Resonances in Dπ and Kπ scattering

INT, Seattle, 10th August 2012

Lattice QCD studies of Excited Resonances and Multi-Hadron systems

Sasa Prelovsek
University of Ljubljana & Jozef Stefan Institute, Slovenia

In collaboration with:
Christian B. Lang, Luka Leskovec, Daniel Mohler, Richard Woloshyn

Outline

- ❑ Motivation

- ❑ Lattice simulation:

- Dπ and D*π scattering & corresponding resonances
- Kπ scattering & corresponding resonances

- the above simulations: $m_1 \neq m_2$, P=p1+p2=0

- ❑ Analitic study:

- scattering of $M_1 M_2$ with $m_1 \neq m_2$, $P=p1+p2 \neq 0$
  generalization of Luscher's formula
  challenges to extract s-wave phase shift in this case
- presented s-wave Dπ, Kπ at P=0 are more reliable

Motivation

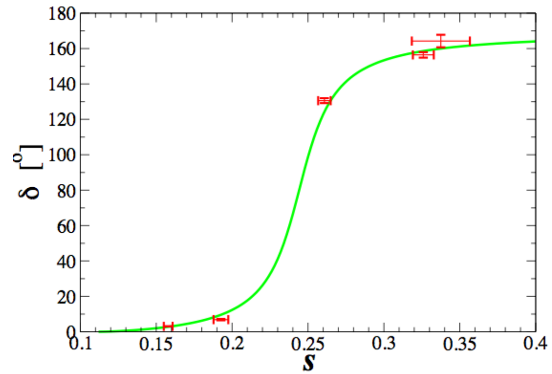
- Most of mesons are hadronic resonances
- Only ρ meson has been treated as resonance on lattice !

  m and Γ from BW-fit of the elastic phase shift δ

  (resonable agreement with exp)

Lang, Mohler, S. P. , Vidmar (2011)

$\pi\,\pi \rightarrow \pi\,\pi$ (I=1, L=1)



Several groups simluated ρ:

Aoki et al. (2007), Gockeler et al (2008), Feng et al, Frison et al. (2010), Feng et al (2011), Aoki et al (2011), Lang et al (2011), Pelissier et al (2011)

- IDEA: follow analogous approach for other meson resonances
  - none yet treated as resonance
  - $m=E_n(L)$ assumed up to now

we study phase shifts in channels

→ $D\pi$ scattering : I = 1/2, s - wave : $D_0^*(2400)$ $[J^P = 0^+]$

$D^*\pi$ scattering : I = 1/2, s - wave : $D_1(2420)$, $D_1(2430)$ $[J^P = 1^+]$

→ $K\pi$ scattering I = 1/2, s - wave : κ, $K_0^*(1430)$ $[J^P = 0^+]$

↗ $K\pi$ scattering I = 1/2, p - wave : $K^*(892)$, $K^*(1410)$, $K^*(1680)$ $[J^P = 1^-]$

→ $K\pi$ scattering : I = 3/2, s - wave (no resonances ?)

$K\pi$ scattering : I = 3/2, p - wave (no resonances ?)

Very few lattice studies up to now:

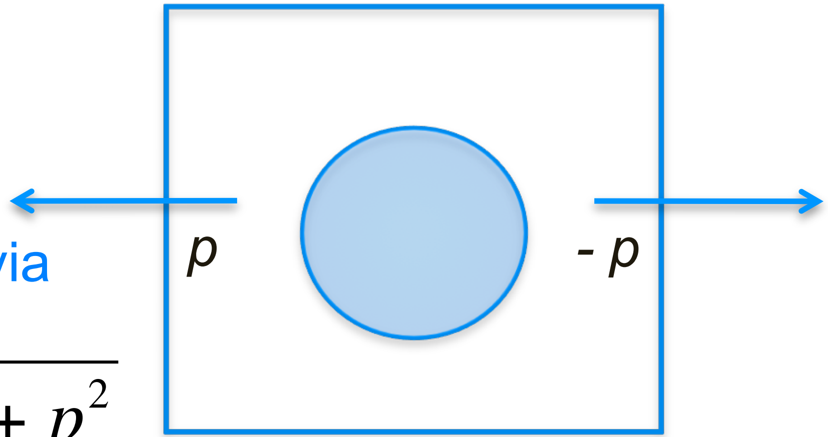
δ directly: →

- only a_0 (δ for small p) for two channels indicated by arrow studied
- other channels *not* studied
- δ away from threshold *not* studied in any of these channels

δ indirectly: →

- from semi-leptonic D→π and K→π form factors f_0 [Flynn & Nieves 2007]

Extracting δ(p) from E_n at $p_1+p_2=0$ [Luscher]



- extract $E_n(L)$
- E_n renders p in "outside" region via

$$E = \sqrt{s} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

- p contains info on δ(p)

$$\tan\delta(s) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)} \qquad q \equiv \frac{L}{2\pi} p$$

$$Z_{00}(1;q^2) \equiv \sum_{\vec{n}\in N^3} \frac{1}{\vec{n}^2 - q^2}$$

Lattice simulation

- 280 gauge config with dynamical u,d quarks (generated by A. Hasenfratz) thanks !!

$$N_f = 2 \quad a = 0.1239 \pm 0.0013\ fm \quad a^{-1} = 1.58 \pm 0.02\ GeV$$

$$N_L^3 \times N_T = 16^3 \times 32 \qquad L \approx 2\ fm \qquad T = 4\ fm \qquad m_\pi \approx 266\ MeV$$

- dynamical u, d , valence u,d,s : Improved Wilson Clover

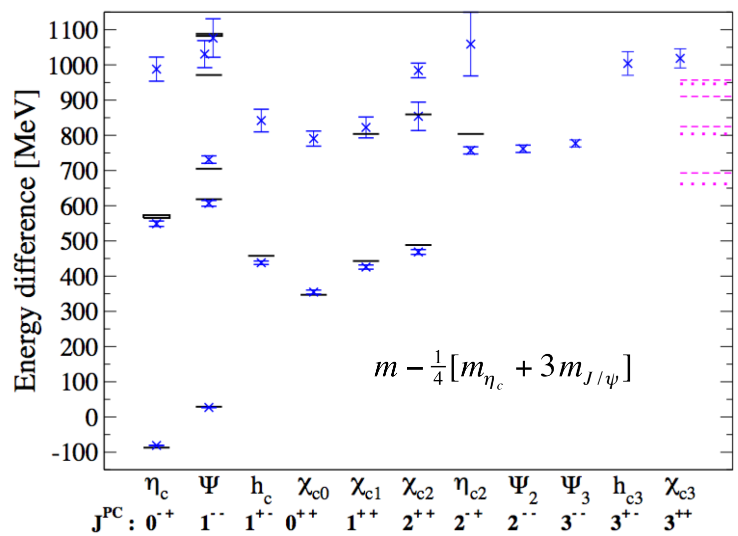
  valence c: Fermilab method [El-Khadra et al. 1997]

[Mohler, S. P., Woloshyn, to be published]

a set using r0

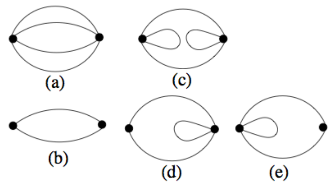
m_s set using ϕ

m_c set using $\frac{1}{4}[M_2(\eta_c) + 3M_2(J/\psi)]_{lat} = \frac{1}{4}[M(\eta_c) + 3M(J/\psi)]_{exp}$



- heavy quark treatment tested on charmonium with satisfactory results

Distillation method for contractions



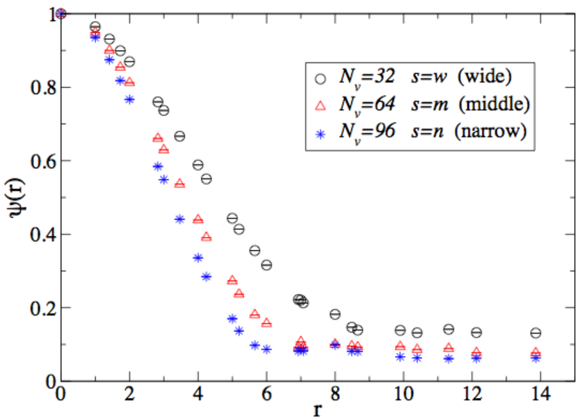
Small N_L , N_T and only one ensemble allow us:

to use the very powerful distillation method [Peardon et al. (2009)]

separable quark smearing, makes backtracking contractions less costly

$$q_s = \sum_{k=1}^{Nv} v^{(k)} v^{(k)+} q \qquad N_v = 96, 64, 32$$

$$\nabla^2 v^{(k)} = \lambda^{(k)} v^{(k)}$$



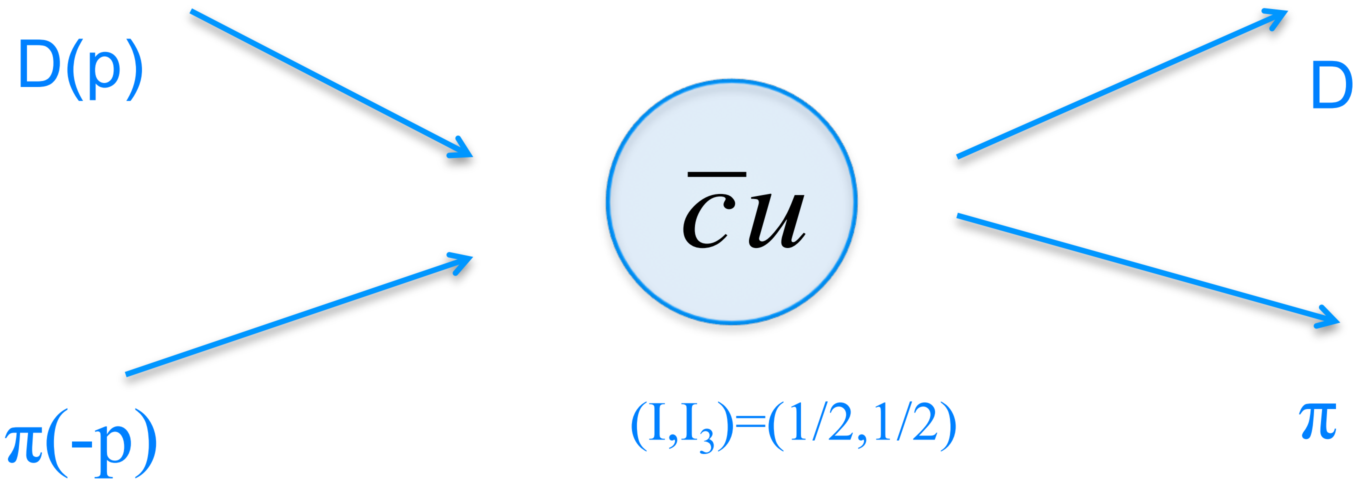
Variational method to extract spectrum

$$C_{ij}(t) = \langle O_i(t) O_j^{+}(0) \rangle$$

$$C(t)\, \vec{\psi}^{(n)}(t) = \lambda^{(n)}(t)\, C(t_0)\, \vec{\psi}^{(n)}(t)$$

$$\lambda^{(n)}(t) \propto e^{-E_n (t - t_0)}$$

Dπ and D*π scattering
&
charm-light resonances



D-meson resonances: brief introduction

- only 1S and 1P $\bar{c}u$ states well established in exp

  for $m_c=\infty$ [Isgur & Wise, 1991] :

