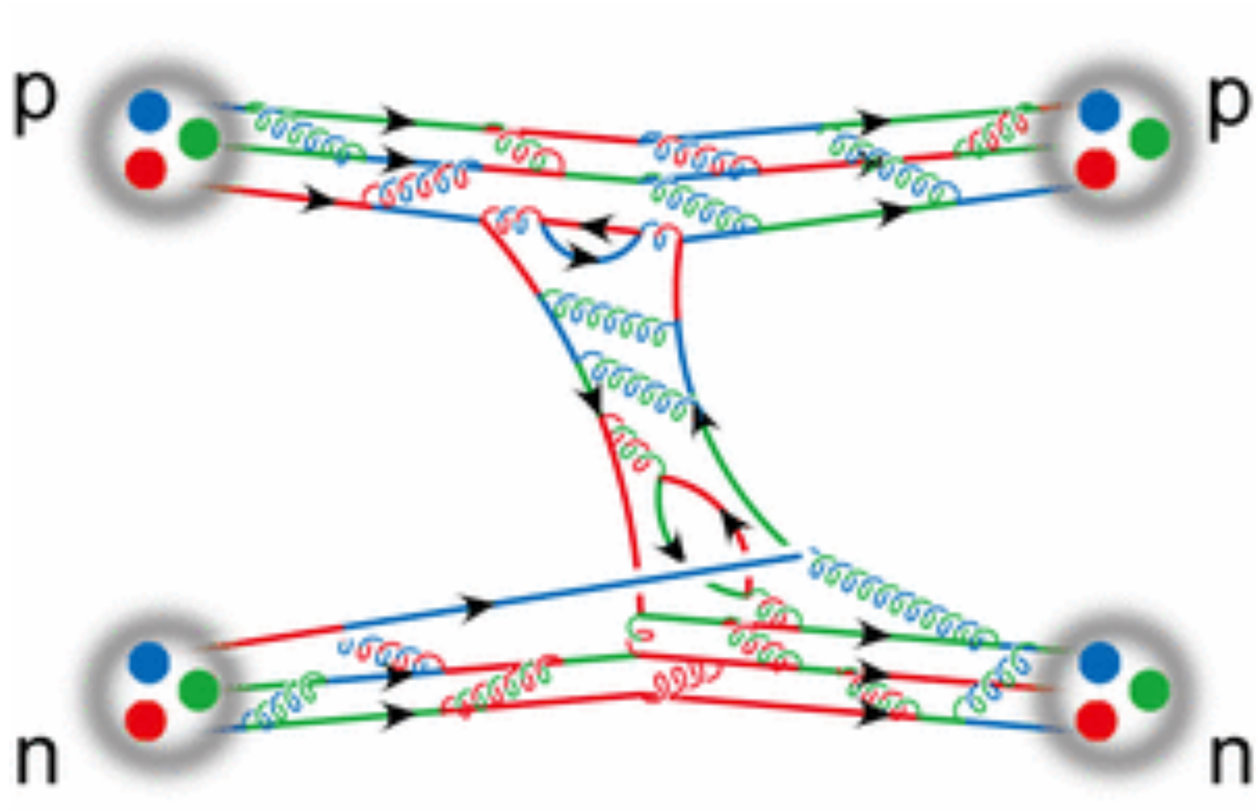
Hadron interactions from lattice QCD

Sinya Aoki
University of Tsukuba

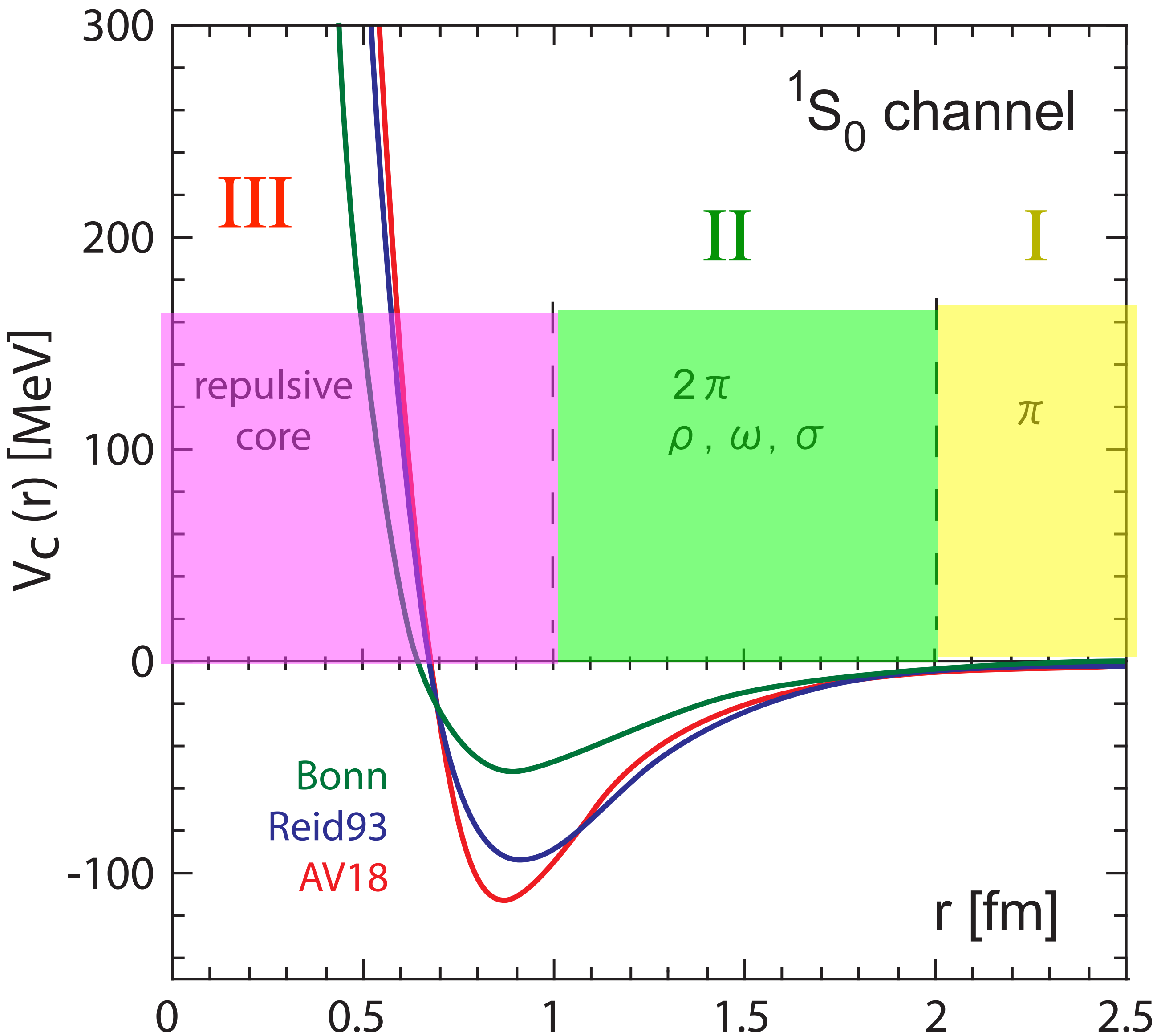


INT program "Lattice QCD Studies of Excited Resonaces and Multi-Hadron Systems"
INT, Seattle, USA, August 6, 2012

1. Introduction

How can we extract hadronic interaction from lattice QCD ?

Ex. **Phenomenological NN potential**
(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

Yiukawa(1935)

II Multi-pions

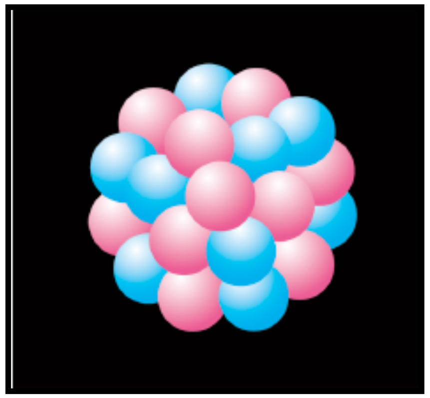
Taketani et al.(1951)

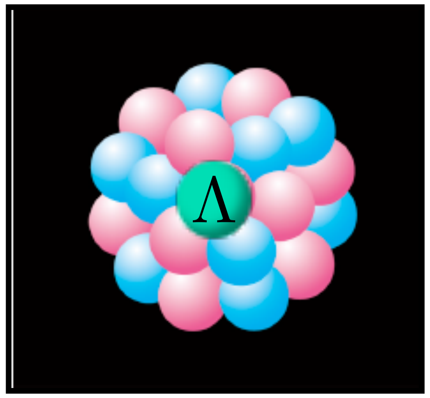
III Repulsive core

Jastrow(1951)

Nuclear force is a basis for understanding ...

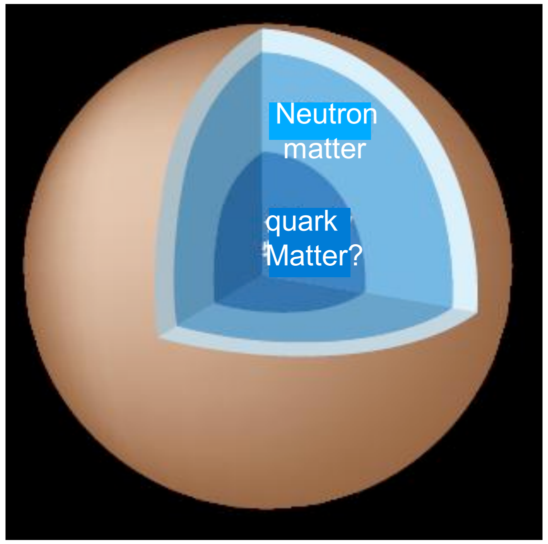
- Structure of ordinary and hyper nuclei



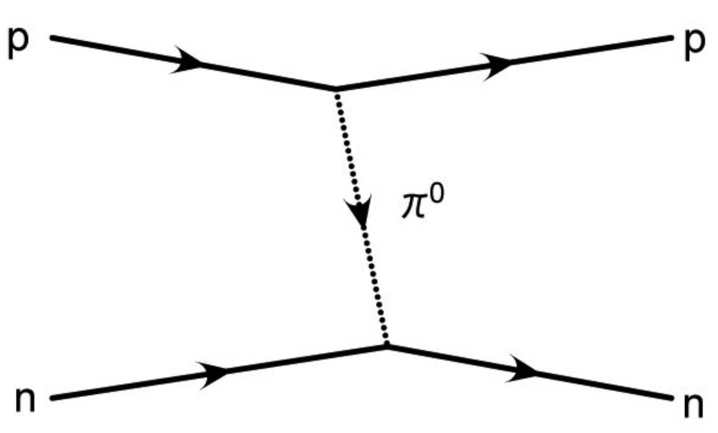


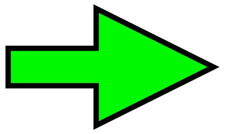
- Ignition of Type II SuperNova

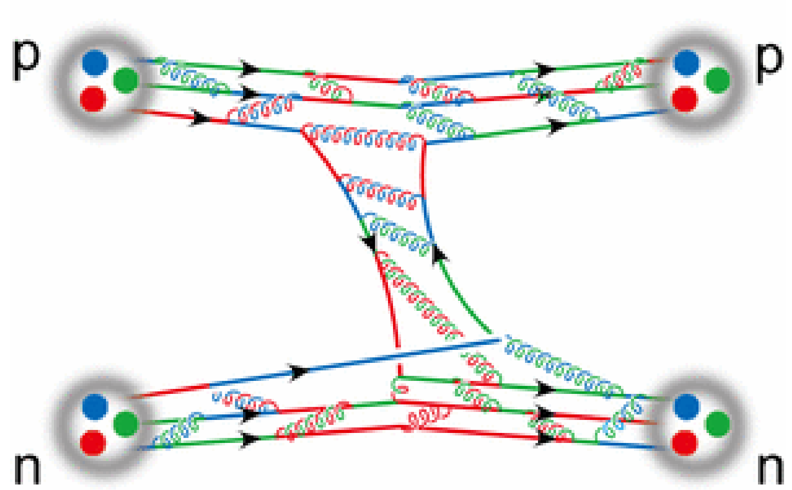
- Structure of neutron star



Can we extract a nuclear force in (lattice) QCD ?







Plan of my talk

1. Introduction
2. Our strategy
3. Nuclear potential
4. Predictions: Hyperon interactions
5. Some applications to nuclear physics
6. Other recent developments
7. Future prospect

2. Our Strategy

Two strategies in lattice QCD

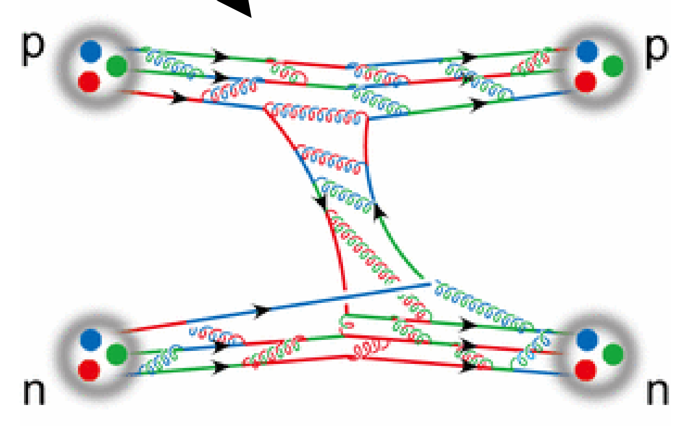
(Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

energy

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

$$W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator



Asymptotic region $\quad r = |\mathbf{r}| \to \infty$

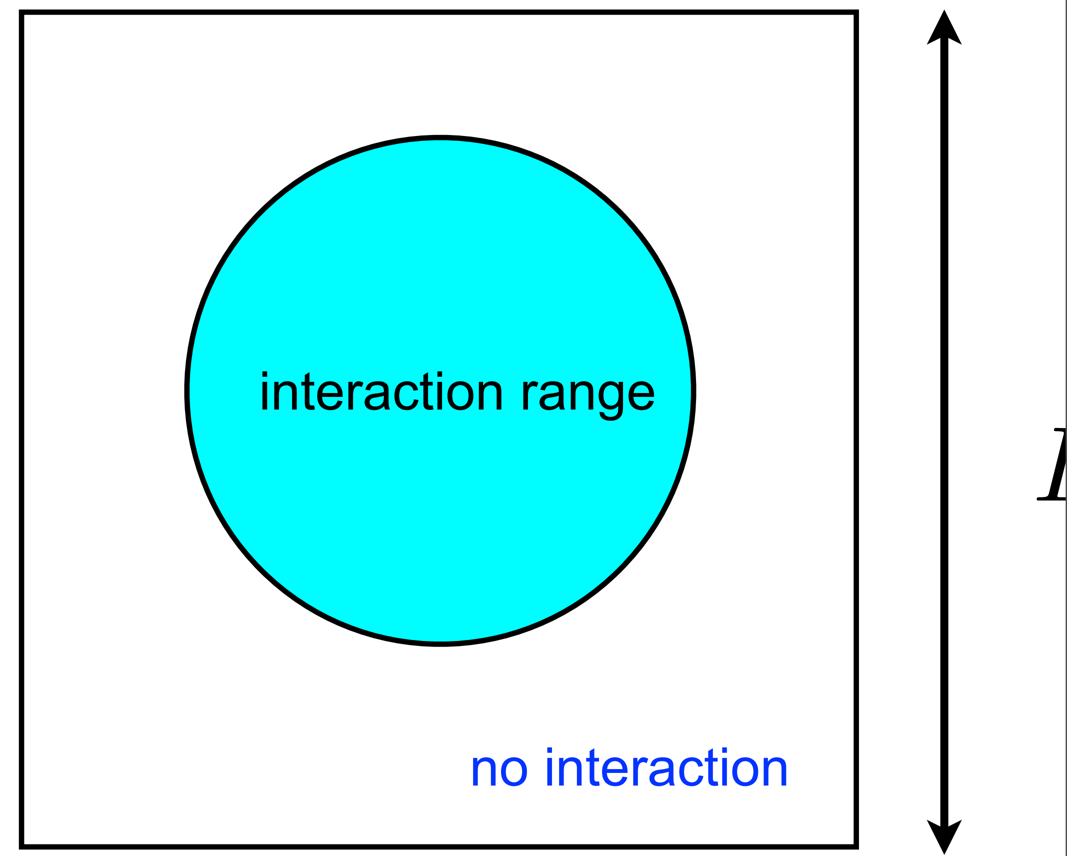
partial wave
$$\varphi^l_{\mathbf{k}} \to A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

$\delta_l(k)$ scattering phase shift (phase of the S-matrix) in QCD !

