The excitation spectrum of charmonium from lattice QCD

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INT Seattle, 30th July 2012

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Charmonium spectroscopy — collaborators:

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Details presented in arXiv:1204.5425
(Accepted for publication in **JHEP**).

Overview

- The “renaissance” in charmonium spectroscopy
- What tools do we need for excited-state spectroscopy?
- New method — “distillation”
- Results — charmonium excitations
  - Dispersion relation
  - Variational analysis and spin identification
  - The excitation spectrum
  - Lattice artefacts
- Scattering with distillation — $I = 2\pi\pi$ as a test
- Summary

- Early 2000's — new discoveries in B-factories of narrow states above the open-charm threshold, the "XYZ"'s
- Provoked substantial phenomenological interest, since they are not explained by quark models
- **X(3872)** — close to $D\bar{D}^*$ threshold and very narrow $\Gamma \approx 0 - 3$ MeV
- **$\mathbf{Z^+}$(4430)** — charged state, so can not be simply $\bar{c}c$
- About 20 more; $Z(3930)$, $X(3940)$, $X(4160)$, $Y(4260)$, $X(4350)$, $Y(4360)$, $Y(4660)$, . . .
- Very little consensus regarding the internal structure of these states

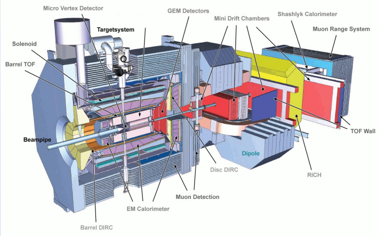
Can we use lattice QCD to study these states?

Panda@FAIR, GSI

- Extensive new construction at GSI Darmstadt
- Expected to start operation 2014?

PANDA: Anti-Proton ANnihilation at DArmstadt

- Anti-proton beam from FAIR on fixed-target.
- Physics goals include charmonium spectroscopy . . .



Methods for excited-state spectroscopy

Field theory on a Euclidean lattice

- Monte Carlo simulations are only practical using **importance sampling**
- Need a non-negative weight for each field configuration on the lattice

Minkowski $\longrightarrow$ Euclidean

- **Benefit:** can isolate lightest states in the spectrum.
- **Problem:** direct information on scattering is lost and must be inferred indirectly.
- For excitations and resonances, must use a **variational method.**

Variational method in Euclidean QFT

- Ground-state energies found from $t \to \infty$ limit of:

Euclidean-time correlation function

$$
\begin{aligned}
C(t) &= \langle 0|\ \Phi(t) \quad \Phi^\dagger(0)\ |0\rangle \\
&= \sum_{k,k'} \langle 0|\ \Phi|k\rangle\langle k|e^{-\hat{H}t}|k'\rangle\langle k'|\Phi^\dagger\ |0\rangle \\
&= \sum_{k} |\langle 0|\ \Phi|k\rangle|^2 \quad e^{-E_k t}
\end{aligned}
$$

- So $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$
- Variational idea: find operator Φ to maximise $C(t)/C(t_0)$ from sum of basis operators $\Phi = \sum_a v_a \phi_a$

[C. Michael and I. Teasdale. NPB215 (1983) 433]

[M. Lüscher and U. Wolff. NPB339 (1990) 222]

Variational method

If we can measure $C_{ab}(t) = \langle 0|\phi_a(t)\phi_b^\dagger(0)|0\rangle$ for all a, b and solve generalised eigenvalue problem:

$$\mathbf{C}(t)\,\underline{v} = \lambda \mathbf{C}(t_0)\,\underline{v}$$

then

$$\lim_{t-t_0\to\infty} \lambda_k = e^{-E_k t}$$

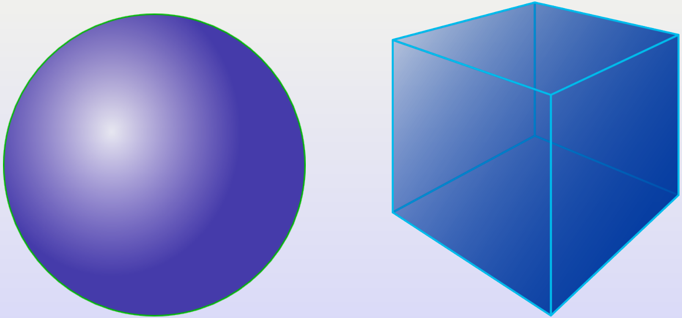
For this to be practical, we need:

- a 'good' basis set that **resembles the states** of interest
- **all elements** of this correlation matrix measured

[see Blossier et.al. JHEP 0904 (2009) 094]

A tale of two symmetries

- Continuum: states classified by J^P irreducible representations of $O(3)$.