  - two 1P states decay only in S-wave ➔ broad

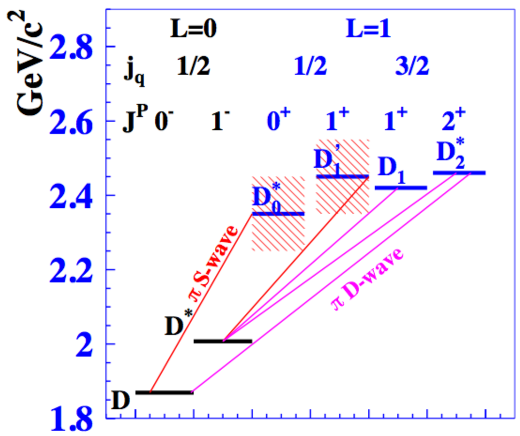
    we treat those two as resonances

  - two 1P states decay only in D-wave ➔ narrow in exp

    ➔ "stable" on our lat

    below D-wave th. D(1)π(-1)

    we treat those as stable: M=E(L)



taken from
Belle PRD(2004)

- radial and orbital excitations [Babar 2010]:

  poorly known in exp (need confirmation), O=quark-antiquark

D π scattering

[Mohler, S. P., Woloshyn, to be published]

Dπ scattering : I=1/2, s-wave, $J^P=0^+$

$D^*_0(2400)$ $\quad \exp: M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u$?

$D_{s0}(2317)$ $\quad \exp: M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s$?

degeneracy between non-strange and strange partners
not naively expected for conventional quark-antiquark

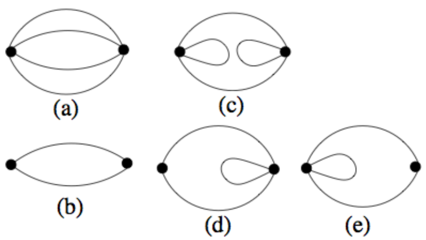
interesting to see if lattice QCD reproduces correct masses
and widths of these two states

[Mohler, S. P., Woloshyn, to be published]

Dπ scattering: I=1/2, s-wave, $J^P=0^+$

exp: $D_0^*(2400)$

interpolators : 4 quark-antiquark, 2 meson-meson



$\bar{q}q$

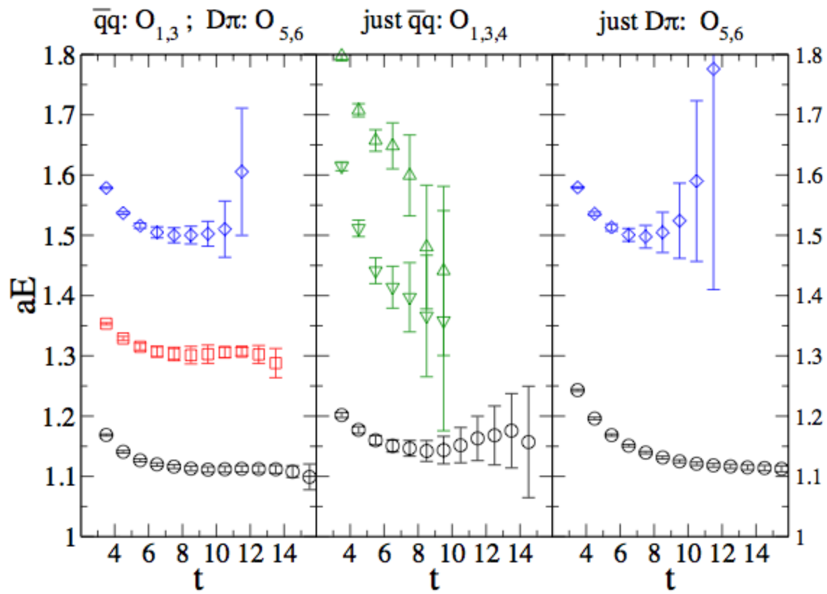
$\bar{q}\gamma_i\overrightarrow{\nabla}_i q$

$\bar{q}\gamma_t\gamma_i\overrightarrow{\nabla}_i q$

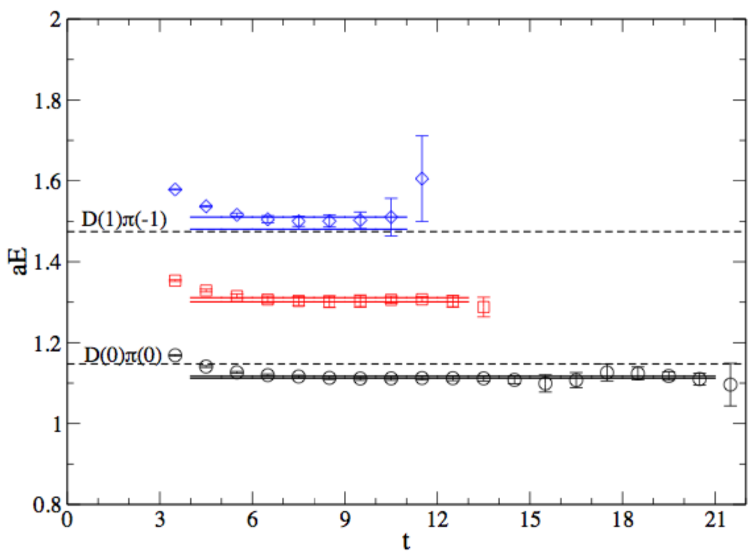
$\bar{q}\overleftarrow{\nabla}_i\overrightarrow{\nabla}_i q$

$$\sqrt{\tfrac{2}{3}}D^-(0)\pi^+(0) + \sqrt{\tfrac{1}{3}}\bar{D}^0(0)\pi^0(0)\,,$$

$$\sum_i \sqrt{\tfrac{2}{3}}D^-(\mathbf{e}_i)\pi^+(-\mathbf{e}_i) + \sqrt{\tfrac{1}{3}}\bar{D}^0(\mathbf{e}_i)\pi^0(-\mathbf{e}_i)$$



Dπ scattering: resulting levels and phase shifts



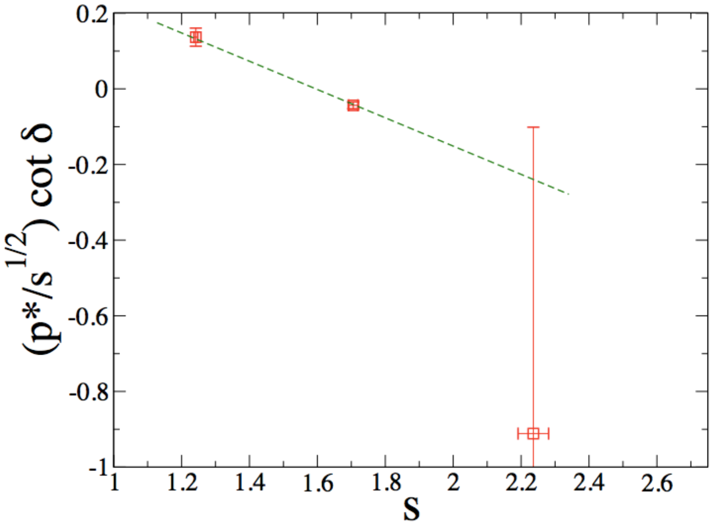
δ~173±12⁰ → $\delta \sim 173 \pm 12^0$

$\delta \sim 103^0$

$\delta \sim 41^0\, i$ → $a_{D\pi}^{I=1/2} = \lim_{p \to 0} \frac{\tan\delta(p)}{p} = 0.81 \pm 0.14\ fm$

[Mohler, S. P., Woloshyn, to be published]

Dπ scattering: extracting resonance parameters for $D_0(2400)$

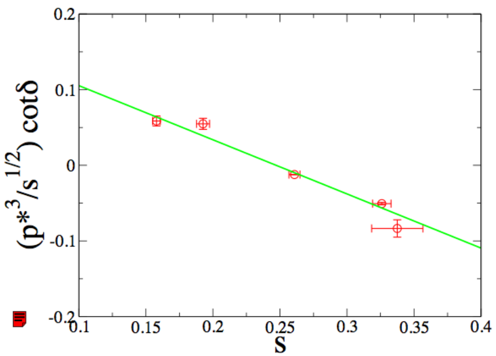


$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta} - 1\right)$$

$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m^2 - s, \quad \Gamma(s) = \frac{p}{s} g^2$$

$$\frac{p}{\sqrt{s}}\cot\delta = \frac{1}{g^2}(m^2 - s)$$

For comparison, our result for rho: there one can check linear behavior.



	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

it would be great to have δ at more values of s, but I will speak about challenges concerning this at the end of my talk

[Mohler, S. P., Woloshyn, to be published]

Dπ scattering : I=1/2, s-wave, $J^P=0^+$

$D^*_0(2400)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u$?

$\bar{c}\bar{s}su \pm \bar{c}\bar{d}du$?

$D_{s0}(2317)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s$?

$\bar{c}\bar{u}us \pm \bar{c}\bar{d}ds$?

Our resulting D0*(2400) mass is in favorable agreement with exp without valence $\bar{s}s$ pair.

