INT Seattle, 8.8.2012

Hadron excitations and decays

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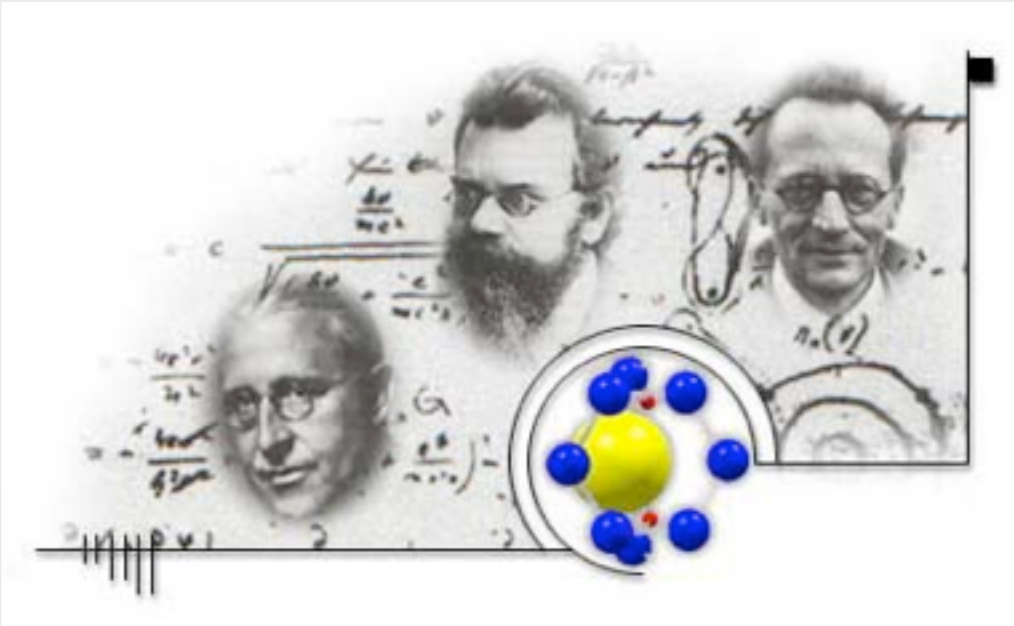
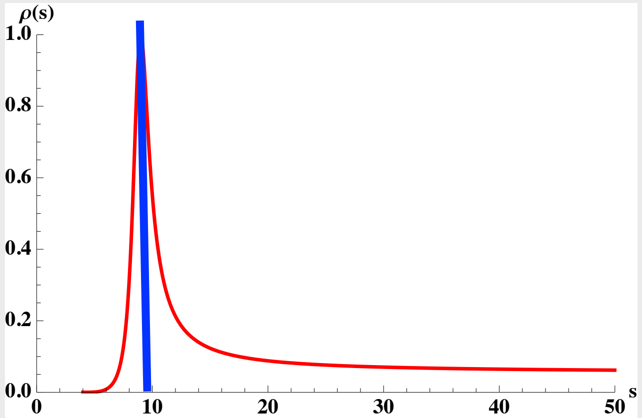
Thanks to my collaborators in related projects:

T. Burch, G. Engel, C. Gattringer, L. Ya Glozman, C. Hagen, L. Leskovec, M. Limmer, T. Maurer, D. Mohler, S. Prelovsek, A. Schäfer, M. Vidmar



Overview

1. Motivation and lattice tools
2. Case 1: Hadron excitations
3. Case 2: Meson decay



Motivation

- Consider only u, d quarks: Only p, n and π are stable (under strong interactions).
- Even lowest 'states' in other quantum channels decay (ρ, N*,...) hadronically: scattering states
- Most hadrons in the PDG tables are resonances
- For many 'particles' the classification is uncertain (multiplet, 'molecular' bound state, glueball)

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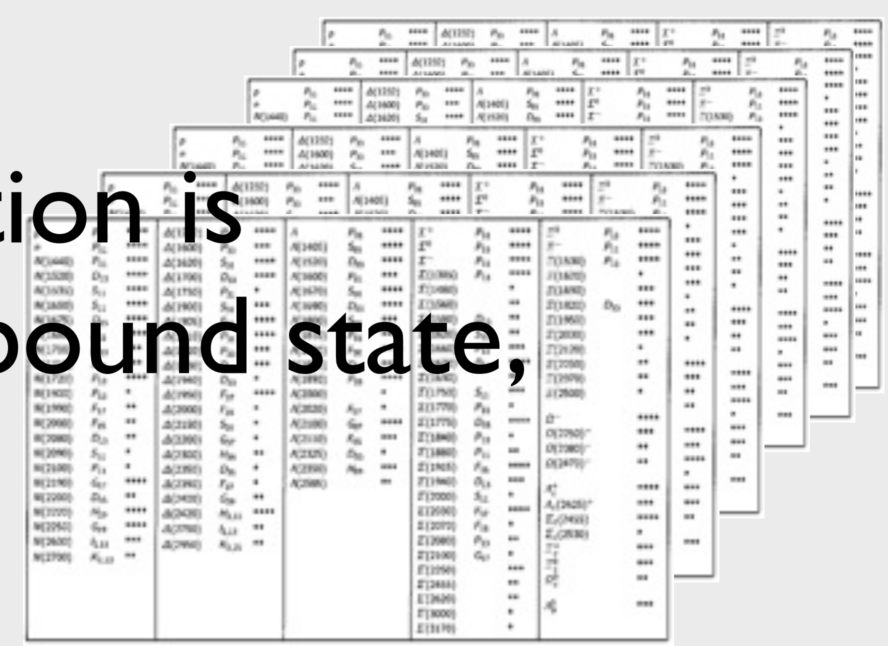
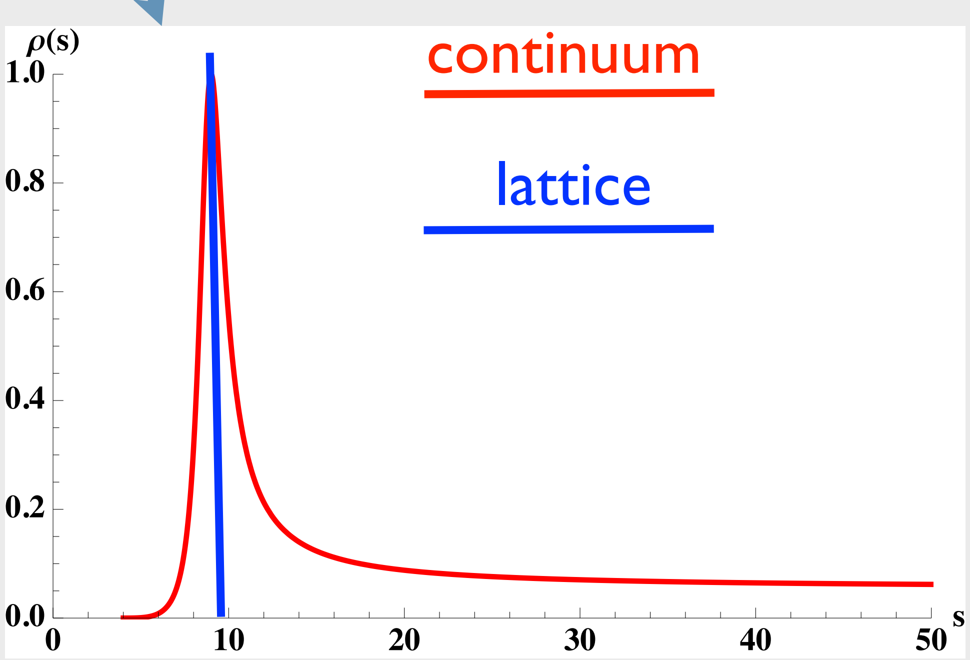


Hadron propagators and spectral function

$$\langle X(t) X^\dagger(0) \rangle \equiv C(t) = \int_{\omega_0}^{\infty} d\omega \; \rho(\omega) \, e^{-\omega t}$$

Finite volume:

Discrete energy levels!



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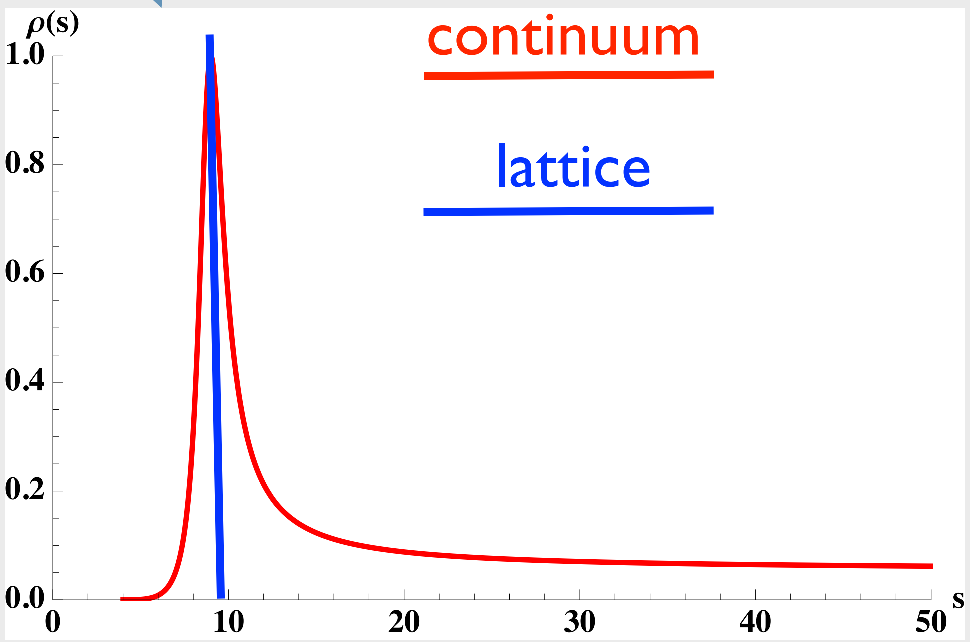

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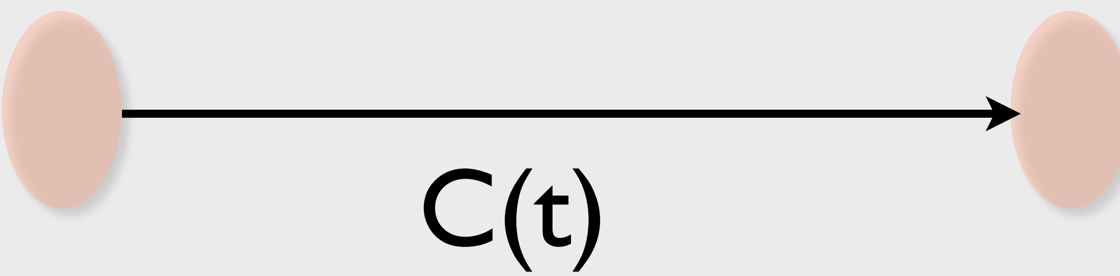

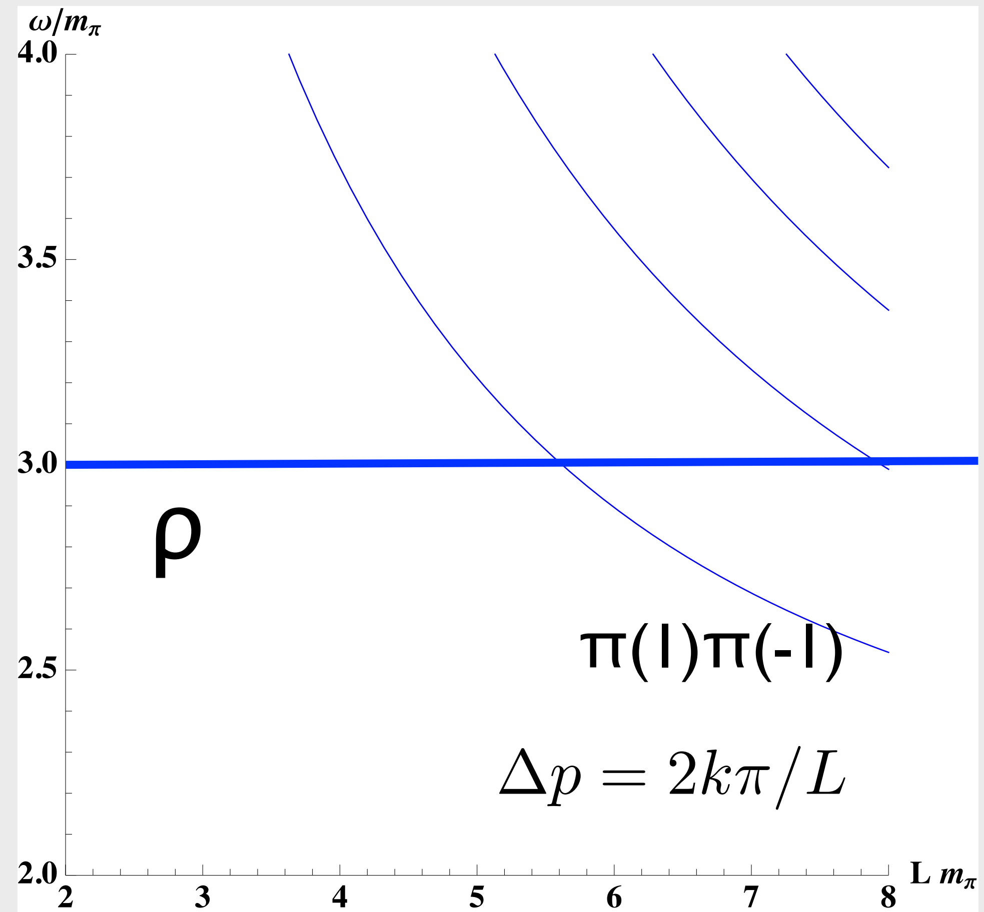

Example: $m_\rho/m_\pi = 3$

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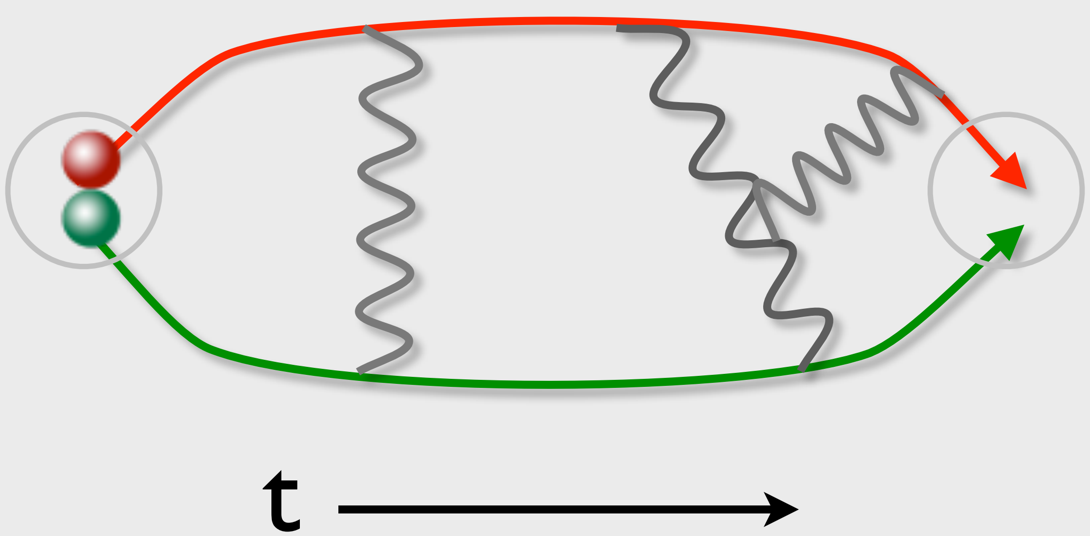

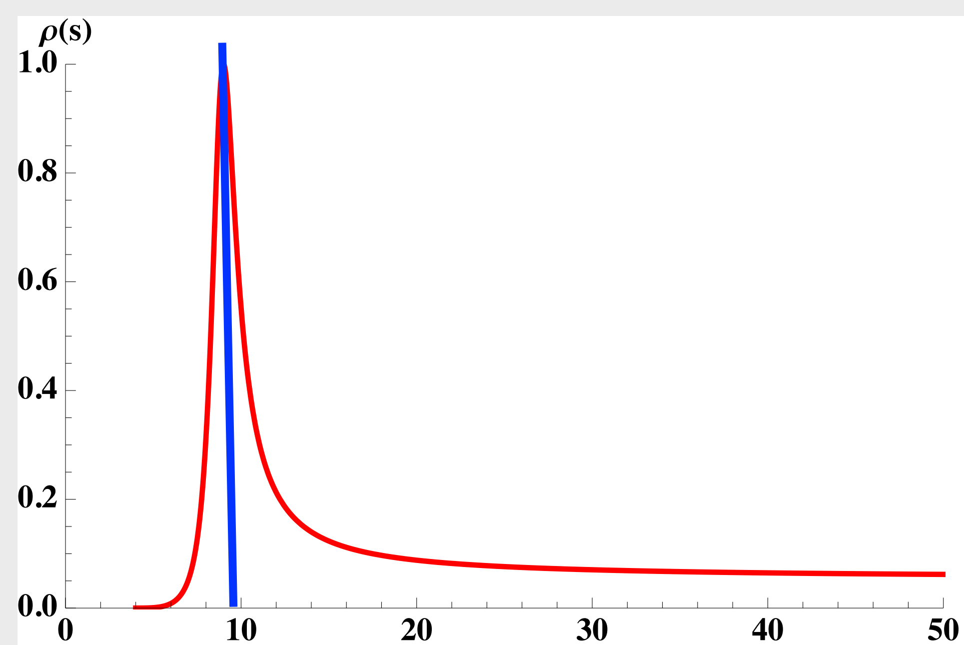

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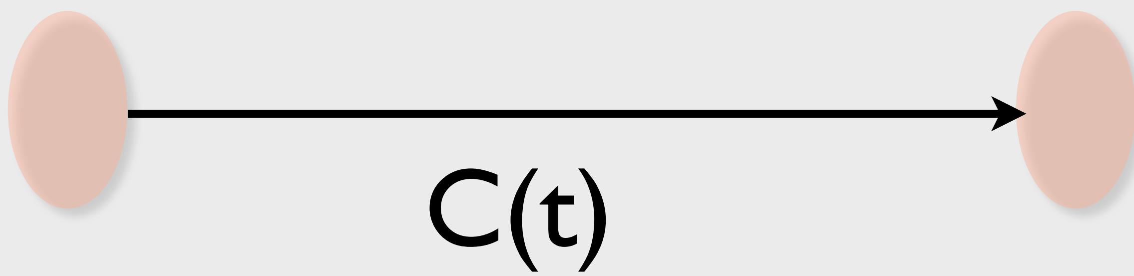

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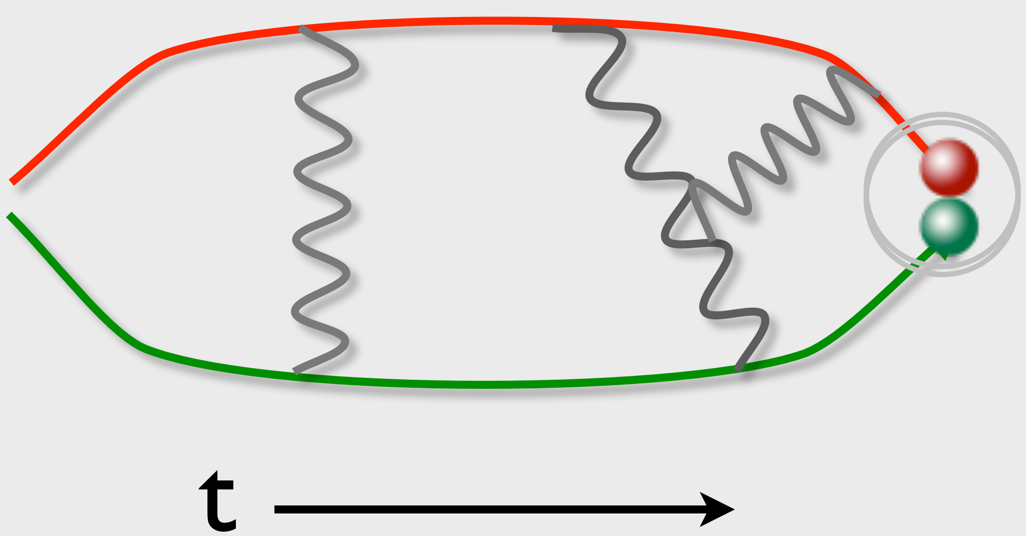

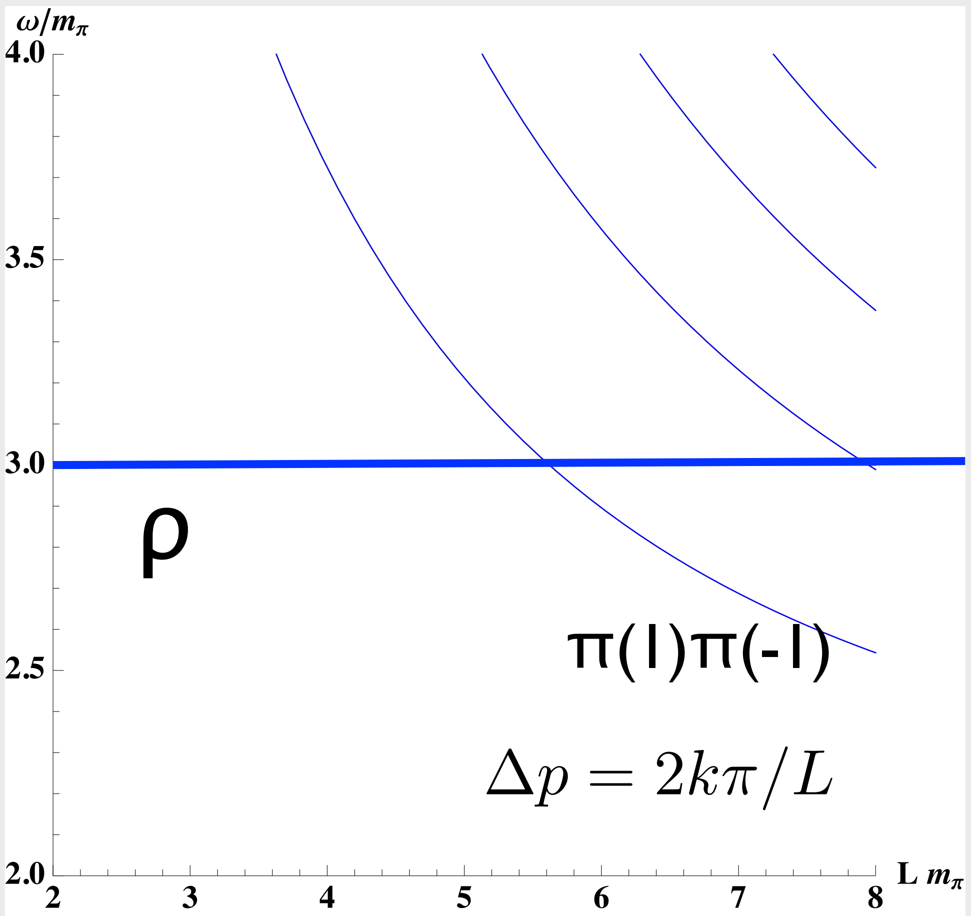

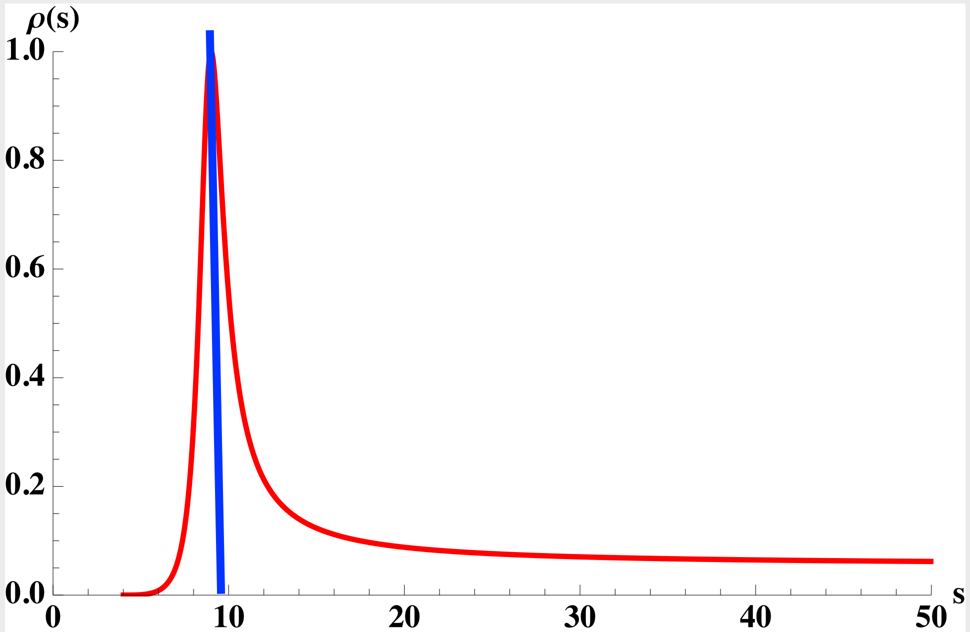

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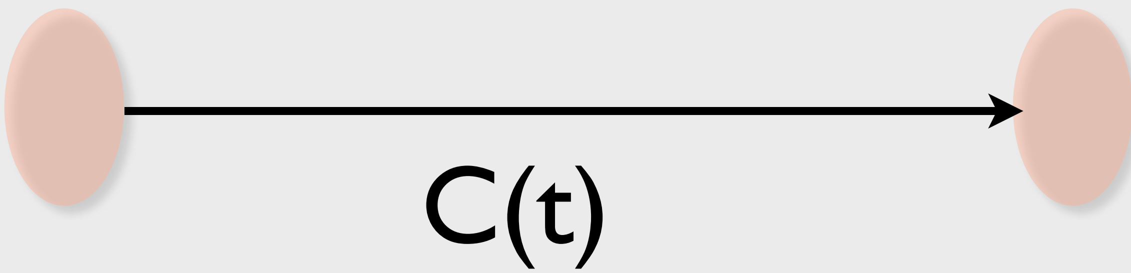

