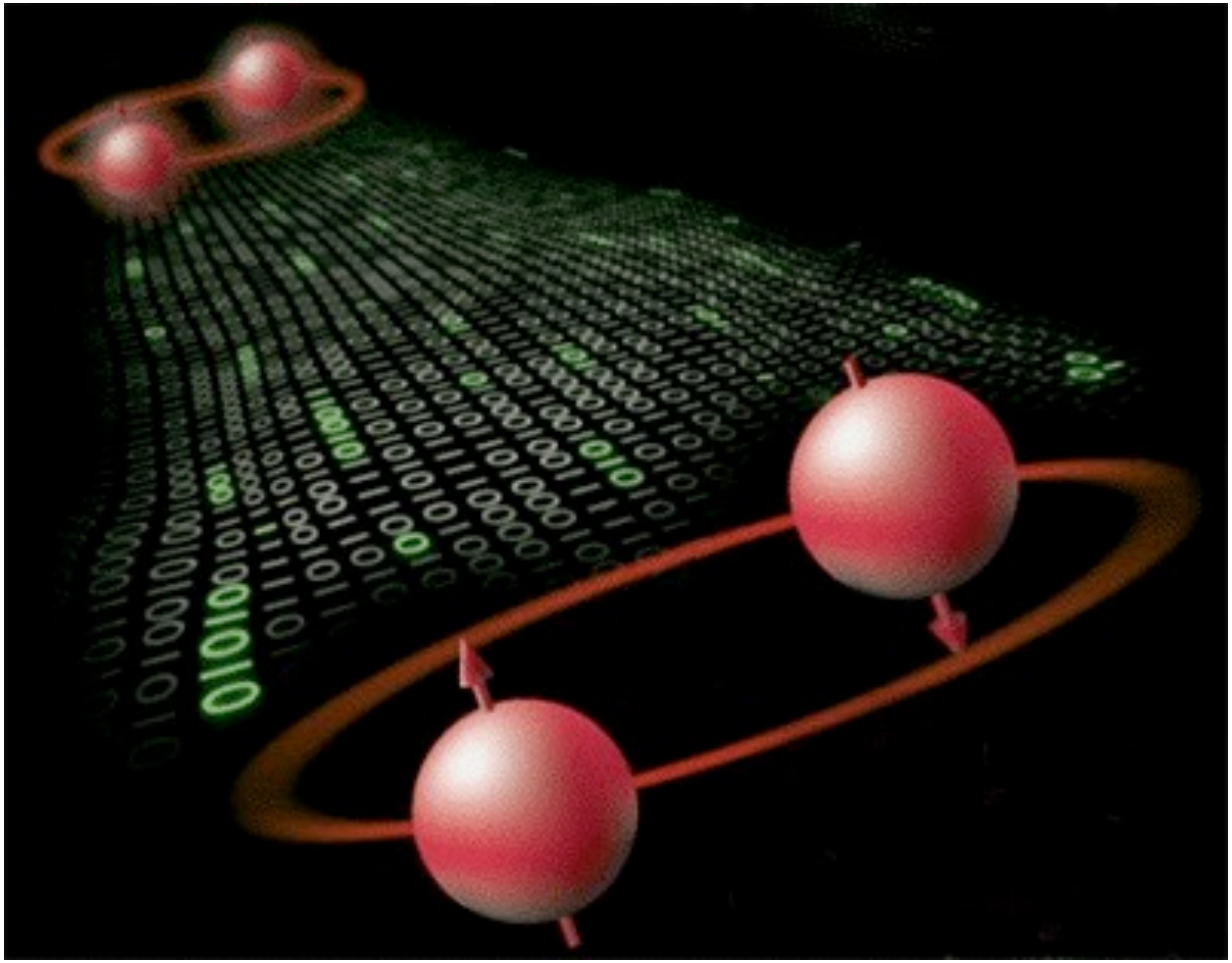
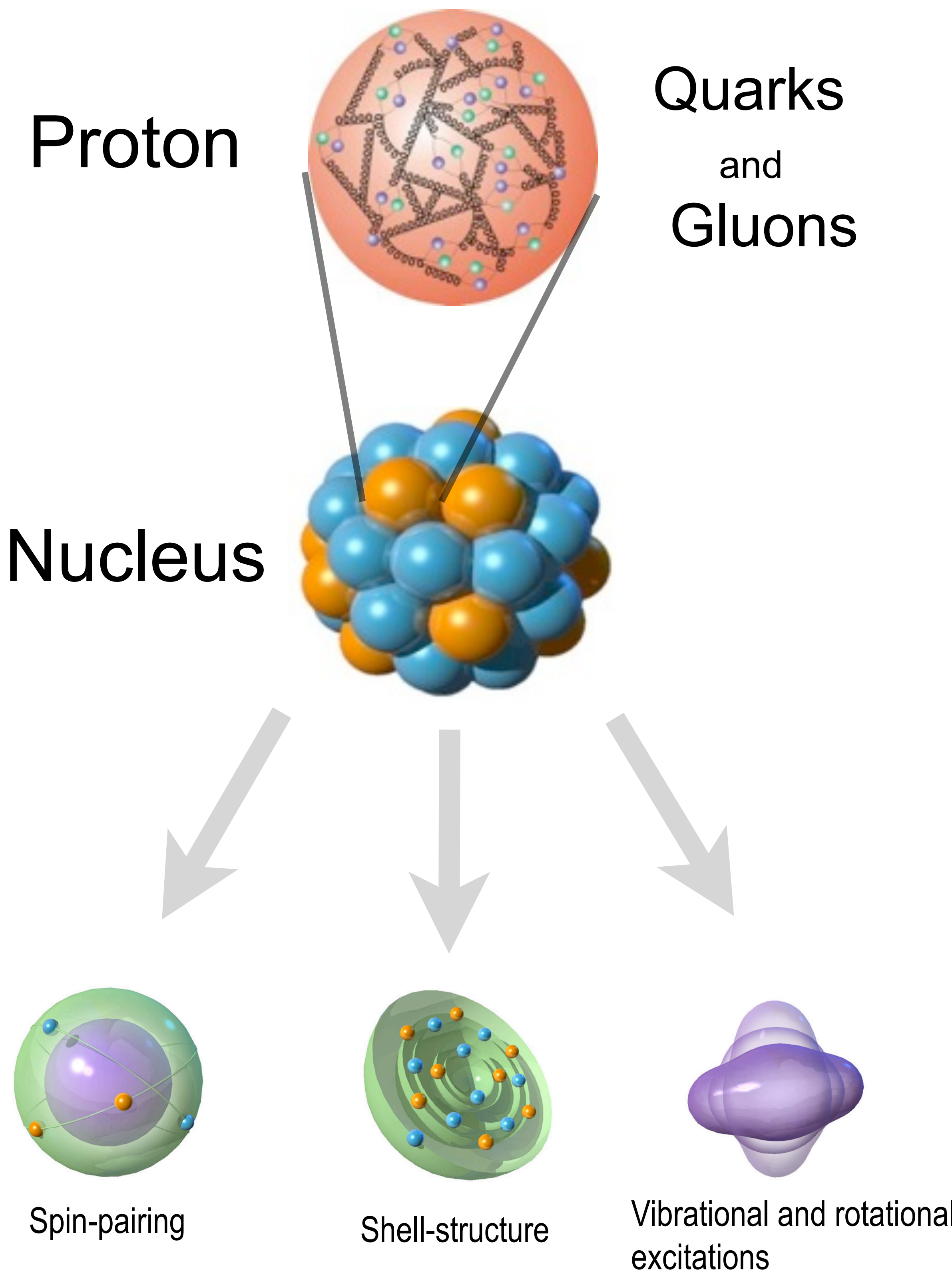


Nuclei

Martin J. Savage
University of Washington
August 2012, INT

The Structure and Interactions of Matter from Quantum Chromodynamics



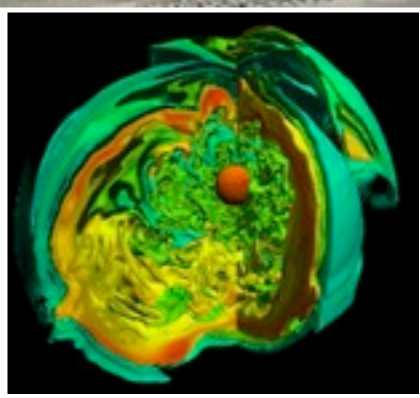
$$\Lambda_{\text{QCD}}$$

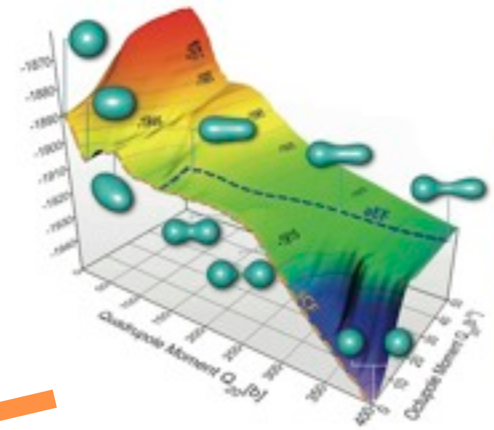
$$\frac{m_u}{\Lambda_{\text{QCD}}} \qquad \frac{m_d}{\Lambda_{\text{QCD}}} \qquad \frac{m_s}{\Lambda_{\text{QCD}}}$$

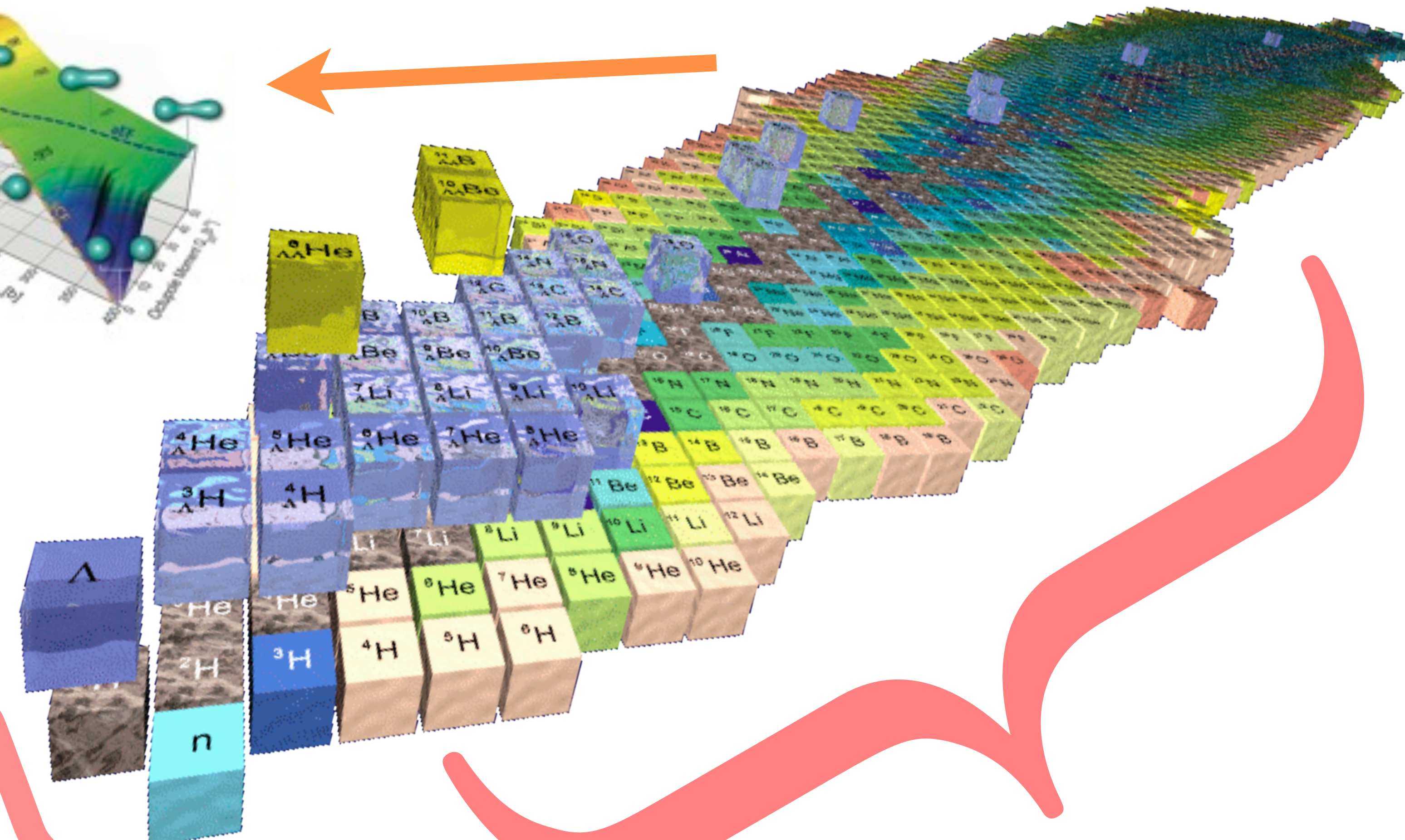
$$\alpha_e$$

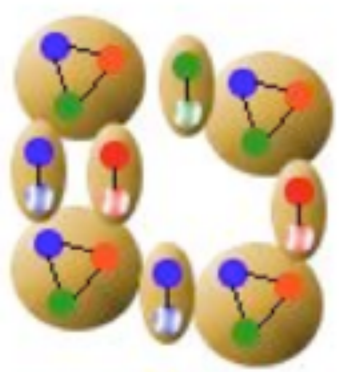
Small number of input parameters responsible for all of strongly interacting matter

(Partial) Unification of Nuclear Physics
- Quantifiable Uncertainties

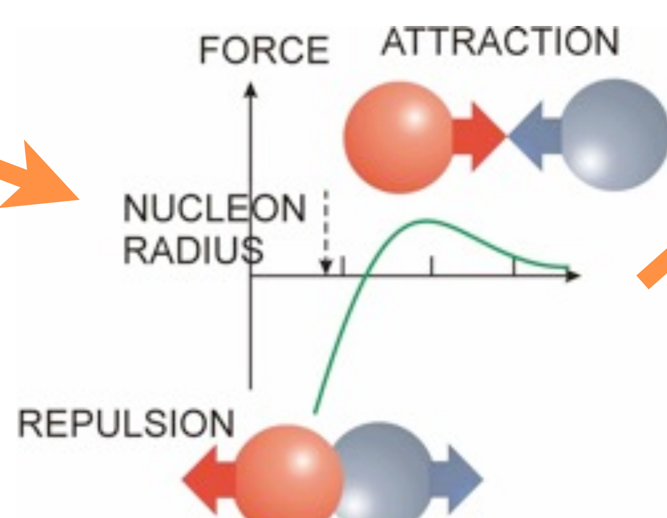








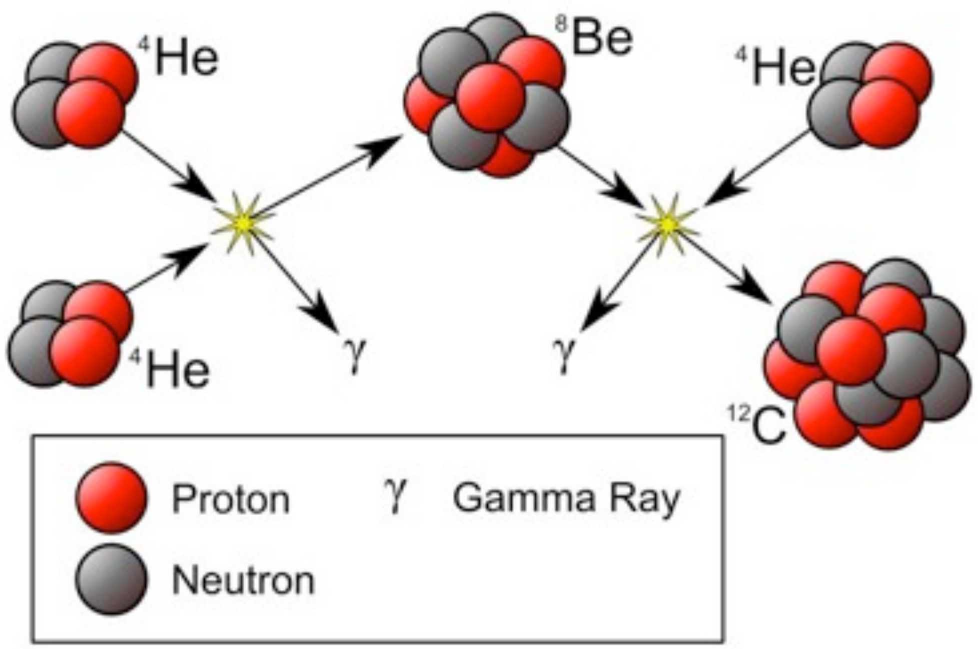
Solve QCD

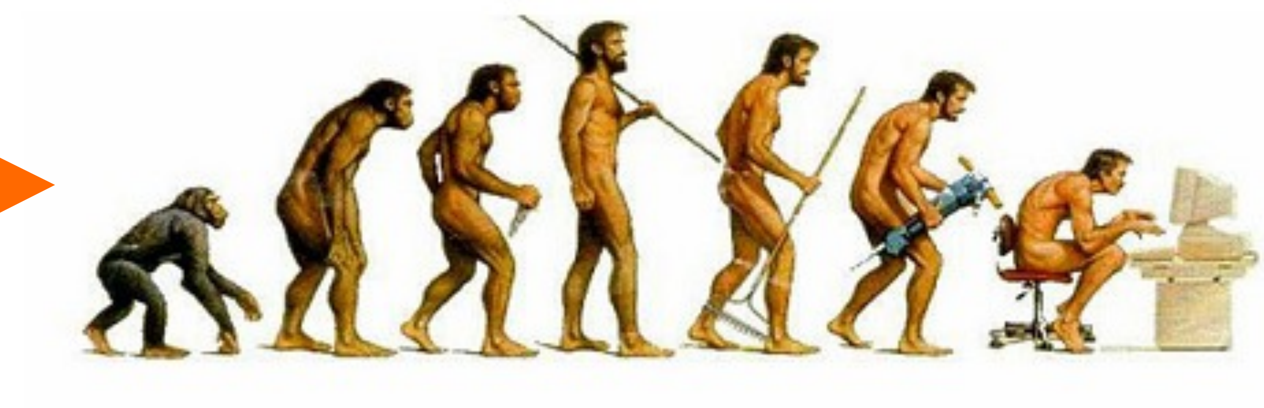


Many-Body Methods
EFT, LatticeEFT,
GFMC, NCSM

Predictions with
Quantifiable Uncertainties

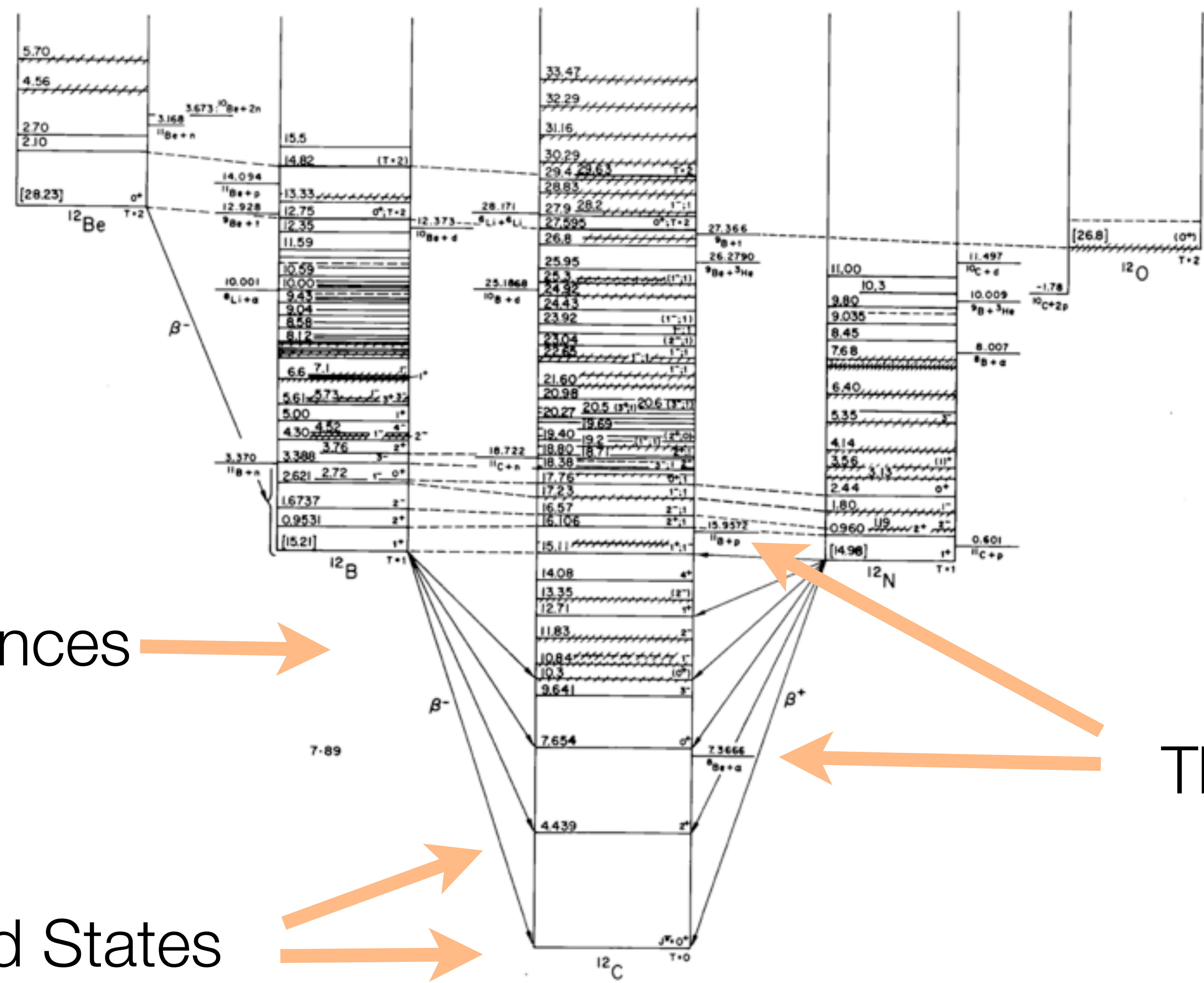
Fine-Tunings define our Universe





- Nuclear physics exhibits fine-tunings
  - *Why ??*
  - *Range of parameters to produce sufficient carbon ?*

