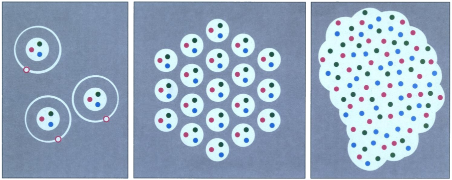
Conformality in many-flavour lattice QCD at strong coupling

arXiv:1208.2148

Philippe de Forcrand (ETH & CERN)
with Seyong Kim (Sejong Univ.) and Wolfgang Unger (ETH)

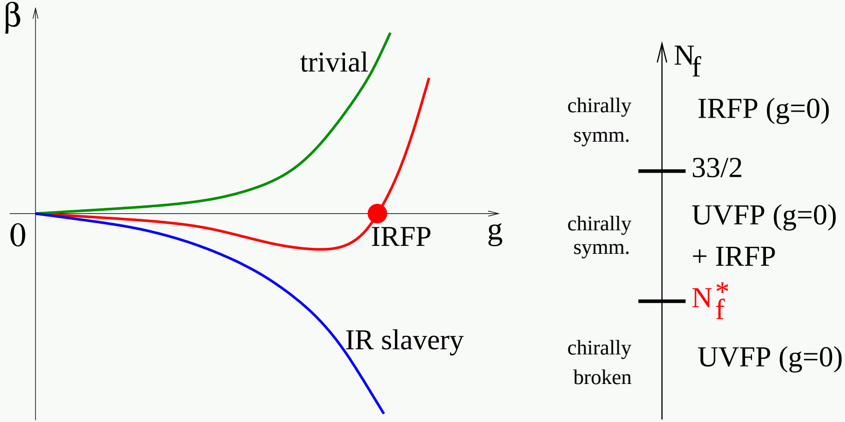
INT Seattle
Aug. 15, 2012



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Question: What happens to QCD when N_f increases?

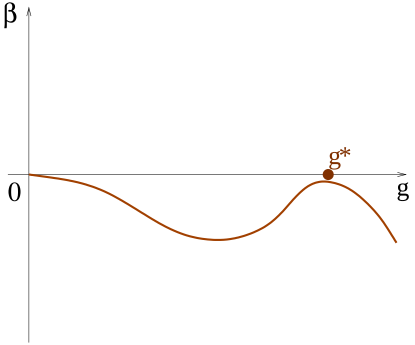
Classification of QCD-like $SU(3)$ theories with N_f fundamental quarks

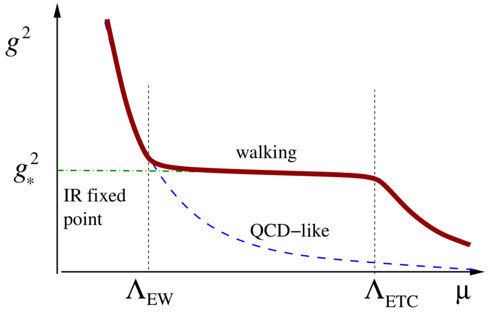


"Conformal window": $N_f \in [N_f^*, \frac{33}{2}[$ $\rightarrow$ non-trivial IRFP

- Upper edge $N_f \lesssim \frac{33}{2}$ $\rightarrow$ IRFP $g^* \ll 1 \rightarrow$ pert. th. (Banks & Zaks)
- **Lower edge?**

"Walking": $N_f = N_f^* - \varepsilon$, just below the conformal window





- $\varepsilon \to 0$, ie. $N_f \to N_f^*$: double zero of β-fct $\to$ Miransky scaling etc..
- scalar Techni-meson: possibility for light composite Higgs!
- pheno OK (FCNCs): walking $\to$ push up the scale of new (ETC) physics

Determining N_f^* on the lattice

Must distinguish between walking and conformal $\rightarrow$ triple difficulty:

- probe extreme infrared
- take continuum limit
- keep quarks massless

Determining N_f^* on the lattice

Must distinguish between walking and conformal $\rightarrow$ triple difficulty:

- probe extreme infrared
- take continuum limit
- keep quarks massless

Models studied

Red: conformal Blue: χSB Black: unclear

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: Appelquist et al; Deuzeman et al; Fodor et al; Jin et al
  - $N_f = 9$: Fodor et al
  - $N_f = 10$: Hayakawa et al; Appelquist et al
  - $N_f = 12$: Hasenfratz; Appelquist et al; Deuzeman et al; Xin and Mawhinney; Fodor et al
  - $N_f = 16$: Damgaard et al; Heller; Hasenfratz; Fodor et al
- SU(2) + fundamental rep fermions:
  - $N_f = 4$: Karavirta et al
  - $N_f = 6$: Del Debbio et al; Karavirta et al; Appelquist et al (unclear)
  - $N_f = 8$: Iwasaki et al
  - $N_f = 10$: Karavirta et al
- SU(2) + $N_f = 2$ adjoint rep: Catterall et al; Bursa et al; Hietanen et al; De Grand et al
- SU(3) + $N_f = 2$ 2-index symmetric rep: DeGrand et al; Sinclair and Kogut; Fodor et al
- SU(4) + $N_f = 2$ 2-index symmetric rep: DeGrand et al

Determining N_f^* on the lattice

Must distinguish between walking and conformal → triple difficulty:

- probe extreme infrared
- take continuum limit
- keep quarks massless

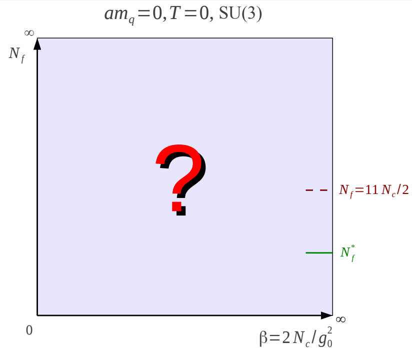
- "time-invariance" violated :-)
  DeGrand, Shamir & Svetitsky, sextet QCD: IRFP → no IRFP → IRFP
  0803.1707 0812.1427 1110.6845
- need to probe infrared → **extremely coarse lattices** → discretization errors ?
  small discretization error can mask physical running behaviour → improved actions ?

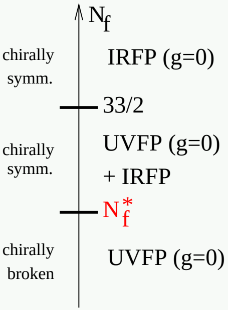
Determining N_f^* on the lattice

Must distinguish between walking and conformal $\rightarrow$ triple difficulty:

- probe extreme infrared
- take continuum limit
- keep quarks massless

Here: **no continuum limit**





Strong coupling limit: $\beta = 0$

Mean Field: chiral symmetry is **always broken** in the strong-coupling limit of staggered fermions at $T = 0$ **for all values of N_f and N_c**

- chiral condensate well known to be independent of N_f and N_c, i.e. in d spatial dimensions: [Kluberg-Stern *et al.*, 1983] $$\langle\bar{\psi}\psi\rangle(T=0) = \frac{\left((1+d^2)^{1/2}-1)/2\right)^{1/2}}{d}$$
- we also found, following [Damgaard *et al.*, 1985]: chiral restoration temperature is $T_c = \frac{d}{4} + \frac{d}{8}\frac{N_c}{N_f} + \mathcal{O}(\frac{1}{N_f^2})$
- mean field expected to work well for large number of d.o.f. per site, e.g. exact results in the Gross-Neveu model for $N_f \to \infty$

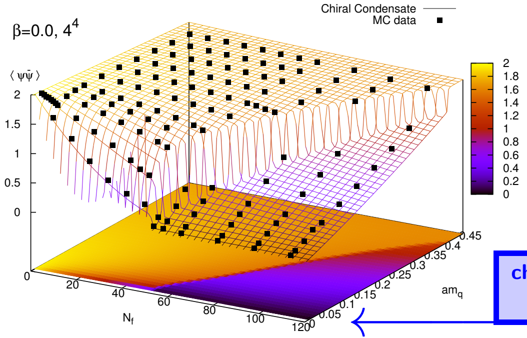
Conventional wisdom: [Poul Damgaard *et al.*, hep-lat/9701008]:
"we see no reasons or numerical indications whatsoever for sensitivity to N_f on the extreme strong-coupling side"

