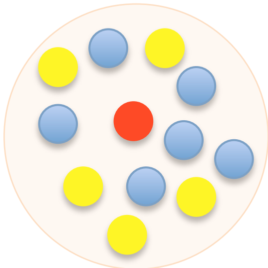
LQCD calculations for hypernuclear physics

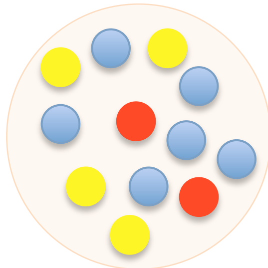
Assumpta Parreño

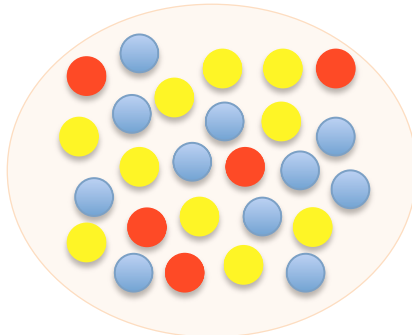
INT program on Lattice QCD studies of excited resonances and multi-hadron systems
Seattle, August 2012

${}^{A}_{Y}Z$



${}^{A}_{YY}Z$





understanding nuclear processes from the underlying theory of strong interactions

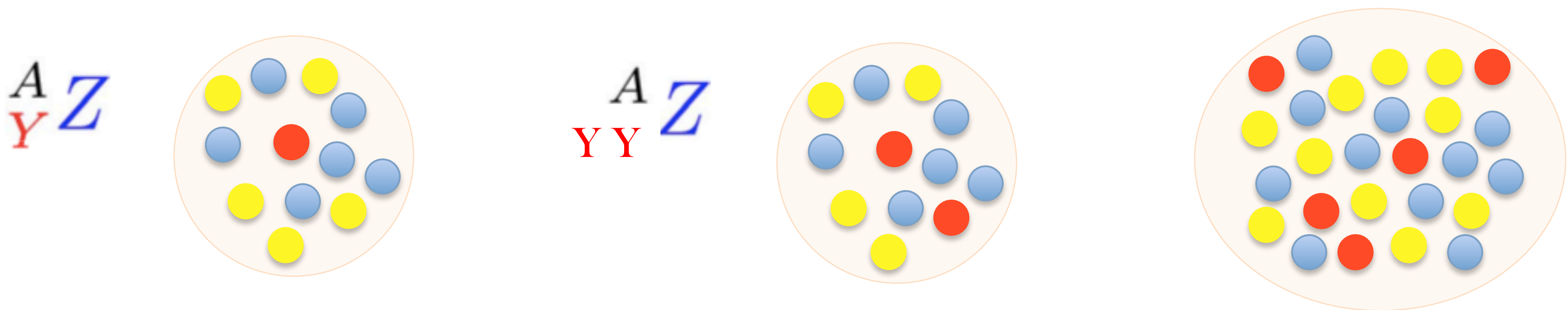
Lüscher 's formalism

motivation

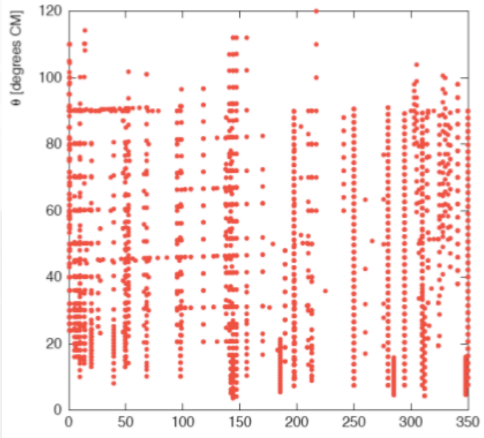
hyperons in nuclei:

✓distinguishible from nucleons
✓glue-like role
✓new spectroscopy
✓source of information about the strong $\Lambda N \rightarrow \Lambda N$ and weak $\Lambda N \rightarrow NN$ interactions

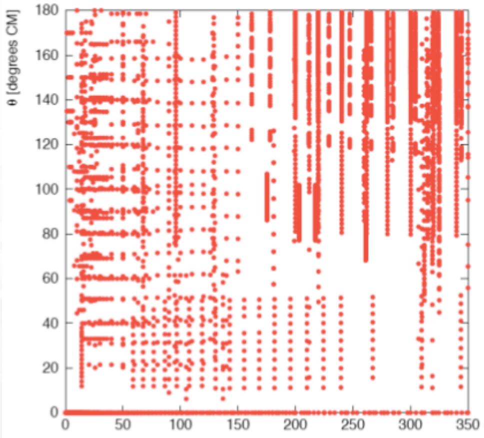
[there are no stable hyperon beams
–unstable against the weak interaction-]



proton-proton



neutron-proton



Strange sector ~ 35 data points
(many pre-1971) with large errors

Λp	# = 12	6.5 MeV < T_{lab} < 50 MeV
$\Sigma^- p \to \Sigma^- p$ Λn $\Sigma^0 n$	# = 6 # = 6 # = 6	9 MeV < Tlab < 12 MeV
$\Sigma^+ p$	# = 4	9 MeV < Tlab < 13 MeV + 3 data from KEK-E289

©Rob Timmermans

Additional information:

YN → Light hypernuclei:

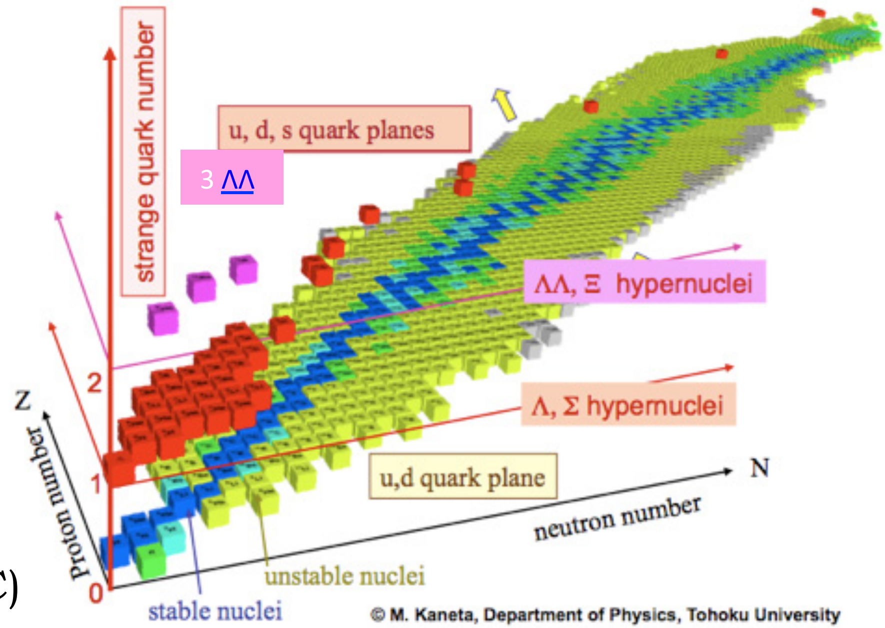
$^3H_\Lambda$, $^4He_\Lambda$, $^4H_\Lambda$, $^5He_\Lambda$

YY → $^6He_{\Lambda\Lambda}$, $^{10}Be_{\Lambda\Lambda}$, $^{13}B_{\Lambda\Lambda}$ …

39 Λ
1 Σ

$pp \to K^+ \Lambda p$ (COSY, Jülich)

$\gamma d \to K^+ n \Lambda$ (CEBAF, ELSA, JLAB, MAMI-C)



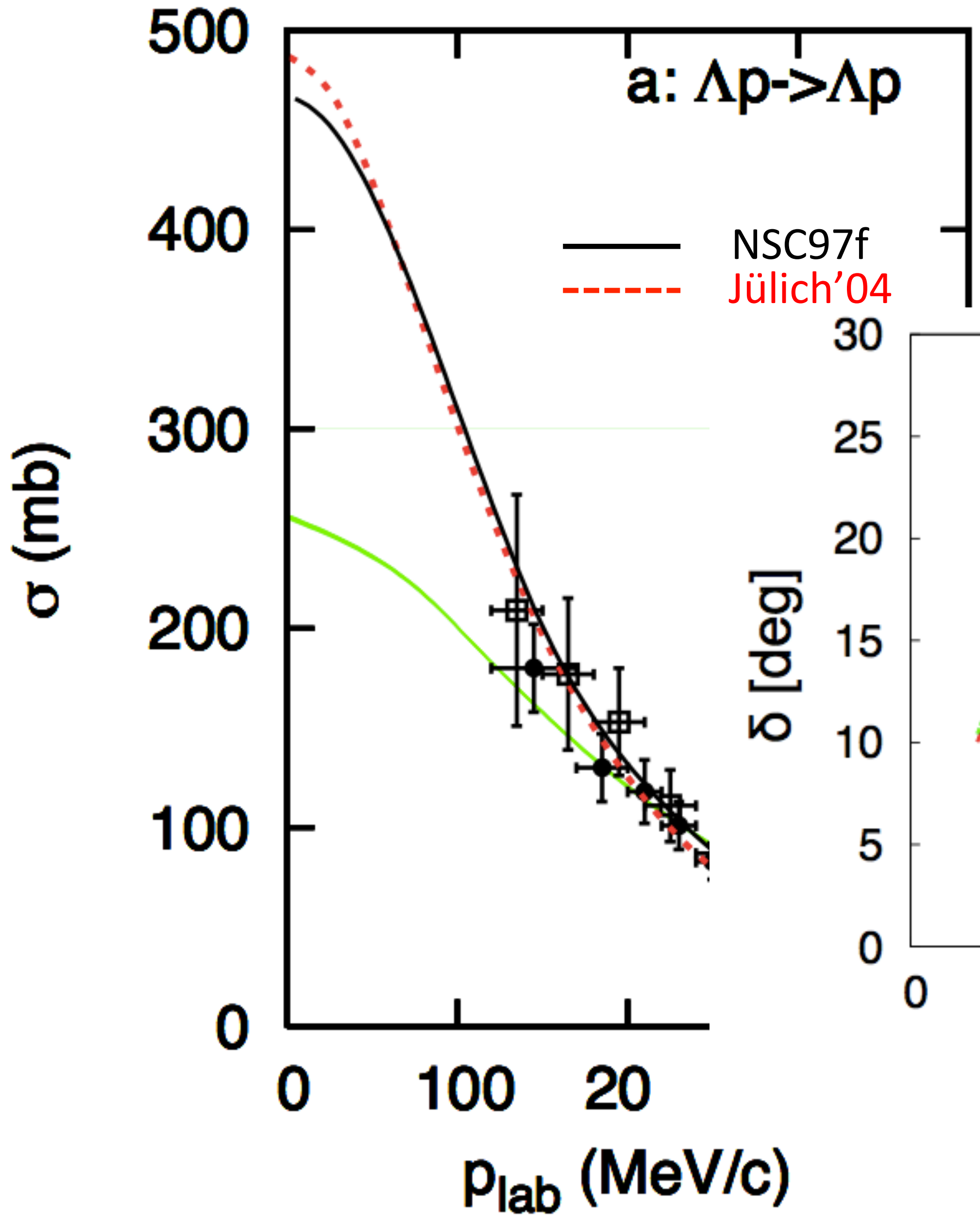
There are not stable hyperon beams.
Where does the information on the strong and weak YN interaction come from?

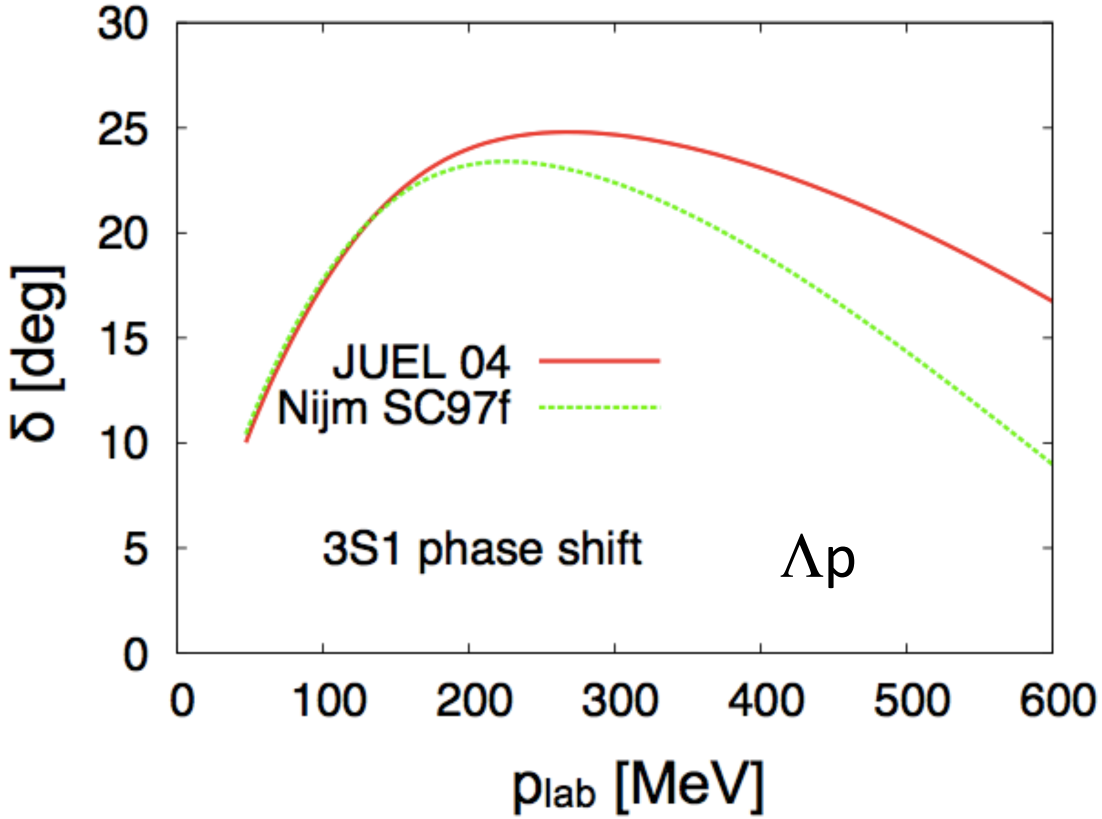
(JPARC, TJNAF, DAΦNE, …)

→ Hypernuclear spectroscopy
Hypernuclear decay

Not clean extraction:
MEDIUM EFFECTS

model dependencies

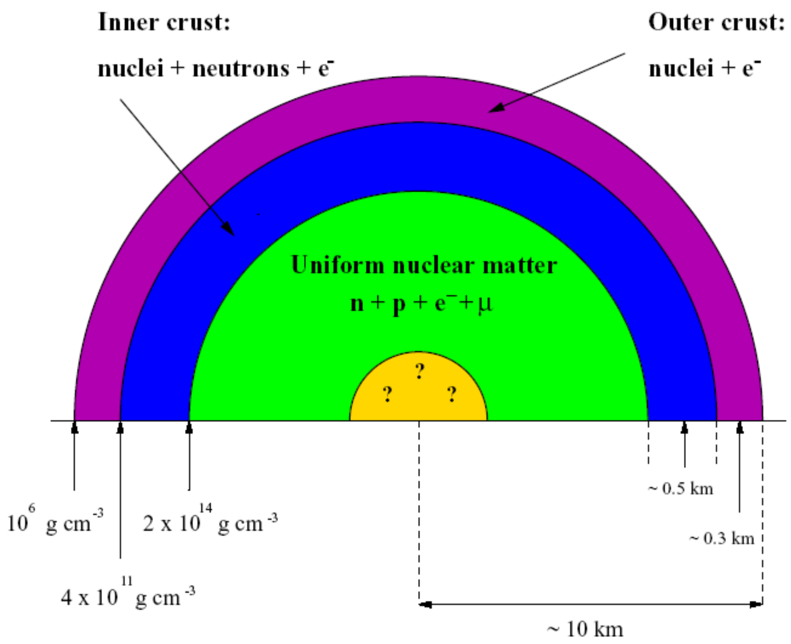


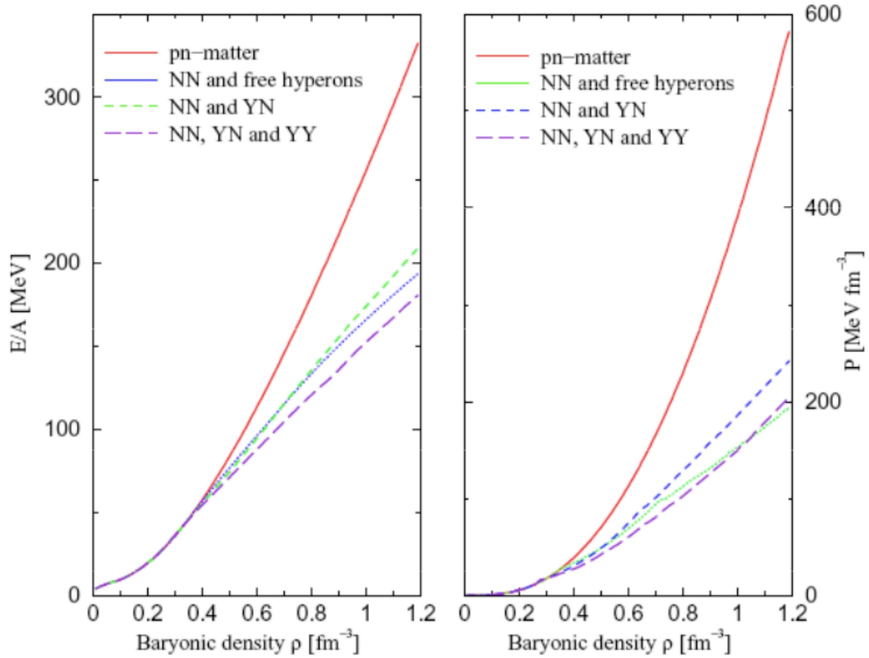


more motivation

Ambartsumyan, Saakyan, 1960

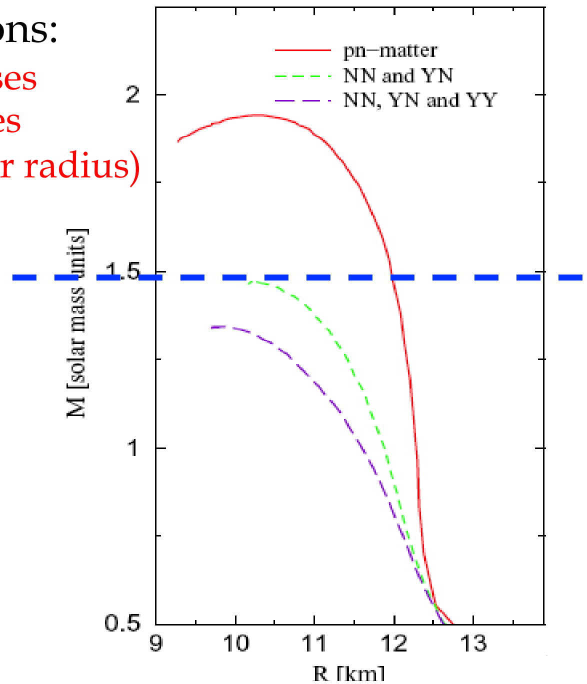
"The core of a neutron star is a fluid of neutron rich matter in equilibrium with respect to the weak interactions (β stable matter)"