$O(3)$ O_h

- Lattice regulator breaks $O(3) \rightarrow O_h$
- Lattice: states classified by R^P **"quantum letter" labelling irrep of O_h**

Irreps of O_h

- O has 5 conjugacy classes (so O_h has 10)
- Number of conjugacy classes = number of irreps
- Schur: $24 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2$
- These irreps are labelled A_1, A_2, E, T_1, T_2

	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$
A_1	1	1	1	1	1
A_2	1	1	-1	-1	1
E	2	-1	0	0	2
T_1	3	0	-1	1	-1
T_2	3	0	1	-1	-1

Spin on the lattice

- O_h has 10 irreps: $\{A_1^{g,u}, A_2^{g,u}, E^{g,u}, T_1^{g,u}, T_2^{g,u},\}$, where $\{g, u\}$ label even/odd parity.
- Link to continuum: subduce representations of $O(3)$ into O_h

	A_1	A_2	E	T_1	T_2
$J = 0$	1				
$J = 1$				1	
$J = 2$			1		1
$J = 3$		1		1	1
$J = 4$	1		1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Enough to search for degeneracy patterns in the spectrum?

$$4 \equiv 0 \oplus 1 \oplus 2$$

Operator basis — derivative construction

- A closer link to the continuum is needed
- Start with continuum operators, built from n derivatives:

$$\Phi = \bar{\psi}\, \Gamma \left(D_{i_1} D_{i_2} D_{i_3} \ldots D_{i_n} \right) \psi$$

- Construct irreps of $SO(3)$, then subduce these representations to O_h
- Now replace the derivatives with lattice finite differences:

$$D_j \psi(x) \rightarrow \frac{1}{a} \Big(U_j(x) \psi(x + \hat{\jmath}) - U_j^\dagger(x - \hat{\jmath}) \psi(x - \hat{\jmath}) \Big)$$

Example: $J^{PC} = 2^{++}$ meson creation operator

- Need more information to discriminate spins. Consider continuum operator that creates a 2^{++} meson:

$$\Phi_{ij} = \bar{\psi}\left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3}\delta_{ij}\gamma \cdot D\right)\psi$$

- Lattice: Substitute gauge-covariant lattice finite-difference D_{latt} for D
- A reducible representation:

$$\Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\}$$

$$\Phi^{E} = \left\{\frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33})\right\}$$

- Look for signature of continuum symmetry:

$$\langle 0|\Phi^{(T_2)}|2^{++(T_2)}\rangle = \langle 0|\Phi^{(E)}|2^{++(E)}\rangle$$

To use all these ideas in a practical calculation, we need access to **all elements**[†] of the quark propagator

[†] not quite - as we will see

New measurement methods for hadron correlation function

Smearing

- **Smeared field:** $\tilde{\psi}$ from ψ, the "raw" quark field in the path-integral:

$$\tilde{\psi}(t) = \Box[U(t)] \ \psi(t)$$

- Extract the essential degrees-of-freedom.
- Smearing should preserve symmetries of quarks.
- Now form creation operator (e.g. a meson):

$$O_M(t) = \bar{\tilde{\psi}}(t)\Gamma\tilde{\psi}(t)$$

- Γ: operator in $\{\underline{s}, \sigma, c\} \equiv \{\text{position,spin,colour}\}$
- Smearing: overlap $\langle n|O_M|0\rangle$ is large for low-lying eigenstate $|n\rangle$

Can redefining smearing help?

- Computing quark propagation in configuration generation and observable measurement is expensive.
- Objective: extract as much information from correlation functions as possible.

Two problems:

1. Most correlators: signal-to-noise falls exponentially
2. Making measurements can be costly:
    - Variational bases
    - Exotic states using more sophisticated creation operators
    - Isoscalar mesons
    - Multi-hadron states

- Good operators are **smeared**; helps with problem 1, can it help with problem 2?

