_L \gamma_\mu V D_L$$

$$J_\mu^{Z^0} = \frac{1}{c_W}\left[\overline{\Psi}_L \gamma_\mu T_3 \Psi_L - s_W^2 \overline{\Psi} \gamma_\mu \mathcal{Q} \Psi\right]$$

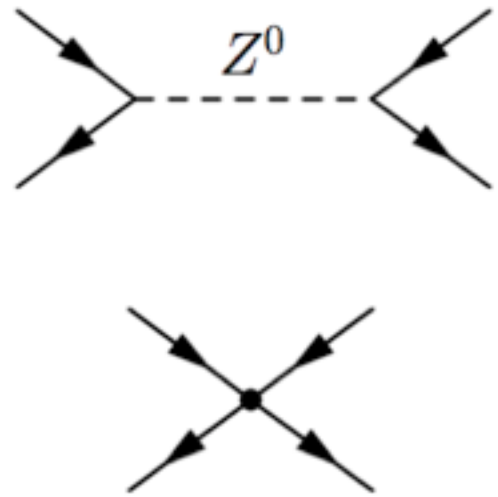
Alleged: 95% probe of hadronic neutral current

QCD Renormalization of Isovector Parity Violation

Alleged: 95% probe of hadronic neutral current



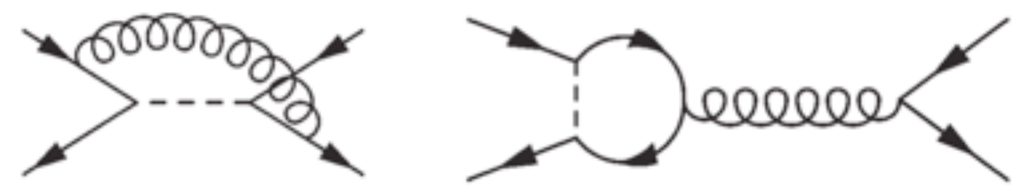
Tree Level



$$\mathcal{L}_{\rm PV}^{I=1} = \frac{G_F}{\sqrt{2}}\frac{1}{3}s_W^2\,(\overline{u}u - \overline{d}d)_A(\overline{u}u + \overline{d}d)_V$$