D* π scattering

[Mohler, S. P., Woloshyn, to be published]

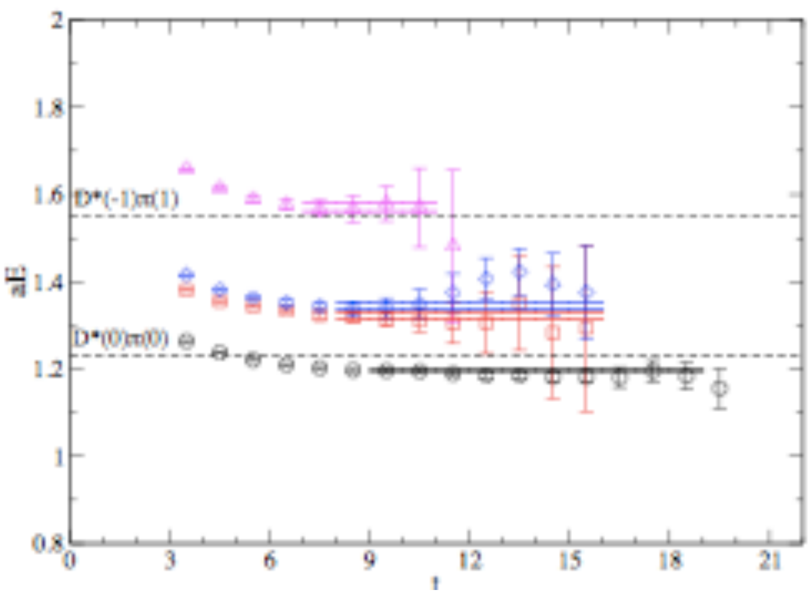
D*π scattering: I=1/2, s-wave, $J^P=1^{++}$

exp:
$D_1(2430)$ broad
$D_1(2420)$ narrow

interpolators : 8 quark-antiquark, 2 meson-meson

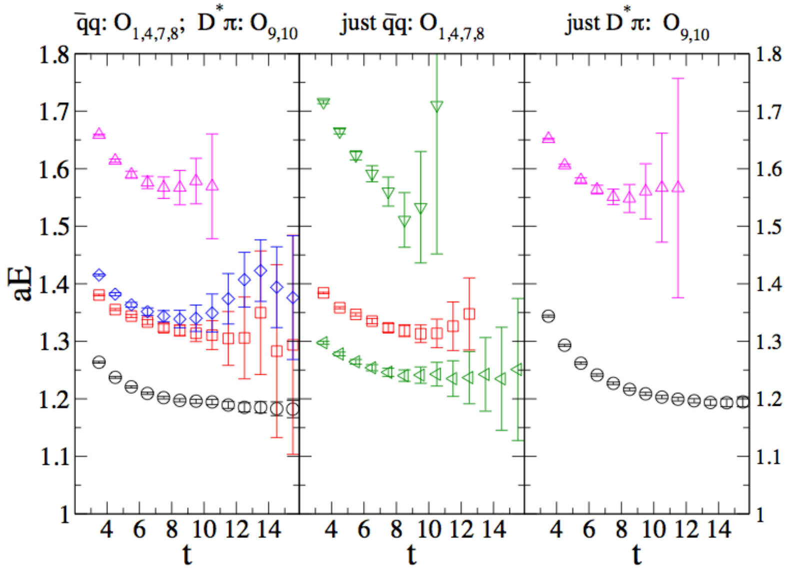
$$\bar{q}\gamma_i\gamma_5 q$$
$$\bar{q}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_i\gamma_5\overrightarrow{\nabla}_i q$$
$$\bar{q}\overleftarrow{\Delta}\gamma_i\gamma_5\overrightarrow{\Delta} q$$
$$\bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q$$
$$\bar{q}|\epsilon_{ijk}|\gamma_5\gamma_j\overrightarrow{D}_k q$$

$$\mathcal{O}_9 = \sqrt{\tfrac{2}{3}}D^{*-}(0)\pi^+(0) + \sqrt{\tfrac{1}{3}}\bar{D}^{*0}(0)\pi^0(0)\,,$$
$$\mathcal{O}_{10} = \sum_i \sqrt{\tfrac{2}{3}}D^{-*}(\mathbf{e}_i)\pi^+(-\mathbf{e}_i) + \sqrt{\tfrac{1}{3}}\bar{D}^{0*}(\mathbf{e}_i)\pi^0(-\mathbf{e}_i)$$



→ $$a_{D^*\pi}^{I=1/2} = \lim_{p\to 0}\frac{\tan\delta(p)}{p} = 0.81 \pm 0.17\ fm$$

analysis/approximation inspired by $m_c=\infty$ limit



[Isgur & Wise, 1991]

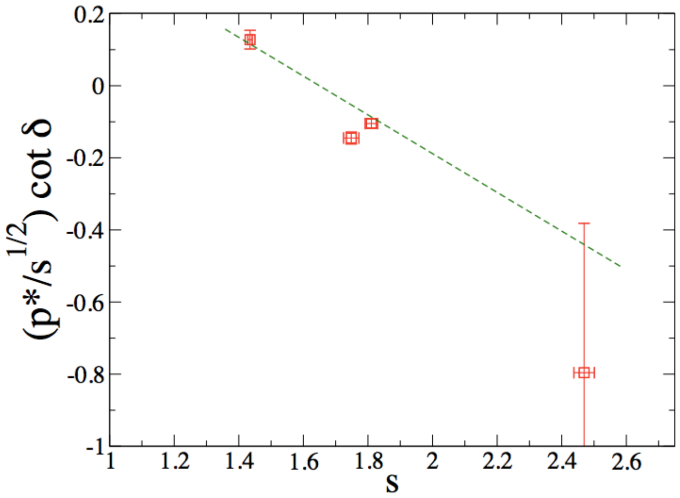
- Blue expected to decay only in S-wave since present only when D(0)pi(0) in the basis.

- Then red expected to decay only in D-wave: stable on our lattice (in HQ limit) as below D-wave threshold

We assume that narrow red state does not affect phase shift of other three levels: BW fit through those three:

blue level: broad $D_1(2430)$

red level: "stable" $D_1(2420)$



$$\Gamma(s) = \frac{p}{s} g^2 \qquad \frac{p}{\sqrt{s}} \cot\delta = \frac{1}{g^2}(m^2 - s)$$

results for $D_1(2430)$

	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

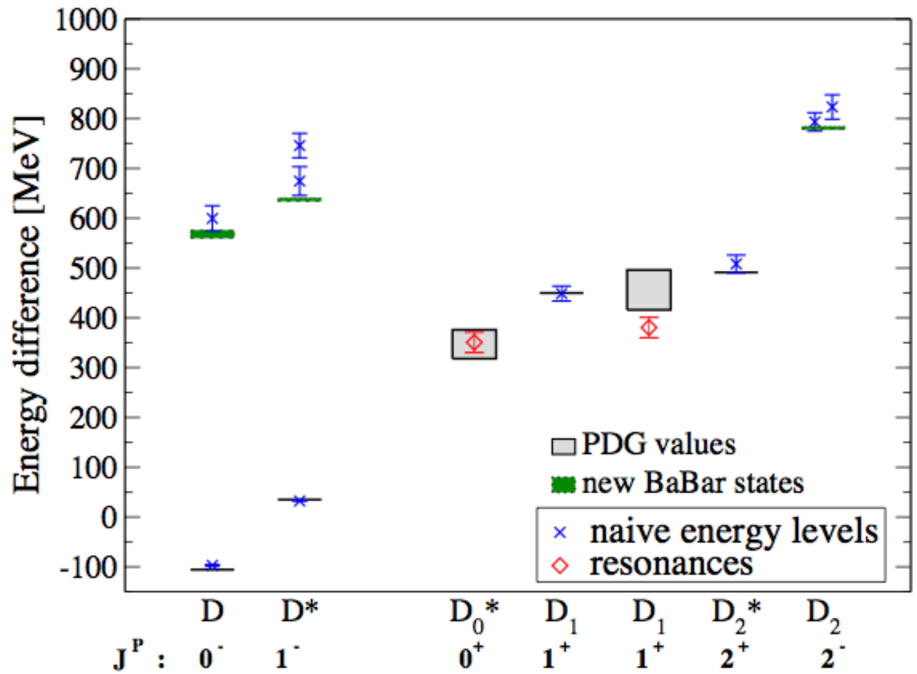
[Mohler, S. P., Woloshyn, to be published]

resulting D-meson spectrum

energy difference ≡

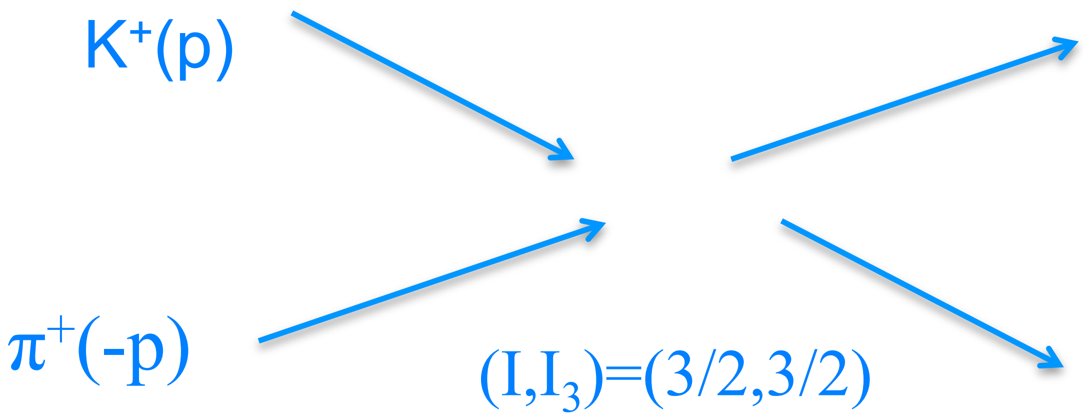
$$m - \tfrac{1}{4}[m(D) + 3m(D^*)] =$$

$$\exp: \tfrac{1}{4}[m(D) + 3m(D^*)] = 1.97\ GeV$$

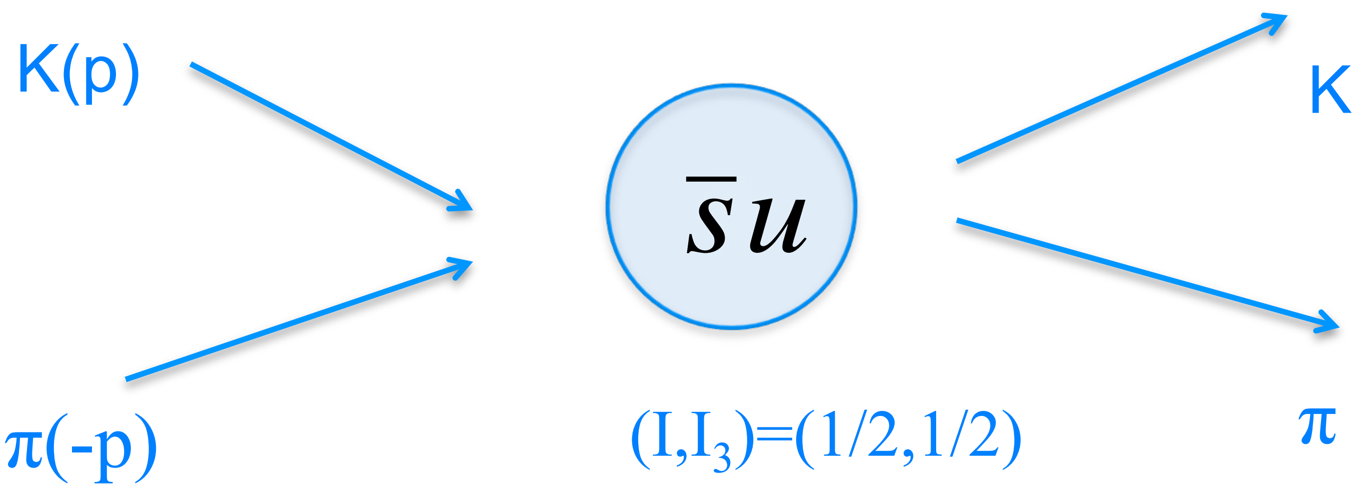


red diamonds: our lat results for resonance masses from scattering study

blue crosses: our lattice results for other resonances: m=E(L), O= qbar q



Kπ scattering & strange resonances



[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204]

