Roper Resonance and 1^{-+} Meson

- Roper resonance: lattice calculations and its nature
- Is 1^{-+} meson a hybrid?

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INT, Aug. 2, 2012

Many Facets of Roper Resonance

- Roper resonance (P_{11} in πN scattering, with I = ½, M = 1440 MeV, Γ ~ 300 MeV) was discovered in pion-nucleon partial wave analysis in 1963.
- πN and γN has large contamination from Δ.
- Confirmation from (α, α') scattering and J/ψ $\rightarrow \bar{N}N\pi$ decay.

(review by L. Alvarez-Ruso)

M. Ablikim, et al., PRL 97, 062001 (2006)

PRL **97,** 062001 (2006) PHYSICAL RE

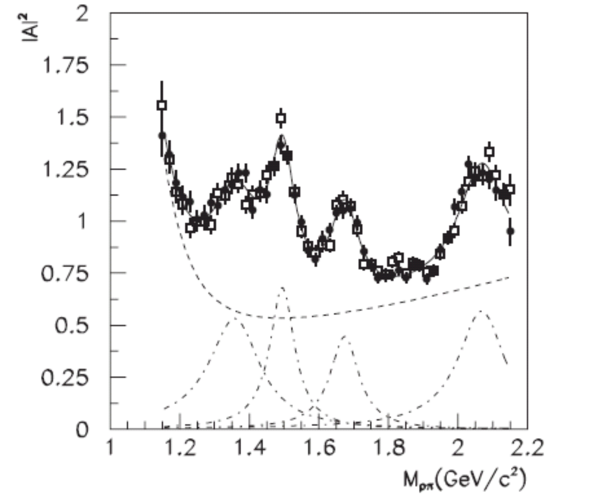
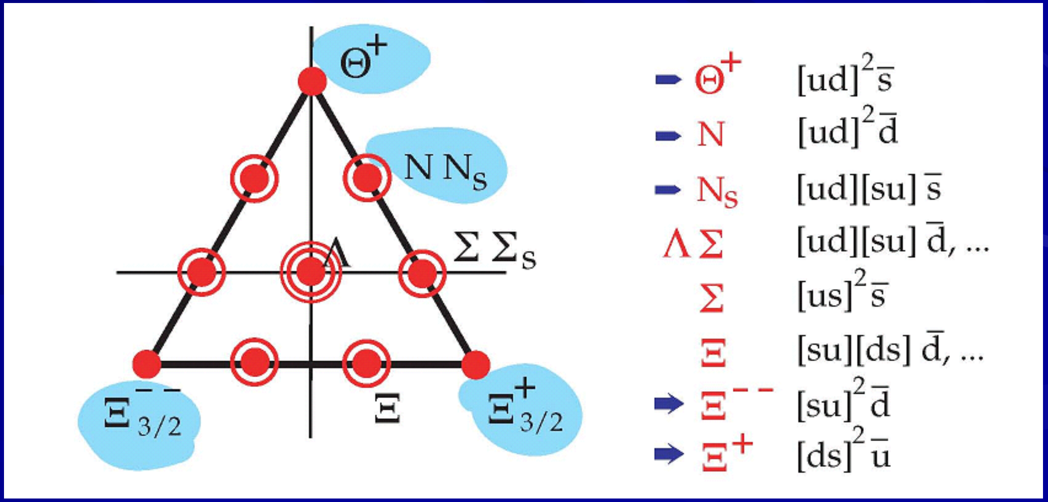


FIG. 6. Data divided by Monte Carlo phase space versus $p\pi$ invariant mass for $J/\psi \rightarrow p\pi^-\bar{n}$ (solid circles) and $J/\psi \rightarrow \bar{p}\pi^+n$ (open squares), compared with our fit (solid curve). The contributions of each resonance peak are shown by the dot-dashed lines in the same figure. The dashed line is the contribution of background terms including the nucleon pole term.

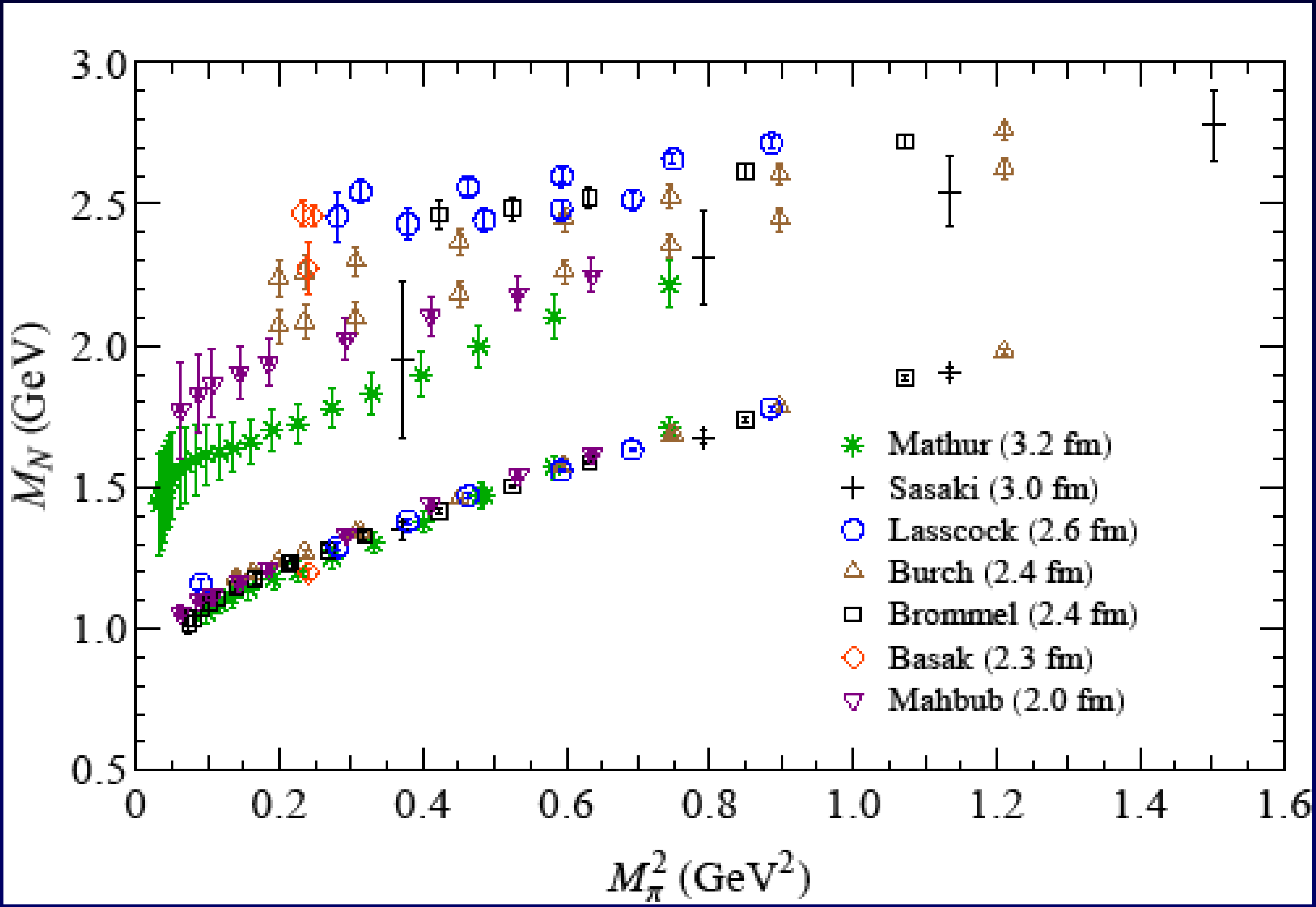
Many Facets of Roper Resonance

Theory:

- Quark potential model prediction is 100-200 MeV too high (Liu and Wong, 1983, Capstick and Isgur, 1986)
- Skyrmion can accommodate it as a radial excitation (J. Breit and C. Nappi, 1984 , Liu, Zhang, Black, 1984; U. Kaulfuss and U. Meissner, 1985)
- Suggestion as a pentaquark (Krewald 2000); as a member of the antidecuplet (Jaffe, Wilczek, 2003)
- Perhaps a hybrid (Barnes, Close, etc. 1983)
- → Lattice calculations



Quenched Lattice Calculations of Roper



Roper on the lattice

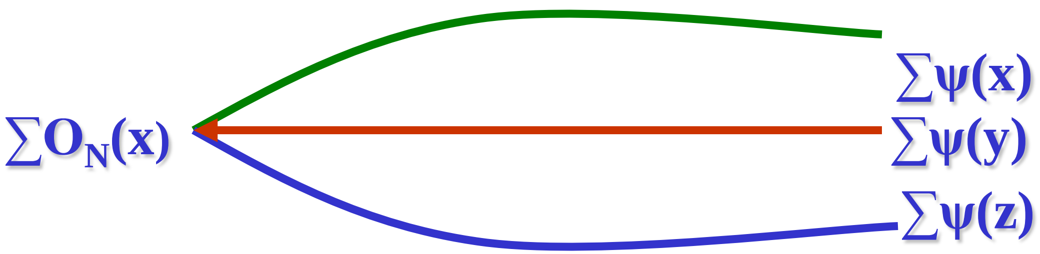
- 4 issues about lattice calculations:
  - Radial excitation or pentaquark state?
  - Dynamical fermions
  - Variation vs Bayesian fitting
  - Chiral dynamics

Roper

Radial excitation? $q^4\underline{q}$ State?

- Roper is seen on the lattice with **three-quark interpolation field.**
- Weight :

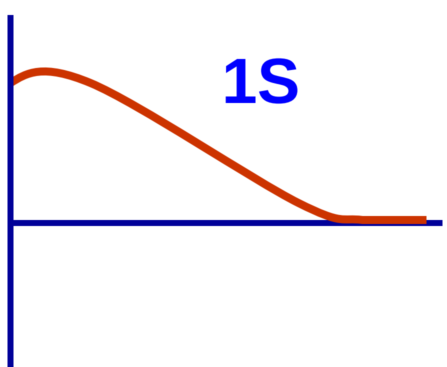
$$|\langle 0|O_N|R\rangle|^2 > |\langle 0|O_N|N\rangle|^2 > 0 \quad \text{(point source, point sink)}$$