A = 12 Energy-Level Diagram

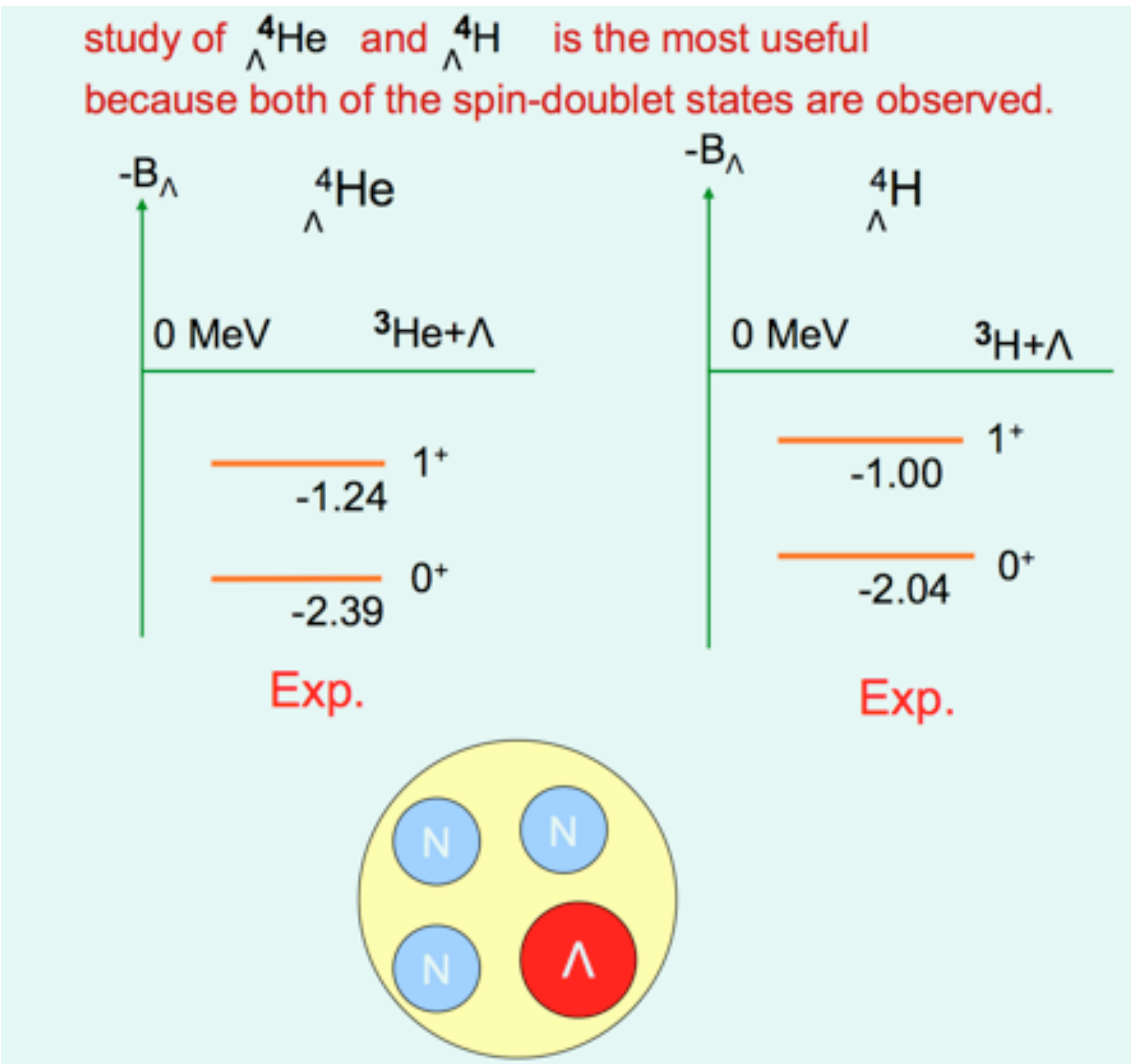


Resonances

Threshold

Bound States

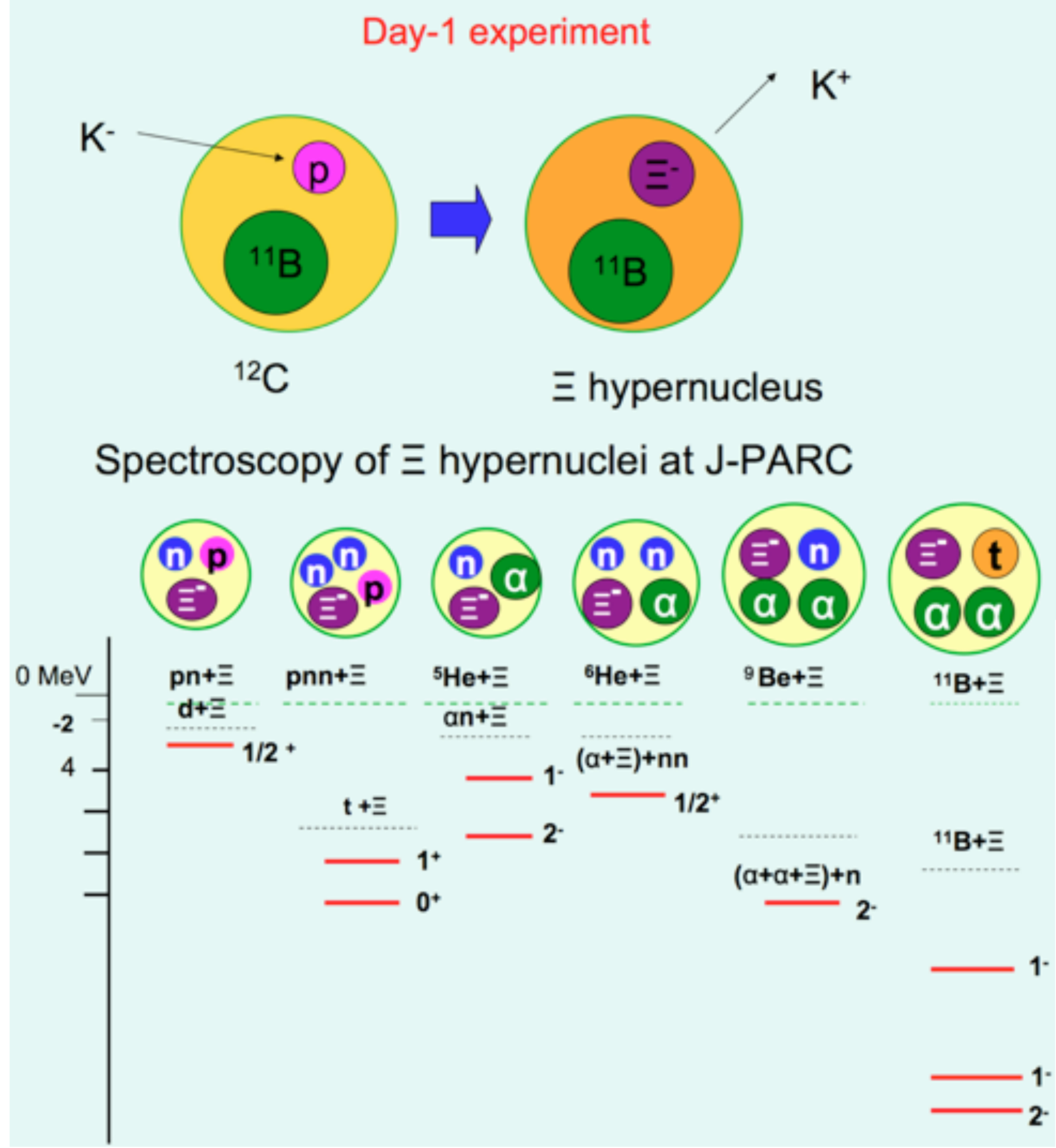
HyperNuclear Programs
J-PARC, FAIR, JLab,



E. Hiyama (2008)

Approved proposal at J-PARC

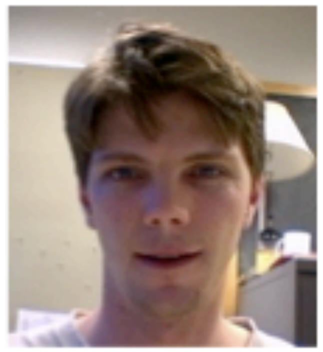
- E05 "Spectroscopic study of Ξ-Hypernucleus, ^{12}Be, via the $^{12}C(K^{-},K^{+})$ Reaction" by Nagae and his collaborators



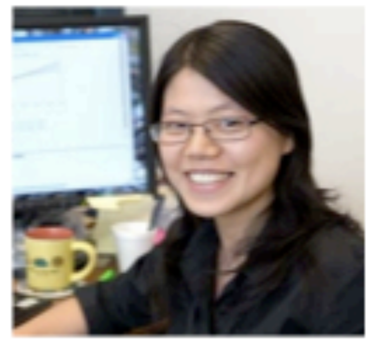
NPLQCD

Silas Beane
New Hampshire

Emmanuel Chang
Barcelona

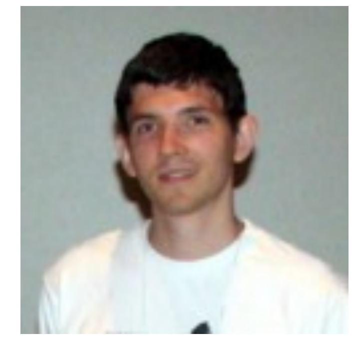


William Detmold
William+Mary / MIT



Huey-Wen Lin
U. of Washington

Tom Luu
LLNL



Saul Cohen
U. of Washington

Pari Junnarkar
New Hampshire

Kostas Orginos
William+Mary

Assumpta Parreno
Barcelona

Martin Savage
U. of Washington

Aaron Torok
ex-New Hampshire

Andre Walker-Loud
LBNL

+

Jefferson Lab

... to make predictions for the structure and interactions of nuclei using lattice QCD.

USQCD

US Lattice Quantum Chromodynamics

Two-Particle Energy Levels
(Luscher's Method)

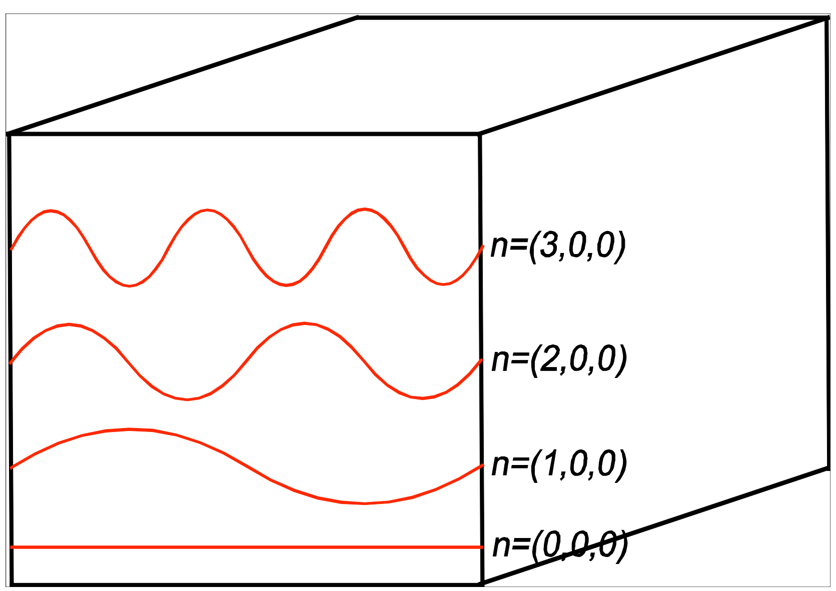
Below Inelastic Thresholds :

Measure on lattice → $\delta E = 2\sqrt{p^2 + m^2} - 2m$

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S}\left(\left(\frac{Lp}{2\pi}\right)^2 \right)$$

A_1^+

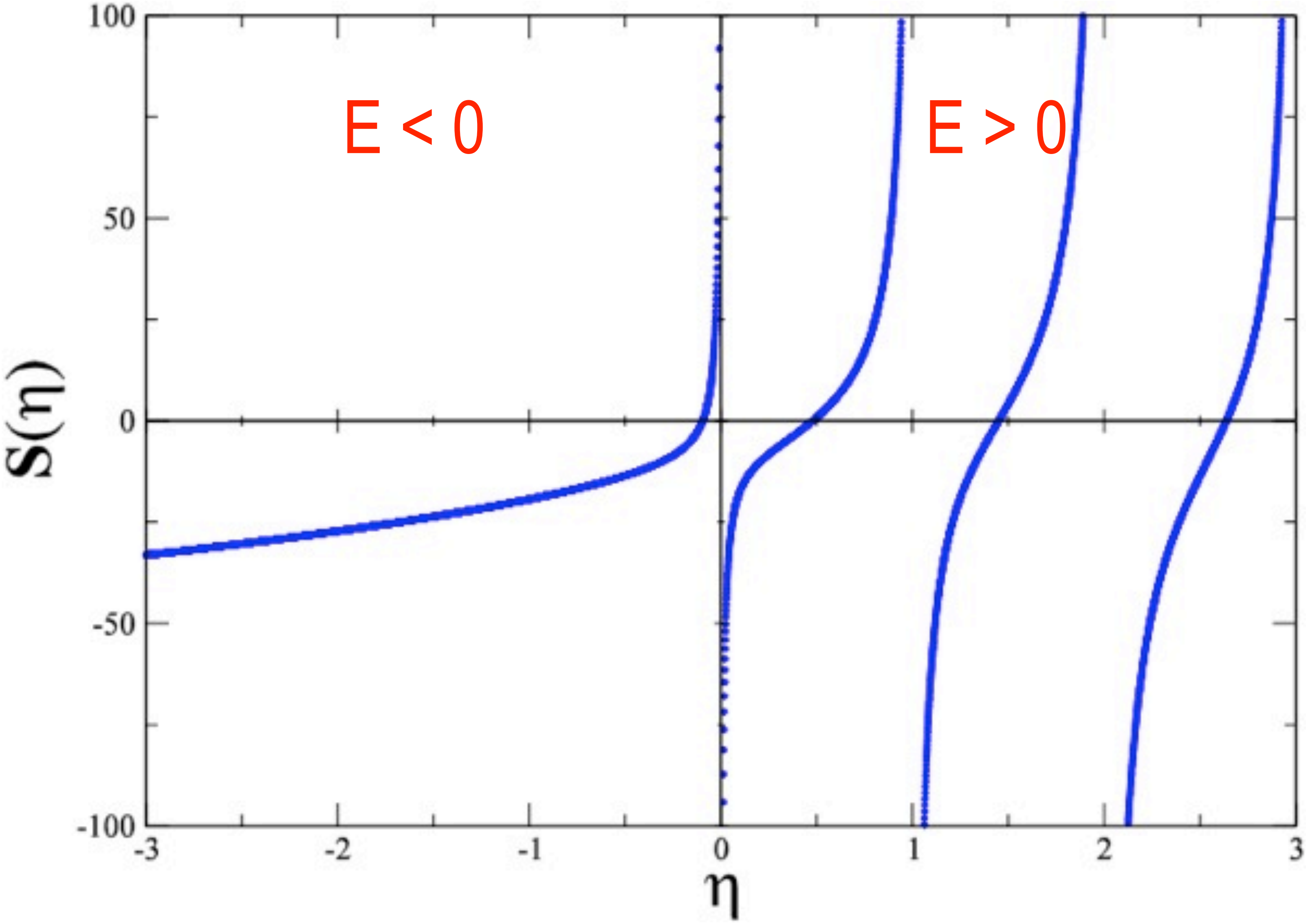
(L=0,4,6,8,..)



$$\mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{\Lambda_j} \frac{1}{|\mathbf{j}|^2 - \eta} - 4\pi\Lambda_j$$

Gives the scattering amplitude at δE

Luscher Eigenvalue Relation



A_1^+

Bound-state or

Scattering state ?

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S}\left(\left(\frac{Lp}{2\pi}\right)^2\right)$$

$$k = \frac{2\pi}{L} n$$

$$n = (nx, ny, nz)$$

Non-interacting particles

$$V = 0 \quad \rightarrow \quad a = r = 0$$

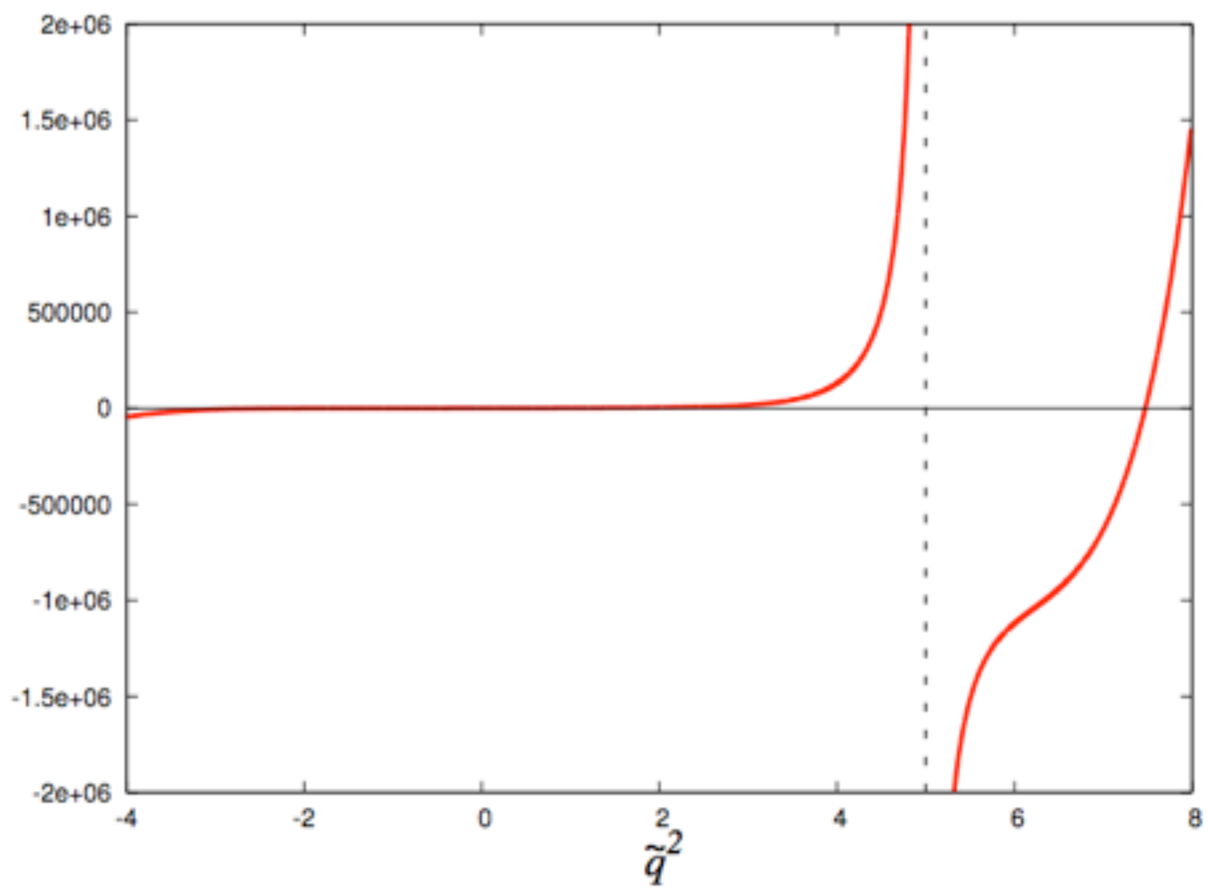
$$S = \infty$$

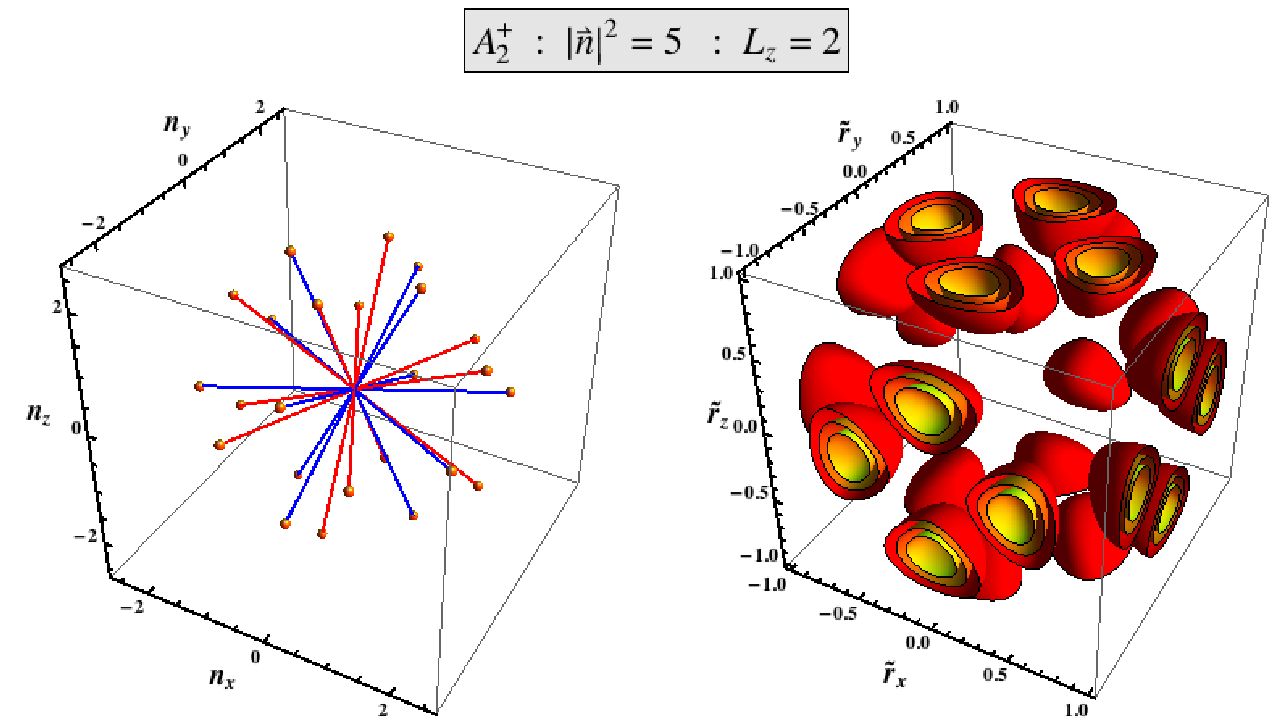
Beyond S-wave

e.g. A_2^+ , L=6

Luscher phenomenology :

Tom Luu *et al*, Phys. Rev. D83 (2011) 114508

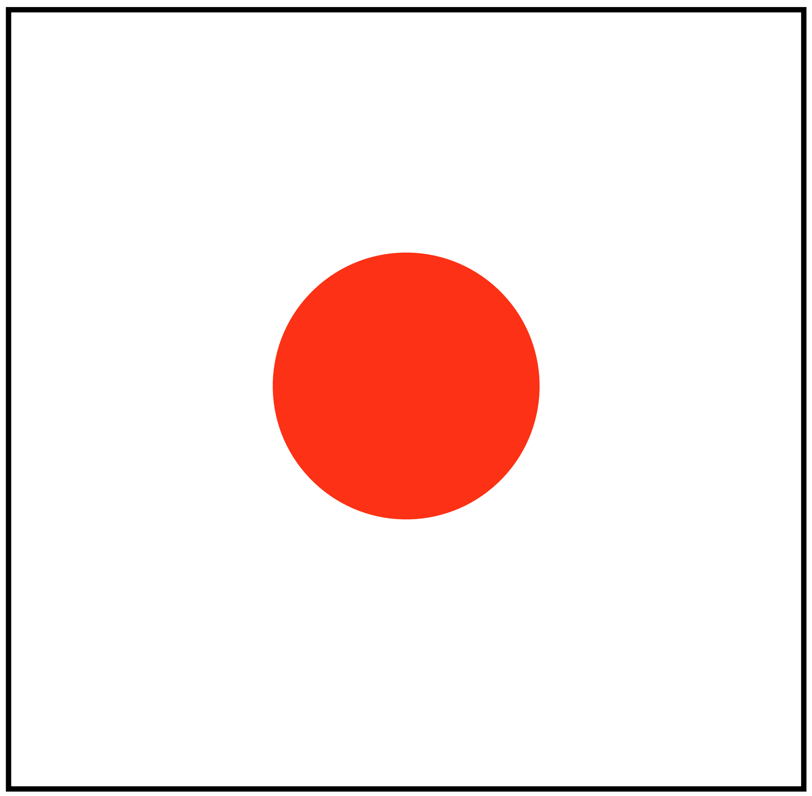




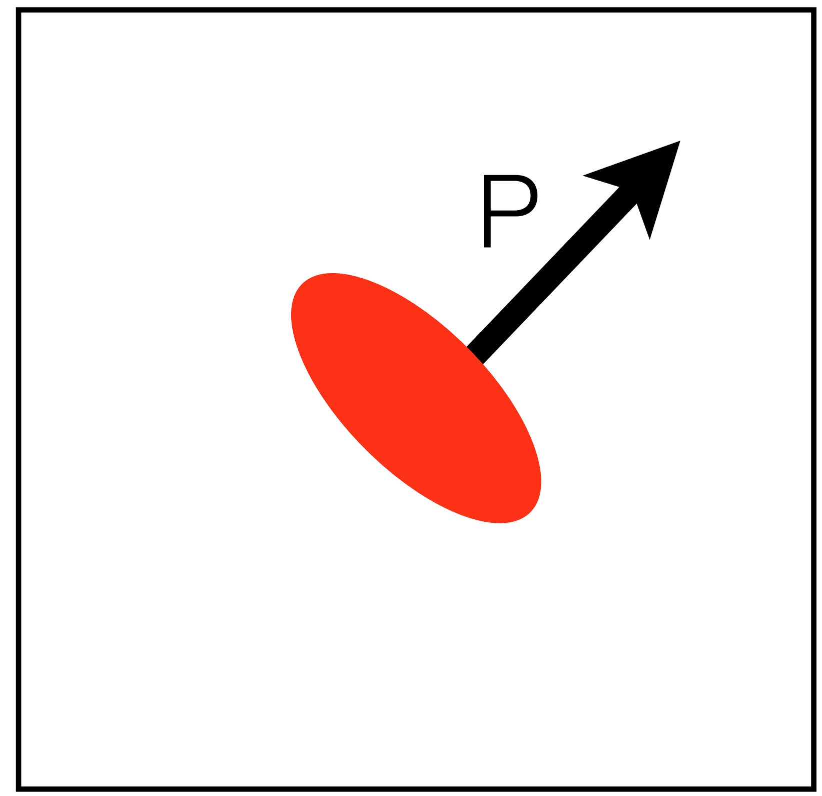
$$
\begin{aligned}
q^{13}\cot\delta_6 \quad &= \left(\frac{2\pi}{L}\right)^{13} \frac{1}{\pi^{3/2}} \times \\
&\left(\tilde{q}^{12}\mathcal{Z}_{0,0}\left(1;\tilde{q}^2\right) + \frac{6\tilde{q}^8\mathcal{Z}_{4,0}\left(1;\tilde{q}^2\right)}{17} - \frac{160\sqrt{13}\tilde{q}^6\mathcal{Z}_{6,0}\left(1;\tilde{q}^2\right)}{323} - \frac{40\tilde{q}^4\mathcal{Z}_{8,0}\left(1;\tilde{q}^2\right)}{19\sqrt{17}} \right. \\
&\left. - \frac{2592\sqrt{21}\tilde{q}^2\mathcal{Z}_{10,0}\left(1;\tilde{q}^2\right)}{7429} + \frac{1980\mathcal{Z}_{12,0}\left(1;\tilde{q}^2\right)}{7429} + \frac{264\sqrt{1001}\mathcal{Z}_{12,4}\left(1;\tilde{q}^2\right)}{7429} \right)
\end{aligned}
$$

Bound-States
At Rest and in Motion

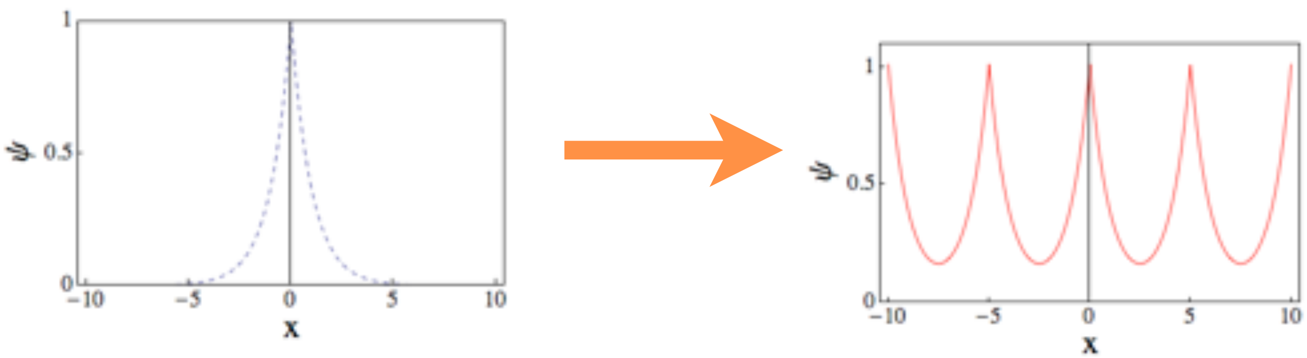
Zohreh Davoudi *et al*, **Phys. Rev. D84 (2011) 114502**