Gaussian smearing

- To build an operator that projects effectively onto a low-lying hadronic state need to use **smearing**
- Instead of the creation operator being a direct function applied to the fields in the lagrangian first smooth out the UV modes which contribute little to the IR dynamics directly.
- A popular gauge-covariant smearing algorithm — Gaussian smearing: Apply the linear operator

$$\Box_J = \exp(\sigma \Delta^2)$$

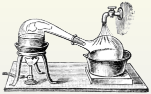
- Δ^2 is a lattice representation of the 3-dimensional gauge-covariant laplace operator on the source time-slice

$$\Delta^2_{x,y} = 6\delta_{x,y} - \sum_{i=1}^{3} U_i(x)\delta_{x+\hat{\imath},y} + U_i^\dagger(x-\hat{\imath})\delta_{x-\hat{\imath},y}$$

- Correlation functions look like $\mathrm{Tr}\ \Box_J M^{-1} \Box_J M^{-1} \Box_J \ldots$

Distillation

"*distill*: to extract the quintessence of" [OED]



- Distillation: **define** smearing to be explicitly a very low-rank operator. Rank is $N_{\mathcal{D}} (\ll N_s \times N_c)$.

Distillation operator

$$\Box(t) = V(t) V^\dagger(t)$$

with $V^a_{\underline{x},c}(t)$ a $N_{\mathcal{D}} \times (N_s \times N_c)$ matrix

- Example (used to date): $\Box_\nabla$ the **projection operator into $\mathcal{D}_\nabla$, the space spanned by the lowest eigenmodes of the 3-D laplacian**
- Projection operator, so idempotent: $\Box_\nabla^2 = \Box_\nabla$
- $\lim_{N_{\mathcal{D}} \to (N_s \times N_c)} \Box_\nabla = I$
- Eigenvectors of ∇^2 not the only choice...

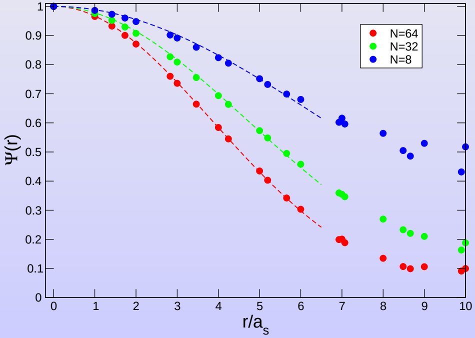
Distillation: preserve symmetries

- Using eigenmodes of the gauge-covariant laplacian **preserves lattice symmetries**

$$U_i(\underline{x}) \xrightarrow{g} U_i^g(\underline{x}) = g(\underline{x})U_i(\underline{x})g^\dagger(\underline{x}+\hat{\imath})$$

$$\Box_\nabla(\underline{x},\underline{y}) \xrightarrow{g} \Box_\nabla^g(\underline{x},\underline{y}) = g(\underline{x})\Box_\nabla(\underline{x},\underline{y})g^\dagger(\underline{y})$$

- Translation, parity, charge-conjugation symmetric
- O_h symmetric
- Close to $SO(3)$ symmetric
- "local" operator



- Consider an isovector meson two-point function:

$$C_M(t_1 - t_0) = \langle\langle \bar{u}(t_1)\Box_{t_1}\Gamma_{t_1}\Box_{t_1} d(t_1) \quad \bar{d}(t_0)\Box_{t_0}\Gamma_{t_0}\Box_{t_0} u(t_0)\rangle\rangle$$

- Integrating over quark fields yields

$$C_M(t_1 - t_0) = \langle \mathrm{Tr}_{\{\underline{s},\sigma,c\}} \left(\Box_{t_1}\Gamma_{t_1}\Box_{t_1} M^{-1}(t_1, t_0)\Box_{t_0}\Gamma_{t_0}\Box_{t_0} M^{-1}(t_0, t_1)\right)\rangle$$

- Substituting the low-rank distillation operator $\Box$ reduces this to a **much smaller** trace:

$$C_M(t_1 - t_0) = \langle \mathrm{Tr}_{\{\sigma,\mathcal{D}\}} \left[\Phi(t_1)\tau(t_1, t_0)\Phi(t_0)\tau(t_0, t_1)\right]\rangle$$