How can we extract it ?

cf. Maiani-Testa theorem
time correlation

Let us consider **Two particles in the finite box**



1st strategy: Luescher's formula for phase shift

extract energy in the finite box: $E = \dfrac{k_n^2}{m_N}$

Finite volume → $\text{allowed value: } k_n^2$

cf. free theory $\mathbf{k_n} = \dfrac{2\pi}{L}\mathbf{n}$

→ two particle energy $E = \dfrac{k_n^2}{m_N}$

different from free theory

→ $\delta_l(k_n)$

Ex. $k \cot \delta_0(k) = \dfrac{2}{\sqrt{\pi} L} Z_{00}(1; q^2)$

$q = \dfrac{kL}{2\pi}$ non-integer

generalize zeta-function $Z_{00}(s; q^2) = \dfrac{1}{\sqrt{4\pi}} \sum_{\mathbf{n} \in \mathbf{Z}^3} (\mathbf{n}^2 - q^2)^{-s}$

2nd strategy: HAL QCD method

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Extract information inside the interaction range as

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

$\mu = m_N/2$

reduced mass

$\epsilon_k = \dfrac{\mathbf{k}^2}{2\mu}$ $\quad H_0 = \dfrac{-\nabla^2}{2\mu}$

non-local potential

→ solve the Schroedinger Eq. in the infinite volume with this “potential”.

→ correct phase shifts (and biding energy) below inelastic threshold by construction

resonance

$W_k < W_{\rm th} = 2m_N + m_\pi$

New method to extract phase shift from QCD
(by-pass Maiani-Testa theorem, using space correlation)

HAL QCD method

Properties & Remarks

1. Potential itself is NOT an observable. Using this freedom, we can construct a non-local but energy-independent potential as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

space w/o zero modes

For $^\forall W_{\mathbf{p}} < W_{\rm th}$

$$\int d^3y\, U(\mathbf{x},\mathbf{y}) \phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(x) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right] \varphi_{\mathbf{p}}(x)$$

Proof of existence (cf. Density Functional Theory)

Of course, potential satisfying this is not unique.

cf. Effective field theories and ChPT

QCD → EFTs $\quad L = \dfrac{1}{f_\pi(m)^2} \mathrm{tr}\, \partial^\mu U^\dagger \partial_\mu U + \cdots$

We can make some parameter mass-independent. → $L = \dfrac{1}{f_0} \mathrm{tr}\, \partial^\mu U^\dagger \partial_\mu U + \cdots$

ChPT

2. In practice, we expand the non-local potential in terms of derivative:
(cf: expansion in ChPT)

$$U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta^3(\mathbf{x} - \mathbf{y})$$

expansion parameter $\frac{W_p - W_k}{W_{\rm th}}$

$$V(\mathbf{x}, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_1 \cdot \mathbf{x})(\sigma_2 \cdot \mathbf{x}) - (\sigma_1 \cdot \sigma_2)$

The expansion agrees with the form of potential proposed by Okubo-Marshak (1958).

$V_A(\mathbf{x})$ local and energy independent coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

If we truncate the expansion, some systematic errors are introduced.

$\varphi_{\mathbf{k}}(\mathbf{x})$ → $V_A(\mathbf{x})$ → $\delta(p)$ not exact

$p \neq k$

$\varphi_{\mathbf{p}}(\mathbf{x})$ → $\tilde{V}_A(\mathbf{x})$ → $\tilde{\delta}(p)$ exact

we can check a size of systematic errors.
(See later).

cf. Expansion in ChPT

(N)LO calculation
+ observables → LOCs → predict other observables with errors.

It is difficult to estimate the convergence in ChPT.

3. (Scheme) Potential depends on the choice of N(x). (cf: running coupling)

4. Non-relativistic approximation is NOT used. We just take the specific (equal-time) flame.

5. Potential $U(\mathbf{x},\mathbf{y})$ can be used at $^\forall L$ and $^\forall W_k < W_{\rm th}$.

angular momentum

6. The method can be extended to inelastic region.

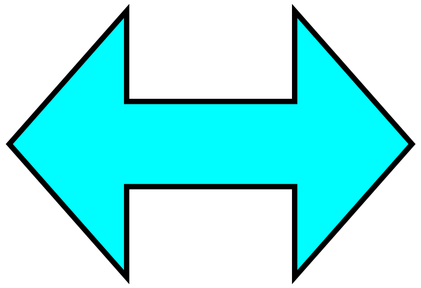
Ex.

$$NN \to NN, NN\pi \qquad \begin{pmatrix} U_{NN,NN}(\mathbf{x};\mathbf{y}) & U_{NN,NN\pi}(\mathbf{x};\mathbf{y},\mathbf{w}) \\ U_{NN\pi,NN}(\mathbf{x},\mathbf{z};\mathbf{y}) & U_{NN\pi,NN\pi}(\mathbf{x},\mathbf{z};\mathbf{y},\mathbf{w}) \end{pmatrix}$$

$$\Lambda\Lambda \to \Lambda\Lambda, N\Xi \qquad \begin{pmatrix} U_{\Lambda\Lambda,\Lambda\Lambda}(\mathbf{x},\mathbf{y}) & U_{\Lambda\Lambda,N\Xi}(\mathbf{x},\mathbf{y}) \\ U_{N\Xi,\Lambda\Lambda}(\mathbf{x},\mathbf{y}) & U_{N\Xi,N\Xi}(\mathbf{x},\mathbf{y}) \end{pmatrix}$$

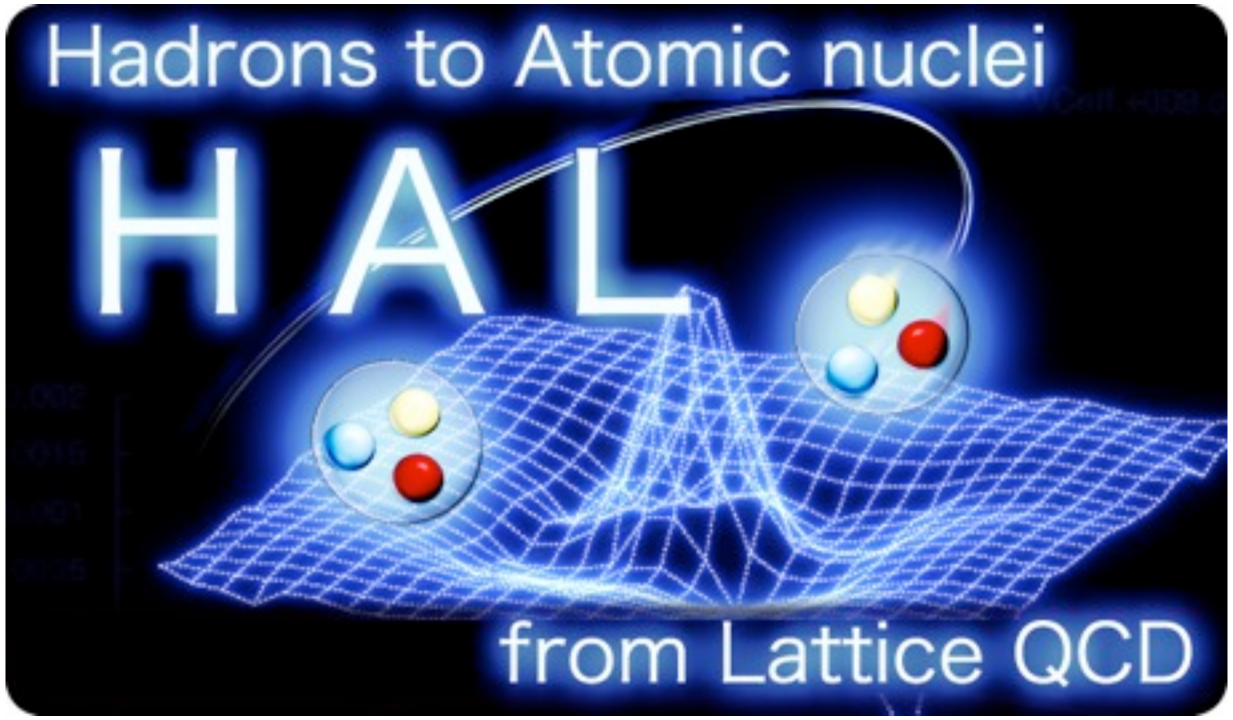
In principle, we can treat all QCD processes.

QFT(QCD) at given energy.



coupled channel quantum mechanics.

HAL QCD Collaboration



Sinya Aoki (U. Tsukuba)
Bruno Charron* (U. Tokyo)
Takumi Doi (Riken)
Tetsuo Hatsuda (Riken/U. Tokyo)
Yoichi Ikeda (TIT)
Takashi Inoue (Nihon U.)
Noriyoshi Ishii (U. Tsukuba)
Keiko Murano (Riken)
Hidekatsu Nemura (U. Tsukuba)
Kenji Sasaki (U. Tsukuba)
Masanori Yamada* (U. Tsukuba)

*PhD Students

Potentials from lattice QCD → Nuclear Physics with these potentials → Neutron stars Supernova explosion

Our strategy

3. Nuclear potential

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \overline{\mathcal{J}}(t_0) | 0 \rangle$$

complete set

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \sum_{n,s_1,s_2} |2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(t_0) | 0 \rangle$$

$$= \sum_{n,s_1,s_2} A_{n,s_1,s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0) | 0 \rangle.$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

normalized 4-pt Correlation function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

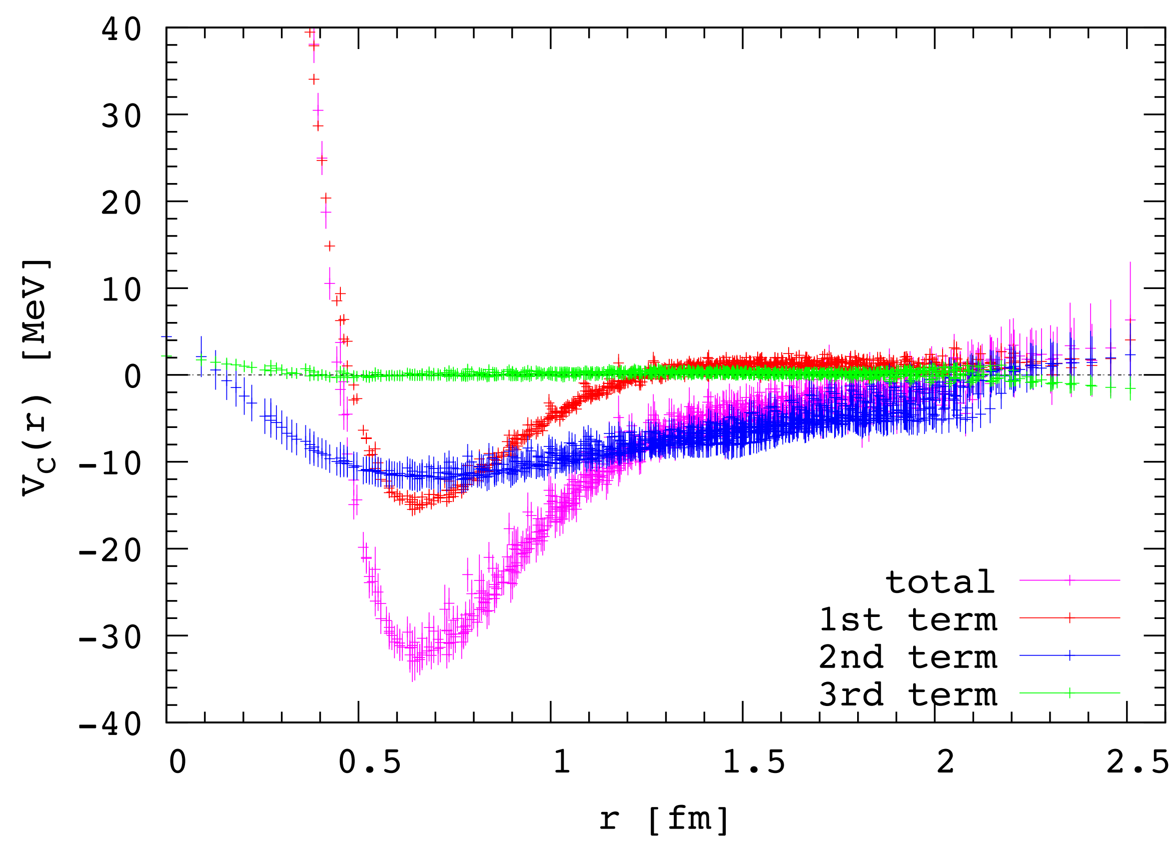
potential

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st **2nd** **3rd** **total**

3rd term(relativistic correction) is negligible.



Ground state saturation is no more required ! (advantage over Luescher's method.)