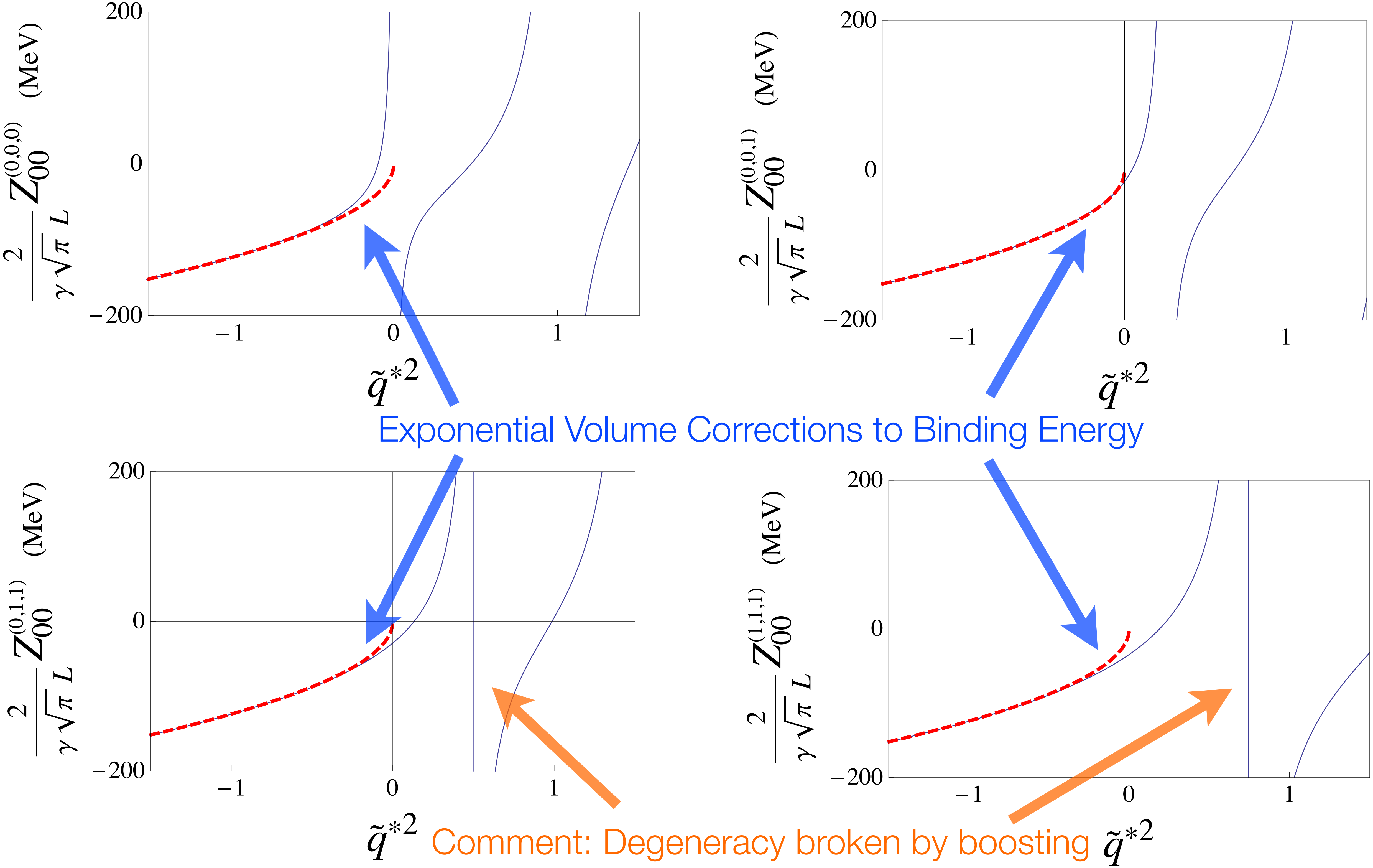
$\sim 3\, e^{-L/R}$



$\sim e^{-L/Rx} + c\, e^{-L/Ry} + d\, e^{-L/Rz}$

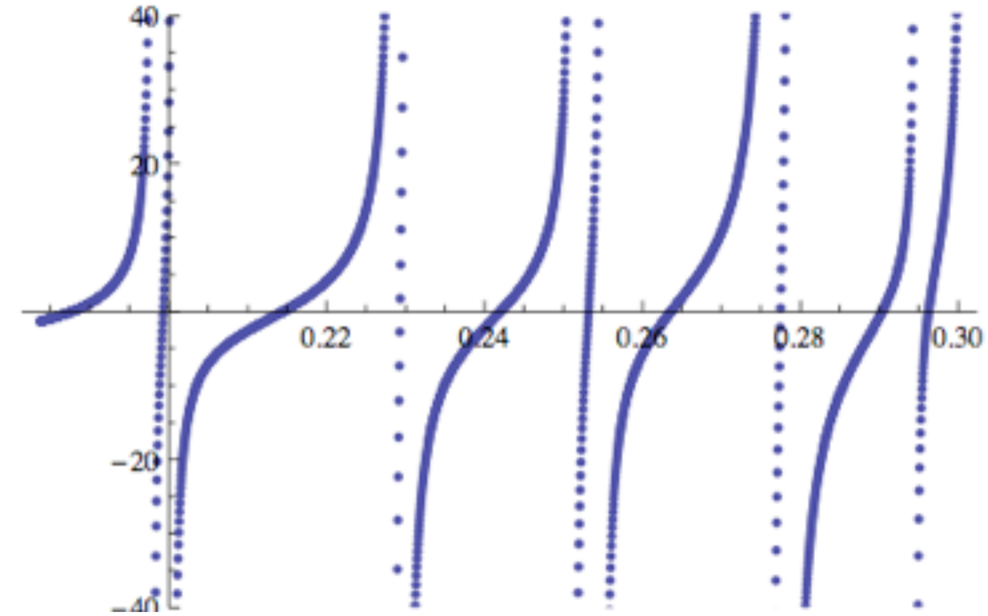


Bound-States in Motion
Equal Masses



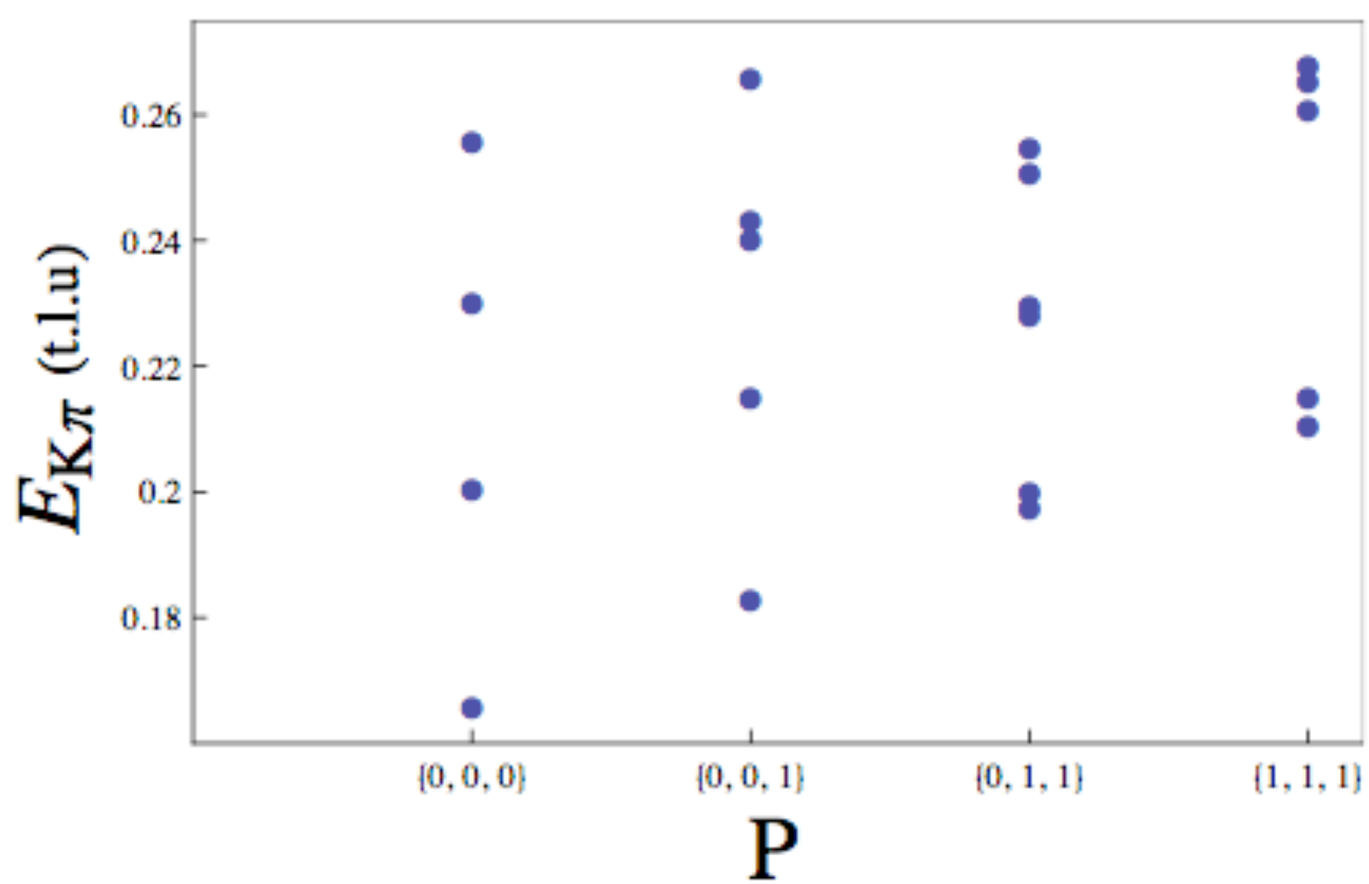
Scattering States in Motion
Nearby States

$$\mathcal{Z}_{00}^{\mathbf{d}=(0,1,1)}$$



non-interacting states

$$\frac{1}{2\gamma^2}\,,\ \frac{1}{2}\,,\ \frac{2\gamma^2+1}{2\gamma^2}\,,\ \frac{3}{2}\,,\ \frac{4\gamma^2+1}{2\gamma^2}\,,\ \frac{4+\gamma^2}{2\gamma^2}\ \cdots\,,$$



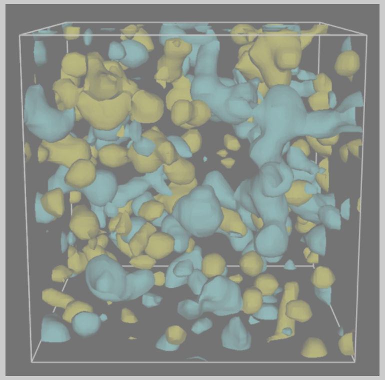
Multi-Volume Anisotropic Clover Study by NPLQCD 2009 - 2011

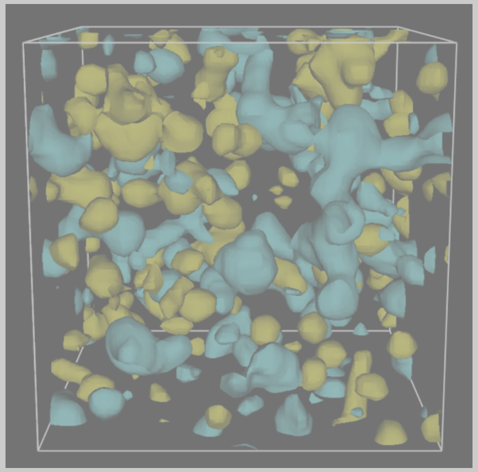
lattice spacing : $b \sim 0.123$ fm

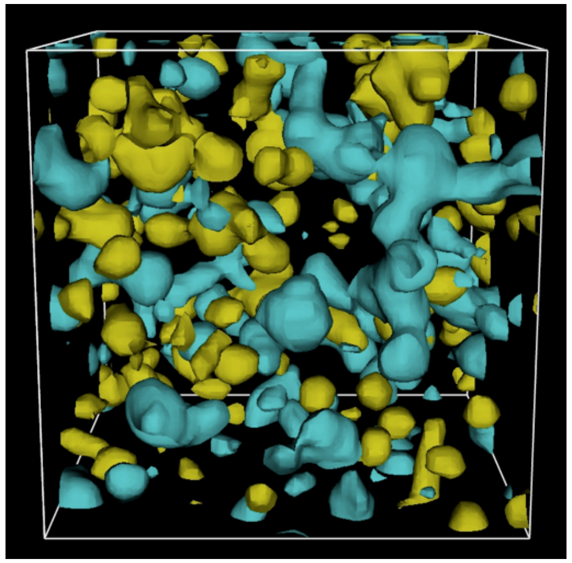
pion mass : $m_\pi \sim 390$ MeV

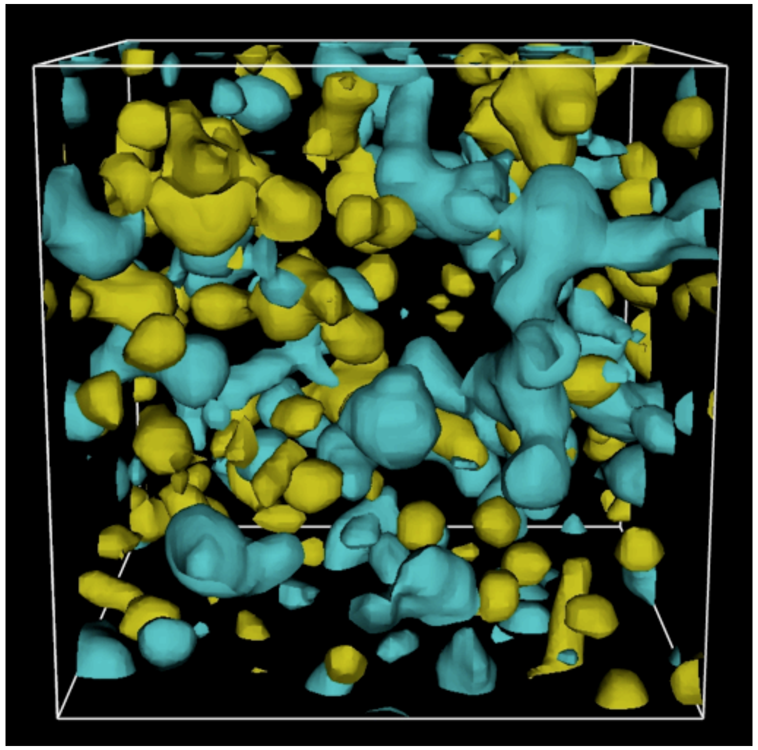
fermion action : Clover

anisotropy : $\xi_t \sim 3.5$


$L \sim 2$ fm


$L \sim 2.5$ fm

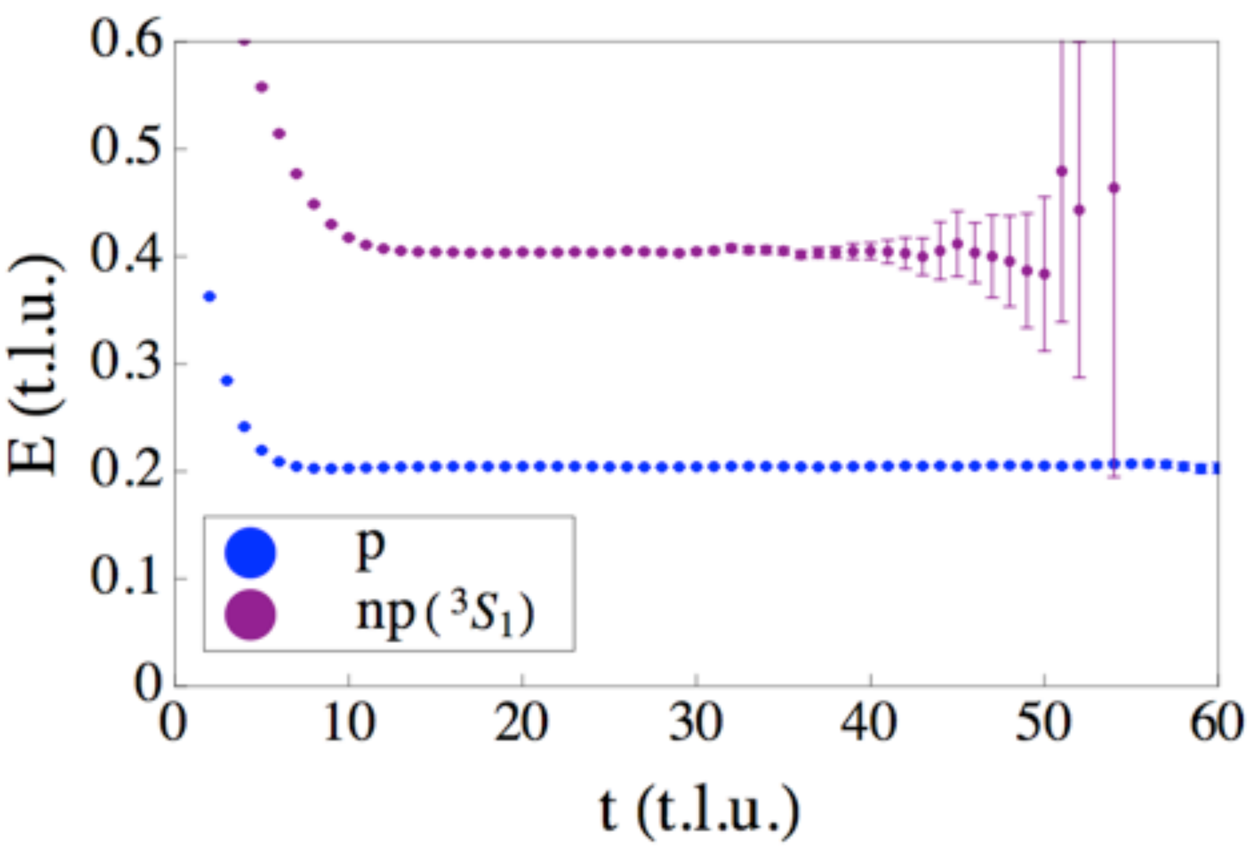

$L \sim 3$ fm


$L \sim 4$ fm

resources $:\sim 80 \times 10^6$ core hrs

$m_\pi L \sim 4 \ , \ 5 \ , \ 6 \ , \ 8 \quad m_\pi T \sim 9 \ , \ 9 \ , \ 9 \ , \ 18$

Anisotropic Clover Multi-Volume Study : Details

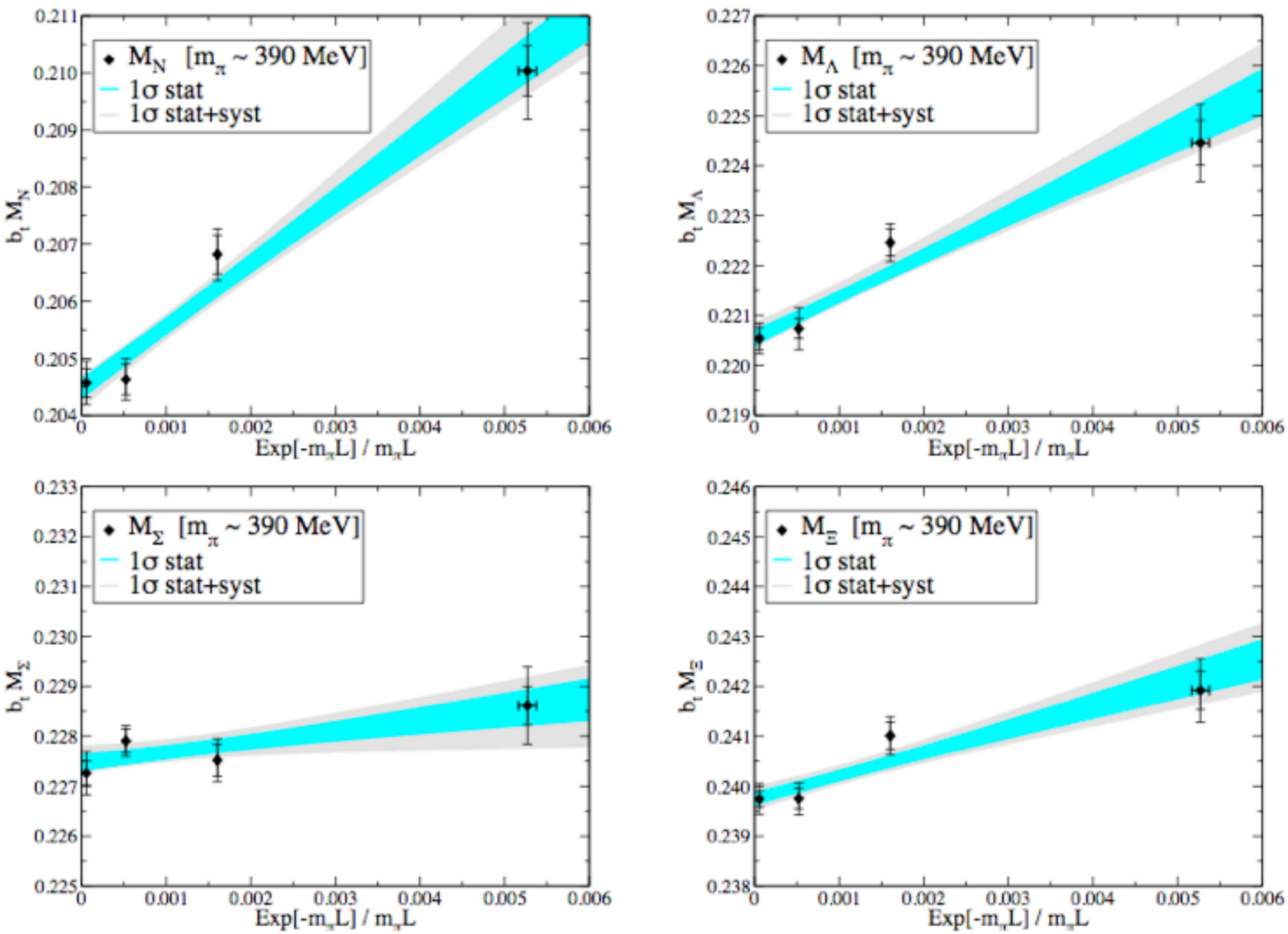


L	cfgs	srcs
24	2215	390,000
32	739	135,000

$L^3 \times T$	$16^3 \times 128$	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 256$	Extrapolation
L (fm)	~ 2.0	$\sim$2.5	$\sim$3.0	$\sim$4.0	∞
$m_\pi L$	3.86	4.82	5.79	7.71	∞
$m_\pi T$	8.82	8.82	8.82	17.64	∞
M_N (t.l.u.)	0.21004(44)(85)	0.20682(34)(45)	0.20463(27)(36)	0.20457(25)(38)	0.20455(19)(17)
M_Λ (t.l.u.)	0.22446(45)(78)	0.22246(27)(38)	0.22074(20)(42)	0.22054(23)(31)	0.22064(15)(19)
M_Σ (t.l.u.)	0.22861(38)(67)	0.22752(32)(43)	0.22791(24)(31)	0.22726(24)(43)	0.22747(17)(19)
M_Ξ (t.l.u.)	0.24192(38)(63)	0.24101(27)(38)	0.23975(20)(32)	0.23974(17)(31)	0.23978(12)(18)

Multi-Volume Study : Finite-Volume Baryon Masses

m L~ 3.9, 4.8, 5.8, 7.7



H-Dibaryon - An Exotic Nucleus
The First QCD Calculation of a Nucleus

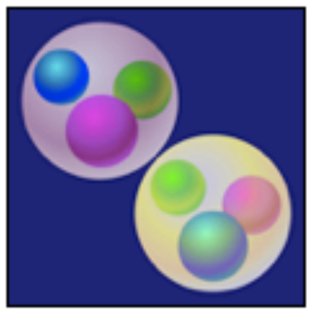
NPLQCD

Hadrons to Atomic nuclei HAL from Lattice QCD

APS » Journals » Physics » Synopses » Binding baryons on the lattice

Physical Review Letters moving physics forward

Binding baryons on the lattice



Credit: Alan Stonebraker

Evidence for a Bound *H* Dibaryon from Lattice QCD

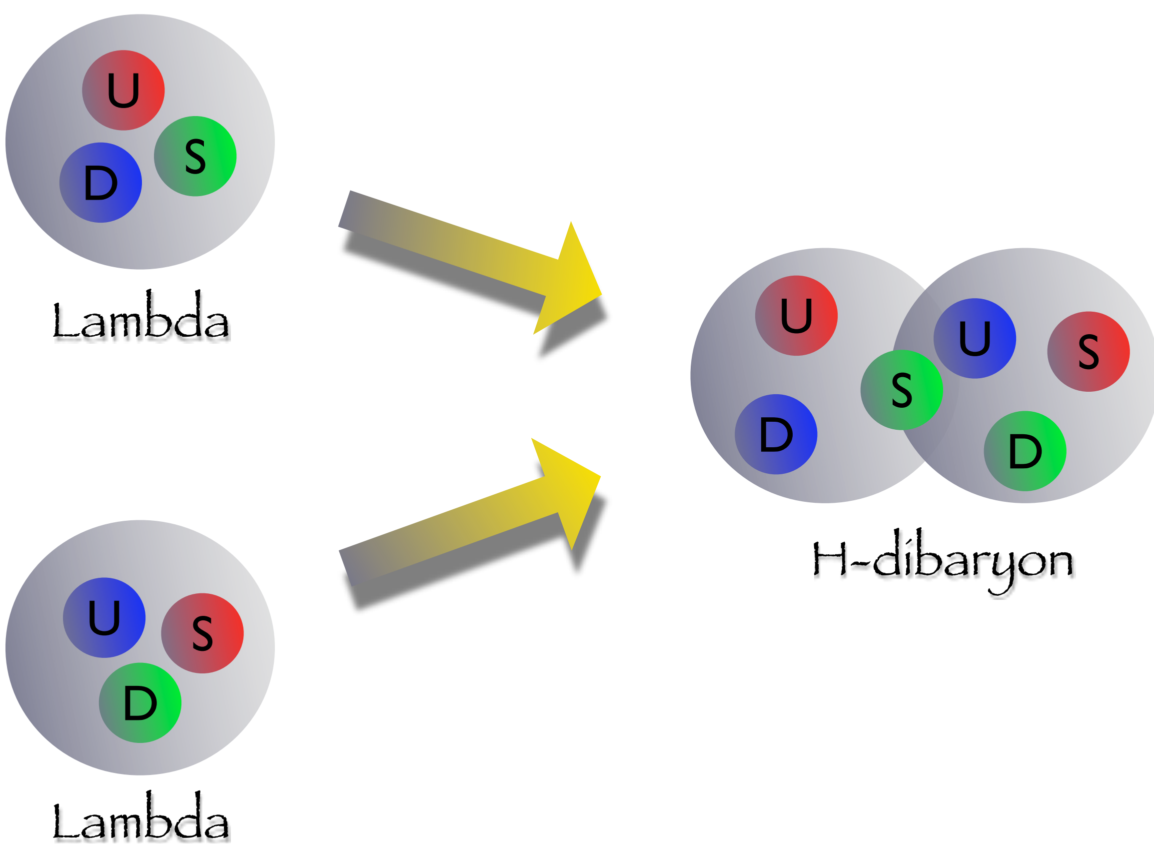
S. R. Beane, E. Chang, W. Detmold, B. Joo, H. W. Lin, T. C. Luu, K. Orginos, A. Parreño, M. J. Savage, A. Torok, and A. Walker-Loud (NPLQCD Collaboration)