The composition of a neutron star depends on the hyperon properties in the medium (i.e. on the YN and YY interactions)

Influence of hyperons:
lower maximum masses
higher central densities
more compact (smaller radius)



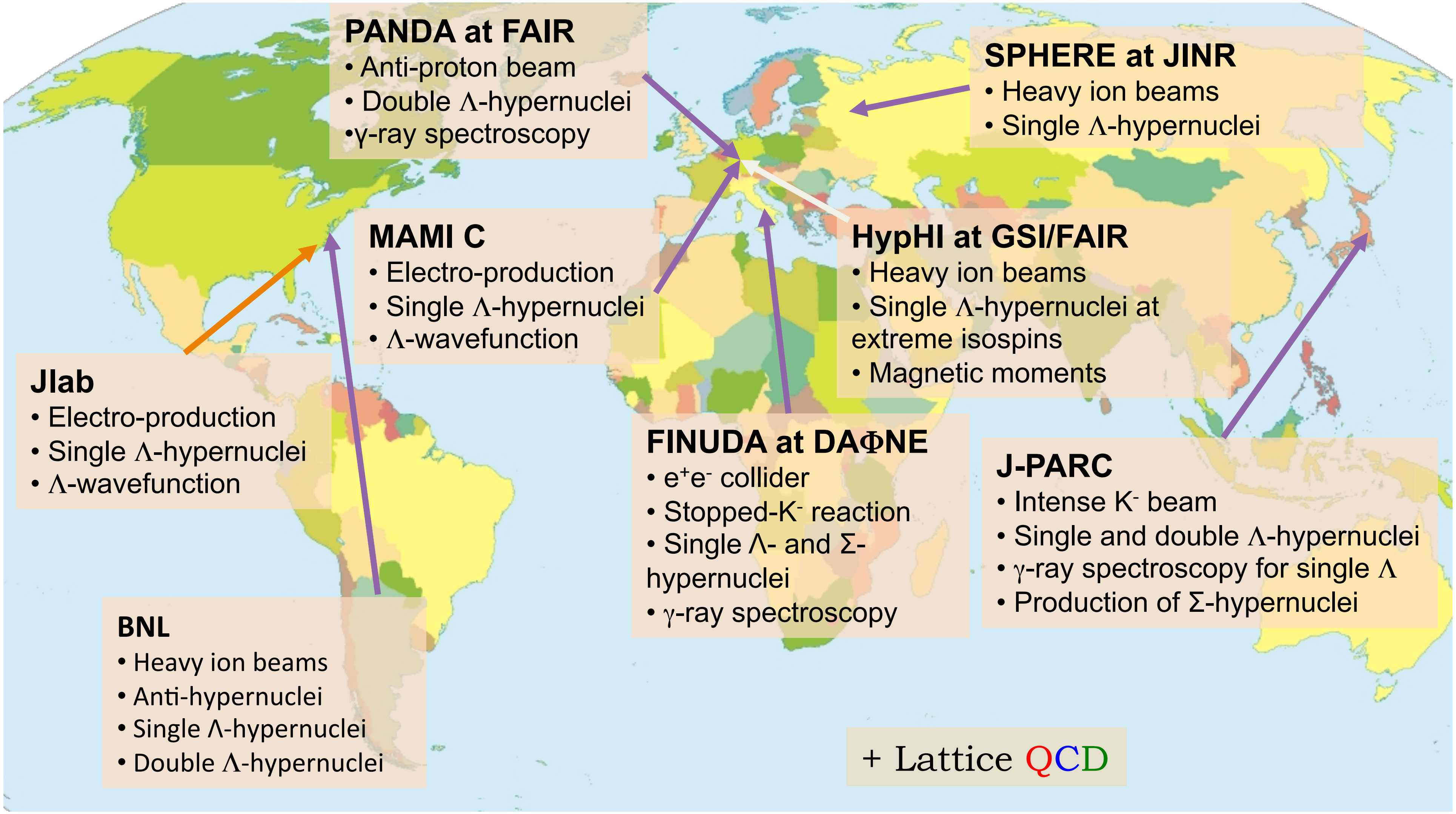
Schulze, Polls, Ramos, Vidaña, PRC73, 058801 (2006)

Summary

Avraham Gal
Lectures on Strangeness Nuclear Physics, Tokai, Japan, Feb. 2012

- ΛN hypernuclear spin dependence deciphered
- How small is Λ spin-orbit splitting and why?
- Role of 3-body ΛNN interactions?
- Repulsive Σ-nuclear interaction; how repulsive?
- Onset of $\Lambda\Lambda$ binding: ${}^{4}_{\Lambda\Lambda}$H or ${}^{5}_{\Lambda\Lambda}$H & ${}^{5}_{\Lambda\Lambda}$He?
- Ξ hyperons bound by ~ 15 MeV in nuclear matter?
  No quasibound Ξ observed yet $\Rightarrow$ J-PARC E05
- Onset of Ξ stability: ${}^{6}_{\Lambda\Xi}$He or ${}^{7}_{\Lambda\Lambda\Xi}$He?
- Is Strange Hadronic Matter $\{N, \Lambda, \Xi\}$ ground state of self-bound strange matter?
  No $\bar{K}$ condensation in self-bound stable matter

"strange" Experimental program



J. Pochodzalla, Int. Journal Modern Physics E, Vol 16, no. 3 (2007) 925-936

The NPLQCD approach

The NPLQCD approach

Determine the low-energy scattering parameters from the energy of the interacting two-hadron system in finite volume

$$E^{(AB)} - m_A - m_B = \sqrt{k^2 + m_A^2} + \sqrt{k^2 + m_B^2} - m_A - m_B = \frac{k^2}{2\mu_{AB}} + \ldots$$

from LQCD simulations

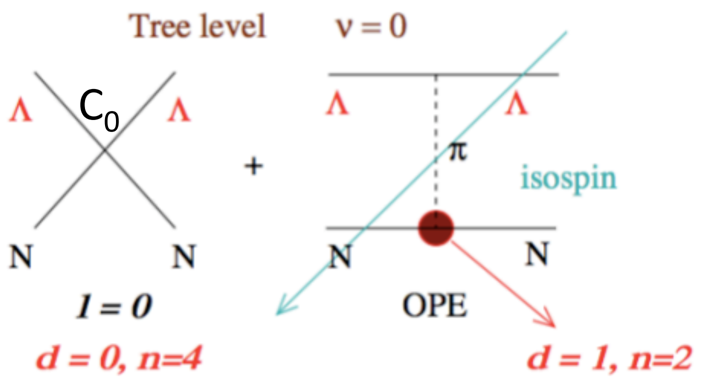
Lüscher

$$k\cot\delta(k) = -\frac{1}{\mathrm{a}} + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{k^2}{\Lambda^2}\right)^{n+1} = -\frac{1}{\mathrm{a}} + \frac{1}{2} r_0 k^2 + \ldots$$

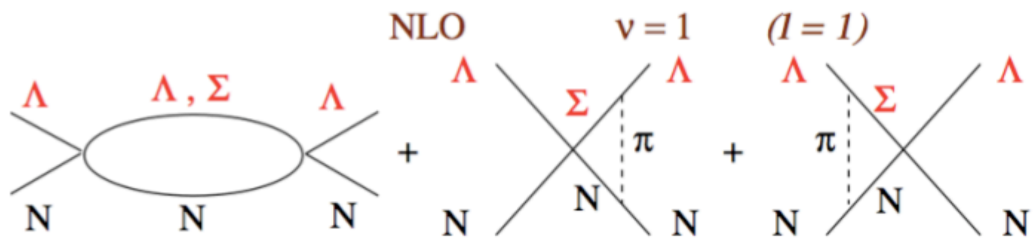
Effective Range Expansion

Effective Field Theory for the Low-Energy ΛN interaction

Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027

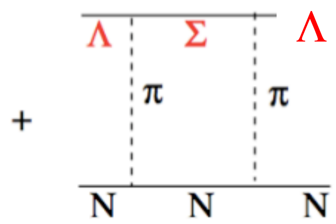


s-wave scattering



2 body diagrams (Weinberg)

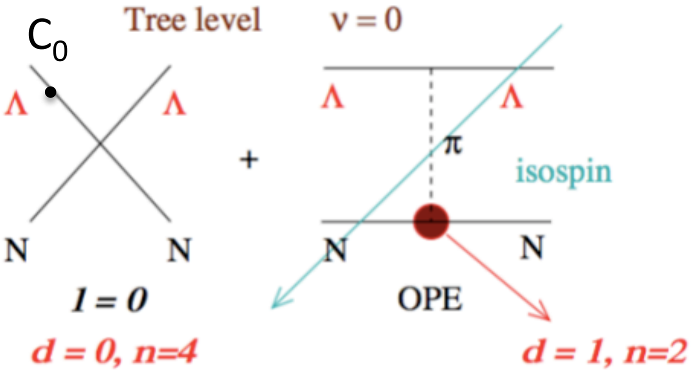
$$\nu = 2L + \sum_i V_i [d_i + \frac{1}{2} n_i - 2]$$

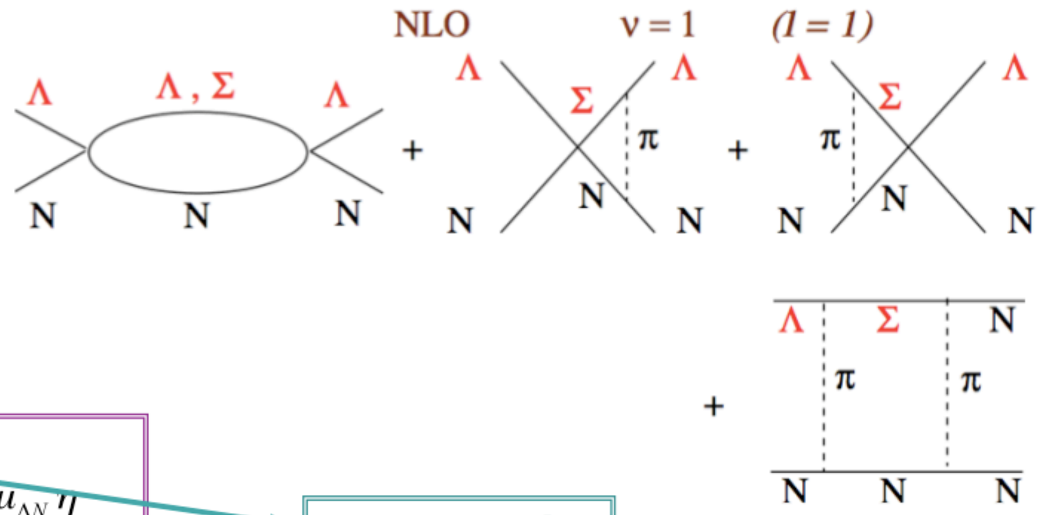


$SU(2)_L \times SU(2)_R$ two-flavor χ PT

Effective Field Theory for the Low-Energy ΛN interaction

Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027





$$a^{(^1S_0)} = -\frac{\mu_{\Lambda N}}{2\pi}\Bigg[{}_{\Lambda\Lambda}C_0^{(^1S_0)} - \frac{3}{4\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2 \mu_{\Lambda N}\,\eta + {}_{\Sigma\Lambda}C_0^{(^1S_0)} \frac{3g_{\Sigma\Lambda}g_A\mu_{\Lambda N}}{2\pi f^2}\frac{\eta^2+\eta\, m_\pi+m_\pi^2}{\eta+m_\pi} - \frac{3g_{\Sigma\Lambda}^2 g_A^2\mu_{\Lambda N}}{4\pi f^4}\frac{2\eta^3+4\eta^2 m_\pi+6\eta m_\pi^2+3m_\pi^3}{2(\eta+m_\pi)^2}\Bigg]$$

Extract LECs

$$r^{(^1S_0)} = -\frac{1}{\mu_{\Lambda N}\pi}\left[\frac{2\pi}{{}_{\Lambda\Lambda}C_0^{(^1S_0)}}\right]^2 \Bigg[\frac{3}{8\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2\frac{\mu_{\Lambda N}}{\eta} + {}_{\Sigma\Lambda}C_0^{(^1S_0)} \frac{3g_{\Sigma\Lambda}g_A\mu_{\Lambda N}}{2\pi f^2}\frac{3\eta^2+9\eta\, m_\pi+8m_\pi^2}{6(\eta+m_\pi)^3} - \frac{3g_{\Sigma\Lambda}^2 g_A^2\mu_{\Lambda N}}{4\pi f^4}\frac{6\eta^3+23\eta^2 m_\pi+28\eta m_\pi^2+7m_\pi^3}{12(\eta+m_\pi)^4}\Bigg]$$

Result of the LQCD simulation

For example, for the singlet channel

$\eta = \sqrt{2\mu_{\Lambda N}\Delta_{\Lambda\Sigma}}$

$g_A \sim 1.27,$

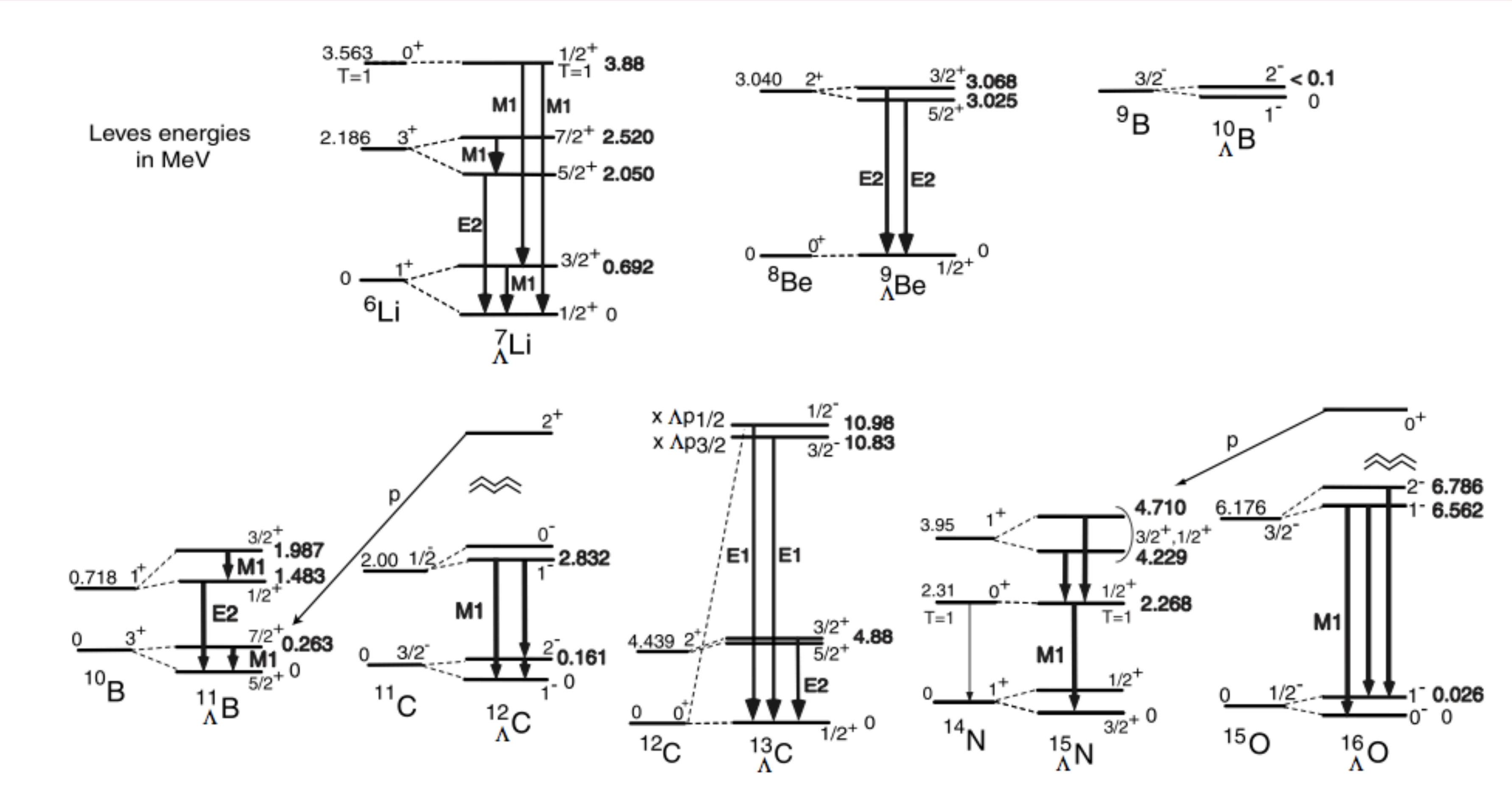
$g_{\Lambda\Sigma} \sim 0.60,$

$0.30 \le g_{\Sigma\Sigma} \le 0.55$

In (hyper)nuclear physics

We need to resolve energy differences of the order of a few hundreds KeV

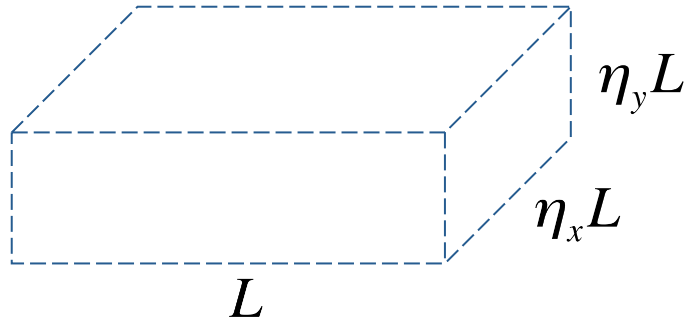
→ **Need high statistics** ~ 400 000 propagators @ m_π ~390 MeV



H. Tamura et al., Nucl. Phys. A 835 (2010) 3 (recent γ-ray measurements)

Anisotropic lattices: $N_t >> N_s$ (N_f=2+1 clover-improved Wilson fermion actions)

higher resolution in the time direction:
- better study of noisy states
- better extraction of excited states
- reduce the systematic due to fitting (confident plateaus)



We use **Gaussian-smeared sources** to optimize the overlap onto the ground-state hadrons.