One Loop

$$\log\frac{M_Z^2}{p^2}$$



$$\log\frac{\mu^2}{p^2}$$

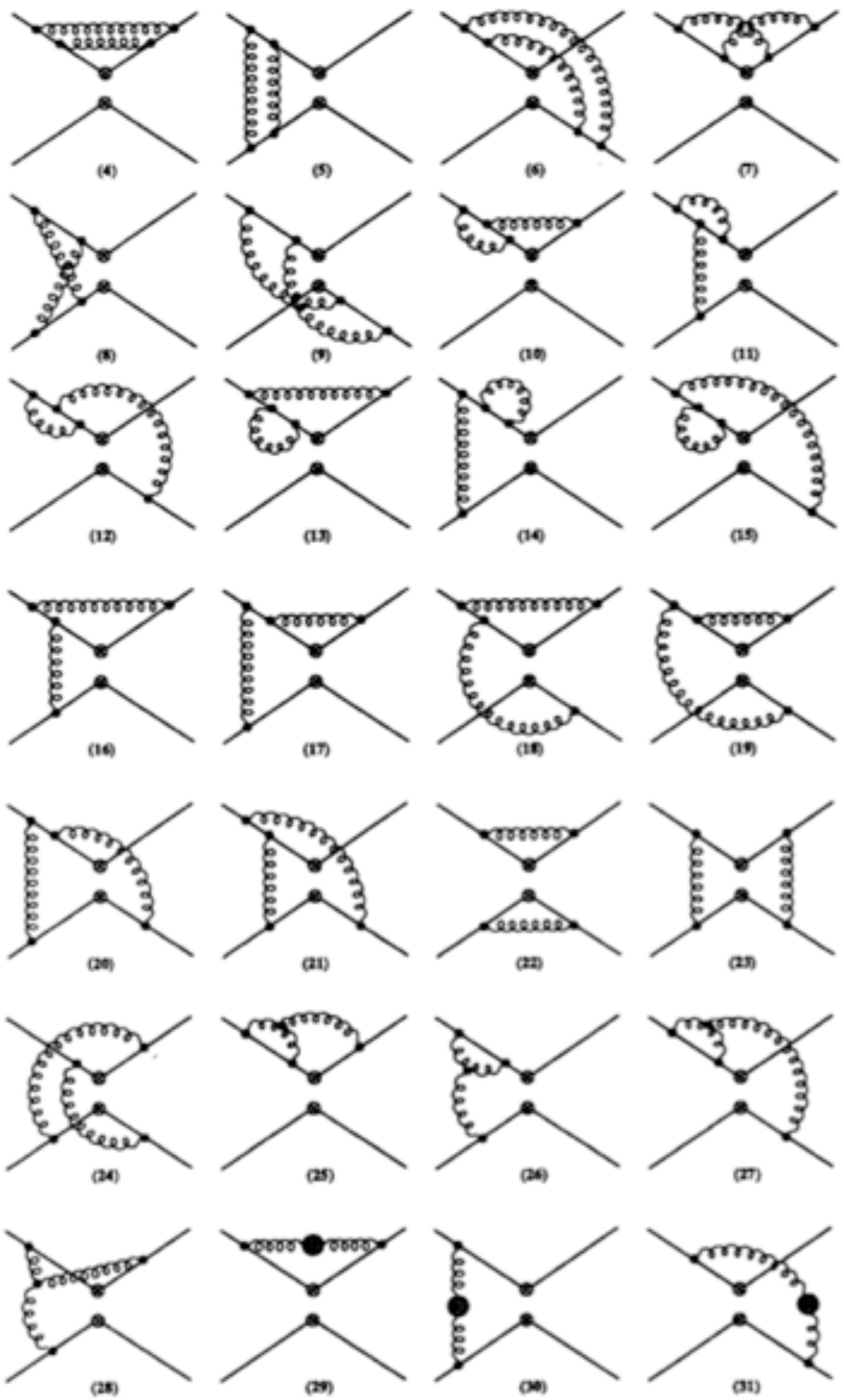
$$\log\frac{M_Z^2}{p^2} = \log\frac{\mu^2}{p^2} - \log\frac{\mu^2}{M_Z^2}$$

$$\mathcal{L}_{\rm PV}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu) \qquad C(\mu) \sim -\log\frac{\mu^2}{M_Z^2}$$

Sum leading logs $\qquad \mu\frac{d}{d\mu}\vec{C} = \frac{\alpha_s}{4\pi}\gamma^T\cdot\vec{C}$