K π : interpolators O

$$P = 0, \quad p_i \equiv \frac{2\pi}{L} e_i$$

I=1/2 s-wave

$$\mathcal{O}_1 = \sum_{\mathbf{x}} \bar{s}(x)\, u(x)\,, \qquad \text{(A6)}$$

$$\mathcal{O}_2 = \sum_{\mathbf{x},i} \bar{s}(x)\, \gamma_i \overrightarrow{\nabla}_i\, u(x)\,,$$

$$\mathcal{O}_3 = \sum_{\mathbf{x},i} \bar{s}(x)\, \gamma_t\, \gamma_i \overrightarrow{\nabla}_i\, u(x)\,,$$

$$\mathcal{O}_4 = \sum_{\mathbf{x},i} \bar{s}(x)\, \overleftarrow{\nabla}_i\, \overrightarrow{\nabla}_i\, u(x)\,,$$

$$\mathcal{O}_5 = \sqrt{\tfrac{1}{3}} K^+(\mathbf{0})\pi^0(\mathbf{0}) + \sqrt{\tfrac{2}{3}} K^0(\mathbf{0})\pi^+(\mathbf{0})$$

$$\mathcal{O}_6 = \sum_i \left[\sqrt{\tfrac{1}{3}} K^+(\mathbf{p}_i)\pi^0(-\mathbf{p}_i) + \sqrt{\tfrac{2}{3}} K^0(\mathbf{p}_i)\pi^+(-\mathbf{p}_i)\right] + (\mathbf{p}_i \leftrightarrow -\mathbf{p}_i)\,,$$

$$\mathcal{O}_7 = \sum_i \left[\sqrt{\tfrac{1}{3}} K_i^{*+}(\mathbf{0})\rho_i^0(\mathbf{0}) + \sqrt{\tfrac{2}{3}} K_i^{*0}(\mathbf{0})\rho_i^+(\mathbf{0})\right]\,,$$

$$\mathcal{O}_8 = \sum_i \left[\sqrt{\tfrac{1}{3}} \mathrm{K_1}_i^+(\mathbf{0})\, a_{1\,i}^0(\mathbf{0}) + \sqrt{\tfrac{2}{3}} \mathrm{K_1}_i^0(\mathbf{0})\, a_{1\,i}^+(\mathbf{0})\right]\,.$$

I=1/2, p-wave

$$\mathcal{O}_{1,i} = \sum_{\mathbf{x}} \bar{s}(x)\, \gamma_i\, u(x)\,,$$

$$\mathcal{O}_{2,i} = \sum_{\mathbf{x}} \bar{s}(x)\, \gamma_t \gamma_i\, u(x)\,,$$

$$\mathcal{O}_{3,i} = \sum_{\mathbf{x},j} \bar{s}(x)\, \overleftarrow{\nabla}_j\, \gamma_i\, \overrightarrow{\nabla}_j\, u(x)\,,$$

$$\mathcal{O}_{4,i} = \sum_{\mathbf{x}} \bar{s}(x)\, \tfrac{1}{2}\left[\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\right] u(x)\,,$$

$$\mathcal{O}_{5,i} = \sum_{\mathbf{x},j,k} \epsilon_{ijk}\, \bar{s}(x)\gamma_j\gamma_5\, \tfrac{1}{2}\left[\overrightarrow{\nabla}_k - \overleftarrow{\nabla}_k\right] u(x)\,,$$

$$\mathcal{O}_{6,i} = \sqrt{\tfrac{1}{3}} K^+(\mathbf{p}_i)\pi^0(-\mathbf{p}_i) + \sqrt{\tfrac{2}{3}} K^0(\mathbf{p}_i)\pi^+(-\mathbf{p}_i) - (\mathbf{p}_i \leftrightarrow -\mathbf{p}_i)$$

I=3/2 s-wave

$$\mathcal{O}_5 = K^+(\mathbf{0})\pi^+(\mathbf{0})$$

$$\mathcal{O}_6 = \sum_i K^+(\mathbf{p}_i)\pi^+(-\mathbf{p}_i) + K^+(-\mathbf{p}_i)\pi^+(\mathbf{p}_i)\,,$$

$$\mathcal{O}_7 = \sum_i K_i^{*+}(\mathbf{0})\rho_i^+(\mathbf{0})\,,$$

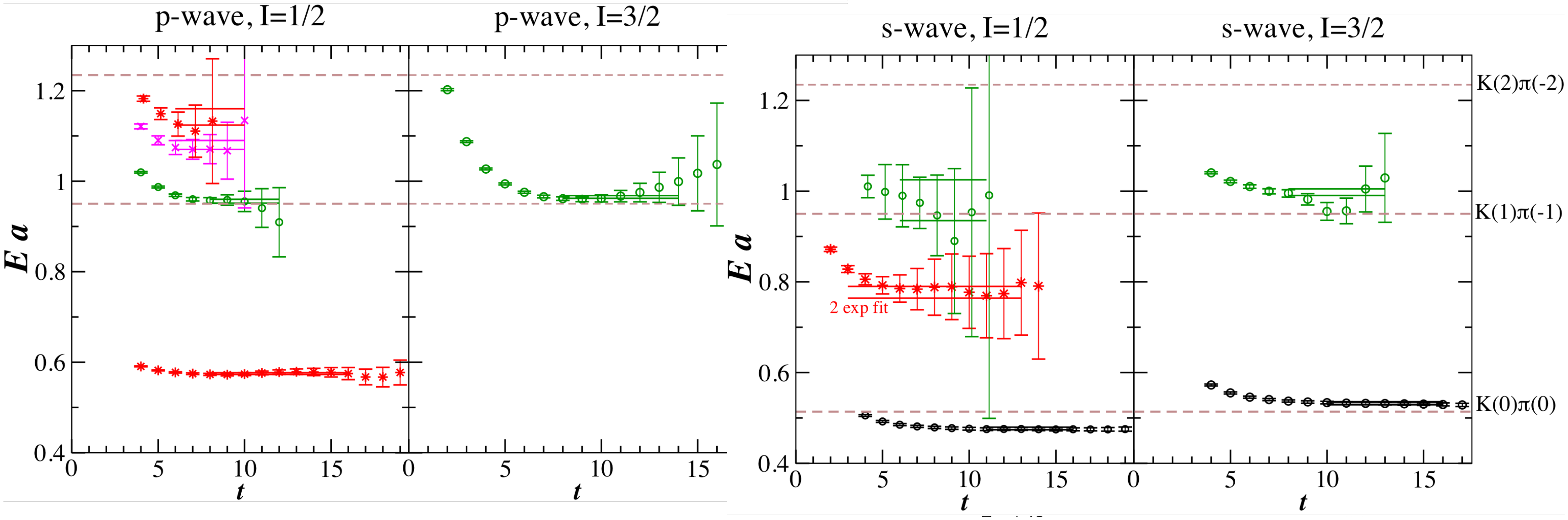
$$\mathcal{O}_8 = \sum_i \mathrm{K_1}_i^+(\mathbf{0})\, a_{1\,i}^+(\mathbf{0})\,.$$

I=3/2 p-wave

$$\mathcal{O}_{6,i} = K^+(\mathbf{p}_i)\pi^+(-\mathbf{p}_i) - K^+(-\mathbf{p}_i)\pi^+(\mathbf{p}_i)$$

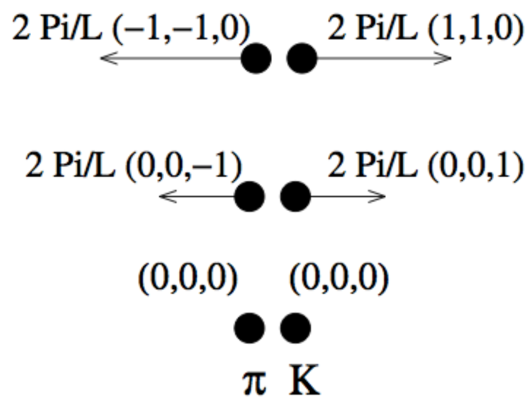
[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204]

K π : energy levels below inelastic threshold



K*(1680)
K*(1410)
K*(892)

K_0*(1430)
no level near κ: discussed later on



1/a ~ 1.6 GeV

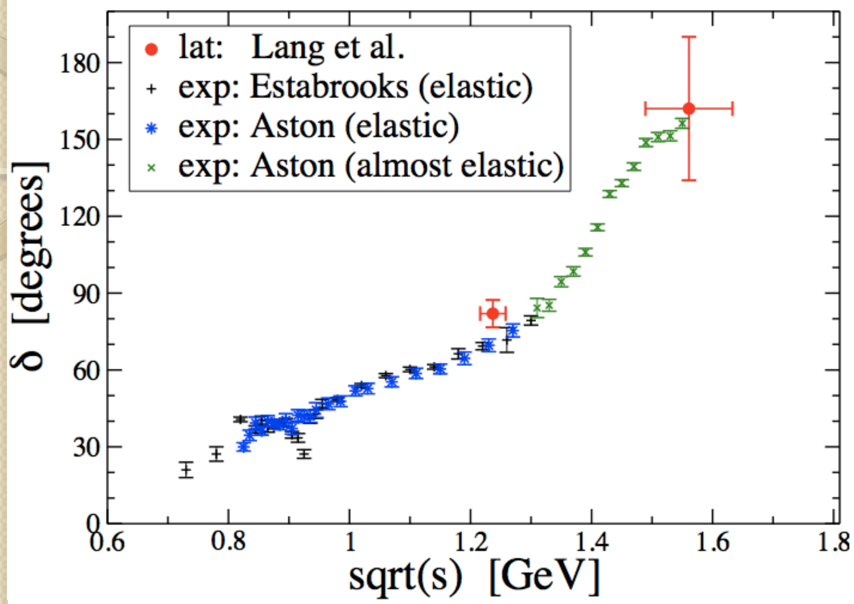
K π phase shifts

[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204]