For the sink, either local or smeared quark field operators are used.

Two-point correlation functions (masses)

$$C(\Gamma,\vec{p},t)=\sum_{\vec{x}_2} e^{-i\vec{p}\vec{x}_2}\Gamma\langle 0|T\{\psi(\vec{x}_2,t_2)\psi(\vec{x}_0,t_0)\}|0\rangle$$

projects onto zero momentum

spin tensor

sink

source

interpolating operators

Masses of (colourless) QCD bound states

(locate the source at t=0)

$$C(t)=\langle 0|\phi(t)\phi^\dagger(0)|0\rangle \xrightarrow[\phi(t)=e^{Ht}\phi e^{-Ht}]{} \langle\phi|e^{-Ht}|\phi\rangle$$

Insert a complete set of energy eigenstates:

$$C(t)=\sum_n \langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle=\sum_n |\langle\phi|n\rangle|^2 e^{-E_n t} \xrightarrow{t\to\infty} Z e^{-E_0 t}$$

mass

i.e. one can obtain the energy of the state provided we see the large time exponential fall-off of the correlation function (Euclidean time evolution suppresses excited states)

$$J^\pi=\left(\frac{1}{2}\right)^+$$

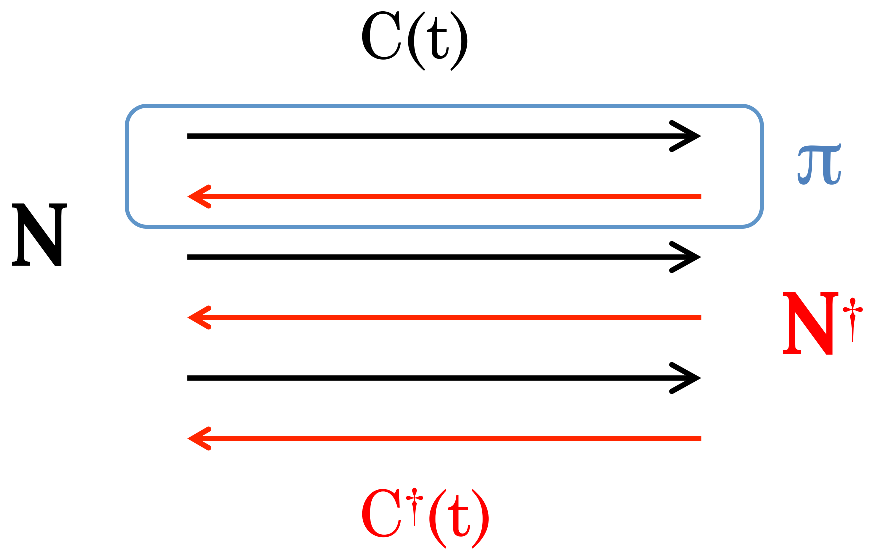
$$p_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Sigma^+_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

$$\Lambda_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Xi^0_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

charge conjugation matrix



Lepage, 1989

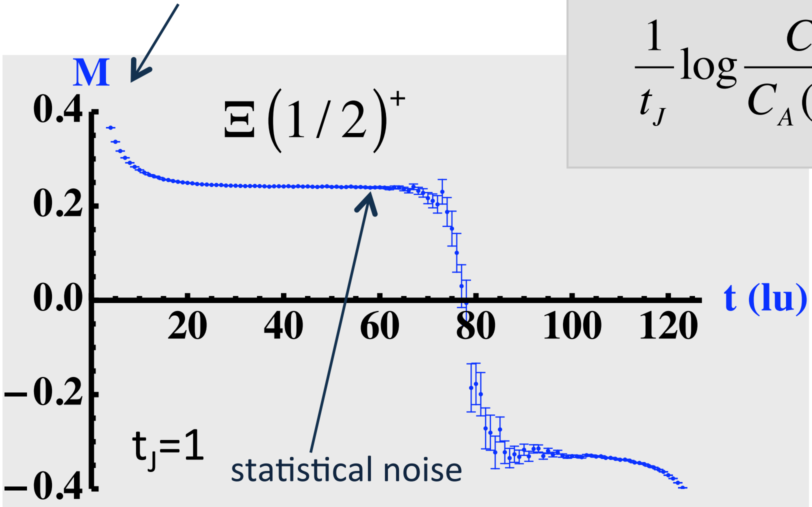
exponential degradation of the signal with time

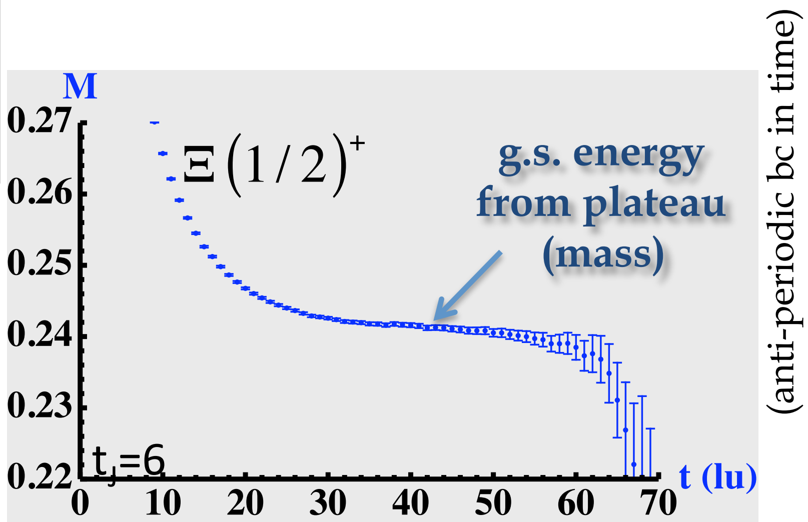
$$\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$

for one nucleon: $\dfrac{\langle C \rangle}{\sigma} \sim \exp\left\{-\left(M_N - \dfrac{3m_\pi}{2}\right)t\right\}$ $\rightarrow$ A nucleons: $\dfrac{\langle C \rangle}{\sigma} \sim \exp\left\{-A\left(M_N - \dfrac{3m_\pi}{2}\right)t\right\}$

effective mass plot

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)} = m_A$$



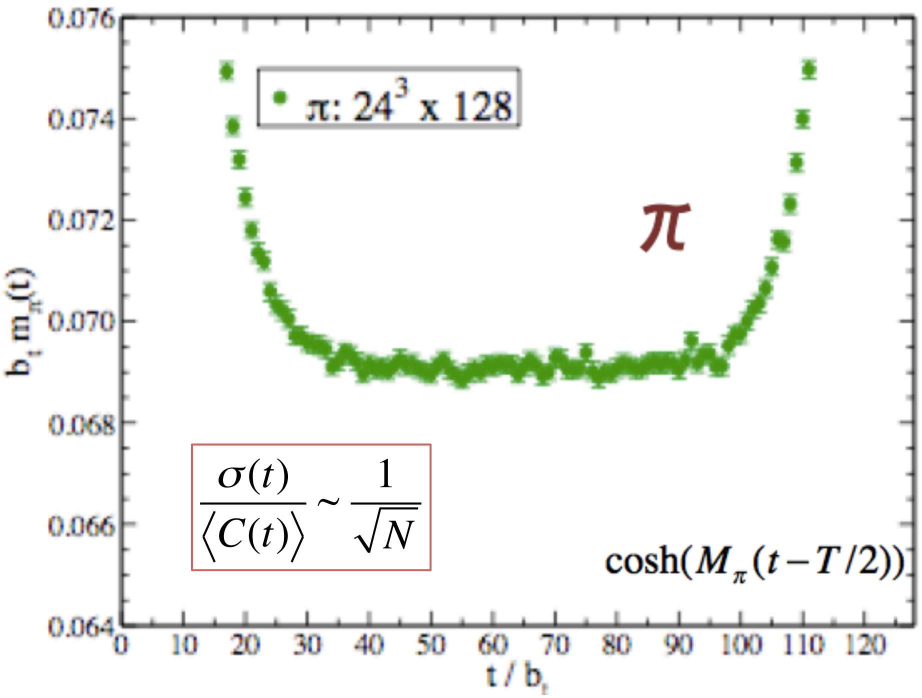


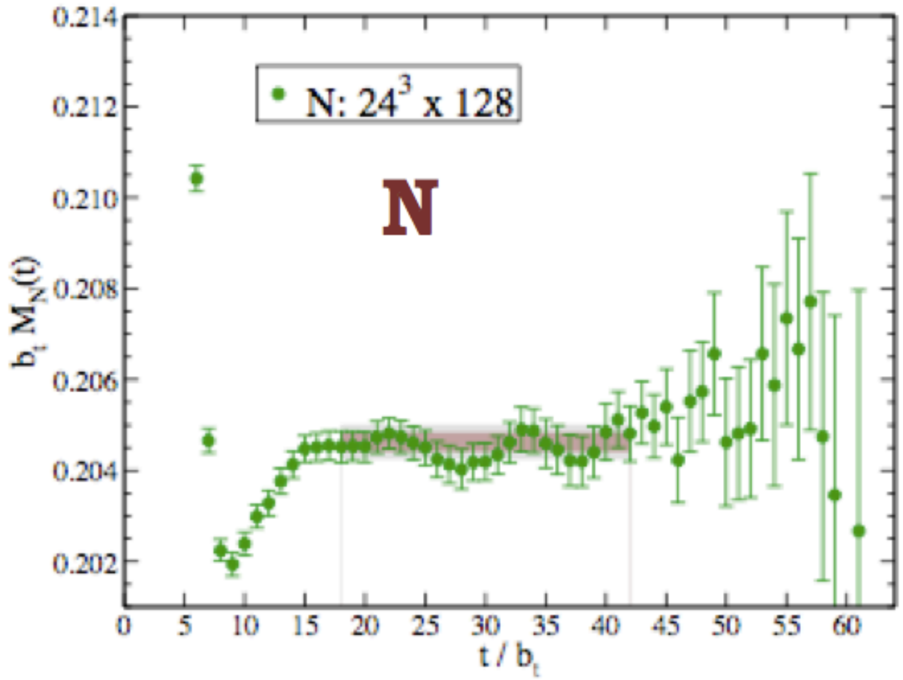
Lepage, 1989

exponential degradation
of the signal with time

$$\sigma^2(C) = \langle CC^+ \rangle - |\langle C \rangle|^2$$

for one nucleon: $\frac{\langle C \rangle}{\sigma} \sim \exp\left\{-\left(M_N - \frac{3m_\pi}{2}\right)t\right\}$ $\rightarrow$ A nucleons: $\frac{\langle C \rangle}{\sigma} \sim \exp\left\{-A\left(M_N - \frac{3m_\pi}{2}\right)t\right\}$





NPLQCD, Phys.Rev. D84 (2011) 14507

Two-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_1H_2,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1}e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_1,\alpha_1}(\vec{x}_1,t)J_{H_2,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_1,\beta_1}(x_0,0)\bar{J}_{H_2,\beta_2}(x_0,0)\right\rangle$$

$$J^{\pi}=\left(\frac{1}{2}\right)^{+}$$

$$p_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Lambda_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Sigma^+_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

$$\Xi^0_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

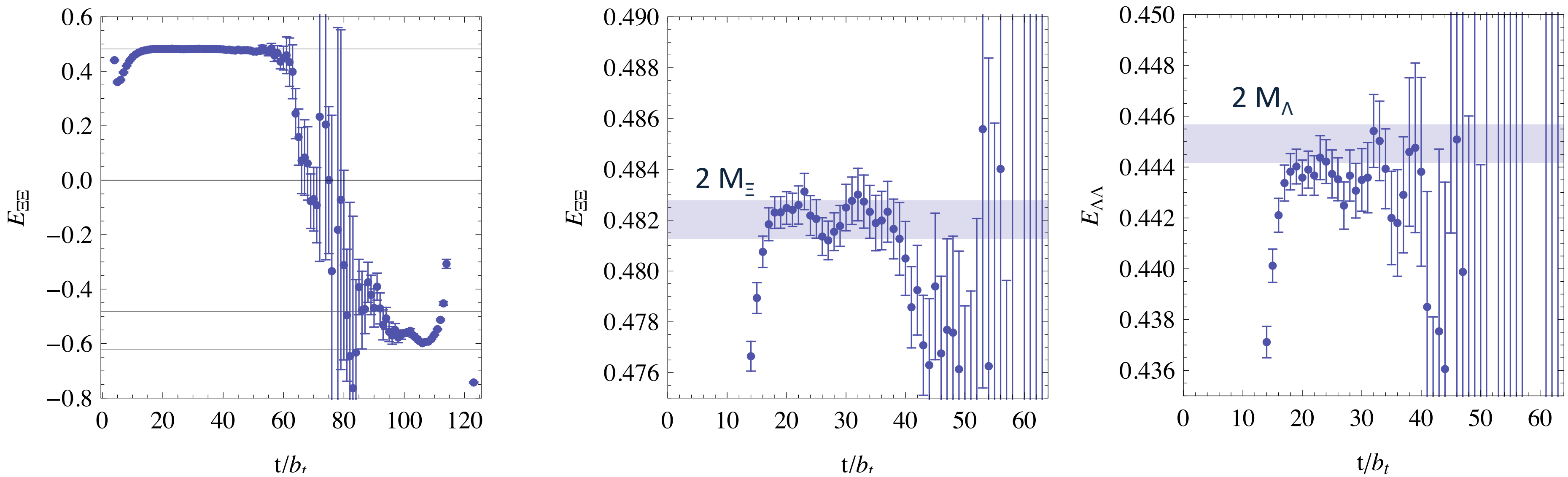
Two-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_1H_2,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1} e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_1,\alpha_1}(\vec{x}_1,t)J_{H_2,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_1,\beta_1}(x_0,0)\bar{J}_{H_2,\beta_2}(x_0,0)\right\rangle$$

away from the source, i.e. at large t

$$C_{H_1H_2}(\vec{p},-\vec{p},t)\sim\sum_n Z^{(i)}_{n;12}(\vec{p})Z^{(f)}_{n;12}(\vec{p})e^{-E^{12}_n(\vec{0})t}$$

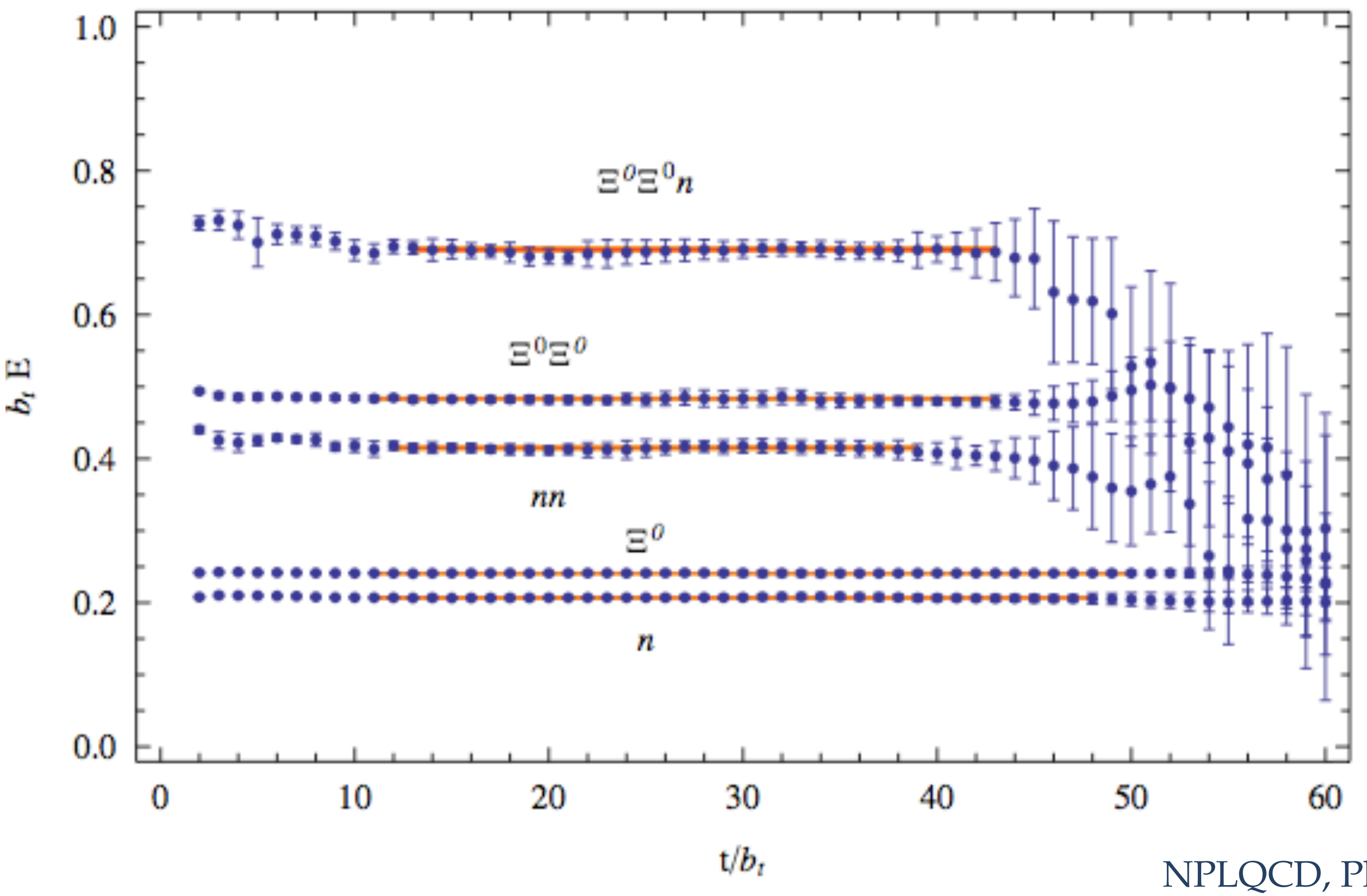
$m_\pi \sim 390$ MeV, $L_s \sim (2.5\ \text{fm})^3$