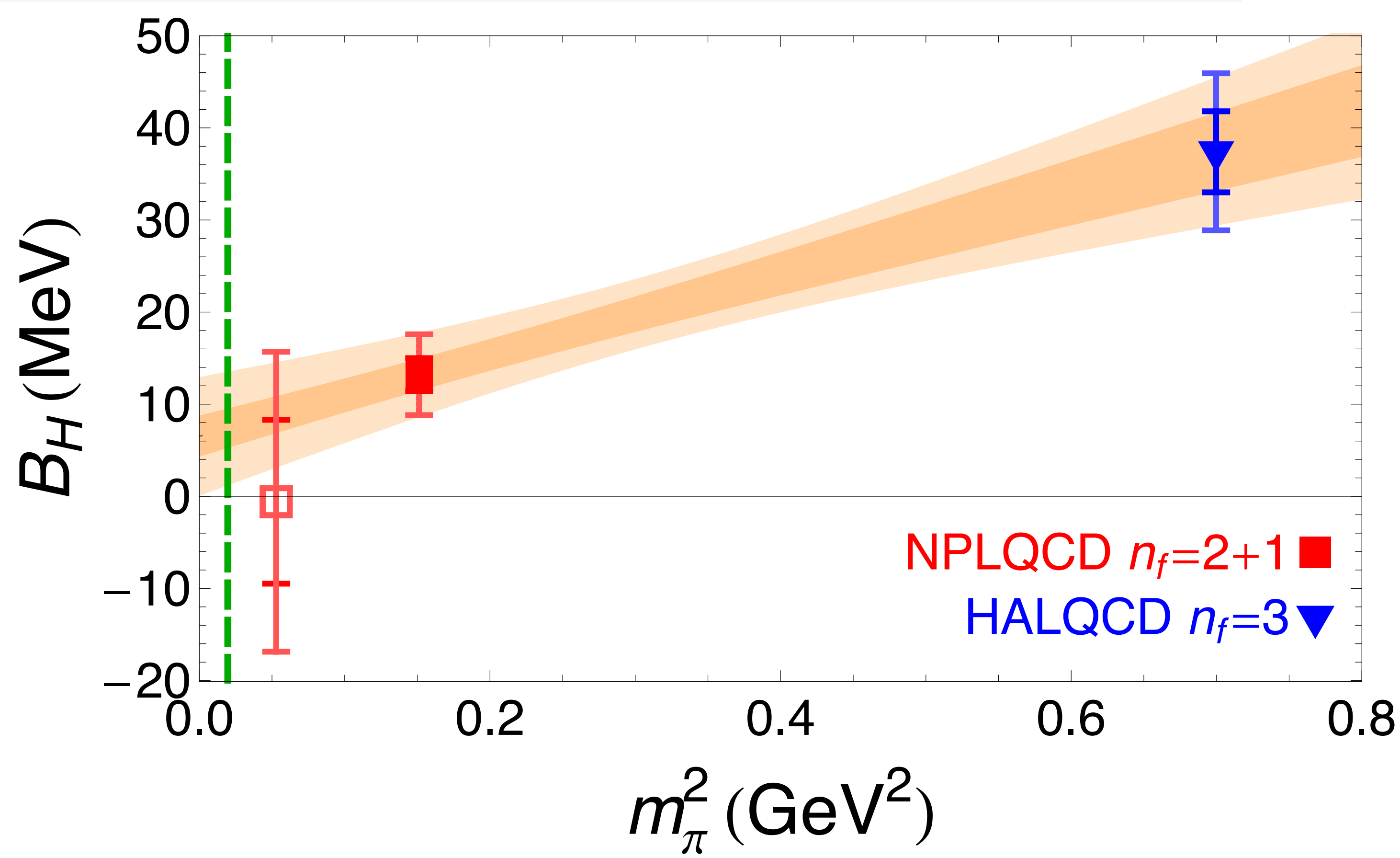
Phys. Rev. Lett. **106**, 162001 (Published April 20, 2011)

Bound *H* Dibaryon in Flavor SU(3) Limit of Lattice QCD

Takashi Inoue, Noriyoshi Ishii, Sinya Aoki, Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda, Keiko Murano, Hidekatsu Nemura, and Kenji Sasaki (HAL QCD Collaboration)

Phys. Rev. Lett. **106**, 162002 (Published April 20, 2011)

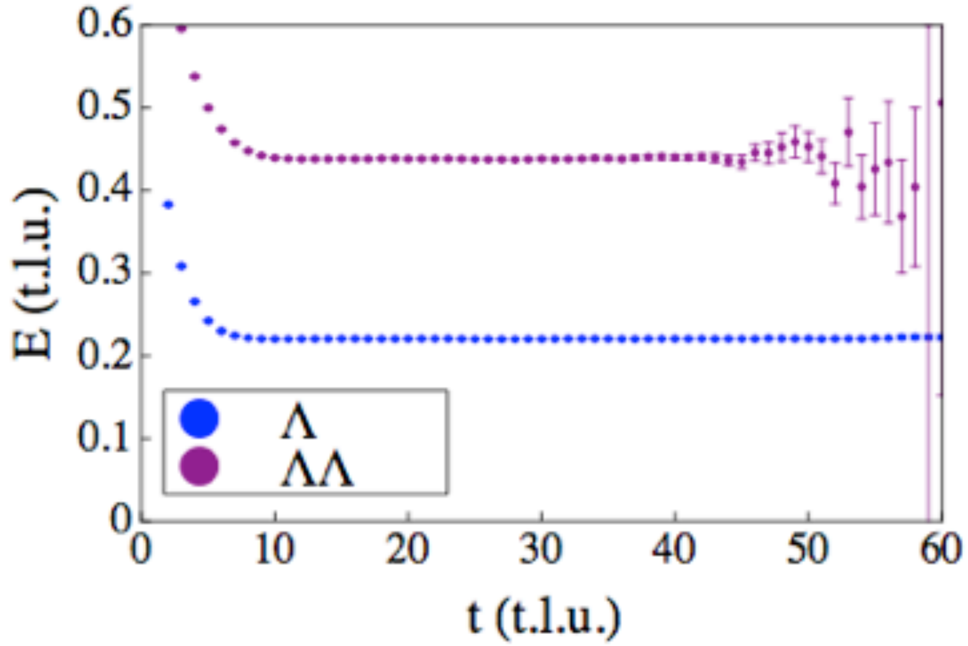


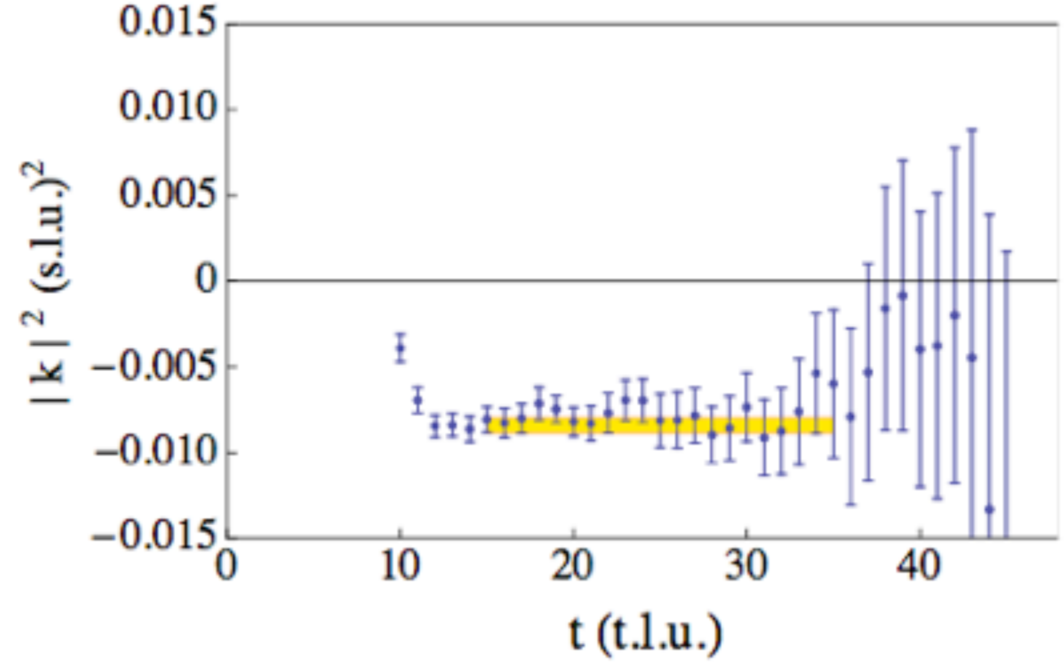


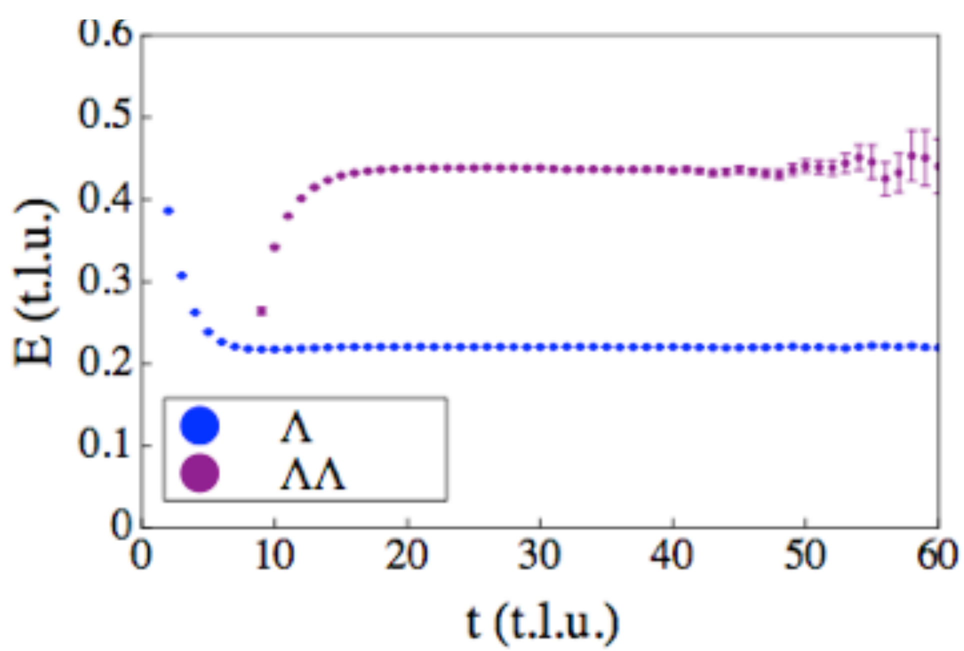
10^8 core-hrs, 1 PB data

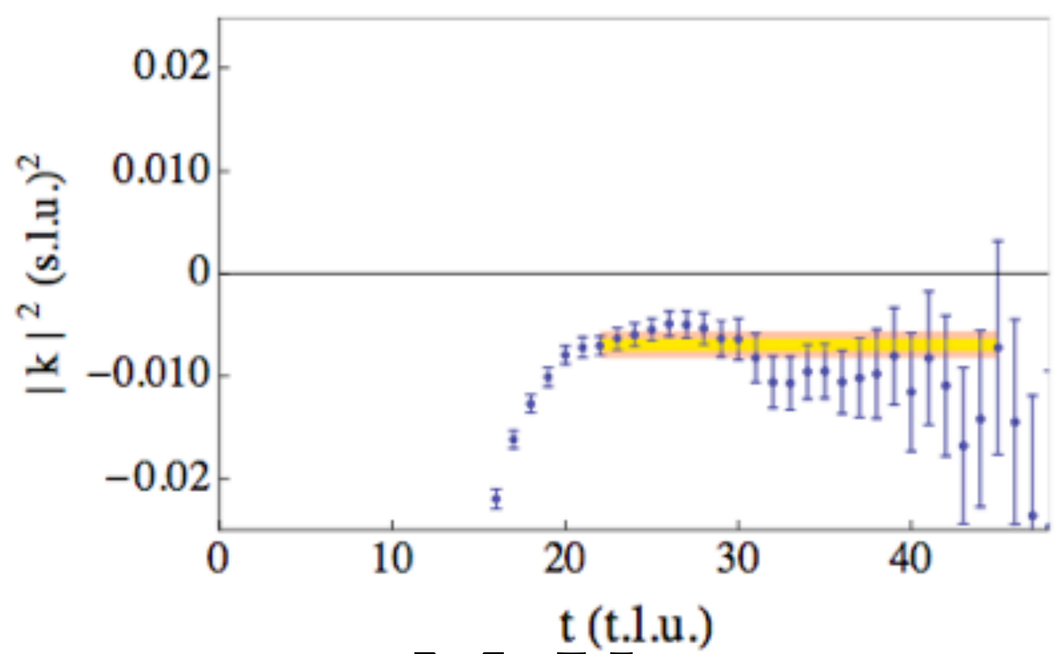
H-Dibaryon
An Exotic Nucleus

H-dibaryon : I=J=0, s=-2 $\Lambda\Lambda - \Sigma\Sigma - N\Xi$



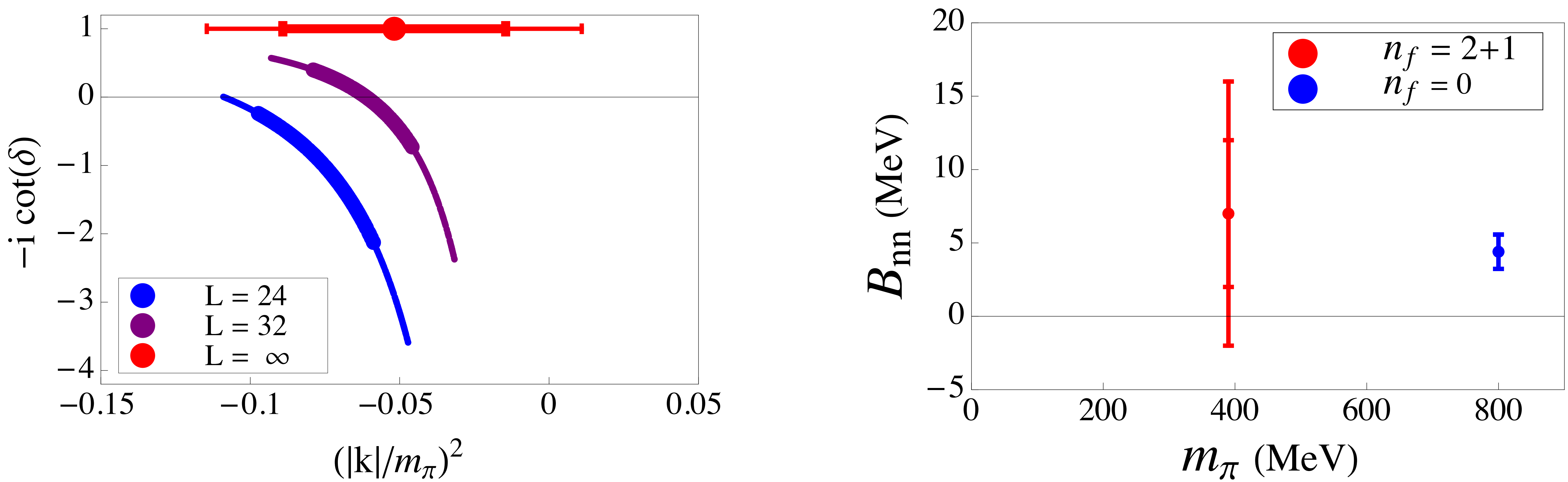




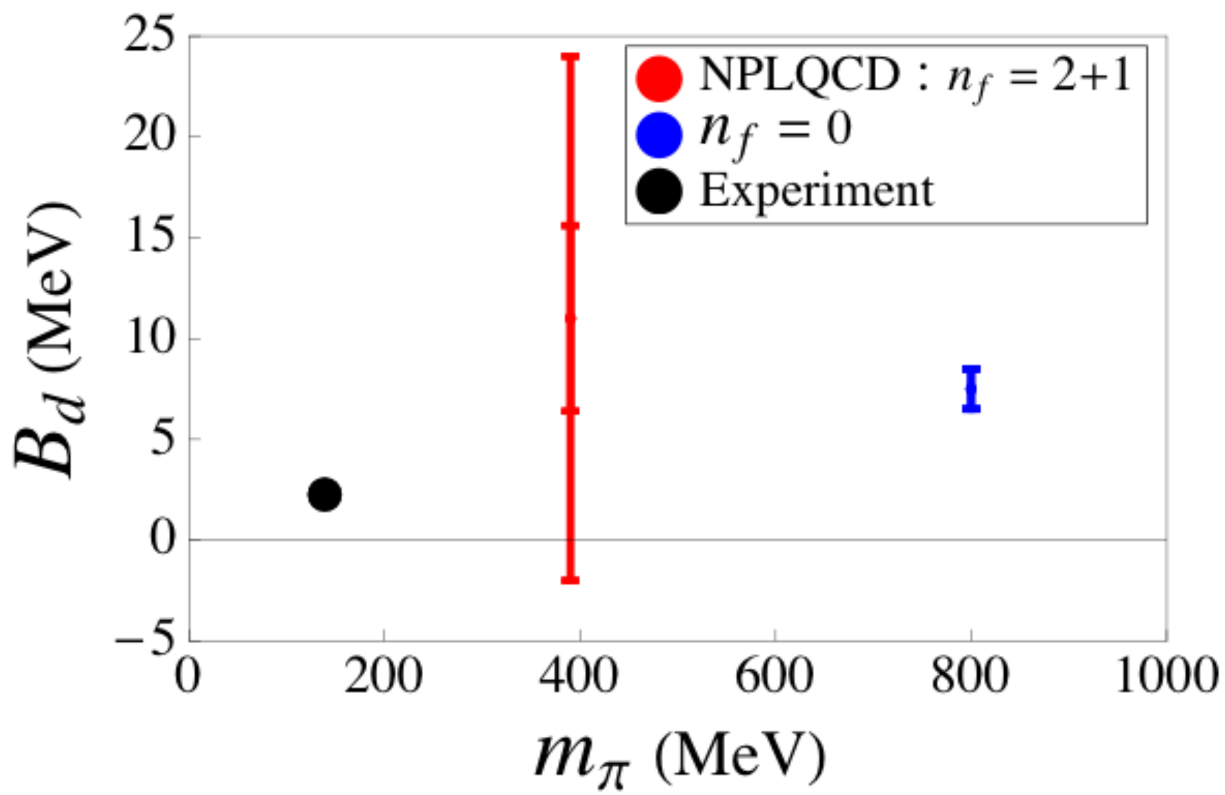


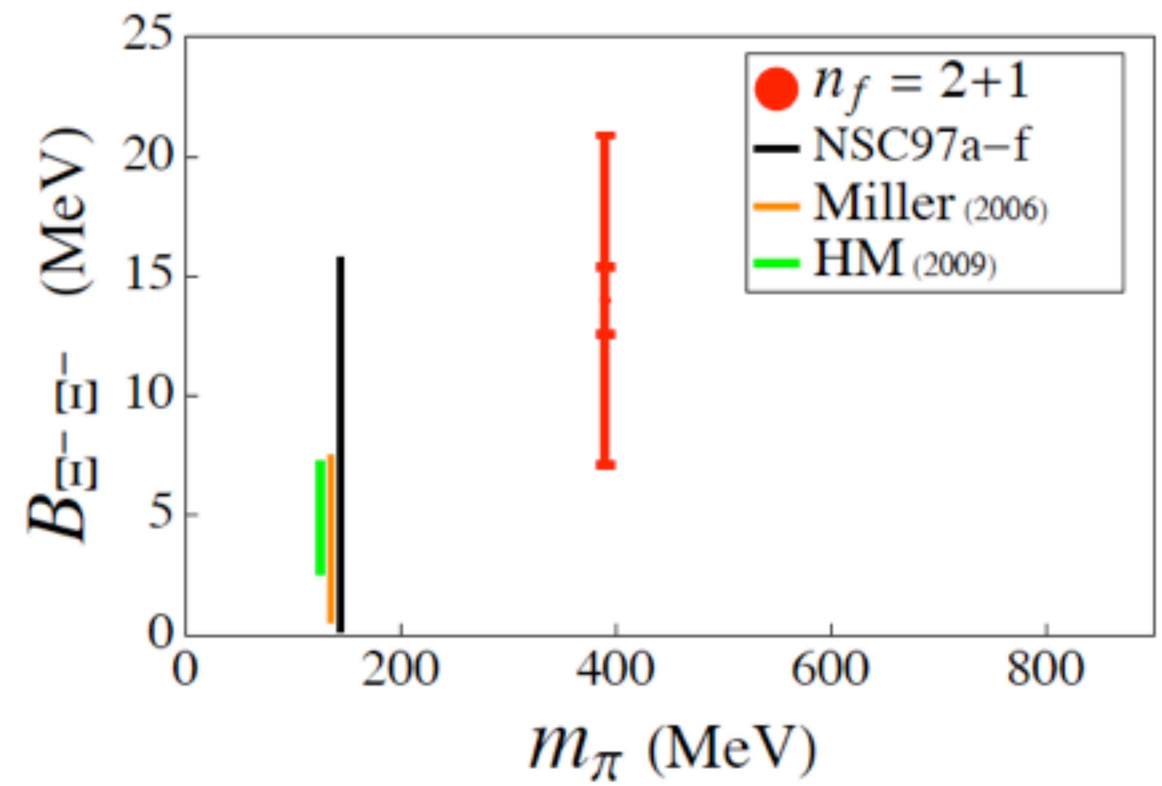
pion mass : $m_\pi \sim 390$ MeV

Two-Body Bound States (I)



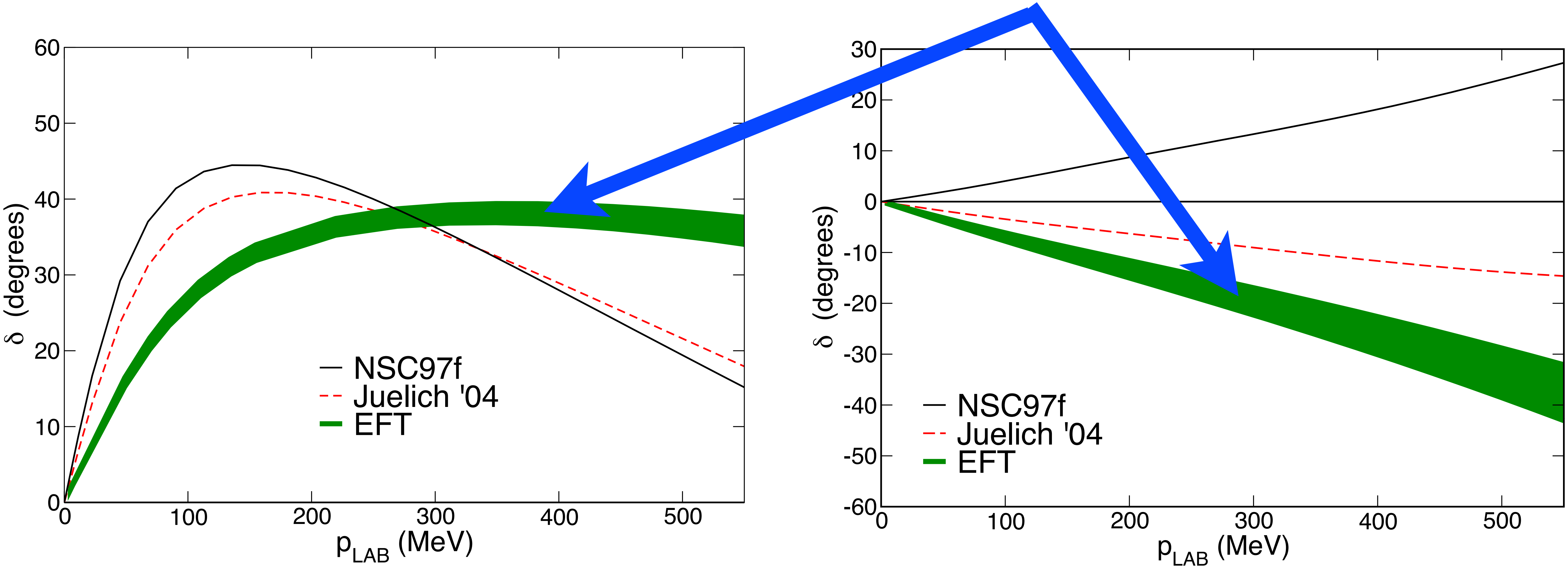
Two-Body Bound States (II)