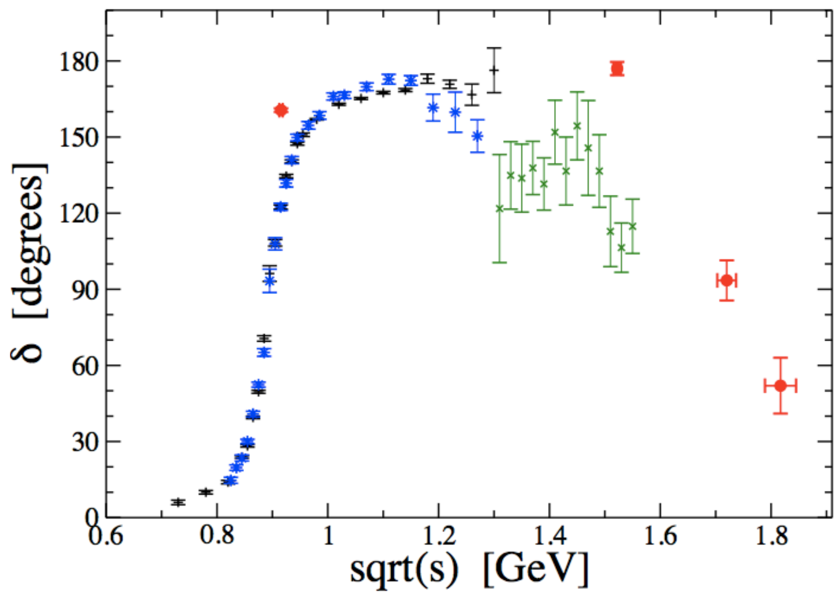
$m_\pi \approx 266\ MeV \quad m_K \approx 552\ MeV$

$$\sqrt{s} = \sqrt{E^2 + P^2} = M_{K\pi}$$

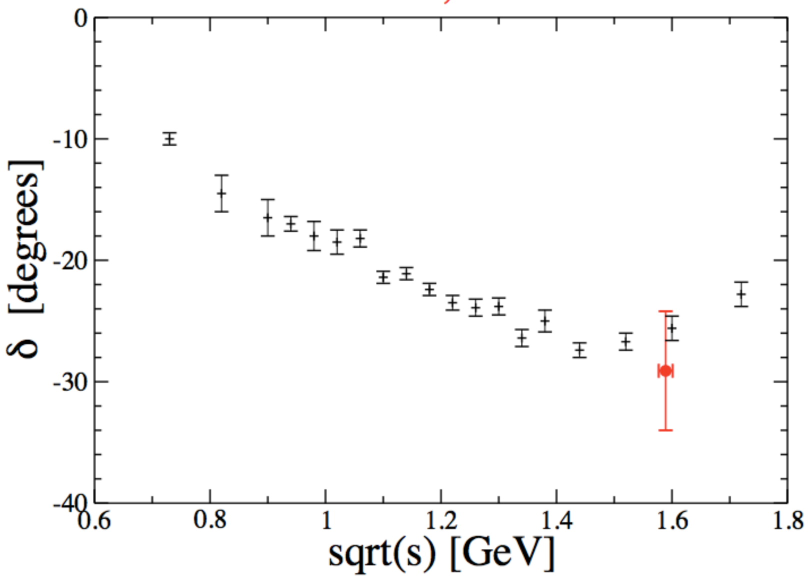
s-wave, I=1/2



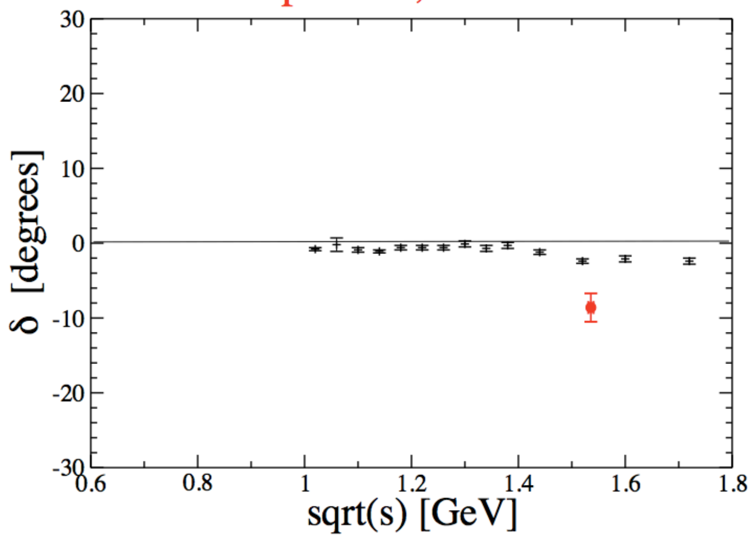
p-wave, I=1/2



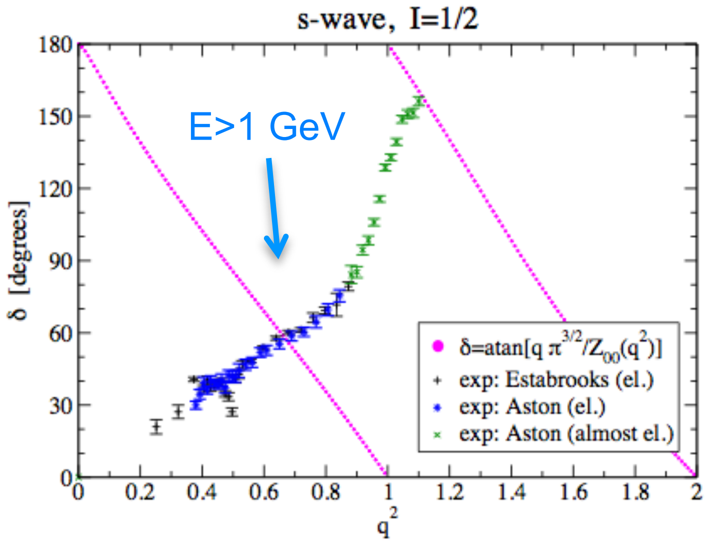
s-wave, I=3/2

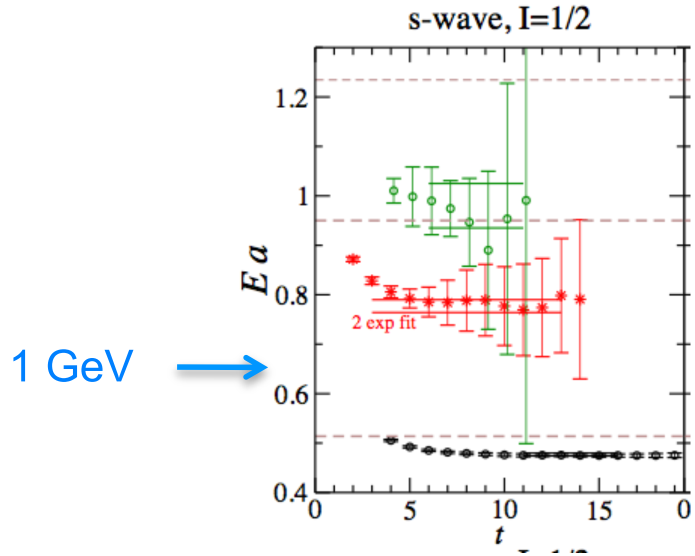


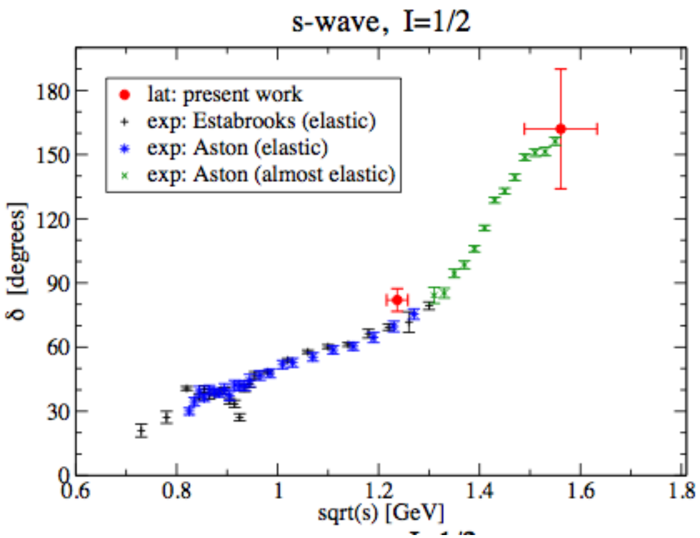
p-wave, I=3/2



cautionary remarks on $K_0^*(800)$ or κ







- we do not see any other level below 1 GeV except for K(0)pi(0)
- so we do not see additional level related to kappa
- this is expected for our lattice L~2 fm assuming experimental δ, since experimental δ does not reach 90^0 below 1 GeV
- conclusion: we qualitatively agree with experimental phase shift but we can not conclude whether kappa pole exists or not

s-wave scattering lengths a_0 for

$K\pi$, $D\pi$, $D^*\pi$

not the main objective of our simulation (since only one mpi used)

[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204]

s-wave scattering lengths

$$a_0 \equiv \lim_{p\to 0} \frac{\tan\delta(p)}{p}$$

at our mpi=266 MeV, mK, mD

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

our lat. sim.	a_0 [fm]	a_0 / μ [GeV^{-2}]
K π , I=3/2	-0.140 ± 0.018	-3.94 ± 0.52
K π , I=1/2	0.636 ± 0.090	17.9 ± 2.5
D π , I=1/2	0.81 ± 0.14	17.7 ± 3.1
D*π , I=1/2	0.81 ± 0.17	17.6 ± 3.6