Three-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_1H_2H_3,\Gamma}(\vec{p}_1,\vec{p}_2,\vec{p}_3;t)=\sum_{\vec{x}_1\vec{x}_2\vec{x}_3} e^{i\vec{p}_1\vec{x}_1}e^{i\vec{p}_2\vec{x}_2}e^{i\vec{p}_3\vec{x}_3}\Gamma^{\beta_1\beta_2\beta_3}_{\alpha_1\alpha_2\alpha_3}$$

$$\left\langle J_{H_1,\alpha_1}(\vec{x}_1,t)J_{H_2,\alpha_2}(\vec{x}_2,t)J_{H_3,\alpha_3}(\vec{x}_2,t)\,\bar{J}_{H_1,\beta_1}(x_0,0)\bar{J}_{H_2,\beta_2}(x_0,0)\bar{J}_{H_3,\beta_3}(x_0,0)\right\rangle$$

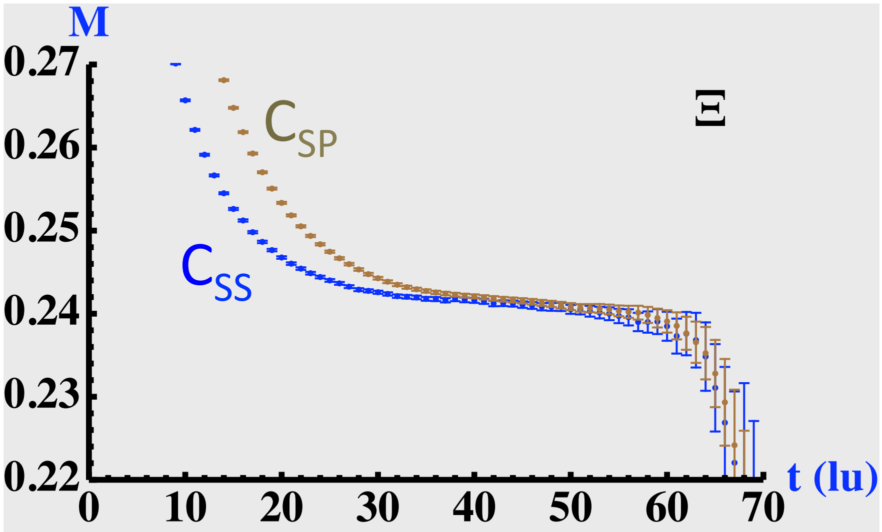


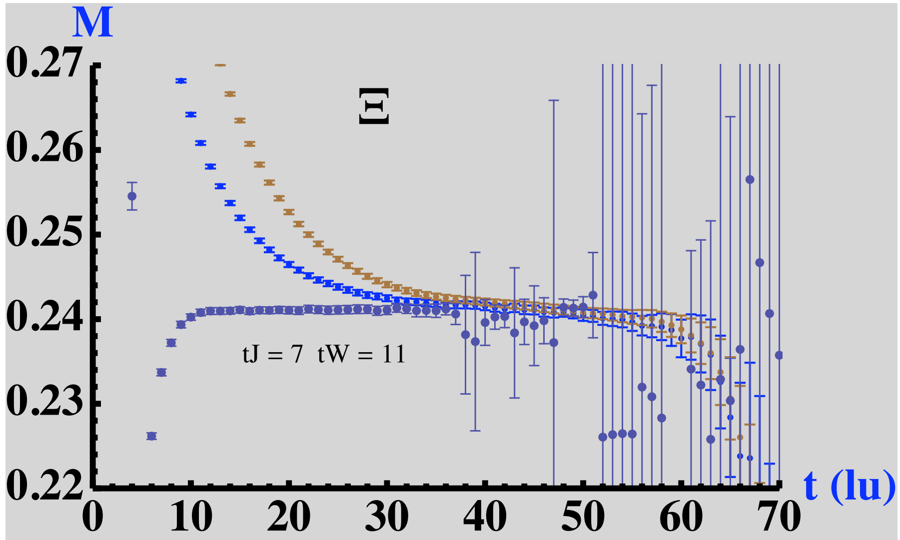
m_π ~390 MeV

L_s ~ (2.5 fm)^3

NPLQCD, Phys. Rev. D80 (2009) 074501

Gaussian smeared sources (S)
Local (P) or Smeared (S) sinks → C_{SS} , C_{SP}



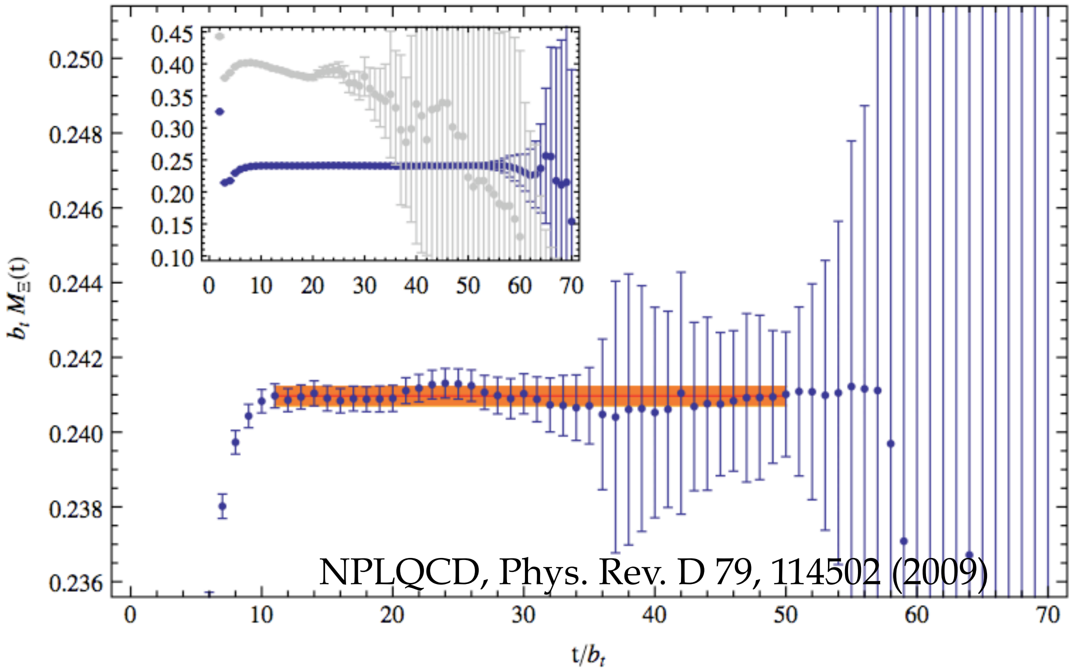


Tracking sub-leading exponential fall-offs can give us excited states

Perform a multiexponential fit

$$\chi^2 = \sum_{t,t',s,s'} [y_s(t) - C_s(t)] (\mathrm{Cov}^{-1})^{s,s'}_{t,t'} [y_{s'}(t') - C_{s'}(t')]$$

$$C_{SS}(t) = \sum_n Z_n^S Z_n^S e^{-E_n t}, \qquad C_{SP}(t) = \sum_n Z_n^S Z_n^P e^{-E_n t}$$



NPLQCD, Phys. Rev. D 79, 114502 (2009)

Two-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_1H_2,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1} e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_1,\alpha_1}(\vec{x}_1,t)J_{H_2,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_1,\beta_1}(x_0,0)\bar{J}_{H_2,\beta_2}(x_0,0)\right\rangle$$

spin tensor

interpolating operators

(source @ t=0)

energy-eigenstates

↕

global quantum numbers:
B, I, I_3, J, s
+
hyper-cubic transformations

$\Lambda\Lambda - \Sigma^{\pm,0}\Sigma^{\mp,0} - \Xi N$

$\Lambda N - \Sigma N$

coupled channel analysis

single channel analysis

Channel	I	$\lvert I_z\rvert$	s
$pp\ (^1S_0)$	1	1	0
$np\ (^3S_1)$	0	0	0
$n\Lambda\ (^1S_0)$	$\frac{1}{2}$	$\frac{1}{2}$	−1
$n\Lambda\ (^3S_1)$	$\frac{1}{2}$	$\frac{1}{2}$	−1
$n\Sigma^-\ (^1S_0)$	$\frac{3}{2}$	$\frac{3}{2}$	−1
$n\Sigma^-\ (^3S_1)$	$\frac{3}{2}$	$\frac{3}{2}$	−1
$\Sigma^-\Sigma^-\ (^1S_0)$	2	2	−2
$\Lambda\Lambda\ (^1S_0)$	0	0	−2
$\Xi^-\Xi^-\ (^1S_0)$	1	1	−4

Two-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_1H_2,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1}e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_1,\alpha_1}(\vec{x}_1,t)J_{H_2,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_1,\beta_1}(x_0,0)\bar{J}_{H_2,\beta_2}(x_0,0)\right\rangle$$

spin tensor

interpolating operators

(source @ t=0)

Energy shift due to the interaction obtained from:

$$\frac{C_{H_1H_2,\Gamma}(\vec{p},-\vec{p},t)}{C_{H_1,\Gamma}(\vec{0},t)\,C_{H_2,\Gamma}(\vec{0},t)}\rightarrow Z^{(i)}_{12}(\vec{p})Z^{(f)}_{12}(\vec{p})e^{-\Delta E^{(12)}_0(0)\,t}$$

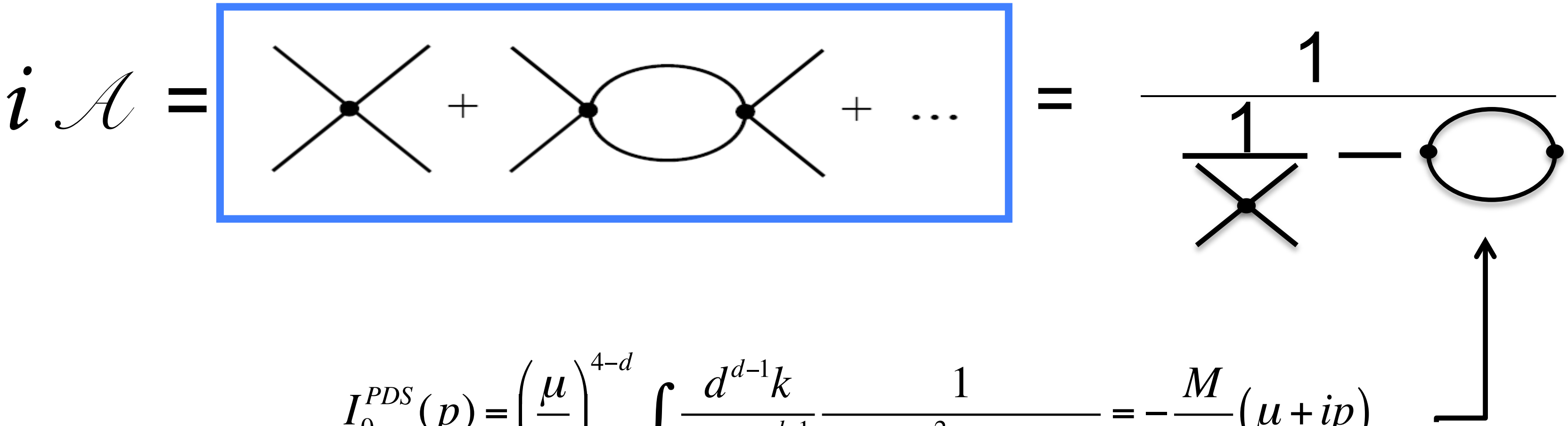
Phase-shifts, scattering lengths, from energies calculated in LQCD

Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)
M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)

Adopted from D. Kaplan

Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)
M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)

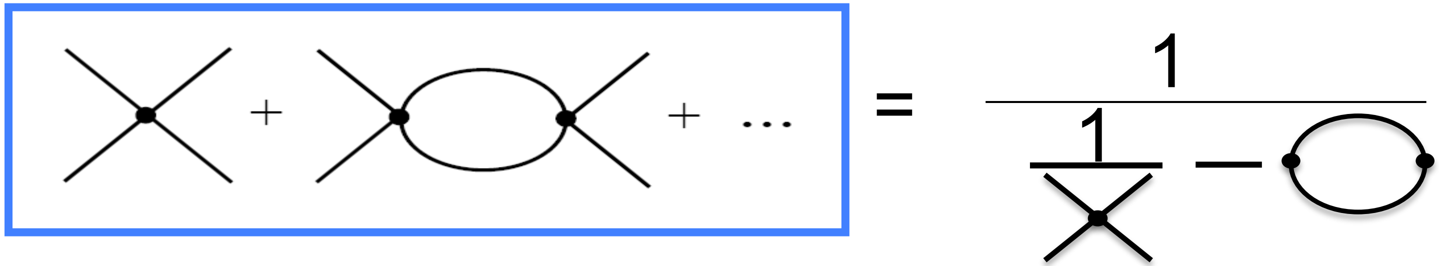
Below inelastic thresholds

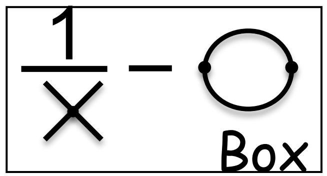


$$I_0^{PDS}(p) = \left(\frac{\mu}{2}\right)^{4-d} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{E - |\vec{k}|^2/M + i\varepsilon} = -\frac{M}{4\pi}(\mu + ip)$$

Below inelastic thresholds

Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)
M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)

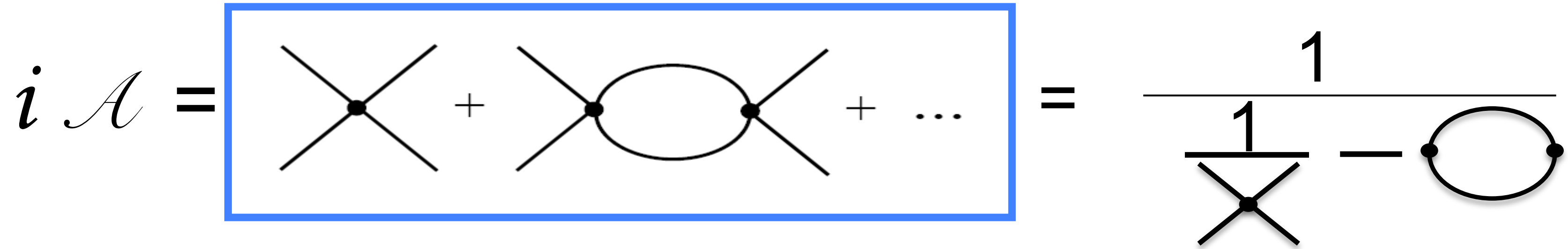
$i\mathcal{A} =$ 

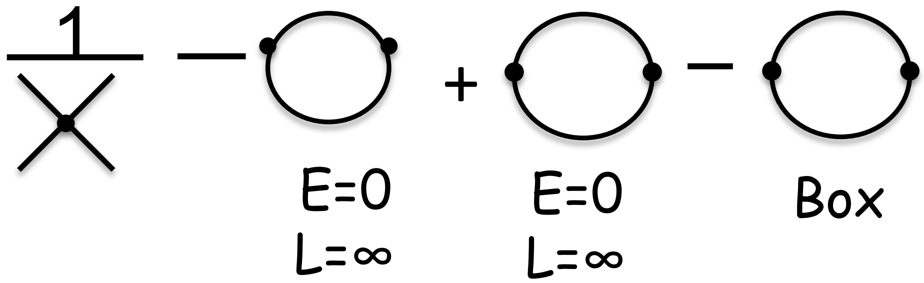
$0 = \mathrm{Re}[(i\mathcal{A})^{-1}] = \mathrm{Re}$ 

$$I_0^{FV}(p) = \frac{1}{L^3} \sum_{k}^{|\vec{k}|<\Lambda} \frac{1}{E - |\vec{k}|^2 / M}$$

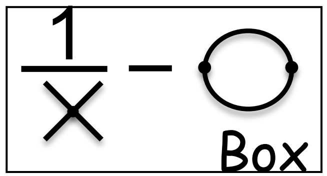
Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)
M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)

Below inelastic thresholds





$0 = \mathrm{Re}[(i\mathcal{A})^{-1}]$ = Re



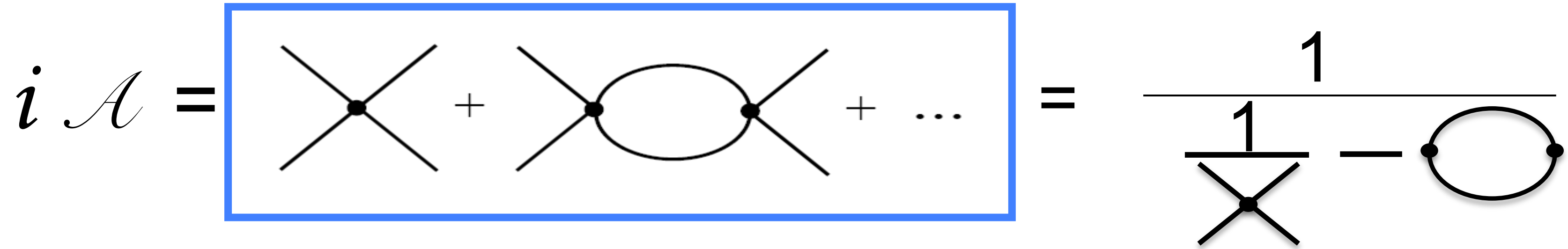
$$\int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{|\vec{k}|^2 / M}$$