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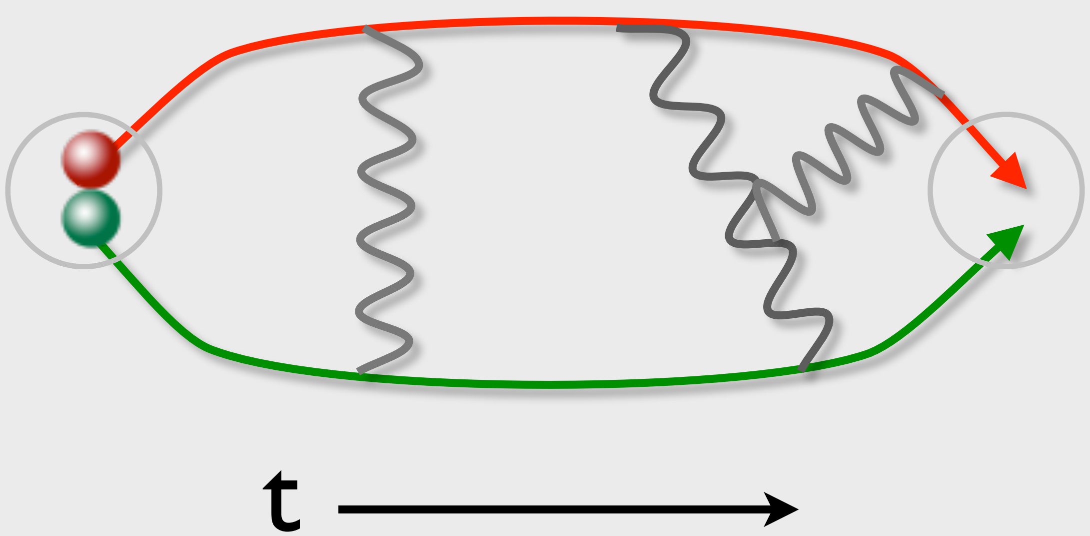
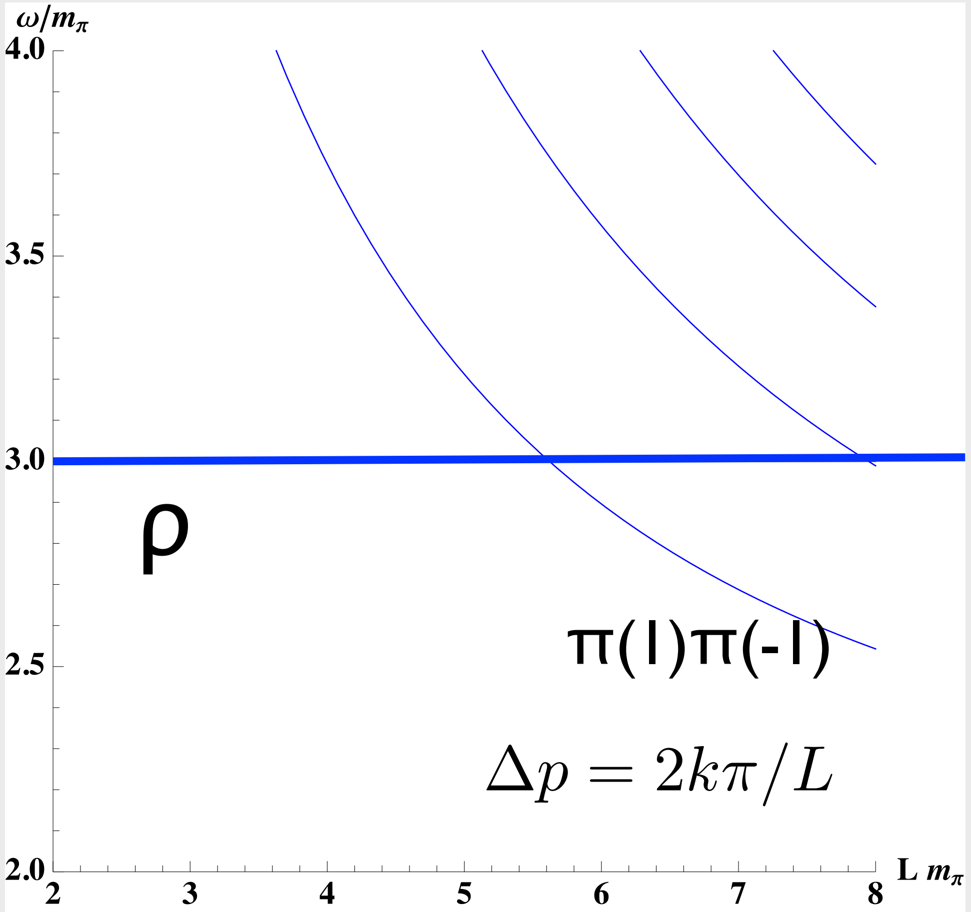
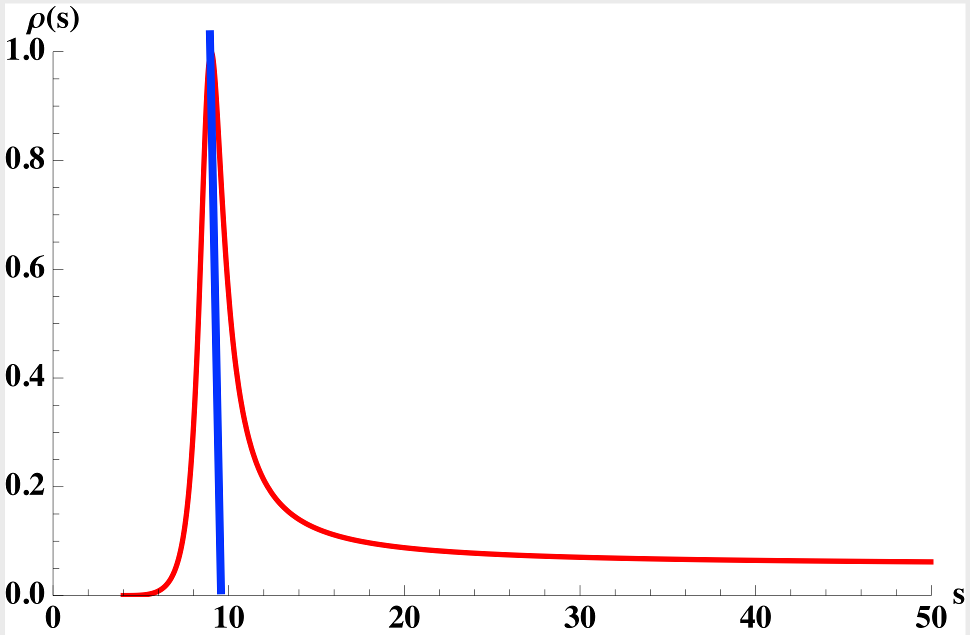
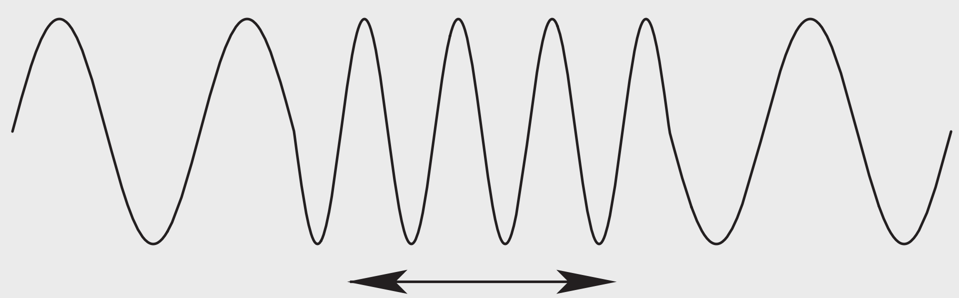






Hadron propagators and spectral function

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237



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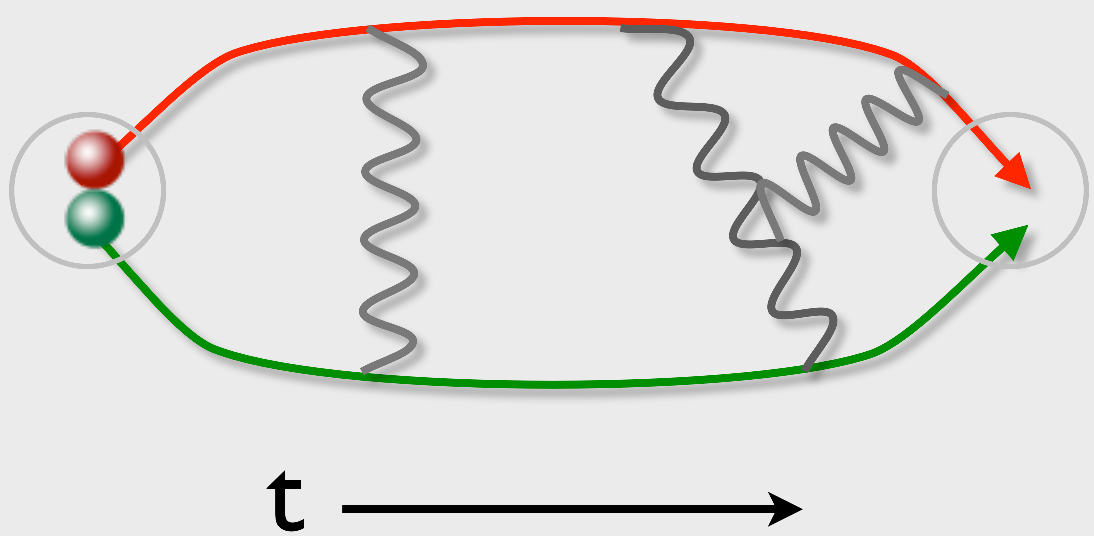

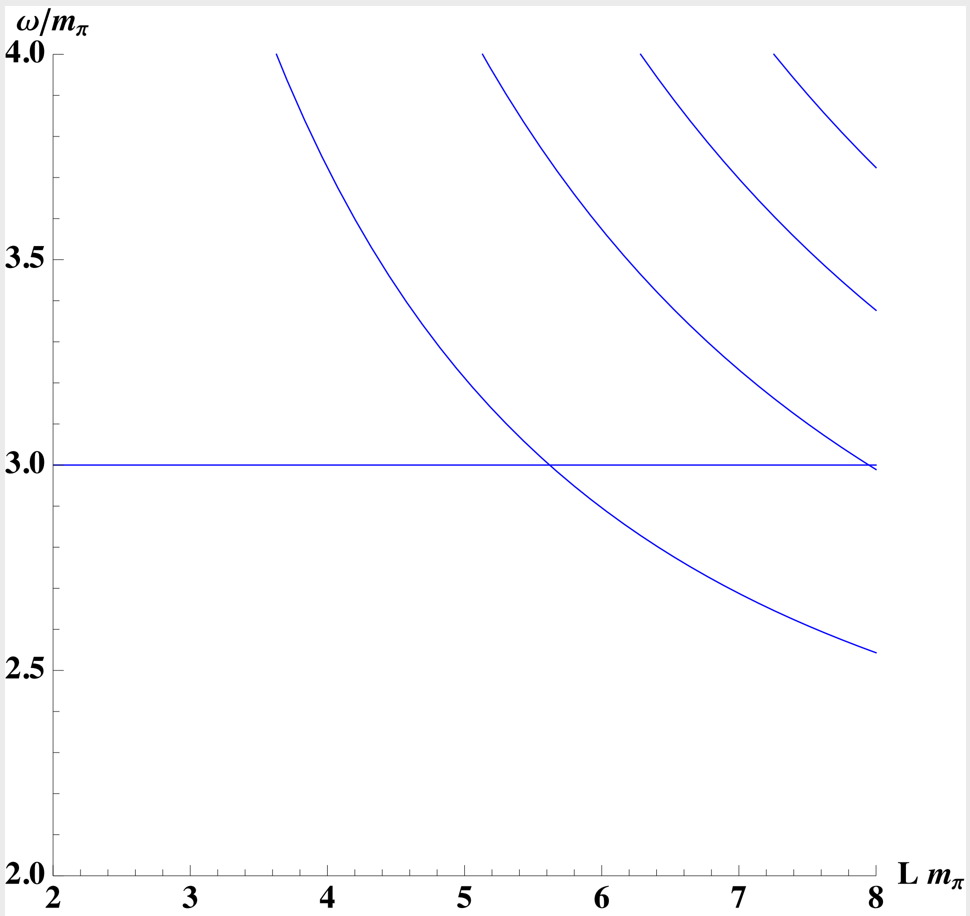

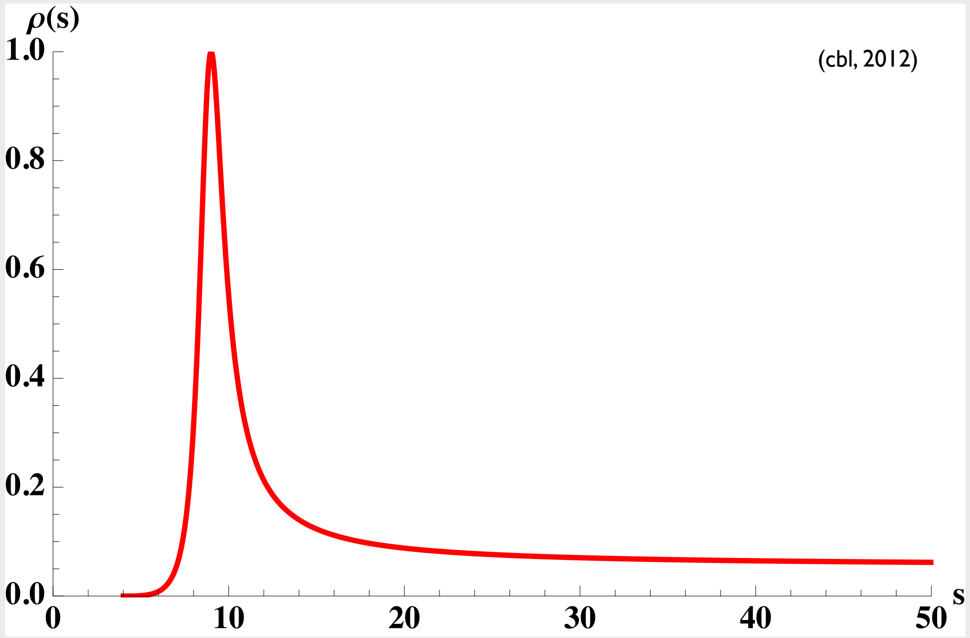

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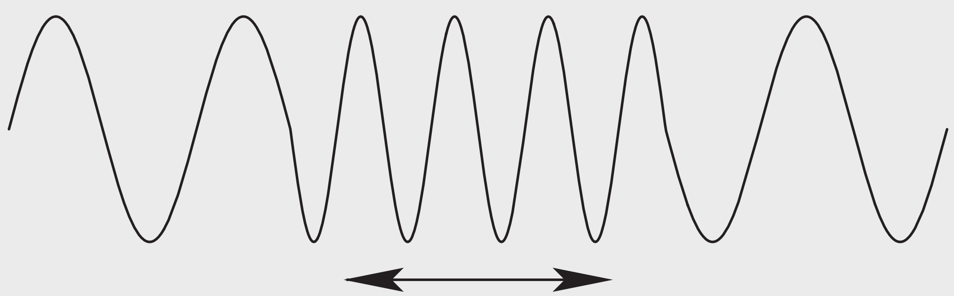

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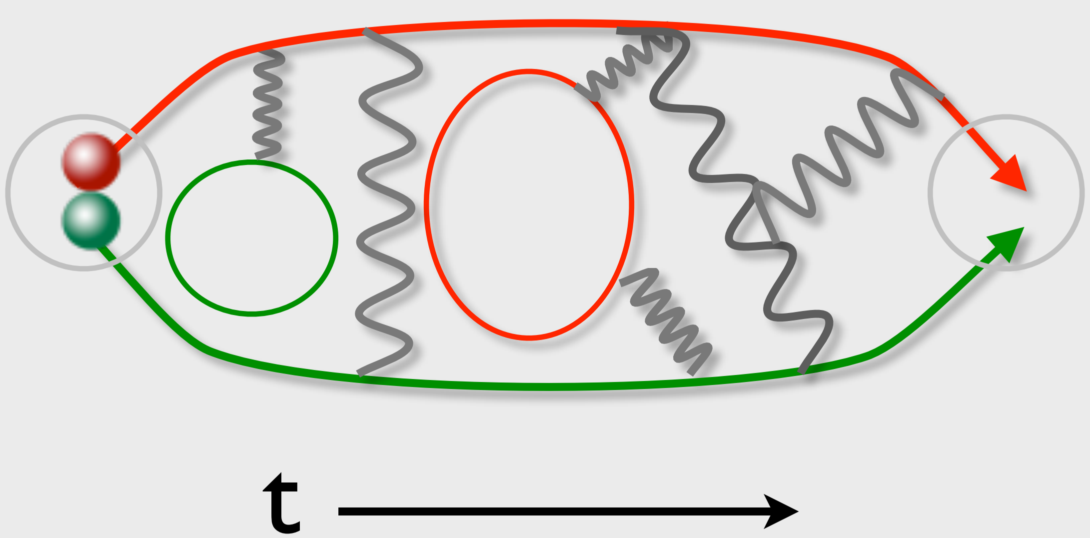

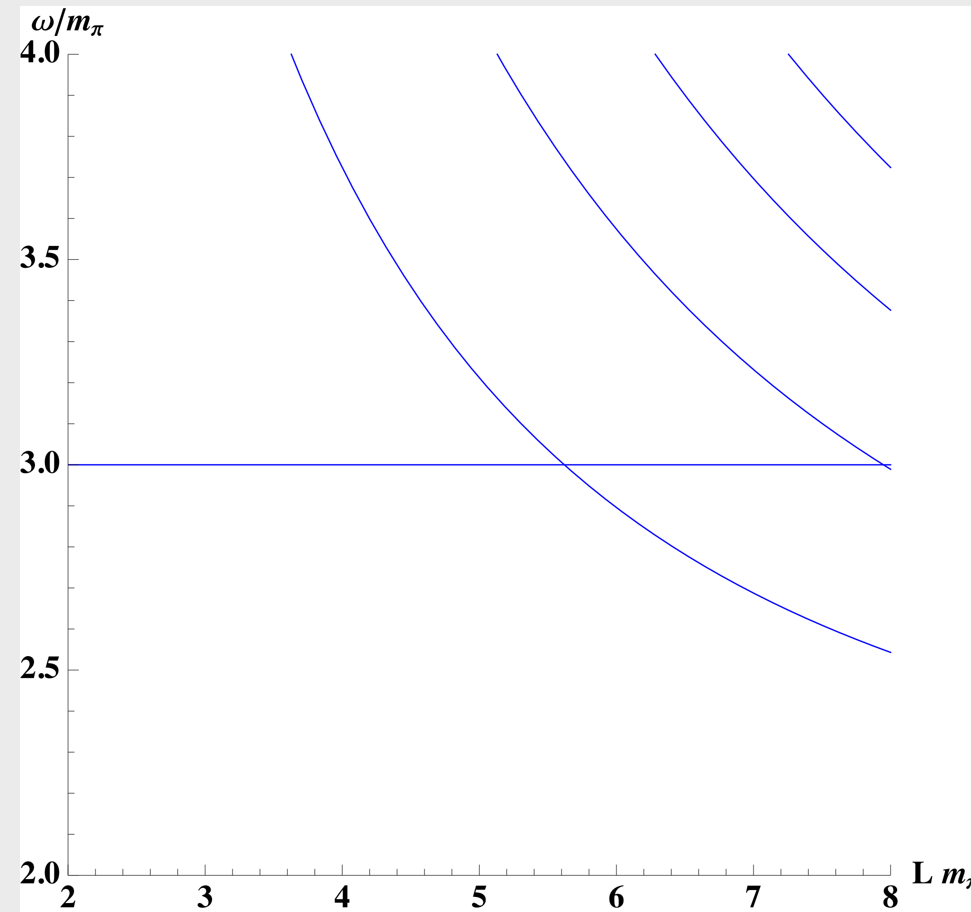

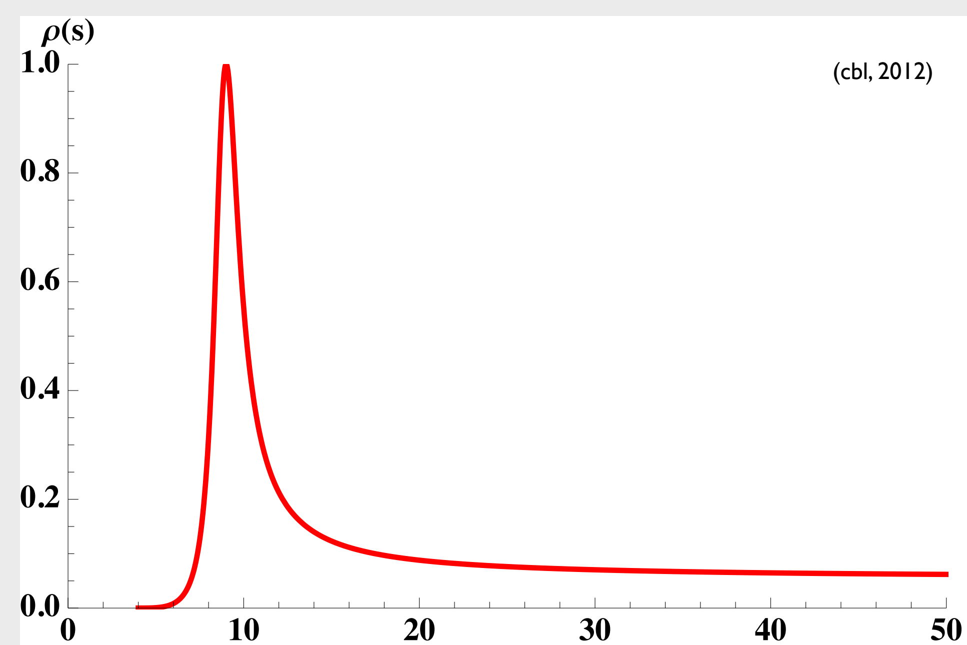

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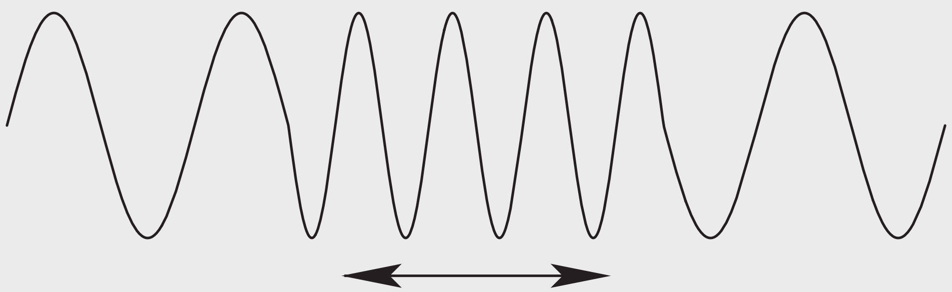

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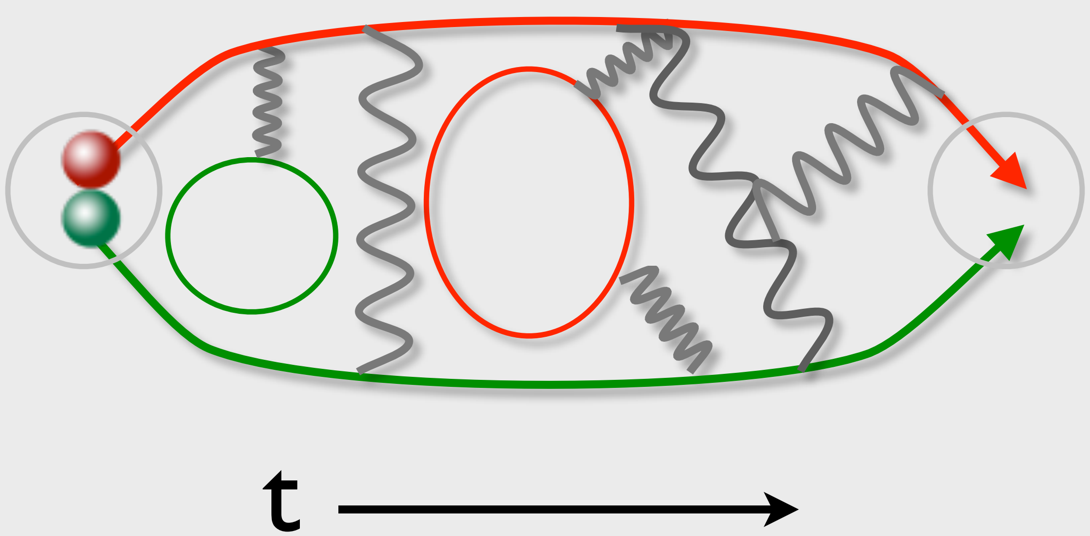