→ $r_{eff} \sim 0$

[Weinberg's current algebra 1966]
scattering of pion on any particle

$$\frac{a_0^{I=1/2}}{\mu} = \frac{1}{2\pi F_\pi^2} \approx 10\ GeV^{-2}$$

$$\frac{a_0^{I=3/2}}{\mu} = -\frac{1}{2} \times \frac{a_0^{I=1/2}}{\mu}$$

a_0 : comparison with others

$$a_0 \equiv \lim_{p\to 0} \frac{\tan\delta(p)}{p}$$

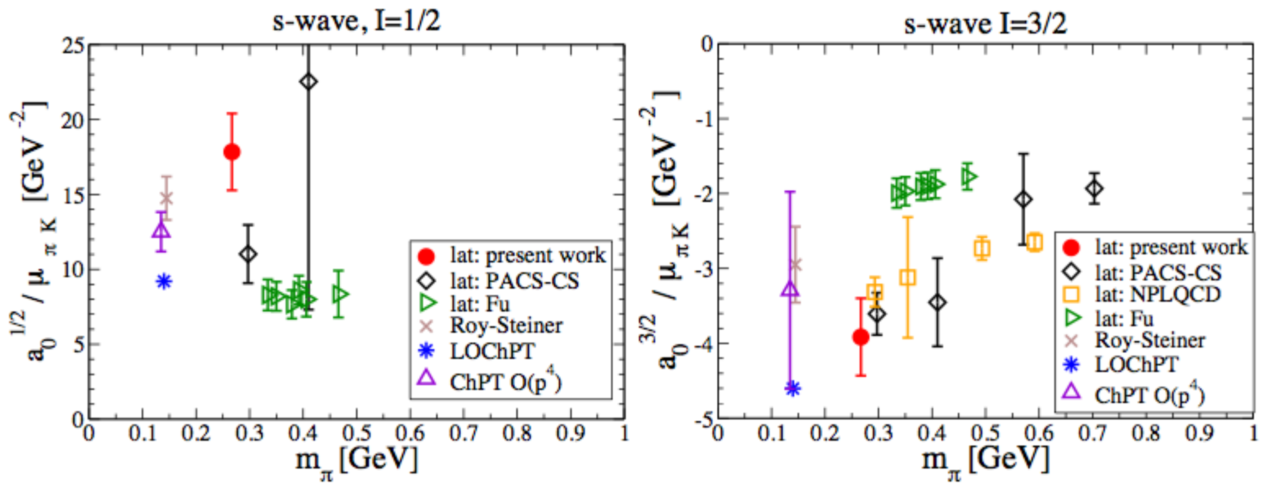
a_0/μ compared as not dependent of mpi in LOChPT

$$\frac{a_0^{I=1/2}}{\mu} = \frac{1}{2\pi F_\pi^2} \approx 10\ GeV^{-2}$$

$$\frac{a_0^{I=3/2}}{\mu} = -\frac{1}{4\pi F_\pi^2}$$

[Weinberg's current algebra 1966]
scattering of pion on any particle

- Kπ



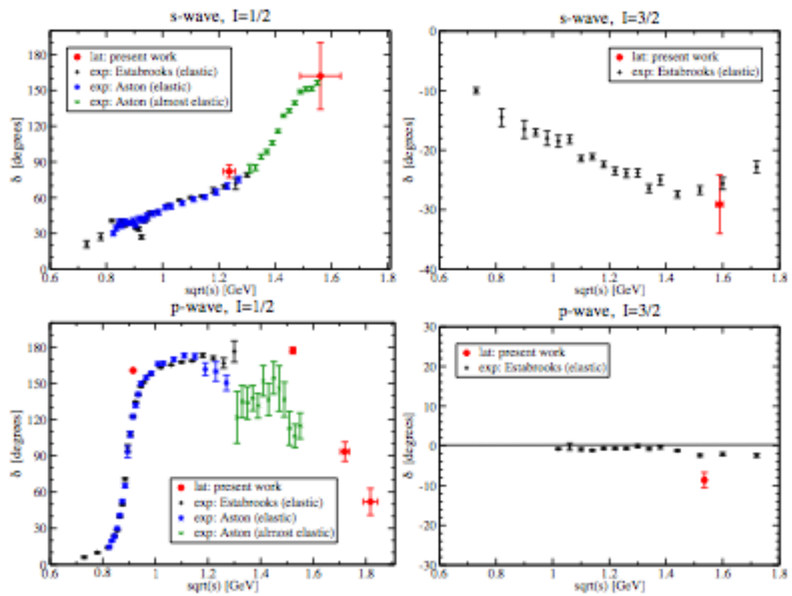
- Dπ

only indirect lattice determination from D➔π semileptonic form factors [Flynn, Nieves 2007]

I=1/2	our result	Flynn & Nieves
a_0/μ [GeV^{-2}]	17.7 ± 3.1	15.9 ± 2.2

Need δ for at more $s=E^2-P^2$!!

- P=0
  $s=E^2$

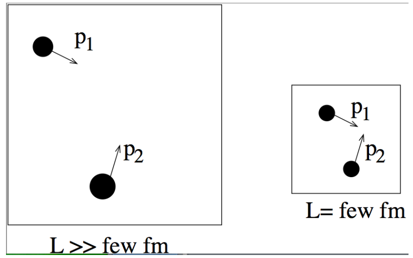


- P≠0
  $s=E^2-P^2$

if m1≠m2: this is good idea for p-wave

brings difficulties for s-wave

Relations between E and δ:



- **P=0** : [Luscher 1986]

P=p1+p2 (total mom.)

- **P≠0 , : m1=m2**

two mesons

[Rummukainen, Gottlieb 1995]

[Kim, Sharpe, Sachrajda 2005]

[Feng, Jansen, Renner 2011]

- **P≠0 , m1≠m2**

[Davoudi & Savage, PRD84, (2011) 114502] : generalized Z_{lm} written

[Fu, PRD85 (2011) 014506]: : A1 irrep

→ [Leskovec & S. P. , PRD85 (2012) 114507] : **P=(0,0,1), (1,1,0)** all irreps

[Doring et al, arXiv: 1205.4838] : P=(1,1,1) and coupled channel

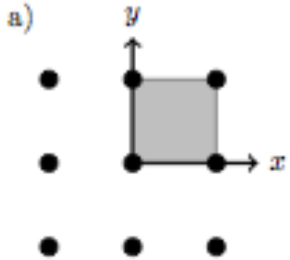
[Gockeler et al, arXiv 1206.4141] : also meson baryon

[Hansen & Sharpe, arXiv: 1204.0826]: coupled ch. LL restricted to s-wave

symmetries of allowed p in CMF for m1≠m2

mesh indicates allowed p in CMF, following from periodic BC

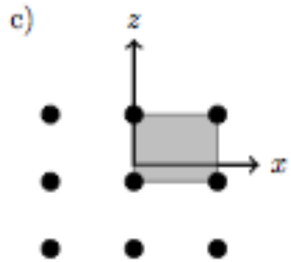
P=(0,0,0)



O_h

assuming d-wave and higher waves negligible:

P=(0,0,1)

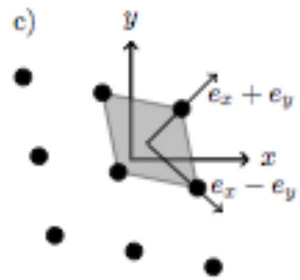


C_{4v}

$$\Gamma^{(0)} = A_1$$
$$\Gamma^{(1)} = A_1 \oplus E$$

inversion not symmetry of the mesh:
s and p wave can mix

P=(1,1,0)



C_{2v}

$$\Gamma^{(0)} = A_1$$
$$\Gamma^{(1)} = A_1 \oplus B_3 \oplus B_2$$

[Leskovec & S. P., PRD85 (2012) 114507]

Recipes for p-wave ($P\neq 0$, $m_1\neq m_2$): Luscher formulae and meson-meson interpolators

P=(0,0,1)

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\,\gamma\, q}{Z^{\mathbf{d}}_{00}(1;q^2) - \frac{1}{\sqrt{5}}\, q^{-2}\, Z^{\mathbf{d}}_{20}(1;q^2)}$$

$\Gamma^{(0)} = A_1$
$\Gamma^{(1)} = A_1 \oplus E$

E irrep

$$(\mathcal{O}_E^{P_1P_2})^I_k = P_1(e_z+e_k)P_2(-e_k) - P_1(e_z-e_k)P_2(e_k)\ , \qquad k=x,\ y$$

$$(\mathcal{O}_E^{P_1P_2})^{II}_k = P_1(e_z+u_k)P_2(-u_k) - P_1(e_z-u_k)P_2(u_k)\ , \qquad u_k = e_x+e_y,\ e_x-e_y$$

P=(1,1,0)

$\Gamma^{(0)} = A_1$
$\Gamma^{(1)} = A_1 \oplus B_3 \oplus B_2$

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\,\gamma\, q}{Z^{\mathbf{d}}_{00}(1;q^2) - \frac{1}{\sqrt{5}}\, q^{-2}\, Z^{\mathbf{d}}_{20}(1;q^2) - \frac{\sqrt{6}}{\sqrt{5}}\, q^{-2}\, Im[Z^{\mathbf{d}}_{22}(1;q^2)]}$$

B2 irrep

$$(\mathcal{O}_{B_2}^{P_1P_2})^I = P_1(e_x)P_2(e_y) - P_1(e_y)P_2(e_x)$$

$$(\mathcal{O}_{B_2}^{P_1P_2})^{II} = P_1(e_x+e_z)P_2(e_y-e_z) - P_1(e_y+e_z)P_2(e_x-e_z) + \{e_z \leftrightarrow -e_z\}$$

P=(1,1,0)

$$\tan\delta_1(p^*) = \frac{\pi^{3/2}\,\gamma\, q}{Z^{\mathbf{d}}_{00}(1;q^2) + \frac{2}{\sqrt{5}}\, q^{-2}\, Z^{\mathbf{d}}_{20}(1;q^2)}$$

B3 irrep

$$(\mathcal{O}_{B_3}^{P_1P_2})^I = P_1(e_x+e_y+e_z)P_2(-e_z) - P_1(e_x+e_y-e_z)P_2(e_z)$$

$$(\mathcal{O}_{B_3}^{P_1P_2})^{II} = P_1(e_x+e_z)P_2(e_y-e_z) + P_1(e_y+e_z)P_2(e_x-e_z) - \{e_z \leftrightarrow -e_z\}$$

Generalized Z_{lm} for P≠0 , m1≠m2

$$Z^{\mathbf{d}}_{lm}(s;q^2) \equiv \sum_{\mathbf{r}\in P_d} \frac{\mathcal{Y}_{lm}(\mathbf{r})}{(\mathbf{r}^2 - q^2)^s}$$

[Davoudi & Savage, PRD84, (2011) 114502]

$$P_d = \{\ \mathbf{r}\ \ |\ \ \mathbf{r} = \hat{\gamma}^{-1}(\mathbf{n} - \tfrac{1}{2}\ A\ \mathbf{d})\ \} \qquad A \equiv 1 + \frac{m_1^2 - m_2^2}{E^{*2}}$$