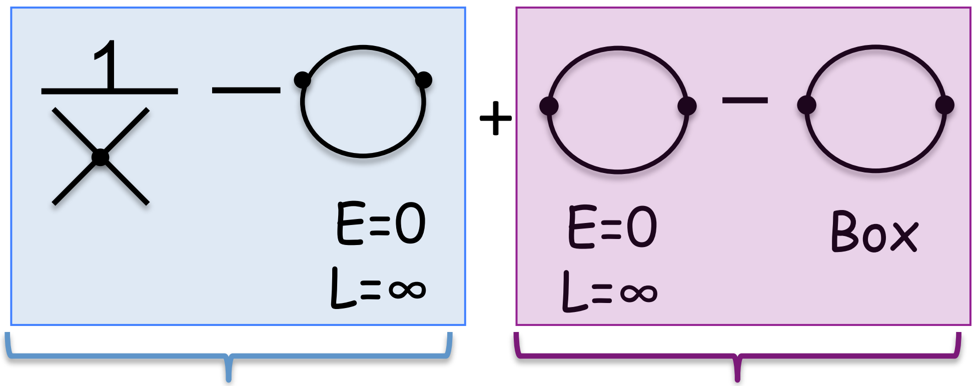
$$\int^{PDS} \frac{d^3k}{(2\pi)^3} \frac{1}{|\vec{k}|^2 / M} = -\frac{M}{4\pi}\mu$$

$$I_0^{FV}(p) = \frac{1}{L^3} \sum_{k}^{|\vec{k}|<\Lambda} \frac{1}{E - |\vec{k}|^2 / M}$$

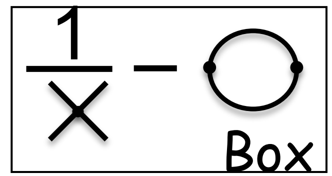
Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)
M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)

Below inelastic thresholds





$0 = \mathrm{Re}[(i\mathcal{A})^{-1}]$ = Re



$$p\cot\delta(p)$$

$$\frac{1}{\pi L}\sum_{\vec{j}}\frac{1}{|\vec{j}|^2-\left(\frac{Lp}{2\pi}\right)^2}-\frac{4\Lambda}{L} \equiv \frac{1}{\pi L}S\left(\frac{p^2L^2}{4\pi^2}\right)$$

$$\Delta E = \frac{p^2}{M} = \frac{4\pi a}{ML^3}\left[1 - c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \ldots\right] \qquad \text{with} \quad c_1 = \frac{1}{\pi}\sum_{\substack{\vec{j}\in Z^3\\ \vec{j}\neq\vec{0}}}\frac{1}{|\vec{j}|^2}, \quad c_2 = c_1^2 - \frac{1}{\pi^2}\sum_{\vec{j}\neq 0}\frac{1}{|\vec{j}|^4}$$

L>>a (recovering Lüscher's formula, M. Lüscher, Commun. Math. Phys. 105, 153 (1986))

$SU(3)_f$ content of the different interaction channels

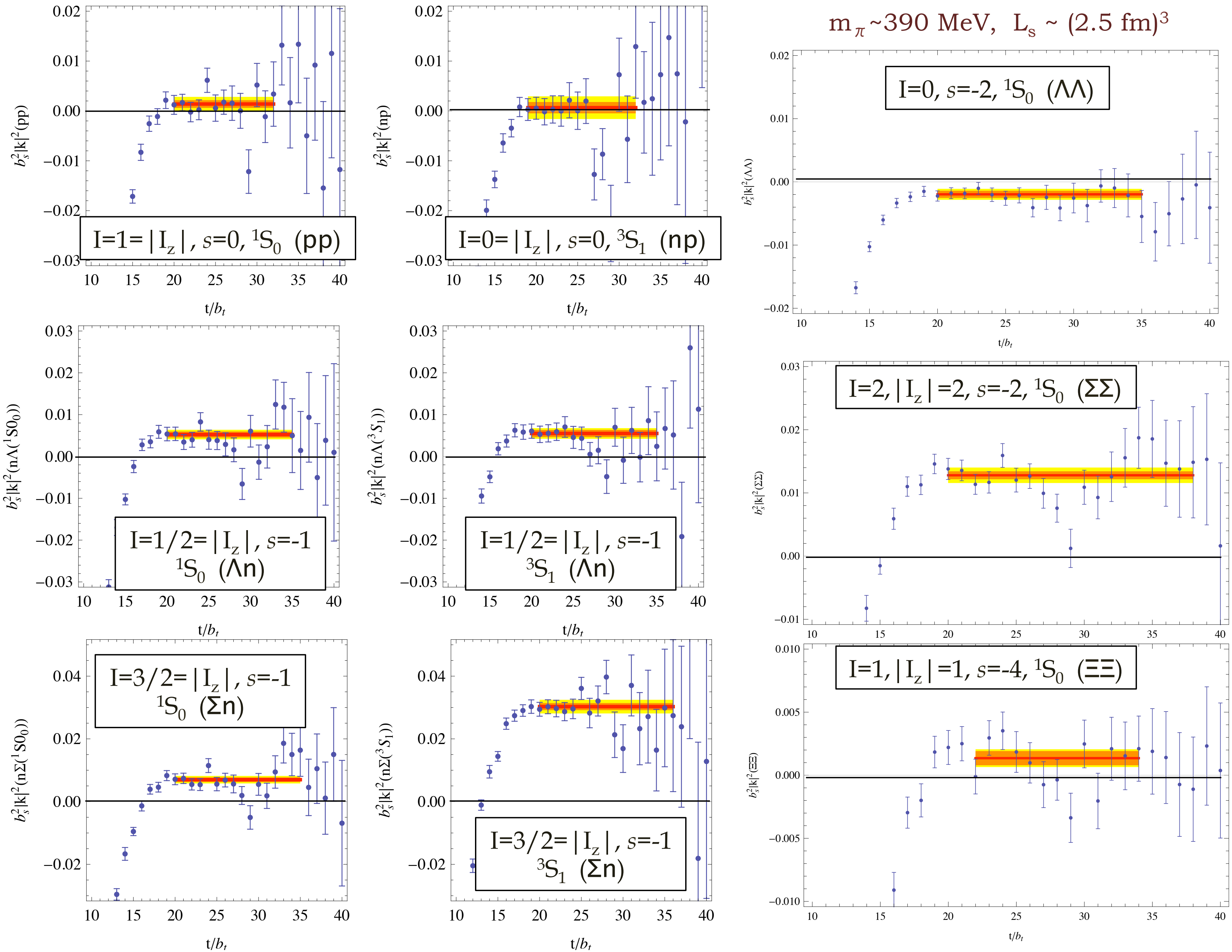
$$SU(3)_f \qquad 8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_S \oplus 8_A \oplus 1$$

space-spin antisymmetric states ($^1S_0$, ^3P, …)

	S	I	Channels	SU(3) irreps
	0	1	NN	{27}
	-1	1/2	Λ N, Σ N	{27}, $\{8\}_s$
		3/2	Σ N	{27}
	-2	0	ΛΛ, ΞN, ΣΣ	{27}, $\{8\}_s$, {1}
		1	Ξ N, Σ Λ	{27}, $\{8\}_s$
(b.s.)		2	Σ Σ	{27}
	-3	1/2	Ξ Λ, Ξ Σ	{27}, $\{8\}_s$
		3/2	Ξ Σ	{27}
(b.s.)	-4	1	Ξ Ξ	{27}

space-spin symmetric states ($^3S_1$, $^1P_1$, …)

S	I	Channels	SU(3) irreps
0	0	NN	{10*}
-1	1/2	Λ N, Σ N	{10*}, $\{8\}_a$
	3/2	Σ N	{10}
-2	0	Ξ N	$\{8\}_a$
	1	Ξ N, Σ Σ	{10}, {10*}, $\{8\}_a$
	1	Σ Λ	{10}, {10*}
-3	1/2	Ξ Λ, Ξ Σ	{10}, $\{8\}_a$
	3/2	Ξ Σ	{10*}
-4	0	Ξ Ξ	{10}



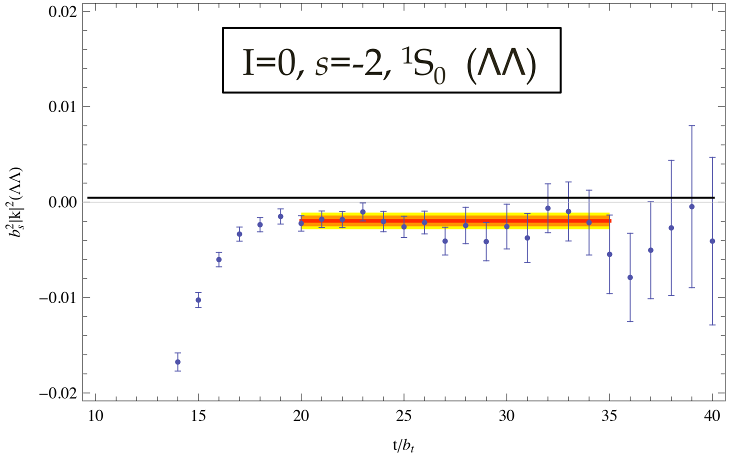
mπ ~390 MeV, Ls ~ (2.5 fm)3
I=1=|Iz|, s=0, 1S0 (pp)
I=0=|Iz|, s=0, 3S1 (np)
I=0, s=-2, 1S0 (ΛΛ)
I=1/2=|Iz|, s=-1
1S0 (Λn)
I=1/2=|Iz|, s=-1
3S1 (Λn)
I=2, |Iz|=2, s=-2, 1S0 (ΣΣ)
I=3/2=|Iz|, s=-1
1S0 (Σn)
I=3/2=|Iz|, s=-1
3S1 (Σn)
I=1, |Iz|=1, s=-4, 1S0 (ΞΞ)
t/bt

$\Lambda\Lambda - \Xi N - \Sigma\Sigma$

$$\Delta E = -4.1(1.2)(1.4)\ \text{MeV}$$

$$-\frac{1}{p\cot\delta} = -0.188^{+0.062\,+0.072}_{-0.072\,-0.085}\ \text{fm}$$

$m_\pi \sim 390$ MeV, $L_s \sim (2.5\ \text{fm})^3$



Bound state?

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

infinite volume

b.s. $p^2 = -\gamma^2$

$$\cot\delta(i\gamma) = i$$

$\longrightarrow$

finite volume:

$$\cot\delta(i\gamma)\big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq 0}\frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

Looking for bound states in the lattice: volume dependence anisotropic clover lattices

$m_\pi \sim 390$ MeV $b_s \sim 0.123$ fm $\xi_t \sim 3.5$

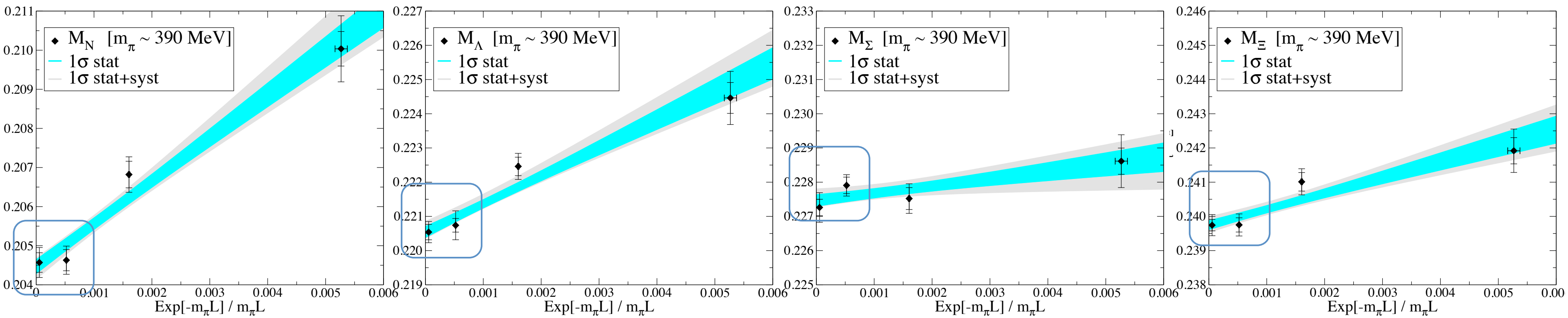
anisotropic clover lattices

$L^3 \times T$	$16^3 \times 128$	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 256$
L (fm)	~ 2.0	~ 2.5	~ 3.0	~ 4.0
$m_\pi L$	3.86	4.82	5.79	7.71
$m_\pi T$	8.82	8.82	8.82	17.64
# props	4.5×10^5	4.3×10^5	3.9×10^5	1.3×10^5

Looking for bound states in the lattice: volume dependence anisotropic clover lattices

$m_\pi \sim 390$ MeV $b_s \sim 0.123$ fm $\xi_t \sim 3.5$

anisotropic clover lattices



$\leftarrow L$

NPLQCD, Phys. Rev. D 84, 014507 (2011)

$L^3 \times T$	$16^3 \times 128$	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 256$
L (fm)	~ 2.0	~ 2.5	~ 3.0	~ 4.0
$m_\pi L$	3.86	4.82	5.79	7.71
$m_\pi T$	8.82	8.82	8.82	17.64
# props	4.5×10^5	4.3×10^5	3.9×10^5	1.3×10^5

bound states

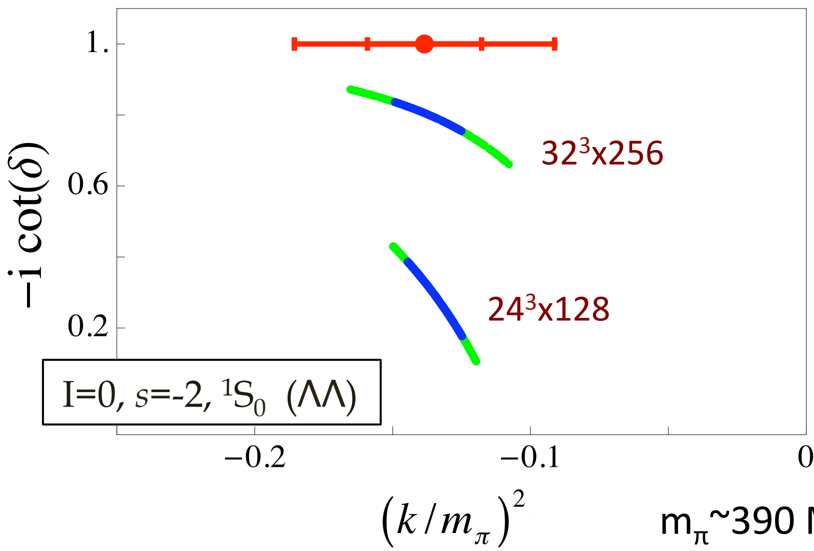
$$k_n \cot\delta(k_n) = \frac{1}{\pi L} S\left(\frac{k_n^2 L^2}{4\pi^2}\right) = \frac{1}{\pi L} \lim_{\Lambda\to\infty}\left(\sum_{j}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \eta^2} - 4\pi\Lambda\right), \qquad \eta = \frac{k_n^2 L^2}{4\pi^2}$$

$$k_{-1} = i\,\kappa$$

$$k\cot\delta(k)\big|_{k=i\kappa} + \kappa = \frac{1}{L}\sum_{\vec{m}\neq 0}\frac{1}{|\vec{m}|} e^{-|\vec{m}|\kappa L} = \frac{1}{L}\left(6e^{-\kappa L} + 6\sqrt{2}e^{-\sqrt{2}\kappa L} + \frac{8}{\sqrt{3}} e^{-\sqrt{3}\kappa L} + \ldots\right)$$

for L large compared to the size of the system, perturbation theory yields

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right) \qquad B_\infty^H = \frac{\gamma^2}{M}$$