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_\mathrm{f}/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

$$S_{\mathrm{eff}} = -N_f \mathrm{Tr} \log(m_q - \not{D}) = N_f \sum_k \frac{1}{k m_q^k} \mathrm{Tr} \not{D}^k \; + \mathrm{const.}$$

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven bulk transition for strong-coupling limit of staggered fermions found

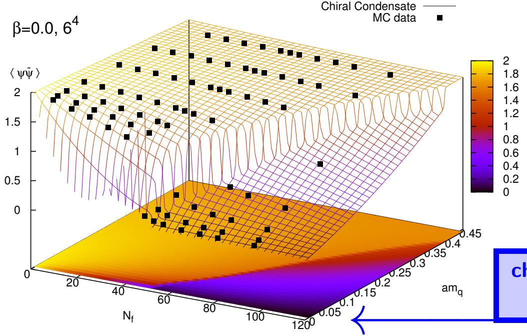


chiral symmetry restored!

- $N_f^c \simeq 52$ *continuum* flavors for $m_q = 0$, N_f^c increases with m_q
  (heavy fermions $\rightarrow$ less ordering)

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven **bulk transition** for strong-coupling limit of staggered fermions found

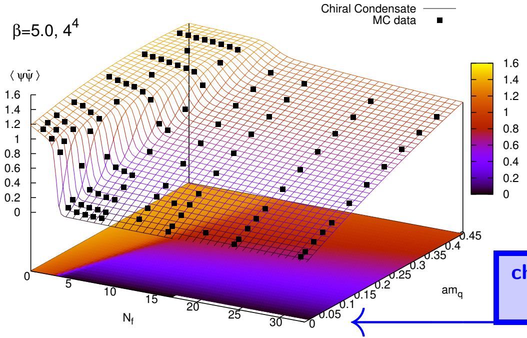


chiral symmetry restored!

- $N_f^c \simeq 52$ *continuum* flavors for $m_q = 0$, N_f^c increases with m_q
(heavy fermions $\rightarrow$ less ordering), almost no finite size effects

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven bulk transition for strong-coupling limit of staggered fermions found

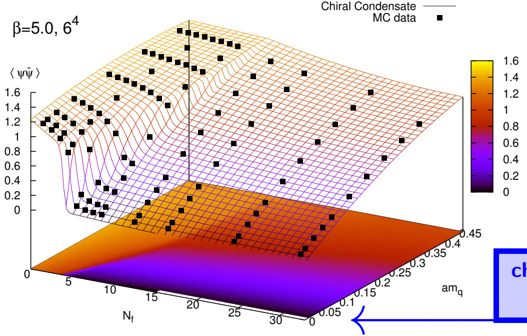


chiral symmetry restored!

- $\beta = 5$ similar but N_f^c smaller

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven bulk transition for strong-coupling limit of staggered fermions found

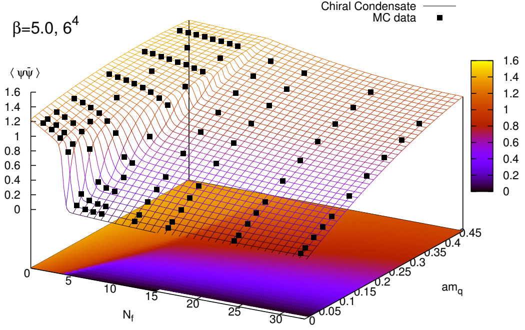


chiral symmetry restored!

- $\beta = 5$ similar but N_f^c smaller, stronger finite size effects

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

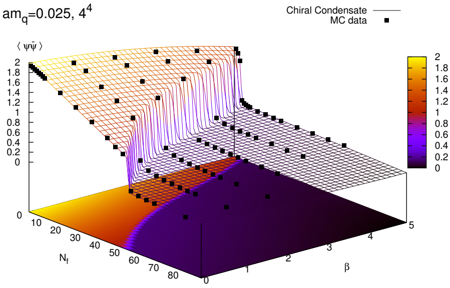
- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven bulk transition for strong-coupling limit of staggered fermions found