[Leskovec & S. P. , PRD85 (2012) 114507]

$$\begin{aligned} Z^{\mathbf{d}}_{lm}(1;q^2) &= \gamma \int_0^1 dt\ e^{tq^2} \sum_{\mathbf{n}\in Z^3,\mathbf{n}\neq 0} (-1)^{A\mathbf{n}\cdot\mathbf{d}}\ (-i)^l\ \mathcal{Y}_{lm}(-\frac{\pi\hat{\gamma}\mathbf{n}}{t})(\frac{\pi}{t})^{3/2} e^{-(\pi\hat{\gamma}\mathbf{n})^2/t} \\ &+ \gamma \int_0^1 dt\ (e^{tq^2} - 1)\left(\frac{\pi}{t}\right)^{3/2} \frac{1}{\sqrt{4\pi}} \delta_{l0}\delta_{m0} - \gamma\pi\delta_{l0}\delta_{m0} \\ &+ \sum_{\mathbf{r}\in P_d} \mathcal{Y}_{lm}(\mathbf{r}) \frac{e^{-(r^2-q^2)}}{r^2 - q^2} \end{aligned}$$

[Leskovec & S. P., PRD85 (2012) 114507]

Difficulty for s-wave ($P\neq0$, $m_1\neq m_2$): Luscher formulae and meson-meson interpolators

P=(1,1,0)

A1 irrep

$$[e^{2i\delta_0(p^*)}(M^B_{00}-i)-(M^B_{00}+i)][e^{2i\delta_1(p^*)}(M^B_{11}-i)-(M^B_{11}+i)] = |M_{10}|^2(e^{2i\delta_0(p^*)}-1)(e^{2i\delta_1(p^*)}-1).$$

$$\Gamma^{(0)} = A_1$$
$$\Gamma^{(1)} = A_1 \oplus B_3 \oplus B_2$$

proportional to $Z_{10} \neq 0$
(since inversion is not sym.)

one Luscher's equation, two unknowns: $\delta_0(p)$, $\delta_1(p)$

some suggestions provided in Leskovec & S.P.;
still remains a serious challenge

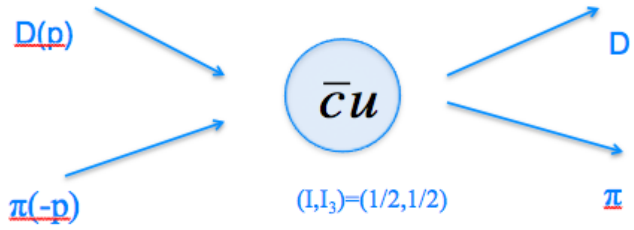
$$(\mathcal{O}^{P_1P_2}_{A_1})^{I} = P_1(e_x+e_y)P_2(0)$$
$$(\mathcal{O}^{P_1P_2}_{A_1})^{II} = P_1(e_x)P_2(e_y)+P_1(e_y)P_2(e_x)$$
$$(\mathcal{O}^{P_1P_2}_{A_1})^{III} = P_1(e_x+e_z)P_2(e_y-e_z)+P_1(e_y+e_z)P_2(e_x-e_z)+\{e_z\leftrightarrow -e_z\}$$
$$(\mathcal{O}^{P_1P_2}_{A_1})^{IV} = P_1(e_x+e_y+e_z)P_2(-e_z)+P_1(e_x+e_y-e_z)P_2(e_z)$$

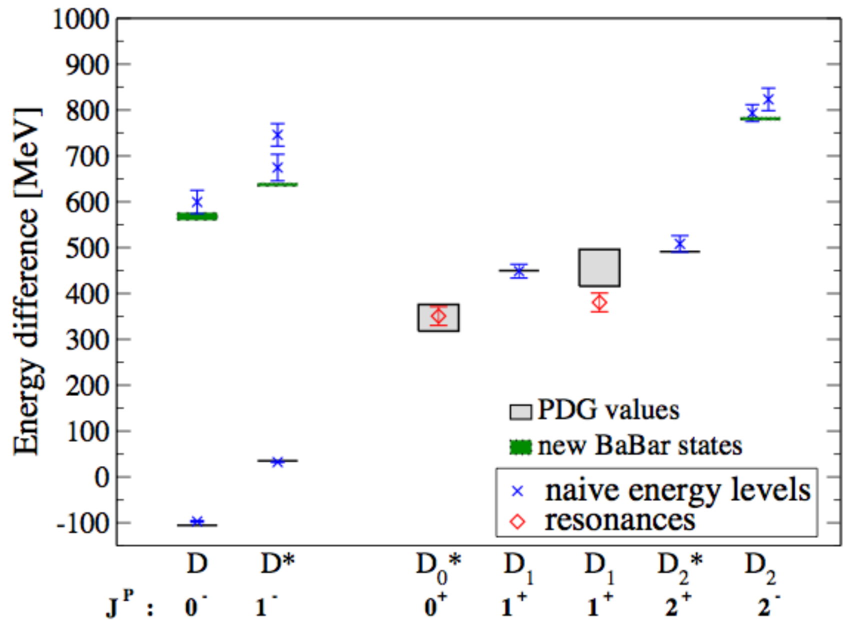
P=(0,0,1) , similar problem

Conclusions

(1) Dπ and D*π scattering at P=0

D-meson resonances





$$\Gamma(s) = \frac{p}{s} g^2$$

D_0^*(2400) resonance
$J^P=0^+$

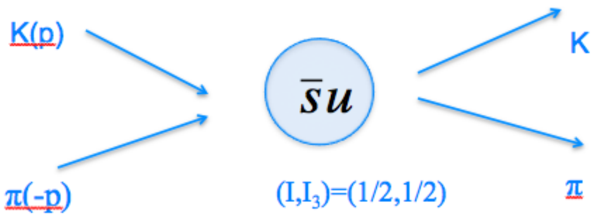
	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

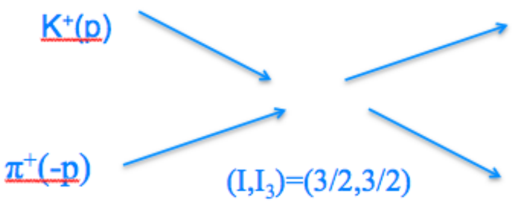
D_1(2430) resonance
$J^P=1^+$

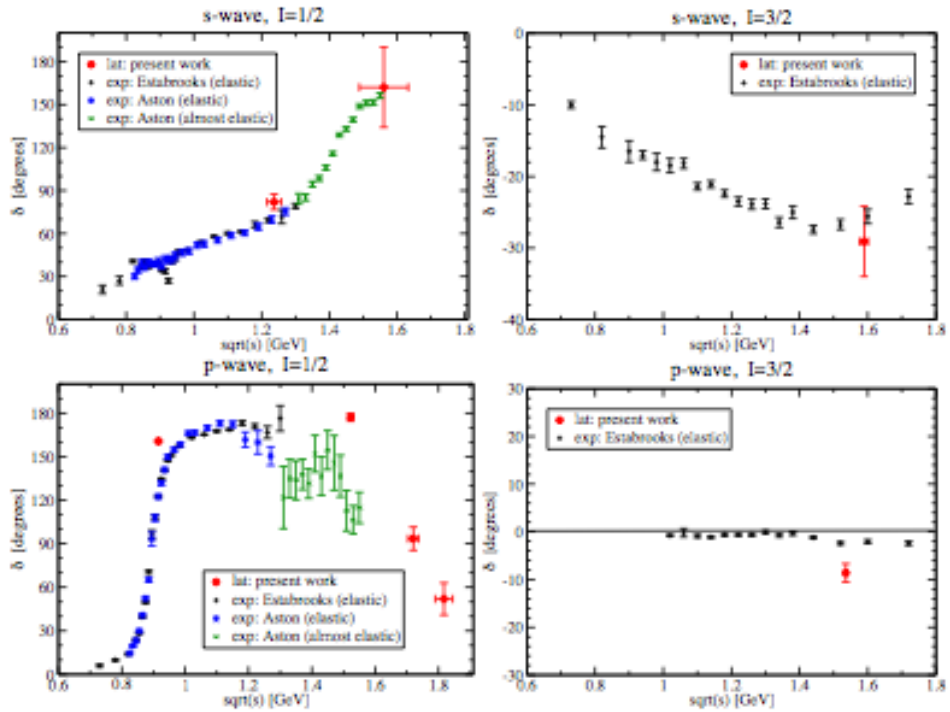
	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

Conclusions (continued)

(2) Kπ scattering at P=0







Conclusions (continued)

(3) meson-meson scattering when **m1≠m2** and **P≠0**

difficulties for <u>s-wave</u>:

there is no irrep where s-wave would not be accompanied by p-wave

good idea for <u>p=wave</u>:

p-wave enters alone (if d-wave and higher partial waves negligible)

I have shown generalized Luscher formula and interpolators

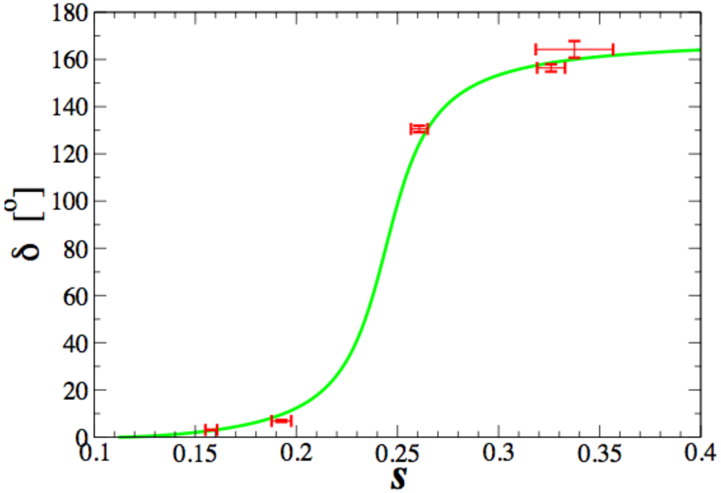
for irreps E, B2, B3 that are useful for P=(0,0,1) and P=(1,1,0)

Backup slides

ρ resonance

[Lang, Mohler, S. P., Vidmar, PRD 2011

π π scattering with L=1 (p-wave) and isospin I=1



$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta} - 1\right)$$

equivalent to real equation

$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m_\rho^2 - s$$

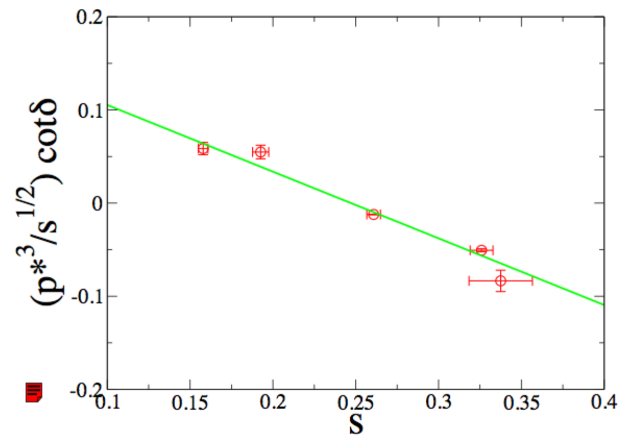
$$\Gamma(s) = \frac{p^3}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}$$