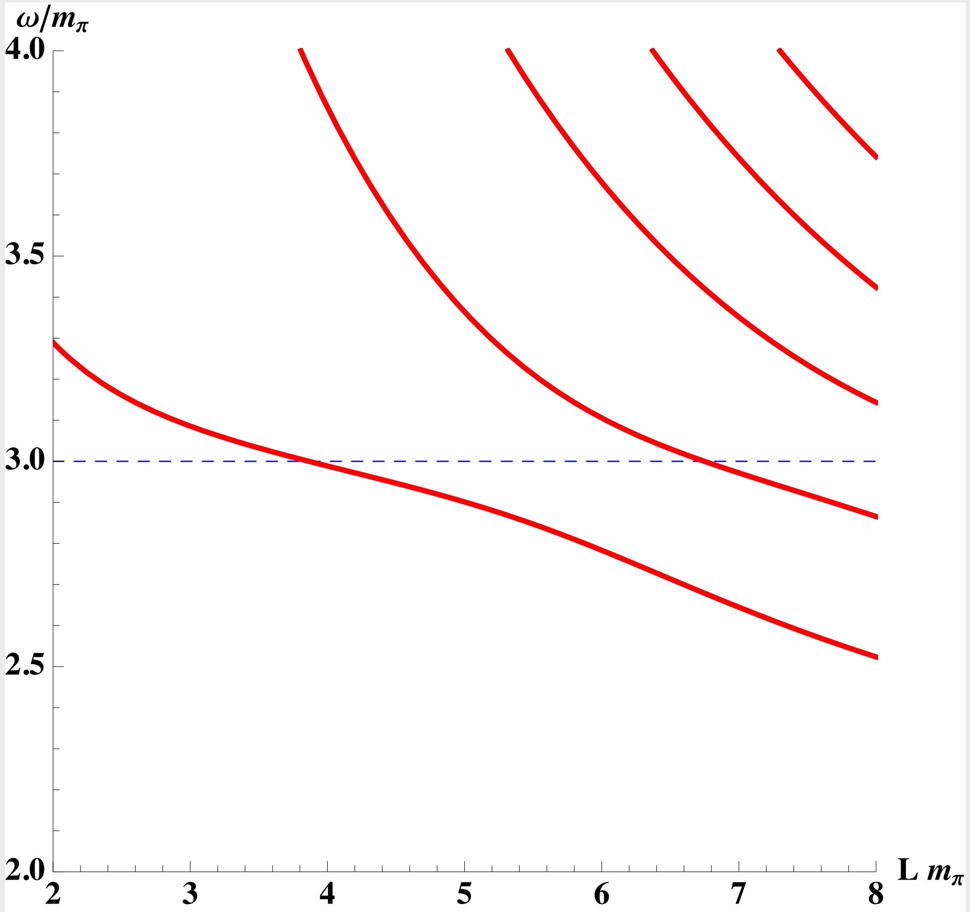

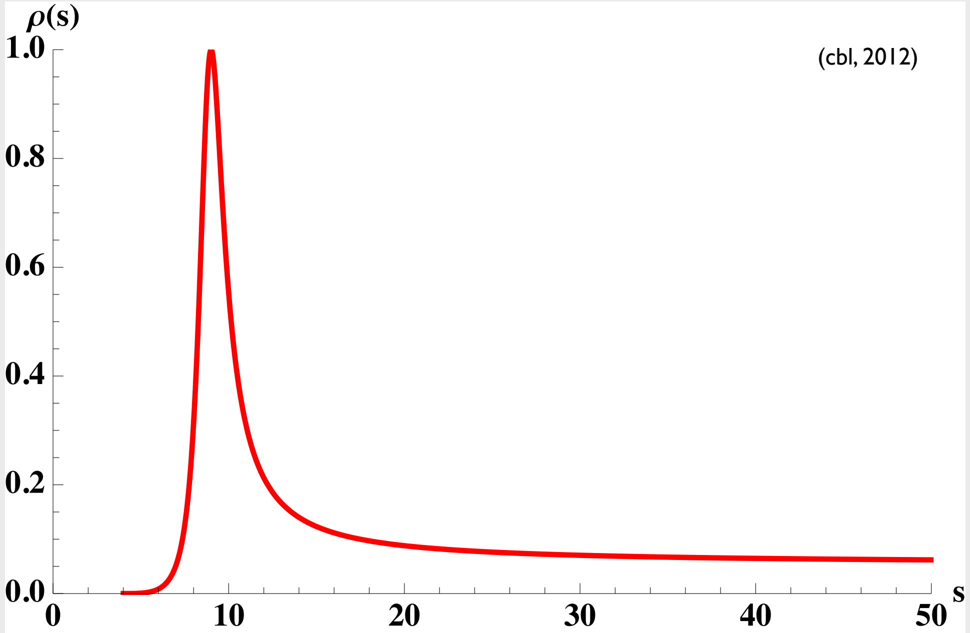

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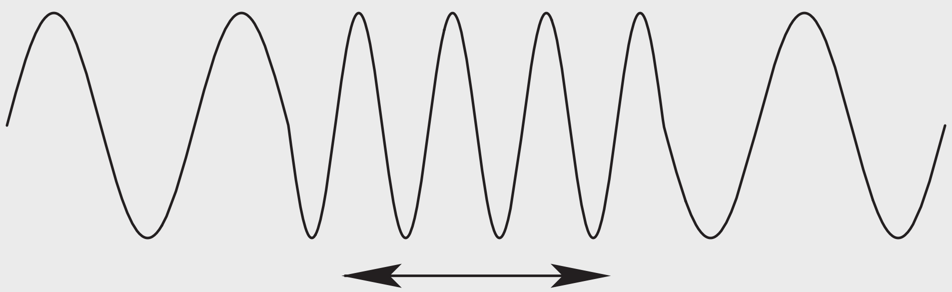

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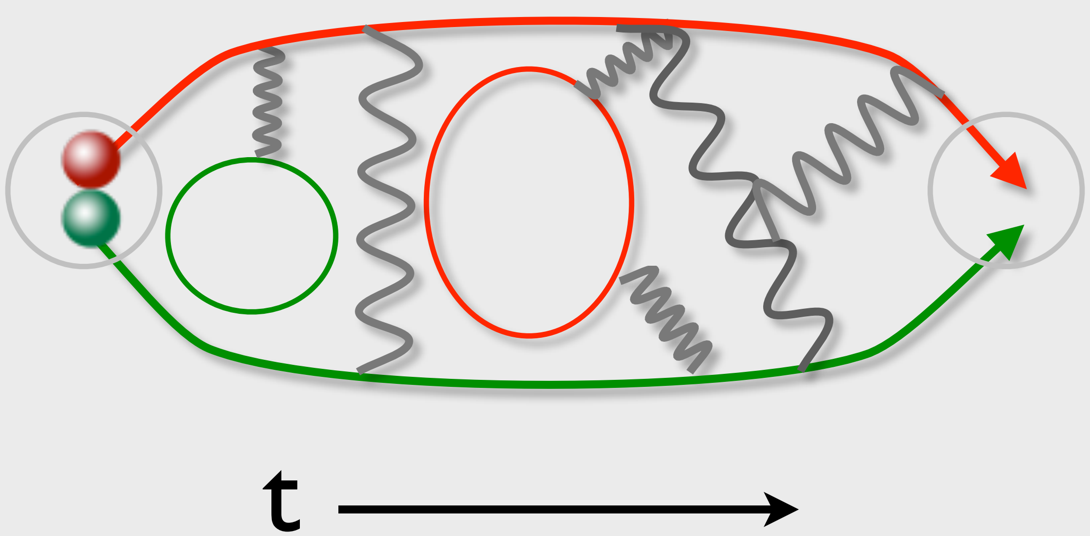

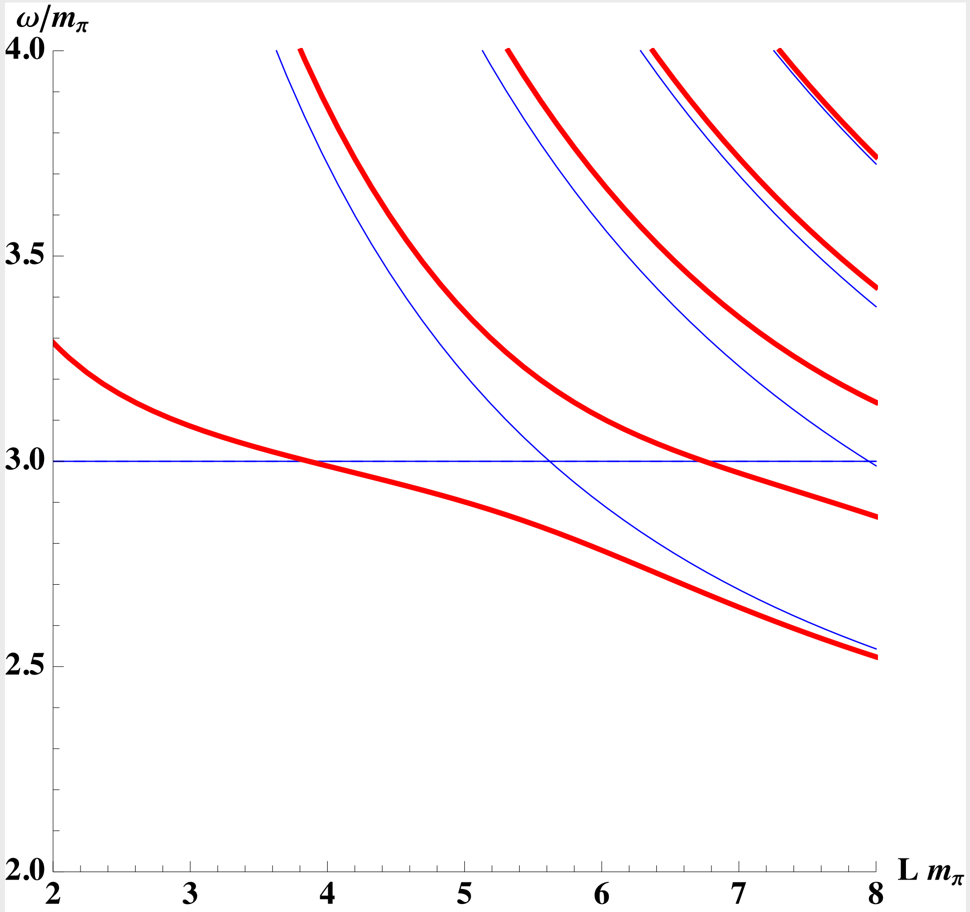

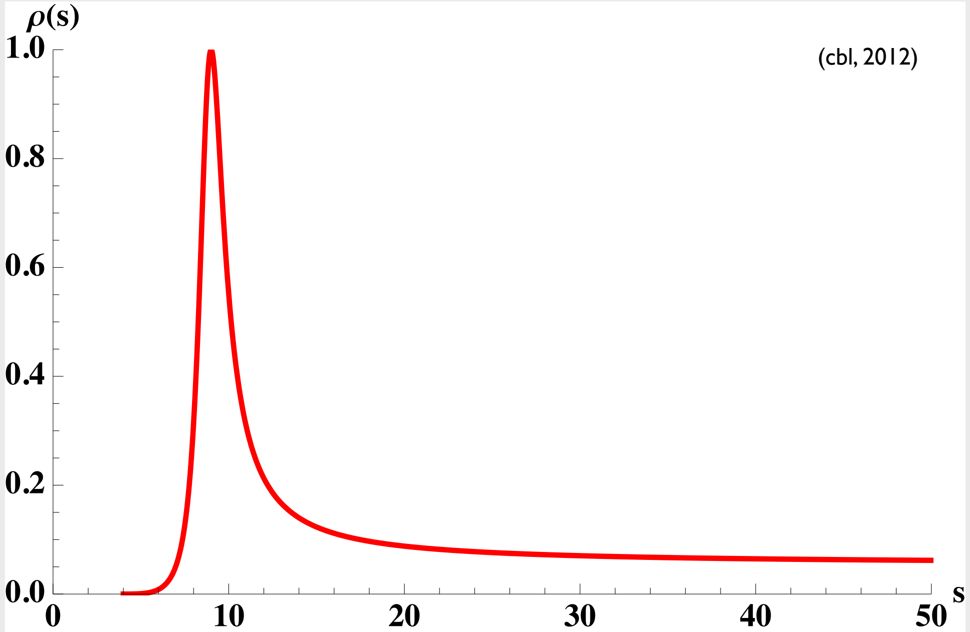

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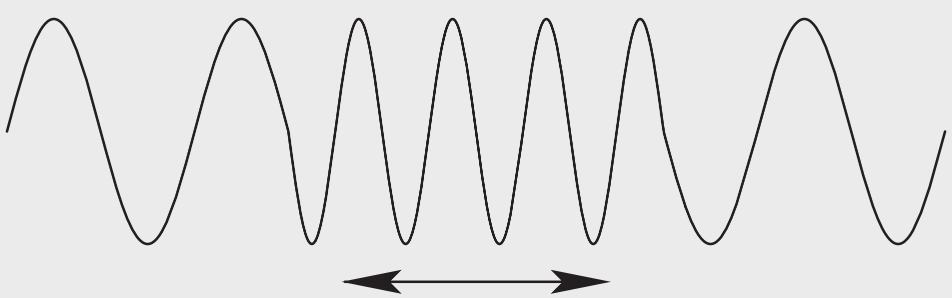

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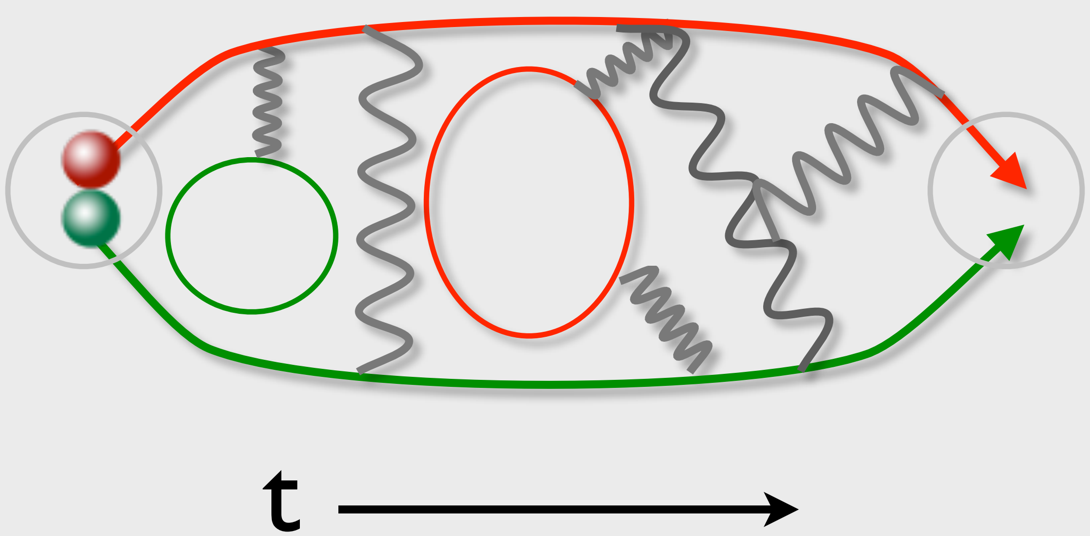

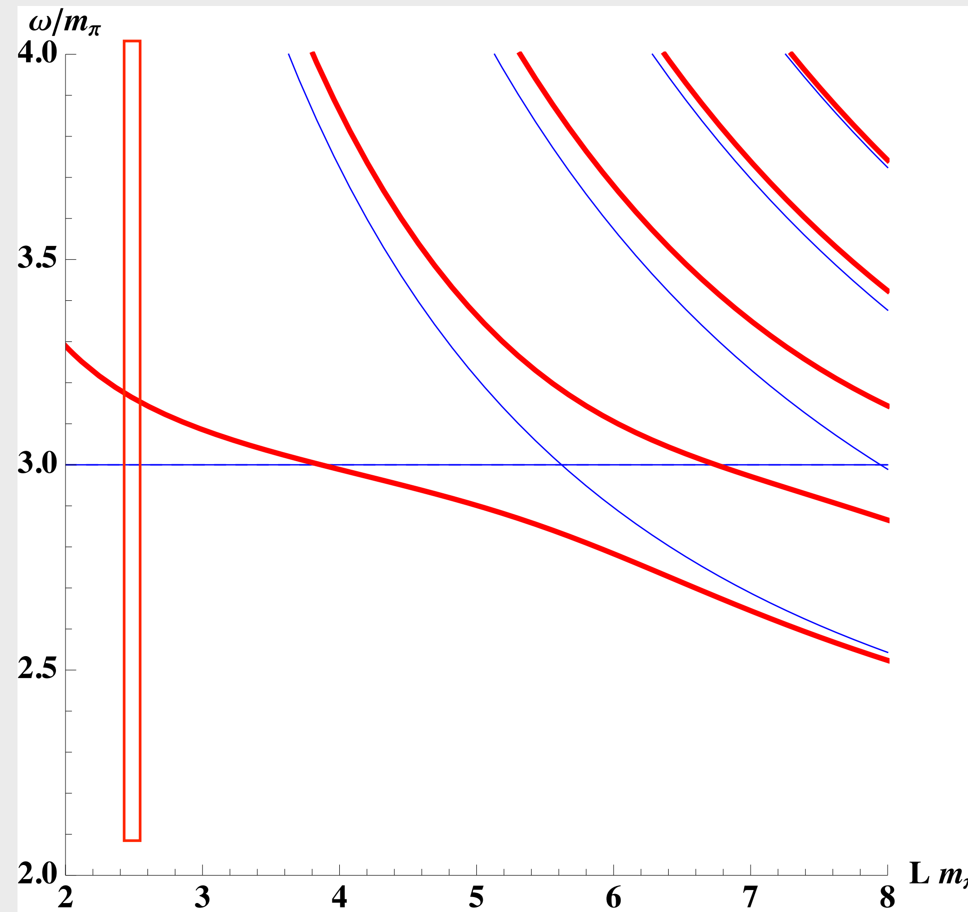

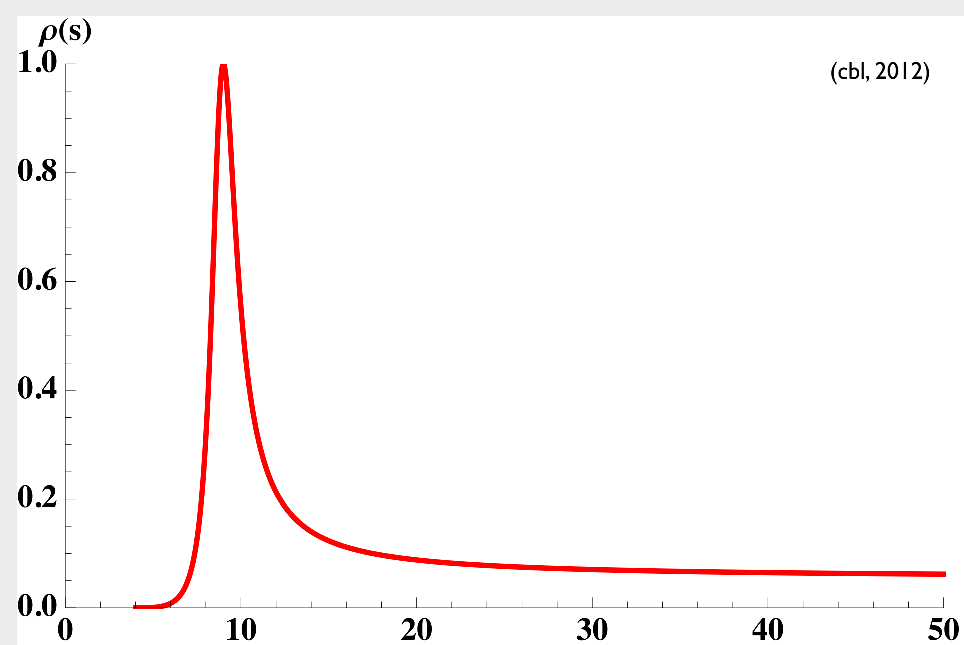

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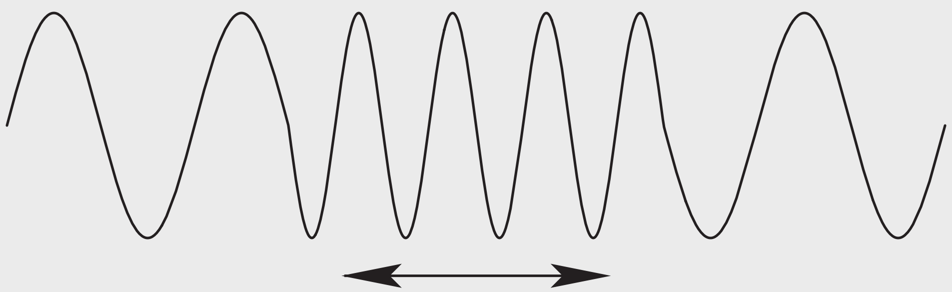

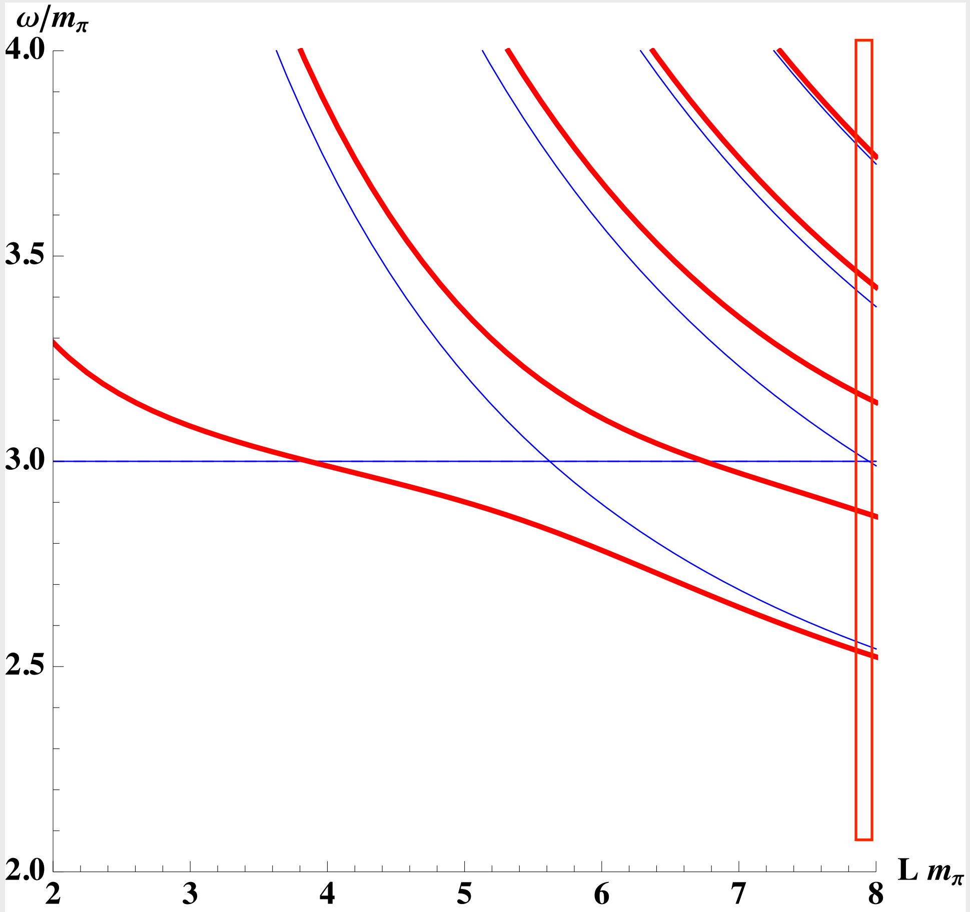

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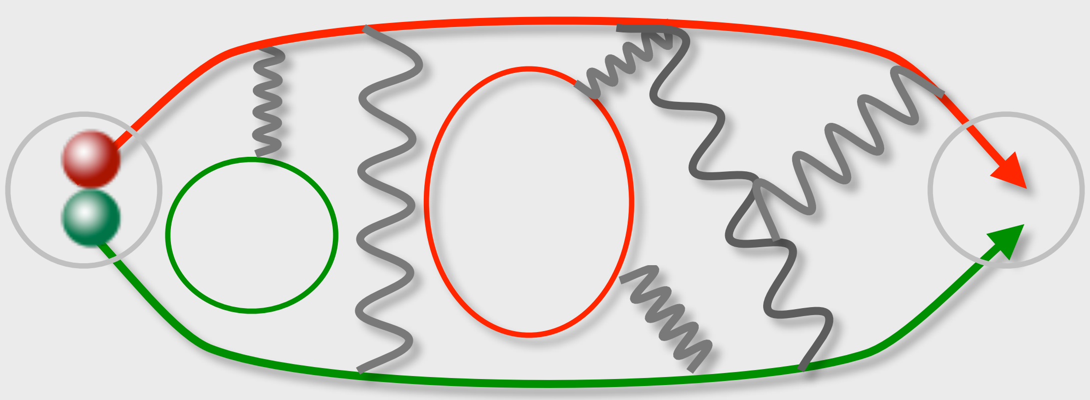