Hyperon Nucleon Interactions

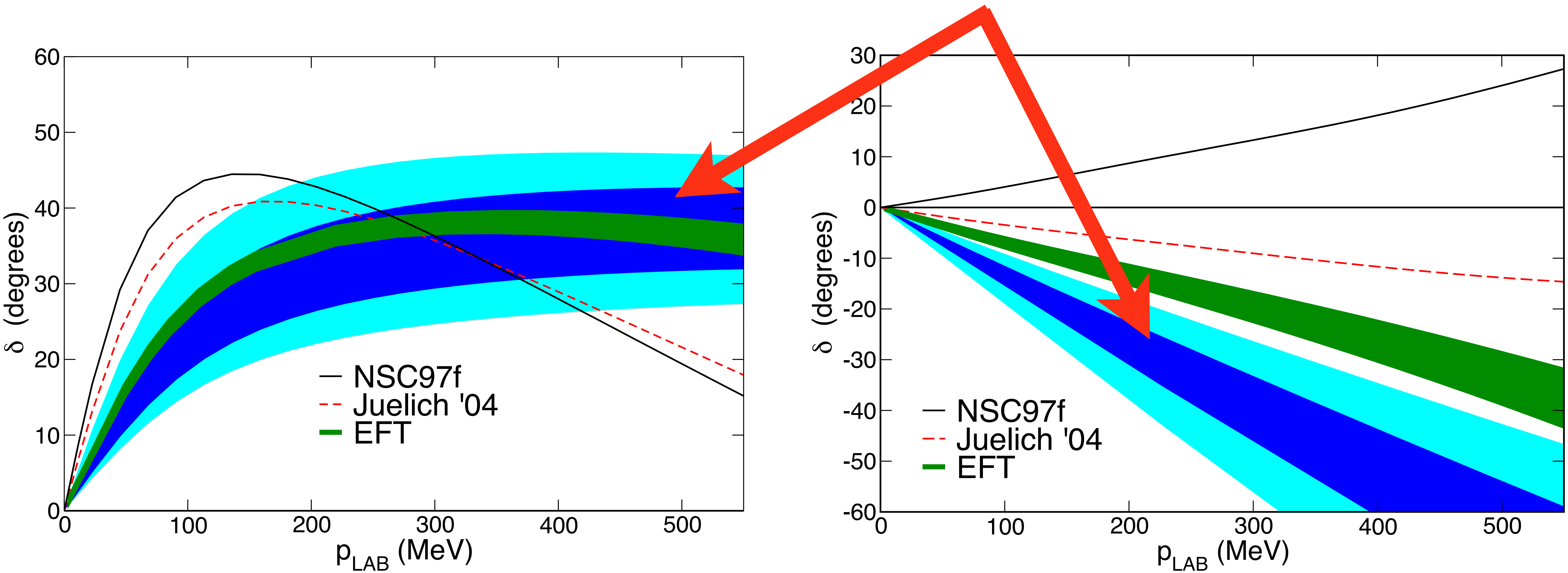
Meissner+Haidenbauer - Experiment + YN-EFT (LO)



Cancellation between channels in dense matter
energy-shift of hyperon

Hyperon Nucleon Interactions

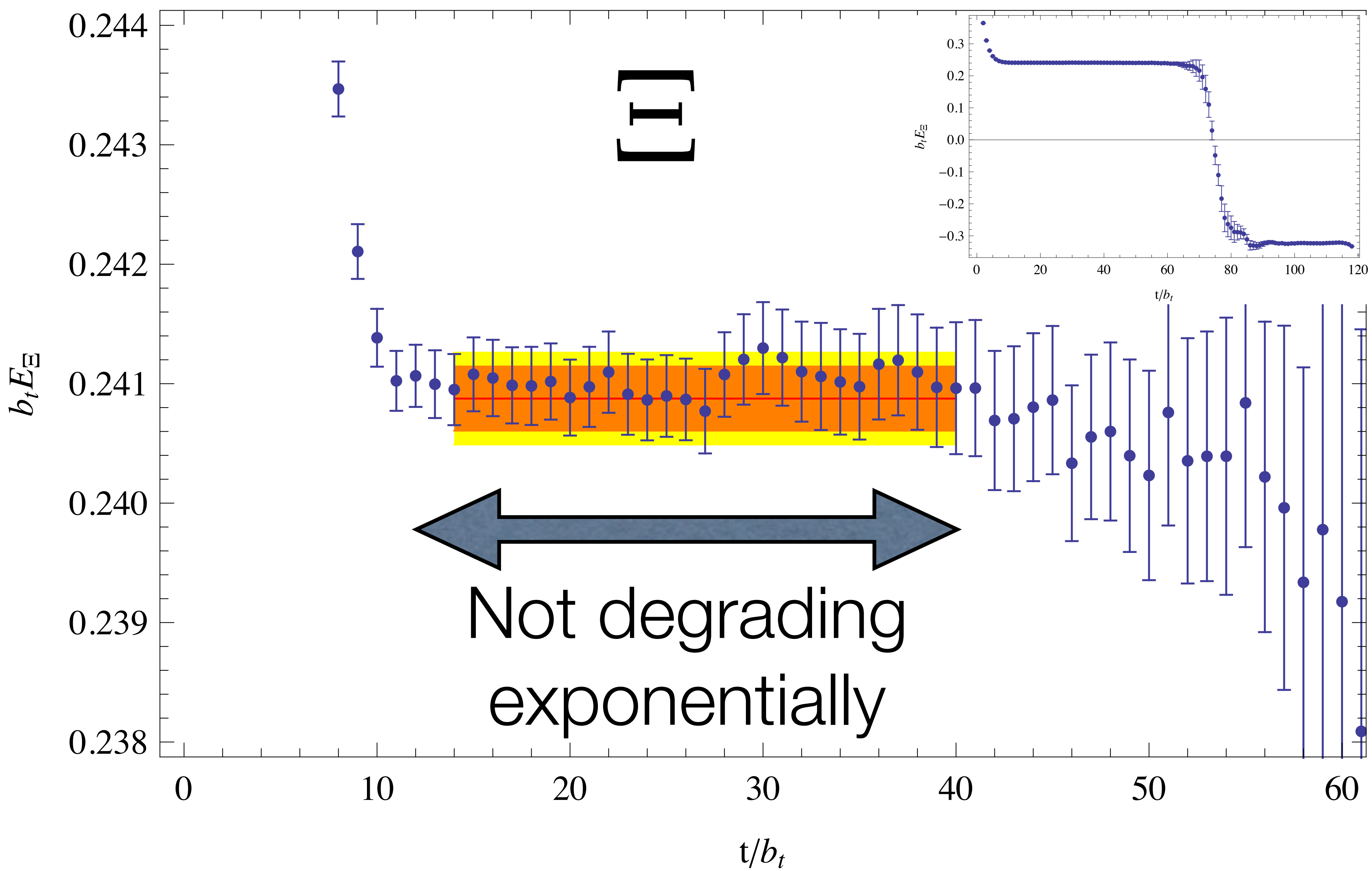
NPLQCD - Lattice QCD + YN-EFT (LO)



Cancellation between channels in dense matter energy-shift of hyperon

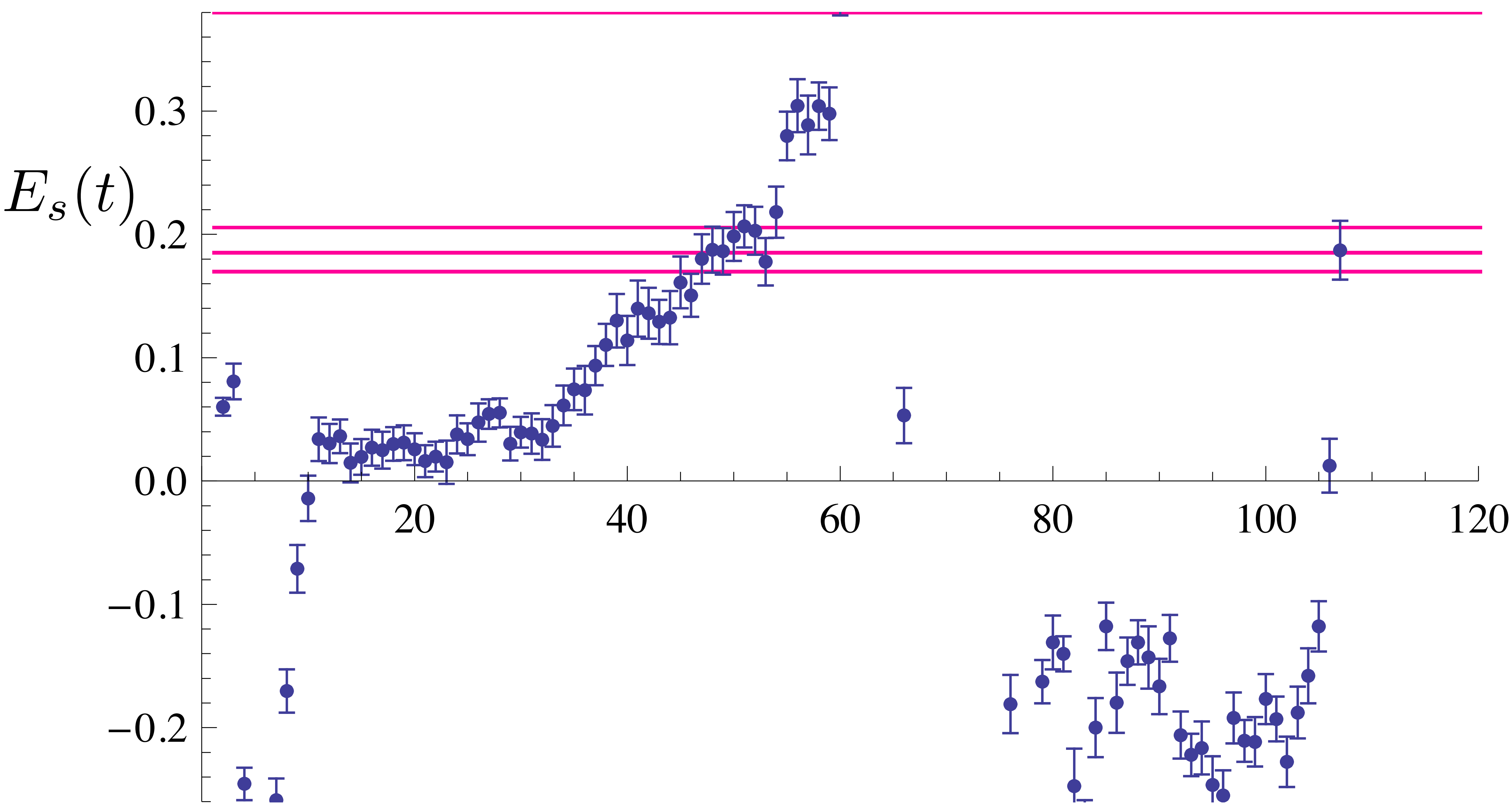
The Golden Window

b~0.123 fm
20x20x20 x 128
pion ~ 390 MeV



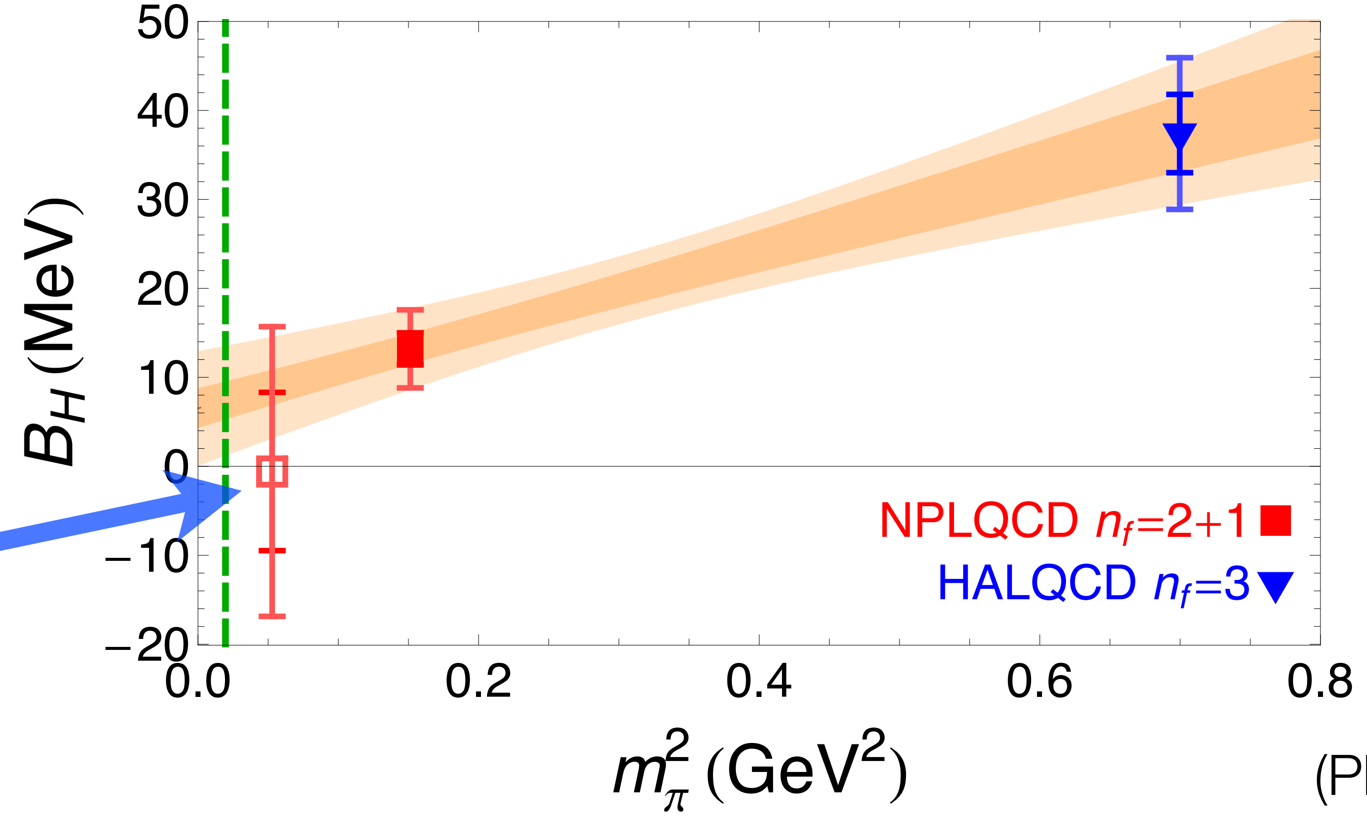
Signal-to-Noise Degradation

$$\frac{\sigma}{\overline{x}} = \alpha \exp\left(E_s(t) t\right)$$



From Anisotropic to Isotropic Clover Configurations

230 MeV,
32^3x256,
mL=4.6 > γL

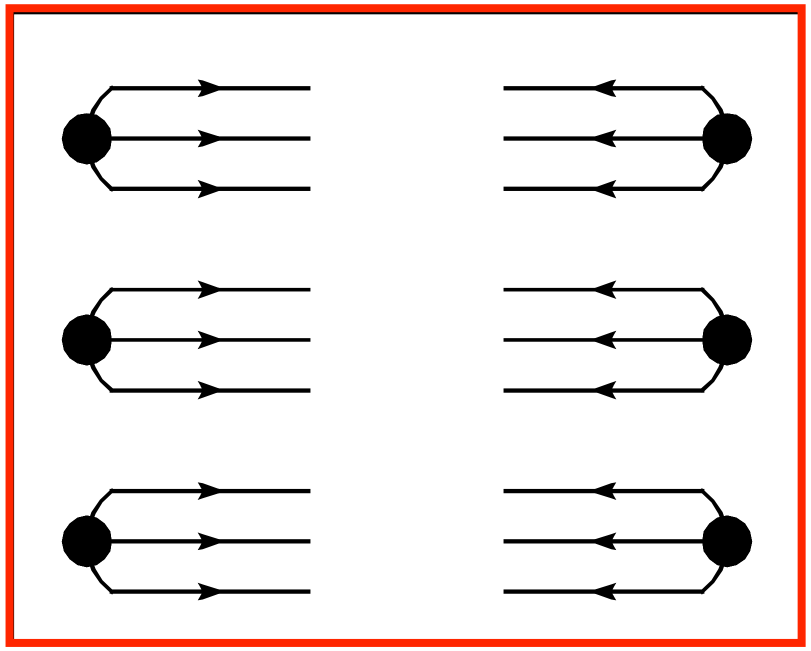


(Physical strange-quark mass)

- Binding becoming smaller at smaller pion mass
  - FV effects getting larger for fixed volume
  - Need larger volumes and better statistics
- Don’t have resources to complete in reasonable time-frame
  - Can’t compete with K-machine head-on
- Only need 1 or 2 levels cleanly - not 10 (at present)

Many Nucleons (Baryons)

Large number of Wick contractions



Proton : N $^{\text{cont}}$ = 2

^{235}U : N $^{\text{cont}}$ = 10^{1494}

$$N_{\text{cont.}} = u!d!s! \quad (\text{Naive})$$
$$= (A+Z)!(2A-Z)!s!$$
$$\sim A^3 \; (\text{Kaplan})$$

Symmetries provide significant reduction

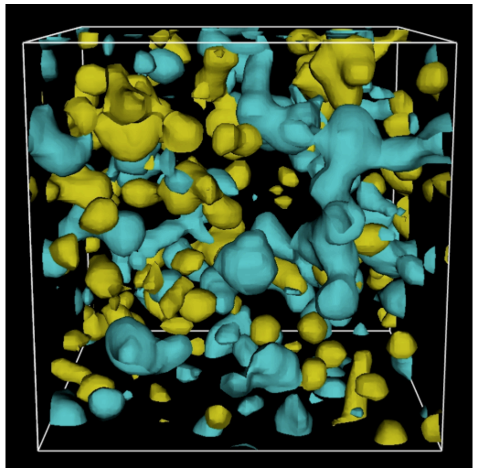
$$^{3}\text{He} \; : \; 2880 \rightarrow 93$$

Recursion Relations

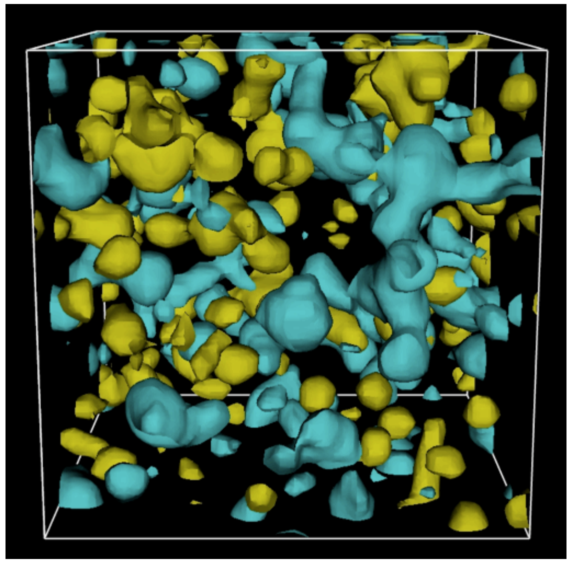
Other Tricks

Multi-Volume Isotropic Clover Study by NPLQCD
In Search of Nuclei

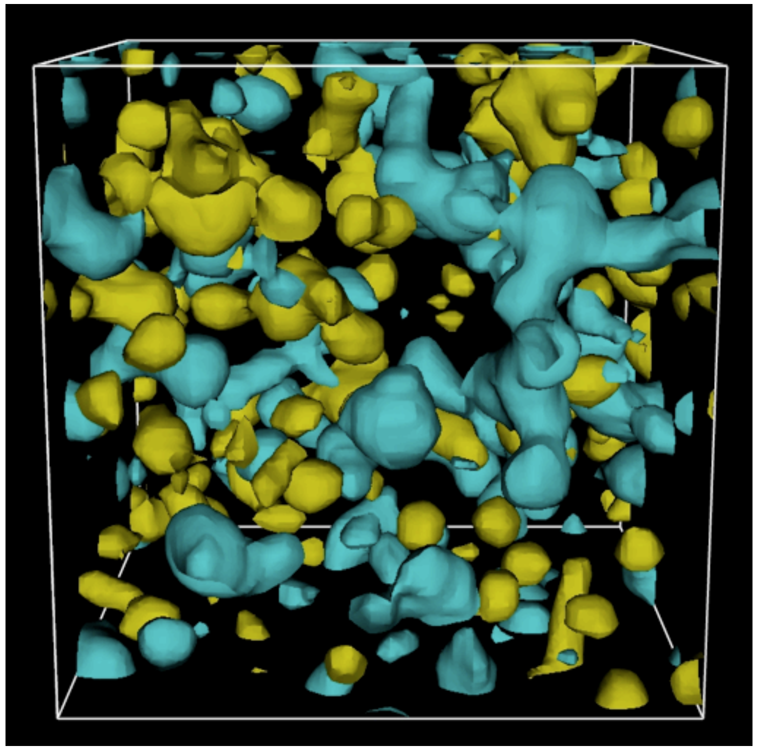
- b ~ 0.145 fm
- SU(3) Symmetry Limit
- m ~ 800 MeV
- Isotropic Clover



L~3.4 fm



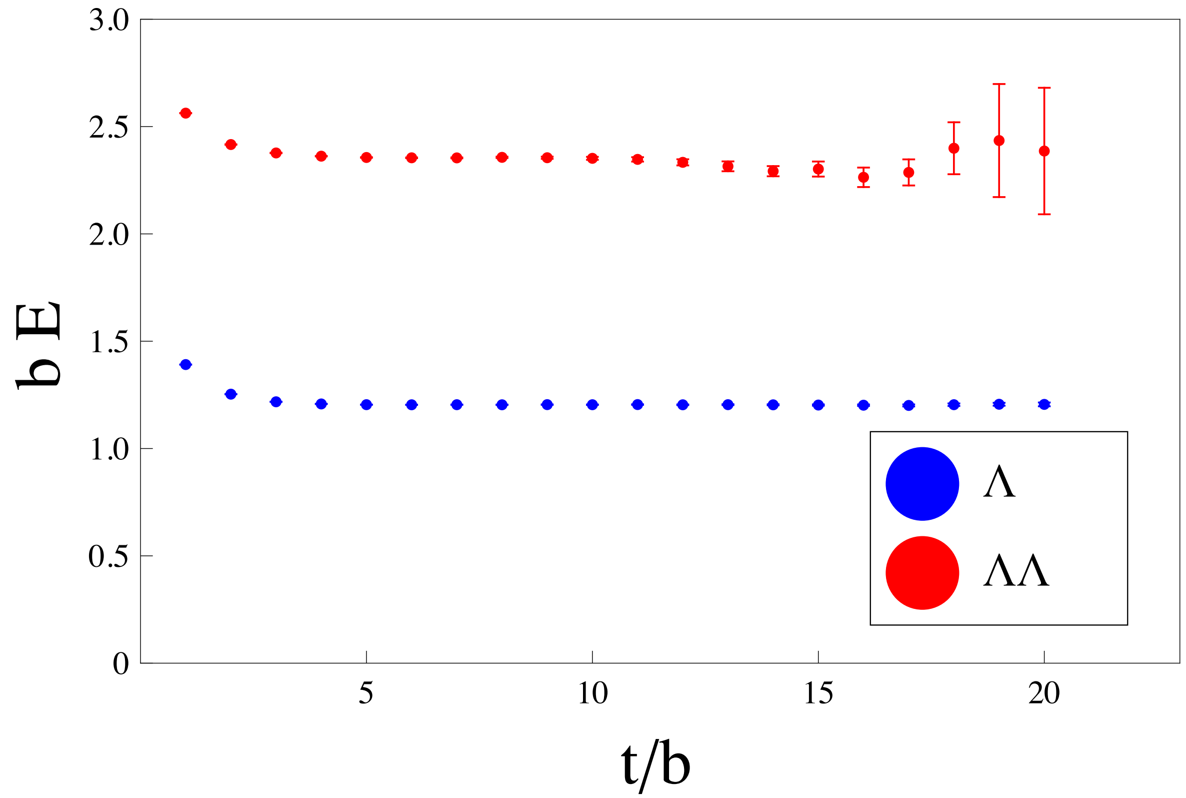
L~4.5 fm



L~6.7 fm

m L ~ 14.3 , 19.0 , 28.5

Isotropic Clover Multi-Volume Study : Details



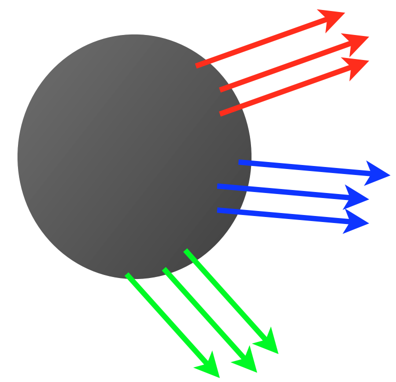
L	cfgs	srcs
24	3822	183 456
32	3050	73 200
48	1212	38784

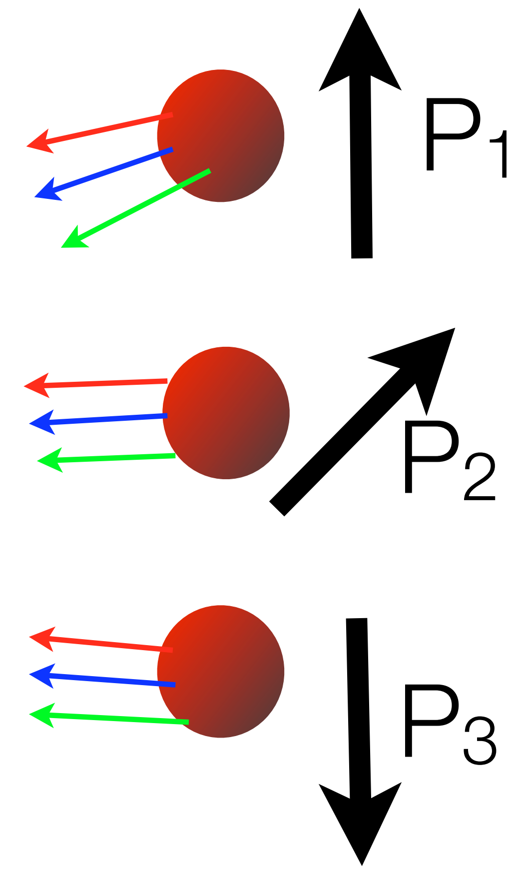
ensemble	$\lvert\mathbf{n}\rvert = 0$	$\lvert\mathbf{n}\rvert^2 = 1$	$\lvert\mathbf{n}\rvert^2 = 2$	$\lvert\mathbf{n}\rvert^2 = 3$	$\lvert\mathbf{n}\rvert^2 = 4$	$\lvert\mathbf{n}\rvert^2 = 5$
$24^3 \times 48$	1.2028(10)(02)	1.2286(11)(01)	1.2538(13)(02)	1.2787(16)(02)	1.3020(20)(03)	1.3256(23)(03)
$32^3 \times 48$	1.2040(15)(06)	1.2188(15)(5)	1.2336(16)(6)	1.2484(17)(8)	1.2624(19)(8)	1.2770(20)(10)
$48^3 \times 64$	1.2022(21)(05)	1.2087(22)(06)	1.2152(23)(06)	1.2216(24)(06)	1.2281(25)(11)	1.2341(26)(12)
$L = \infty$	1.2034(13)(03)					