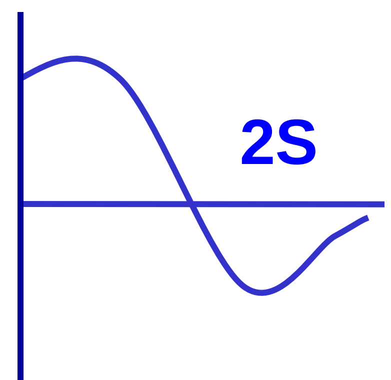


Point sink **Wall source**

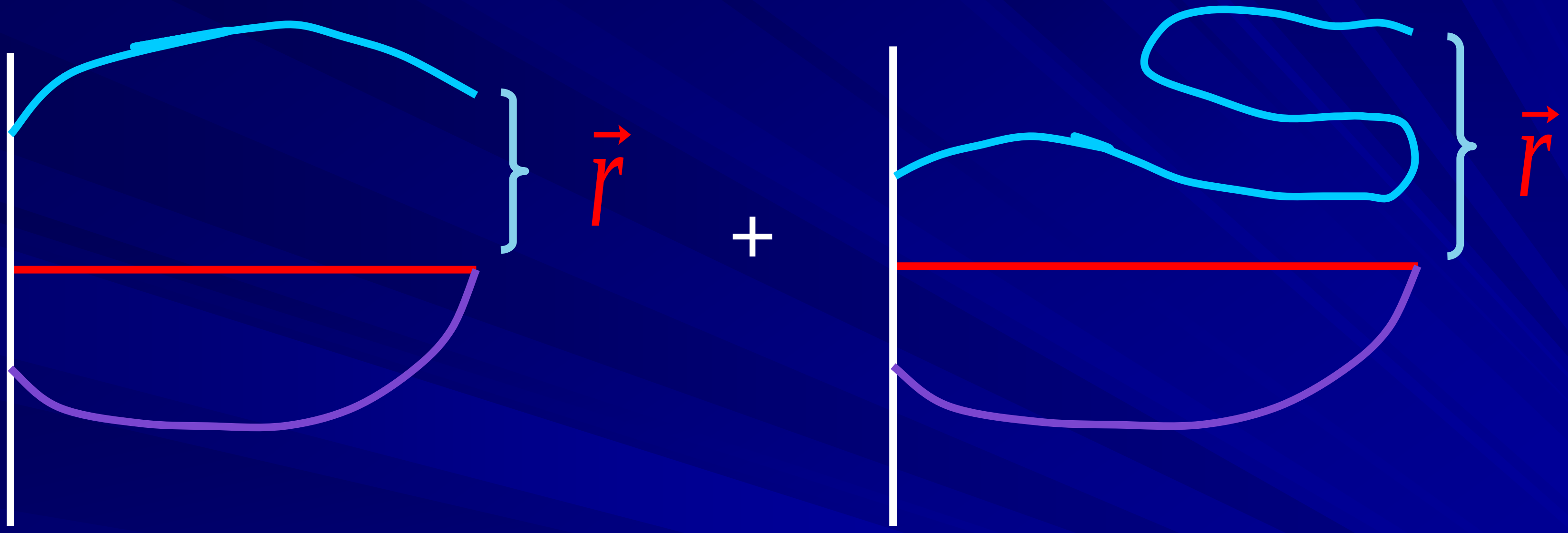
$$\langle 0|O_N(0)|N\rangle \langle N|\sum\psi(x)\sum\psi(y)\sum\psi(z)|0\rangle > 0$$

However, $\langle 0|O_N(0)|R\rangle \langle R|\sum\psi(x)\sum\psi(y)\,|\sum\psi(z)|0\rangle < 0$





Bethe-Salpeter Wavefunction

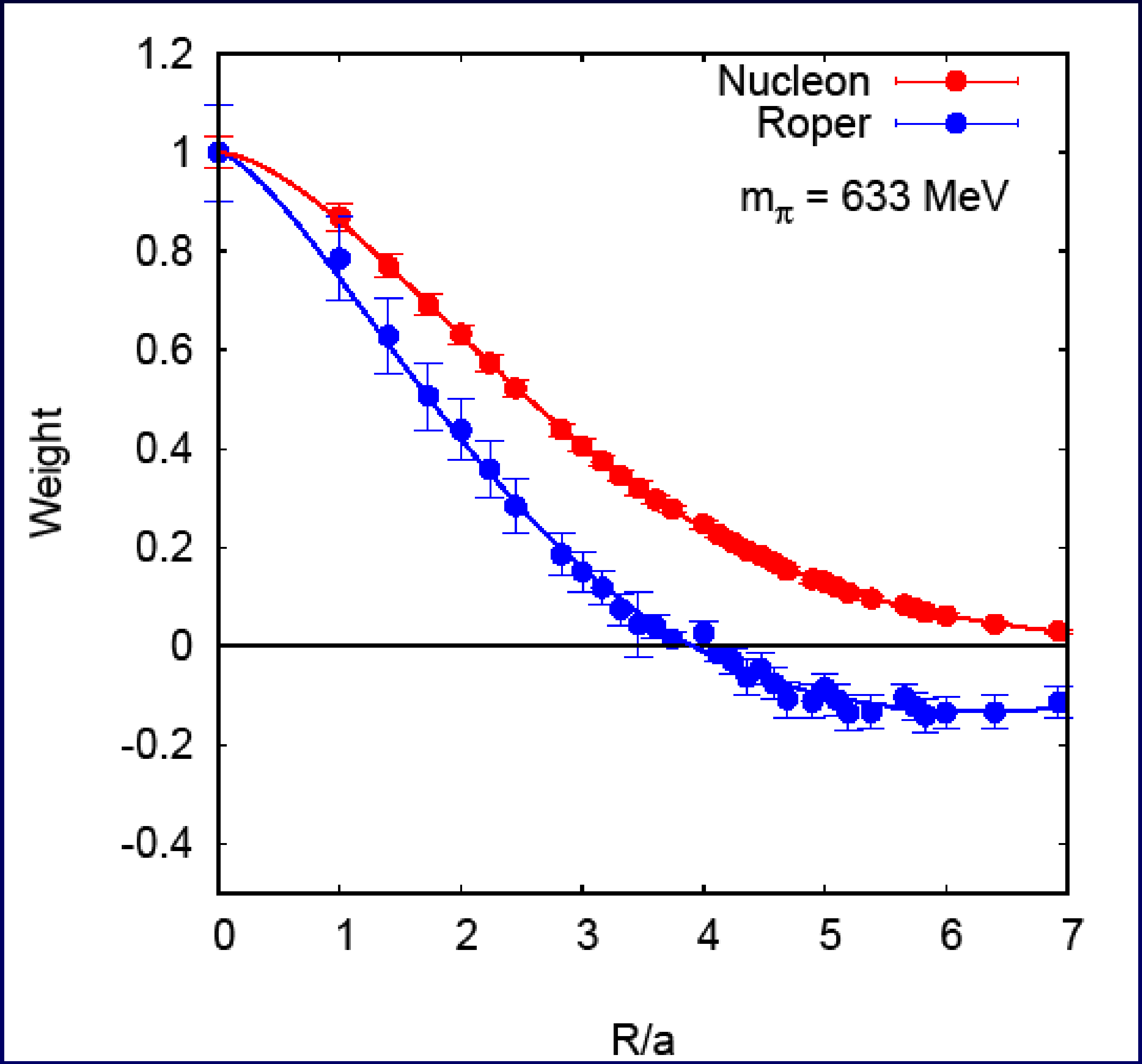


$$O_{RN} = \int dr\, \Psi^*_R(r)\Psi_N(r) = 0 \text{ at non-relativistic limit,}$$

$$O_{RN} = \int dr\, \Psi^*_R(r)\Psi_N(r) \uparrow \text{ as } m_q \downarrow$$

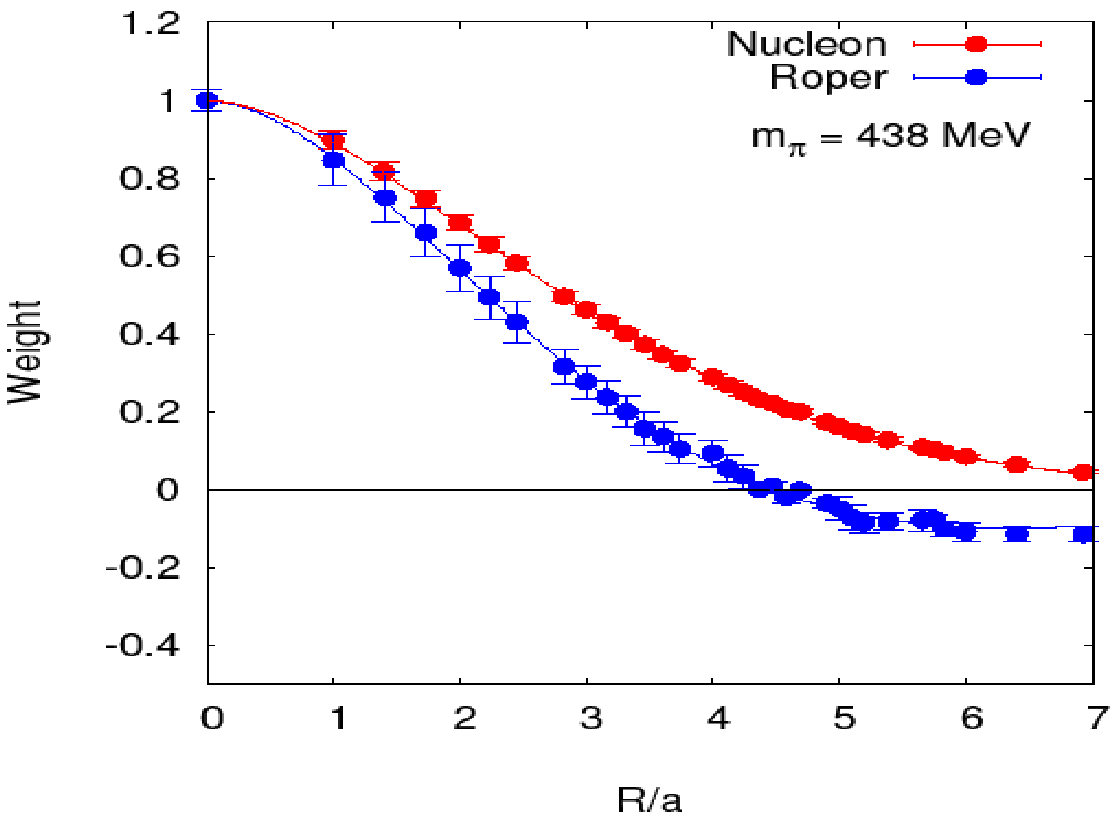
Nucleon and Roper wavefunctions for m_π = 633 MeV