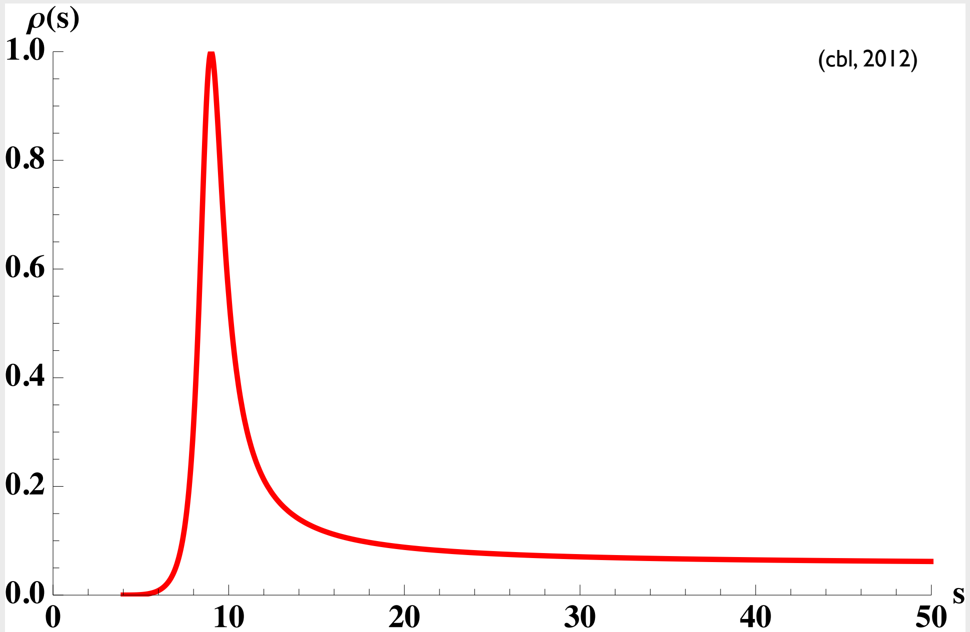

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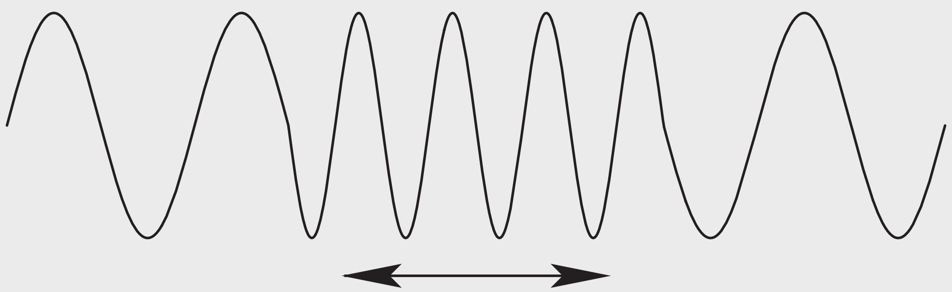

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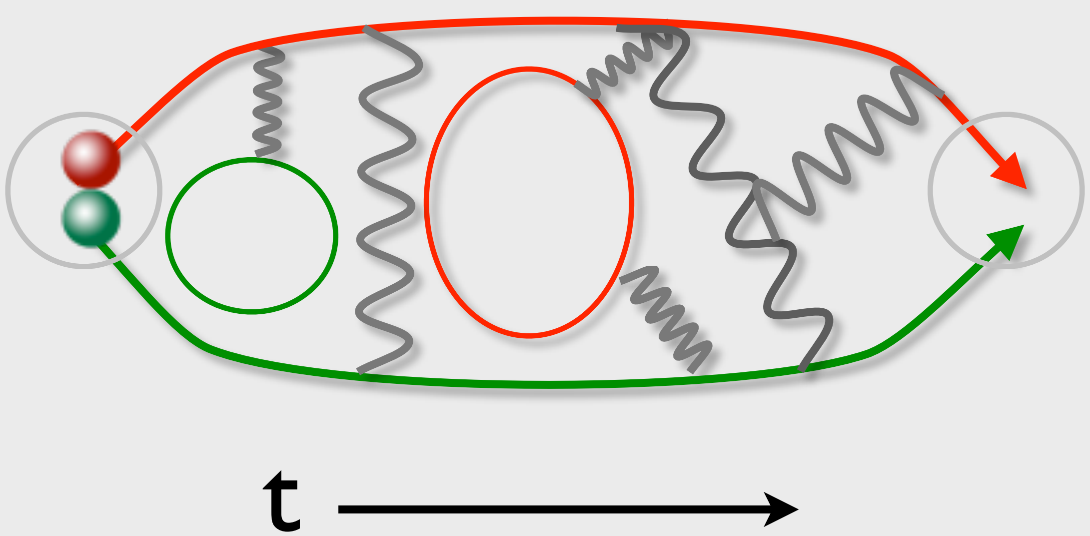

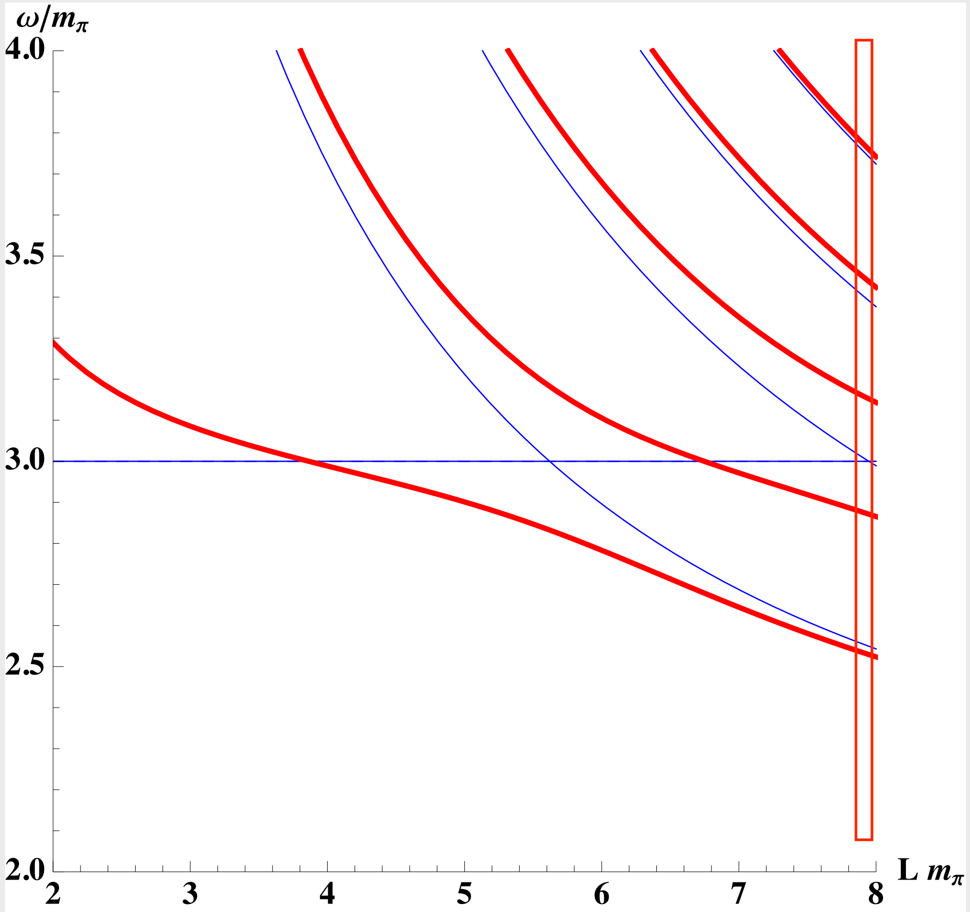

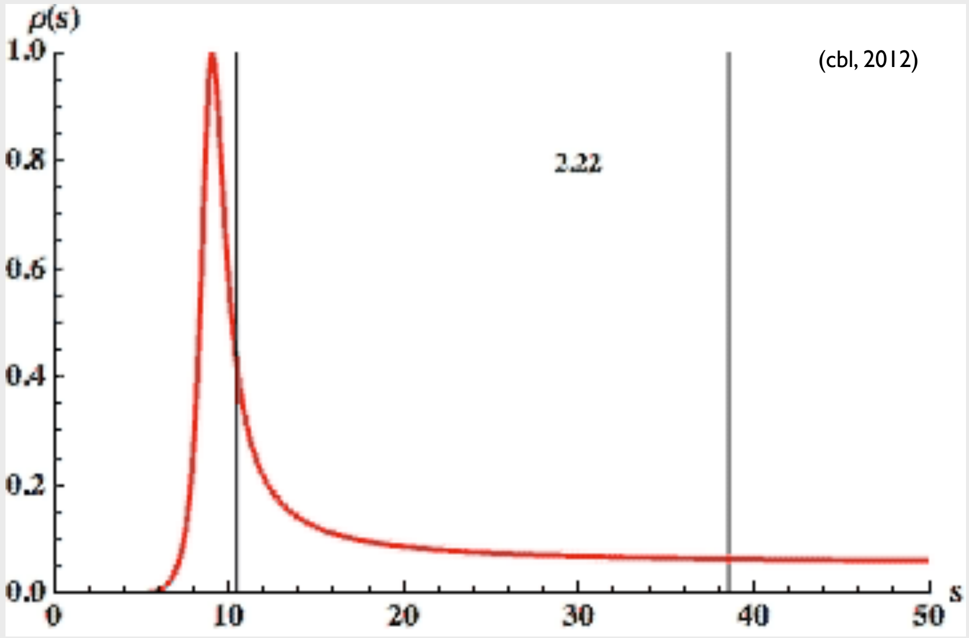

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- Energy values: masses of hadrons?
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How to extract several energy levels from correlation functions?

How to interpret the (hopefully) observed values?

What do we need?

- Gauge configurations (with dynamical quarks)
- Quark propagators
- Hadron interpolators and propagators
- A method to extract higher energy levels
- Interpretation of the obtained energy levels

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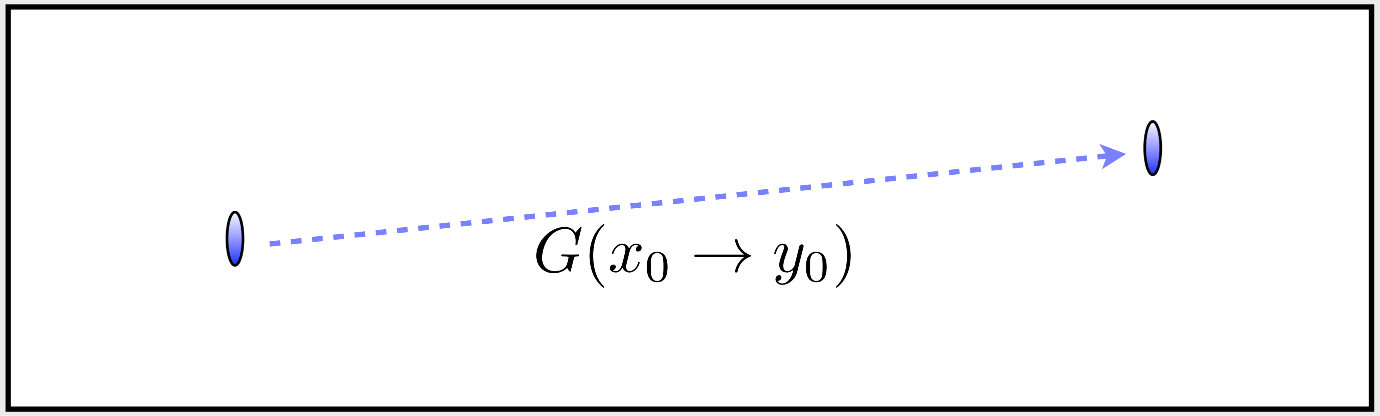

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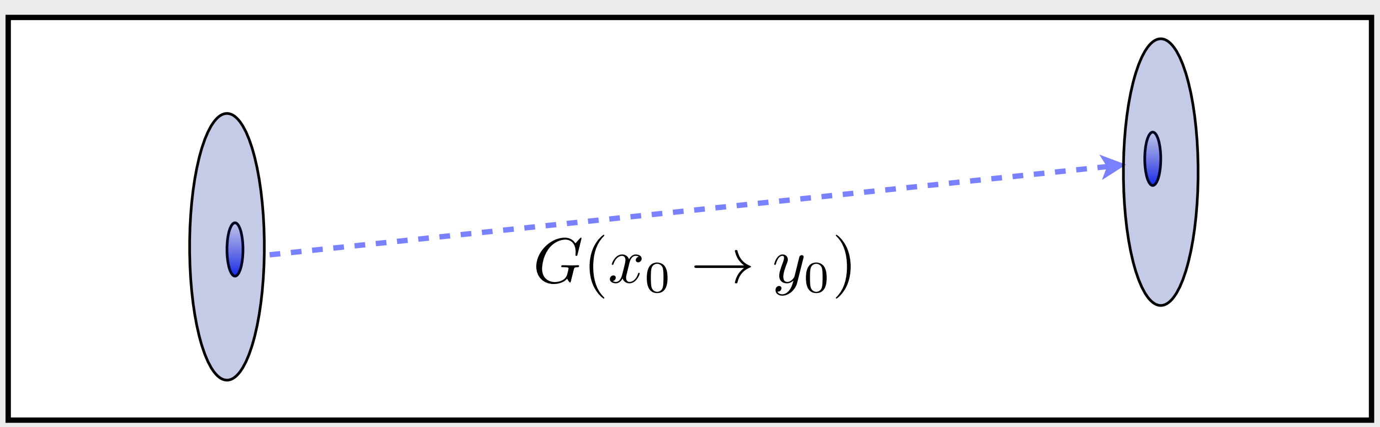

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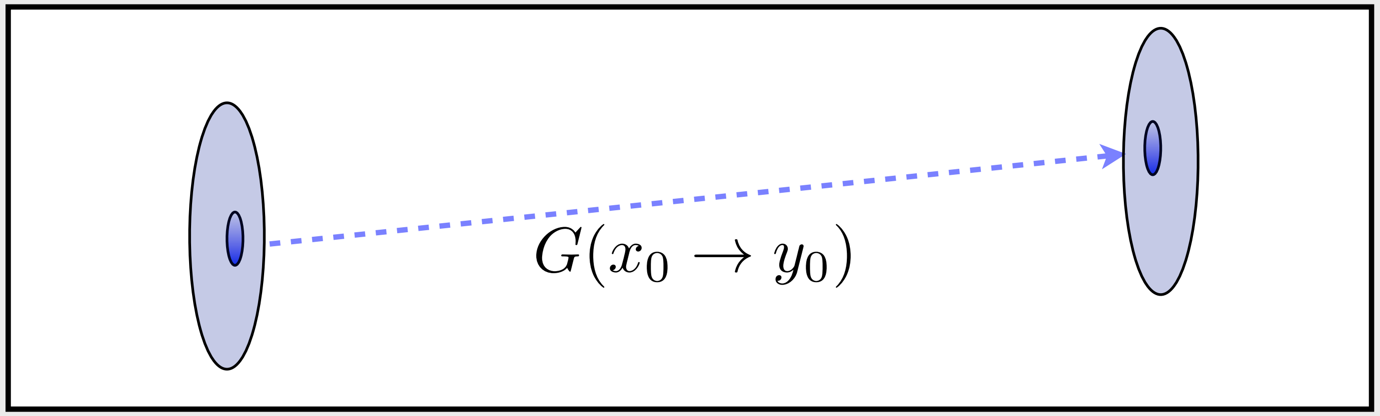

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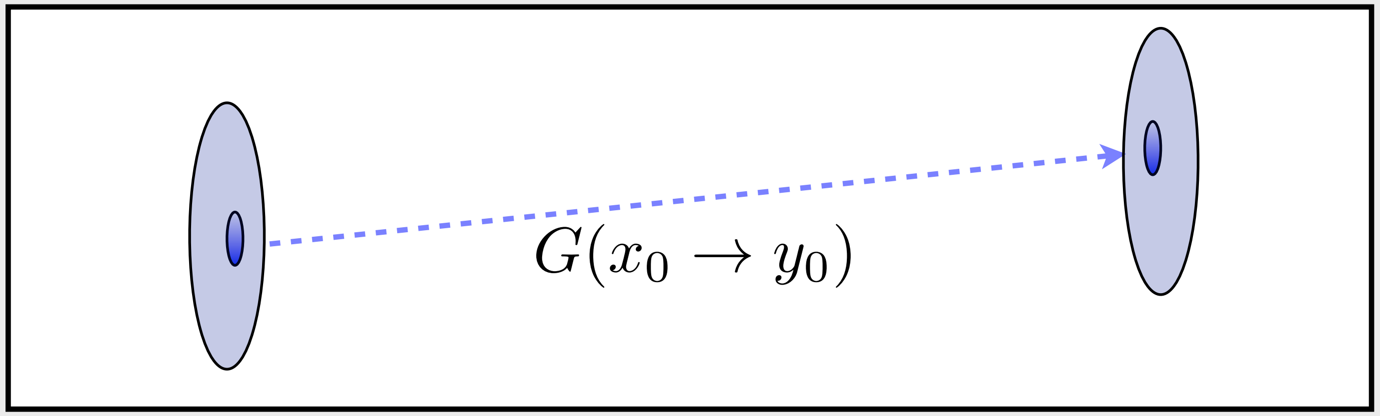

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- Variational method (Michael, Lüscher/Wolff)

Burch (03/06)
Basak (05)

Energy levels: Disentangle the states

- Use several interpolators X_i
- Compute all cross-correlations

$$C_{ij}(t) = \langle X_i(t) X_j^\dagger(0) \rangle$$

- Solve the generalized eigenvalue problem:

$$C(t)\, u^{(n)} = \lambda^{(n)}\, C(t_0)\, u^{(n)}$$

- The eigenvalues give the energy levels (masses):

$$\lambda^{(n)}(t) \propto e^{-t\, E_n} \left(1 + \mathcal{O}(e^{-t \Delta E_n})\right)$$

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Lüscher, Wolff: NPB339(90)222
Michael, NPB259(85)58
See also Blossier et al., JHEP0904(09)094

Hadron operators

We need several hadron interpolators to allow a good representation of the hadronic states!

- Several Dirac structures, e.g

Pion $\overline{u}\gamma_5 d,\ \overline{u}\gamma_t\gamma_5 d,\ \ldots$

$$N^{(i)} = \epsilon_{abc}\,\Gamma_1^{(i)}\,u_a\,\left(u_b^T\,\Gamma_2^{(i)}\,d_c - d_b^T\,\Gamma_2^{(i)}\,u_c\right)$$

$$\Delta_\mu = \epsilon_{abc}u_a(u_b^T C\gamma_\mu u_c)$$

	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$
$i=1$	1	$C\gamma_5$
$i=2$	γ_5	C
$i=3$	i	$C\gamma_4\gamma_5$

(projected to definite parity)

- Extended operators (cf. HSC)

Quark sources

Different quark source shapes:

- Point
- Wall
- Stochastic
- Separable sources (see: distillation)
- Spatially smeared quarks (Jacobi smearing)
- Derivative sources

$$S_0 = \delta(m - m_0)\, \delta_{\alpha\alpha_0}\, \delta_{aa_0}$$

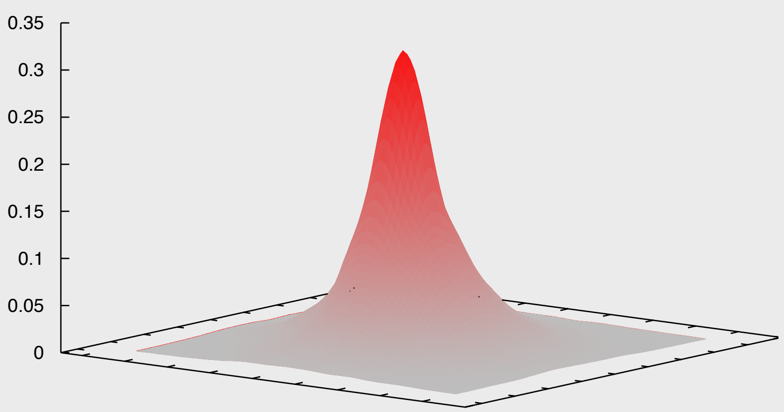
$$G = D^{-1}\, S_0 \to D\, G = S_0$$

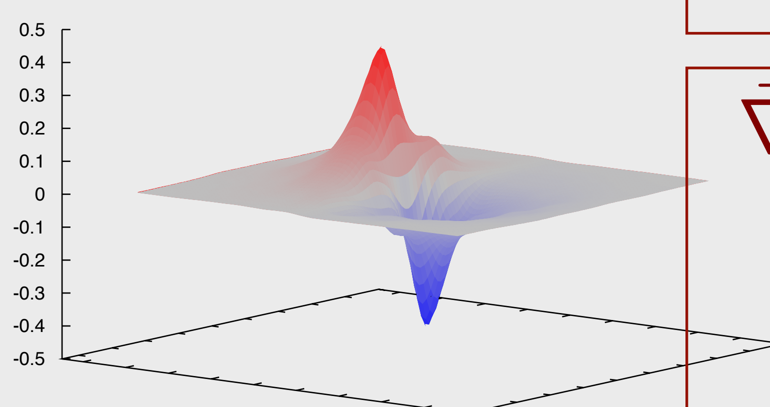
$$S = \sum_{n=0}^{N} \kappa^n\, H^n S_0$$

$$H(\vec{n}, \vec{m}\,) = \sum_{j=1}^{3} \Big[U_j(\vec{n}, 0)\, \delta(\vec{n} + \hat{j}\,, \vec{m}\,) + U_j(\vec{n} - \hat{j}\,, 0)^\dagger\, \delta(\vec{n} - \hat{j}\,, \vec{m}\,) \Big]$$

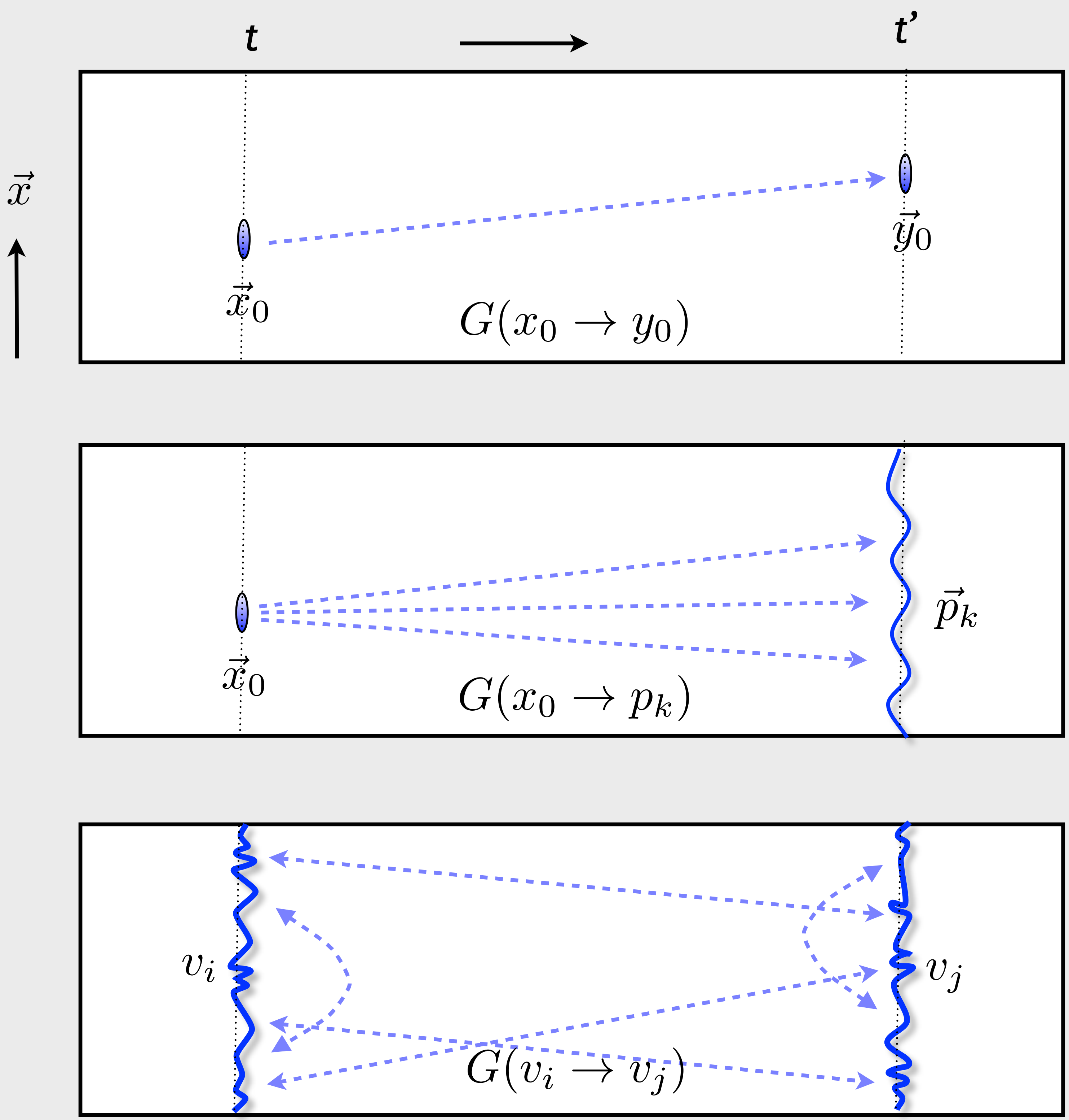
$$\vec{\nabla}_i(\vec{x}, \vec{y}) = U_i(\vec{x}, 0)\delta_{\vec{x}+\hat{i},\vec{y}} - U_i(\vec{x} - \hat{i}, 0)^\dagger \delta_{\vec{x}-\hat{i},\vec{y}}$$

$$S_{\partial_i} = \vec{\nabla}_i S$$





Correlators



point → point
smeared → smeared

smeared → momentum

eigenmode ↔ eigenmode

Separable sources

LapH smearing and distillation *Peardon et al. PRD80(09)054506*

e.g. meson: $\boldsymbol{M} = \overline{u}_x \, D_{x,y} \, d_y$

Separable sources

LapH smearing and distillation *Peardon et al. PRD80(09)054506*

e.g. meson: $M = \overline{u}_x \qquad D_{x,y} \qquad d_y$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

e.g. meson: $M = \bar{u}_x S^\dagger(x, x') D_{x', y'} \Gamma S(y', y) d_y$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

e.g. meson:

$$M = \bar{u}_x S^\dagger(x,x') D_{x',y'} \Gamma S(y',y) d_y$$

$$\sum_i^N g_i(x) g_i^\dagger(x') \qquad \sum_i^N g_i(y') g_i^\dagger(y)$$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

e.g. meson:

$$M = \bar{u}_x S^\dagger(x,x') D_{x',y'} \Gamma S(y',y) d_y$$

$$S^\dagger(x,x') = \sum_i^N g_i(x) g_i^\dagger(x') \qquad S(y',y) = \sum_i^N g_i(y') g_i^\dagger(y)$$

$$\langle M(0) M(t) \rangle = \sum_{ijkn} \langle \bar{u}\, g_i\, g_i^\dagger\, D\Gamma\, g_j\, g_j^\dagger\, d\, \bar{d}\, g_k\, g_k^\dagger\, D\Gamma\, g_n\, g_n^\dagger\, u \rangle$$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