Remark

Another construction of energy-independent and non-local potential

generalized 4-pt Correlation function

$$R(\mathbf{x},\mathbf{y},t) = \frac{1}{e^{-2m_N t}} \int d^3x_1\, d^3y_1\, \langle 0|T\{N(\mathbf{x_1}+\mathbf{x},t)N(\mathbf{x_1},t)\bar{N}(\mathbf{y_1}+\mathbf{y},0)\bar{N}(\mathbf{y_1},0)\}|0\rangle$$

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right) R(\mathbf{x},\mathbf{y},t) = \int d^3z\, U(\mathbf{x},\mathbf{z}) R(\mathbf{z},\mathbf{y},t)$$

$$U(\mathbf{x},\mathbf{y}) = \int d^3z \left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right) R(\mathbf{x},\mathbf{z},t)\cdot \tilde{R}^{-1}(\mathbf{z},\mathbf{y},t)$$

truncated "inverse"

$$R^{-1}(\mathbf{z},\mathbf{y},t) = \sum_{\lambda_n(t)\neq 0} \frac{1}{\lambda_n(t)} v_n(\mathbf{x},t) v_n^\dagger(\mathbf{y},t)$$

without zero modes

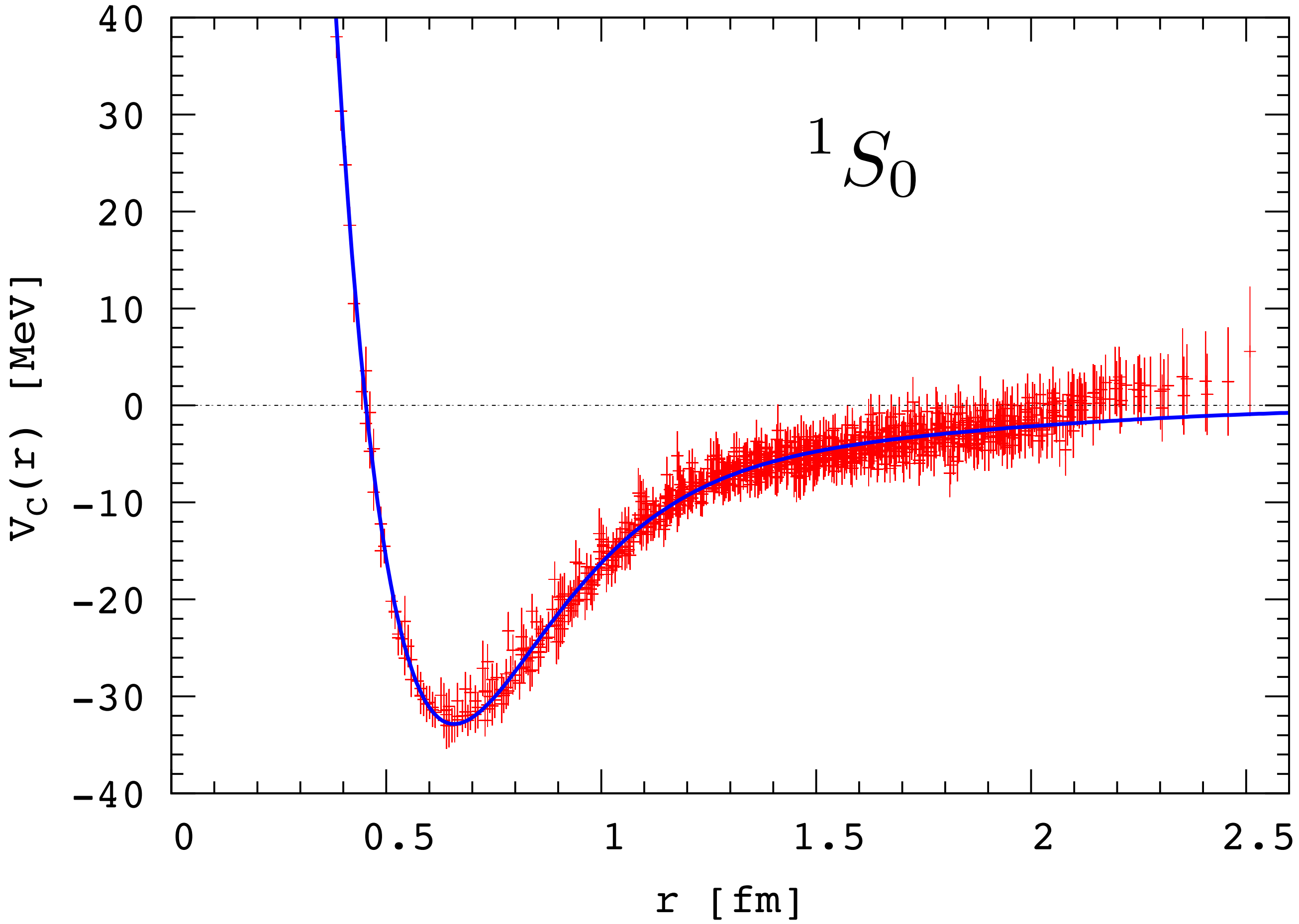
$\lambda_n(t), v_n(\mathbf{x},t)$: eigenvalue and eigenfunction of hermitian operator $R(\mathbf{x},\mathbf{y},t)$

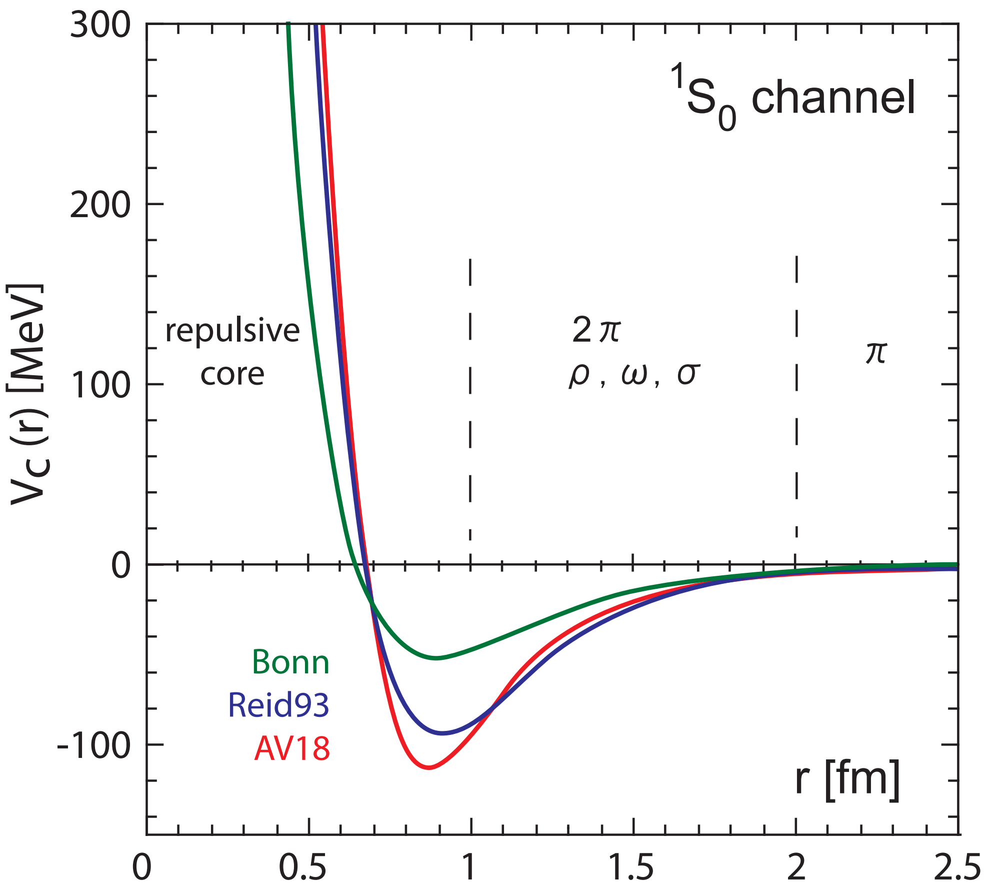
NN potential

2+1 flavor QCD, spin-singlet potential (in preparation)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV

phenomenological potential





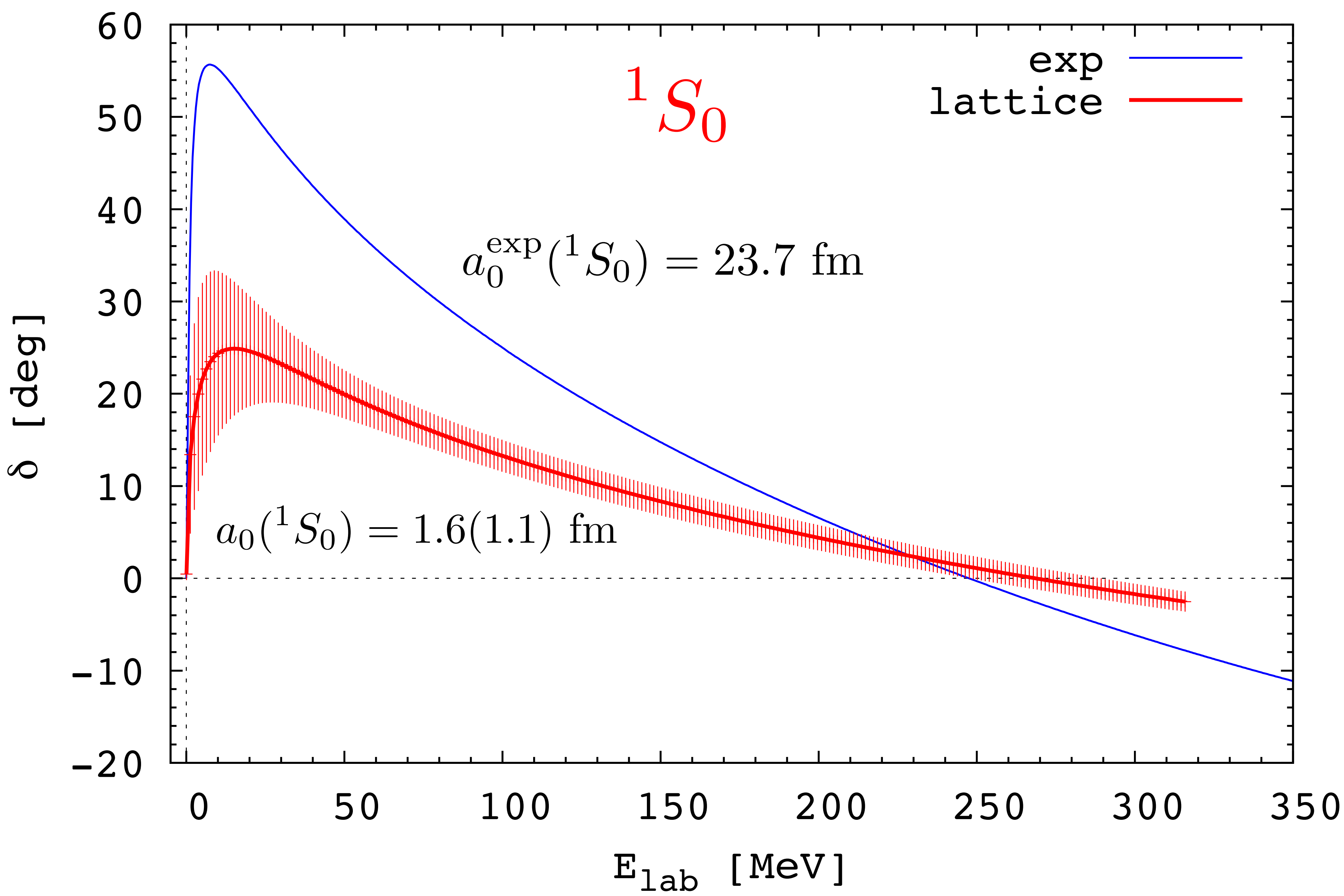
Qualitative features of NN potential are reproduced !

(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007.

NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

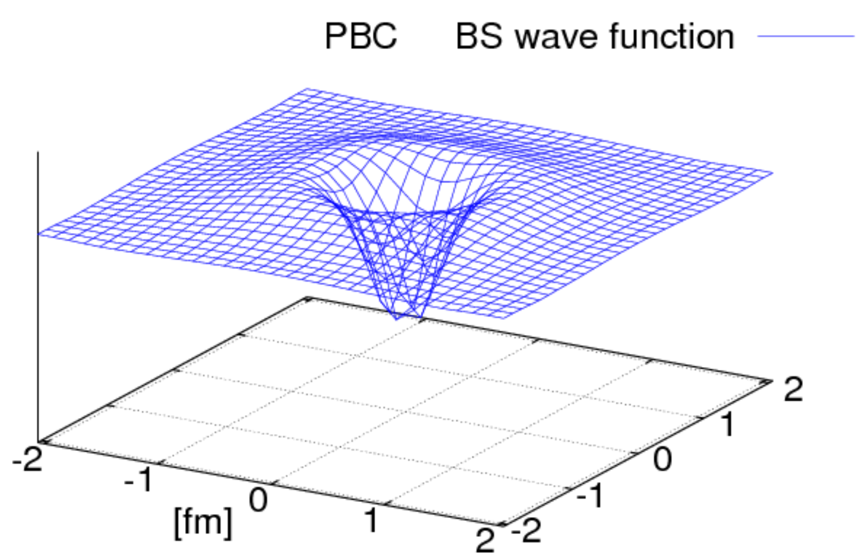
$m_\pi \simeq 0.53$ GeV

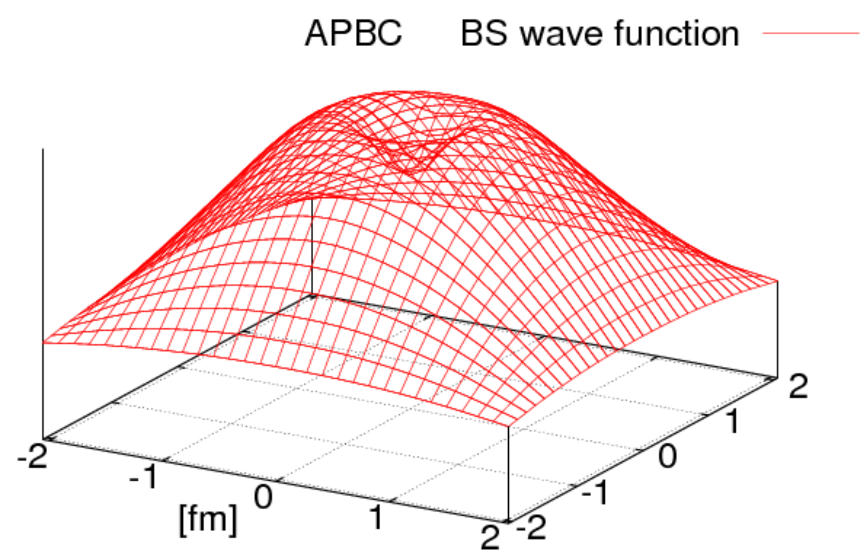
a=0.137fm

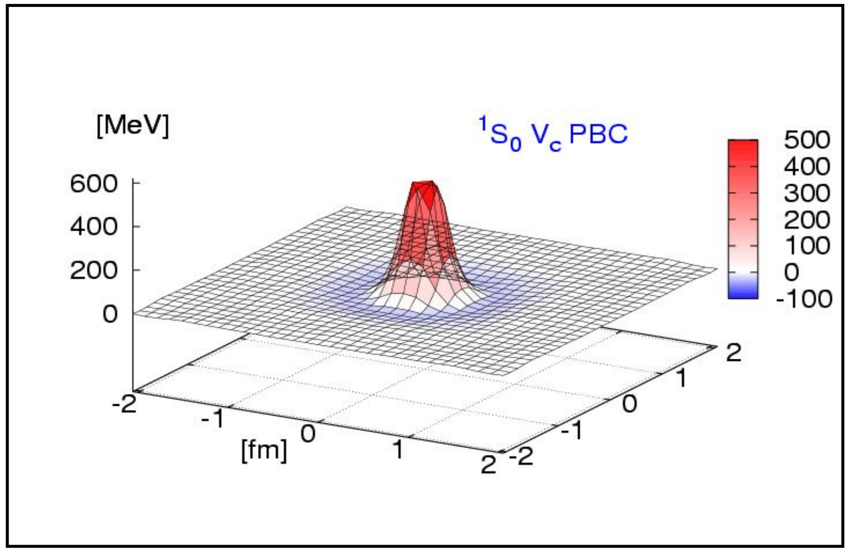
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