O_{RN} = 0.30

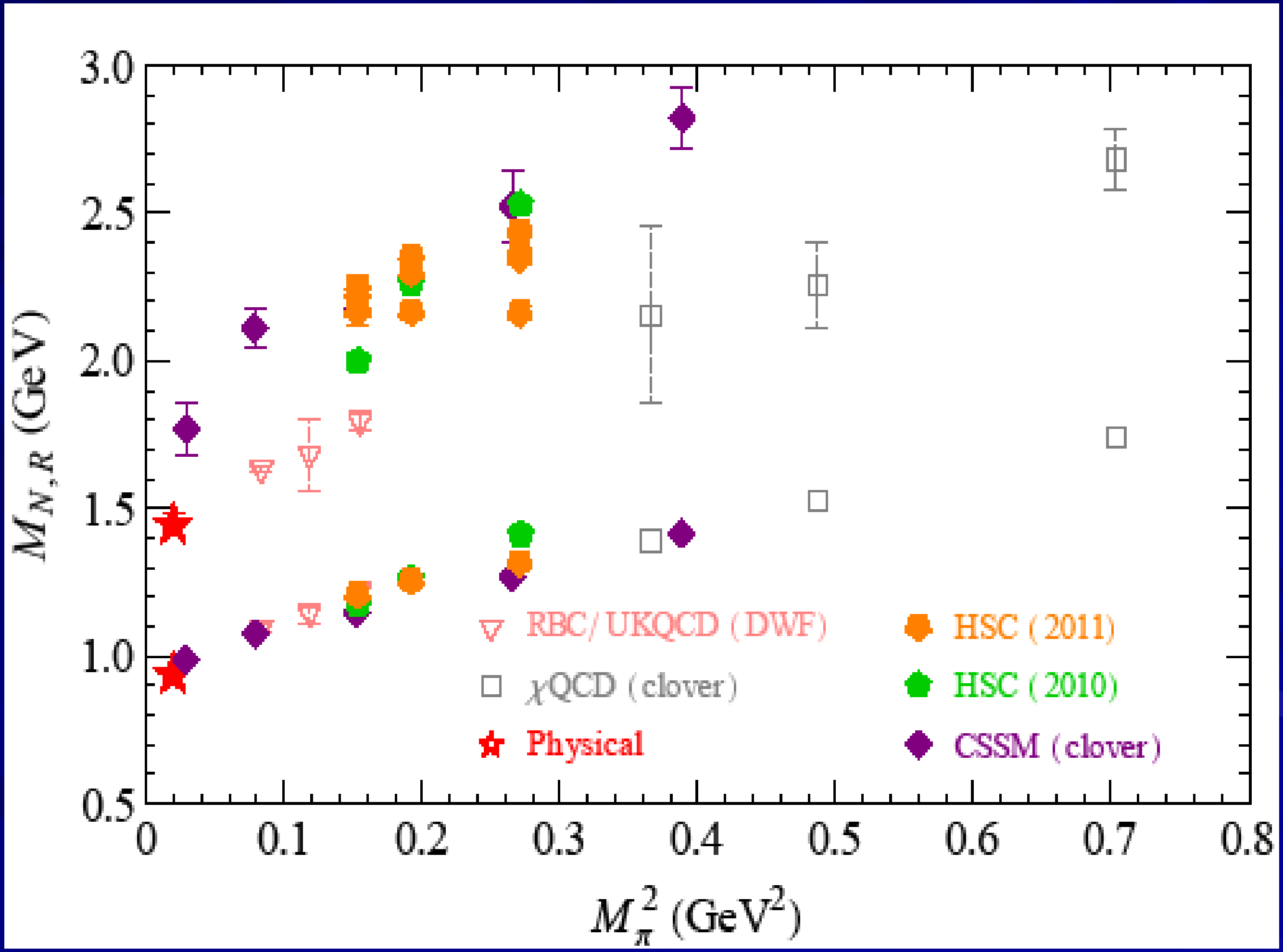


Roper and Nucleon Wavefunctions at m_π = 438 MeV

O_{RN} = 0.59

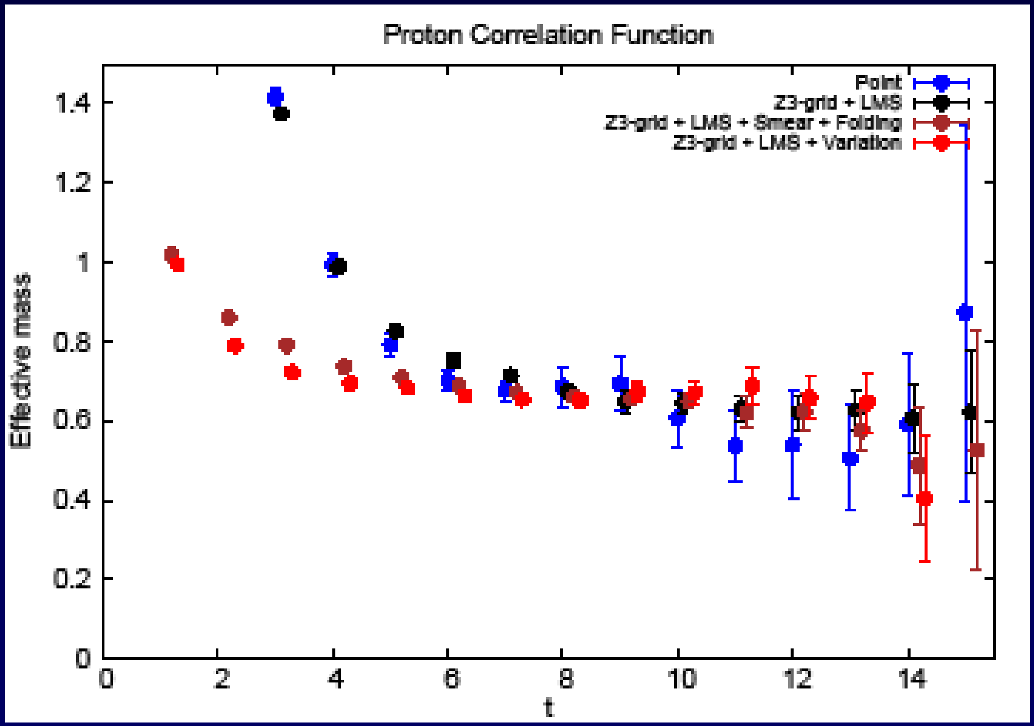


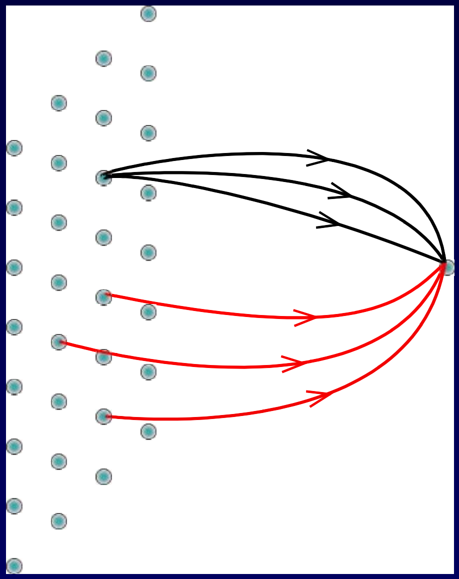
Dynamical Fermions



Dynamical Fermions (Overlap on DWF Configurations)

- Improvement of nucleon correlator with low-mode substitution





$24^3 \times 64$ lattice with $m_\pi = 331$ MeV, a = 1.73 GeV^{-1}

47 configurations

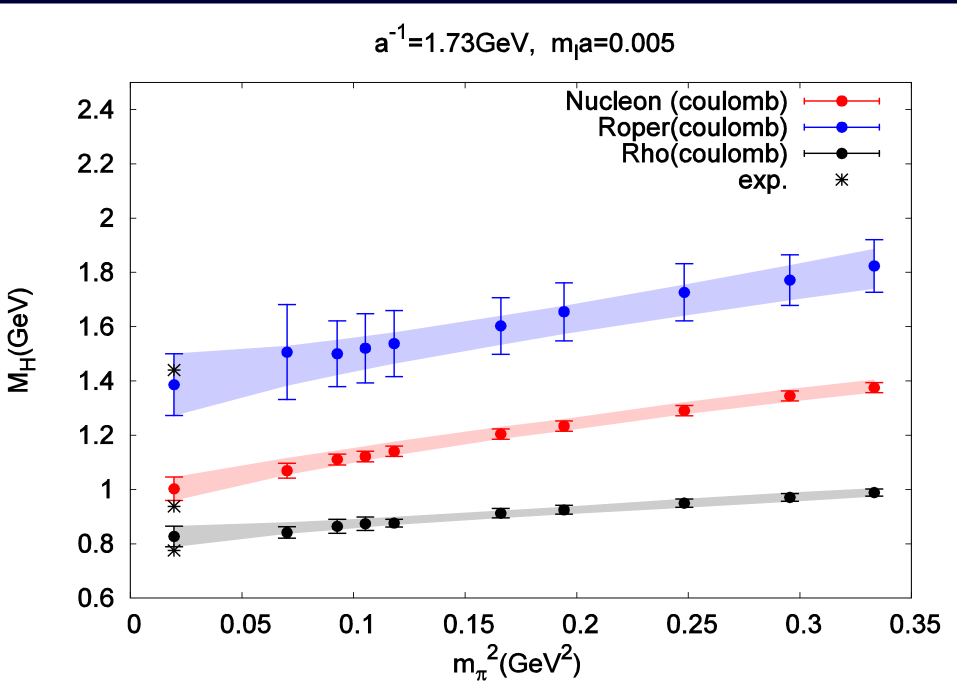
Point source: $m_N = 1.13(14)$ GeV;

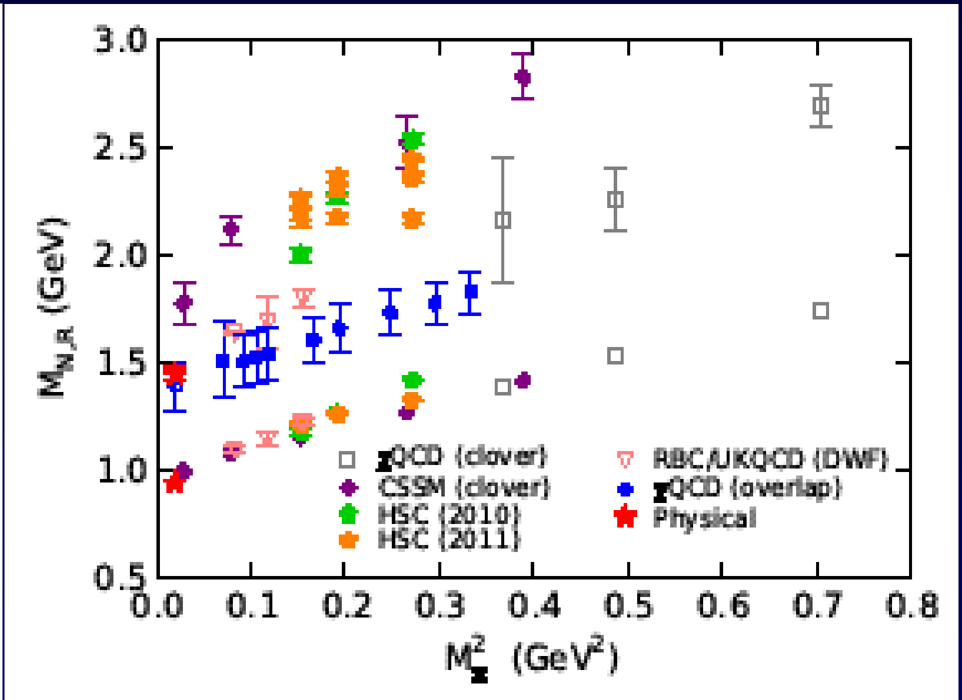
Z_3 grid source: $m_N = 1.08(5)$ GeV;

Z_3 grid smeared source: $m_N = 1.14(2)$ GeV;