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$$M = \bar{u}_x S^\dagger(x,x') D_{x',y'} \Gamma S(y',y) d_y$$

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$$= \sum_{ijkn} \phi_{ij}(0)\, \tau_{jk}(0,t)\, \phi_{kn}(t)\, \tau_{ni}(t,0)$$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

e.g. meson: $M = \bar{u}_x S^\dagger(x,x') D_{x',y'} \Gamma S(y',y) d_y$

$S^\dagger(x,x') = \sum_i^N g_i(x) g_i^\dagger(x')$ $\quad$ $S(y',y) = \sum_i^N g_i(y') g_i^\dagger(y)$

$$\langle M(0) M(t) \rangle = \sum_{ijkn} \langle \; g_i^\dagger \, D\Gamma \, g_j \; g_j^\dagger \, d \, \bar{d} \, g_k \; g_k^\dagger \, D\Gamma \, g_n \; g_n^\dagger \, u \, \bar{u} \, g_i \rangle$$

$$= \sum_{ijkn} \phi_{ij}(0) \, \tau_{jk}(0,t) \, \phi_{kn}(t) \, \tau_{ni}(t,0)$$

Separable sources

LapH smearing and distillation

Peardon et al. PRD80(09)054506

e.g. meson: $M = \bar{u}_x S^\dagger(x,x') D_{x',y'} \Gamma S(y',y) d_y$

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$$= \sum_{ijkn} \phi_{ij}(0) \, \tau_{jk}(0,t) \, \phi_{kn}(t) \, \tau_{ni}(t,0)$$

Perambulator τ: Propagator from source i to sink n

“Laplacian Heaviside smearing”

Perambulator: Propagator from source i to sink j

Distillation operator: Spectral representation in terms of eigenvectors of the 3D Laplacian

$$S(x,y) = \sum_i^N c_i g_i(x) g_i^\dagger(x')$$

e.g., for

Advantage:
High flexibility in interpolator def.;
Disconnected contributions

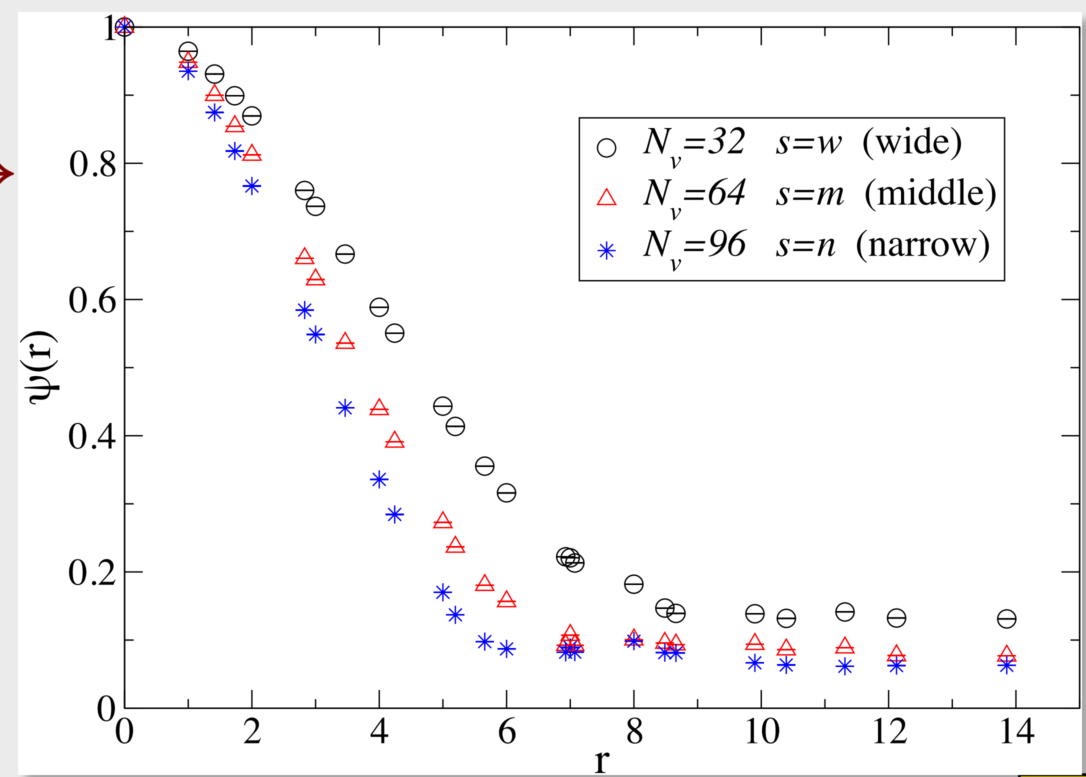
“Laplacian Heaviside smearing”

Perambulator: Propagator from source i to sink j

Distillation operator: Spectral representation in terms of eigenvectors of the 3D Laplacian

$$S(x,y) = \sum_i^N c_i g_i(x) g_i^\dagger(x')$$

e.g., for $c_i = 1,\ N = 32,\ 64,\ 96$ $\rightarrow$



Advantage:

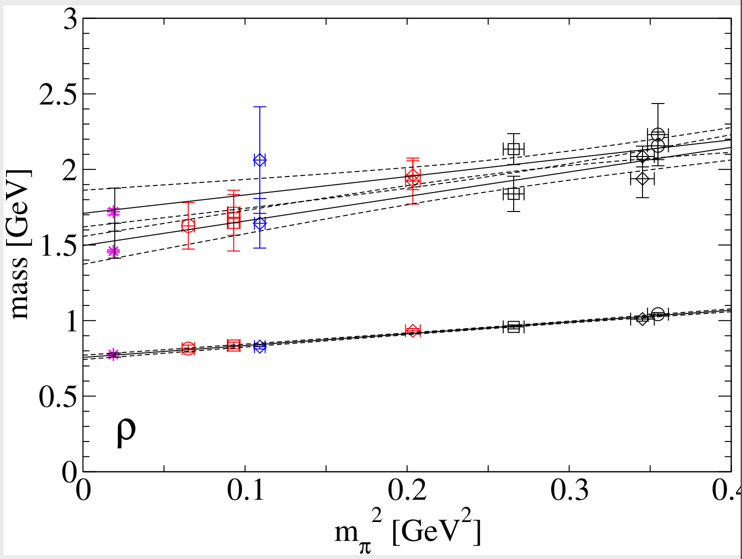
High flexibility in interpolator def.;
Disconnected contributions

Plus/minus

- All hadron-hadron correlators (and 3-point functions) can be constructed from the perambulators.
- High flexibility for interpolator structure: $\Gamma,\ \vec{\nabla}_i,\ \exp(\mathrm{i}\,\vec{p}\cdot\vec{x})$
- Needs many (NxN$_T$) Dirac operator inversions (perambulators)!
- Volume scaling! Stochastic dilution ?

Overview

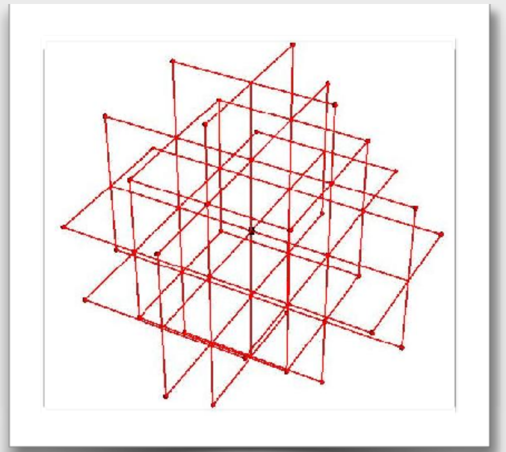
1. Motivation and lattice tools
2. Case 1: Hadron excitations
3. Case 2: Meson decay



Case 1: Single hadron interpolators for baryons and mesons

Gattringer et al. PRD 79 (2009) 054501
Engel et al. PRD 82 (2010) 034505;
PRD 85 (2012) 034508

Simulation with 2 sea quarks:



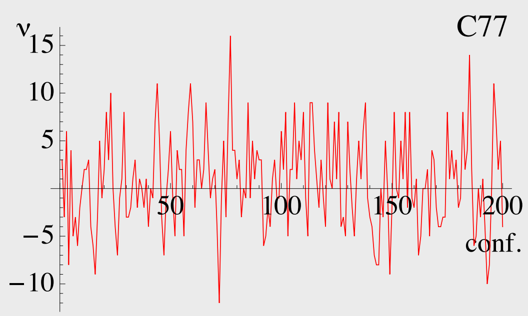
(Gattringer, PRD63(2001)114501)

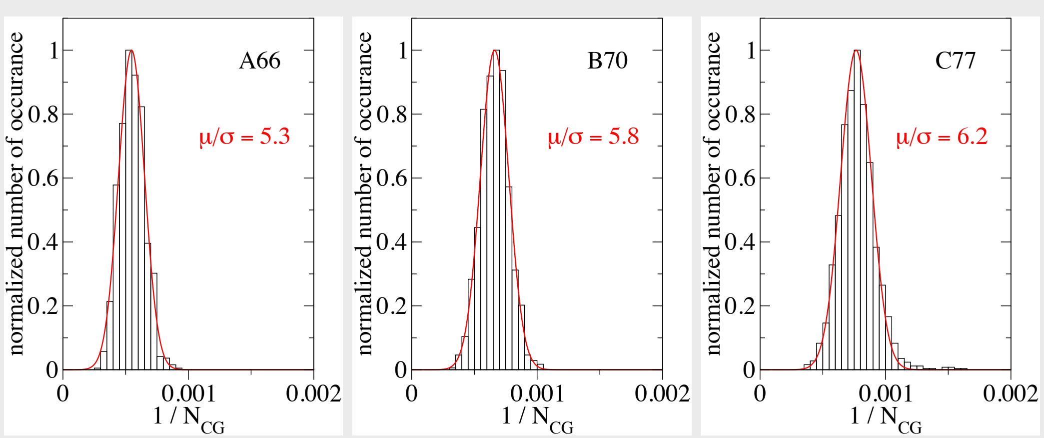
- Chirally improved (approximate GW) action + stout smearing
- Lüscher-Weisz gauge action
- 7 ensembles of 200-300 configurations
- 16^3x32 (size 2.4 fm), 24^3x48 (size 3.6 fm)
- Pion masses 260..540 MeV
- Smeared sources, single hadron interpolators

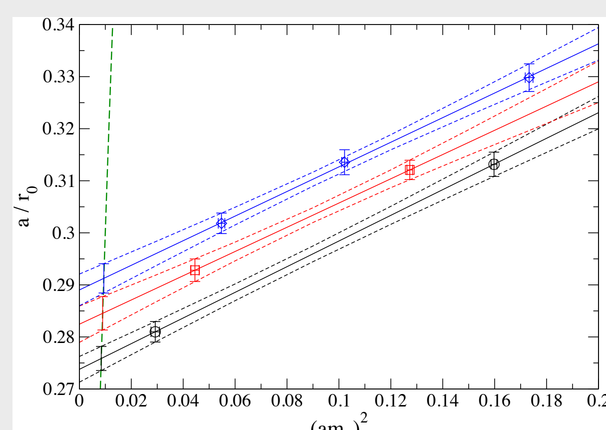
see, e.g., also other collab.s:
Edwards et al., arXiv:1104.5152
and citations in the review
Lin, arXiv:1106.1608

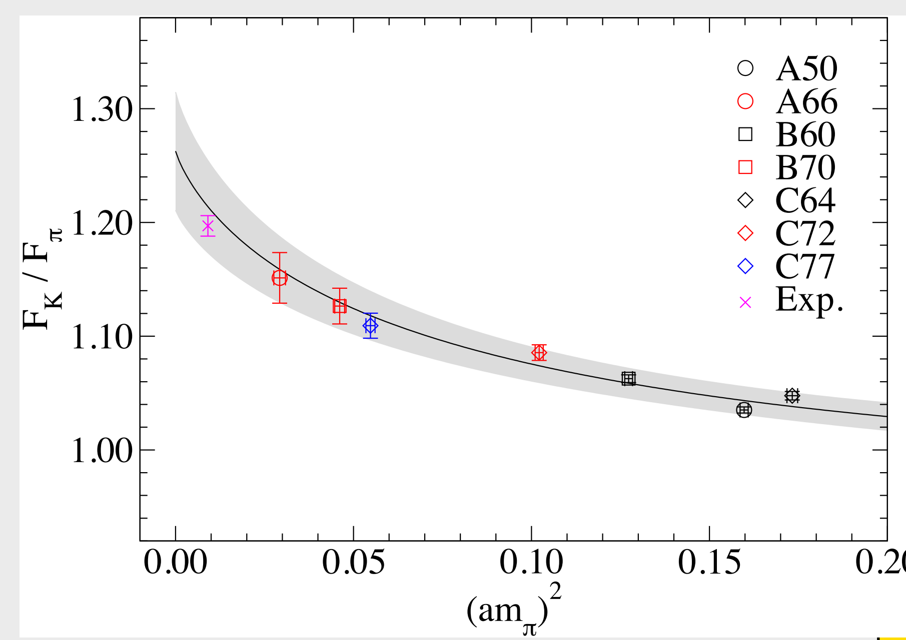
Details

set	β_{LW}	m_0	m_s	configs.	$L^3 \times T\ [a^4]$	$m_\pi L$	a [fm]	m_π [MeV]	$a\, m_{\mathrm{AWI}}$	m_{AWI} [MeV]
A50	4.70	-0.050	-0.020	200	$16^3 \times 32$	6.40	0.1324(11)	596(5)	0.03027(8)	45(1)
A66	4.70	-0.066	-0.012	200	$16^3 \times 32$	2.72	0.1324(11)	255(7)	0.00589(40)	9(1)
B60	4.65	-0.060	-0.015	300	$16^3 \times 32$	5.72	0.1366(15)	516(6)	0.02356(13)	34(1)
B70	4.65	-0.070	-0.011	200	$16^3 \times 32$	3.38	0.1366(15)	305(6)	0.00836(23)	12(1)
C64	4.58	-0.064	-0.020	200	$16^3 \times 32$	6.67	0.1398(14)	588(6)	0.02995(20)	42(1)
C72	4.58	-0.072	-0.019	200	$16^3 \times 32$	5.11	0.1398(14)	451(5)	0.01728(16)	24(1)
C77	4.58	-0.077	-0.022	300	$16^3 \times 32$	3.74	0.1398(14)	330(5)	0.01054(19)	15(1)
LA66	4.70	-0.066	-0.012	97	$24^3 \times 48$	4.08	0.1324(11)			
SC77	4.58	-0.077	-0.022	600	$12^3 \times 24$	2.81	0.1398(14)			
LC77	4.58	-0.077	-0.022	153	$24^3 \times 48$	5.61	0.1398(14)			