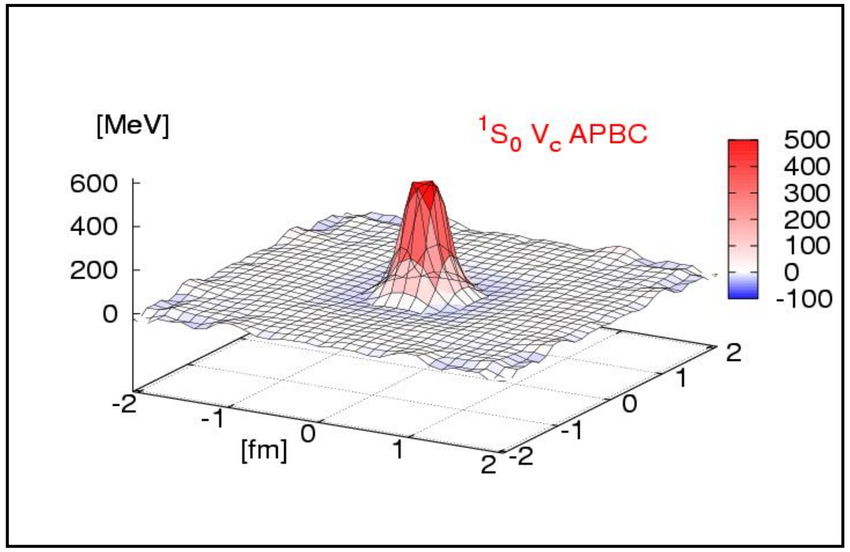
PTP 125 (2011)1225.

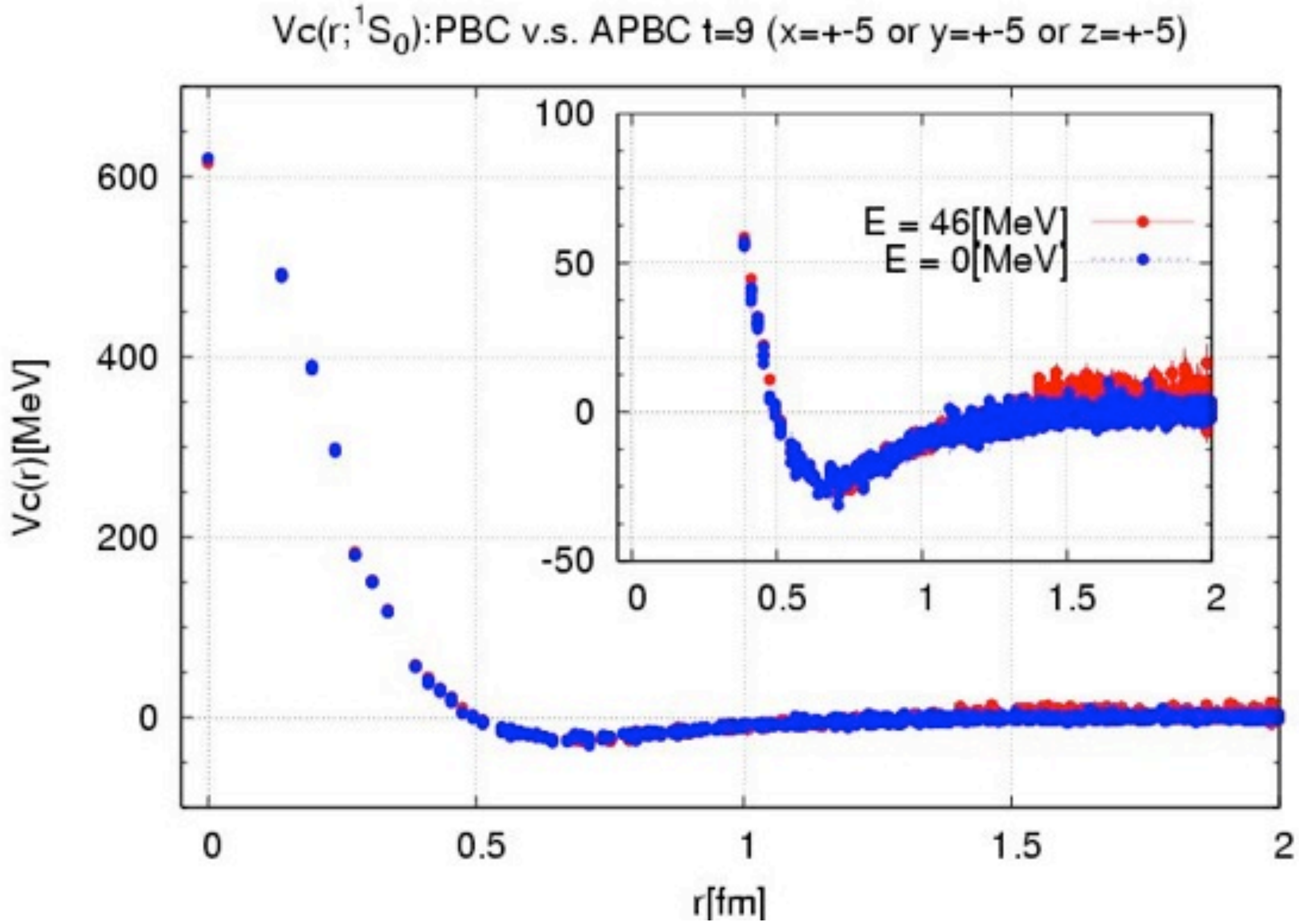
● PBC (E～0 MeV) ● APBC (E～46 MeV)

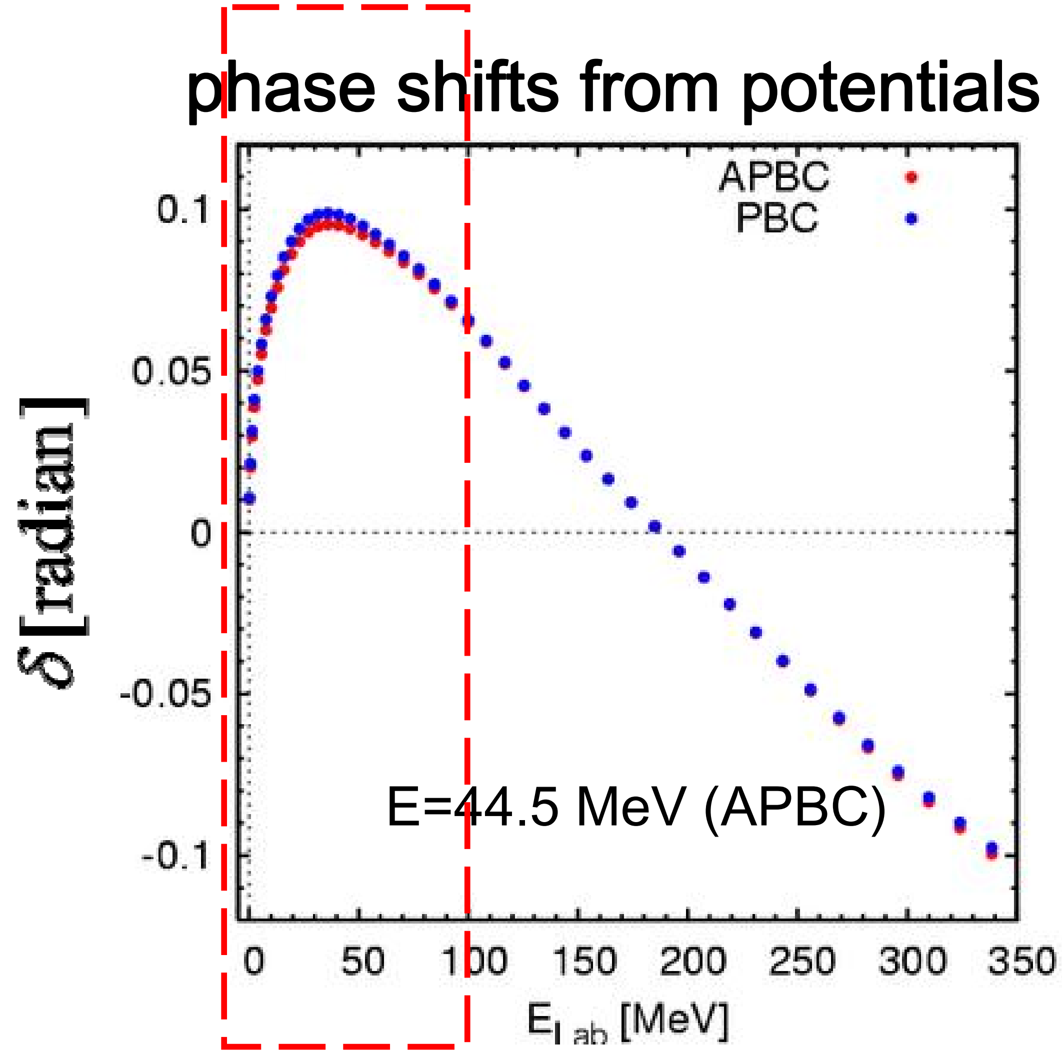












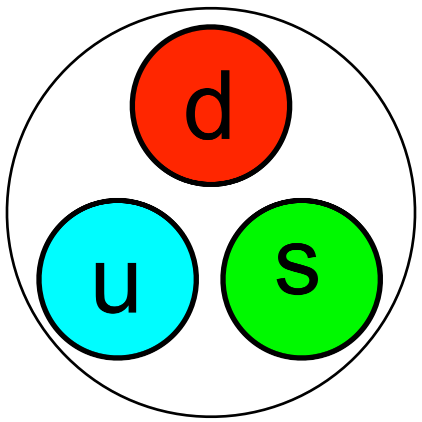
Higher order terms turn out to be very small at low energy in HAL scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(cf. convergence of ChPT, convergence of perturbative QCD)

4. Predictions: Hyperon interactions



$$p = (uud), n = (udd)$$

nucleon(N)

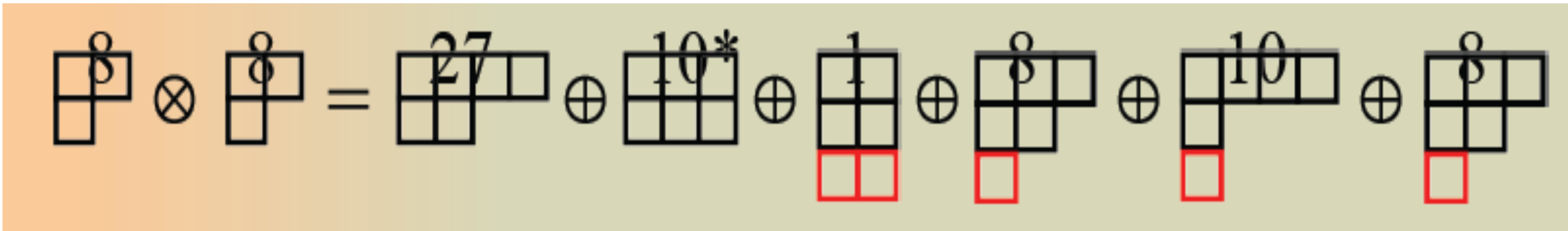
$$\Lambda = (uds)_{I=0}$$

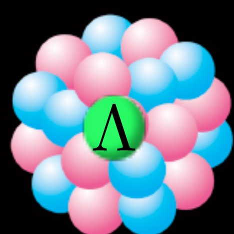
$$\Sigma^+ = (uus), \Sigma^0 = (uds)_{I=1}, \Sigma^- = (dds)$$

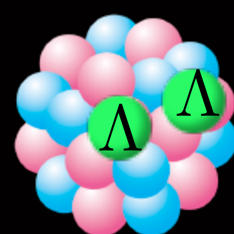
hyperon(Y)

$$\Xi^0 = (uss), \Xi^- = (dss)$$

Octet Baryon interactions







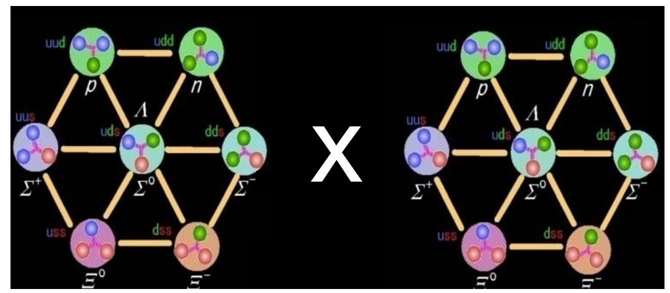
- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN ?