Variation: $m_N = 1.16(1)$ GeV

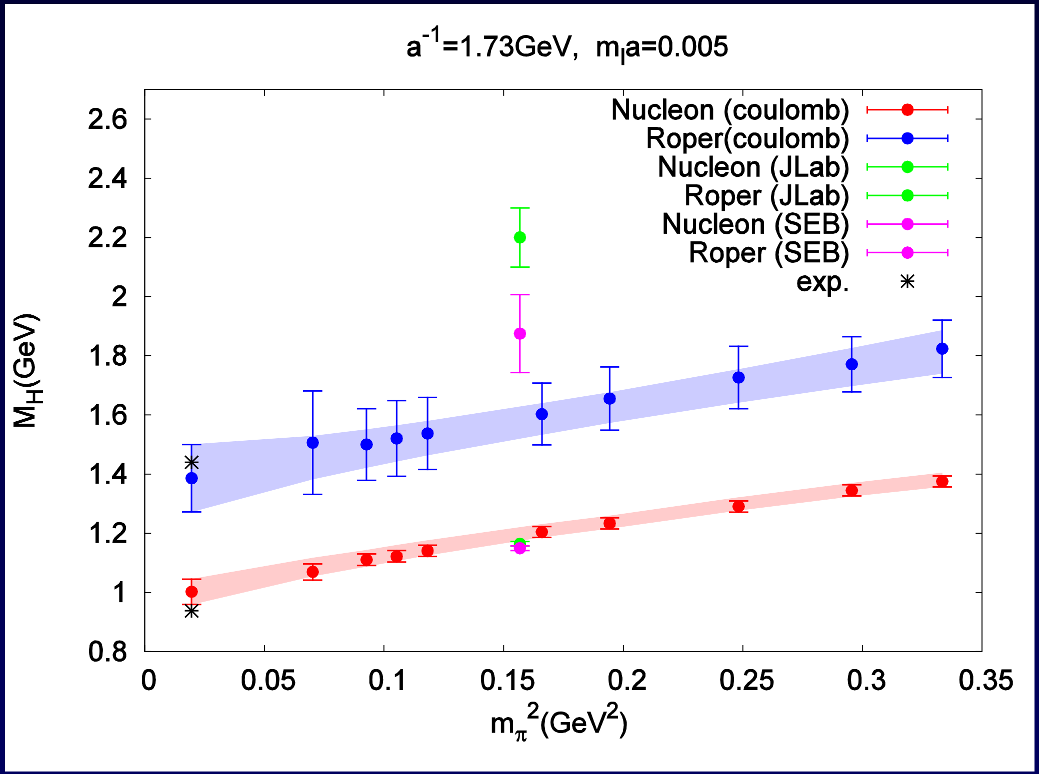
Roper resonance from Coulomb wall source





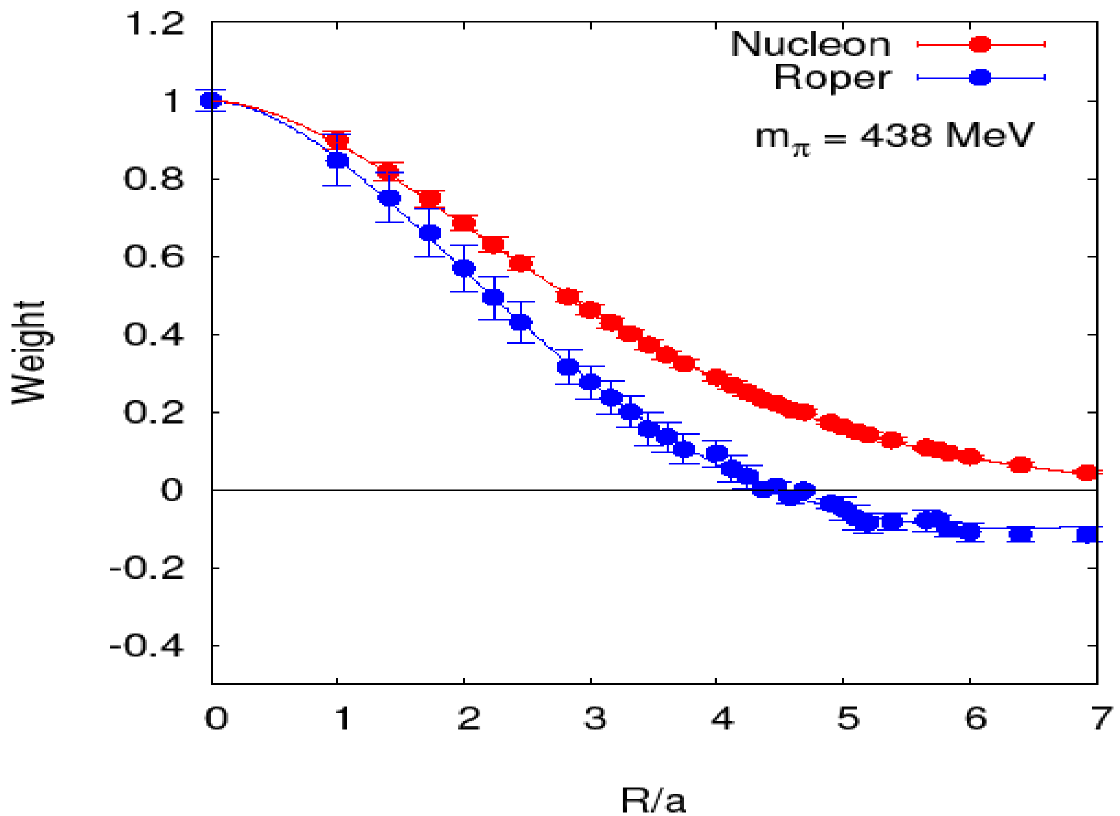
$$m_N = m_0 + c_1 m_\pi^2 + c_2 m_\pi^3 (\text{mixed}) + \text{c}_3 m_\pi^4 \ln(m_\pi^2 / \mu^2) + \ldots$$

$24^3 \times 64$ lattice with $m_\pi = 331$ MeV(sea), a = 1.73 GeV^{-1}


a^{-1}=1.73GeV, $m_l a$=0.005
M_H(GeV)
m_π^2(GeV^2)
Nucleon (coulomb)
Roper(coulomb)
Nucleon (JLab)
Roper (JLab)
Nucleon (SEB)
Roper (SEB)
exp.
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

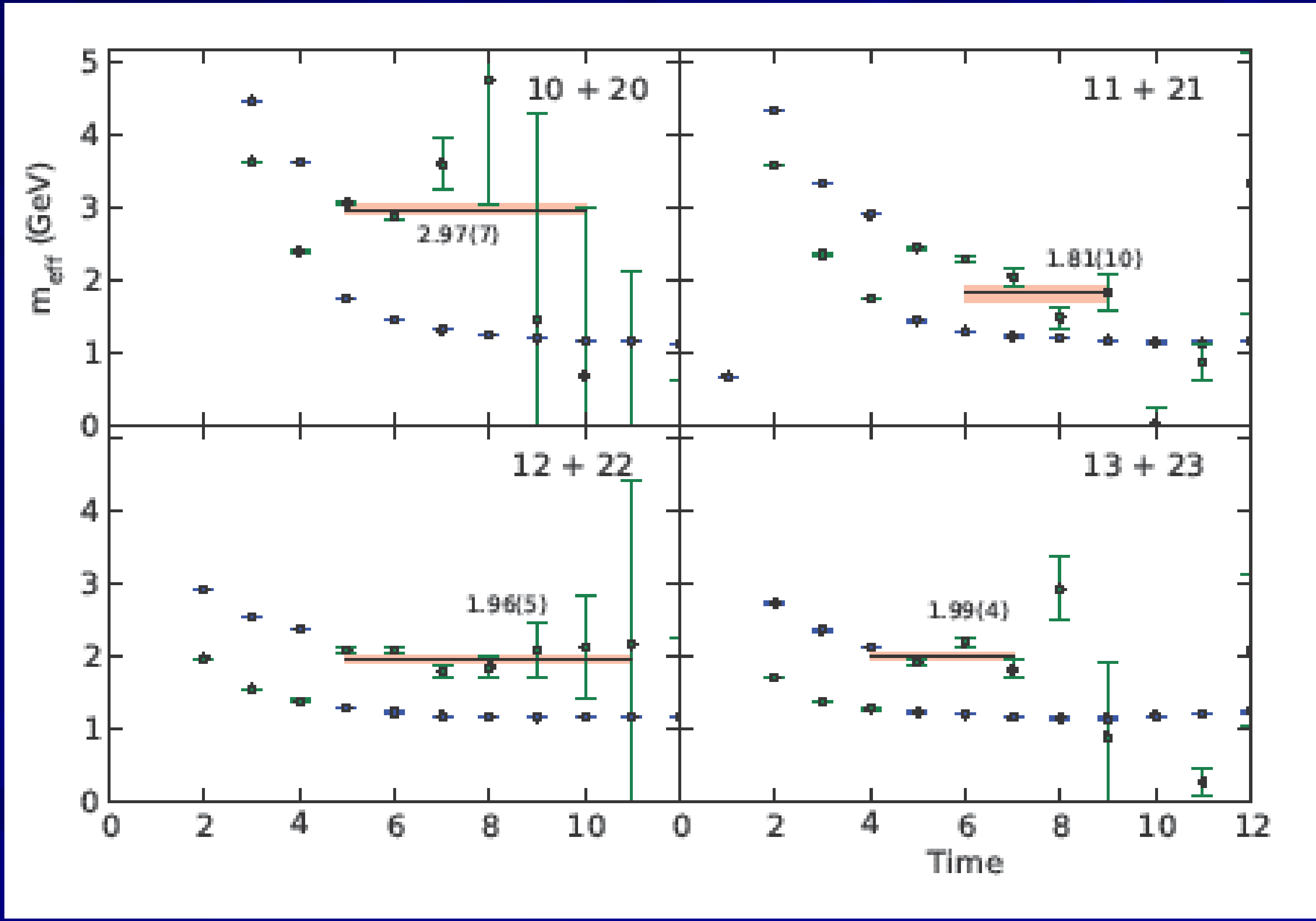
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$O_{RN} = 0.59$



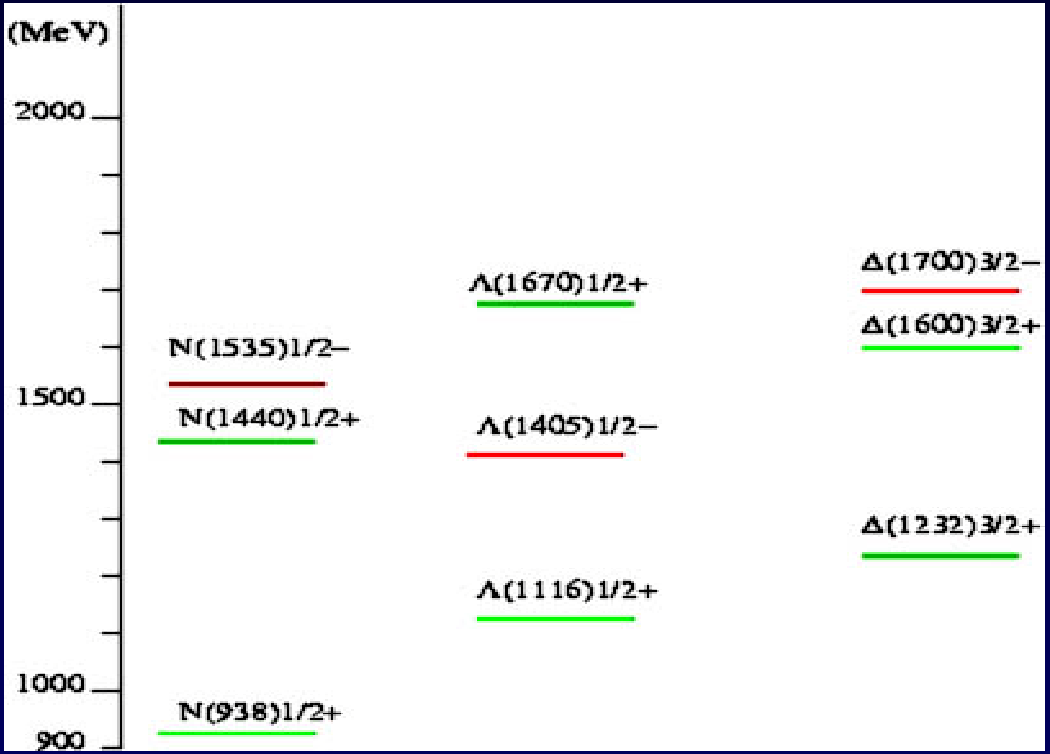
Variation with 2 operators

(10 – operator 1, no smearing, 23 – operator 2, 3 smearing)



Variation with wall and point sources?