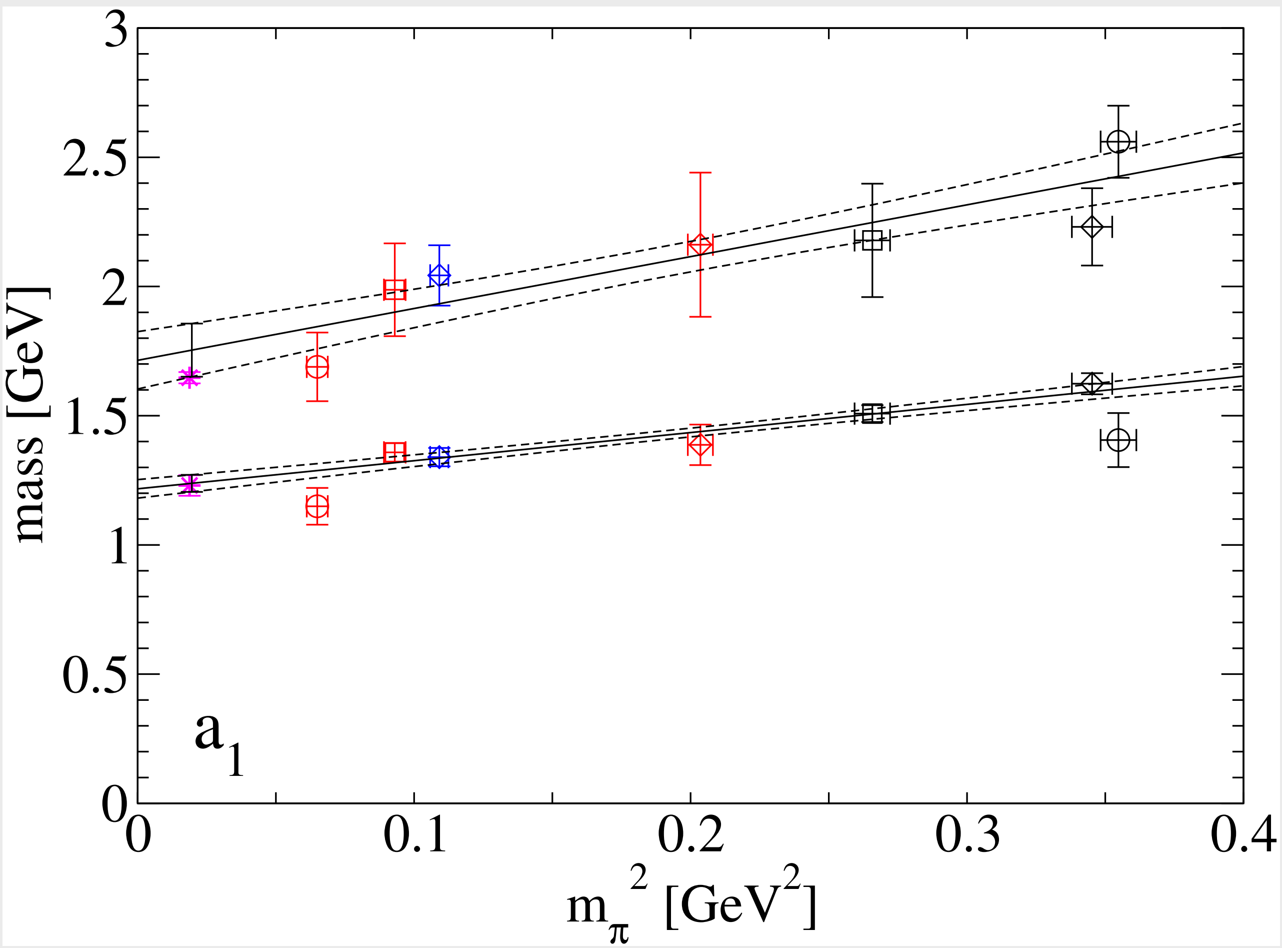


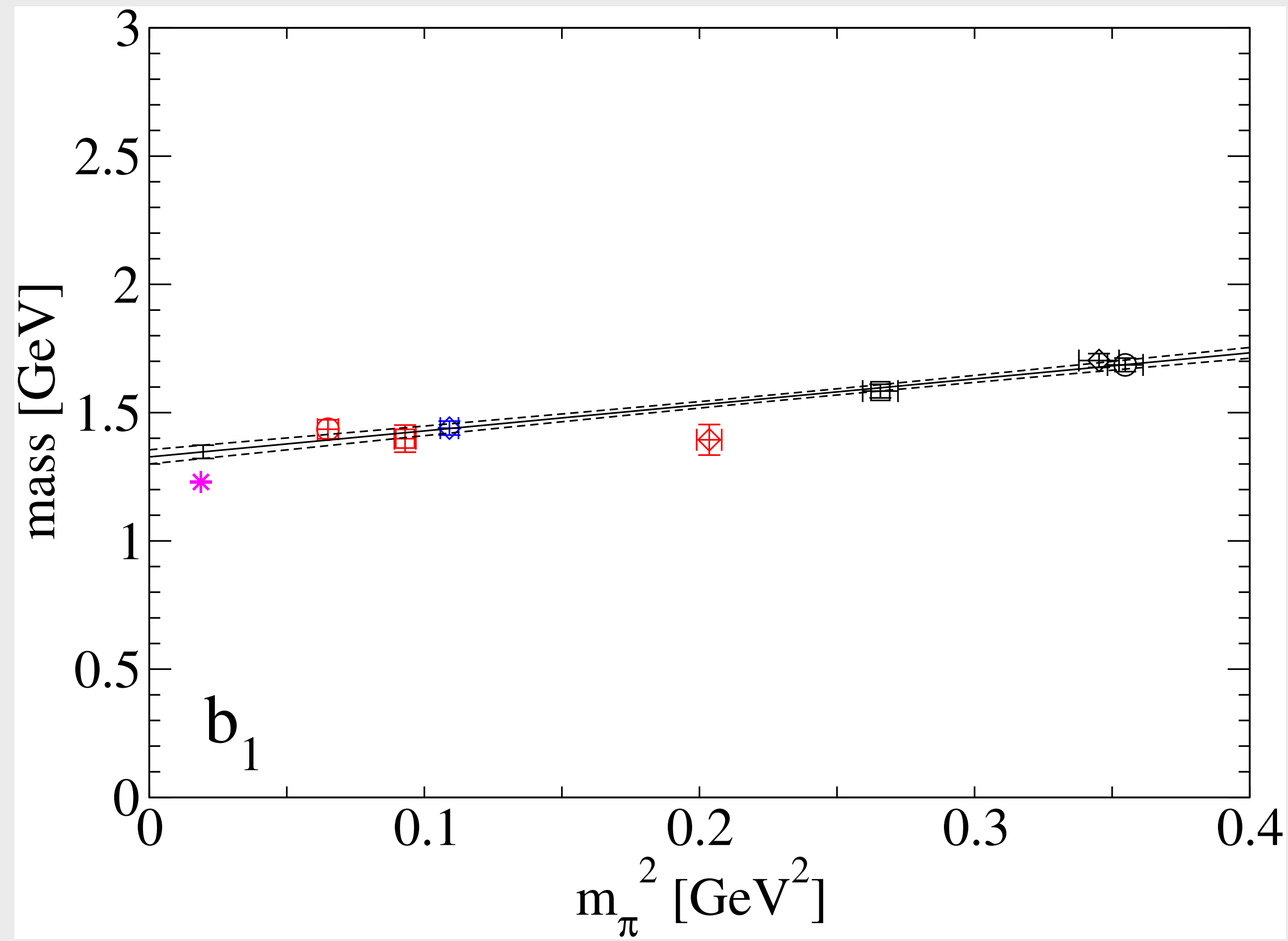






1^{++}: $a_1(1260)$, 1^{+-}: $b_1(1235)$

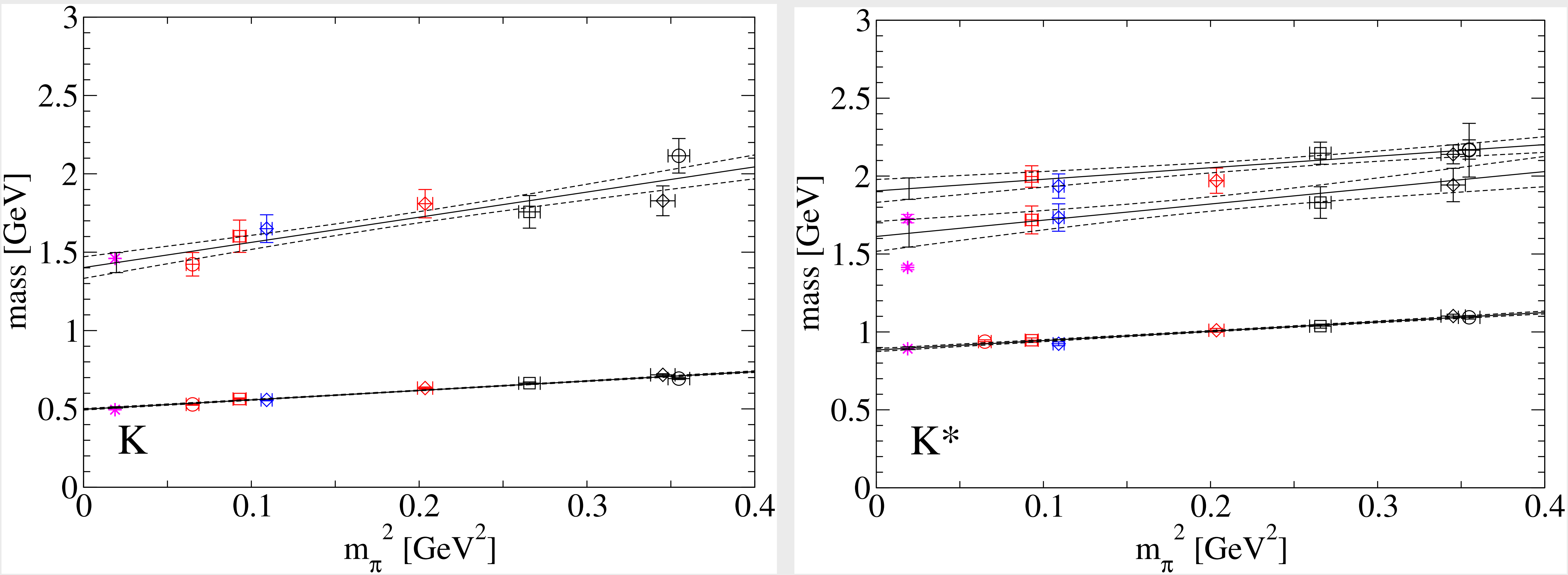




Good signal for ground state needs interpolators with derivative sources

Engel et al. PRD 85 (2012) 034508

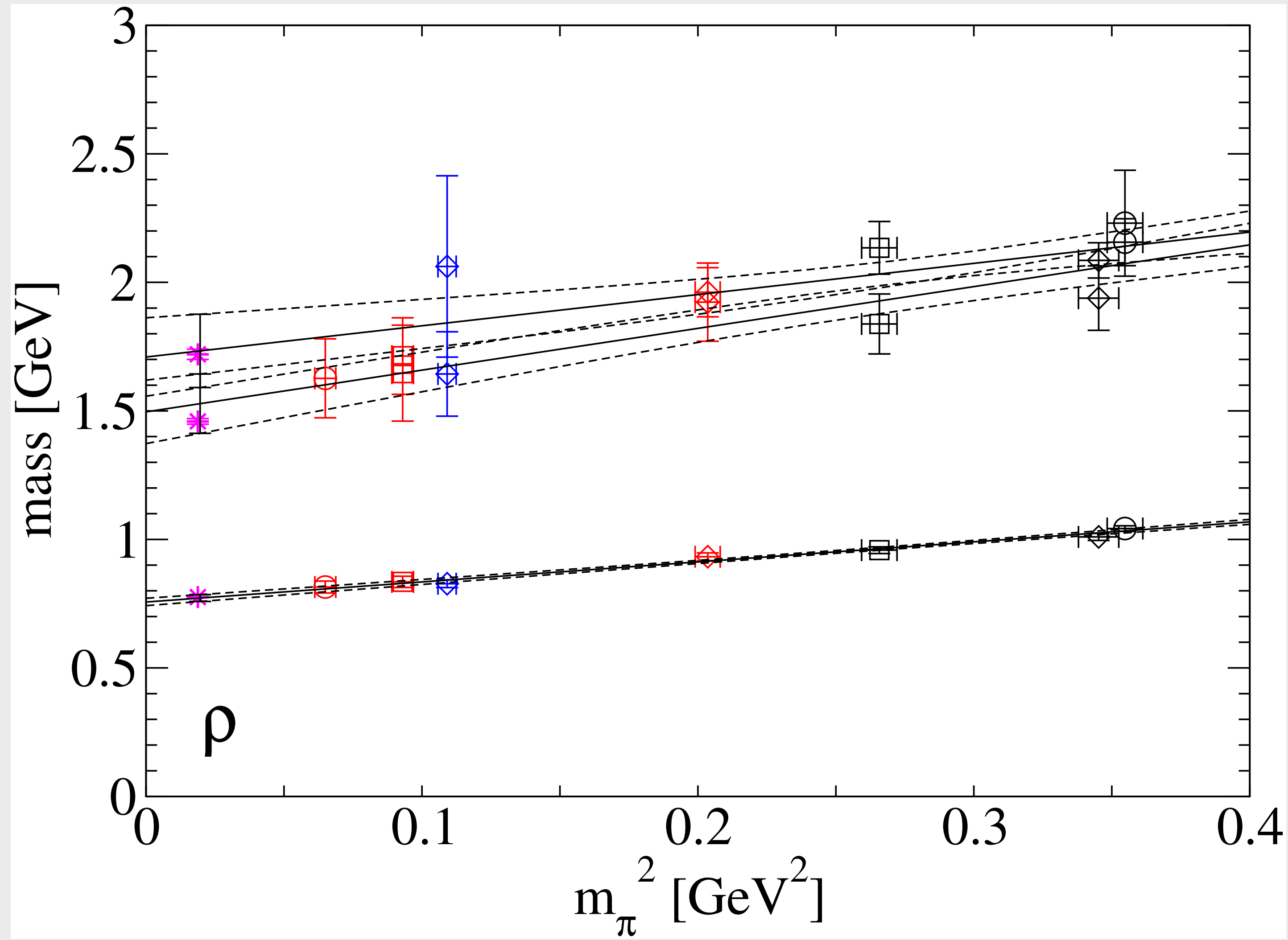
Strange mesons: 1/2(0^-)K(495), 1/2(1^-) K*(892)



Engel et al. PRD 85 (2012) 034508

1^{--}: ρ(770), ρ'(1450)

- No decay yet (p-wave!)
- 1st excitation ρ(1450)
- 2nd excitation ρ(1570/1720) signal is seen for some combinations of interpolators

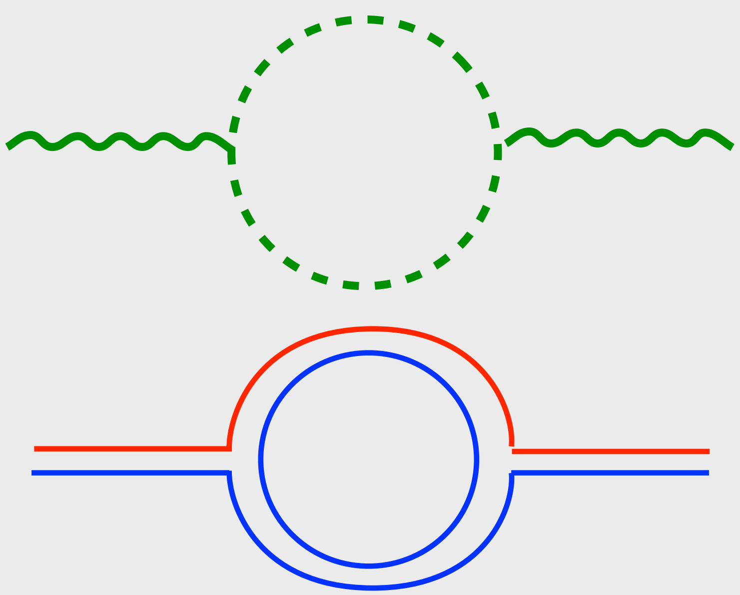


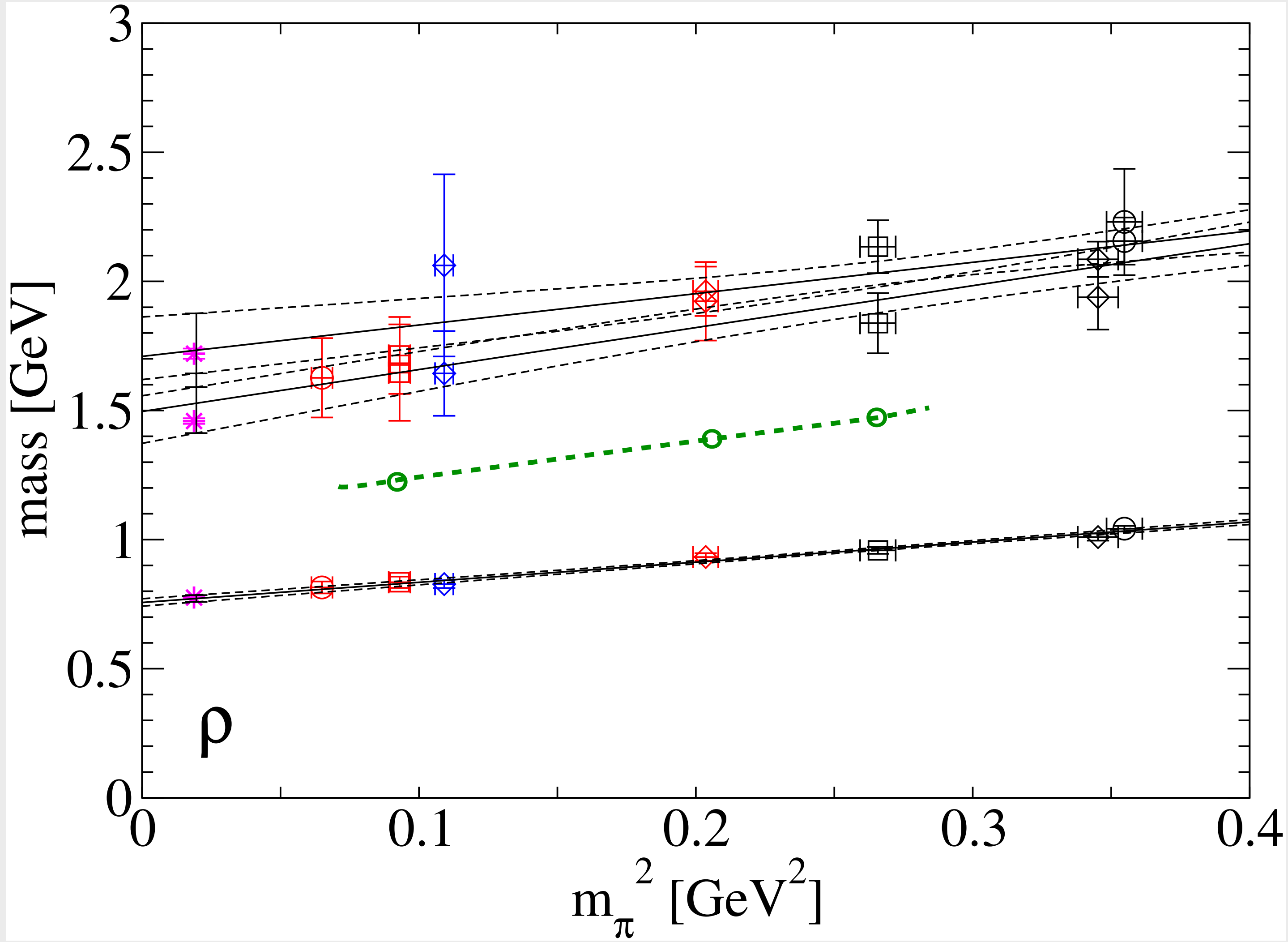
Engel et al. PRD 85 (2012) 034508

1⁻⁻: ρ(770), ρ‘(1450)

- No decay yet (p-wave!)
- 1st excitation ρ(1450)
- 2nd excitation ρ(1570/1720) signal is seen for some combinations of interpolators

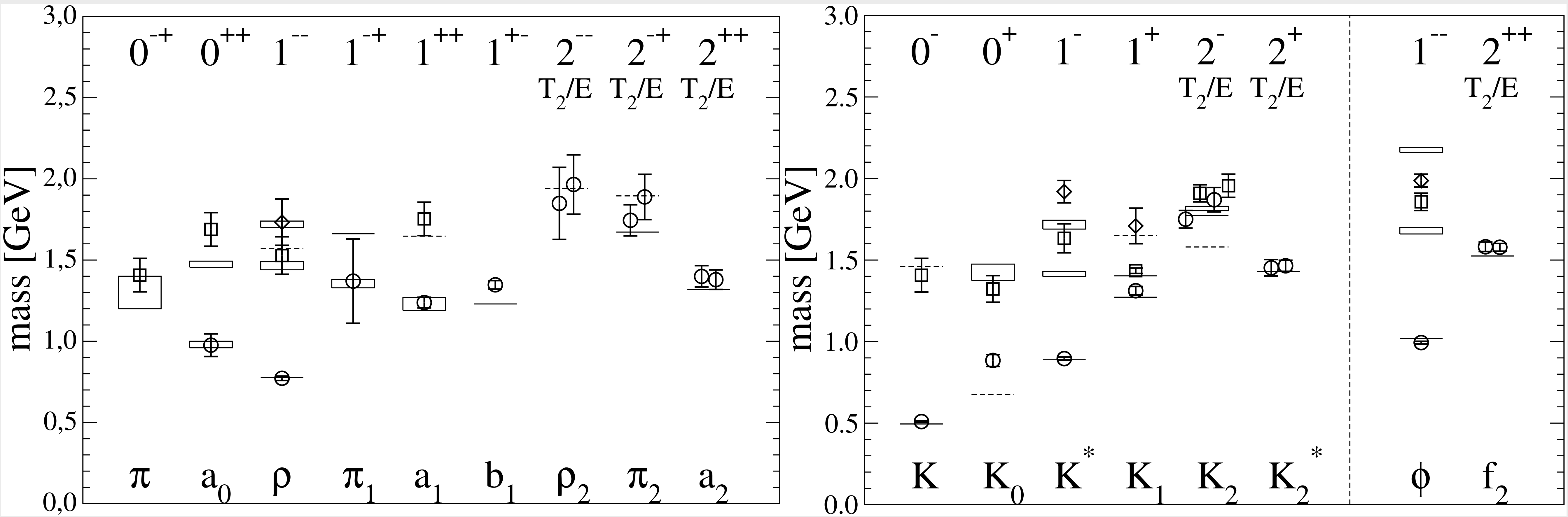
- Challenge: Where is the ππ state?





Engel et al. PRD 85 (2012) 034508

Meson summary



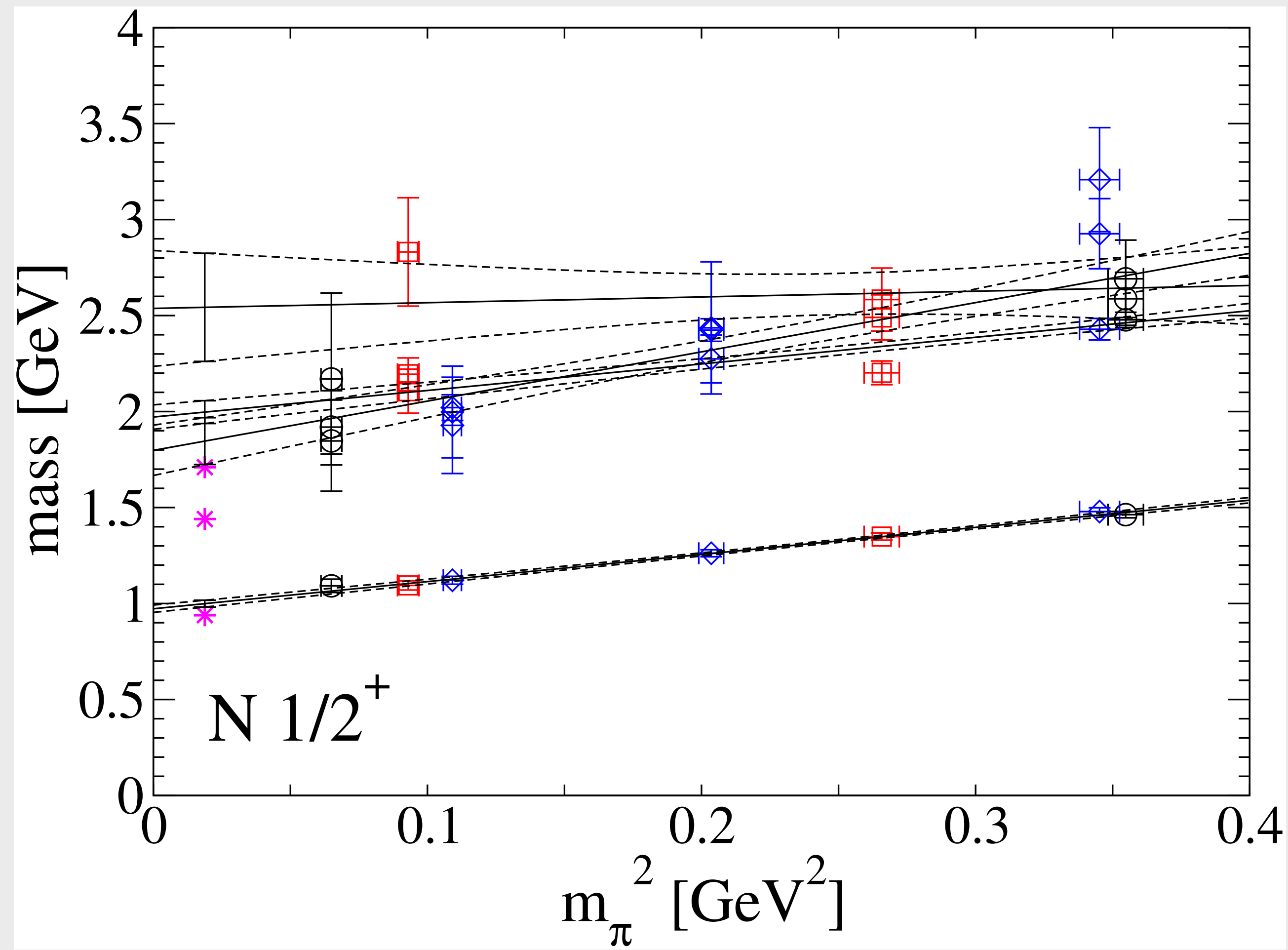
Engel et al. PRD 85 (2012) 034508

1/2$^+$: N(940), N(1440), N(1710)

Similar to quenched results! Two excitations (higher one vague), too high up!

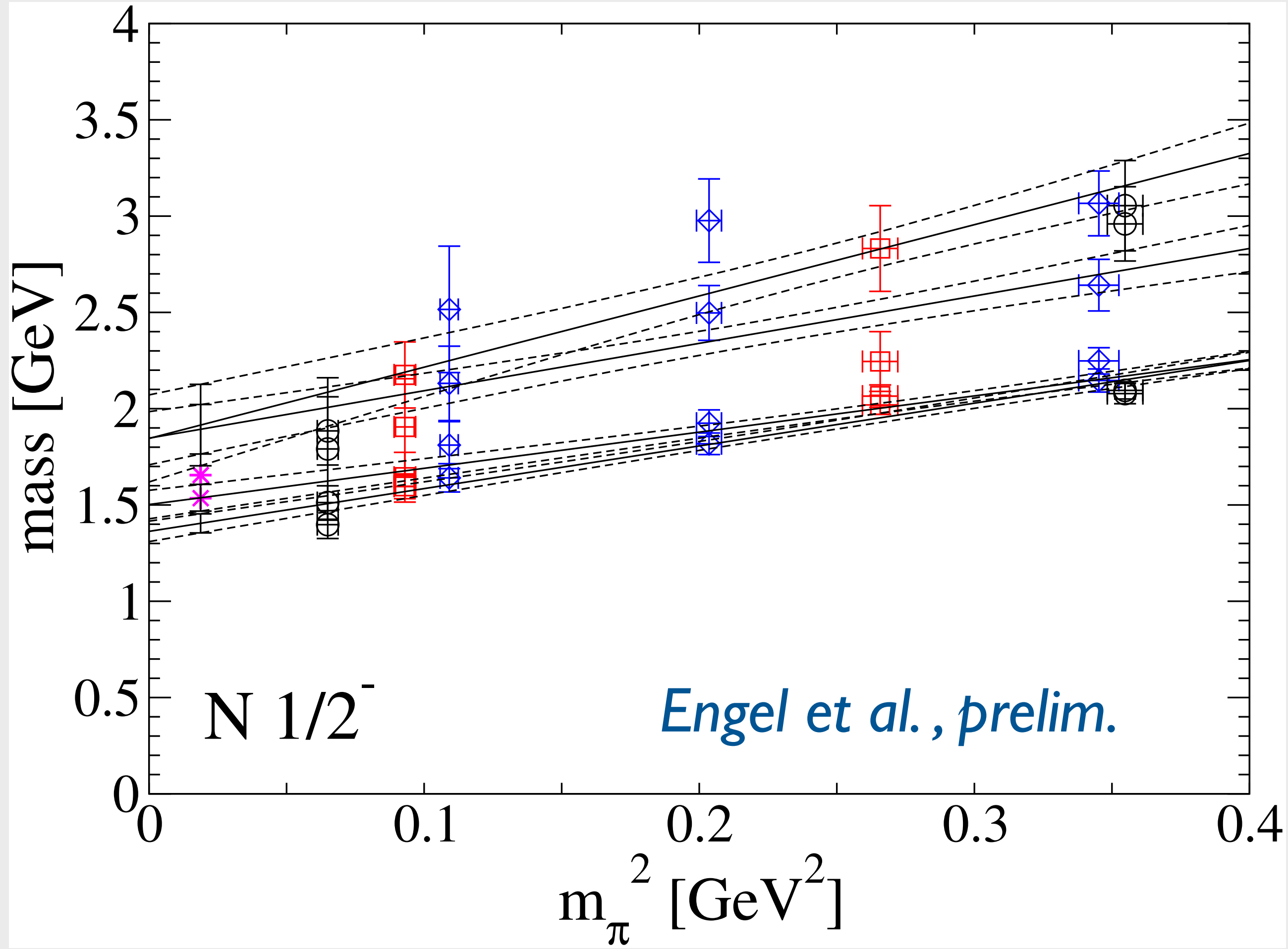
Roper?

*(cf. CSSM, Mahbub et al. , Phys.Lett. **B707**, 389-393 (2012); Hall et al., arXiv 1207.3562; cf. K.-F. Liu's talk last week)*



Engel et al. , prelim.

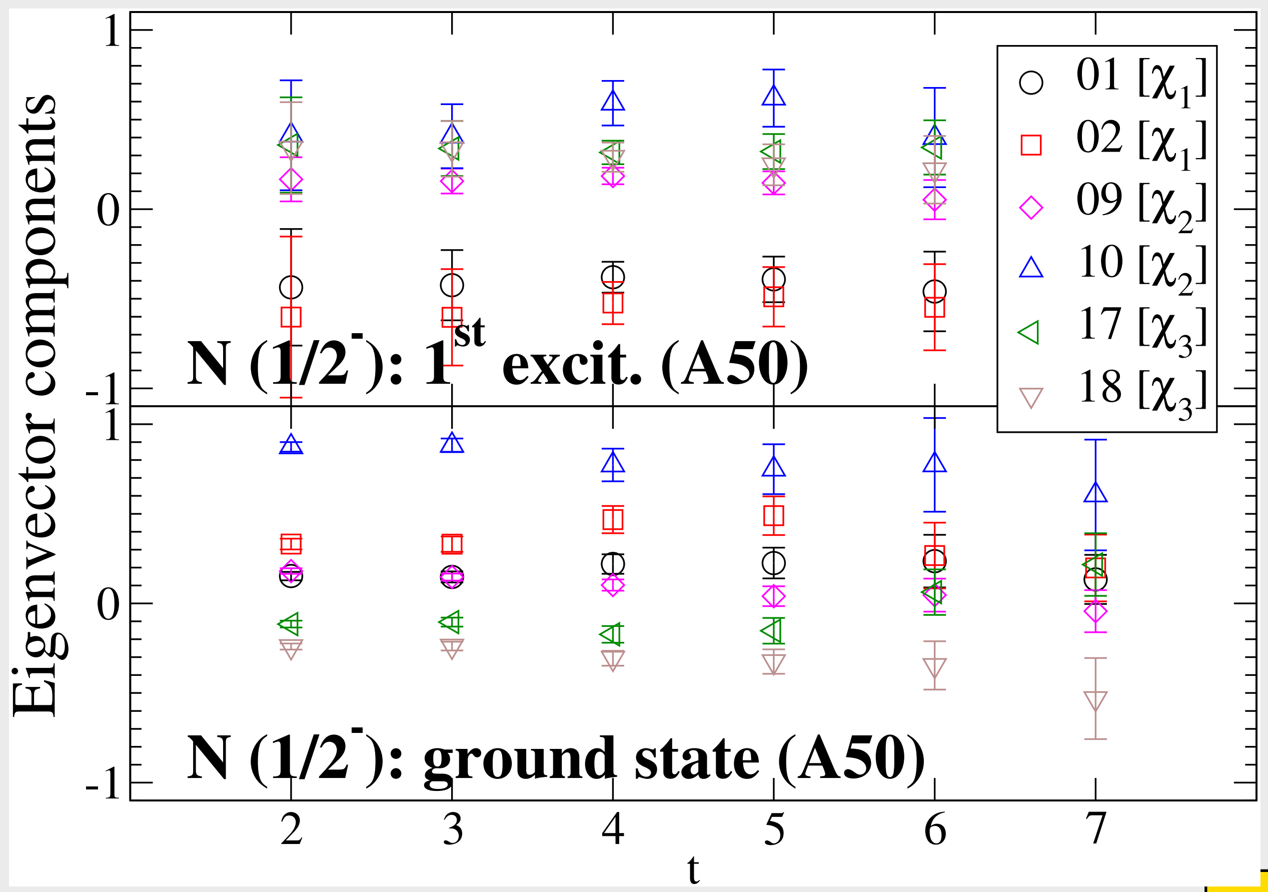
$1/2^-$: N(1535), N(1650)



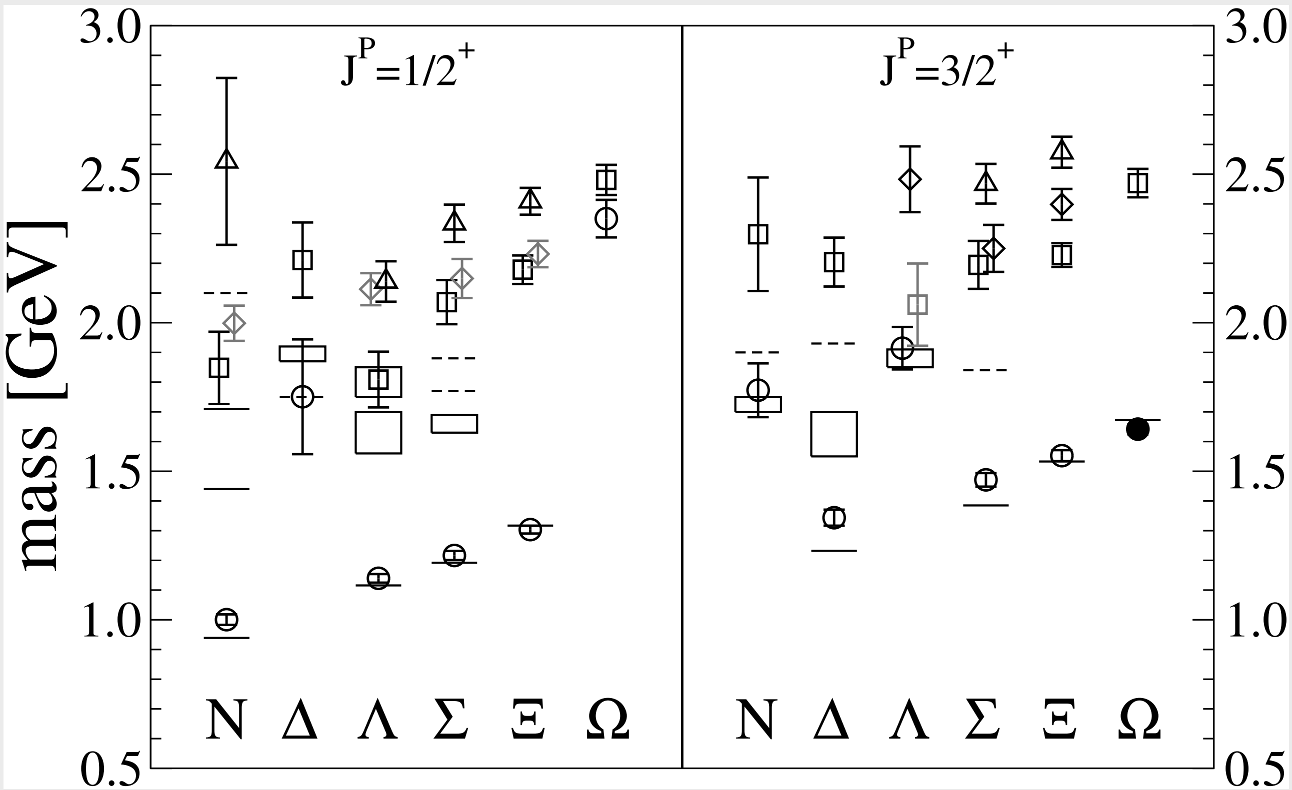
Two states seen, but not clearly resolvable; lower level dominated by χ_2