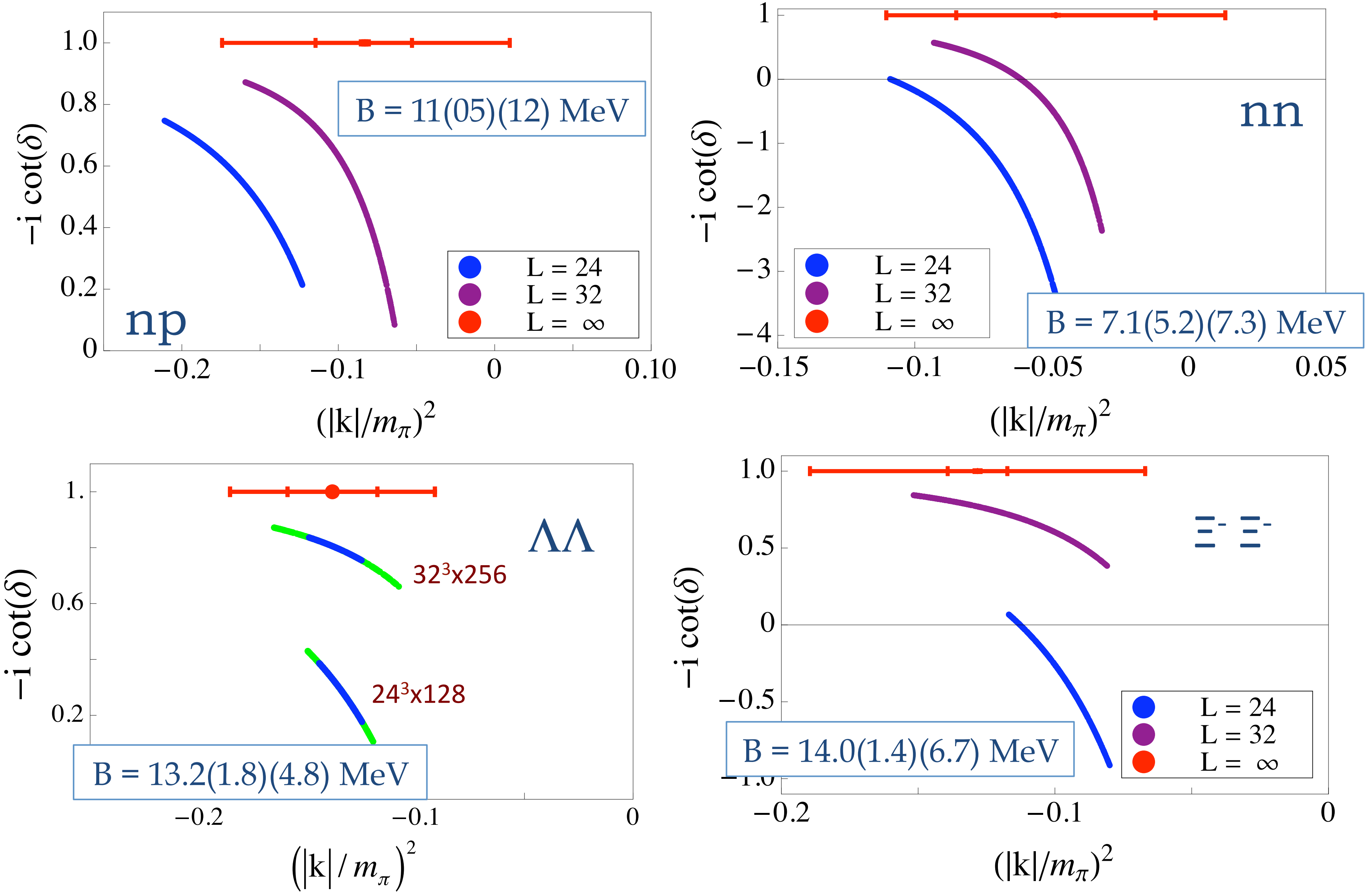
m_π~390 MeV

$$-i\cot\delta(k)\big|_{k=i\gamma} = 1$$

$$B_\infty^H = 13.2 \pm 4.4 \text{ MeV}$$

updated from PRL 106 (2011) 162001

bound states



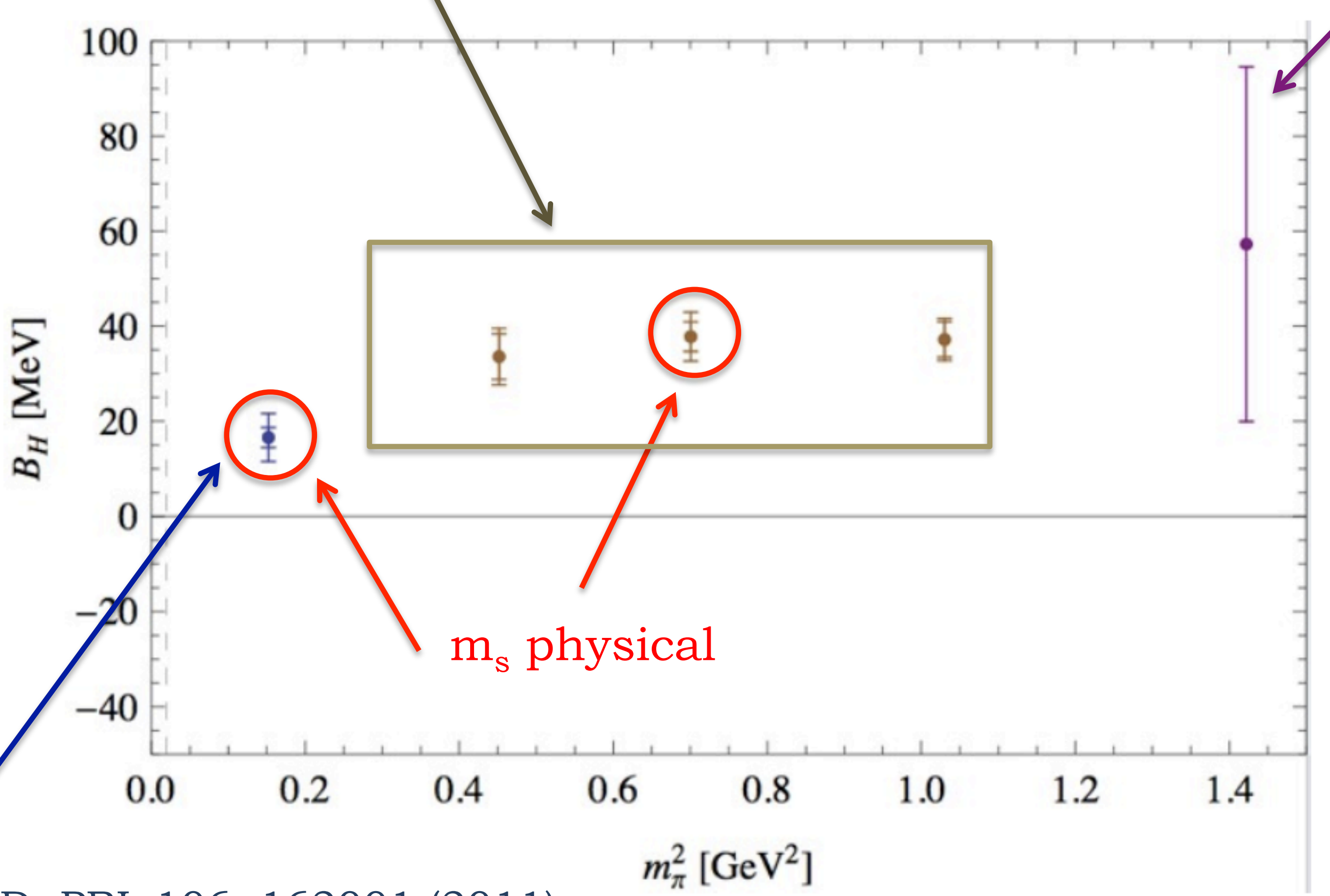
NPLQCD, Phys.Rev. D85 (2012) 054511

$\Lambda\Lambda - \Xi N - \Sigma\Sigma$

The I=0, s=-2, $^1S_0$ channel

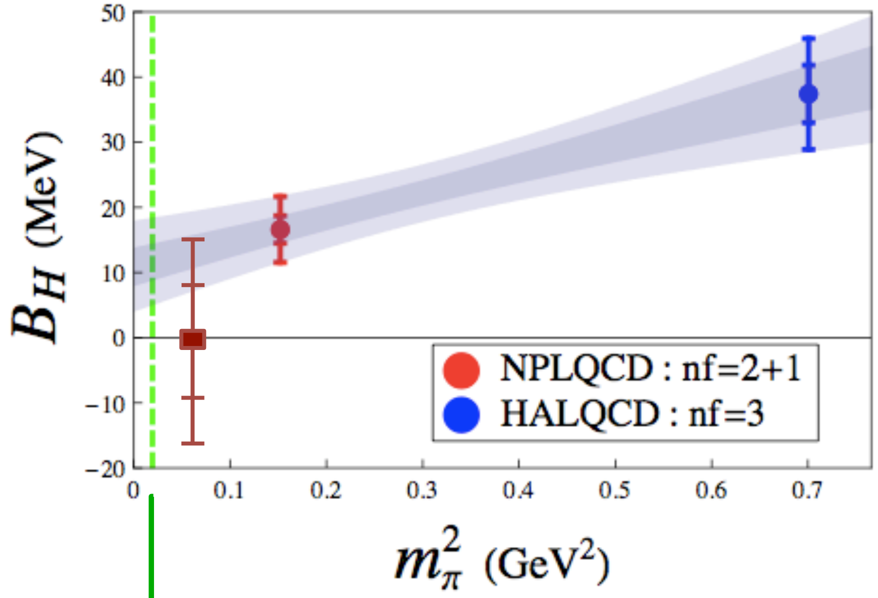
HALQCD, PRL 106, 162002 (2011)
n_f=3, b_s = 0.12 fm, L: 2, 3, 3.9 fm
m_π = 670, 830, 1015 MeV

Luo, Loan & Liu arXiv:1106.1945
n_f=0, 0.2 < b < 0.4 fm,
m_π > 1 GeV, L: 2.4 – 4.8 fm

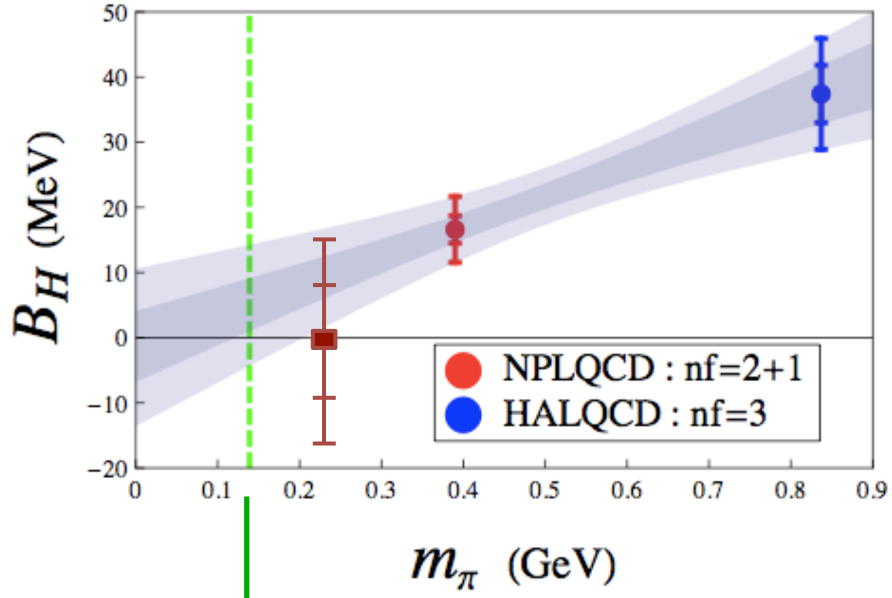


NPLQCD, PRL 106, 162001 (2011)
n_f=2+1, b_s = 0.12 fm, L: 2, 2.5, 3, 3.9 fm
m_π = 390 MeV

Extrapolation on the pion mass of the lattice results



Mod. Phys. Lett. A26 (2011) 2587



physical pion mass

$$B(m_\pi) = B_0 + d_1\, m_\pi^2$$

$$B_H^{quad} = +11.5 \pm 2.8 \pm 6 \text{ MeV}$$

$$B(m_\pi) = B_0 + c_1\, m_\pi$$

$$B_H^{lin} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV}$$

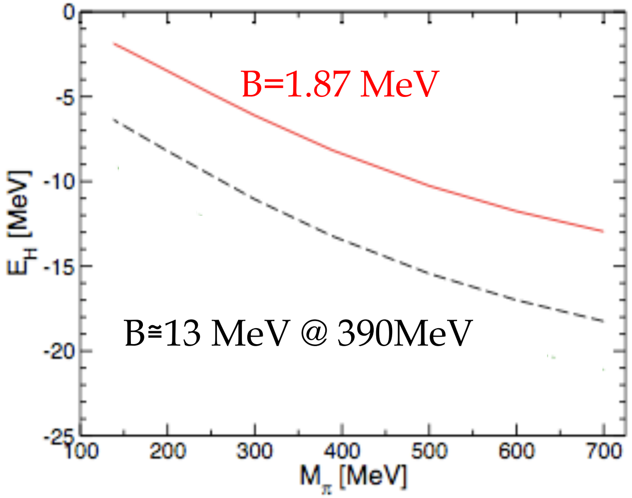
no definite conclusion of the “H” b.e.

We need more resources to perform simulations at lighter quark masses (m_π~200–250 MeV)

Other extrapolations on the pion mass

Quark mass dependence of the H-dibaryon b.e. in the framework of Chiral EFT and effects of SU(3) breaking

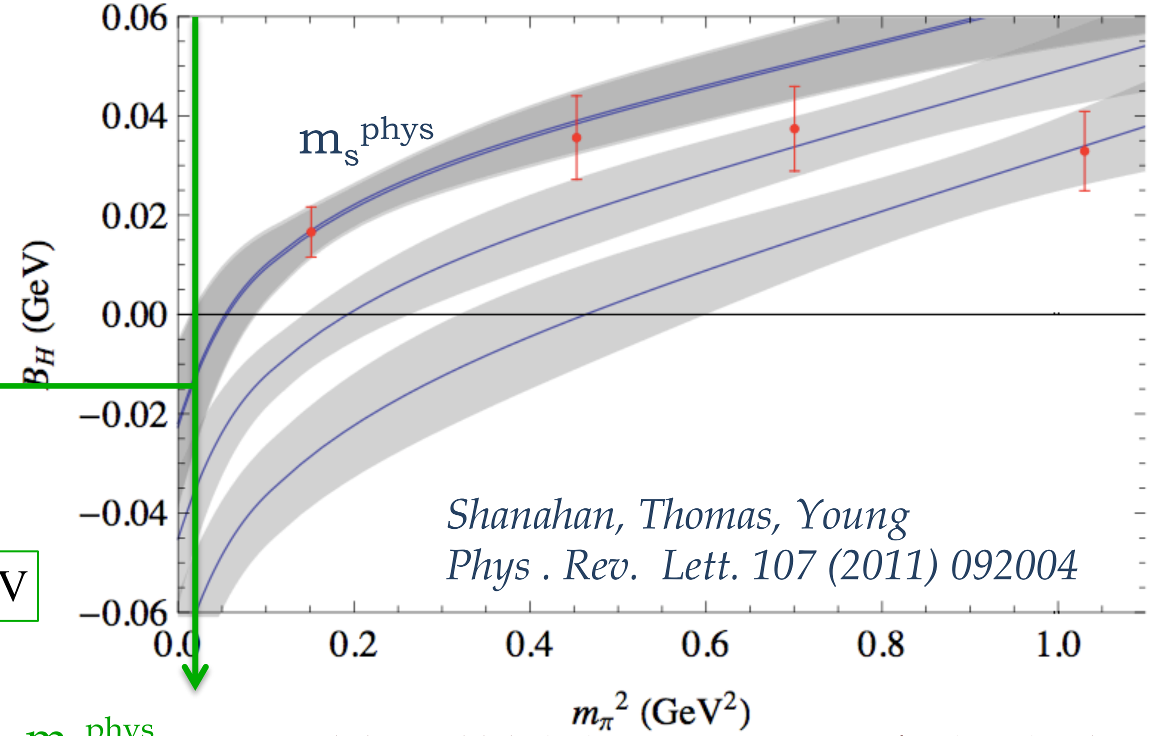
Haidenbauer, Meissner, Phys. Lett. B 706 (2011)



NPLQCD $m_N = 1151.3$ MeV $m_\Lambda = 1241.9$ MeV $m_\Sigma = 1280.3$ MeV

$m_\Xi = 1349.6$ MeV $m_\pi = 389$ MeV $\underset{C_1}{\longleftrightarrow}$ $E_H = -13.2$ MeV

→ the b.s. moves upwards but remains below the ΞN threshold

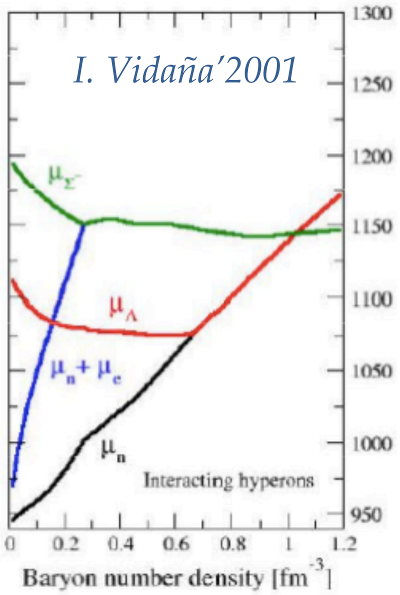


$B_H = -13 \pm 14$ MeV

m_π^{phys}

SU(3) χ PT (Finite Range Regularization)

The $^1S_0$ and $^3S_1$ $\Sigma^- n$ interactions



$SU(3)_f$ content of the different interaction channels

$$SU(3)_f \qquad 8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_S \oplus 8_A \oplus 1$$

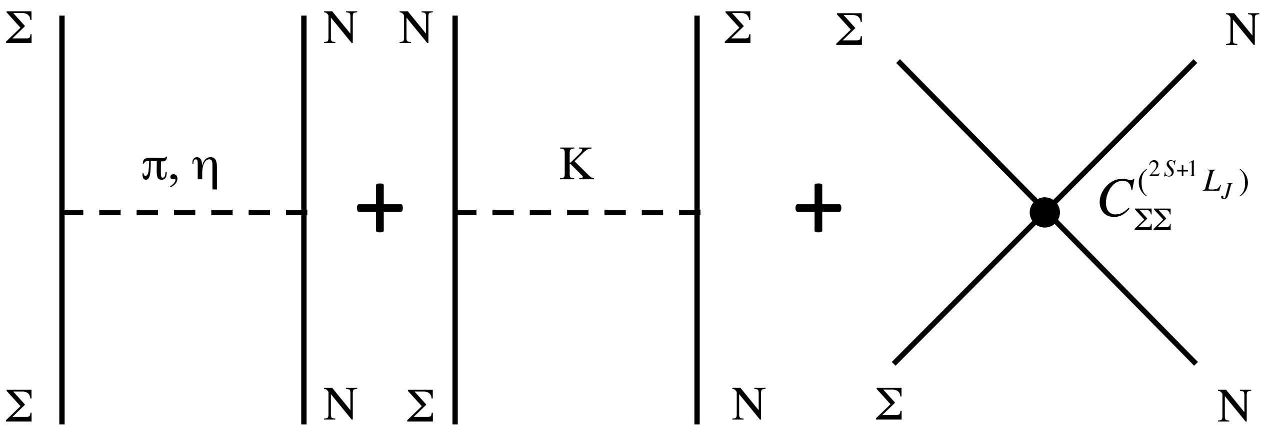
space-spin antisymmetric states ($^1S_0$, ^3P, …)

	S	I	Channels	SU(3) irreps
	0	1	NN	{27}
	-1	1/2	ΛN, ΣN	{27}, $\{8\}_s$
		3/2	ΣN	{27}
	-2	0	ΛΛ, ΞN, ΣΣ	{27}, $\{8\}_s$, {1}
		1	ΞN, ΣΛ	{27}, $\{8\}_s$
(b.s.)		2	ΣΣ	{27}
	-3	1/2	ΞΛ, ΞΣ	{27}, $\{8\}_s$
		3/2	ΞΣ	{27}
(b.s.)	-4	1	ΞΞ	{27}

space-spin symmetric states ($^3S_1$, $^1P_1$, …)

S	I	Channels	SU(3) irreps
0	0	NN	{10*}
-1	1/2	ΛN, ΣN	{10*}, $\{8\}_a$
	3/2	ΣN	{10}
-2	0	ΞN	$\{8\}_a$
	1	ΞN, ΣΣ	{10}, {10*}, $\{8\}_a$
	1	ΣΛ	{10}, {10*}
-3	1/2	ΞΛ, ΞΣ	{10}, $\{8\}_a$
	3/2	ΞΣ	{10*}
-4	0	ΞΞ	{10}