Glozman and Riska Challenge Phys. Rep. 268, 263 (1996)



Hyperfine Interaction of Quarks in Baryons

- Color-spin

$$\lambda_1^C \bullet \lambda_2^C \, \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

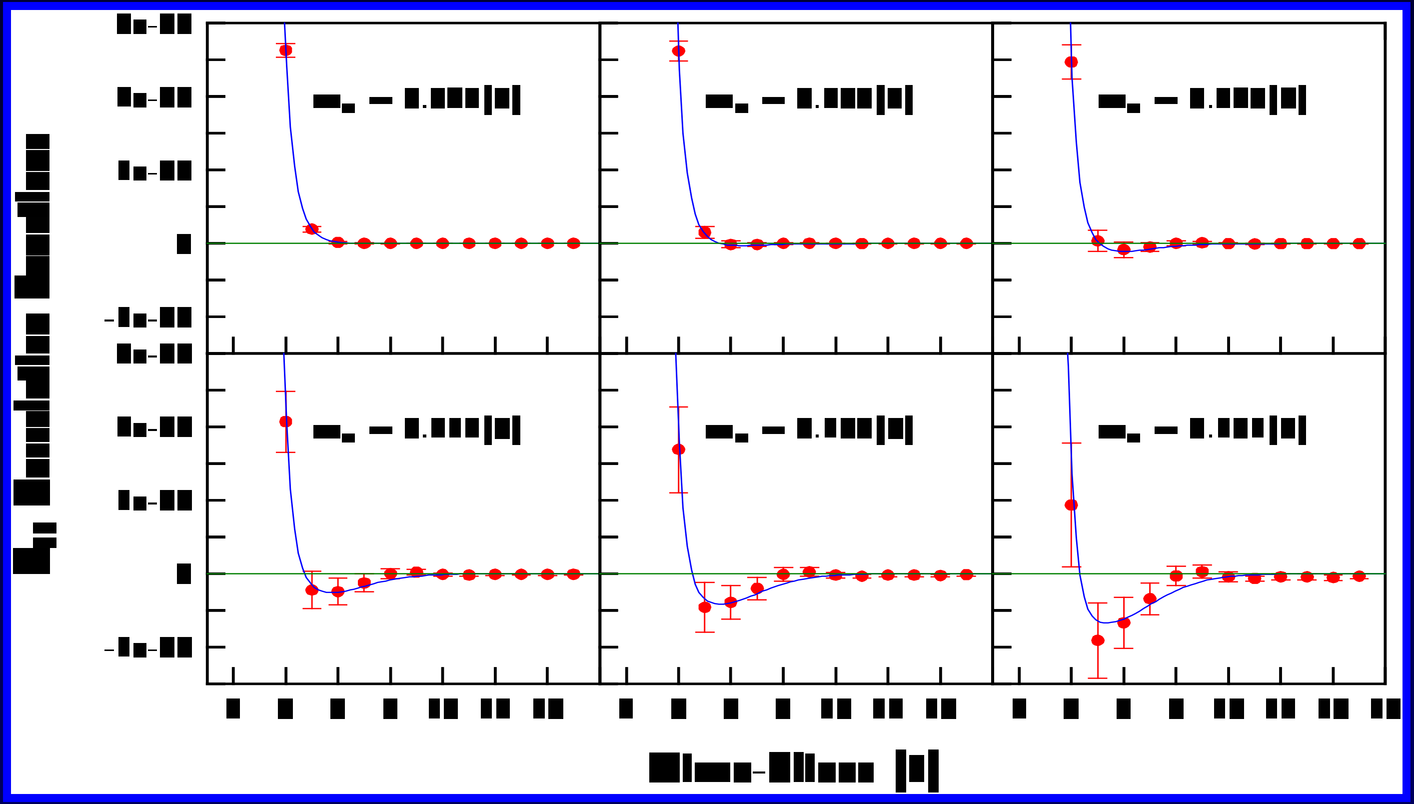
- One-gluon exchange

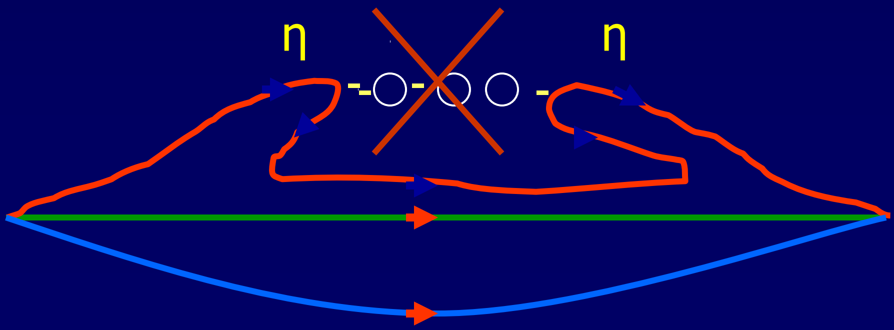
- Flavor-spin

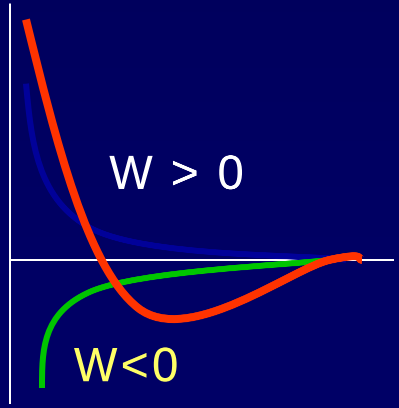
$$\lambda_1^F \bullet \lambda_2^F \, \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

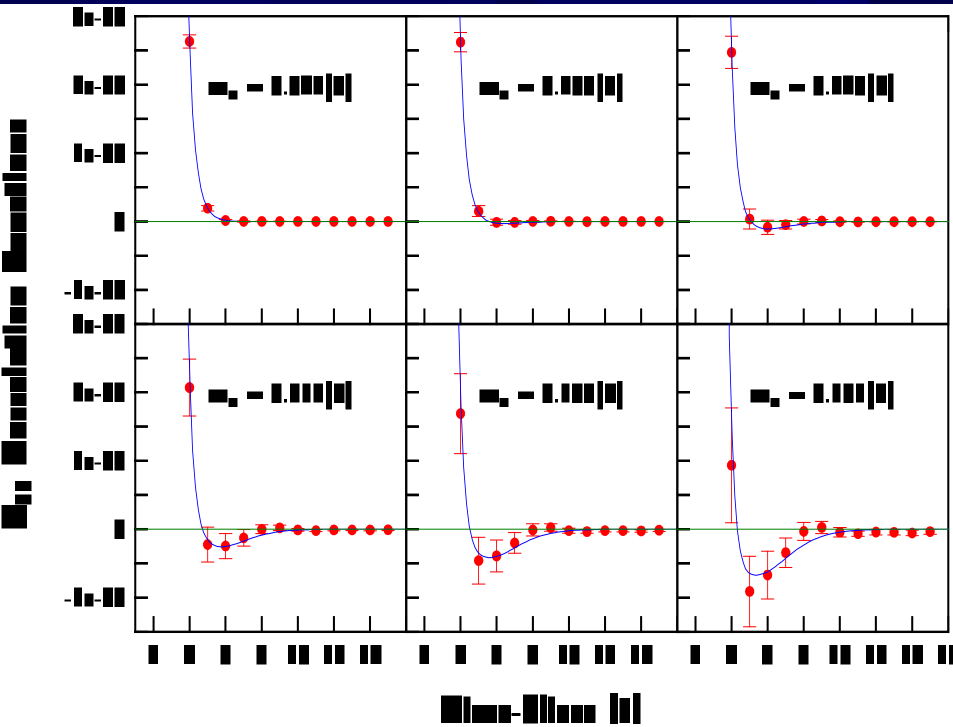
- Goldstone boson exchange

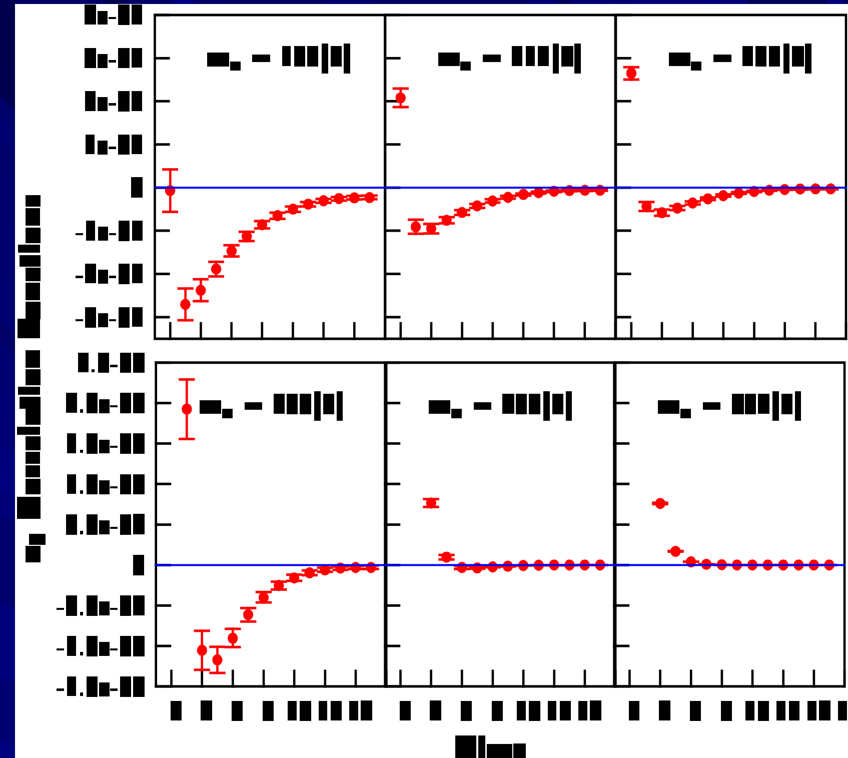
Evidence of η'N GHOST State in S_{11} (1535) Channel