Baryon Potentials in the flavor SU(3) symmetric limit

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric　　Anti-symmetric

6 independent potentials in flavor-basis

$$V^{(27)}(r),\ \ V^{(8s)}(r),\ \ V^{(1)}(r) \longleftarrow {}^1S_0$$

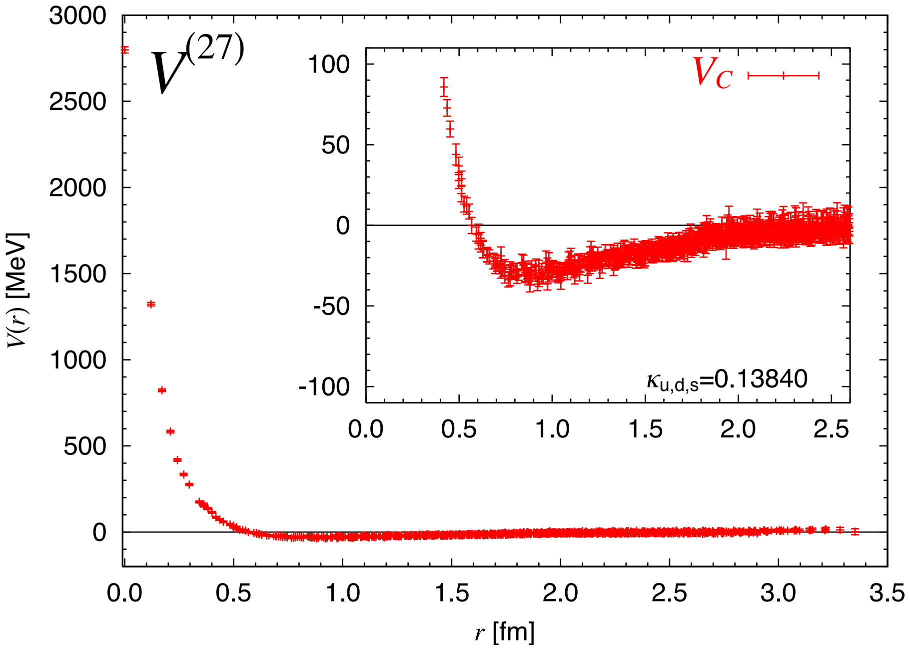
$$V^{(10^*)}(r),\ \ V^{(10)}(r),\ \ V^{(8a)}(r) \longleftarrow {}^3S_1$$

3-flavor QCD　　a=0.12 fm

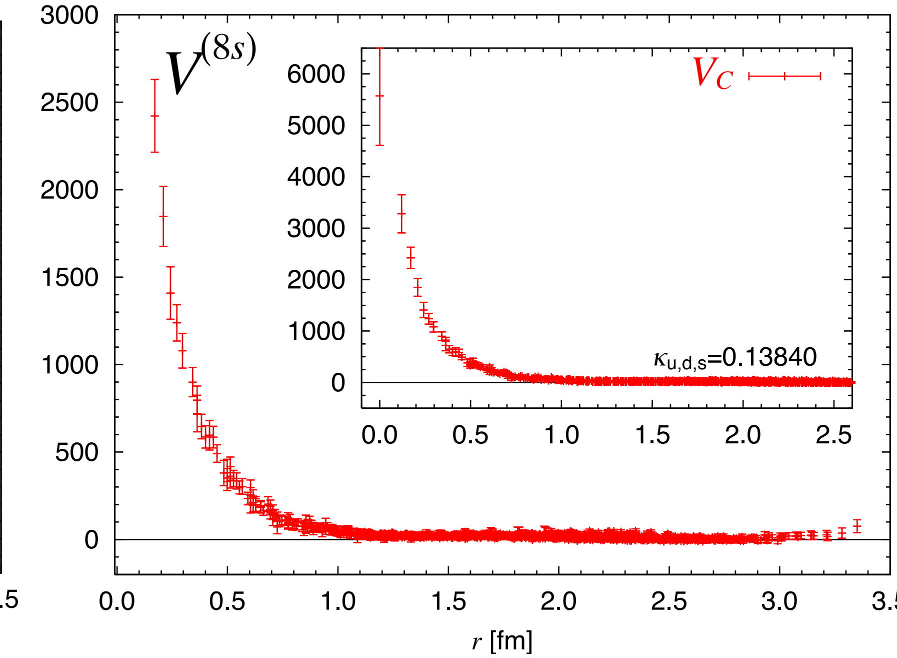
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591　　L=2 fm

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]　　L=2-4 fm

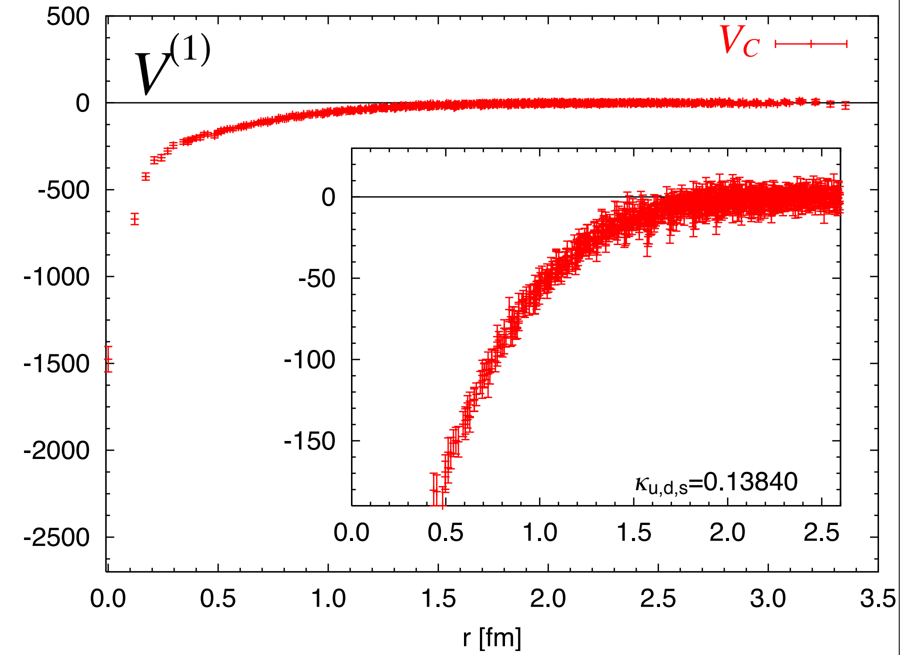
$L \simeq 4$ fm, $m_\pi \simeq 470$ MeV



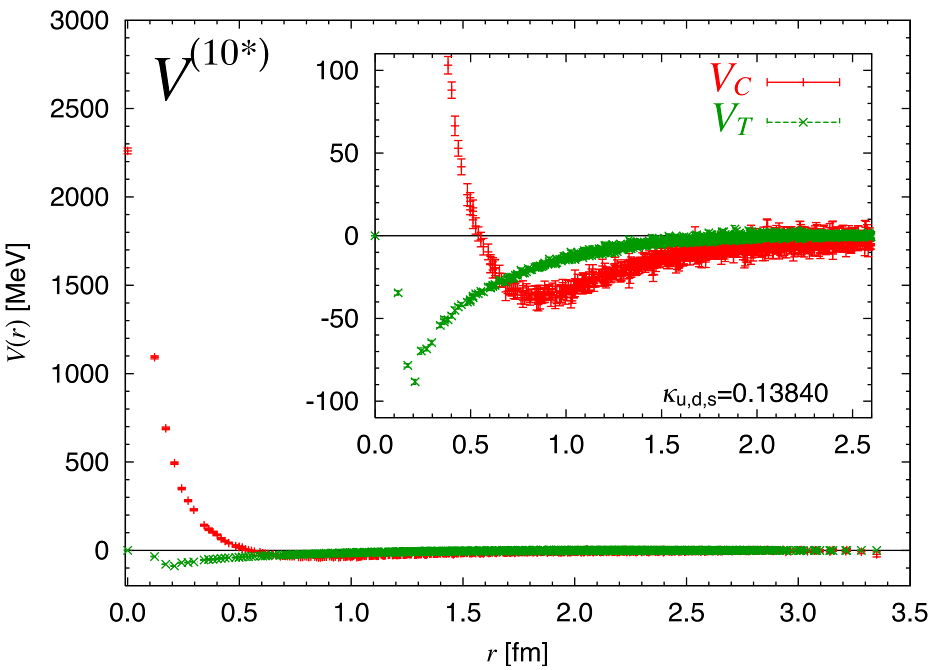
same as NN



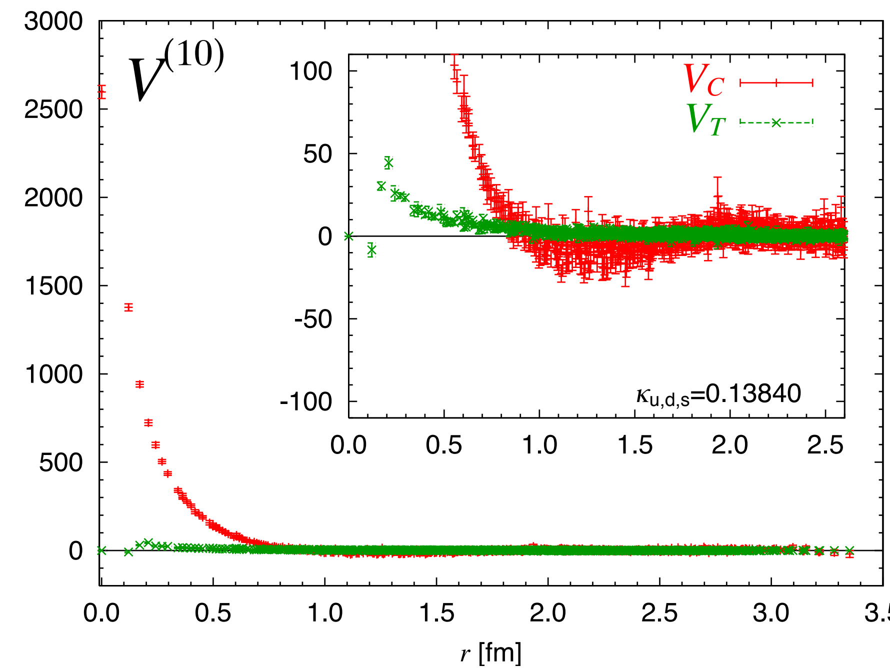
8s: strong repulsive core. repulsion only.



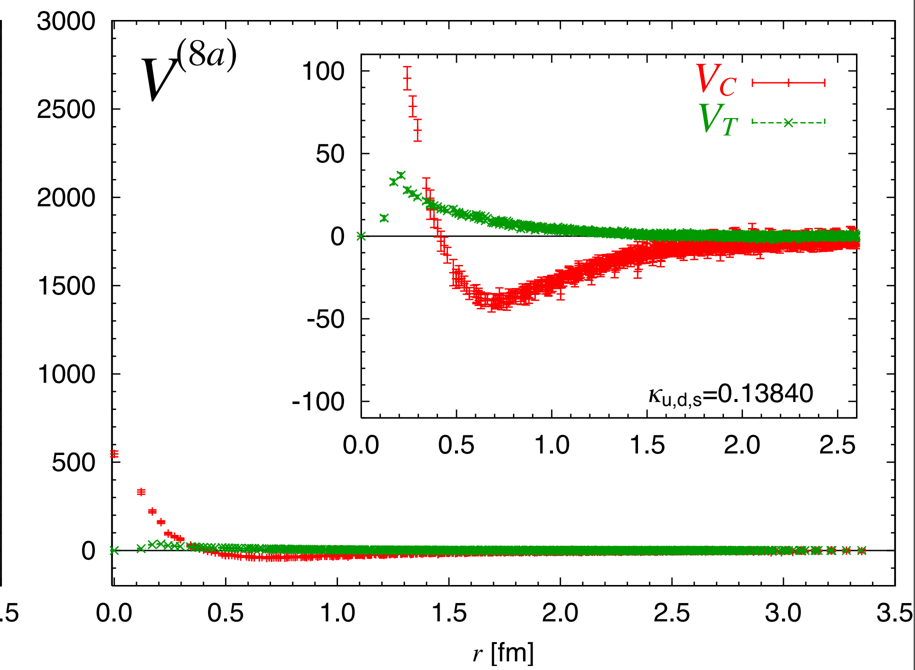
1: attractive instead of repulsive core ! attraction only .



same as NN



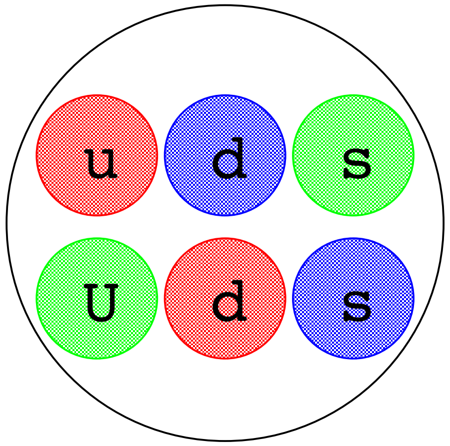
10: strong repulsive core. weak attraction.



8a: weak repulsive core.
strong attraction.

Flavor dependences of BB interactions become manifest in SU(3) limit !