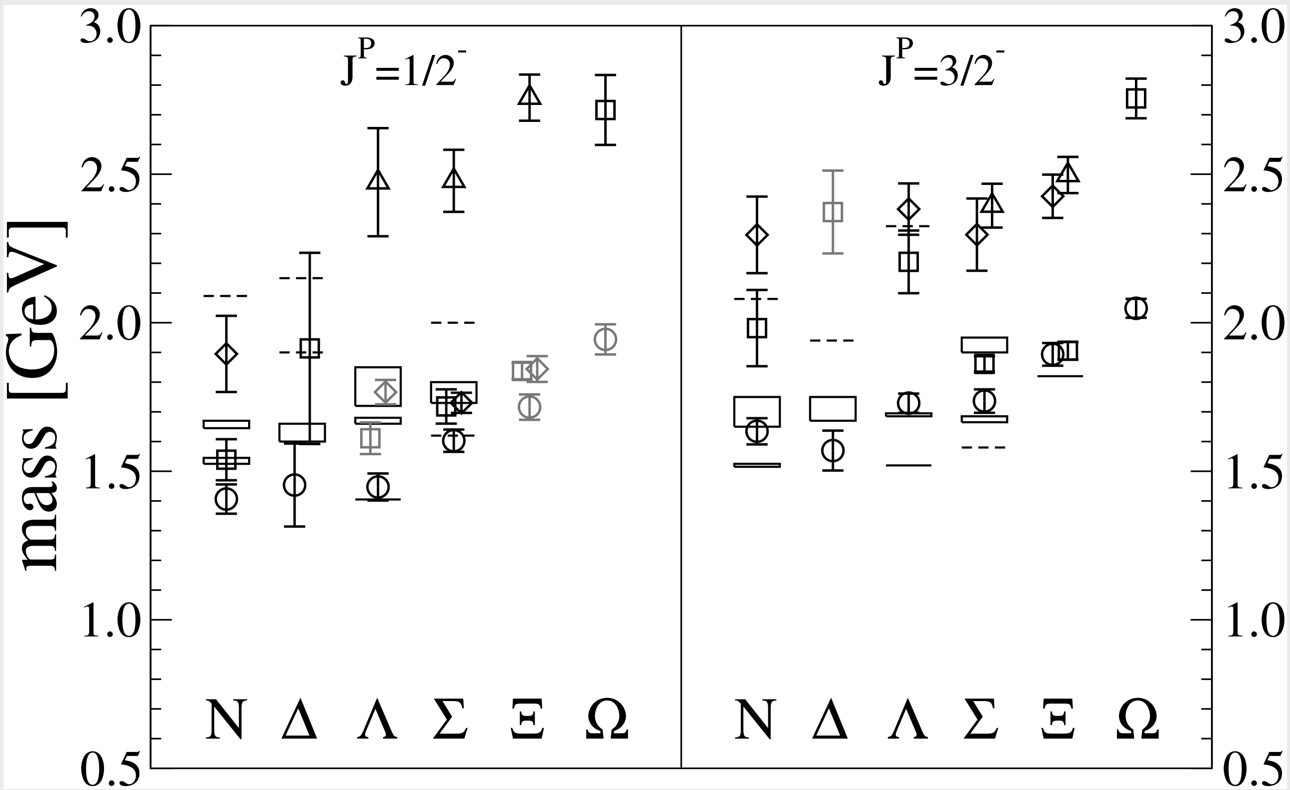
Is one level a πN in s-wave signal?

(pro/con: eigenvectors are stable for A,B,C: no level crossing, no change of splitting towards higher valence masses? But: g_A?)



Baryon summary (finite volume)

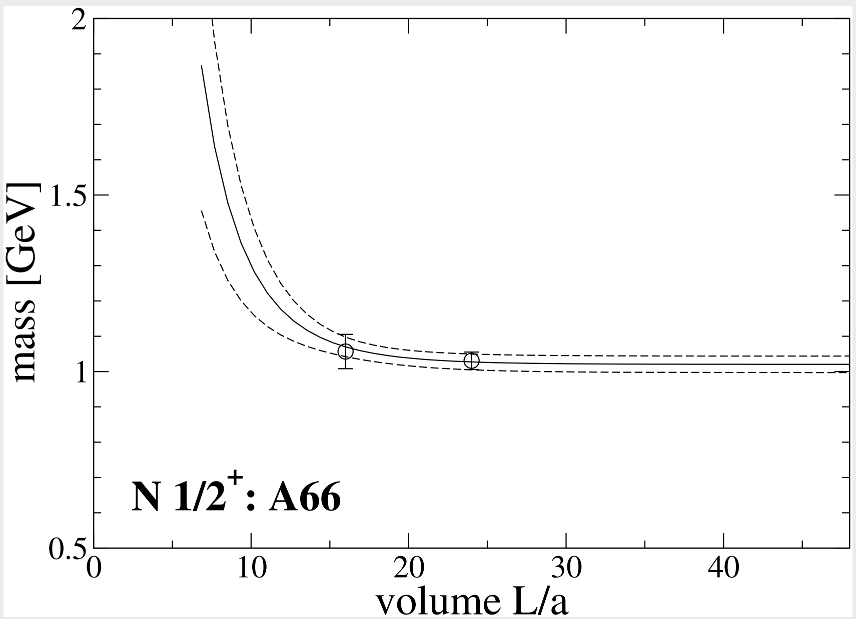


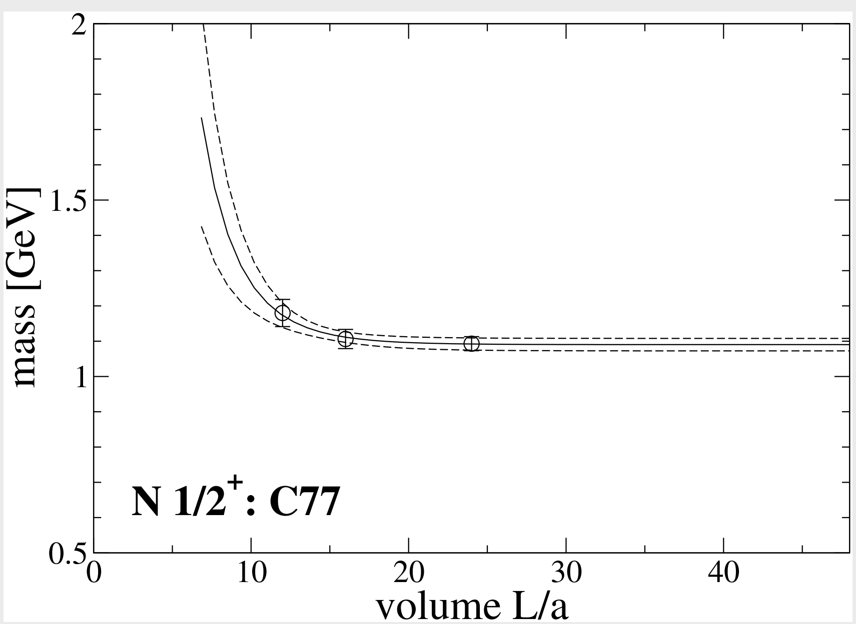


Engel et al., prelim.

Extrapolation to infinite volume

$12^3 \times 24 \rightarrow 16^3 \times 32 \rightarrow 24^3 \times 48$

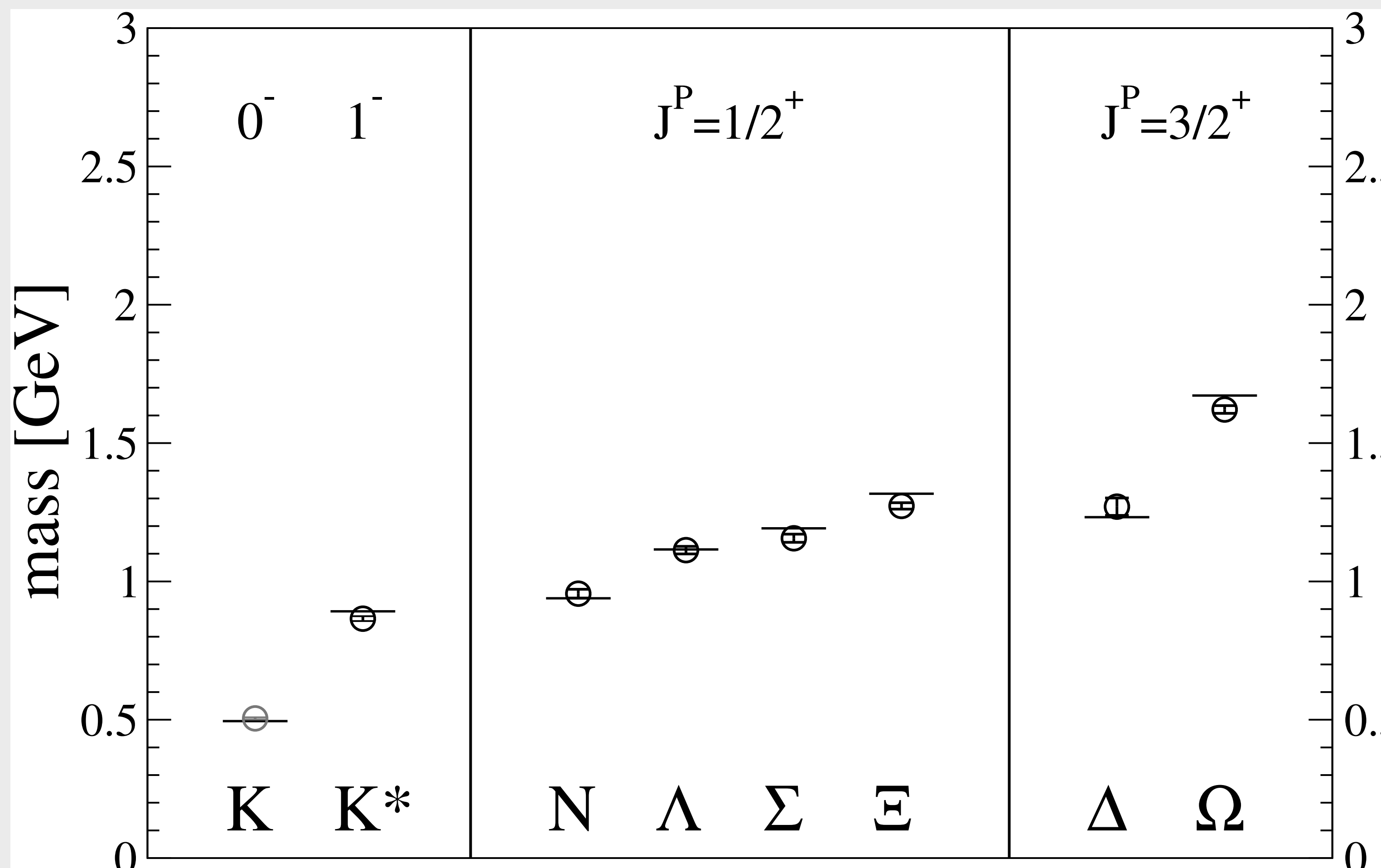




$$E(L) = E_\infty + c(m_\pi) \mathrm{e}^{-m_\pi L}$$

(cf. Dürr et al., Science 322(2008) 1224)

Only for ground states!



Engel et al., prelim.

Challenge

Why do we not see the meson-meson and meson-baryon intermediate states?

We need to include these in the set of hadron interpolators!

see also: Bulava et al. PRD82(10)014507

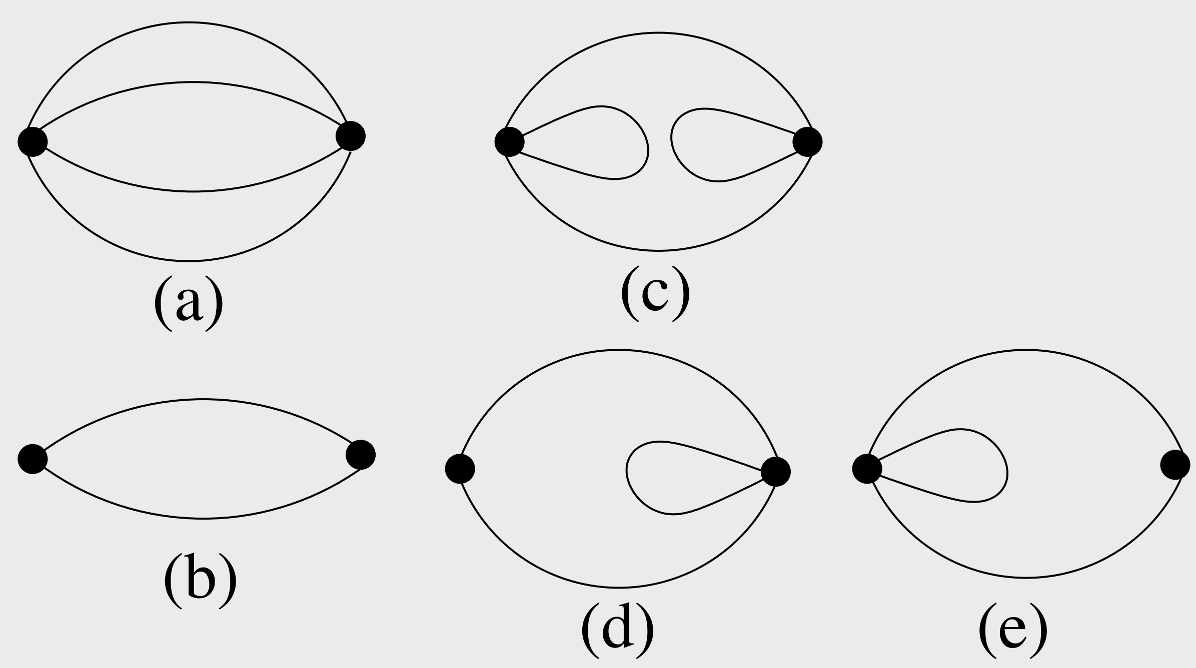
Challenge

Why do we not see the meson-meson and meson-baryon intermediate states?

We need to include these in the set of hadron interpolators!

see also: Bulava et al. PRD82(10)014507

But: These involve (partially) disconnected contractions!



Overview

1. Motivation and lattice tools

2. Case 1: Hadron excitations

3. Case 2: Meson decay

see also D. Mohler, LAT12

$\rho(770)$ [j] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.49 \pm 0.34$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

$\rho(770)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$\pi\pi$		$\sim$ 100 %		363
$\rho(770)^{\pm}$ decays				
$\pi^{\pm}\gamma$		$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2	375
$\pi^{\pm}\eta$		$< 6 \times 10^{-3}$	CL=84%	153
$\pi^{\pm}\pi^{+}\pi^{-}\pi^{0}$		$< 2.0 \times 10^{-3}$	CL=84%	254
$\rho(770)^{0}$ decays				
$\pi^{+}\pi^{-}\gamma$		$(9.9 \pm 1.6) \times 10^{-3}$		362
$\pi^{0}\gamma$		$(6.0 \pm 0.8) \times 10^{-4}$		376
$\eta\gamma$		$(3.00 \pm 0.20) \times 10^{-4}$		194
$\pi^{0}\pi^{0}\gamma$		$(4.5 \pm 0.8) \times 10^{-5}$		363
$\mu^{+}\mu^{-}$	[k]	$(4.55 \pm 0.28) \times 10^{-5}$		373
$e^{+}e^{-}$	[k]	$(4.72 \pm 0.05) \times 10^{-5}$		388

Rho decay

CBL, Mohler, Prelovsek, Vidmar
PR D84 (2011)054503 (1105.5636)
PoS LAT11, 137 (1111.0409)

- Study $\pi\pi \to \rho \to \pi\pi$ scattering (p wave)
- N_f=2, improved Wilson fermions
  (m_π=266 MeV);
  280 configurations from A. Hasenfratz et al.
  (Thanks! See *Hasenfratz et al., PRD78(08)014515,054511*)
- Up to 18 interpolators
- Non-zero momentum states
- Determine p-wave phase shift

also:
Aoki et al., PoS LAT10,108 +
LAT11(1106.5385+1111.0337)
Feng et al., PoS LAT10,104
Frison et al. PoS LAT10,139

Interpolators

$$\mathcal{O}_1^s(t) = \sum_{\mathbf{x},i} \frac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_2^s(t) = \sum_{\mathbf{x},i} \frac{1}{\sqrt{2}}\ \bar{u}_s(x)\ \gamma_t A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_3^s(t) = \sum_{\mathbf{x},i,j} \frac{1}{\sqrt{2}}\ \bar{u}_s(x)\overleftarrow{\nabla}_j\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_j u_s(x)\ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_4^s(t) = \sum_{\mathbf{x},i} \frac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}] u_s(x)\ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_5^s(t) = \sum_{\mathbf{x},i,j,k} \frac{1}{\sqrt{2}}\ \epsilon_{ijl}\ \bar{u}_s(x)\ A_i\gamma_j\gamma_5\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}] u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s=n,m,w)\ ,$$

$$\mathcal{O}_6^{s=n}(t) = \frac{1}{\sqrt{2}}[\pi^+(\mathbf{p_1})\pi^-(\mathbf{p_2}) - \pi^-(\mathbf{p_1})\pi^+(\mathbf{p_2})]\ , \qquad \pi^\pm(\mathbf{p_i}) = \sum_{\mathbf{x}} \bar{q}_n(x)\gamma_5\tau^\pm \mathrm{e}^{\mathrm{i}\mathbf{p_i}\mathbf{x}} q_n(x)\ .$$

Interpolators

$$\mathcal{O}_1^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{Px}}\ u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

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$$\mathcal{O}_3^s(t) = \sum_{\mathbf{x},i,j} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\overleftarrow{\nabla}_j\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{Px}}\ \overrightarrow{\nabla}_j u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_4^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{Px}}\ \overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i \mathrm{e}^{\mathrm{i}\mathbf{Px}}]u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_5^s(t) = \sum_{\mathbf{x},i,j,k} \tfrac{1}{\sqrt{2}}\ \epsilon_{ijl}\ \bar{u}_s(x)\ A_i\gamma_j\gamma_5\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{Px}}\overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l \mathrm{e}^{\mathrm{i}\mathbf{Px}}]u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s=n,m,w)\ ,$$

$$\mathcal{O}_6^{s=n}(t) = \tfrac{1}{\sqrt{2}}[\pi^+(\mathbf{p_1})\pi^-(\mathbf{p_2}) - \pi^-(\mathbf{p_1})\pi^+(\mathbf{p_2})]\ , \qquad \pi^\pm(\mathbf{p_i}) = \sum_{\mathbf{x}} \bar{q}_n(x)\gamma_5\tau^\pm \mathrm{e}^{\mathrm{i}\mathbf{p_i x}} q_n(x)\ .$$

... include ππ operator

Interpolators

$$\mathcal{O}_1^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s = n, m, w)\ ,$$

$$\mathcal{O}_2^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ \gamma_t A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s = n, m, w)\ ,$$

$$\mathcal{O}_3^s(t) = \sum_{\mathbf{x},i,j} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\overleftarrow{\nabla}_j\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_j u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s = n, m, w)\ ,$$

$$\mathcal{O}_4^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}]u_s(x)\ \ - \{u_s \leftrightarrow d_s\} \qquad (s = n, m, w)\ ,$$

$$\mathcal{O}_5^s(t) = \sum_{\mathbf{x},i,j,k} \tfrac{1}{\sqrt{2}}\ \epsilon_{ijl}\ \bar{u}_s(x)\ A_i\gamma_j\gamma_5\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}]u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s = n, m, w)\ ,$$

$$\mathcal{O}_6^{s=n}(t) = \tfrac{1}{\sqrt{2}}[\pi^+(\mathbf{p_1})\pi^-(\mathbf{p_2}) - \pi^-(\mathbf{p_1})\pi^+(\mathbf{p_2})]\ , \qquad \pi^\pm(\mathbf{p_i}) = \sum_{\mathbf{x}} \bar{q}_n(x)\gamma_5\tau^\pm \mathrm{e}^{\mathrm{i}\mathbf{p_i}\mathbf{x}} q_n(x)\ .$$

... include ππ operator

... and three quark widths (s, m, w)

Energy levels and phase shift

Only 2 (3?) levels can be determined reliably for given volume!

Use different momenta ("moving frame")!

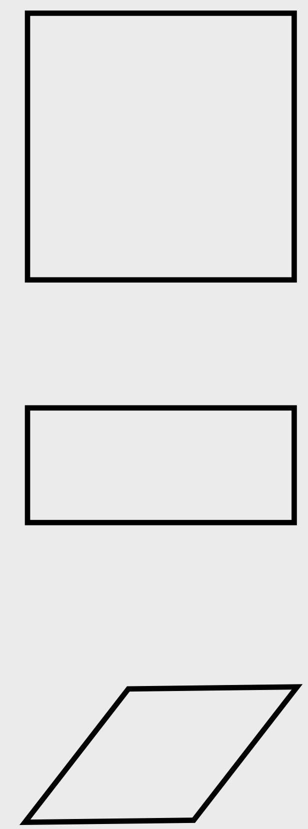
Rummukainen, Gottlieb: NP B 450(1995) 397
Kim, Sachrajda,Sharpe: NP B 727 (2005) 218
Feng, Jansen, Renner: PoS LAT10 (2010) 104
Fu, PR D85 (2012) 014506
Leskovec, Prelovsek, PR D85(2012)114507
Göckeler et al., arXiv:1206.4141

Rho momenta

$$\vec{p} = (0,0,0) \quad (\mathrm{units} 2\pi/L)$$

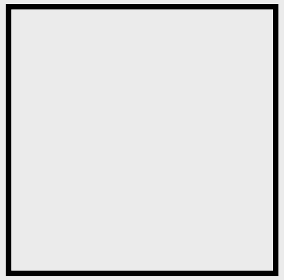
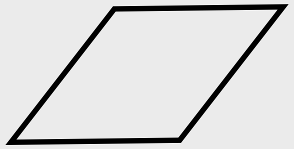
$$\vec{p} = (0,0,1)$$

$$\vec{p} = (1,1,0)$$



Relativistic distortion

Rho momenta

	Relativistic distortion	Symmetry group	Irrep for ρ
$\vec{p} = (0,0,0) \quad (\text{units} 2\pi/L)$		O_h	T_1^-
$\vec{p} = (0,0,1)$		D_{4d}	A_2^-
$\vec{p} = (1,1,0)$		D_{2d}	B_1^-

Energy levels give phase shift values

$$E,\, m_\pi \;\rightarrow\; E_{CM} \;\rightarrow\; q \;\rightarrow\; \delta(q)$$

(0,0,0) : $$\tan\delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

(0,0,1) : $$\tan\delta(q) = \frac{\gamma\pi^{3/2} q^3}{q^2\,\mathcal{Z}_{00}^{\vec{d}}(1;q^2) + \sqrt{\frac{4}{5}}\;\mathcal{Z}_{20}^{\vec{d}}(1;q^2)}$$

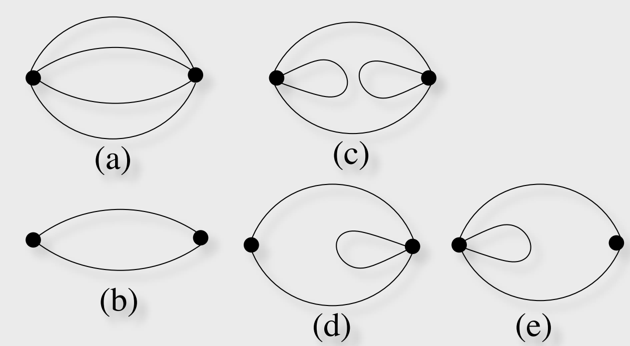
(1,1,0) : $$\tan\delta(q) = \frac{\gamma\pi^{3/2} q^3}{q^2\,\mathcal{Z}_{00}^{\vec{d}}(1;q^2) - \sqrt{\frac{1}{5}}\;\mathcal{Z}_{20}^{\vec{d}}(1;q^2) + \mathrm{i}\sqrt{\frac{3}{10}}\;\left(\mathcal{Z}_{22}^{\vec{d}}(1;q^2) - \mathcal{Z}_{2\bar{2}}^{\vec{d}}(1;q^2)\right)}$$