Leading Order ΣN interactions



$$C_{\Sigma\Sigma}^{(^{2S+1}L_J)}$$

quark-mass dependence

$$V_{REG}(r) \sim g e^{-\Lambda^2 r^2}, \quad g e^{-\Lambda^4 r^4}$$

$$V_{\Sigma N}^{^3S_1}(r) = \frac{\alpha}{6}\left(\frac{g_A}{f_\pi}\right)^2 m_\pi^2 \frac{e^{-m_\pi r}}{4\pi r} + \frac{(1-2\alpha)^2}{6}\left(\frac{g_A}{f_\pi}\right)^2 m_K^2 \frac{e^{-m_K r}}{4\pi r} + \frac{(1-\alpha)(4\alpha-1)}{18}\left(\frac{g_A}{f_\pi}\right)^2 m_\eta^2 \frac{e^{-m_\eta r}}{4\pi r}$$

$$V_{\Sigma N}^{^1S_0}(r) = -\frac{\alpha}{2}\left(\frac{g_A}{f_\pi}\right)^2 m_\pi^2 \frac{e^{-m_\pi r}}{4\pi r} - \frac{(1-2\alpha)^2}{2}\left(\frac{g_A}{f_\pi}\right)^2 m_K^2 \frac{e^{-m_K r}}{4\pi r} - \frac{(1-\alpha)(4\alpha-1)}{6}\left(\frac{g_A}{f_\pi}\right)^2 m_\eta^2 \frac{e^{-m_\eta r}}{4\pi r}$$

(only S-wave)

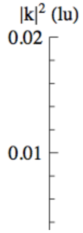
$f_\pi = 92.4\,\text{MeV}$ $\quad \alpha = \frac{F}{F+D}$

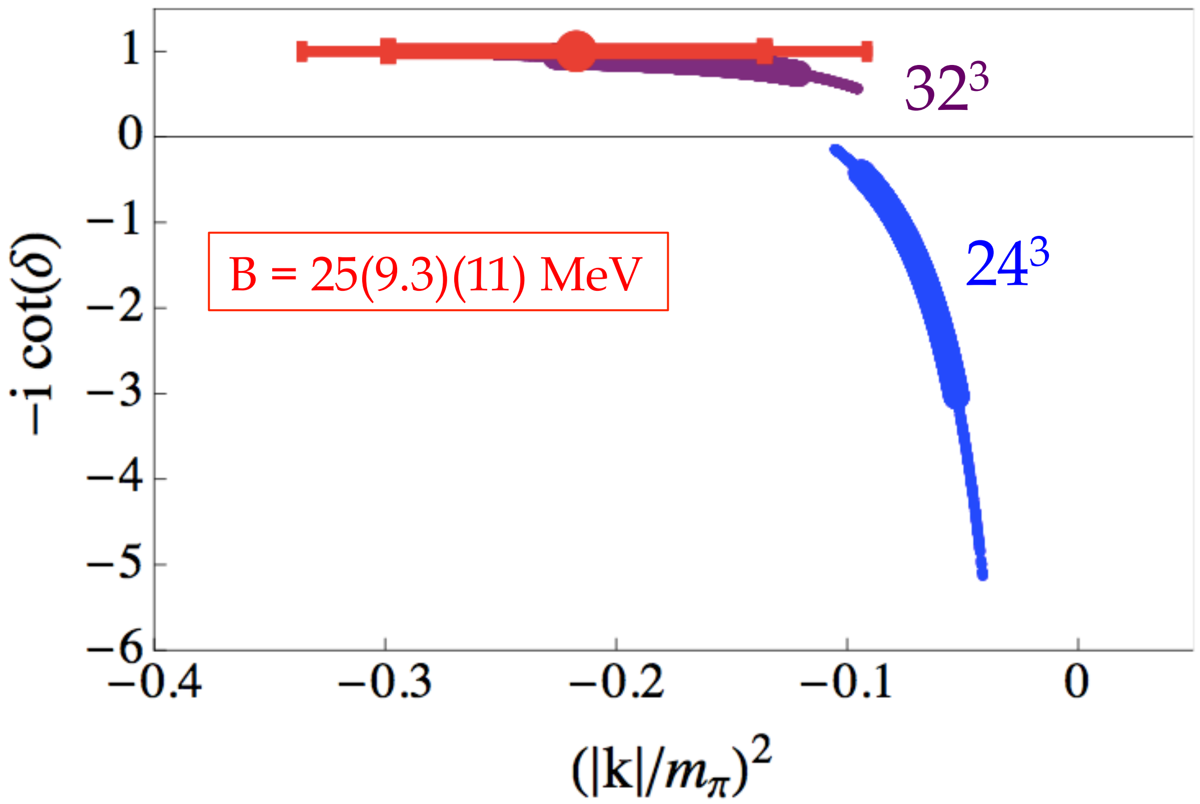
$^1S_0$ $\Sigma^- n$

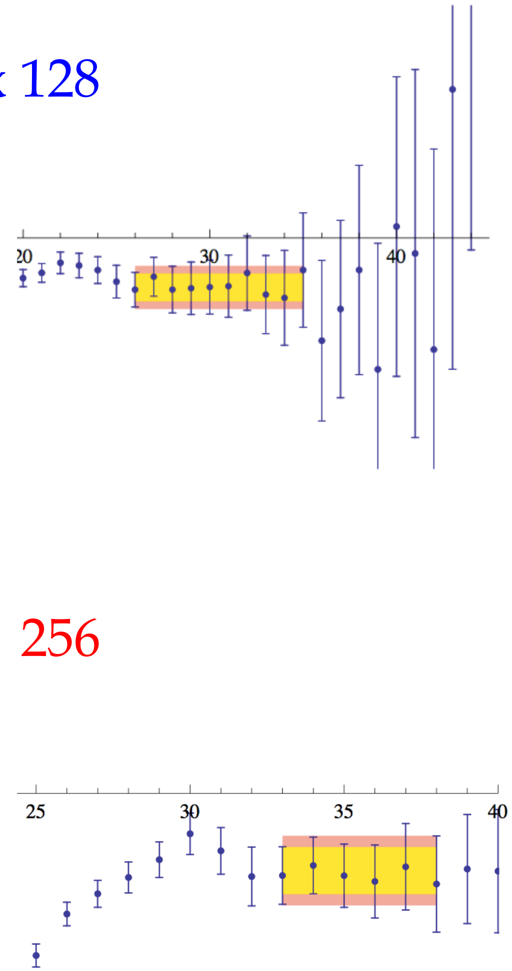
m_π ~390 MeV

Attractive channel

i.e. consistent with $SU(3)_f$

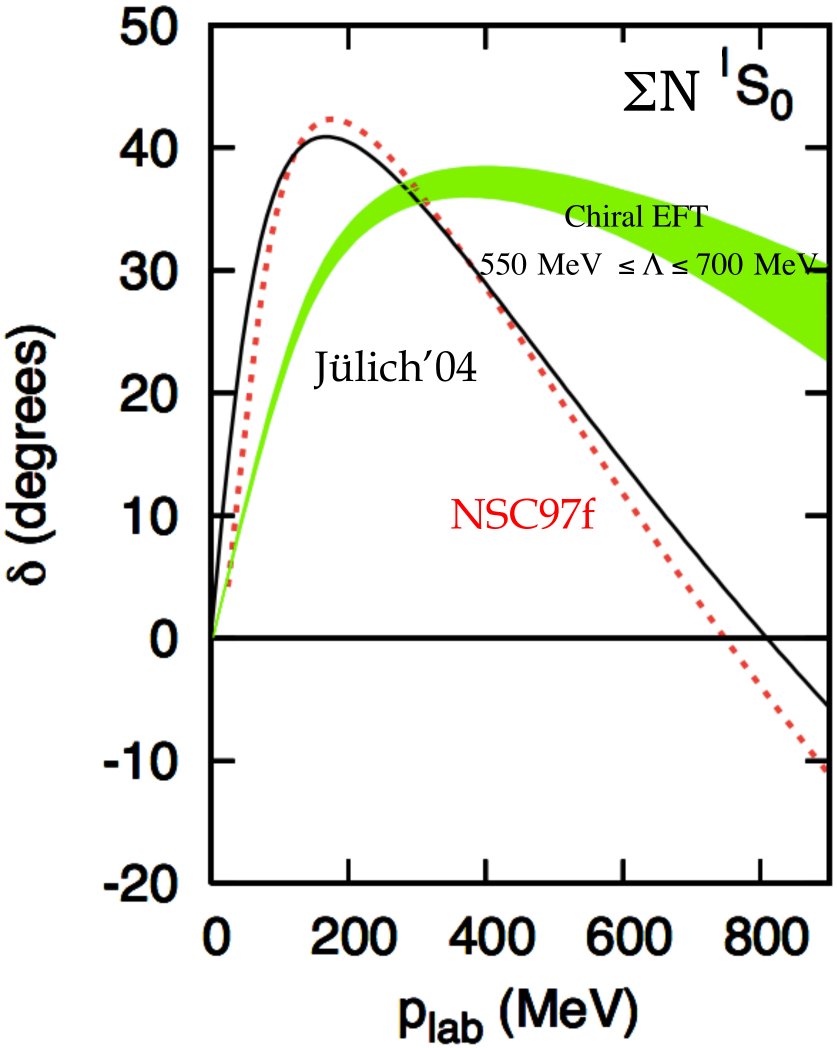




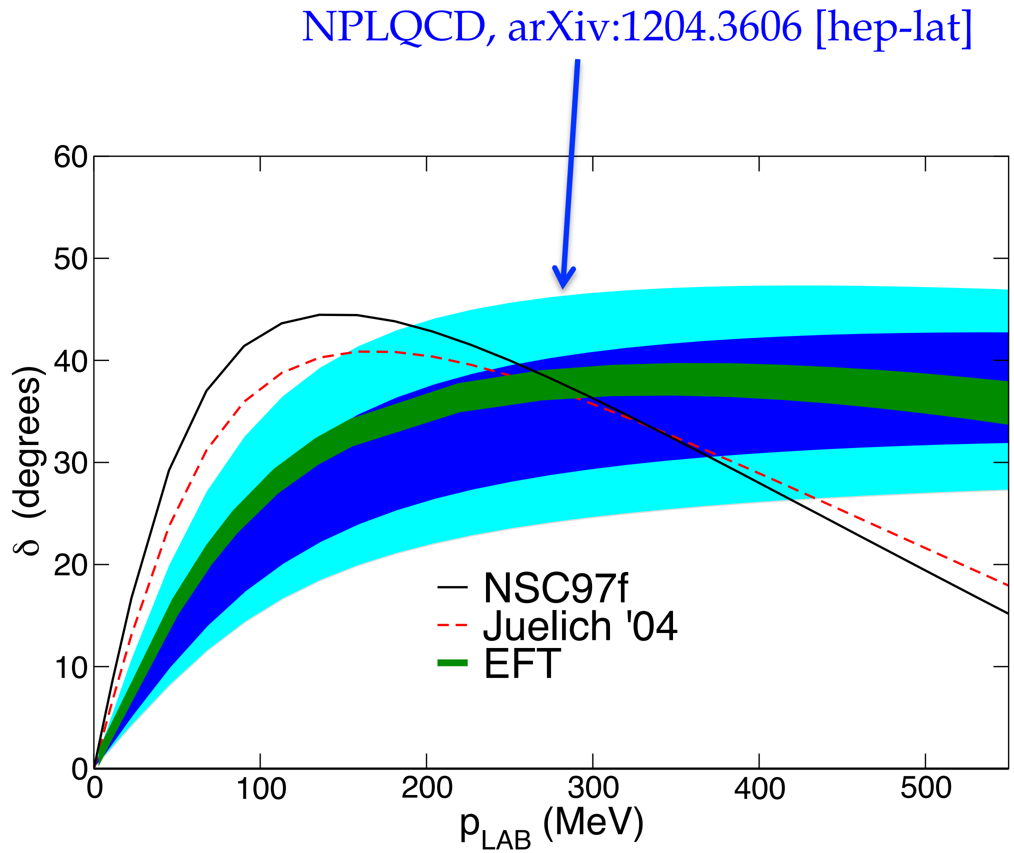


The $C_{\Sigma\Sigma}^{(^1S_0)}$ coefficient is fit to reproduce the B.E calculated in the lattice

$^1S_0\ \Sigma^- n$



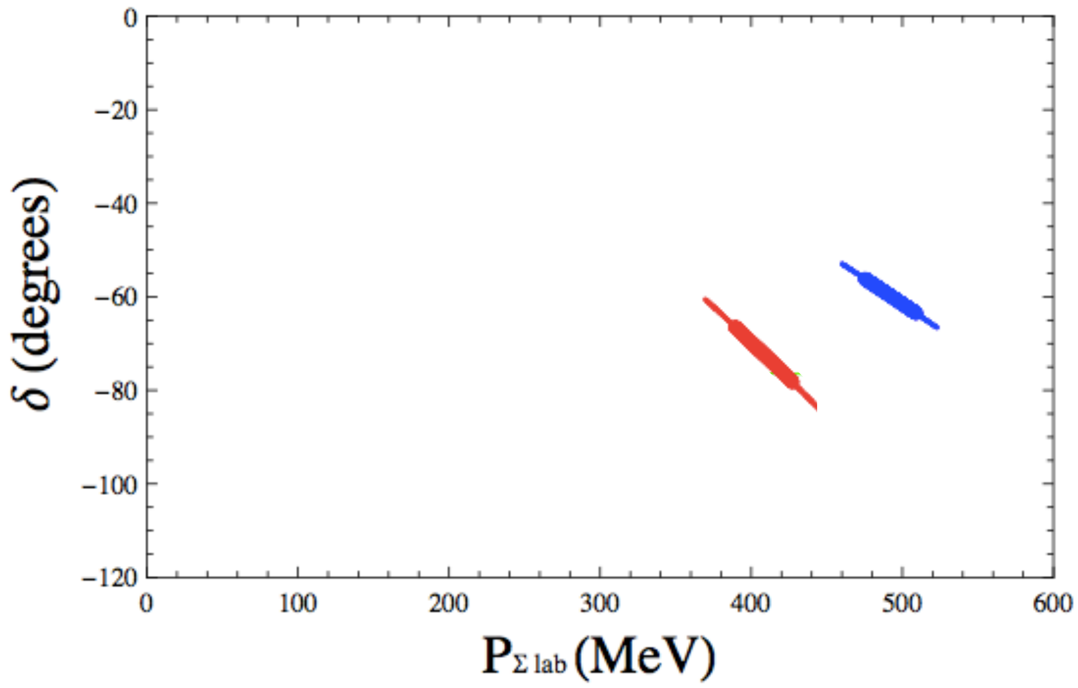
LQCD predicition @ m_π^{phys}

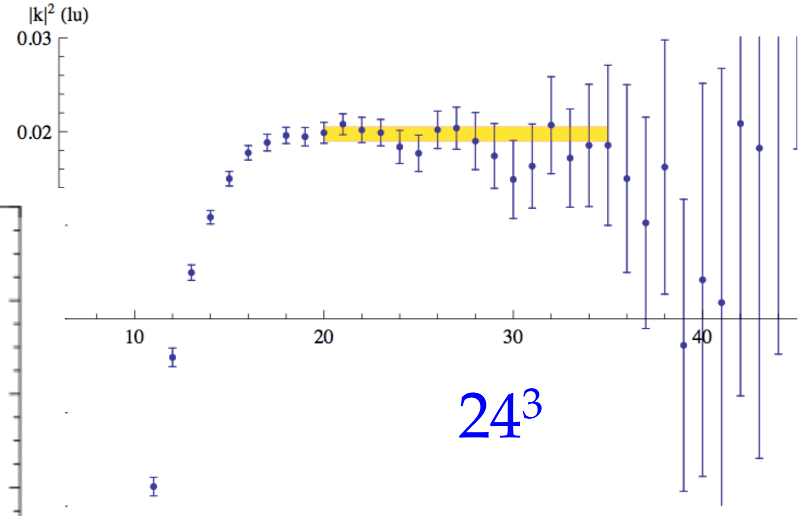


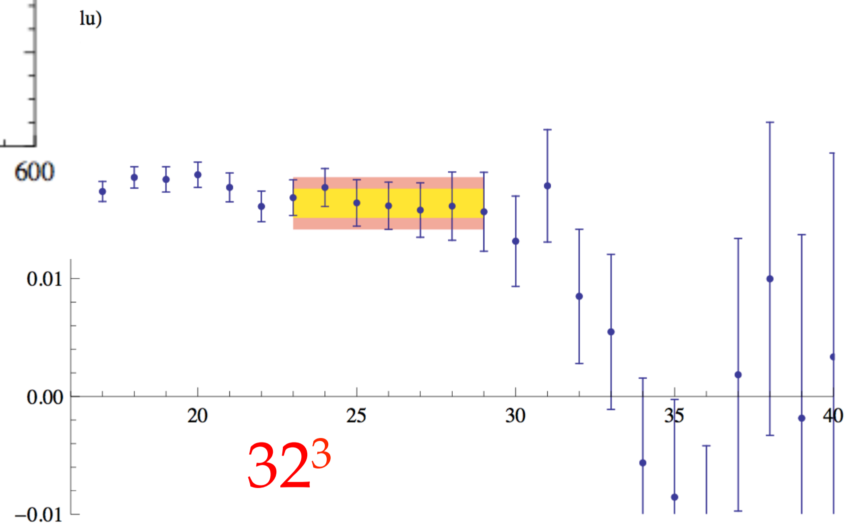
J. Haidenbauer, U.-G. Meissner, A. Nogga and H. Polinder, Lect. Notes Phys. **724** (2007) 113