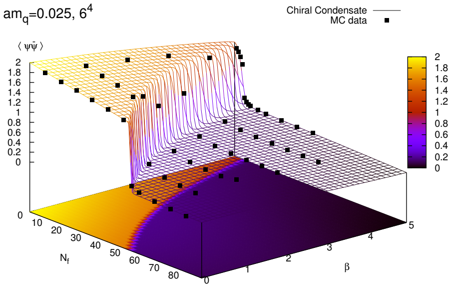


- Explanation for **failure of mean field:** terms of $\mathcal{O}(\frac{N_f}{N_c}, \frac{N_f}{d^2})$ are neglected (hopping of two mesons, baryon loops)

The Chirally Restored Phase for large β

- smooth variation with $\beta \rightarrow N_f$-driven transition extends to weak coupling
- $N_f^c \simeq \mathcal{O}(10)$ at weaker coupling
- connection with N_f-driven transition to **conformal window?**





Characterizing the chirally restored phase

Chirally symmetric yet “confining” ($\beta = 0$)

Conformal or not ? If conformal, trivial (IRFP $g^* = 0$) or not?

Characterizing the chirally restored phase

Chirally symmetric yet "confining" ($\beta = 0$)

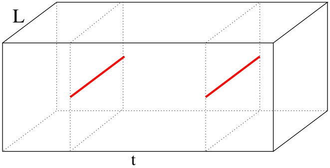
Conformal or not ? If conformal, trivial (IRFP $g^* = 0$) or not?

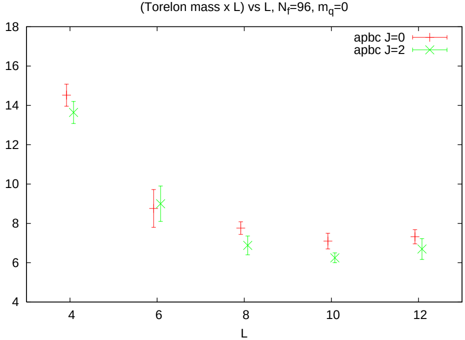
- Numerical simulations: $N_f = 56$ & 96, $\beta = 0$, $m_q = 0$, max. $12^3 \times 24$
- $\langle \mathrm{Plaq} \rangle \approx 0.35$ & 0.52, similar to weak-coupling; $\beta = 0$ not special
- Observables: - torelon mass (gluon flux tube)
  - Dirac eigenvalue spectrum
  - meson masses

Characterizing the chirally restored phase: I. Torelon masses

Energy of spatially-wrapping loop
$E(L) \propto \sigma L$ in confining theory
Here:

- $E(L)$ **decreases as L increases**
- $E(L) \sim 1/L$

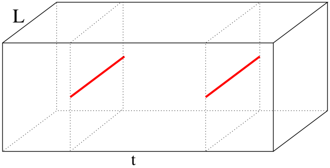


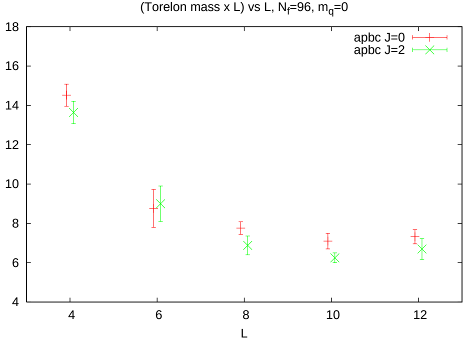


Characterizing the chirally restored phase: I. Torelon masses

Energy of spatially-wrapping loop
$E(L) \propto \sigma L$ in confining theory
Here:

- $E(L)$ **decreases as L increases**
- $E(L) \sim 1/L$



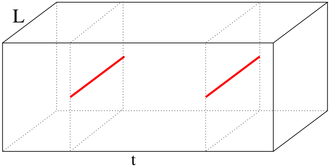


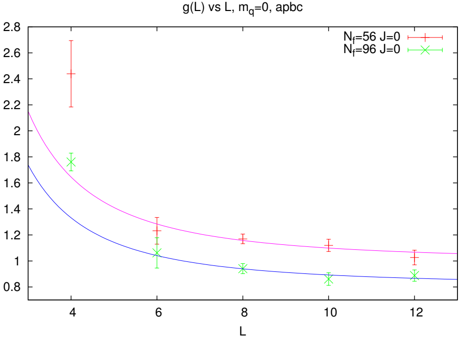
Explanation: no string tension, no flux tube
Torelon is simply glueball with mass $\sim 1/L$ $\rightarrow$ IR conformal