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

A-body SU(3) Structure

- Local operators
- Spin Projection





$$\mathbf{8} \otimes \mathbf{8} \;=\; \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{1}$$

A=2

$$(\;\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}\;)_{J^\pi=1/2^+} \;\rightarrow\; \mathbf{35} \oplus \overline{\mathbf{35}} \oplus \mathbf{27} \oplus \mathbf{8}$$

$$(\;\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}\;)_{J^\pi=3/2^+} \;\rightarrow\; \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \;\mathbf{1}$$

A=3

$$(\;\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}\;)_{J^\pi=0^+} \;\rightarrow\; \mathbf{1} \oplus \mathbf{27} \oplus \overline{\mathbf{28}}$$

$$(\;\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}\;)_{J^\pi=1^+} \;\rightarrow\; \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \overline{\mathbf{35}}$$

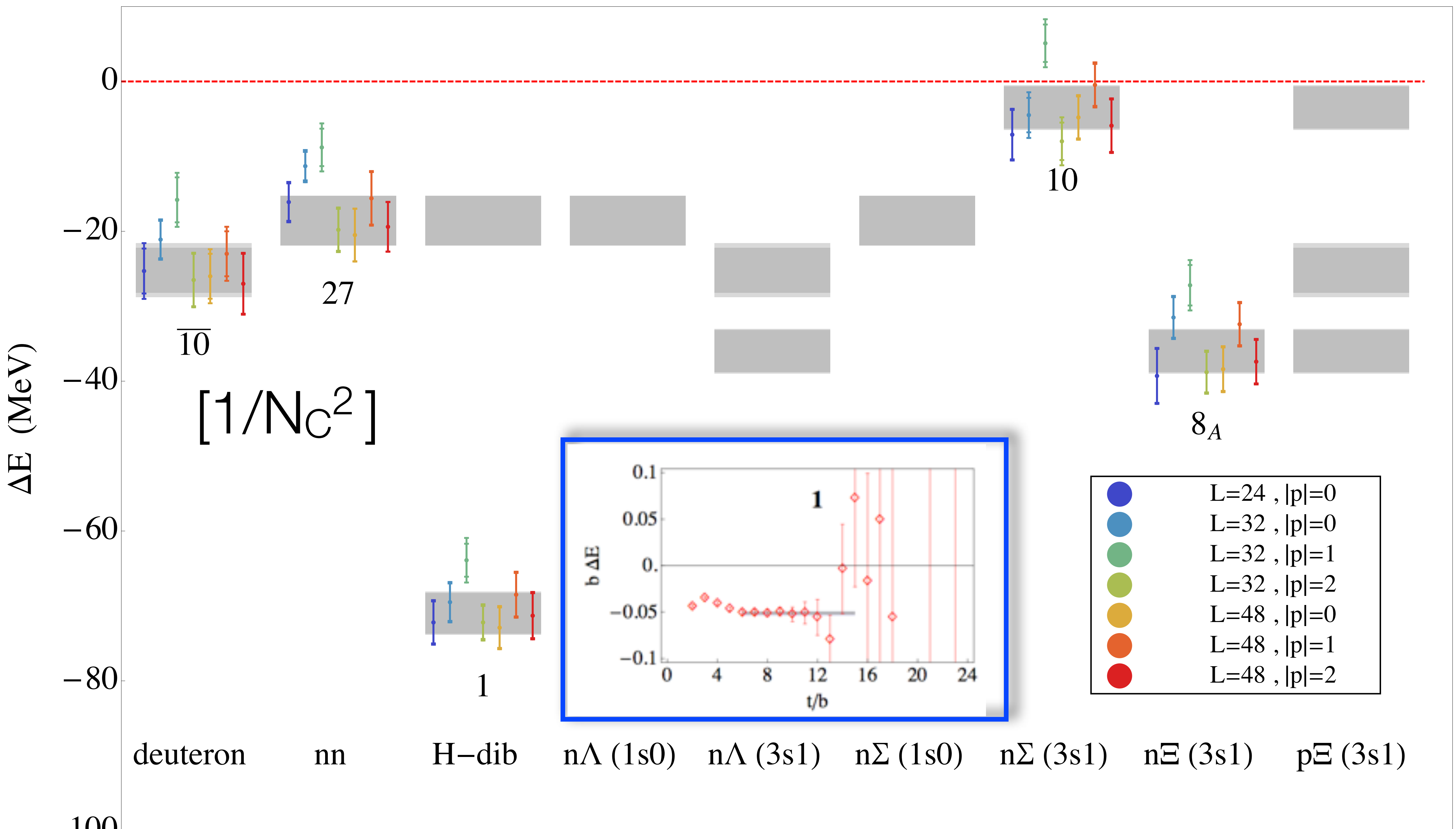
$$(\;\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}\;)_{J^\pi=2^+} \;\rightarrow\; \mathbf{8} \oplus \mathbf{27} \quad ,$$

A=4

A-body SU(3) Structure

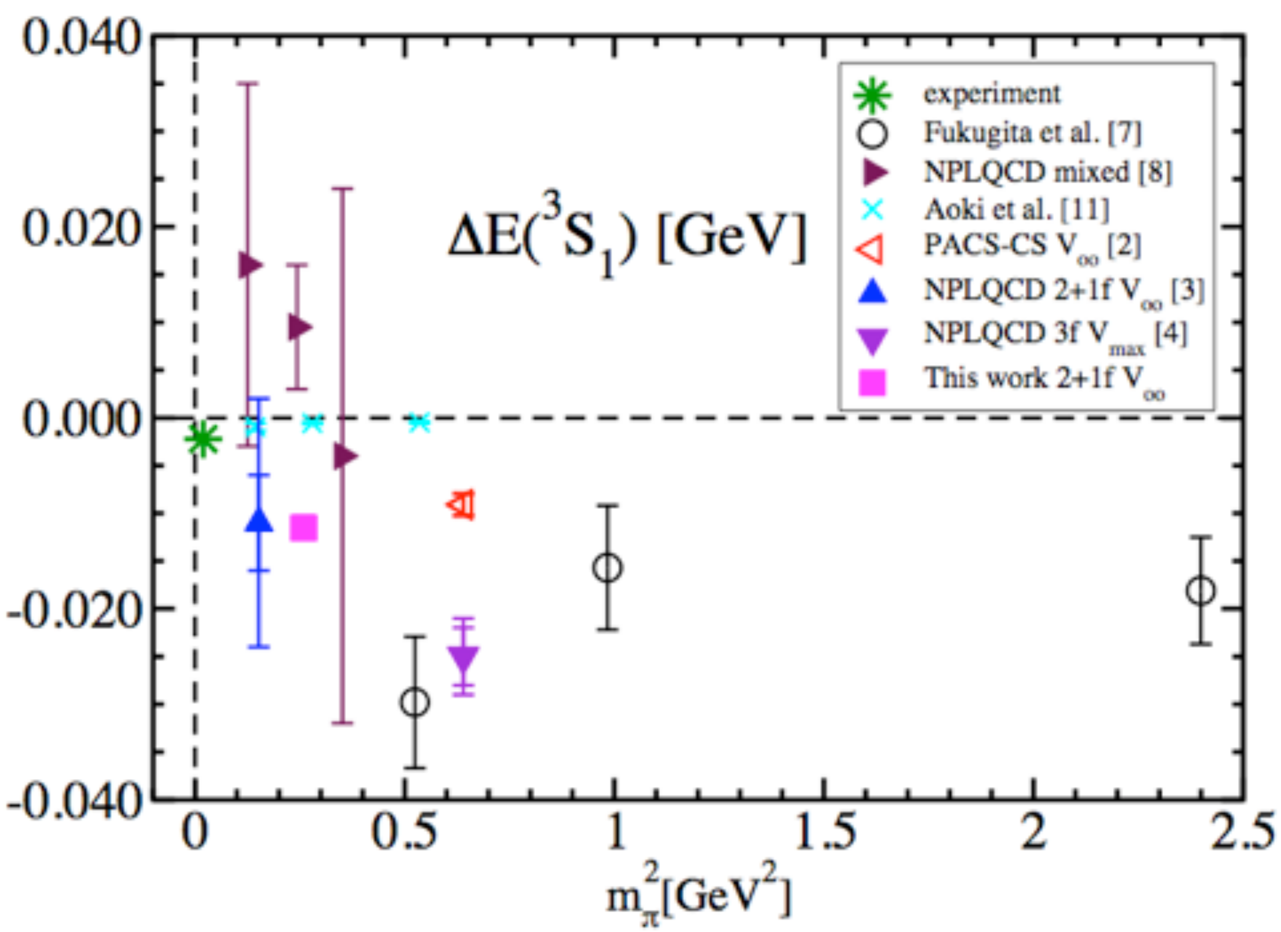
Label	A	s	I	J^π	Local SU(3) irreps	This work
N	1	0	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Λ	1	-1	0	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Σ	1	-1	1	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Ξ	1	-2	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
d	2	0	0	1^+	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
nn	2	0	1	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	1^+	$\mathbf{8}_A$, $\overline{\mathbf{10}}$	—
$n\Sigma$	2	-1	3/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Sigma$	2	-1	3/2	1^+	$\mathbf{10}$	$\mathbf{10}$
$n\Xi$	2	-2	0	1^+	$\mathbf{8}_A$	$\mathbf{8}_A$
$n\Xi$	2	-2	1	1^+	$\mathbf{8}_A$, $\mathbf{10}$, $\overline{\mathbf{10}}$	—
H	2	-2	0	0^+	$\mathbf{1}$, $\mathbf{27}$	$\mathbf{1}$, $\mathbf{27}$
^{3}H, ^{3}He	3	0	1/2	$1/2^+$	$\overline{\mathbf{35}}$	$\overline{\mathbf{35}}$
$^{3}_{\Lambda}$H$(1/2^+)$	3	-1	0	$1/2^+$	$\overline{\mathbf{35}}$	—
$^{3}_{\Lambda}$H$(3/2^+)$	3	-1	0	$3/2^+$	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
$^{3}_{\Lambda}$He, $^{3}_{\Lambda}\tilde{\text{H}}$, $nn\Lambda$	3	-1	1	$1/2^+$	$\mathbf{27}$, $\overline{\mathbf{35}}$	$\mathbf{27}$, $\overline{\mathbf{35}}$
$^{3}_{\Sigma}$He	3	-1	1	$3/2^+$	$\mathbf{27}$	$\mathbf{27}$
^{4}He	4	0	0	0^+	$\overline{\mathbf{28}}$	$\overline{\mathbf{28}}$
$^{4}_{\Lambda}$He, $^{4}_{\Lambda}$H	4	-1	1/2	0^+	$\overline{\mathbf{28}}$	—
$^{4}_{\Lambda\Lambda}$He	4	-2	0	0^+	$\mathbf{27}$, $\overline{\mathbf{28}}$	$\mathbf{27}$, $\overline{\mathbf{28}}$
$\Lambda\Xi^0 pnn$	5	-3	0	$3/2^+$	$\overline{\mathbf{10}}$ + ...	$\overline{\mathbf{10}}$

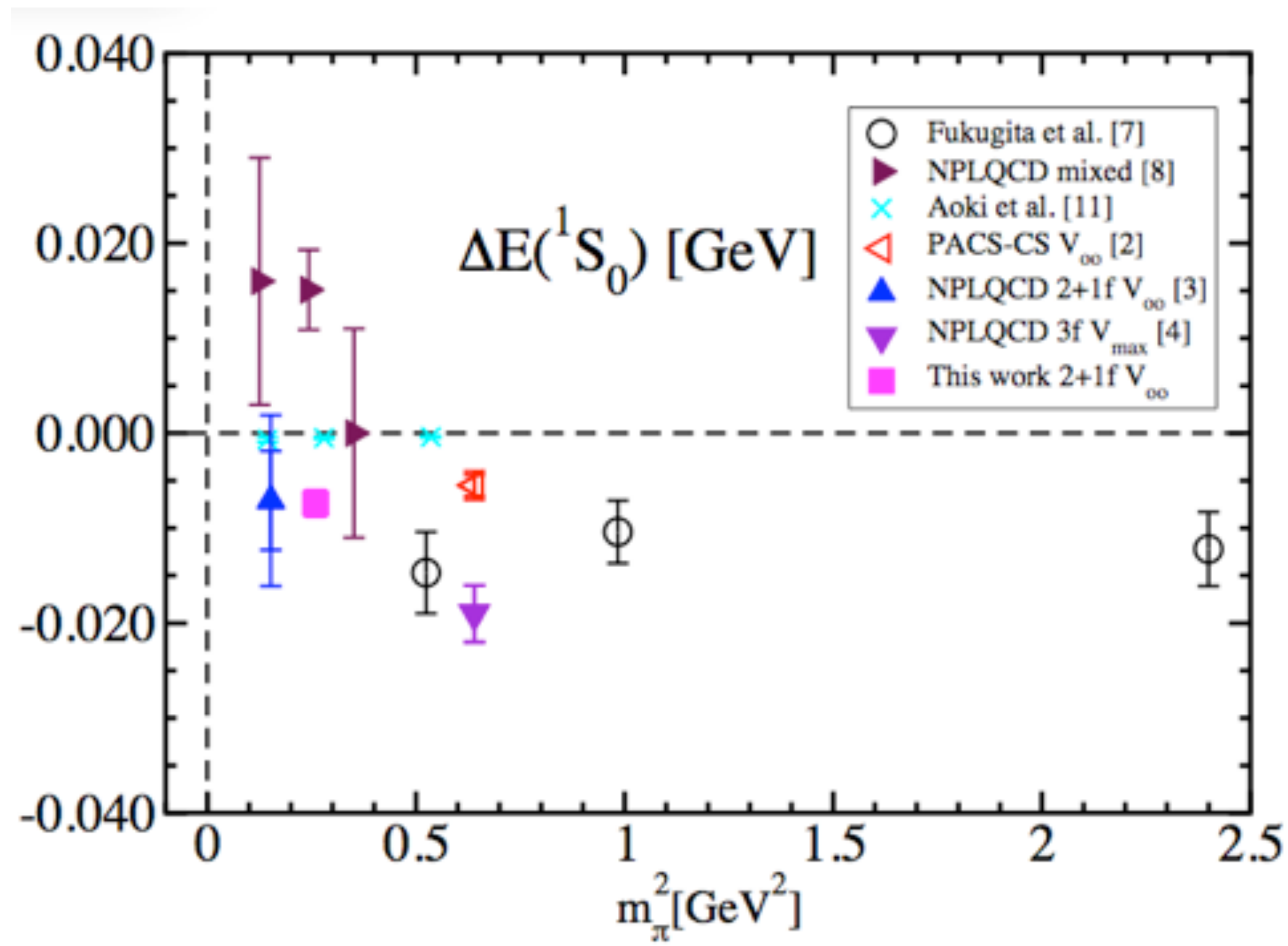
SU(3) Symmetry
2-Body Spectrum



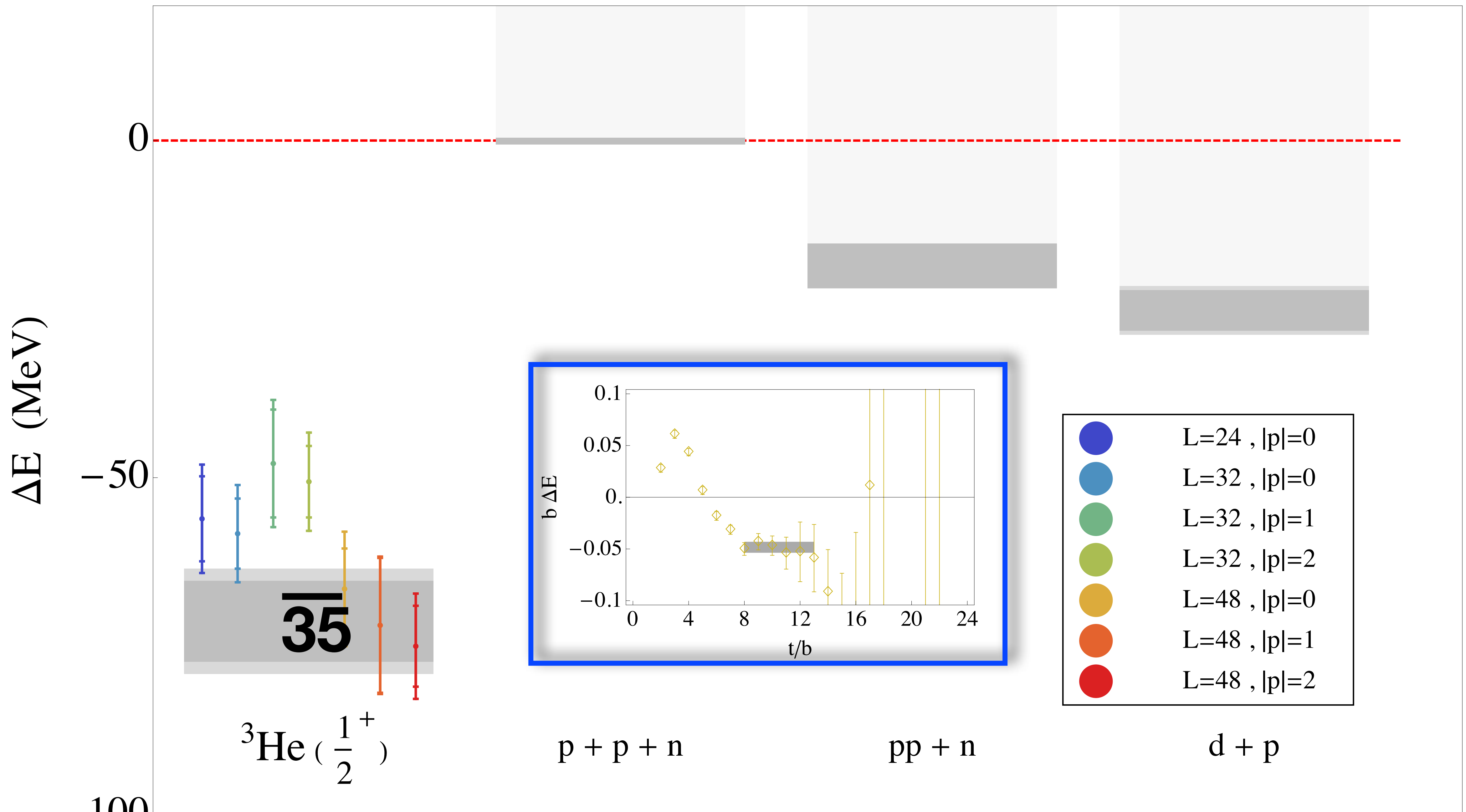
NPLQCD : e-Print: **arXiv:1206.5219 [hep-lat]**

Deuteron and nn : All Lattice Calculations



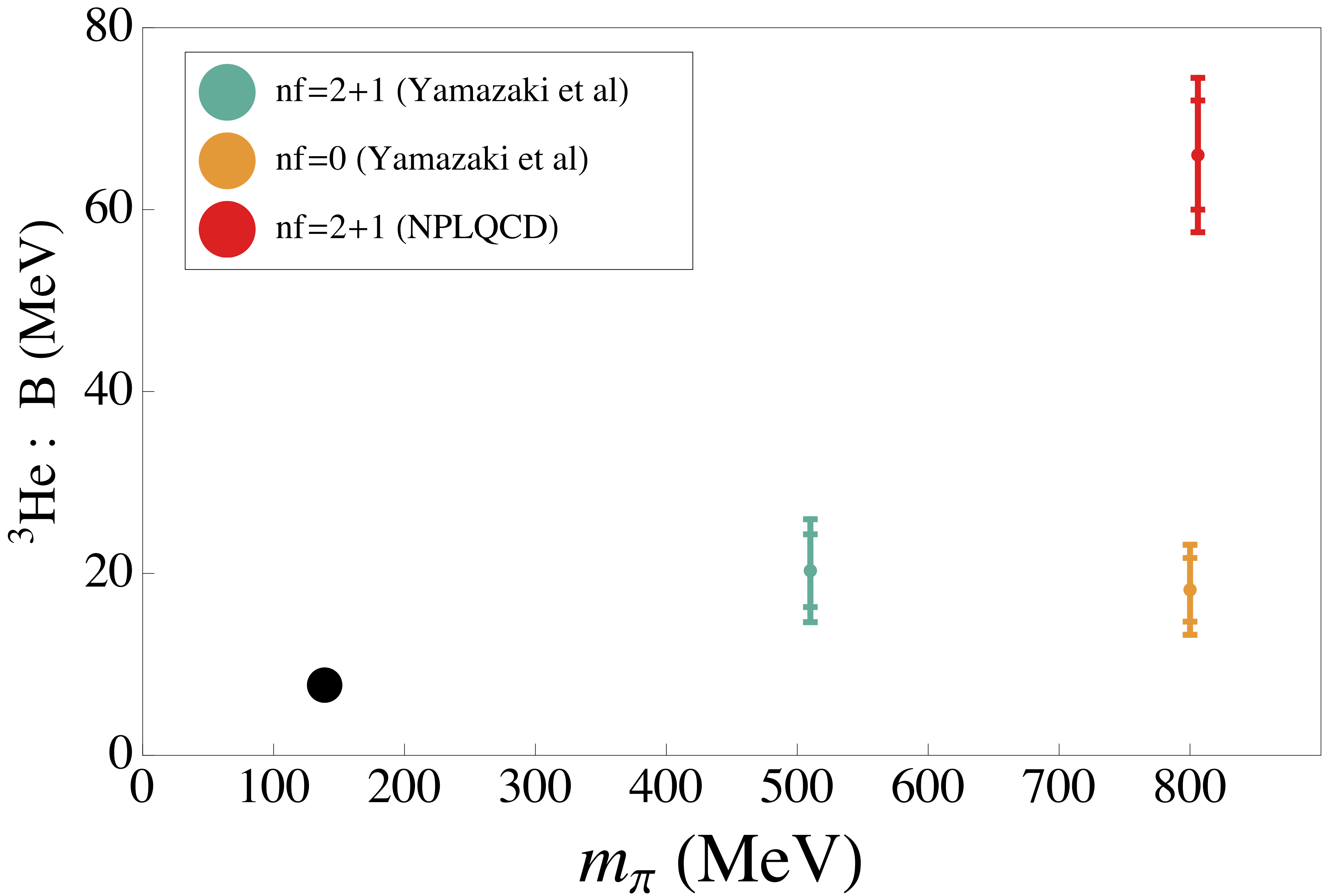


SU(3) Symmetry
3-Body Nuclear Spectrum

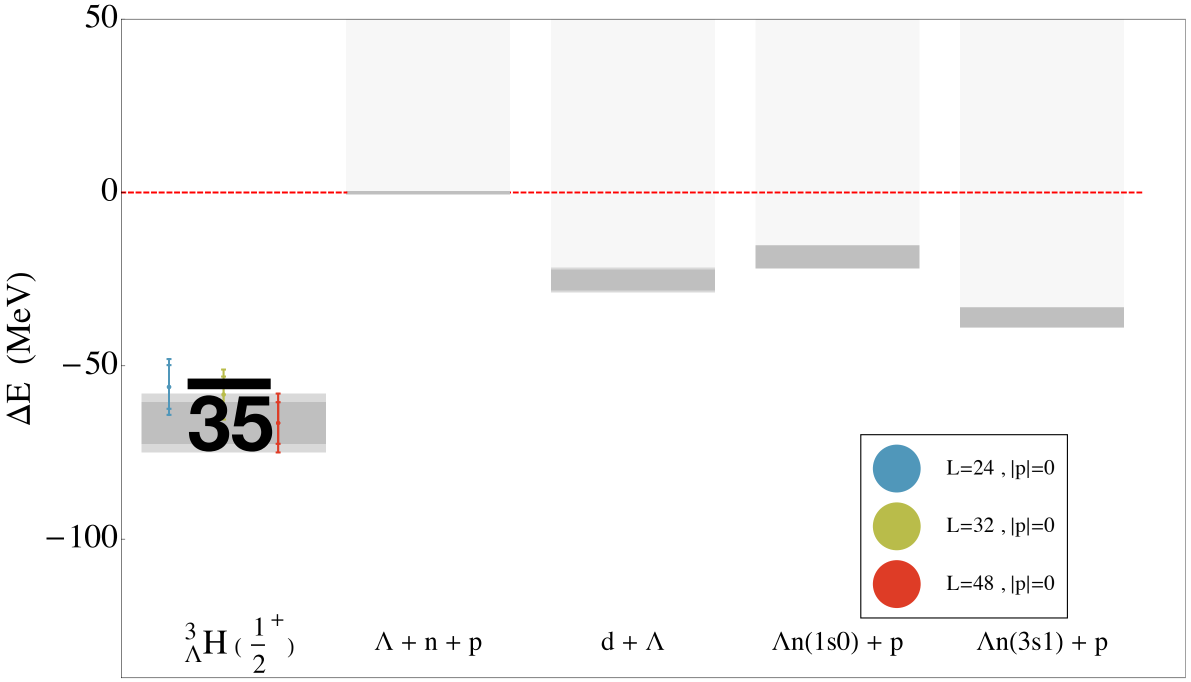


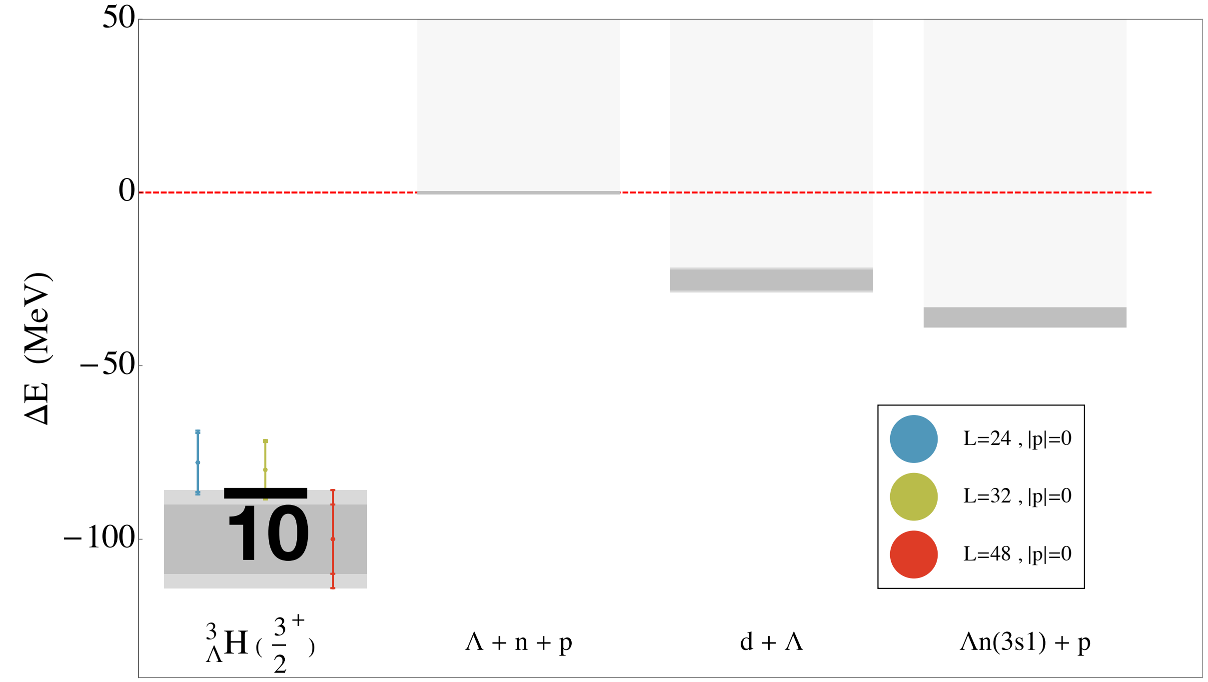
NPLQCD : e-Print: **arXiv:1206.5219 [hep-lat]**