- $\Phi^{\alpha,a}_{\beta,b}$ and $\tau^{\alpha,a}_{\beta,b}$ are $(N_\sigma \times N_\mathcal{D}) \times (N_\sigma \times N_\mathcal{D})$ matrices.

$$\Phi(t) = V^\dagger(t)\Gamma_t V(t) \qquad \tau(t, t') = V^\dagger(t)M^{-1}(t, t')V(t')$$

The "perambulator"

Results

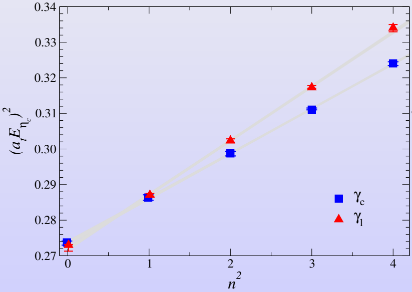
HadSpec lattices

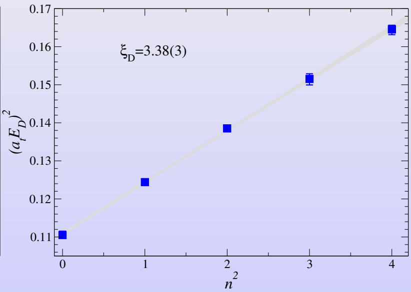
- **Anisotropic** lattice: $a_s \neq a_t$
- Gauge action is tree-level $\mathcal{O}(a_s^2)$-improved
- Quarks: tree-level $\mathcal{O}(a)$-improved SW action
- Non-perturbative tuning of action parameters such that $\xi = a_s/a_t = 3.5$
- $N_f = 2 \oplus 1$ dynamical flavours
- $m_\pi \approx 400$ MeV
- Scale set from Ω baryon mass
- $16^3 \times 128$ and $24^3 \times 128$ volumes used here
- 96 and 552 samples in each ensemble
- $N_{\mathcal{D}} = 64$ for 16^3, $N_{\mathcal{D}} = 162$ for 24^3 volumes

Charmonium

Dispersion relations - η_c and D mesons

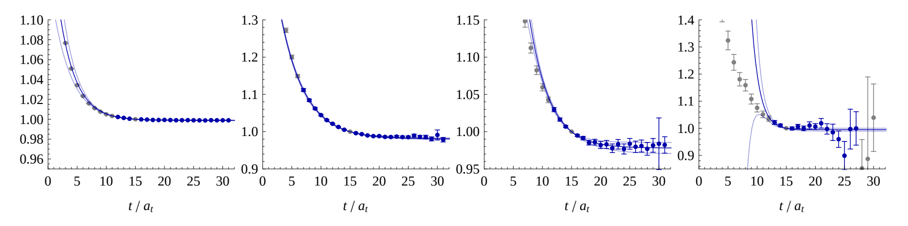
- Action parameters for charm quark tuned to ensure dispersion relation for η_c is relativistic
- Using these tuned parameters, D meson also has relativistic dispersion relation





Fits to $\lambda_k(t)$

- Variational basis, so can access excited states
- Fit $\lambda_k(t)$ to one or two exponentials
- Second exponential to stabilise some fits - value not used
- Plots show $\lambda_k(t) \times e^{E_k(t-t_0)}$



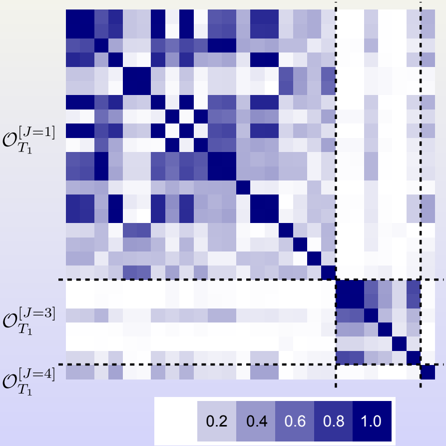
- Data from T_1^{--} channel $(J = 1, 3, 4, \ldots)$

Subduction of derivative-based operators

- T_1^{--} variational basis
- 26 operators, up to $D_i D_j D_k$
- Correlation matrix at $t/a_t = 5$, normalised:

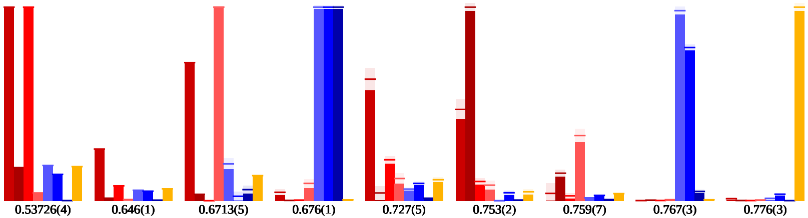
$$Q_{ij} = \frac{C_{ij}}{\sqrt{C_{ii} C_{jj}}}$$

- Reasonable **spin separation** seen



Spin identification

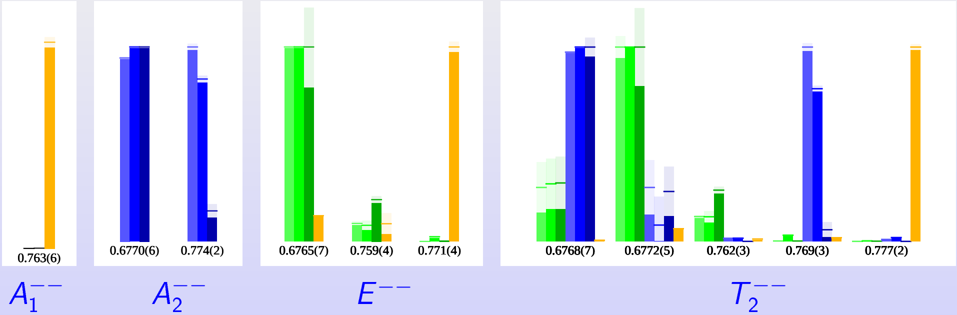
- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.



- Can help identify glue-rich states, using operators with $[D_i, D_j]$

. . . the rest of the spin-4 state

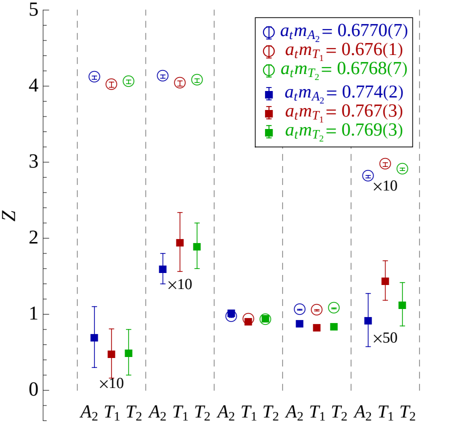
- All polarisations of the spin-4 state are seen
- Spin labelling: **Spin 2**, **Spin 3** and **Spin 4**.



Identifying spin - operator overlaps

- Example — 3^{--} continuum
- Look for remnant of continuum symmetry:

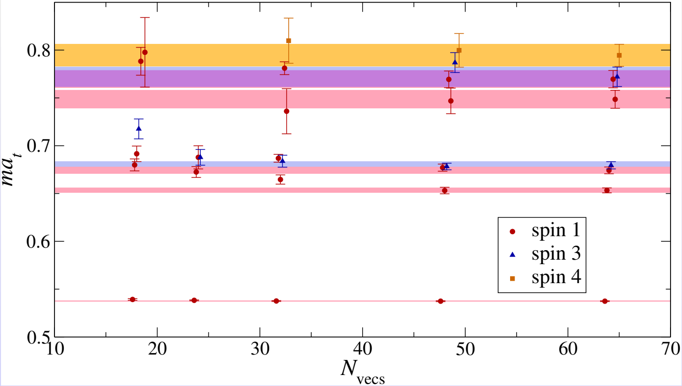
$$\langle 0|\Phi^{[J=3]}_{A_2^{--}}|k\rangle = \langle 0|\Phi^{[J=3]}_{T_1^{--}}|k\rangle = \langle 0|\Phi^{[J=3]}_{T_2^{--}}|$$



- Can identify two spin-3 states.