The final relation has two parameters:
m(ρ) and g(ρππ)

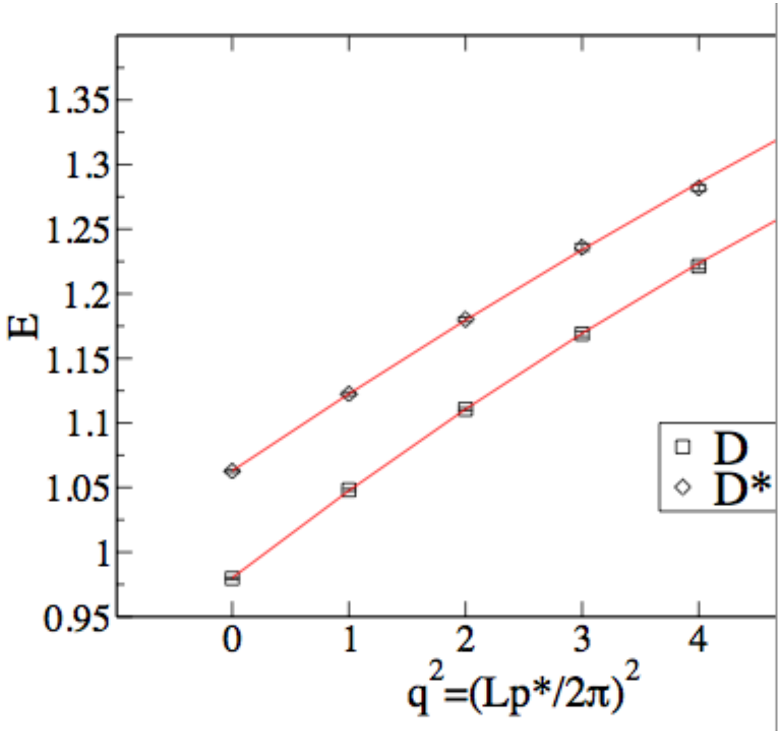
$$L_{\rm eff} = g_{\rho\pi\pi}\sum_{abc}\epsilon_{abc}(k_1 - k_2)_\mu\ \rho^a_\mu(p)\pi^b(k_1)\pi^c(k_2)$$

our final result

lat ($m_\pi = 266\,MeV$)	exp ($m_\pi = 140\,MeV$)
$m_\rho \approx 792 \pm 12$ MeV	$m_\rho = 775$ MeV
$g_{\rho\pi\pi} = 5.13 \pm 0.20$	$g_{\rho\pi\pi} = 5.97$

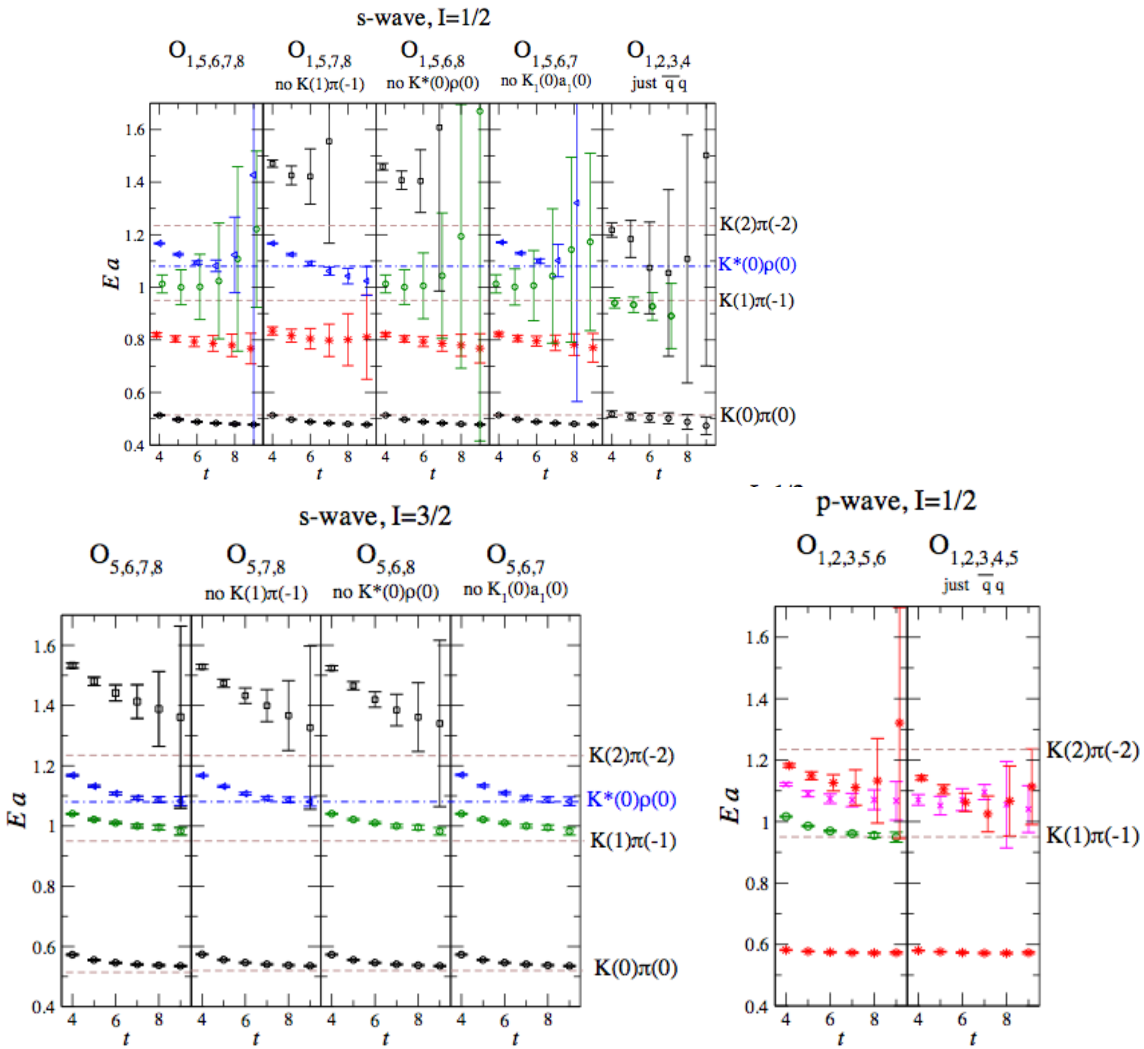


Dispersion relation for D and D*



$$E(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{a^3 W_4}{6}\sum_i p_i^4 - \frac{(\mathbf{p}^2)^2}{8M_4^3} + \ldots$$

K-pi energy levels: dependence on interpolator basis



effective range for K-pi I=3/2 s-wave

p cot(δ)= $\frac{1}{a_\ell^I} + \frac{1}{2} r_\ell^I p^{*2} + \mathcal{O}(p^{*4})$

from ground state

$$a_0^{I=3/2} = -1.13 \pm 0.15\, a = -0.140 \pm 0.018 \text{ fm} \qquad (18)$$

$$\frac{a_0^{I=3/2}}{\mu_{K\pi}} = -3.94 \pm 0.52 \text{ GeV}^{-2} \quad \text{at} \quad m_\pi \simeq 266 \text{ MeV} \ .$$

from two states

$$a_0^{3/2} = -1.12 \pm 0.15\, a = -0.139 \pm 0.018 \text{ fm}$$

$$r_0^{I=3/2} = 1.5 \pm 2.0\, a = 0.19 \pm 0.25 \text{ fm}$$

respresent.	dim	Id	$C_2(e_x+e_y)$	$\sigma(e_x-e_y)$	$\sigma(e_z)$	polynom.	vector $\mathbf{u}$
irred. A_1	1	1	1	1	1	$1\ ,\ x+y$	$\mathbf{0}$, e_x+e_y
irred. A_2	1	1	1	-1	-1	$(l>1)$	$(l>1)$
irred. B_3	1	1	-1	1	-1	z	e_z
irred. B_2	1	1	-1	-1	1	$x-y$	e_x-e_y
$\Gamma^{l=0}$	1	1	1	1	1	Y_{00}	
$\Gamma^{l=1}$	3	3	-1	1	1	Y_{10}, Y_{11}, Y_{1-1}	

Table 1: Characters $\chi(\hat{R}) = \sum_{i=1}^{dim} D(\hat{R})_{ii}$ of representations D for transformations $\hat{R} \in C_{2v}$ (with principal axis e_x+e_y), that leave the mesh P_d in Fig. 2 for $\mathbf{d} = e_x+e_y$ invariant. Representations $A_{1,2}$ and $B_{2,3}$ are irreducible while the representation $\Gamma^{l=1}$ is reducible. Example of polynomials and vectors $\mathbf{u}$ that transform according to these representations are given on the right.

$$\Gamma^{(0)} = A_1$$
$$\Gamma^{(1)} = A_1 \oplus B_3 \oplus B_2$$
$$\Gamma^{(2)} = 2A_1 \oplus A_2 \oplus B_3 \oplus B_2$$

respresent.	dim	Id	$C_4(e_z)$ $C_4^{-1}(e_z)$	$C_2(e_z)$	$\sigma(e_x)$ $\sigma(e_y)$	$\sigma(e_x+e_y)$ $\sigma(e_x-e_y)$	polynom.	vector $\mathbf{u}$
irred. A_1	1	1	1	1	1	1	$1\ ,\ z$	$\mathbf{0}$, e_z
irred. E	2	2	0	-2	0	0	x, y or Y_{11}, Y_{1-1}	e_x, e_y
$\Gamma^{l=0}$	1	1	1	1	1	1	Y_{00}	
$\Gamma^{l=1}$	3	3	1	-1	1	1	Y_{10}, Y_{11}, Y_{1-1}	

Table 2: Characters for transformations $R \in C_{2v}$ (with principal axis e_z), that leave the mesh P_d for $d = e_z$ in Fig. 3 invariant. In addition to irreps A_1 and E, C_{4v} has also A_2 and $B_{1,2}$ but they do not appear for $l = 0, 1$ so we omit them. Example of simple objects that transform according to these representations are given on the right.

$$\Gamma^{(0)} = A_1$$
$$\Gamma^{(1)} = A_1 \oplus E$$
$$\Gamma^{(2)} = A_1 \oplus B_1 \oplus B_2 \oplus E\ .$$