Characterizing the chirally restored phase: I. Torelon masses

Energy of spatially-wrapping loop
$E(L) \propto \sigma L$ in confining theory
Here:

- $E(L)$ **decreases as L increases**
- $E(L) \sim 1/L$





Explanation: no string tension, no flux tube
Torelon is simply glueball with mass $\sim 1/L$ $\rightarrow$ IR conformal

Torelon mass is Debye mass m_D after relabeling axes

Remember $m_D = 2gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$ $\rightarrow$ Define running coupling $g(L) = m_D(L)L/2\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$

In that scheme, $g(L) = \mathrm{const.} + \mathcal{O}(1/L^2) \Longrightarrow$ **non-trivial IRFP !** $(g^* \searrow$ as $N_f \nearrow)$

Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$
- measures the fraction of eigenvalues smaller than λ
- derivative gives $\rho(\lambda)$

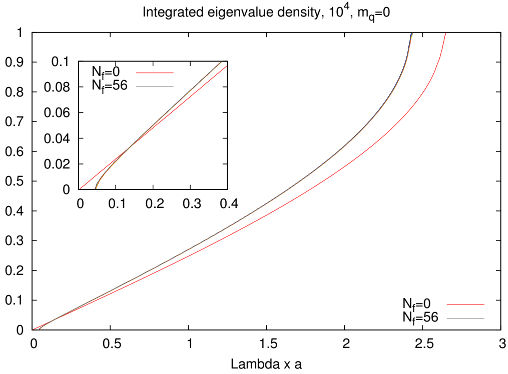
Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$

Compare $N_f = 0$ (quenched configurations) and $N_f = 56$ (chirally symmetric phase)

- similar for large eigenvalues (UV)
- the $N_f = 56$ curve shows a **gap** for small eigenvalues (IR), consistent with chiral symmetry restoration: $\rho(0) = 0$



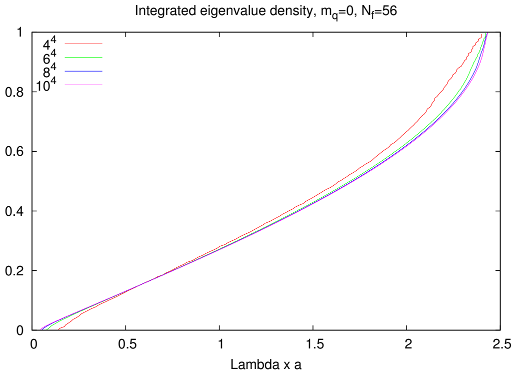
Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$

Compare different volumes for $N_f = 56$:

- large eigenvalues (UV) are L-independent,
- the IR spectral gap shrinks as L increases



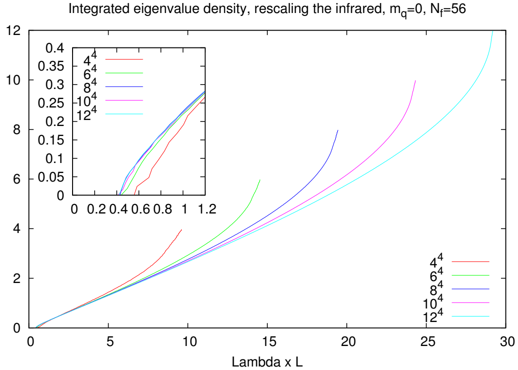
Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar\lambda) d\bar\lambda = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$

Compare different volumes for $N_f = 56$:

- IR spectrum invariant after rescaling by L: spectral gap $\propto 1/L$
- IR physics only depends on L, while the UV physics depends on a
- no other scale in the system $\Rightarrow$ Dirac spectrum consistent with **IR-conformal theory!**