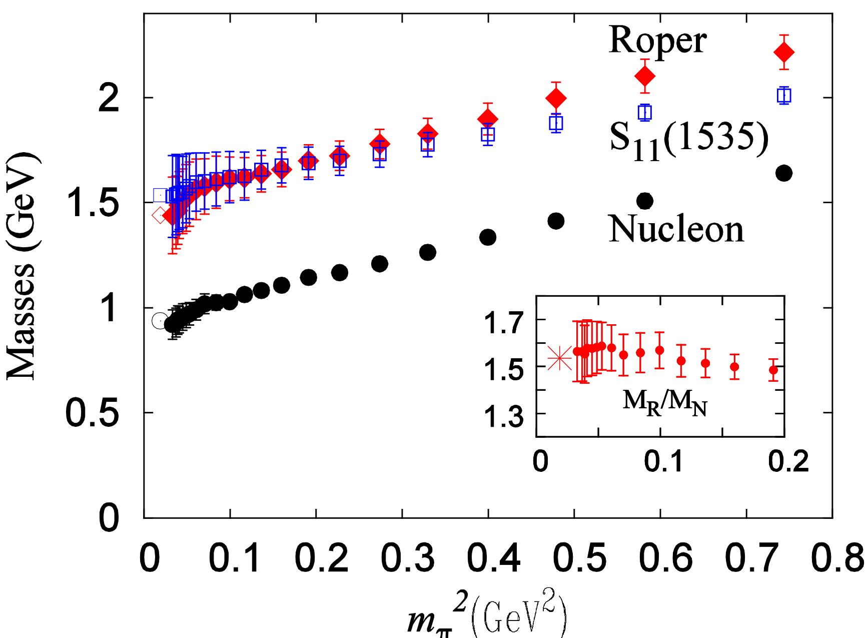




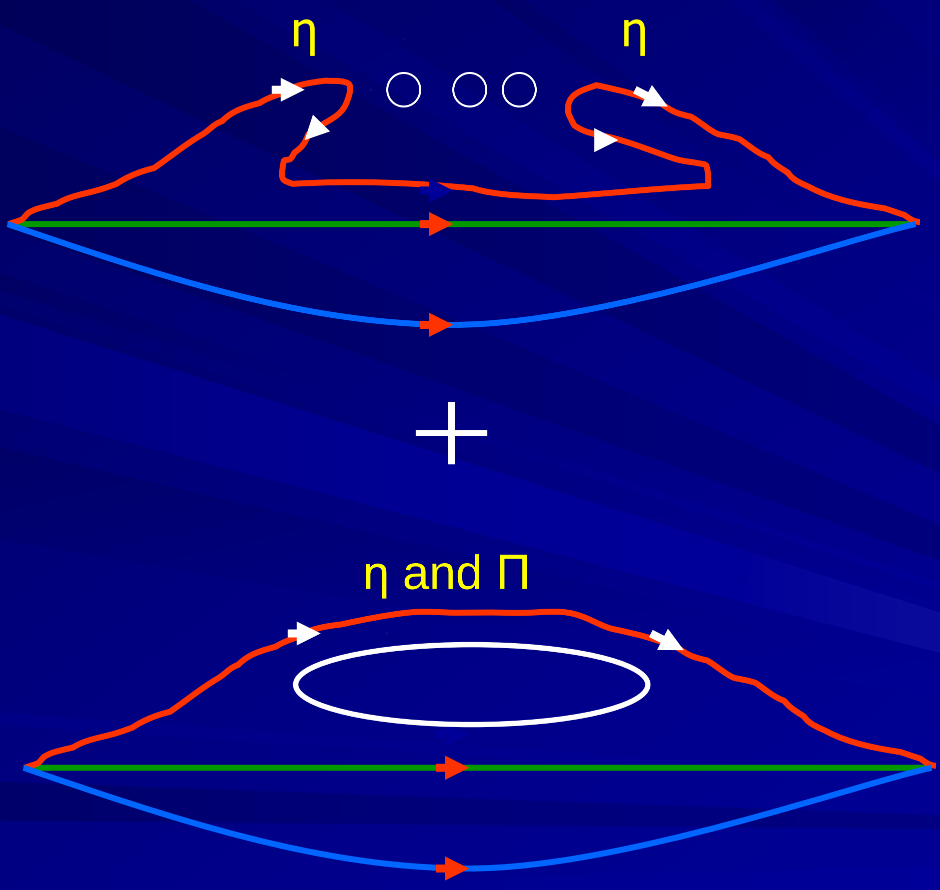




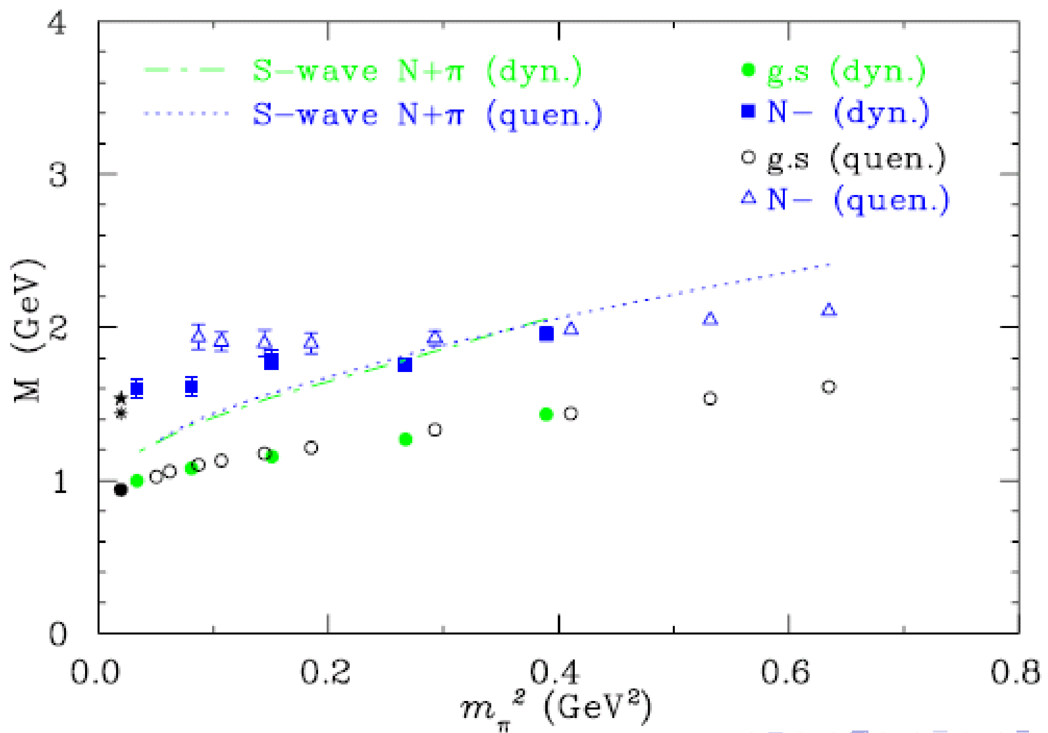



Roper
S11(1535)
Nucleon
Masses (GeV)
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
m_π^2 (GeV^2)
1.7
1.5
1.3
0 0.1 0.2
M_R/M_N

Dynamical Fermions

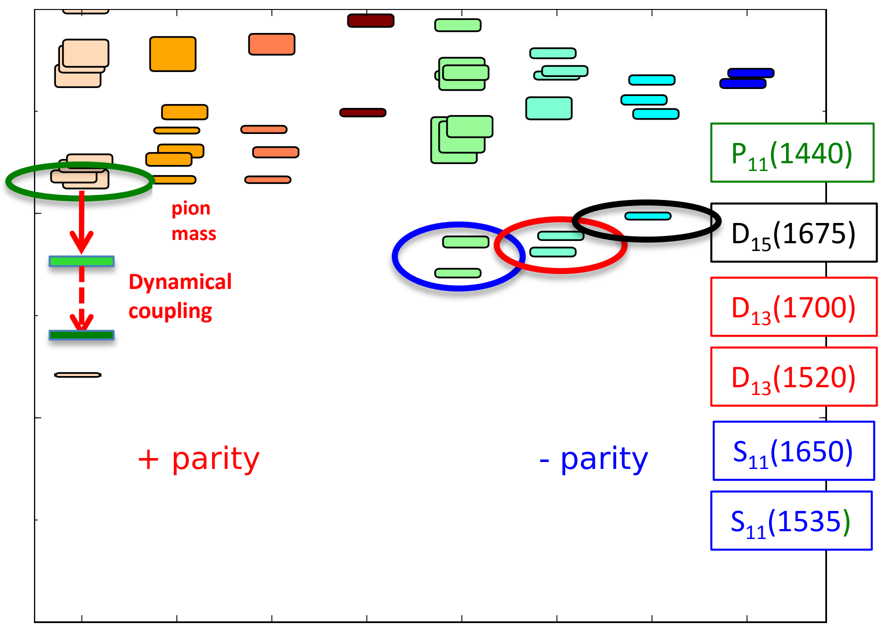


Quenched Vs Dynamical $N\frac{1}{2}^-$ (1535) (Sommer scale)



N* spectrum in LQCD & dynamical coupling

Lattice N* states (m_π=396MeV)

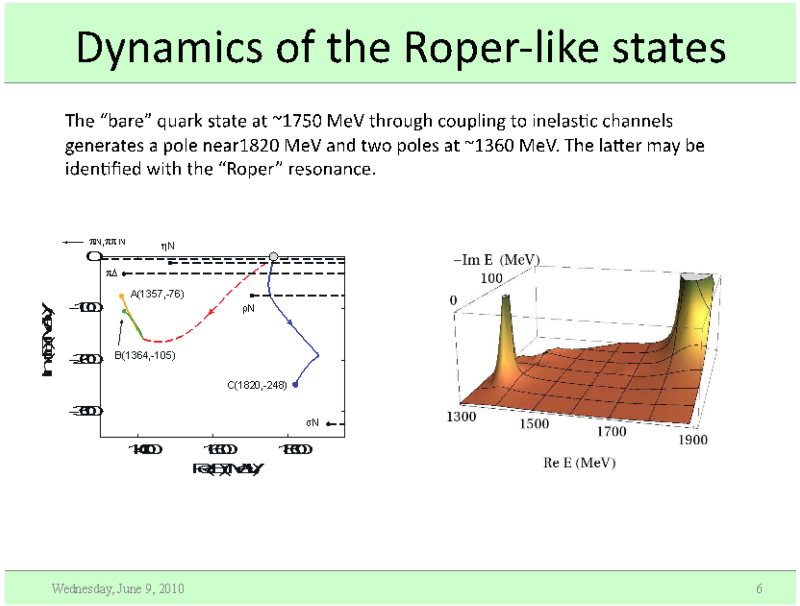


LQCD finds states as predicted in SU(6)xO(3)

R. Edwards, J. Dudek, D. Richards,
S. Wallace, PRD84, 074508 (2011)

Dynamics of P_{11}-states:

The bare state at ~1750 MeV through coupling to inelastic channels generates 2 poles below 1400 MeV. They are identified with the "Roper" resonance.



N. Suzuki et al. (JLab/EBAC),
Phys.Rev.Lett.104:042302,2010

Nature of Roper Resonance
--- current understanding

- Roper is the radial excitation of nucleon with large couplings to Nη and Nπ. The real part of the Nη and Nπ loops pushes down the pole of radial excitation and the imaginary part gives the width of Roper, much the same way the Nπ coupling changes the Δ mass and gives rise of its width.
- Issues with lattice calculations:
  - Variation vs Bayesian fitting: the size of the operators.
  - Chiral dynamics of the fermion action.

Is 1^{-+} Meson a Hybrid?