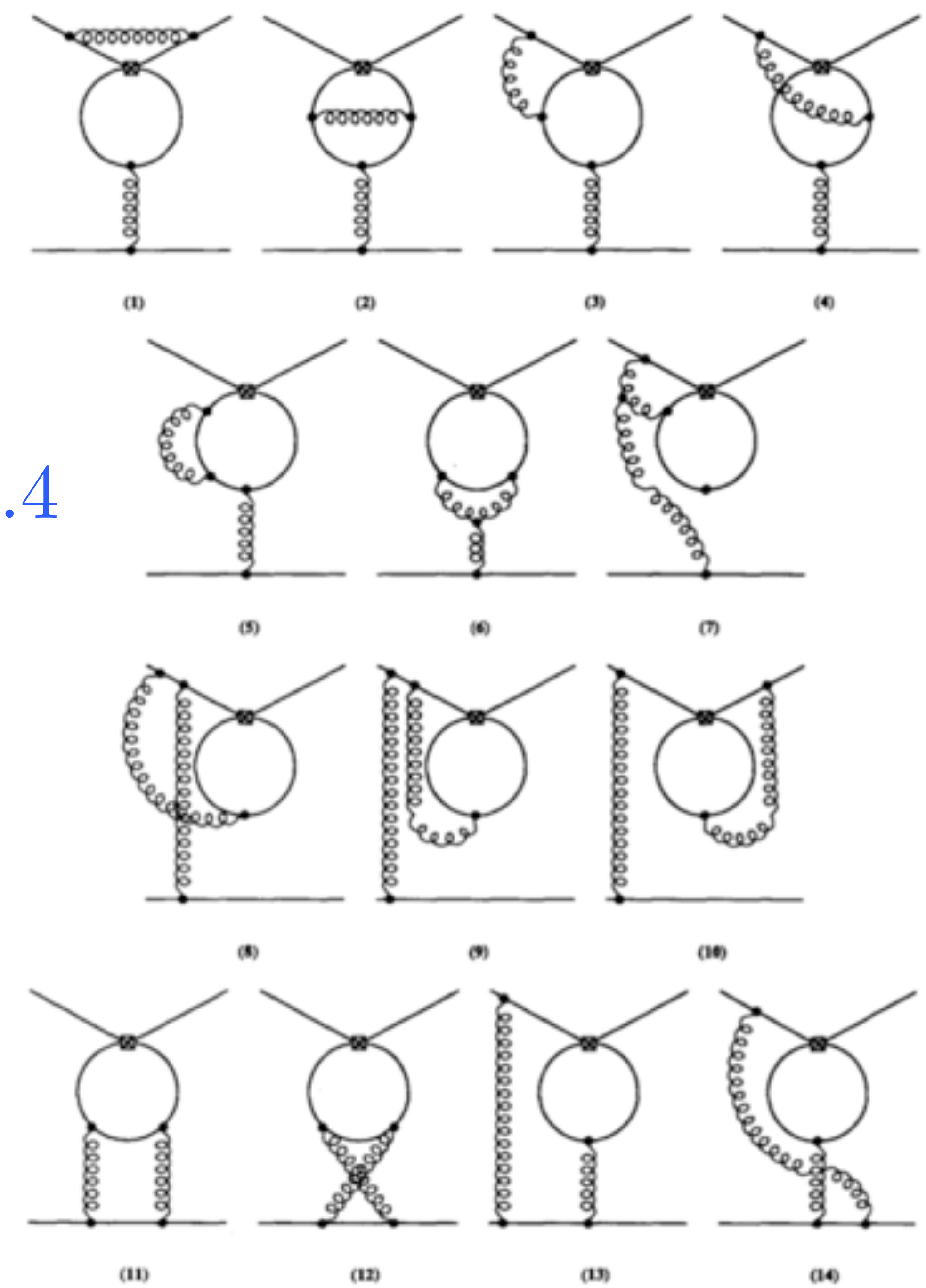
70's Donoghue, McKellar, . . . , 90's Dia Savage Liu Springer

QCD Renormalization of Isovector Parity Violation



$\alpha_s(1\,\texttt{GeV}) \sim 0.4$

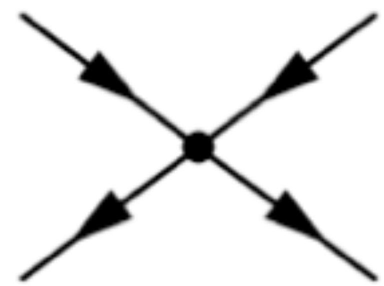
Sum NLL



... renormalization scheme dependence (Good!)

QCD Renormalization of Isovector Parity Violation

Renormalization scheme: dimensional regularization



$\gamma_\mu \gamma_5 \otimes \gamma^\mu$

$\gamma_\nu \gamma_\rho \gamma_\mu \gamma_5 \otimes \gamma^\nu \gamma^\rho \gamma^\mu$

Four dimensions affords simplification $\gamma_\nu \gamma_\rho \gamma_\mu = g_{\nu\rho}\gamma_\mu - g_{\nu\mu}\gamma_\rho + g_{\rho\mu}\gamma_\nu - i\varepsilon_{\nu\rho\mu\sigma}\gamma^\sigma \gamma_5$

Enlarge operator basis to include mixing with evanescent operators

$\Delta F \neq 0$ Experts: Buras Jamin Lautenbacher Weisz, Ciuchini Franco Lubicz Martinelli Reina Scimemi Silvestrini

(PV in Lattice QCD requires different scheme than dim. reg. ... one-loop matching = *bookkeeping*)

QCD Renormalization of Isovector Parity Violation

Renormalization scheme: dimensional regularization

E.g.

$\Delta S = 1$ $\quad Q_1 = (\overline{s}d)_{V-A}(\overline{u}u)_{V-A}$ Mass independent scheme & QCD flavor blind!