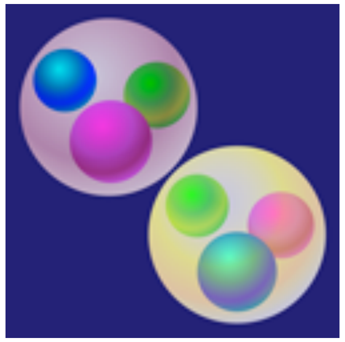
H-dibaryon:
a possible six quark state(uuddss)
predicted by the model but not observed yet.



http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.106.162001

Binding baryons on the lattice

April 26, 2011



H-dibaryon in the flavor SU(3) symmetric limit

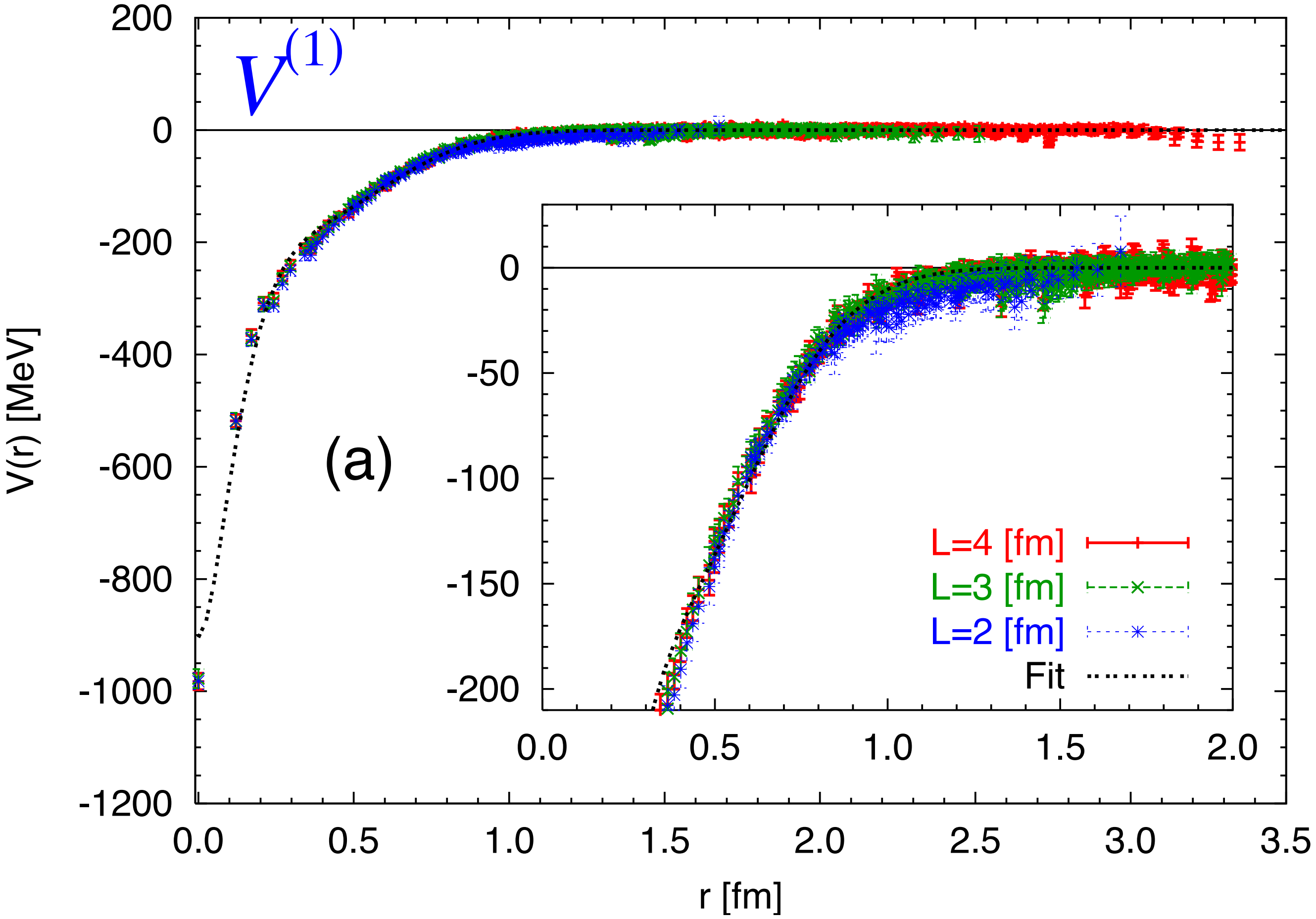
a=0.12 fm

Inoue *et al.* (HAL QCD Coll.), PRL106(2011)162002

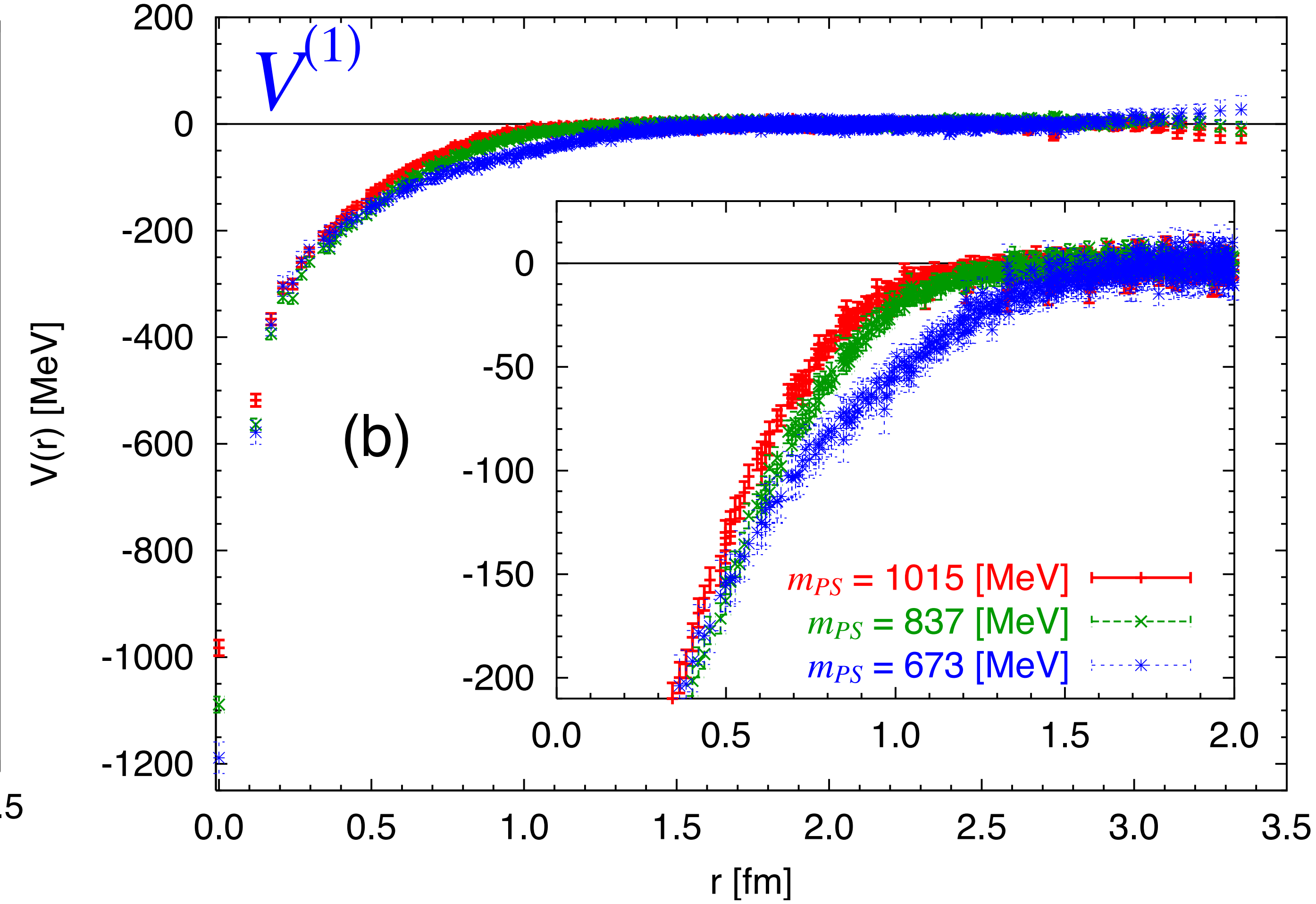
Attractive potential
in the flavor singlet channel → possibility of a bound state (H-dibaryon)

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

volume dependence



pion mass dependence

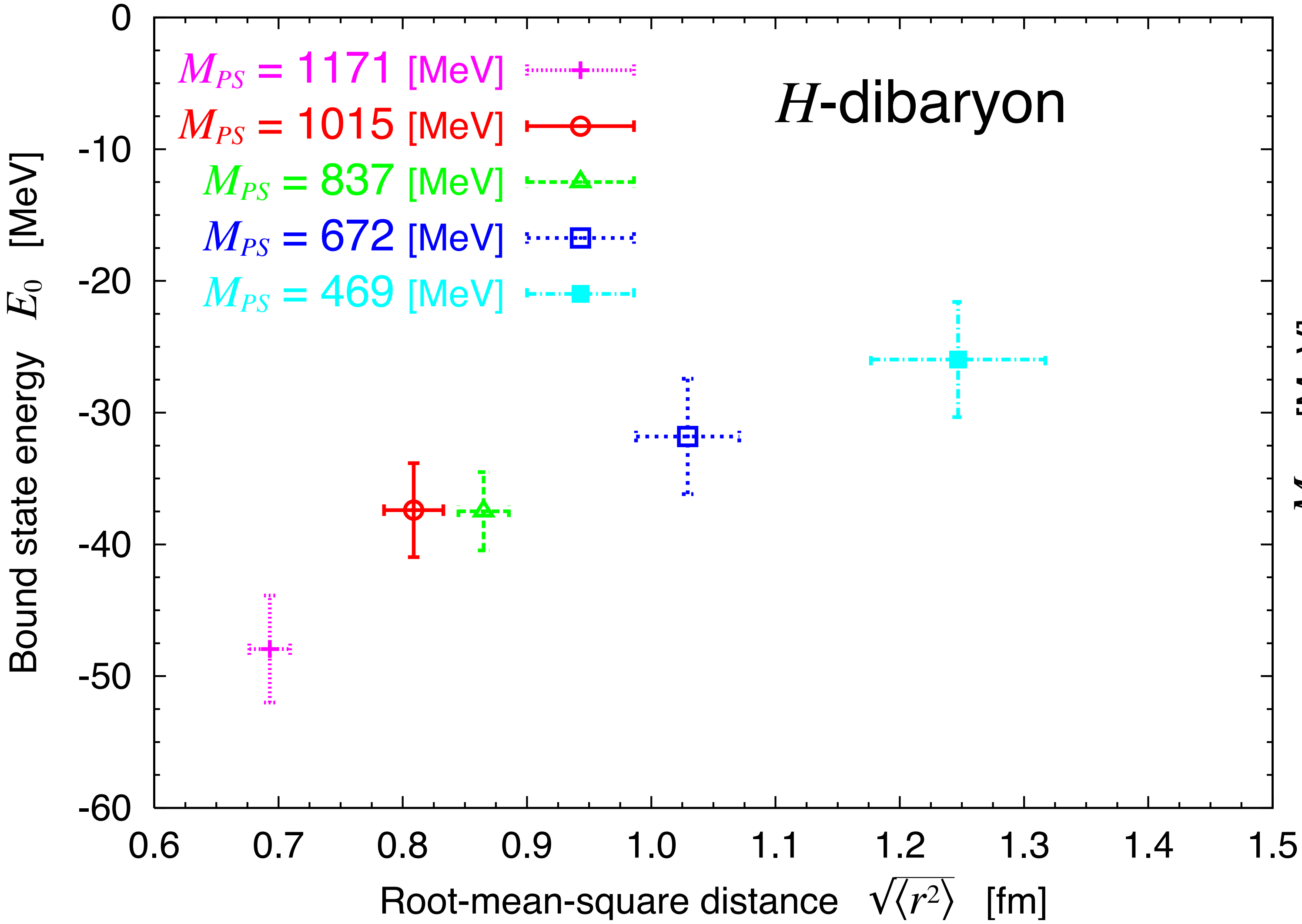


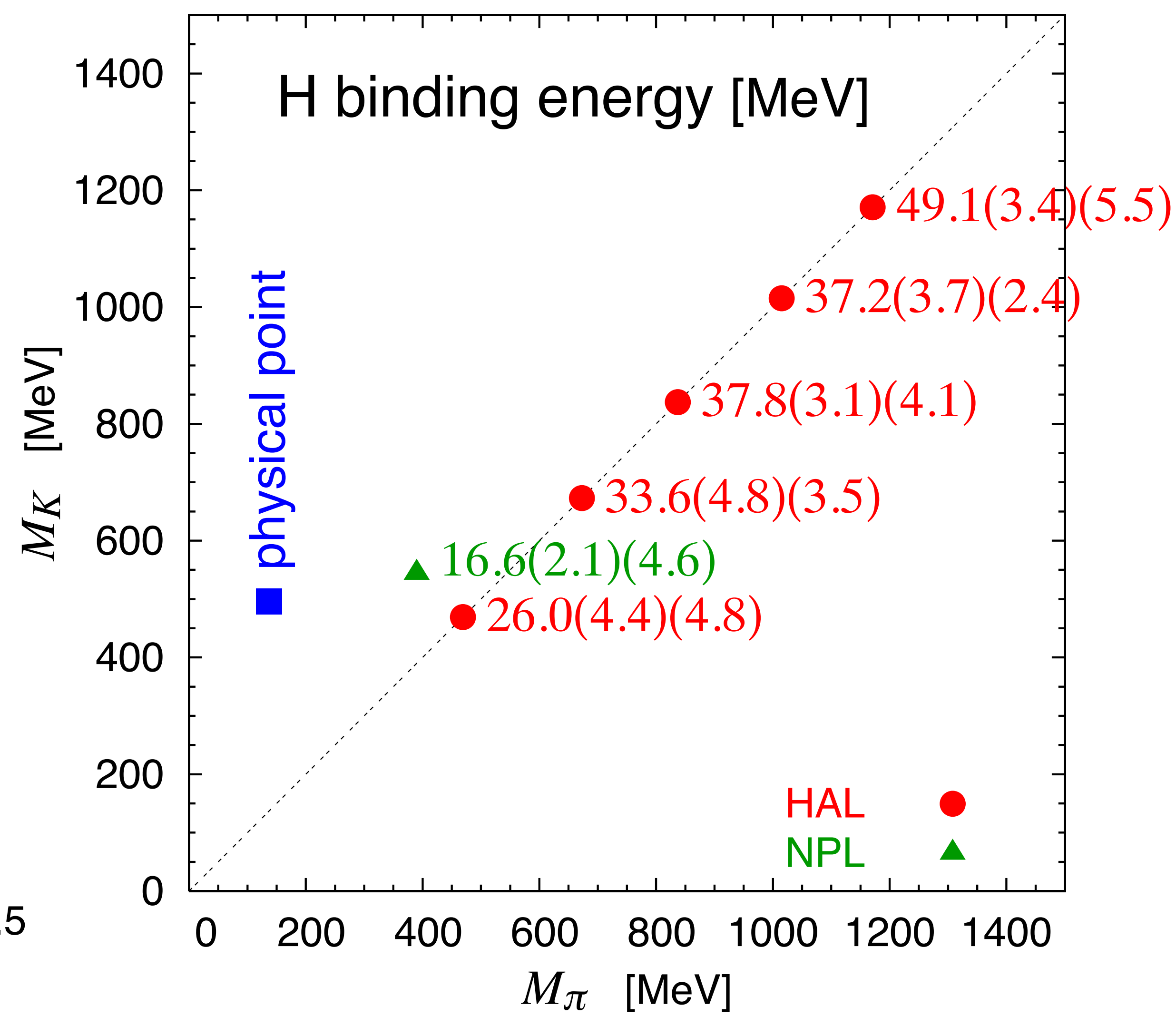
L=3 fm is enough for the potential.

lighter the pion mass, stronger the attraction

fit potentials at L=4 fm by $V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$

Solve Schroedinger equation in the infinite volume → One bound state (H-dibaryon) exists.