Recipe

$$E, m_\pi \rightarrow E_{CM} \rightarrow q \rightarrow \delta(q)$$

- Up to 6 pion interpolators, var. analysis → pion mass
- Up to 18 ρ interpolators,var. analysis →energy levels E

  - the distillation method allows to include

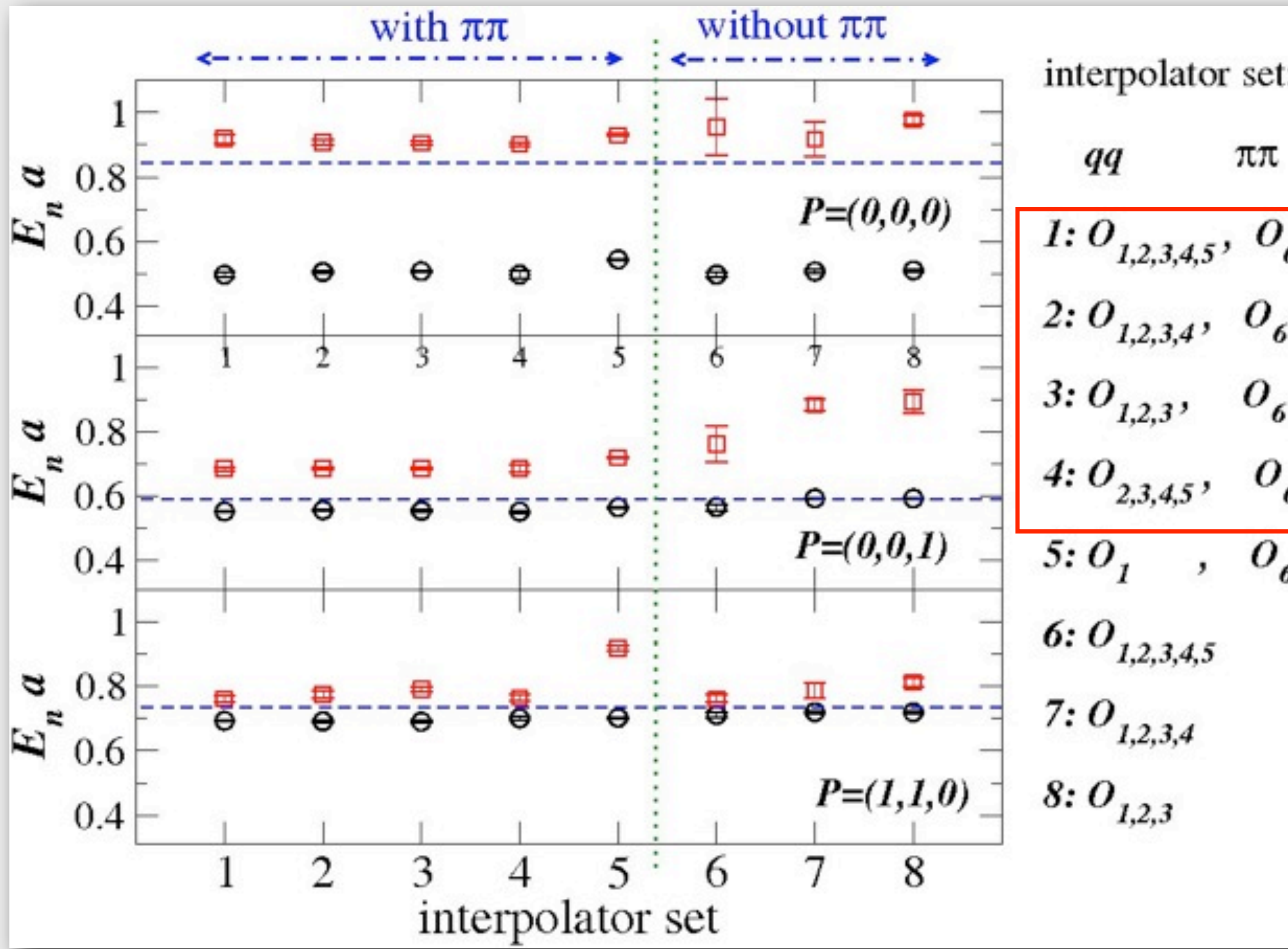


- Compute E_{CM} and q
- Compute from q the values of the phase shift
- Repeat for each momentum set → total of 6 energy values

Tests - how many do we need?

Lowest two levels (for selected submatrices)

t_0=4
fit range 7-10



Tests - how many do we need?

Lowest two levels
(for selected
submatrices)

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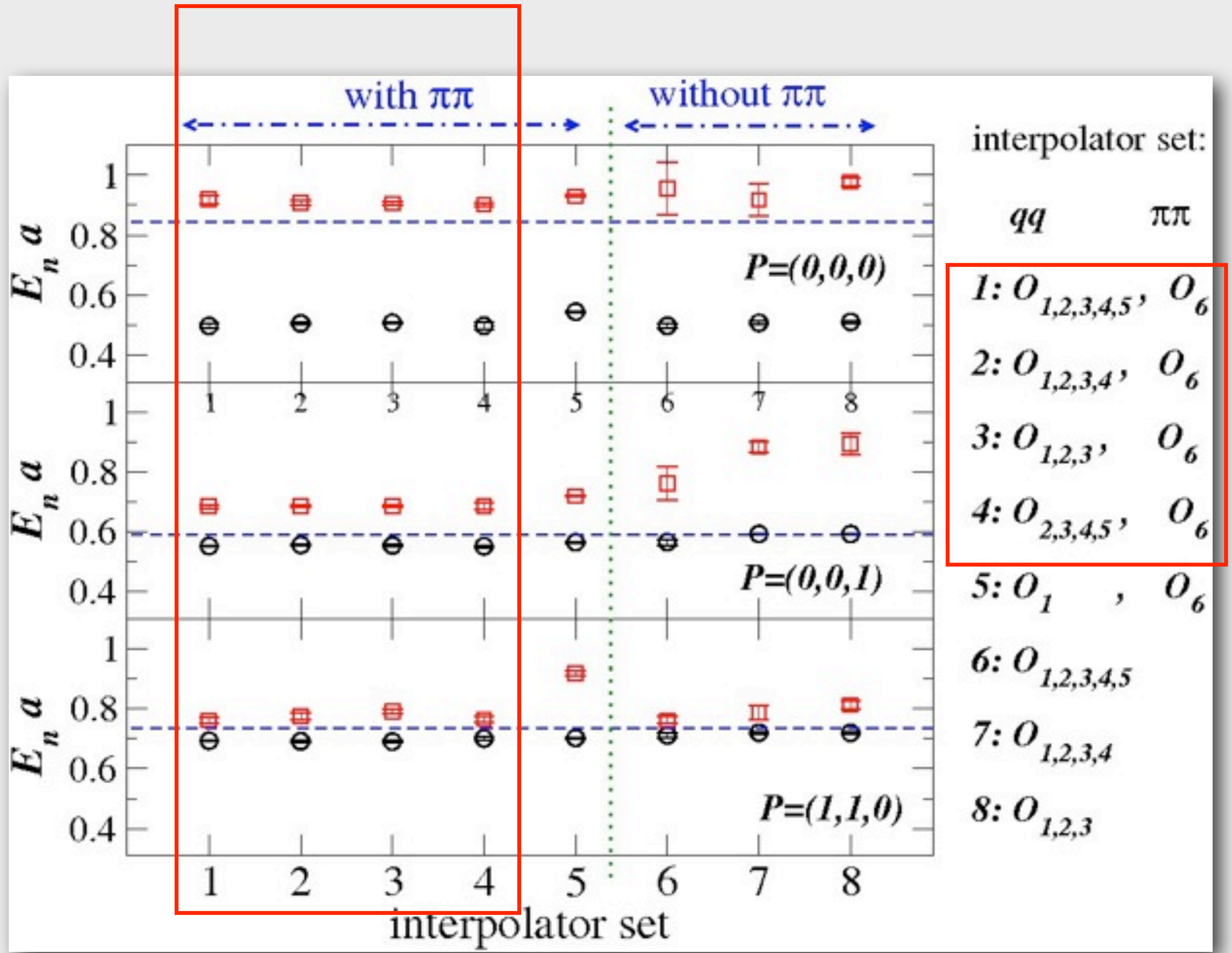
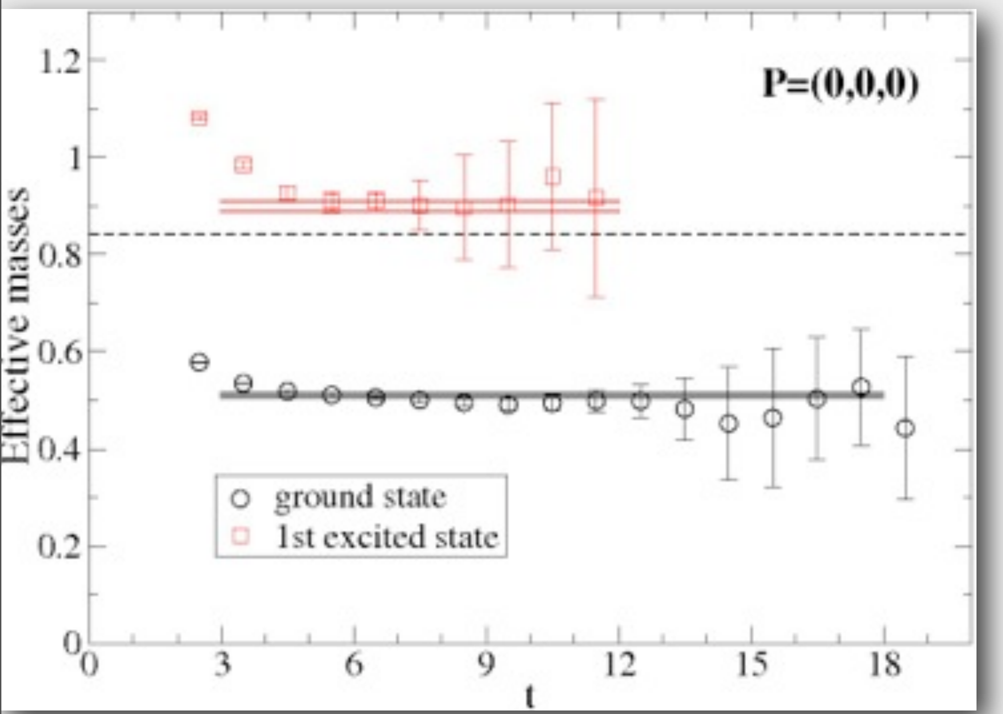
fit range 7-10



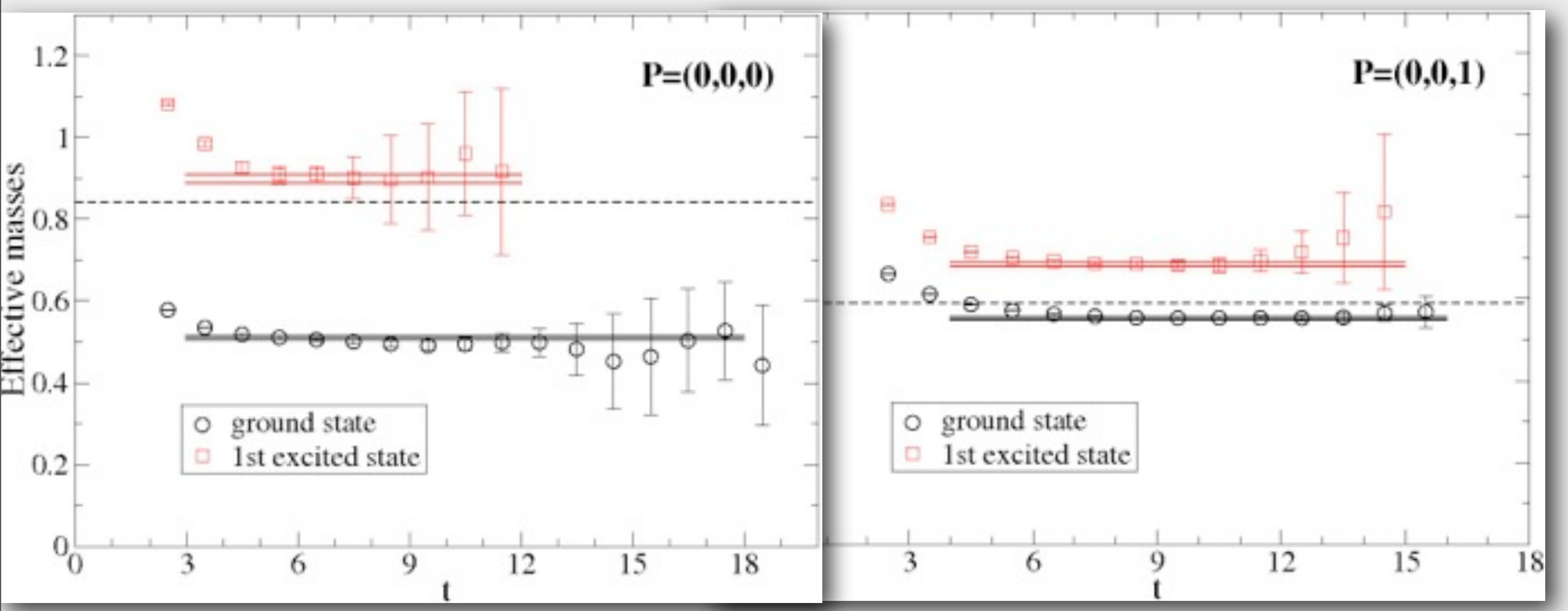
Lowest two energy levels



Bands: Fit range for $\lambda(t)$ - 2 exp fits

----- noninteracting $\pi\ \pi$ energy

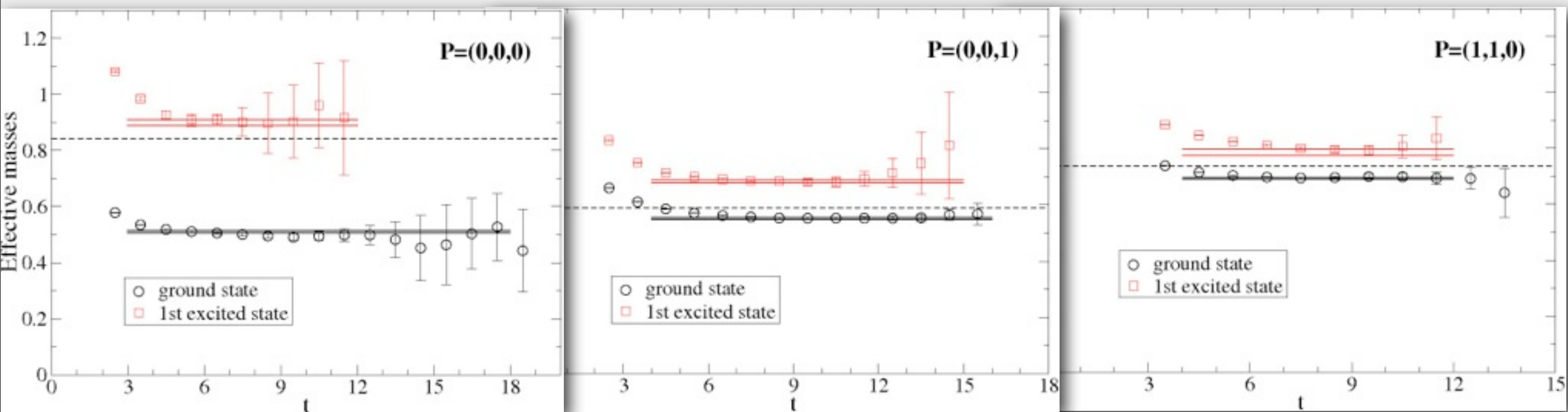
Lowest two energy levels



Bands: Fit range for λ(t) - 2 exp fits

----- noninteracting π π energy

Lowest two energy levels



Bands: Fit range for λ(t) - 2 exp fits

----- noninteracting π π energy

ππ→ππ scattering amplitude

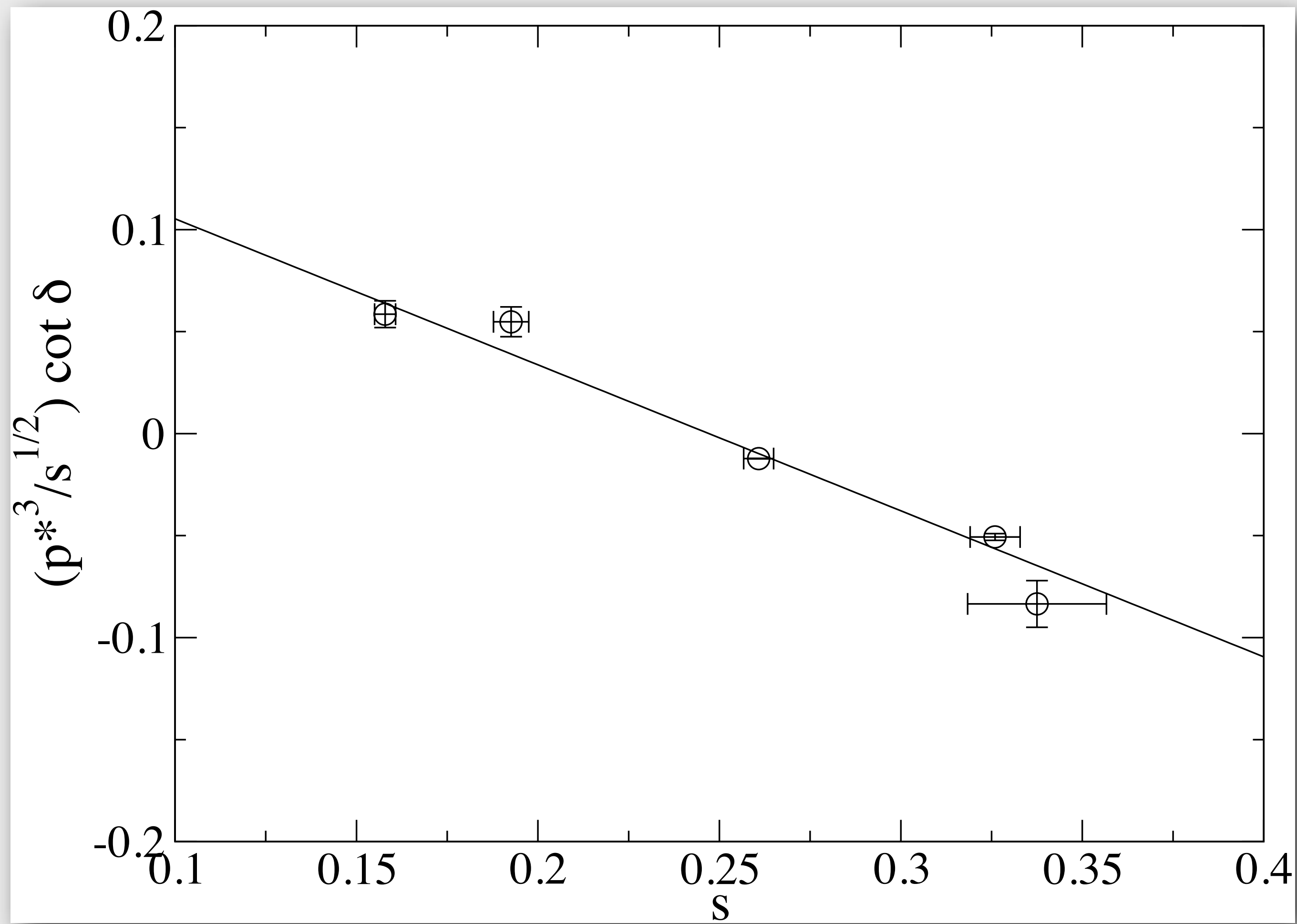
$$a_1 = \frac{-\sqrt{s}\,\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\,\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)} \sin\delta(s)$$

$$(s = E_{CM}^2)$$

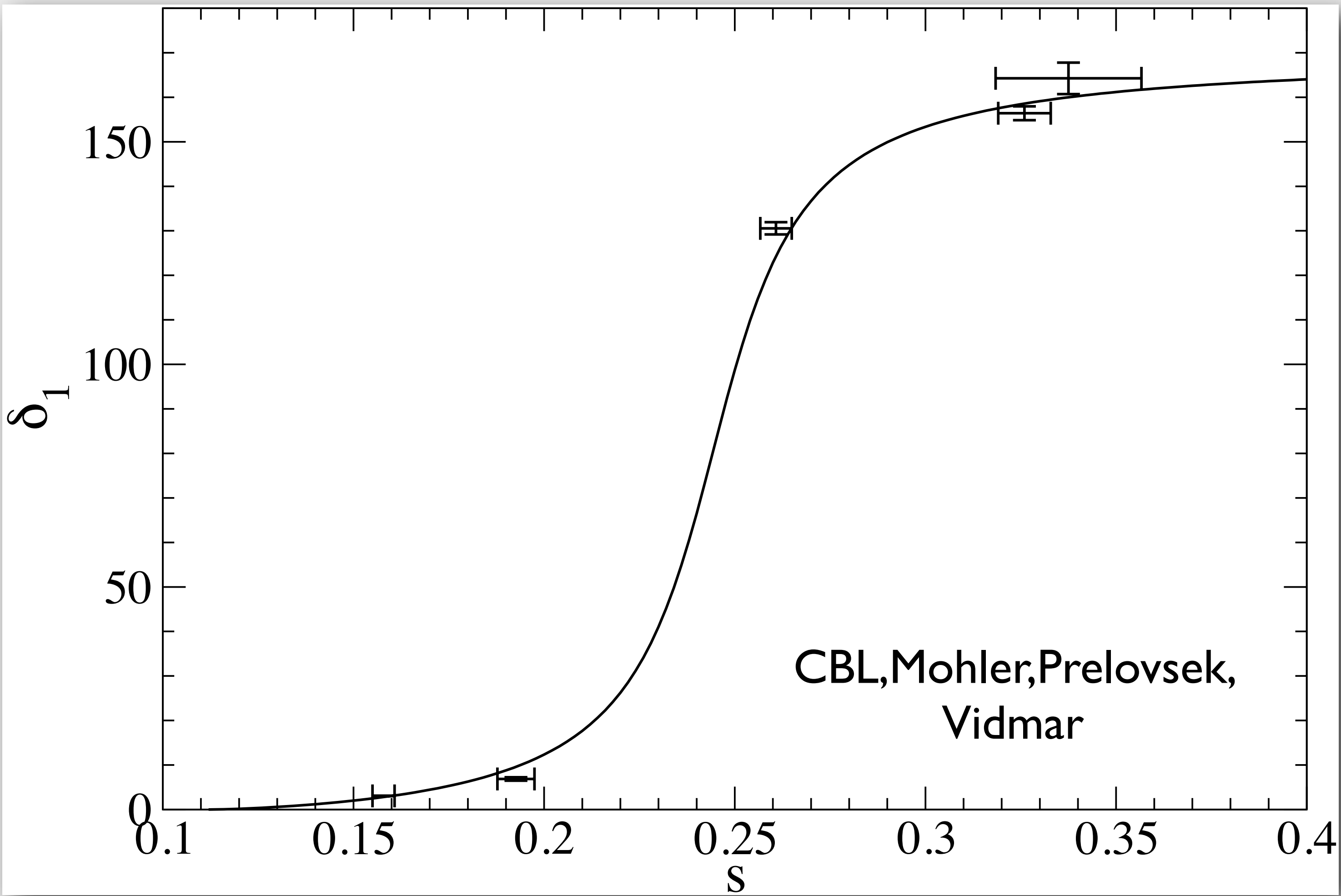
(in units of a=.1239 fm)

$$\sqrt{s}\,\Gamma(s)\ \cot\delta(s) = m_\rho^2 - s$$

with $\Gamma(s) = \frac{p^3}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}$



Phase shift



$$g_{\rho\pi\pi} = 5.13(20)$$
$$m_\pi = 266(3)(3)\ \text{MeV}$$
$$m_\rho = 792(7)(8)\ \text{MeV}$$

$$g_{\rho\pi\pi,exp} = 5.96$$

Feng et al. (ETMC)
PoS LAT10(10)104

$$g_{\rho\pi\pi} = 6.77(67)$$
$$m_\pi = 290\ \text{MeV}$$
$$m_\rho = 980\ \text{MeV}$$

Frison et al. (BMW)
PoS LAT10(10)139

$$g_{\rho\pi\pi} = 5.5(2.9)/6.6(3.4)$$
$$m_\pi = 200/340\ \text{MeV}$$

Aoki et al. (PACS-CS)
PoS LAT10(10)108

$$g_{\rho\pi\pi} = 5.24(51)$$
$$m_\pi = 410\ \text{MeV}$$
$$m_\rho = 891\ \text{MeV}$$

Aoki et al. (PACS-CS)
PRD84(11)094505

$$g_{\rho\pi\pi} = 5.52(40)/5.98(56)$$
$$m_\pi = 410/300\ \text{MeV}$$
$$m_\rho = 893/863\ \text{MeV}$$

Further results

πρ - a_1 ($J^{PC}=1^{++}$)

Prelovsek et al.,
PoS LAT2011, 137
arXiv:1111.0409

πK - s-wave and p-wave

L., Leskovec, Mohler, Prelovsek
arXiv:1207.3204

Unequal mass mesons in moving frames

Leskovec, Prelovsek
PRD85(2012)114507
arXiv:1202.2145

→ talk by Sasa Prelovsek on friday!

Summary

One needs to bring together several sophisticated tools:

- Dynamical fermions
- Many hadron interpolators
- Variational analysis
- Momentum states
- Methods for disconnected graphs
- Phase shift methods

First results are being obtained:

- Excited hadrons, lowest levels
- Meson decay

There is a lot to do:

- Volume study
- Further hadronic channels (like scalar mesons or meson-baryon states)
- Method improvement (more levels)
- Extension to inelastic region (e.g. Rusetsky et al.(09), Bernard et al.(10))

Hadron excitations and decays

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