W-exchange

U-spin

$$(\overline{s}s - \overline{d}d)_{V-A}(\overline{u}u)_{V-A}$$

V-spin

$$(\overline{u}u - \overline{d}d)_{V-A}(\overline{s}s)_{V-A}$$

Parity invariance

$\Delta I = 1$ $\quad \mathcal{O} = (\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d)_L(\overline{s}\gamma^\mu s)_L - (\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d)_R(\overline{s}\gamma^\mu s)_R$

Z-exchange

$\Delta I = 1$ follows from $\Delta S = 1$ including QED penguins, and BSM operators

. . . just different initial conditions in evolution

QCD Renormalization of Isovector Parity Violation

Results ('t Hooft-Veltman scheme)

$$\mathcal{L}_{PV}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$O_1 = (\bar{u}u - \bar{d}d)_A(\bar{u}u + \bar{d}d)_V,$

$O_2 = (\bar{u}u - \bar{d}d]_A[\bar{u}u + \bar{d}d)_V,$

$O_3 = (\bar{u}u - \bar{d}d)_V(\bar{u}u + \bar{d}d)_A,$

$O_4 = (\bar{u}u - \bar{d}d]_V[\bar{u}u + \bar{d}d)_A,$

Non-Strange vs. Strange

$O_5 = (\bar{u}u - \bar{d}d)_A(\bar{s}s)_V,$ $O_6 = (\bar{u}u - \bar{d}d]_A[\bar{s}s)_V,$

$O_7 = (\bar{u}u - \bar{d}d)_V(\bar{s}s)_A,$ $O_8 = (\bar{u}u - \bar{d}d]_V[\bar{s}s)_A.$

Fierz constraint

- 2 ops in chiral basis

$L \otimes L - R \otimes R$

$L \otimes R - R \otimes L$

Alleged: 95% probe of hadronic neutral current

$C_i(\mu = 1\,\texttt{GeV})$

i	LO [18]	LO	NLO (Z)	NLO (Z + W)
1	0.403	0.264	−0.054	−0.055
2	0.765	0.981	0.803	0.810
3	-0.463	−0.592	−0.629	−0.627
4	0	0	0	0
5	5.61	5.97	4.85	5.09
6	−1.90	−2.30	−2.14	−2.55
7	4.74	5.12	4.27	4.51
8	−2.67	−3.29	−2.94	−3.36

$\sin^2\theta_W$ Non-Strange

vs.

1 Strange

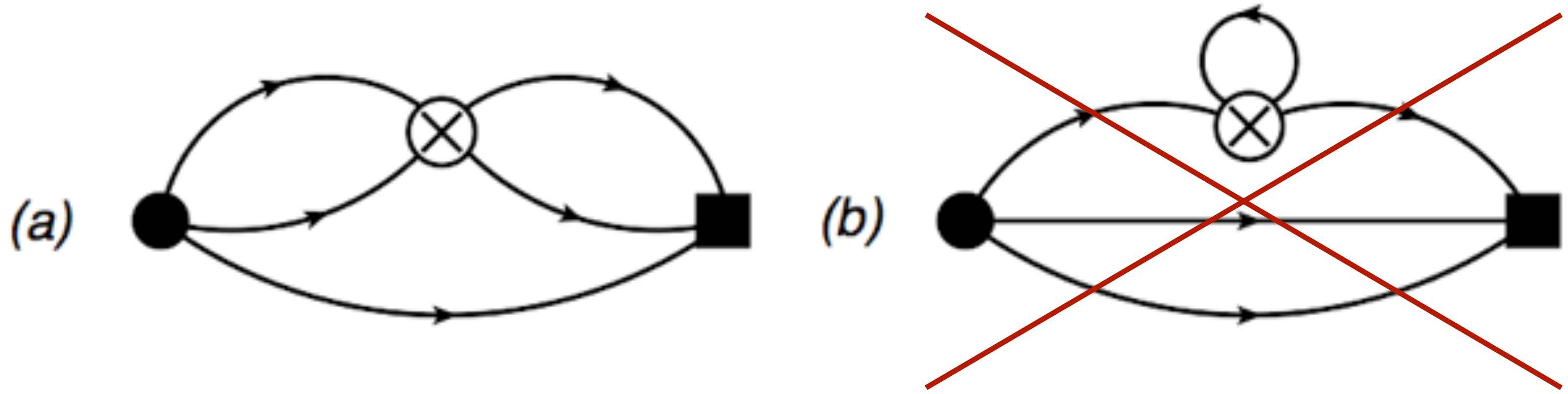
80 - 100%
Dynamical Question!