Y. Yang, Y. Chen, KFL, arXiv:1202.2205

Exotics:

- Glueballs
- Hybrids $(q\bar{q}g)$
- Tetraquark mesoniums $(q\bar{q}q\bar{q})$; pentaquark baryons $(q^4\bar{q})$

How to identify glueballs and hybrids in experiments and lattice calculations and distinguish them from ordinary $q\bar{q}$ mesons?

→ exotic quantum numbers, e.g.

$$J^{PC} = 0^{+-}, 1^{-+}, 2^{-+}$$

Meson Interpolation Fields

- dim 3 $\overline{\Psi}\Gamma\Psi$
- dim 4 $\overline{\Psi}\Gamma\overleftrightarrow{D}\Psi,\ GG$
- dim 5 $\overline{\Psi}\Gamma\Psi G,\ \overline{\Psi}\Gamma DD\Psi,\ GDG$
- dim 6 $\overline{\Psi}\Gamma\Psi\overline{\Psi}\Gamma\Psi,\ GGG,\ GDDG,\ \text{etc}$

Dim 3 operators $\overline{\Psi}\Gamma\Psi$ do not generate exotic mesons with $J^{PC} = 1^{-+}, 0^{--}, 0^{+-}, 2^{+-}$ but they can be produced with dim 5 $\overline{\Psi}\Gamma\Psi G$ ops.

→ they are hybrids.

Experiments and Lattice Results

- Expts on 1^{-+} :
  - $\pi_1(1400)$, M= 1376±17 MeV, Γ=300±40 MeV
  - $\pi_1(1600)$, M=1653±17 MeV, Γ=225±38 MeV (?)

- Lattice calculations (Quenched):
  - UKQCD (1997): 2.0(2) GeV
  - MILC (1997): 2.0(1) GeV, 2.1(1) GeV
  - Lacock and Schilling (1998): 1.9(2) GeV
  - MILC (2003): ~ 1.6 GeV
  - Adelaide (2004): ~ 2.4 GeV; ~1.6 GeV
  - HSC (2010)
  - J. Dudek (2011)

$$\vec{J} = \vec{L} + \vec{S}, \quad P = (-)^{L+1}, \quad C = (-)^{L+S}$$

are non-relativistic definitions!

- For example:

	Pauli Spinor	Dirac Spinor
$L=1, S=1, J=0$	$\overline{\psi}\vec{\sigma}\cdot\vec{\Delta}\psi$	$\overline{\Psi}\Psi$
$L=1, S=1, J=1$	$\overline{\psi}\vec{\sigma}\times\vec{\Delta}\psi$	$\overline{\Psi}\gamma_i\gamma_5\Psi$
$L=1, S=1, J=2$	$\overline{\psi}\vec{\sigma}\otimes\vec{\Delta}\psi$	$\overline{\Psi}\gamma\otimes\overleftrightarrow{D}\Psi$
$L=1, S=0, J=1$	$\overline{\psi}\vec{\Delta}\psi$	$\overline{\Psi}\sigma_{ij}\Psi$

- Lower component of Dirac spinor has a different parity from the upper one.

$$\Psi_{free} \propto \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{\mathrm{p}}}{E+m} \end{pmatrix} \chi\, \mathrm{e}^{ip.x}$$

1^{-+} Meson

- Exotic:
  - $\vec{J} = \vec{L} + \vec{S}, \quad P = (-)^{L+1}, \quad C = (-)^{L+S}$
  - P → L = even
  - J → S = 1
  - C = −
  - Cannot be $q\bar{q}$ meson

- Yet dim 4 $\bar{\Psi}\gamma_4 \overleftrightarrow{D}_i \Psi$ is 1^{-+} (B.A. Li, 1975)
  - $P: \quad \Psi(\vec{x}) \to \gamma_4 \Psi(-\vec{x})$
  - $C: \quad \Psi \to \mathscr{C} \bar{\Psi}$
  - Exotic $q\bar{q}$?
  - Note: there is no dim 3 op. for 2^{++}

It is produced with $\bar{\Psi}\gamma_i \overleftrightarrow{D}_j \Psi$.

Content and Interpolation Field

- One cannot always judge the content of a hadronic state by its interpolation field. For example
  - the lowest state from $\bar{\Psi}\Psi$ is $\eta\pi$ and $\pi\pi$, not a_0 and f_0.
  - the lowest P-wave state from χ_N is πN not S_{11}.
- The lowest pseudoscalars from

$G\tilde{G}$ is η and η' not glueball.

$$<0|G\tilde{G}|\eta>, <0|G\tilde{G}|\eta> \;\geq\; <0|G\tilde{G}|G>$$

– will need $<0|\bar{\Psi}\gamma_5\Psi|\eta>, <0|\bar{\Psi}\gamma_5\Psi|\eta> \;>>\; <0|\bar{\Psi}\gamma_5\Psi|G>$

to help decide which one is a glueball.

(Cheng, Li and Liu, PRD 79, 014024 (2009))

Criteria for a meson to be a hybrid:

Compare the matrix elements of both the dim 4 and dim 5 operators of 1^{-+} against other ordinary mesons, particularly the 2^{++}

- Dim 4 m.e. $<0|\bar{\Psi}\gamma_4 \overleftrightarrow{D}_i \Psi|1^{-+}> \ll <0|O_4|0^{-+},1^{--},0^{++},1^{+\pm},2^{++}>$
- Dim 5 m.e. $<0|\bar{\Psi}\varepsilon_{ijk}\gamma_j B_k \Psi|1^{-+}> \gg <0|O_5|0^{-+},1^{--},0^{++},1^{+\pm},2^{++}>$

Dim 3, 4 (D-type) and dim 5 (B-type) operators

Table 1: Interpolation operators $\bar{\psi}\Gamma\psi$ (dimension 3, Γ-type), $\bar{\psi}\Gamma \times \overleftrightarrow{D}\psi$ (dimension 4, D-type), and $\bar{\psi}\Gamma \times B\psi$ (dimension 5, B-type). $\Sigma_i \equiv \frac{1}{2}\varepsilon_{ijk}\sigma_{jk}$ and repeated indices are summed over.

	Γ	D	B
0^{-+}	γ_5	$\Sigma_i \overleftrightarrow{D}_i$	$\gamma_i B_i$
1^{--}	γ_i	$\overleftrightarrow{D}_i$	$\gamma 5 B_i$
0^{++}	$\mathbb{I}$	$\gamma_i \overleftrightarrow{D}_i$	$\Sigma_i B_i$
1^{++}	$\gamma_5\gamma_i$	$\varepsilon_{ijk}\gamma_j \overleftrightarrow{D}_k$	$\varepsilon_{ijk}\Sigma_j B_k$
1^{+-}	Σ_i	$\gamma_5 \overleftrightarrow{D}_i$	B_i
2^{++}		$\lvert\varepsilon_{ijk}\rvert\gamma_j \overleftrightarrow{D}_k$	$\lvert\varepsilon_{ijk}\rvert\Sigma_j B_k$
1^{-+}		$\gamma_4 \overleftrightarrow{D}_i$	$\varepsilon_{ijk}\gamma_j B_k$
		$\varepsilon_{ijk}\Sigma_j \overleftrightarrow{D}_k$	

Non-relativistic Reduction

- why no such operators in quark model

Table 2: Non-relativistic form for the three kinds of operators (Γ, D and B) as shown in Table 1. Here we list the operators $\mathcal{O}$ in the interpolation field $\chi^\dagger \mathcal{O} \phi$. Repeated indices are summed over.

	Γ	D	B
0^{-+}	$\mathbb{I}$	$\frac{1}{2m_c} \overleftrightarrow{D}_i \overleftrightarrow{D}_i$	$i\sigma_i B_i$
1^{--}	σ_i	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j \overleftrightarrow{D}_i$	B_i
0^{++}	$\frac{1}{2m_c} \overleftrightarrow{D}_i \sigma_i$	$\sigma_i \overleftrightarrow{D}_i$	$\frac{1}{2m_c} \overleftrightarrow{D}_i B_i$
1^{++}	$\frac{1}{2m_c} \varepsilon_{ijk} \overleftrightarrow{D}_j \sigma_k$	$\varepsilon_{ijk} \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} (\varepsilon_{ijk} \overleftrightarrow{D}_j B_k + i\partial_i(\sigma_j B_j))$
1^{+-}	$\frac{1}{2m_c} \overleftrightarrow{D}_i$	$\overleftrightarrow{D}_i$	$\frac{1}{2m_c} \sigma_j \overleftrightarrow{D}_j B_i$
2^{++}		$\lvert\varepsilon_{ijk}\rvert \sigma_j \overleftrightarrow{D}_k$	$\frac{1}{2m_c} \lvert\varepsilon_{ijk}\rvert (\overleftrightarrow{D}_j B_k + i\varepsilon_{jmn} \sigma_m \partial_n (B_k))$
1^{-+}		$\frac{1}{2m_c} (\sigma \cdot \overleftarrow{D} \overleftrightarrow{D}_i + \overleftrightarrow{D}_i \sigma \cdot \overrightarrow{D})$ $\frac{1}{2m_c} (\overleftarrow{D}_i \sigma_j \overleftrightarrow{D}_j + \sigma_j \overleftrightarrow{D}_j \overrightarrow{D}_i)$	$\varepsilon_{ijk} \sigma_j B_k$

Note: $\vec{D} + \overleftarrow{D}$ is the $q\bar{q}$ center of mass momentum which is not a dynamical d.o.f. in the constituent quark model.

Charmoniums

Anisotropic 12^3 x 96 lattice with Wilson action, $\beta = 2.8$, $\zeta = 5$, $a_s = 0.138$ fm.

Table 3: Masses of charmonium states from Γ- and B-type sources and point sinks.

	$\Gamma_w \to \Gamma_p$	$B_w \to \Gamma_p$	$B_w \to B_p$	$B_w \to D_p$	PDG
0^{-+}	3000 ± 3	3000 ± 3	2999 ± 3	3000 ± 3	2980.3 ± 1.2
1^{--}	3096 ± 3	3095 ± 3	3093 ± 3	3094 ± 3	3096.916 ± 0.011
0^{++}	3458 ± 30	3485 ± 18	3485 ± 21	3476 ± 18	3414.75 ± 0.31
1^{++}	3497 ± 21	3491 ± 10	3492 ± 28	3492 ± 28	3510.66 ± 0.07
1^{+-}	3489 ± 30	3475 ± 21	3486 ± 12	3494 ± 6	3525.42 ± 0.29
2^{++}	–	–	3529 ± 40	3501 ± 13	3556.20 ± 0.09
1^{-+}	–	–	4205 ± 84	4234 ± 42	–

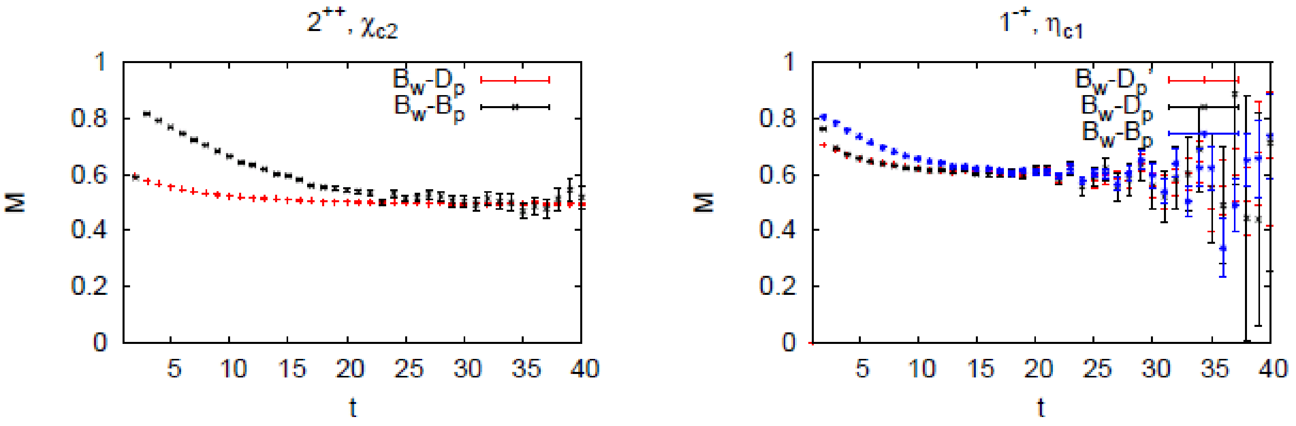


Figure 4: The same as Fig. 2 for $\chi_{c2}(2^{++})$ and $\eta_{c1}(1^{-+})$

Table 2: Non-relativistic form for the three kinds of operators (Γ, D and B) as shown in Table 1. Here we list the operators $\mathcal{O}$ in the interpolation field $\chi^\dagger \mathcal{O} \phi$. Repeated indices are summed over.