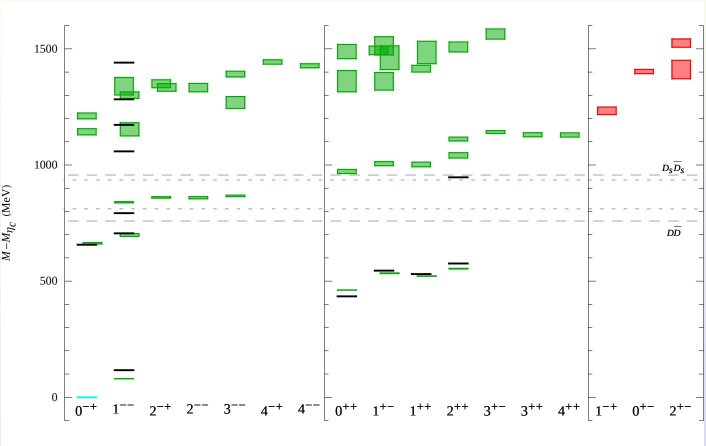
Spectrum - dependence on distillation basis

- 16^3 lattice - vary $N_{\mathcal{D}}$
- Calculation done on smaller ensemble



- Stable spectrum for $N_{\mathcal{D}} > 48$

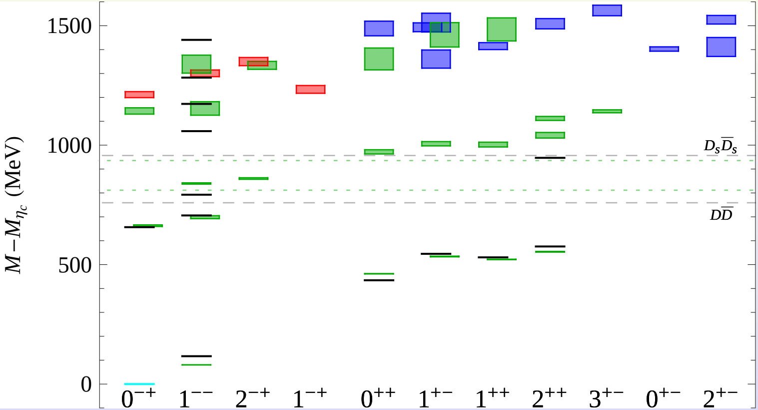
Excitation spectrum of charmonium



- Quark model: $1S, 1P, 2S, 1D, 2P, 1F, 2D, \ldots$ all seen.
- Not all fit quark model: spin-exotic (and non-exotic) hybrids seen

[Liu et.al. arXiv:1204.5425]

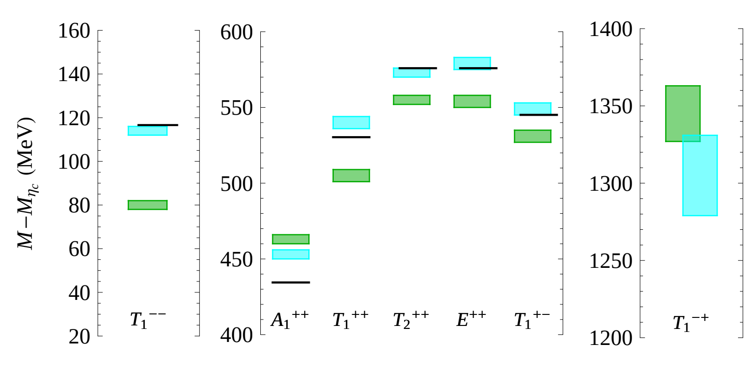
Gluonic excitations in charmonium?



- See states created by operators that excite intrinsic gluons
- two- and three-derivatives create states in the open-charm region.

[Liu et.al. arXiv:1204.5425]

Lattice artefacts in charmonium



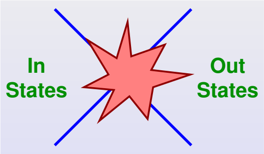
- Hyperfine structure sensitive to lattice artefacts. Boost co-efficient of action term to suppress these.
- green → light blue. Shifts are ≈ 40 MeV.

[Liu et.al. arXiv:1204.5425]

Measuring scattering properties using distillation

Scattering matrix elements not directly accessible from Euclidean QFT [Maiani-Testa theorem]

- Scattering matrix elements: asymptotic $|\text{in}\rangle, |\text{out}\rangle$ states.
  $\langle \text{out}\ |e^{i\hat{H}t}|\ \text{in}\rangle \rightarrow \langle \text{out}\ |e^{-\hat{H}t}|\ \text{in}\rangle$
- Euclidean metric: project onto ground-state



- **Lüscher's formalism:** information on elastic scattering inferred from **volume dependence** of spectrum
- Requires precise data, resolution of two-hadron and excited states.