$^3S_1\ \Sigma^- n$

m_π~390 MeV







The interaction in this channel is very repulsive

→ Hard repulsive potential core of extended size

Soving the 3D Schrödinger Equation $-\frac{\hbar^2}{2\mu}\nabla^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\psi(\vec{r})$

$$\psi(\vec{r})=\int\frac{d^3k}{(2\pi)^3}\exp\left(i\vec{k}\vec{r}\right)\tilde{\psi}(\vec{k}) \;\rightarrow\; \frac{1}{L^3}\sum_{\vec{n}}\exp\left(\frac{2\pi}{L}\vec{n}\vec{r}\right)\tilde{\psi}\left(\frac{2\pi}{L}\vec{n}\right)$$

$$V(\vec{r})=\int\frac{d^3k}{(2\pi)^3}\exp\left(i\vec{k}\vec{r}\right)\tilde{V}(\vec{k}) \;\rightarrow\; \frac{1}{L^3}\sum_{\vec{n}}\exp\left(\frac{2\pi}{L}\vec{n}\vec{r}\right)\tilde{V}\left(\frac{2\pi}{L}\vec{n}\right)$$

$$\psi(\vec{r})=\psi(\vec{r}+\vec{m}L)$$
$$V(\vec{r})=V(\vec{r}+\vec{m}L)$$

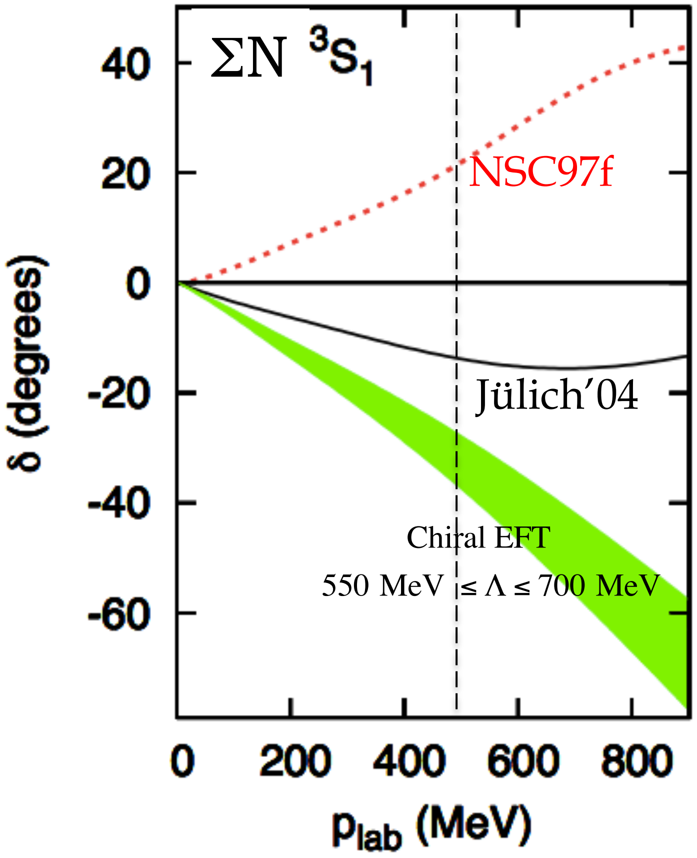
$$\hat{H}_{\vec{n},\vec{n}'}\tilde{\psi}_L\left(\frac{2\pi}{L}\vec{n}\right)=E_L\tilde{\psi}_L\left(\frac{2\pi}{L}\vec{n}\right)$$

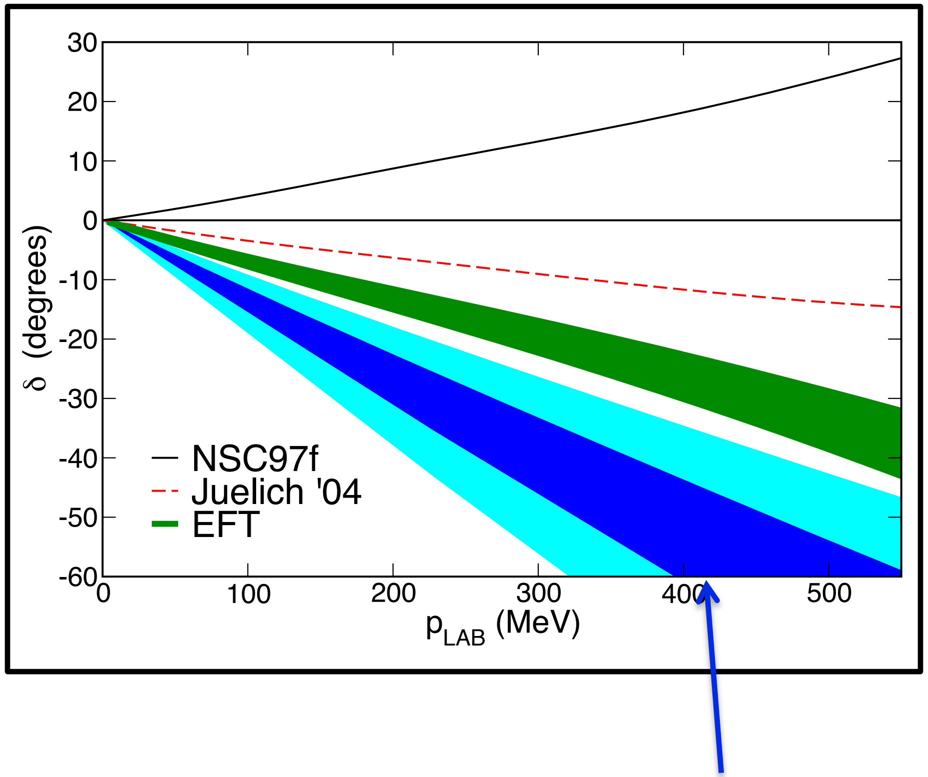
$$\hat{H}_{\vec{n},\vec{n}'}=-\frac{2\pi^2\hbar^2}{\mu L^2}\left|\vec{n}\right|^2\delta_{\vec{n},\vec{n}'}+\tilde{V}\left(\frac{2\pi}{L}(\vec{n}-\vec{n}')\right)$$

Reproduce the energy levels obtained in our LQCD calculations

$^3S_1 \ \Sigma^- n$

m_π^{phys}





J. Haidenbauer, U.-G. Meissner, A. Nogga and H. Polinder,
Lect. Notes Phys. **724** (2007) 113

A simple estimation of the Energy-shift of Σ^-'s in dense neutron matter using Fumi's theorem *G.D. Mahan, Many-Particle Physics, Plenum Press, NY (1981)*

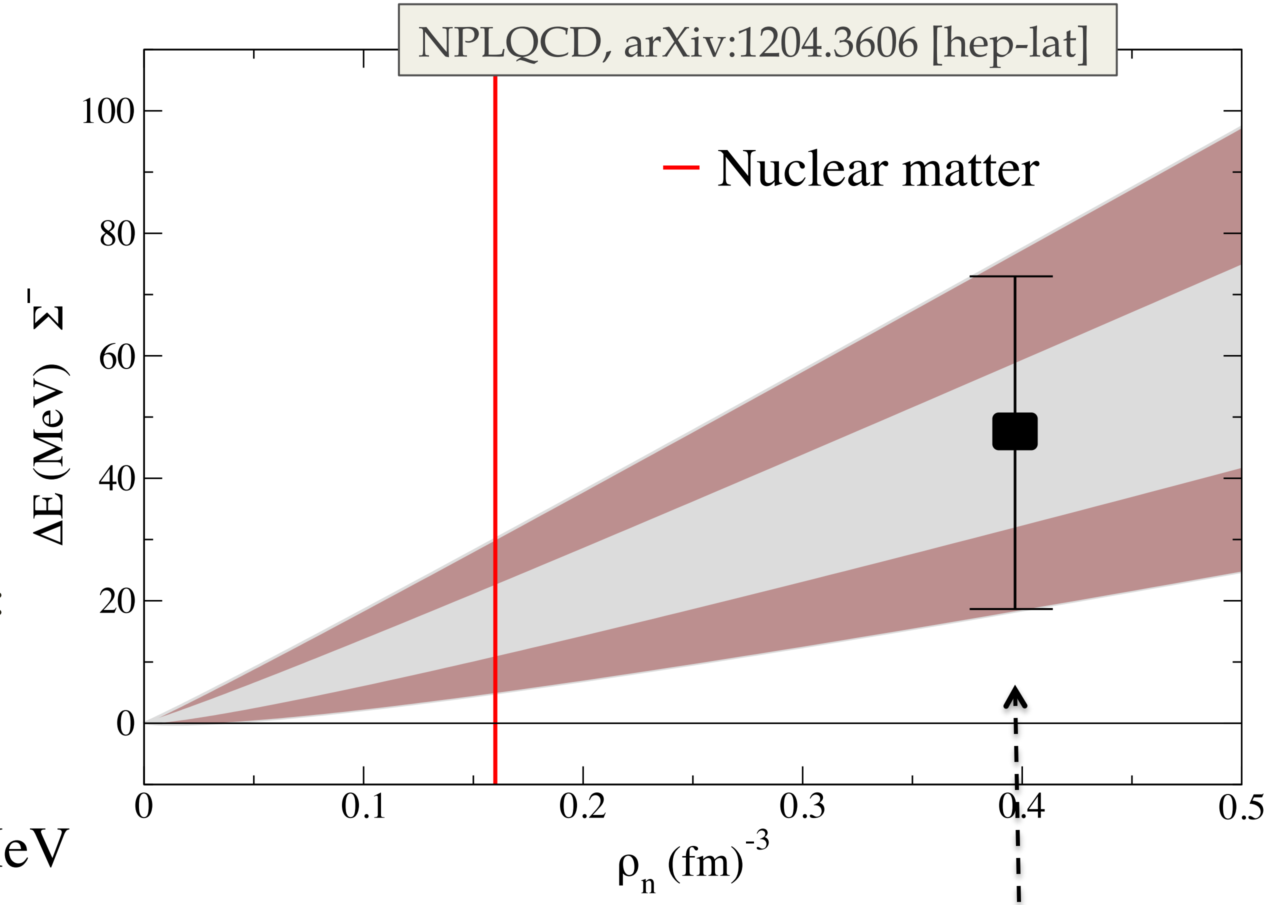
$$\Delta E = -\frac{1}{\pi\mu}\int_0^{k_F} dk\, k\left[\frac{3}{2}\,\delta_{^3S_1}(k)+\frac{1}{2}\,\delta_{^1S_0}(k)\right]$$

$$\rho_n \sim 0.4\ \text{fm}^{-3} \rightarrow \mu_{e^-} \sim 200\ \text{MeV}$$
$$\Rightarrow \mu_n + \mu_{e^-} \sim 1290\ \text{MeV}$$
$$(\mu_n \sim M_N + 150\ \text{MeV})$$

Σ^- would be a relevant dof in dense neutron matter if:

$$\mu_{\Sigma^-} = M_{\Sigma^-} + \Delta E \quad \leq \quad 1290\ \text{MeV}$$
$$(\Delta E \leq 100\ \text{MeV})$$

$$\Delta E = 46 \pm 13 \pm 24\ \text{MeV}$$



Lattice QCD is a field rapidly growing that can play an important role in the determination of quantities relevant to nuclear physics processes, and in particular, in hypernuclear physics

Our high statistics dynamical simulations of BB systems @ $m_\pi \sim 390$ MeV and at L ~ 2 , 2.5 , 3 , 4 fm allowed us to distinguish scattering states from bound states

We have found evidence of bound NN ($^1S_0$ and $^3S_1$), ΛΛ and ΞΞ ($^1S_0$) systems

We need more resources in order to undertake simulations at lighter quark masses (and large volumes) to constrain the chiral extrapolations

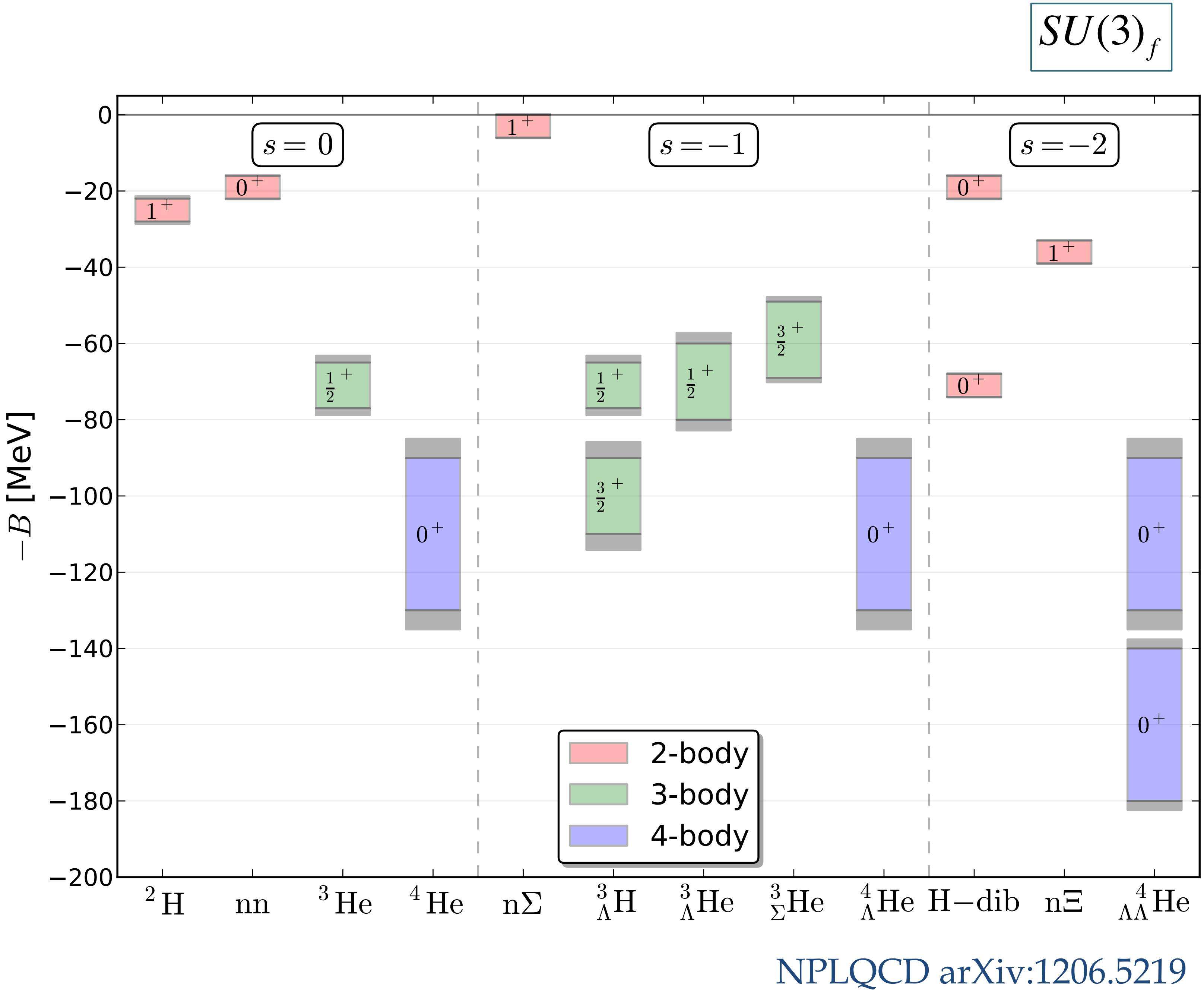
We have obtained the first LQCD predictions for hypernuclear physics:

Scattering phase-shifts for the $^1S_0$ and $^3S_1$ n Σ^- channels

At present:

- Analyzing the ΛN - ΣN system (I=1/2)
- Simulations at the SU(3) point
- Multibaryon systems

} Martin J. Savage's talk last Thursday

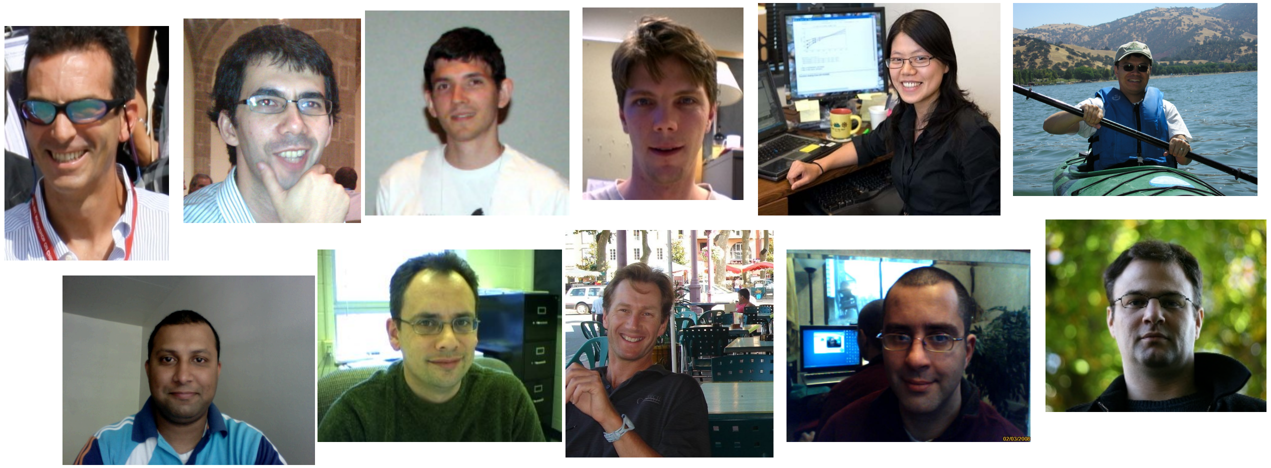


$SU(3)_f$
$s=0$
$s=-1$
$s=-2$
$-B$ [MeV]
0
−20
−40
−60
−80
−100
−120
−140
−160
−180
−200
1^+
0^+
$\frac{1}{2}^+$
$\frac{3}{2}^+$
2-body
3-body
4-body
^{2}H
nn
^{3}He
^{4}He
nΣ
$^3_\Lambda$H
$^3_\Lambda$He
$^3_\Sigma$He
$^4_\Lambda$He
H−dib
nΞ
$^4_{\Lambda\Lambda}$He
NPLQCD arXiv:1206.5219

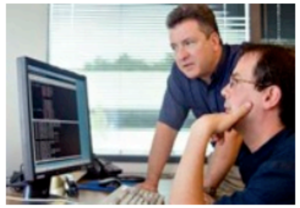
Acknowledgments

NPLQCD Collaboration

Silas R. Beane (New Hampshire), Emmanuel Chang (Barcelona), Saul Cohen (Washington), William Detmold (MIT), Huey-Wen Lin (Washington), Thomas Luu (LLNL), Parikshit Junnakar (New Hampshire), Kostas Orginos (William and Mary & JLab), Martin J. Savage (Washington), Aaron Torok (Indiana), André Walker-Loud (LBNL)



+



R. Edwards, B. Joó
JLab

Resources/Institutions