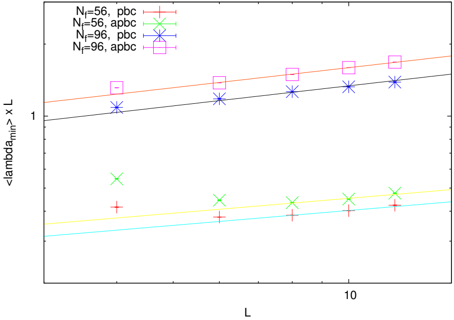
Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$

Compare different volumes for $N_f = 56$:

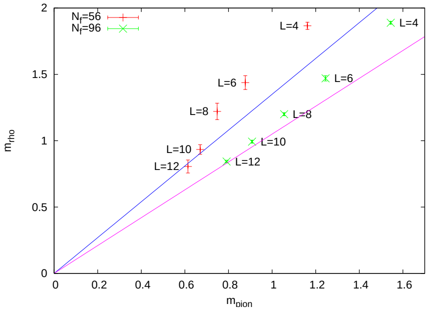
- IR spectrum invariant after rescaling by L: spectral gap $\propto 1/L$
- IR physics only depends on L, while the UV physics depends on a
- no other scale in the system $\Rightarrow$ Dirac spectrum consistent with **IR-conformal theory!**
- Tiny deviations from $1/L$ scaling $\rightarrow$ anomalous mass dimension γ^* ($\sim$ 0.26 and 0.38)



Characterizing the chirally restored phase: III. hadron masses

Hadron spectrum obtained from simulations with $N_f = 56$ and $N_f = 96$ at **zero quark mass**

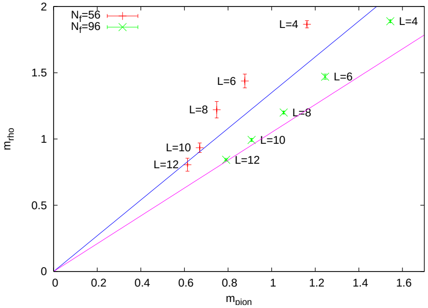
- hadron masses measured for $m_q = 0$ are non-zero
- but masses decrease (a lot) as the lattice size L is increased
- parity partners degenerate (c.f. chiral symmetry restoration)
- mass ratios $\sim$ **independent of L ?**:

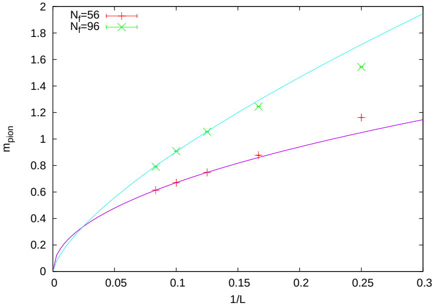


Characterizing the chirally restored phase: III. hadron masses

Hadron spectrum obtained from simulations with $N_f = 56$ and $N_f = 96$ at **zero quark mass**

- hadron masses measured for $m_q = 0$ are non-zero
- but masses decrease (a lot) as the lattice size L is increased
- parity partners degenerate (c.f. chiral symmetry restoration)
- mass ratios $\sim$ **independent of L ?**: $M_H \propto (1/L)^{\frac{1}{1+\gamma^*}}$ $(\gamma^* \sim 1.0\,\&\,0.4)$





Conjecture: $\beta = 0$ IR-conformal phase is analytically connected with the weak-coupling, continuum IR-conformal phase

Study of **continuum limit** is much more difficult:

- for a given lattice size L^4, the scales are ordered as $a \ll 1/\Lambda \ll L$
- at strong-coupling the hierarchy is $a \simeq 1/\Lambda \ll L$
- range of conformal invariance ($L\Lambda$) maximized at $\beta = 0$ for given lattice size L/a

weak coupling: strong coupling:

Conjecture: $\beta = 0$ IR-conformal phase is analytically connected with the weak-coupling, continuum IR-conformal phase

Study of **continuum limit** is much more difficult:

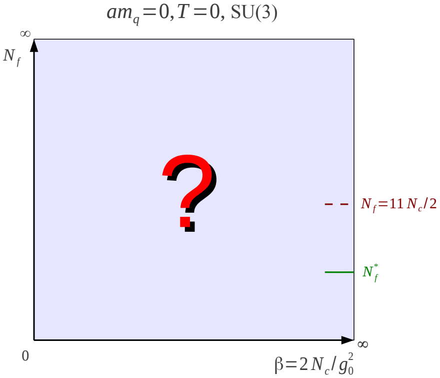
- for a given lattice size L^4, the scales are ordered as $a \ll 1/\Lambda \ll L$
- at strong-coupling the hierarchy is $a \simeq 1/\Lambda \ll L$
- range of conformal invariance ($L\Lambda$) maximized at $\beta = 0$ for given lattice size L/a

weak coupling:

strong coupling:

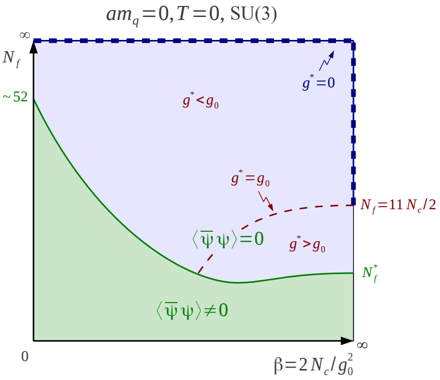
strong-coupling limit is the laboratory of choice to study a 4d IR-conformal gauge theory

Conjectured phase diagram



Conjectured phase diagram

- IF $\beta = 0$ chirally symmetric phase is non-trivial

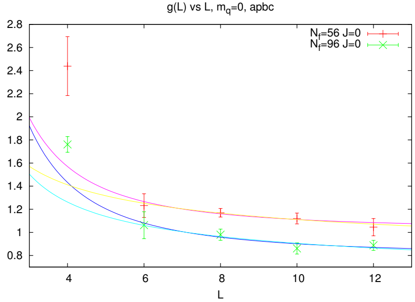


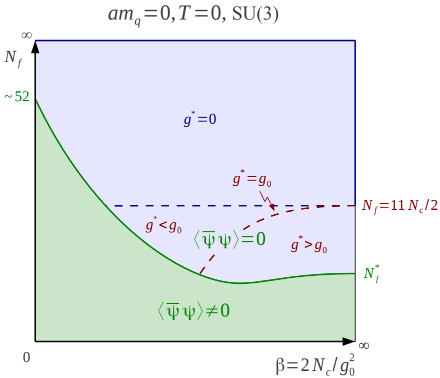
Dashed line $g^* = g_0$ is NOT a phase transition (scheme-dependent)

Conjectured phase diagram

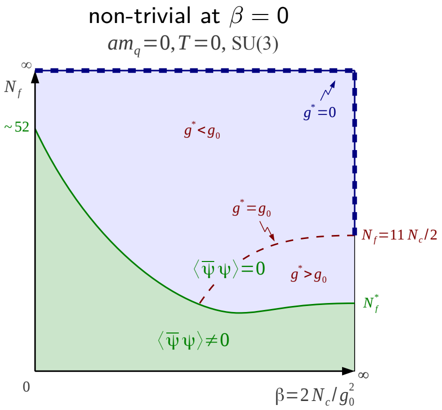
- Or IF $\beta = 0$ chirally symmetric phase is trivial

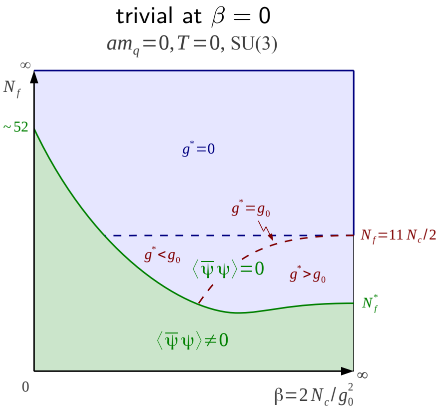
$$g(L) \sim 1/\log(L)$$





Conjectured phase diagram





Either way, single phase transition (chiral symmetry):
if all first-order → "jumping" dynamics (Sannino) no walking!