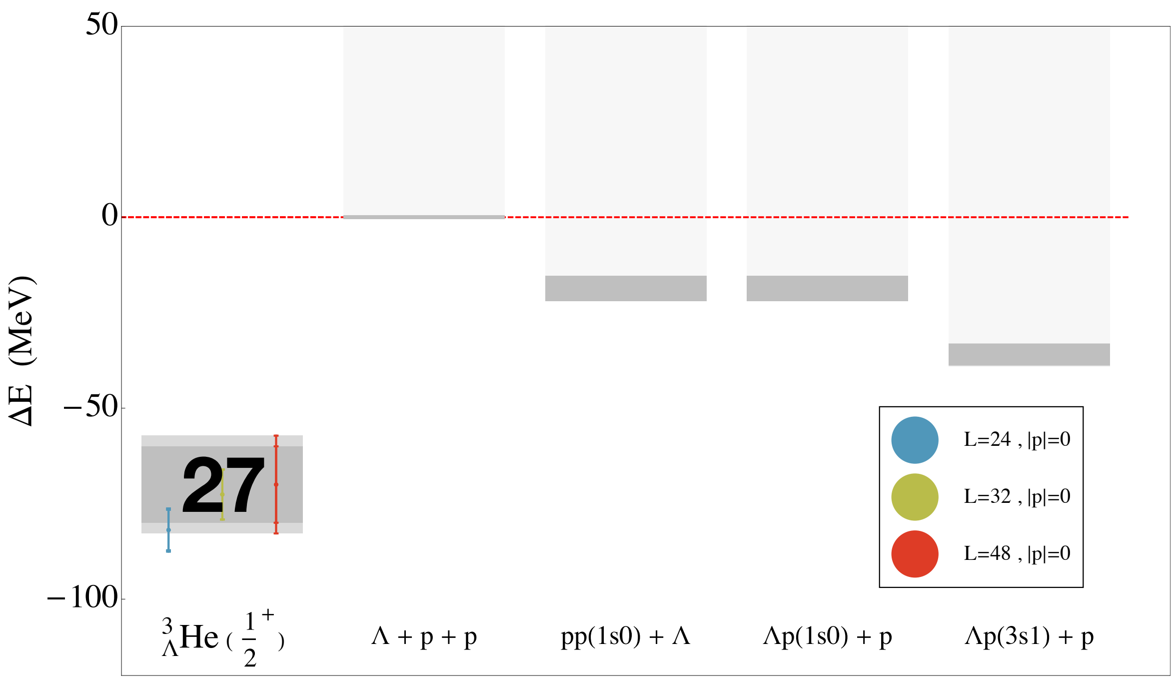
^{3}He : All Calculations

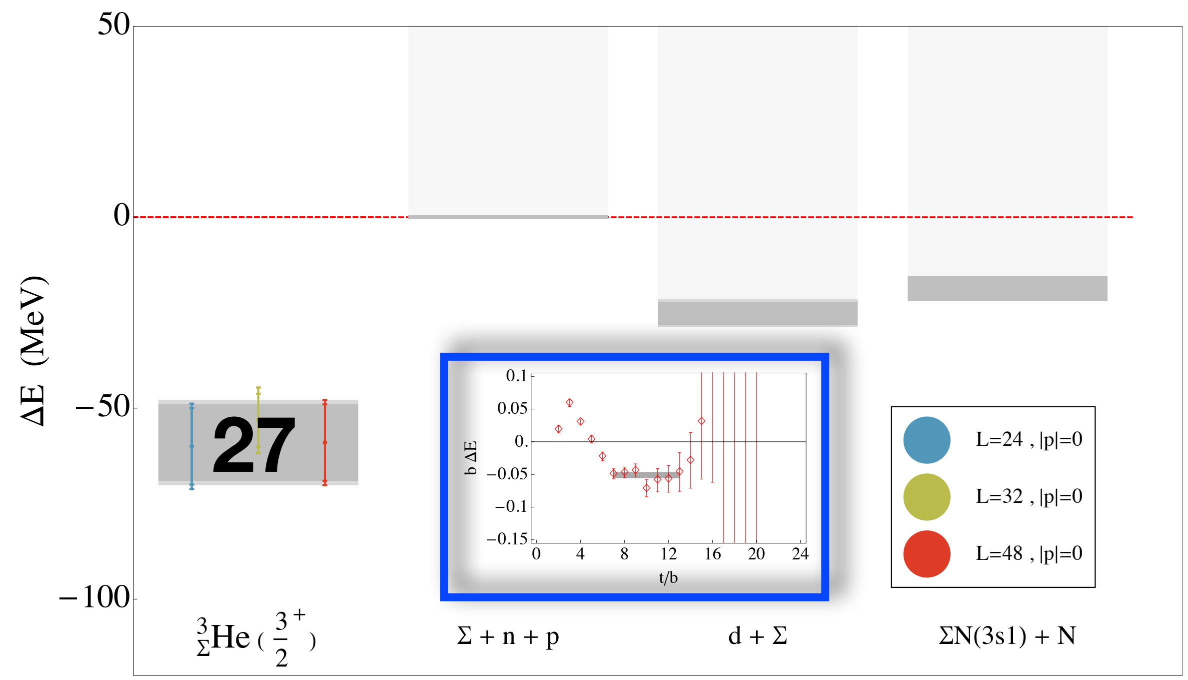


SU(3) Symmetry
3-Body Hypernuclear Spectrum

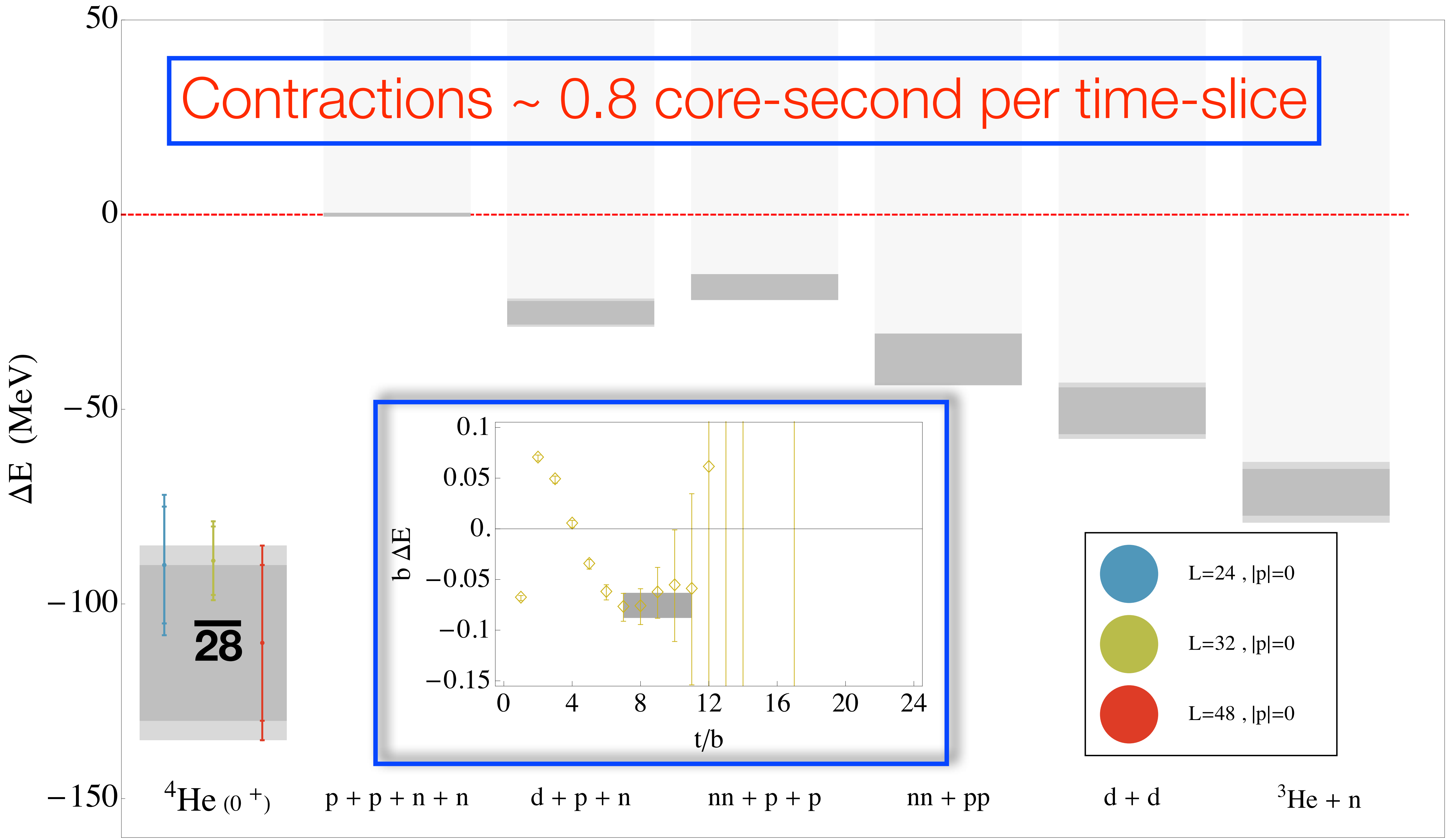






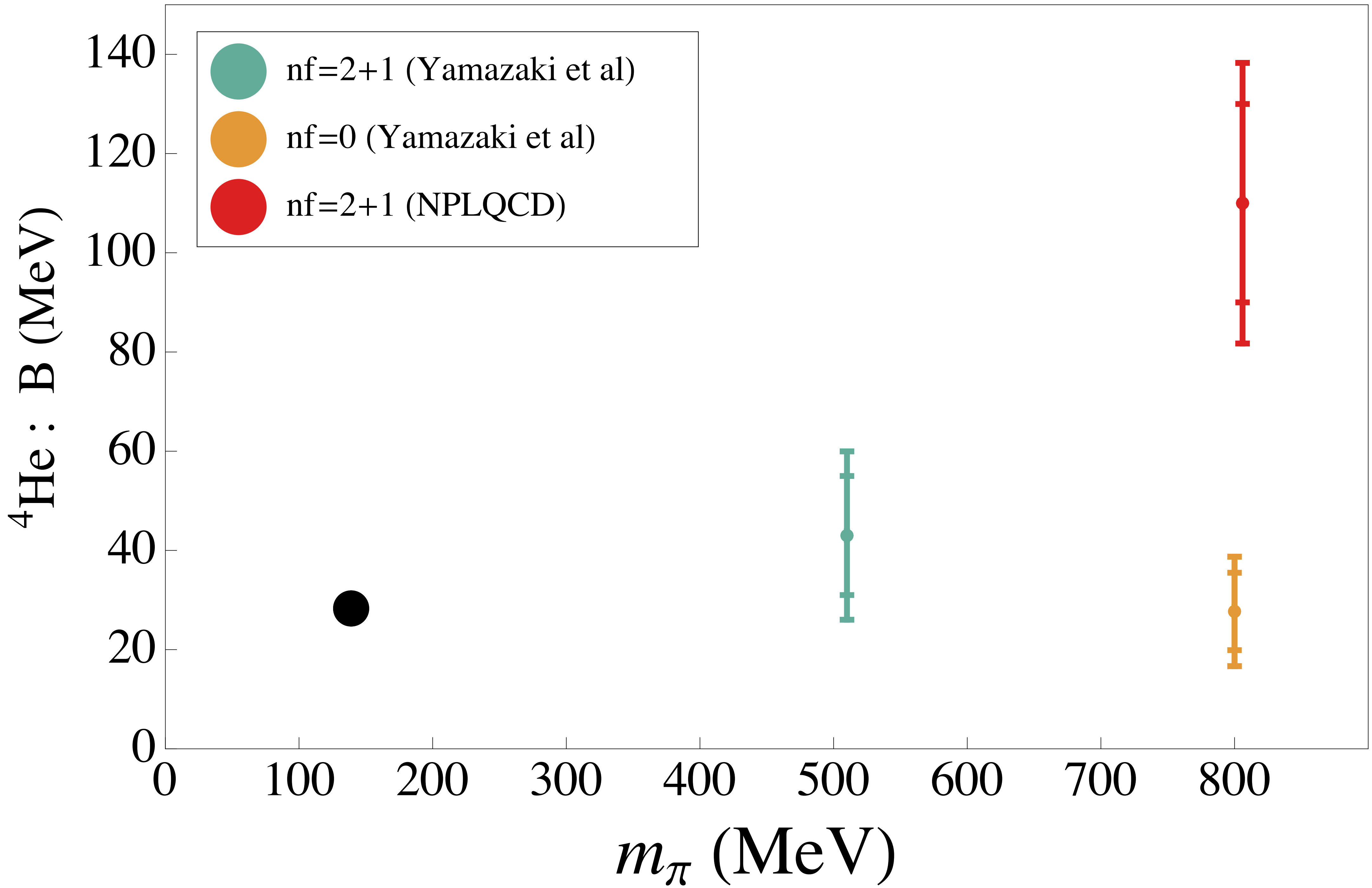


SU(3) Symmetry
4-Body Nuclear Spectrum

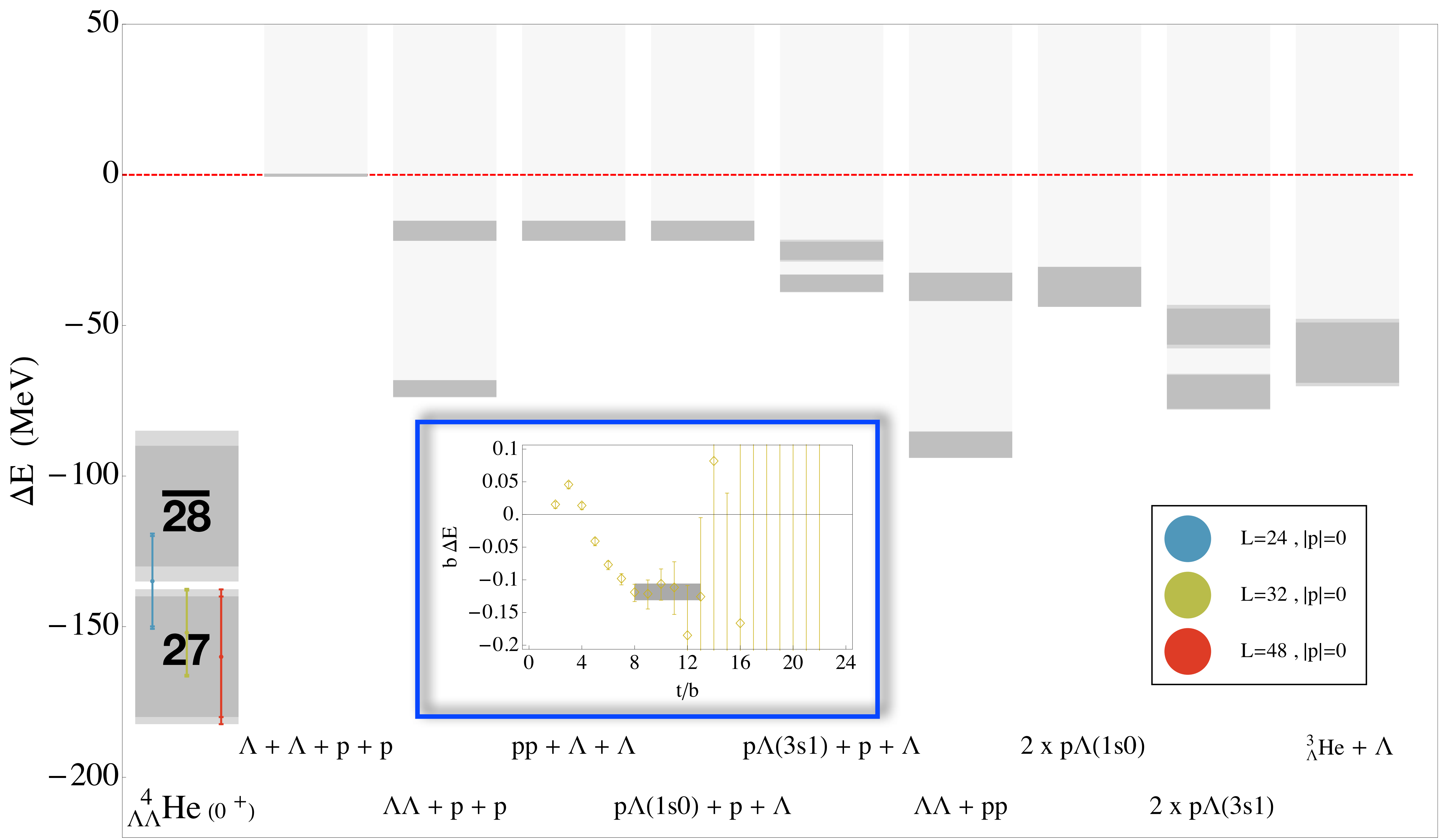


NPLQCD : e-Print: **arXiv:1206.5219 [hep-lat]**

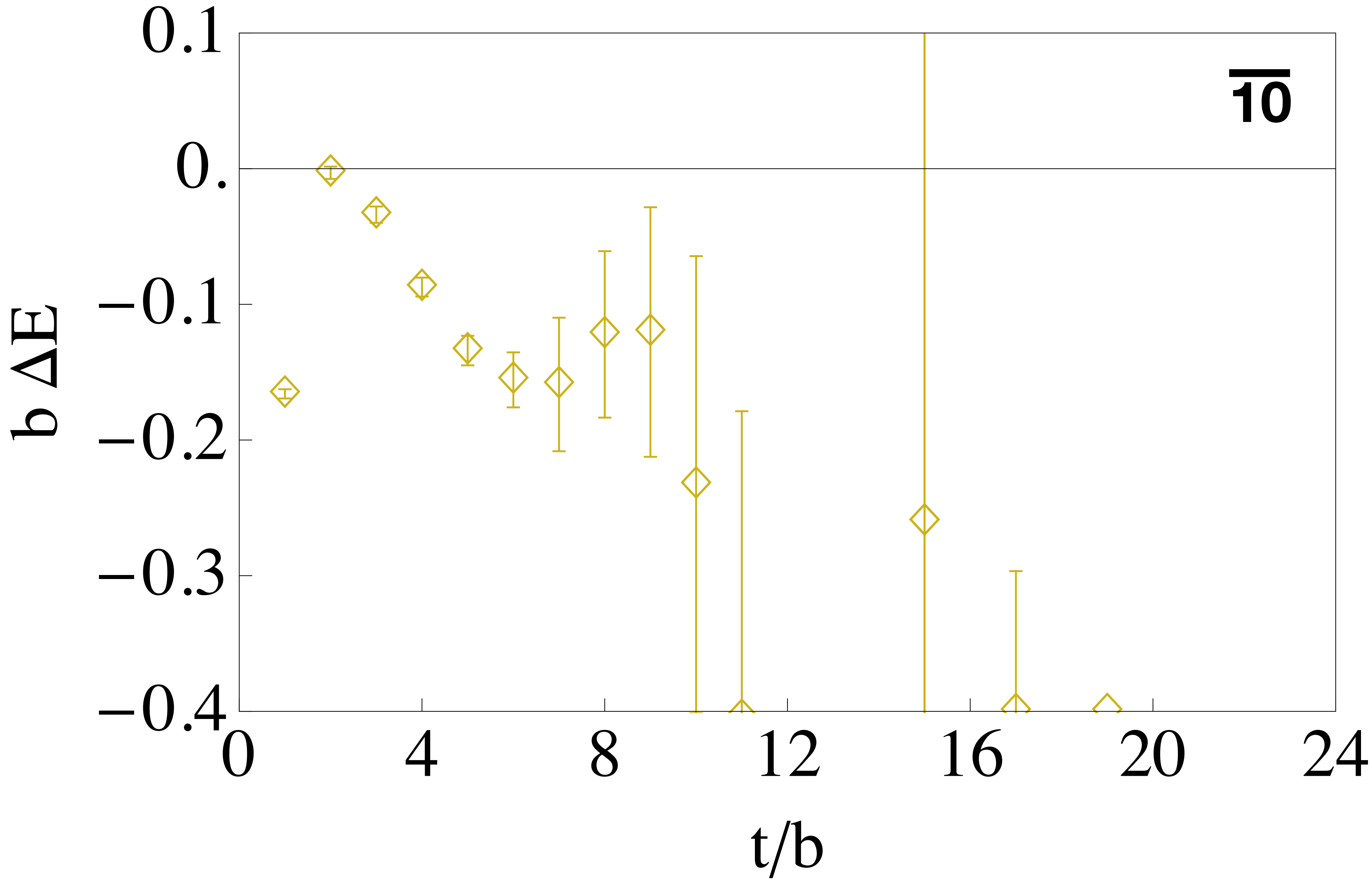
^{4}He :
All Calculations



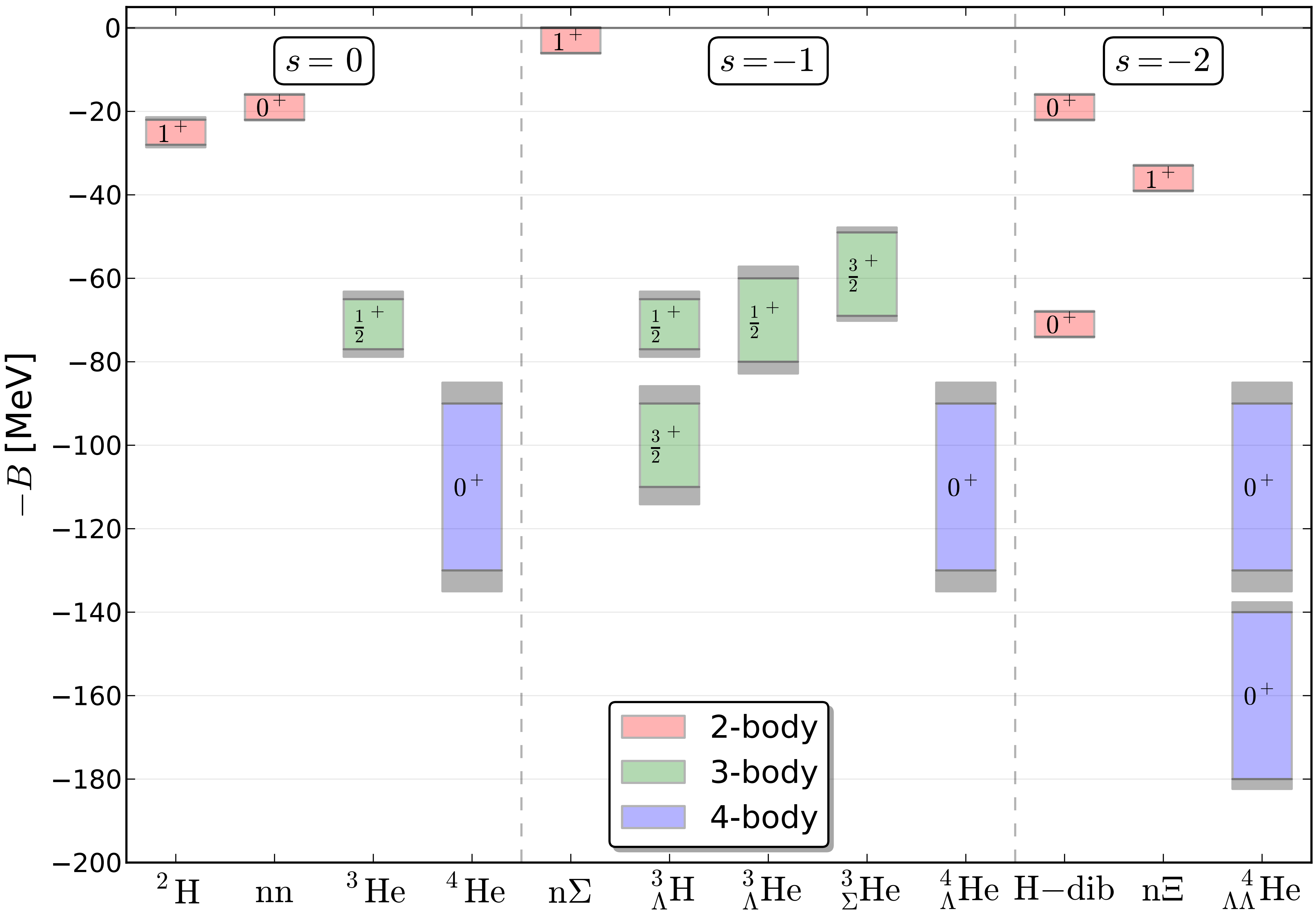
SU(3) Symmetry
4-Body Hypernuclear Spectrum