	Γ	D	B
0^{-+}	$\mathbb{I}$	$\frac{1}{2m_c}\overleftrightarrow{D}_i\overleftrightarrow{D}_i$	$i\sigma_i B_i$
1^{--}	σ_i	$\frac{1}{2m_c}\sigma_j\overleftrightarrow{D}_j\overleftrightarrow{D}_i$	B_i
0^{++}	$\frac{1}{2m_c}\overleftrightarrow{D}_i\sigma_i$	$\sigma_i\overleftrightarrow{D}_i$	$\frac{1}{2m_c}\overleftrightarrow{D}_i B_i$
1^{++}	$\frac{1}{2m_c}\varepsilon_{ijk}\overleftrightarrow{D}_j\sigma_k$	$\varepsilon_{ijk}\sigma_j\overleftrightarrow{D}_k$	$\frac{1}{2m_c}(\varepsilon_{ijk}\overleftrightarrow{D}_j B_k + i\partial_i(\sigma_j B_j))$
1^{+-}	$\frac{1}{2m_c}\overleftrightarrow{D}_i$	$\overleftrightarrow{D}_i$	$\frac{1}{2m_c}\sigma_j\overleftrightarrow{D}_j B_i$
2^{++}		$\lvert\varepsilon_{ijk}\rvert\sigma_j\overleftrightarrow{D}_k$	$\frac{1}{2m_c}\lvert\varepsilon_{ijk}\rvert(\overleftrightarrow{D}_j B_k + i\varepsilon_{jmn}\sigma_m\partial_n(B_k))$
1^{-+}		$\frac{1}{2m_c}(\sigma\cdot\overleftarrow{D}\overleftrightarrow{D}_i + \overleftrightarrow{D}_i\sigma\cdot\overrightarrow{D})$	$\varepsilon_{ijk}\sigma_j B_k$
		$\frac{1}{2m_c}(\overleftarrow{D}_i\sigma_j\overleftrightarrow{D}_j + \sigma_j\overleftrightarrow{D}_j\overrightarrow{D}_i)$	

$$\frac{1}{ma} = \frac{v}{m_c a_t \zeta} \sim 0.7048;$$

$$\varepsilon = \frac{1}{2ma} \sim 0.352,$$

$$\int d^3x \bar{\psi}\gamma_4 \overleftrightarrow{D}_i \psi \xrightarrow[N.R.]{} \int d^3x \frac{-\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi;$$

$$\int d^3x \varepsilon_{ijk} \bar{\psi}\Sigma_j \overleftrightarrow{D}_k \psi \xrightarrow[N.R.]{} \int d^3x \frac{\varepsilon_{ijk}}{m} \chi^\dagger \sigma_j B_k \phi$$

$$v_{c\bar{c}} = \frac{\vec{D} + \overleftarrow{D}}{2m} \sim v_c \sim 0.3c;$$

$$v_{rev} = \frac{\vec{D} - \overleftarrow{D}}{2m} \sim 0.3c$$

Table 4: Matrix elements $< 0|\mathcal{O}_p|J^{PC} >$ for charmoniums.

	Γ_p	D_p	B_p
0^{-+}	0.0697 ± 0.0014	0.0503 ± 0.0007	0.0251 ± 0.0006
1^{--}	0.0502 ± 0.0005	0.0149 ± 0.0001	0.0075 ± 0.0002
0^{++}	0.035 ± 0.005	0.075 ± 0.015	0.009 ± 0.003
1^{++}	0.020 ± 0.003	0.062 ± 0.005	0.0023 ± 0.0002
1^{+-}	0.014 ± 0.002	0.045 ± 0.005	0.0019 ± 0.0002
2^{++}		0.044 ± 0.003	0.00080 ± 0.00008
1^{-+}		0.0059 ± 0.0005	0.0082 ± 0.0006
		0.0054 ± 0.0004	

$$\times \frac{1}{v_{rel}} \sim 0.027(3)$$

$$\times \frac{1}{ma} = 0.0058(4)$$

$$\times \frac{1}{v_{c\bar{c}}} = \begin{cases} 0.020(2) \\ 0.018(1) \end{cases}$$

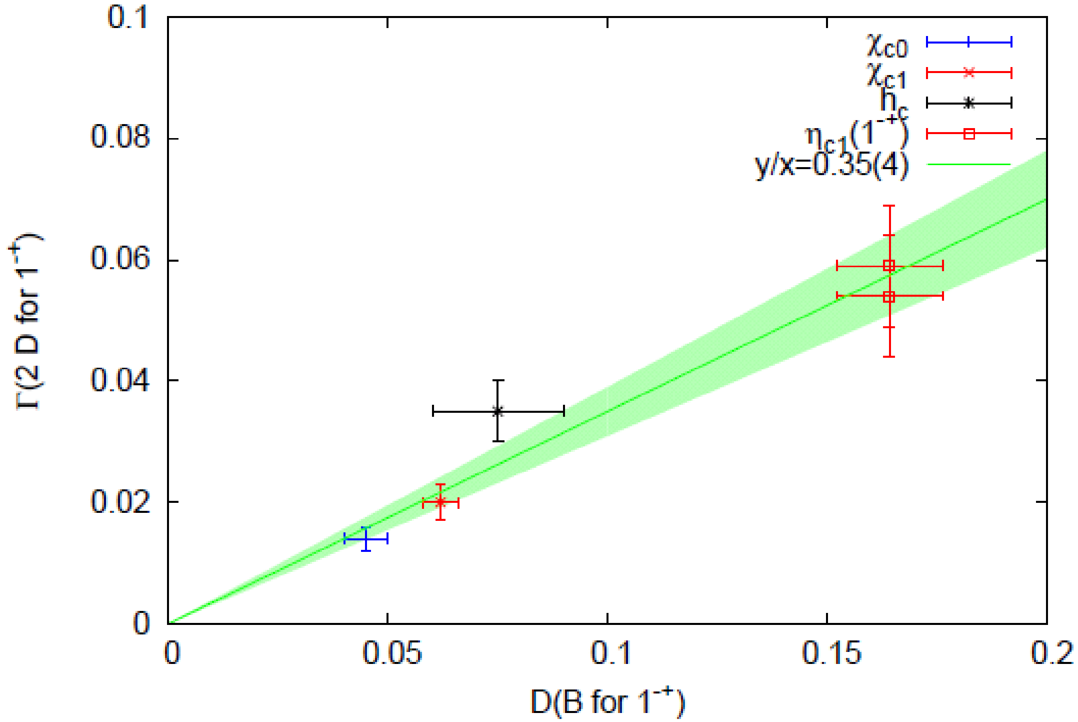


Figure 5: Global fit for the ratios of Γ for χ_{c0}, χ_{c1} and h_c (2 D for 1^{-+}) m.e. to the corresponding m.e.of D (B for 1^{-+}).

$$0^{++}, 1^{++}, 1^{+-}: \; \Gamma \text{ m.e. } \approx \frac{1}{2m} \text{ D m.e.}$$

$$1^{-+}: \; \text{D m.e. } \approx \frac{1}{m} \text{ B m.e.}$$

$$\varepsilon = \frac{1}{2m} \sim 0.352$$

- Leading NR reduction is reasonably good.
- Mixing of operators with different dimensions seems to be small.

Strangeness mesons

Table 6: The matrix elements $< 0|\mathcal{O}_p|J^{PC} >$ for strange quarkoniums.

	Γ_p	D	B
0^{-+}	0.0247 ± 0.0002	0.021 ± 0.002	0.005 ± 0.0001
1^{--}	0.0141 ± 0.0002	0.0113 ± 0.0005	0.0025 ± 0.0001
0^{++}	0.043 ± 0.006	0.033 ± 0.005	0.017 ± 0.004
1^{++}	0.029 ± 0.004	0.034 ± 0.004	0.0018 ± 0.0002
1^{+-}	0.019 ± 0.006	0.029 ± 0.005	0.0019 ± 0.0004
2^{++}		0.010 ± 0.007	0.0003 ± 0.0001
1^{-+}		0.007 ± 0.001	0.004 ± 0.001
		0.006 ± 0.002	

- D and B m.e. of 1^{-+} are comparable in size to those of other ordinary mesons.
- No evidence for 1^{-+} in the charmonium and strangeonium regions to be hybrids.

Exotic Quantum Numbers

- NR reduction shows that 1^{-+} involves a center of mass motion of the $q\bar{q}$ pair.

- MIT bag model (Jaffe and Johnson, 1976; DeGrand and Jaffe, 1976)

$$\Psi(2^{+\pm}) = \frac{1}{\sqrt{2}}(S_{1/2}\bar{P}_{3/2} \mp P_{3/2}\bar{S}_{1/2}) \rightarrow \text{cm motion for } 2^{+-}$$

$\Rightarrow$ Spectrum is doubled. Simiarly (hamonic oscillator wf),

$$\Psi(1^{-\pm}) = \frac{1}{\sqrt{2}}(1S_{1/2}2\bar{S}_{1/2} \mp 2S_{1/2}1\bar{S}_{1/2})$$

- The `exotic' q.n. can be accommodated by

$$C = (-)^{l+s}$$
$$\vec{J} = \vec{L} + \vec{l} + \vec{s}$$
$$P = L + l + 1$$

$$1^{-+}: \quad L = l = 1, \; s = 1$$

Conclusion

- By examining m.e. of dim 4 and dim 5 operators of 1^{-+} against those of ordinary mesons, we find no evidence for it to be a hybrid in the $c\bar{c}$ and $s\bar{s}$ regions.
- NR reduction shows that it involves a center of mass AM of the $q\bar{q}$ pair.
- These `exotic' q.n. are accessible in chiral quark models, bag models, flux-tube models, and QCD.
- To accommodate these q.n., the parity and AM rules need to be modified to

$$C = (-)^{l+s},\ \vec{J} = \vec{L} + \vec{l} + \vec{s},\ P = L + l + 1$$