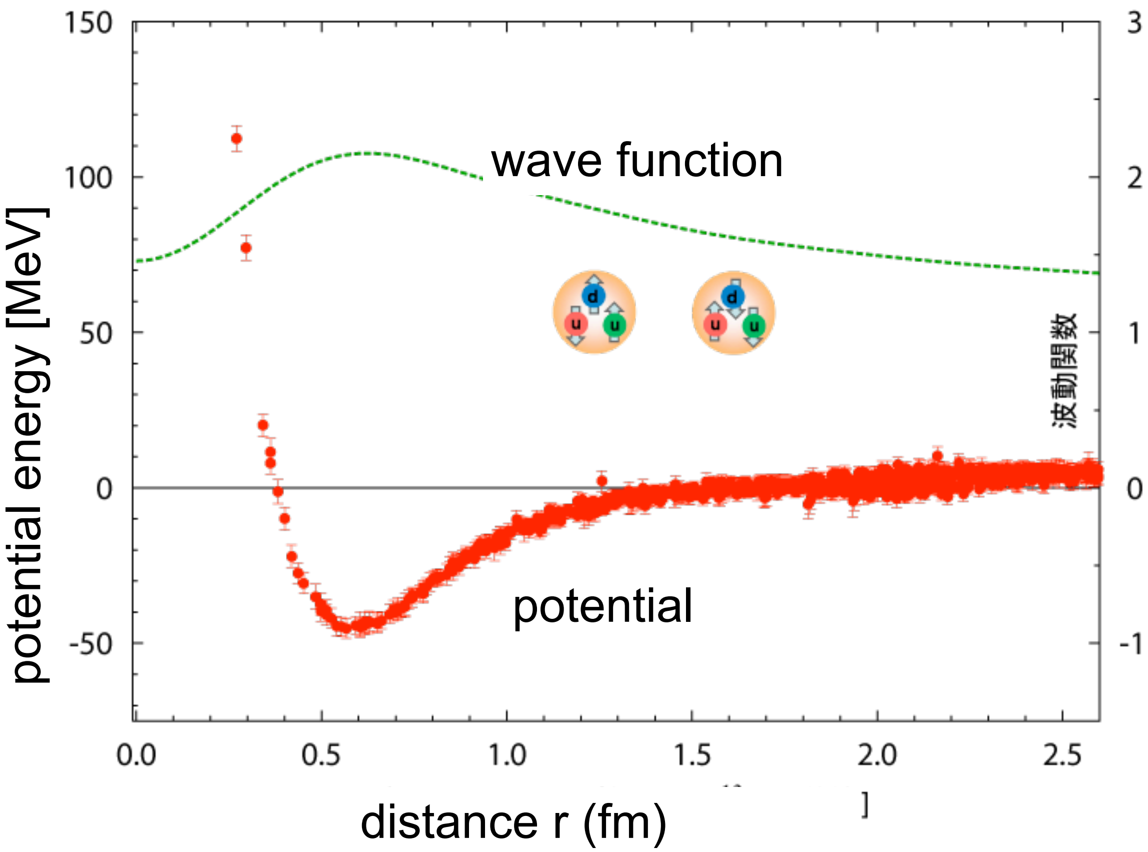




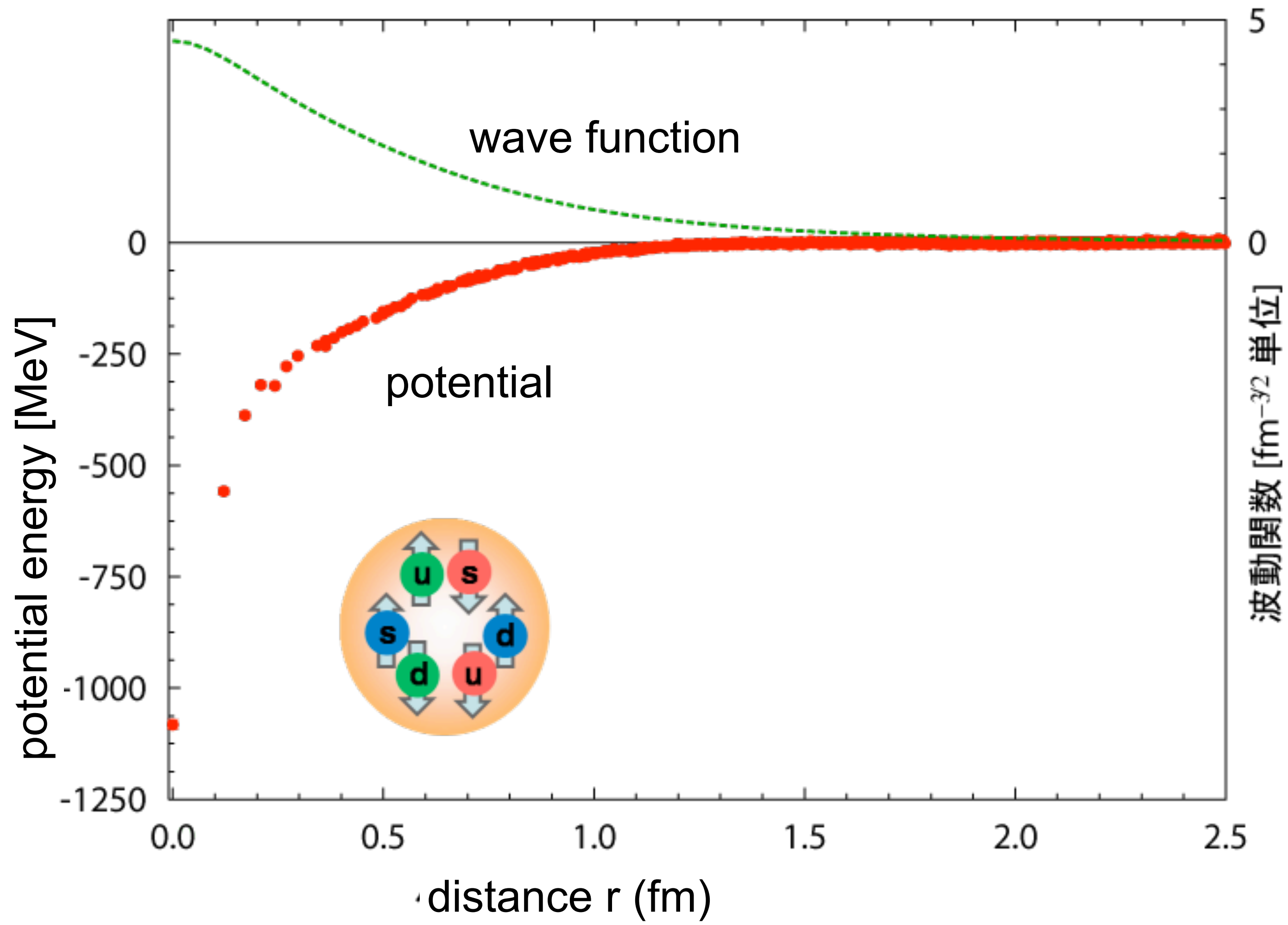
An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?