[18] Kaplan Savage, NuPhA **556** (1993)

B Tiburzi, PRD **85** 054020 (2012)

First Lattice Computation (**J Wasem**) PRC **85** 022501(R) (2012)

$p \sim \mathcal{O}_p(x)$ $(\pi n)_{\text{s-wave}} \sim \gamma_5 \mathcal{O}_p(x)$ $\langle p | \mathcal{L}_{\text{PV}}^{I=1} | \pi n \rangle = h_\pi^1$



$$h_{\pi NN}^{1,con} = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7}$$

Easy to pick on a first calculation (much harder to have done it, or improve it!)

- Multi-hadron overlap?
- No Lellouch-Lüscher factor
- Ignores regularization scheme
- Inexact kinematics ∂_τ^2 vs. m_π^2
- Noisy operator self-contractions

Wilson coefficients of strange operators are enhanced

$5\times$

Strangeness not-so suppressed

$N_c^{-1/2}$

Auxiliary Fields for Isovector Parity Violation

- Perhaps only a Gedankenexperiment until exascale computers materialize

E.g. $\mathcal{O} = \left(\bar{q}\gamma_\mu\gamma_5\tau^3 q\right)\left(\bar{q}\gamma_\mu q\right) \longrightarrow -\left[\bar{q}\gamma_\mu\left(\gamma_5\tau^3 - 1\right) q\right]^2$

$P \otimes \tau^1$

τ^3–chiral symmetry

Introduces PC and PV four-quark operators

Integrate in auxiliary field $\Delta\mathcal{L} = \sigma^2 + i\sigma\left[\bar{q}\gamma_\mu\left(\gamma_5\tau^3 - 1\right) q\right]$

No sign problem $\gamma_5 \otimes \tau^1$–Hermiticity

- Can implement all isovector PV operators in sign-problem-free ways
  Continuum limit (!?!?)

$$\langle p|\mathcal{L}_{\rm PV}^{I=1}|\pi n\rangle = h_\pi^1 \quad \rightarrow \langle p|\pi^+(x)|n\rangle_\sigma$$

Nucleon anapole moment: just calculate anapole form factor

PV NN interactions from PV two-point functions

Bodies buried in gauge field generation

Isotensor Parity Violation

$$\mathcal{O} = (\overline{q}\tau^3 q)_A(\overline{q}\tau^3 q)_V - \frac{1}{3}(\overline{q}\vec{\tau}\, q)_A \cdot (\overline{q}\vec{\tau}\, q)_V$$

- Only **one** operator & **without** self-contractions

$$\mathcal{L}_{PV}^{\Delta I=2} = \frac{G_F}{\sqrt{2}} C(\mu)\mathcal{O}(\mu)$$

Renormalization by bookkeeping

B Tiburzi, arXiv:1207.4996

LO	$C(1\,\mathtt{GeV})/C^{(0)}$
LO [15]	0.79
LO	0.70
NLO	$C(1\,\mathtt{GeV})/C^{(0)}$
't Hooft-Veltman	0.58
Naïve Dim. Reg.	0.74
RI/MOM	0.77
RI/SMOM($\gamma_\mu, \not{q}$)	0.67
RI/SMOM(γ_μ, γ_μ)	0.75
RI/SMOM($\not{q}, \not{q}$)	0.73
RI/SMOM($\not{q}, \gamma_\mu$)	0.81

[15] Kaplan Savage, NuPhA **556** (1993)

Better proving ground for Lattice QCD?

$$\mathcal{L}_{NN} = [\vec{\nabla} p^\dagger \cdot \vec{\sigma}\,\sigma_2\, p^*] \cdot [n^T \sigma_2\, n] + \ldots$$

s- to p-wave **NN** interaction

Operator matrix element between two hadrons (... bound states currently!)

πN interactions

$$\mathcal{L}_{\pi\pi N} + \mathcal{L}_{\pi\gamma N}$$

External fields could "substitute" for hadrons

πPV

Isotensor three-pion vertex exists (πPV very suppressed in other channels)

Anatomy of Parity Violation

- New neutron experiments will constrain PV in few-body systems
- Connecting few-body PV to many-body PV stringent test of methods NMB/NNEFT
- Connecting PV four quark operators to PV couplings between hadrons: test of non-pQCD
- Connection of nuclear PV to Standard Model

"Parity Violation"