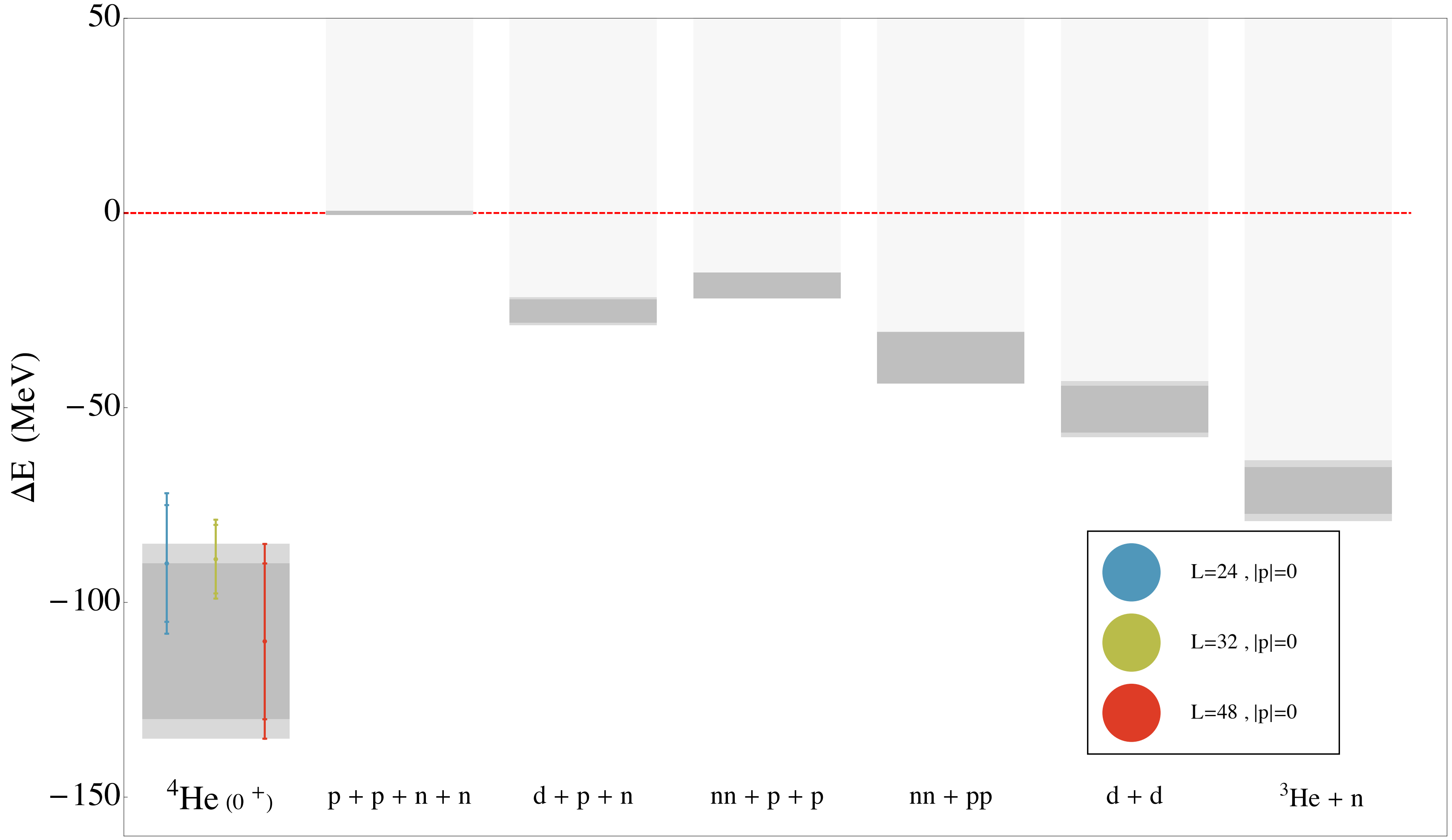
SU(3) Symmetry
5-Body Hypernuclear Spectrum



SU(3) Symmetry
Summary Spectrum from Isotropic



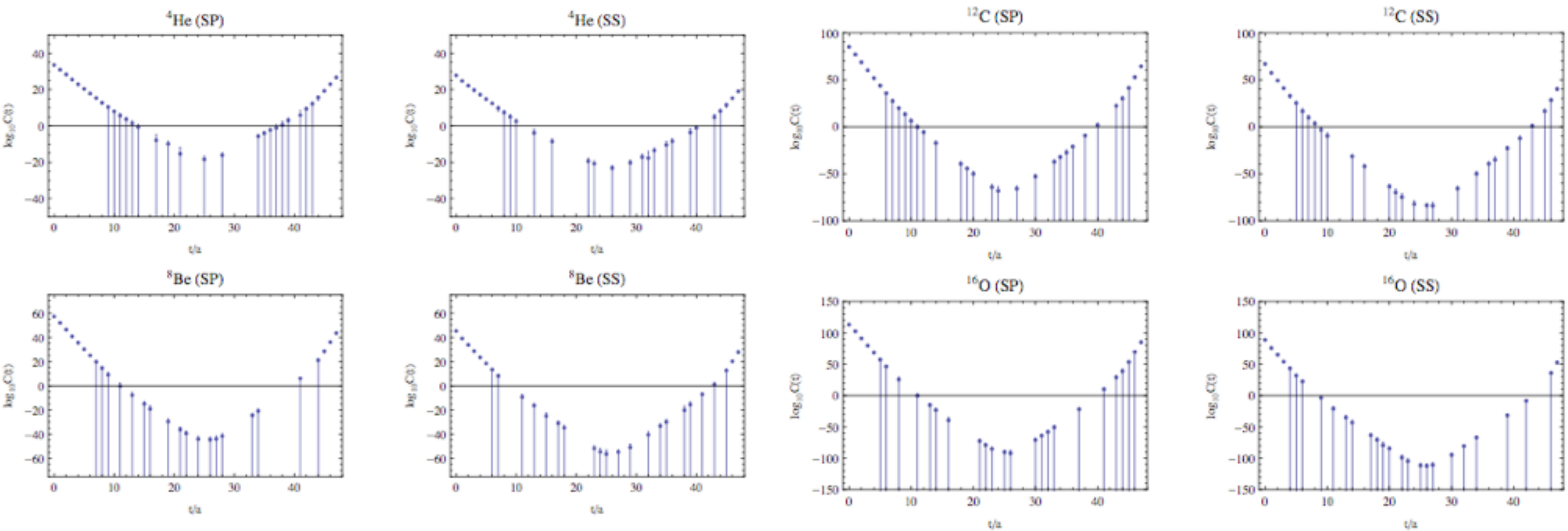
Volume Dependence ?



A>2 : Volume dependence unknown

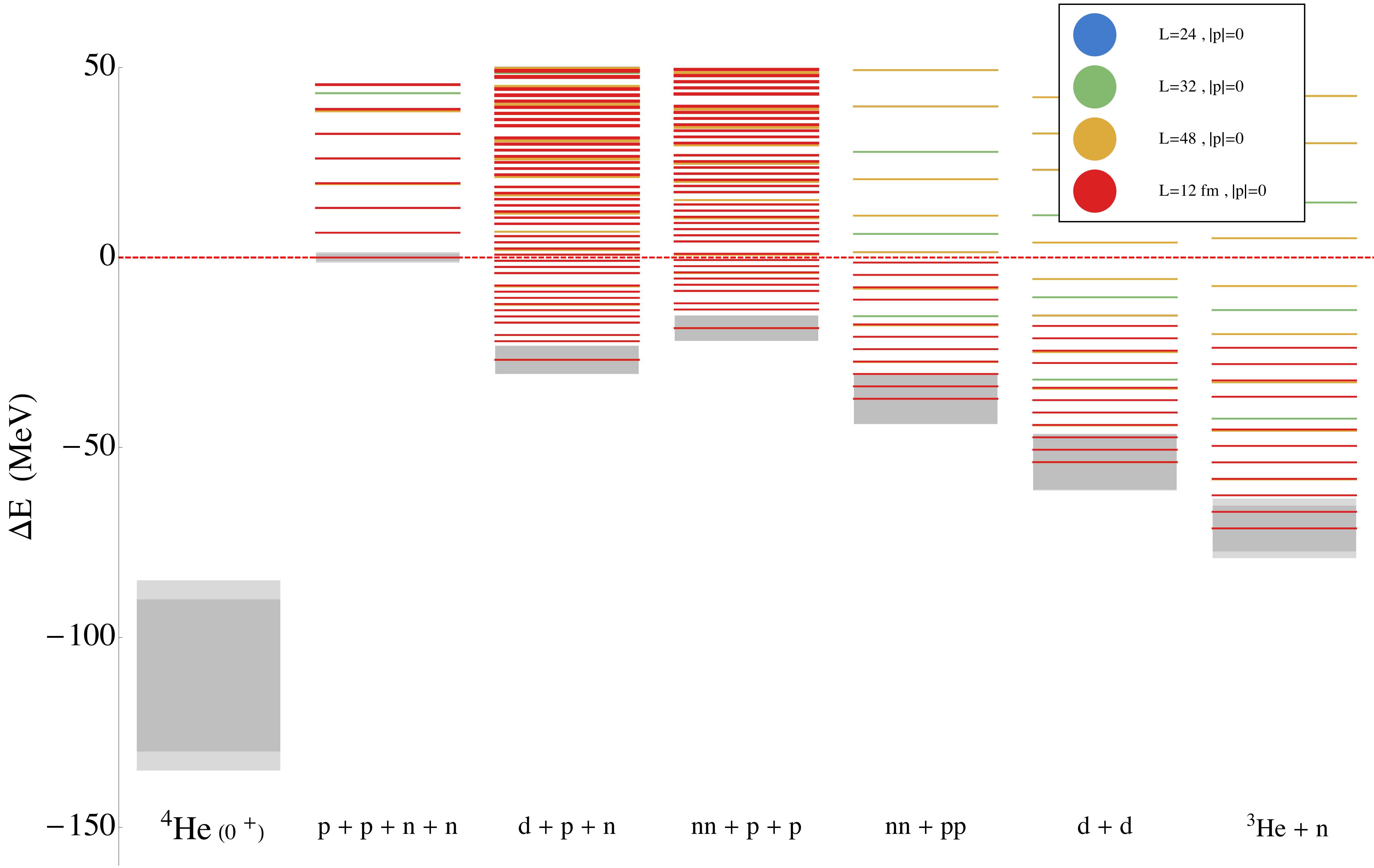
^{4}He : R << 3.5 fm

Medium A Nuclei : Developments in Contractions

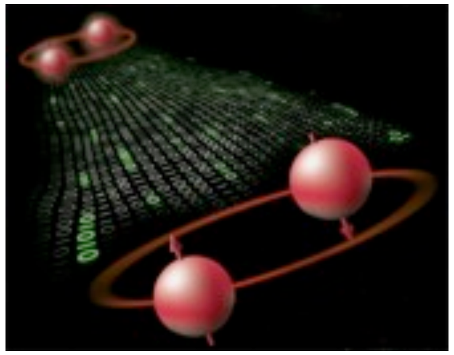


(Detmold and Orginos, **arXiv:1207.1452)**

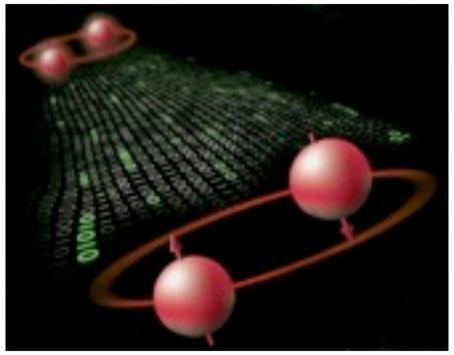
A Challenge



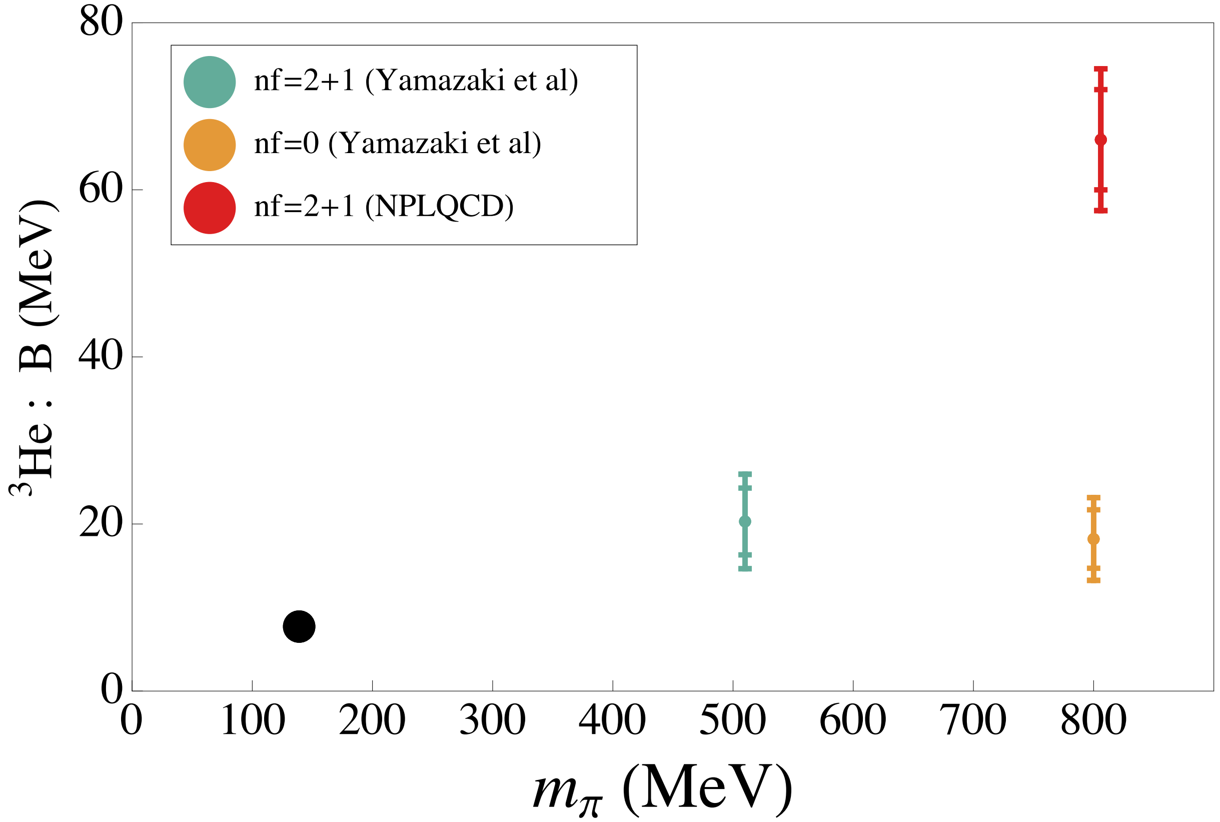
Nuclear Wavefunctions?

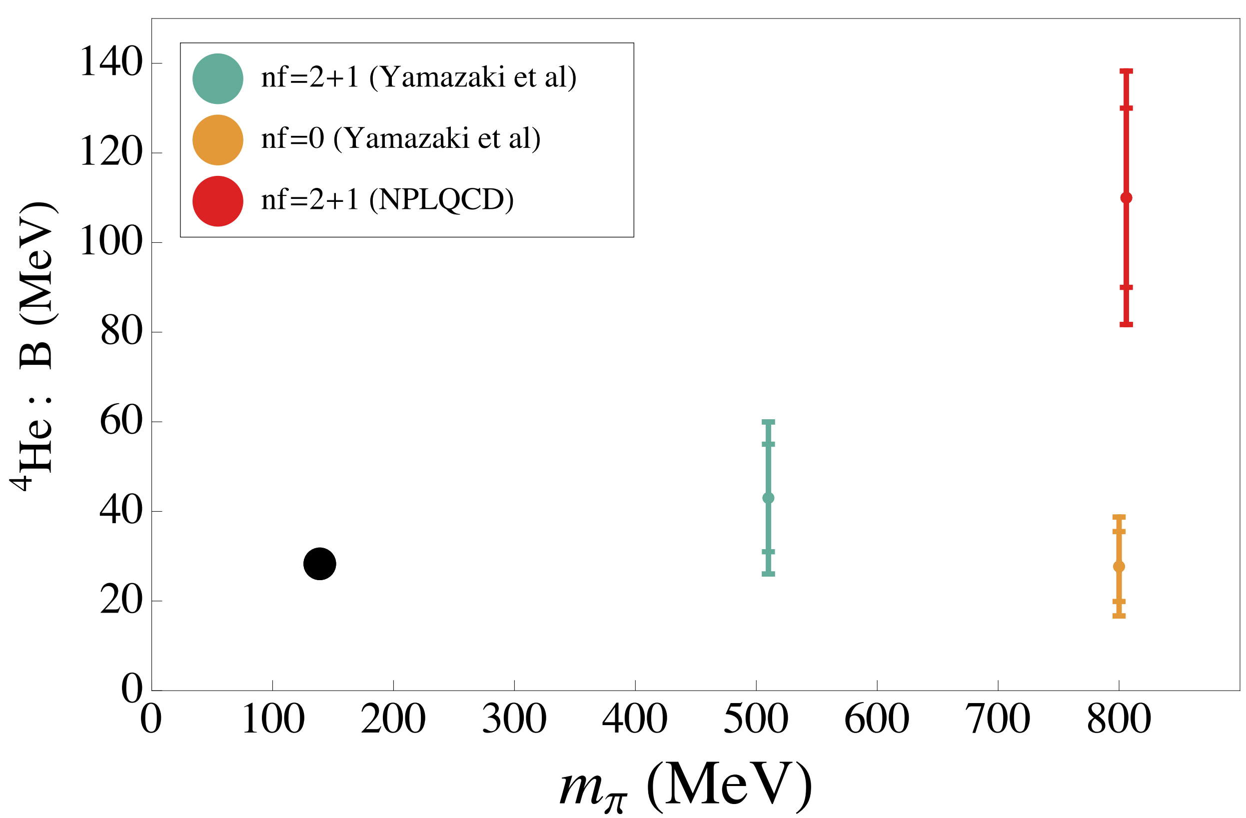


Summary



- Algorithmic progress in contractions
  - s-shell nuclei and beyond
  - ongoing
- A < 5 ground state energies (not physical pion mass)
- Encouraging progress in A > 4 systems





END

An Unnecessary Diversion

A Primer - 1990 : Luscher says ....

Explicitly, the stationary effective Schrödinger equation in the centre-of-mass frame reads

$$-\frac{1}{2\mu}\Delta\psi(\mathbf{r}) + \frac{1}{2}\int d^3r'\, U_E(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}') = E\psi(\mathbf{r}), \qquad (7.1)$$

where the parameter E is related to the true energy W of the system through

$$W = 2\sqrt{m^2 + mE}. \qquad (7.2)$$

The "potential" $U_E(\mathbf{r},\mathbf{r}')$ is the Fourier transform of the modified Bethe-Salpeter kernel $\hat{U}_E(\mathbf{k},\mathbf{k}')$ introduced in ref.[3]. It depends analytically on

38

E in the range $-m < E < 3m$ and is a smooth function of the coordinates $\mathbf{r}$ and $\mathbf{r}'$, decaying exponentially in all directions †. Furthermore, the potential

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)ESY 90-131
)ctober 1990

Two-Particle States on a Torus and their Relation to the Scattering Matrix

M. Lüscher

Deutsches Elektronen-Synchrotron DESY, Hamburg

It therefore follows that....

Taking U to be energy-independent is a model-dependent assertion and not a QCD prediction

A Reminder
2005 : Aoki says ....

The static two-pion wave function $\phi(\vec{x};k)$ with the energy eigenvalue $E = 2\sqrt{k^2 + m_\pi^2}$ in the center of mass system on a finite periodic box of volume L^3 satisfies the effective Schrödinger equation [14, 16] :

$$(\Delta + k^2)\phi(\vec{x};k) = \int \mathrm{d}^3y \; U_k(\vec{x},\vec{y})\phi(\vec{y};k) \; , \qquad (1)$$

where $\vec{x}$ and $\vec{y}$ are the relative coordinate of the two pions. $U_k(\vec{x},\vec{y})$ is the Fourier transform of the modified Bethe-Salpeter kernel for the two-pion interaction on the finite volume [14], and is related to the off-shell two-pion scattering amplitude (see Appendix A). It is generally non-local and depends on the two-pion energy. It should be noticed that k^2 in (1) is not a

$I = 2$ Pion Scattering Length from Two-Pion Wave Functions

S. Aoki,[1] M. Fukugita,[2] K-I. Ishikawa,[3] N. Ishizuka,[1,4] Y. Iwasaki,[1] K. Kanaya,[1]
T. Kaneko,[5] Y. Kuramashi,[1,4] M. Okawa,[3] A. Ukawa,[1,4] T. Yamazaki,[1,*] and T. Yoshié[1,4]

(CP-PACS Collaboration)

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[2] *Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277 8582, Japan*
[3] *Department of Physics, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan*
[4] *Center for Computational Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan*
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(Dated: February 1, 2008)

arXiv:hep-lat/0503025v2 14 May 2005

Abstract

We calculate the two-pion wave function in the ground state of the $I = 2$ S-wave system and find the interaction range between two pions, which allows us to examine the validity of the necessary condition for the finite-volume method for the scattering length proposed by Lüscher. We work in the quenched approximation employing a renormalization group improved gauge action for gluons and an improved Wilson action for quarks at $1/a = 1.207(12)$ GeV on $16^3 \times 80$, $20^3 \times 80$ and $24^3 \times 80$ lattices. We conclude that the necessary condition is satisfied within the statistical errors for the lattice sizes $L \geq 24$ (3.92 fm) when the quark mass is in the range that corresponds to $m_\pi^2 = 0.273 - 0.736$ GeV^2. We obtain the scattering length with a smaller statistical error from the wave function than from the two-pion time correlator.

PACS numbers: 12.38.Gc, 11.15.Ha

*Present address : RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

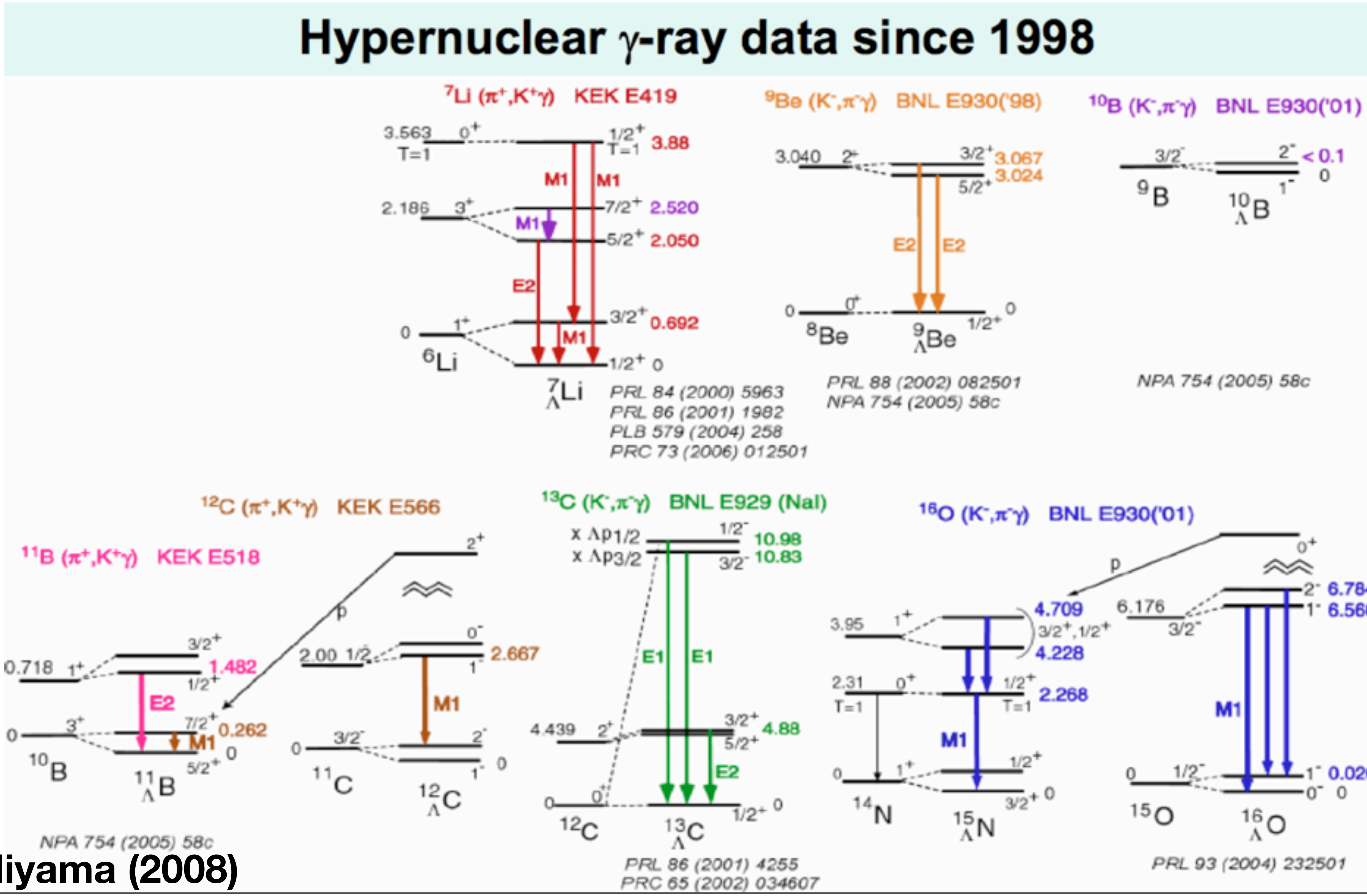
1

It therefore follows that....

Taking U to be energy-independent is a model-dependent assertion and not a QCD prediction

Back To Reality

HyperNuclear Programs
EM results



E. Hiyama (2008)