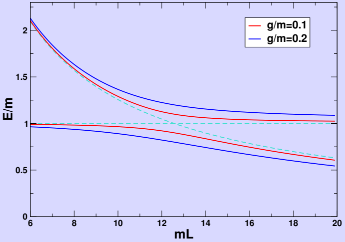
Hadrons in a finite box: scattering

- On a finite lattice with periodic b.c., hadrons have quantised momenta; $\underline{p} = \frac{2\pi}{L}\{n_x, n_y, n_z\}$
- Two hadrons with total $P = 0$ have a discrete spectrum
- These states can have same quantum numbers as those created by $\bar{q}\Gamma q$ operators and QCD can mix these

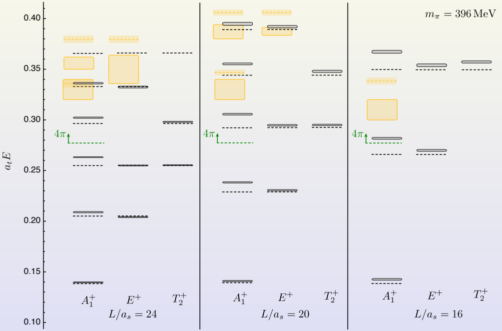
- This leads to shifts in the spectrum in finite volume
- This is the same physics that makes resonances in an experiment
- Lüscher's method - relate elastic scattering to energy shifts

Toy model

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$



$I = 2$ $\pi - \pi$ phase shift

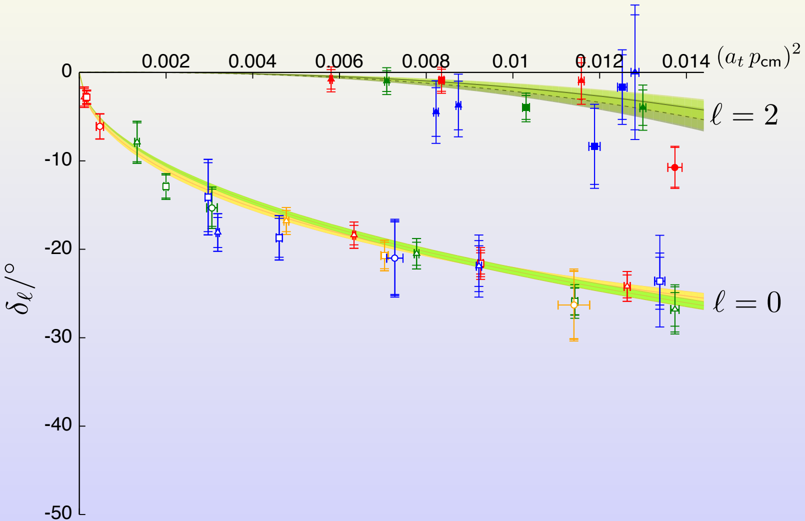


- Lüscher's method: first determine energy shifts as volume changes
- Data for $L = 16a_s, 20a_s, 24a_s$
- Small energy shifts are resolved

- Measured δ_0 and δ_2 (δ_4 is very small)
- $I = 2$ a useful first test - simplest Wick contractions

  Dudek *et.al.* [Phys.Rev.D83:071504,2011, arXiv:1203.6041]
- See Christopher Thomas' talk later in this workshop

$I = 2$ $\pi - \pi$ phase shift



Dudek *et.al.* [Phys.Rev.D83:071504,2011, `arXiv:1203.6041`]

Summary

Progress

- Variational methods very effective in constructing states up to ≈ 4.5 GeV
- Spin identification possible
- Hybrid excitations emerge
- "Distillation" method works well for charmonium

Challenges

- Include multi-hadron operators to study scattering and resonance behaviour.
- Molecules, tetraquarks, ...
- Precision needs better control of $a \longrightarrow 0$
- m_π closer to physical value
- Molecular states will need very (too) large lattices.

The lattice should help us to understand the nature of the new charmonium states, although many problems remain unsolved