Conclusions

Shown: for $\beta = 0$, a strong first order **bulk transition** exists which is N_f**-driven** to a chirally symmetric phase

- in the chiral limit: $N_f^c = 52(4)$ *continuum* flavors
- finding in contrast to meanfield prediction (go back to meanfield ?)
- chirally restored phase extends towards weak coupling

Argued: for $\beta = 0$, **"large-N_f QCD" is IR-conformal with [perhaps] non-trivial IRFP**

- strong-coupling limit allows economical study of a 4d IR-conformal gauge theory
- large N_f, $\mathbf{m_q = 0}$ simulations can be performed without too much computer effort
  $\rightarrow$ single IR scale L

Conjectured: strong coupling chirally symmetric, IR-conformal phase is analytically connected with the **continuum IR-conformal phase**

Conclusions

Shown: for $\beta = 0$, a strong first order **bulk transition** exists which is **N_f-driven** to a chirally symmetric phase

- in the chiral limit: $N_f^c = 52(4)$ *continuum* flavors
- finding in contrast to meanfield prediction (go back to meanfield ?)
- chirally restored phase extends towards weak coupling

Argued: for $\beta=0$, **"large-N_f QCD" is IR-conformal with [perhaps] non-trivial IRFP**

- strong-coupling limit allows economical study of a 4d IR-conformal gauge theory
- large $N_f, \mathbf{m_q = 0}$ simulations can be performed without too much computer effort

  $\rightarrow$ single IR scale L

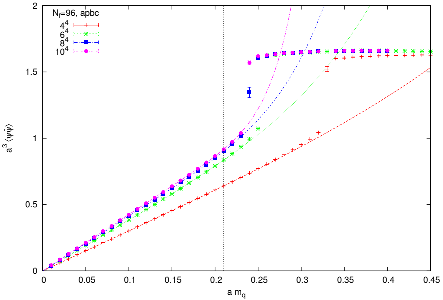
Conjectured: strong coupling chirally symmetric, IR-conformal phase is analytically connected with the **continuum IR-conformal phase**

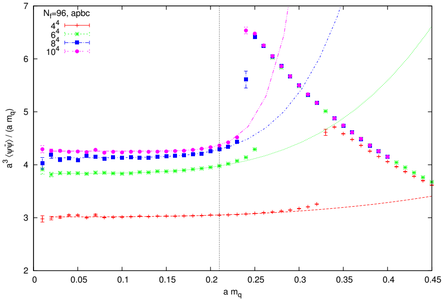
Questions:

- larger L at $\beta = 0$ $\rightarrow$ trivial or non-trivial ?
- follow transition line to weak coupling $\rightarrow$ first-order ?
- other ETC theories (esp. adjoint fermions) ?
- non-zero m_q, non-zero T ?

Mass deformation: can one determine γ^* ?

$$\langle\bar{\psi}\psi\rangle = c_1 m_q + c_2 m_q^{\frac{3-\gamma^*}{1+\gamma^*}} + c_3 m_q^3$$

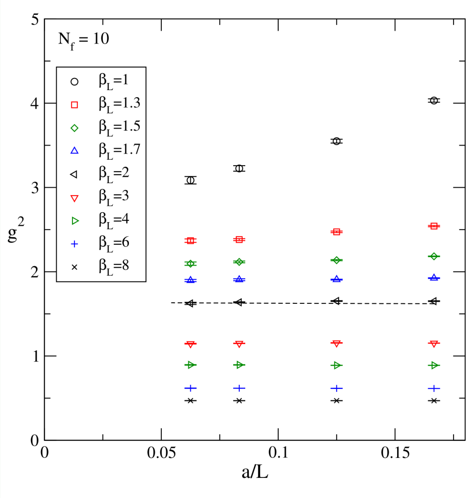




- heavier quarks have less ordering effect $\rightarrow$ transition to chirally broken phase
- systematic error from finite-size effects, fitting range and analytic ansatz $(\gamma^* < 0)$

For a given N_f, does g^* depend on β ?

Rummukainen et al, arXiv:1111.4104: $SU(2)$ with $N_f = 10 \rightarrow$ Banks-Zaks perturbative IRFP



Does $g^2(\beta, a/L)$ go to $(g^2)^*$ when $a/L \rightarrow 0$ $\forall\beta$?