Deuteron

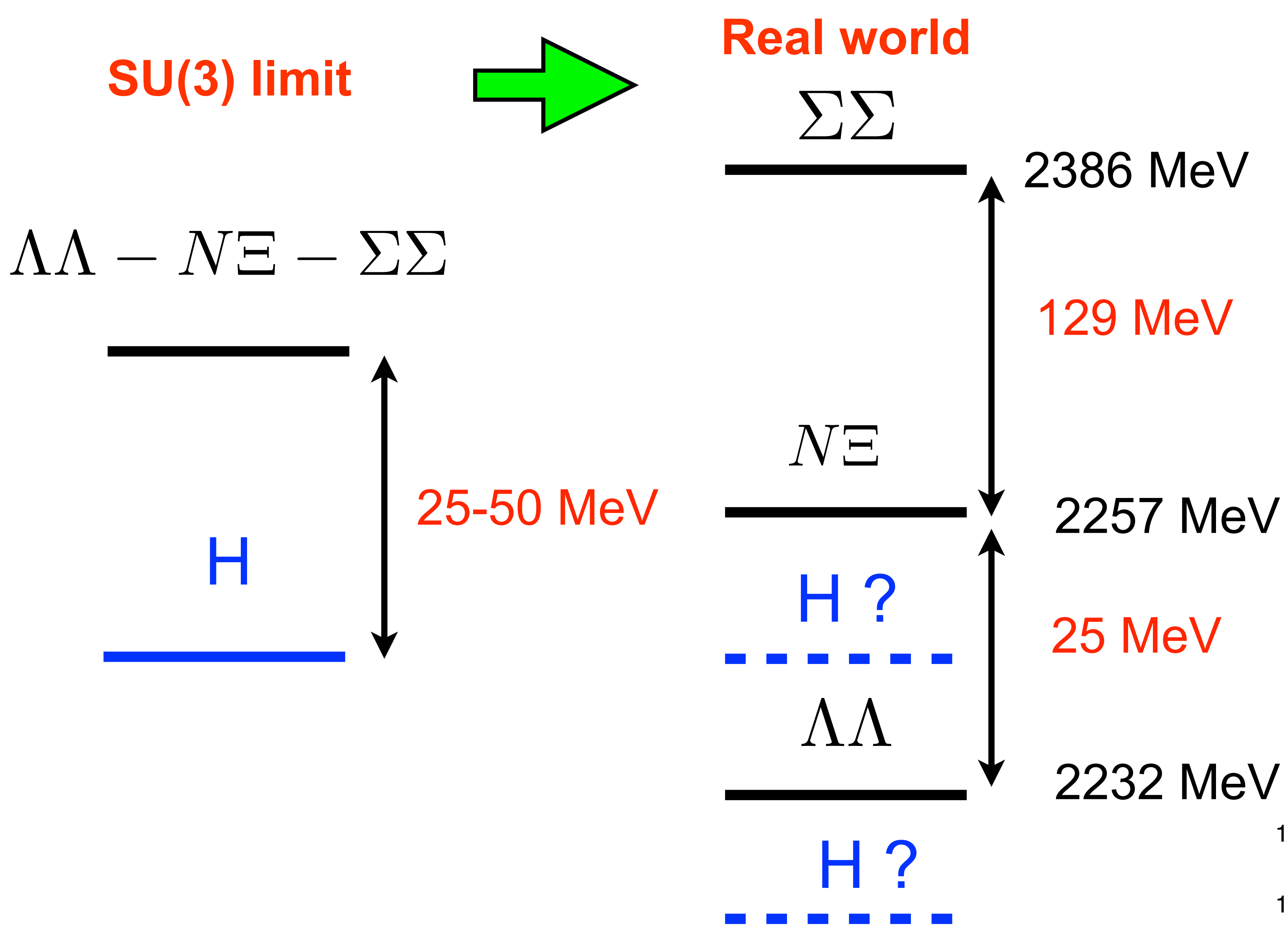


H-dibayon

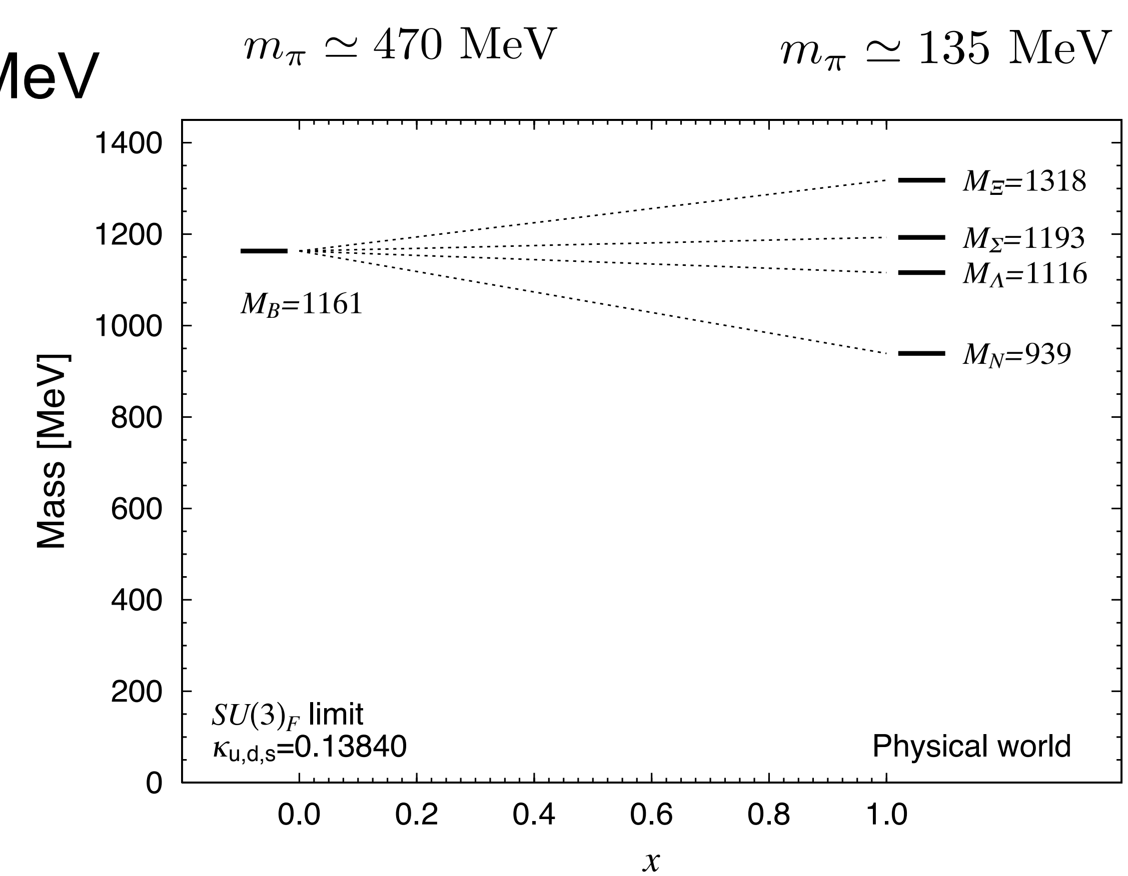


5. Some applications to nuclear physics

H-dibaryon with the flavor SU(3) breaking



$$m_u = m_d \neq m_s$$



Our approximation for SU(3) breaking

1. Linear interpolation of octet baryon masses

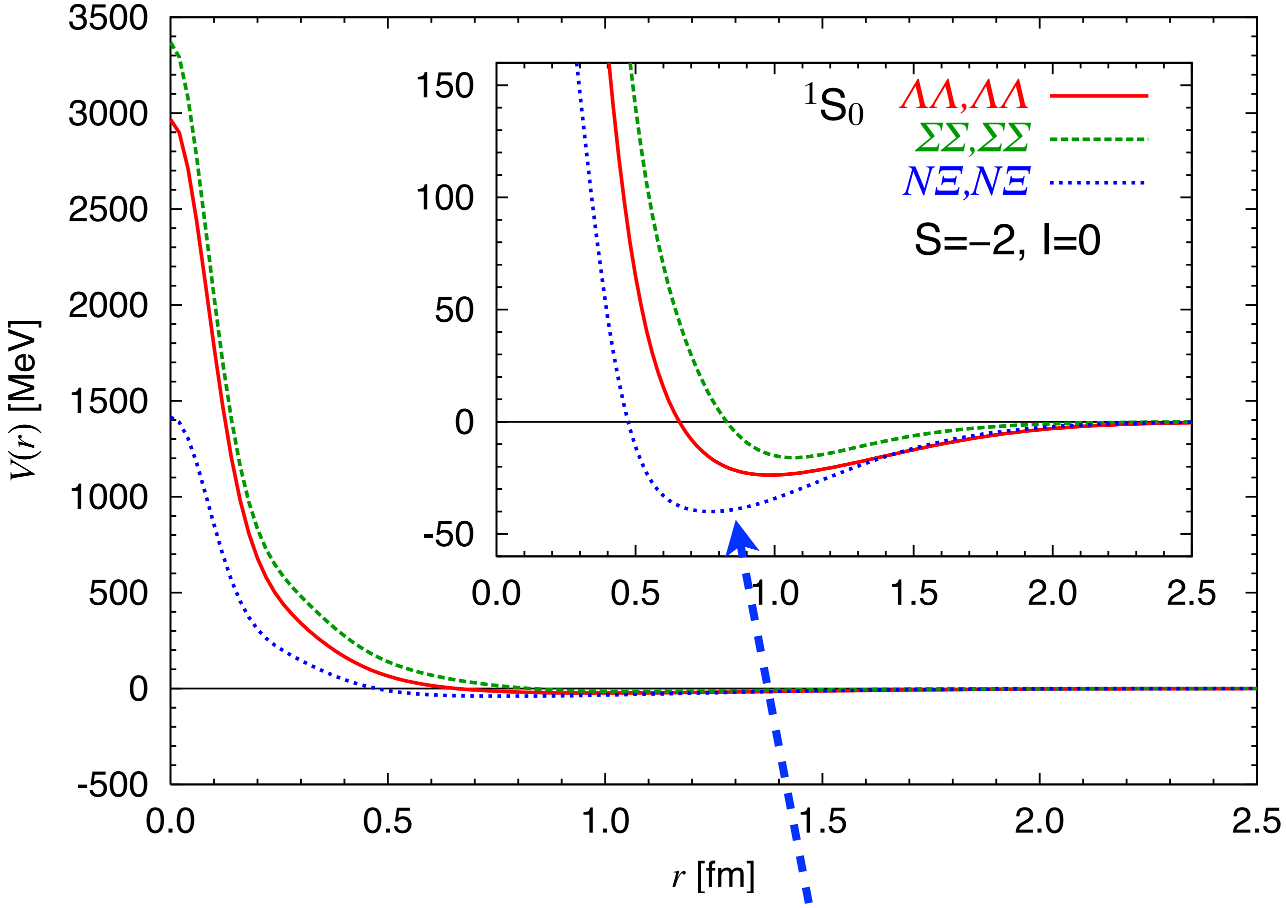
$$M_Y(x) = (1-x)M_Y^{\mathrm{SU(3)}} + xM_Y^{\mathrm{Phys}}$$

2. Potentials in particle basis in SU(3) limit

Potentials in particle basis in SU(3) limit (S=-2, I=0)

$m_\pi \simeq 470$ MeV

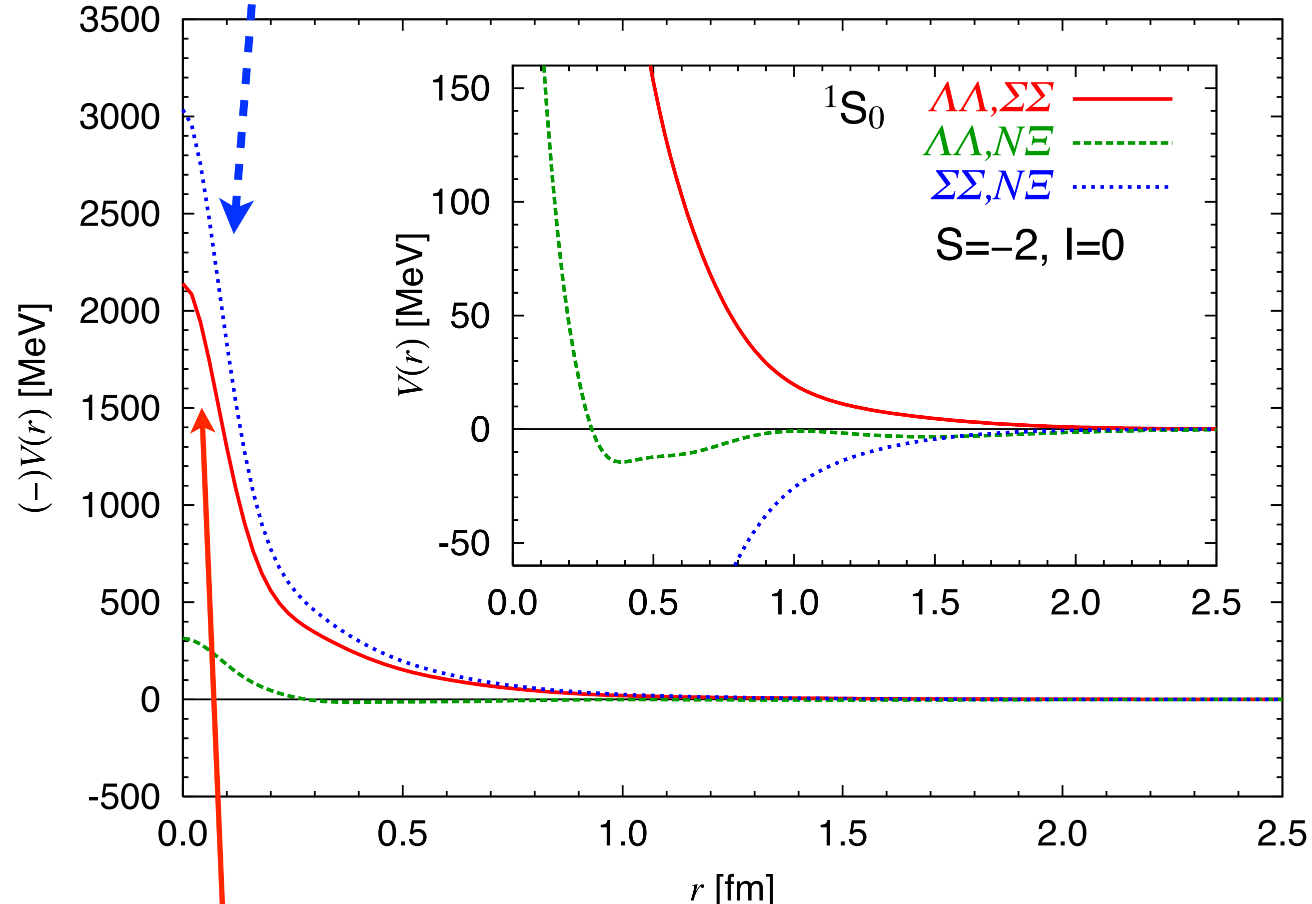
diagonal potential



most attractive

sizable

transition potential



sizable

This part needs to be improved.

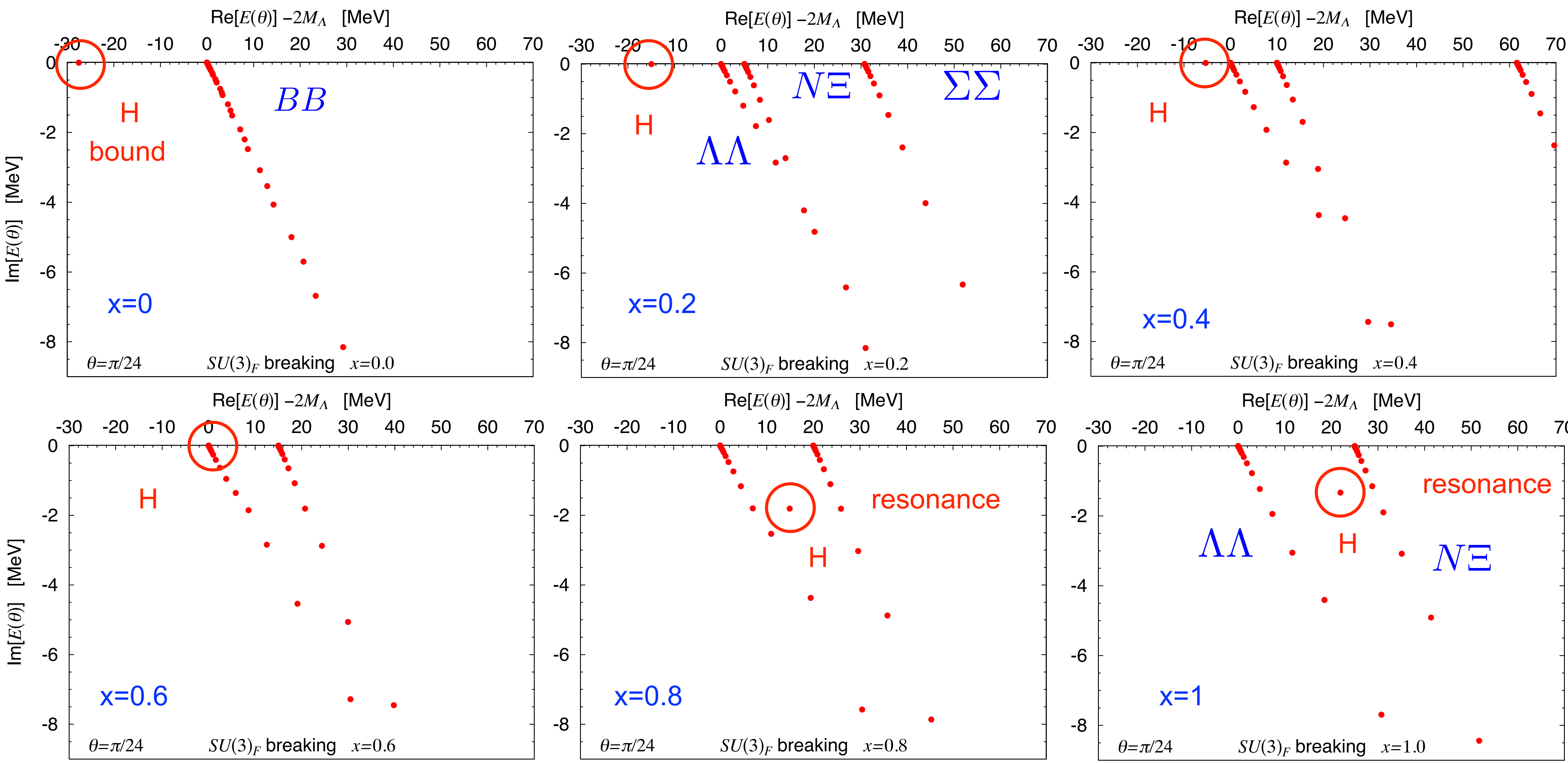
The direct calculation of potentials in 2+1 flavor QCD is in progress.

K. Sasaki *et al.* (HAL QCD Coll.), Lat 2012

Energy eigenvalues

complex scaling method

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

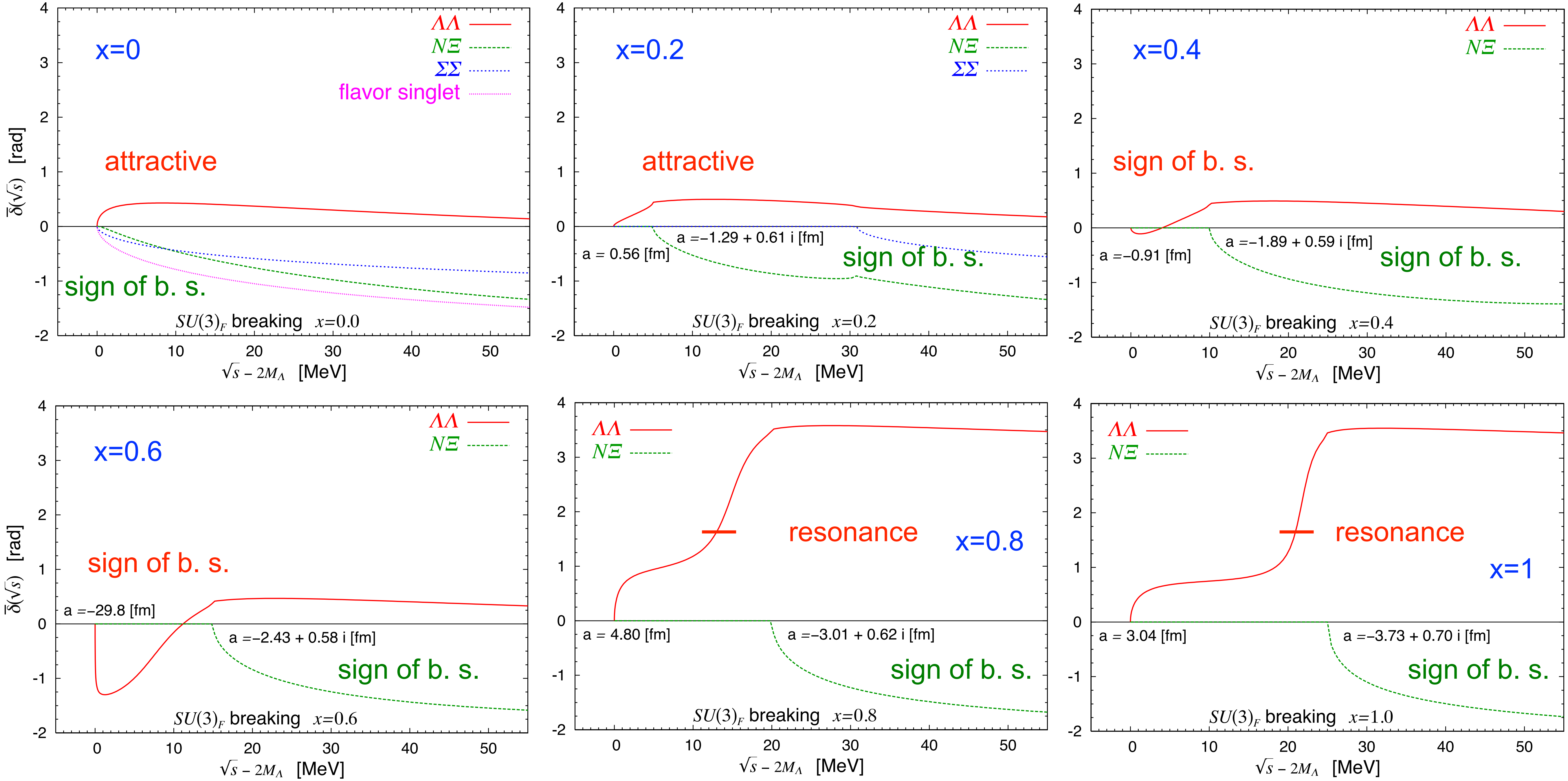


H-dibaryon seems to become resonance at physical point.

This needs a direct confirmation by 2+1 flavor QCD.

Phase shift

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]



H couples most strongly $N\Xi$.

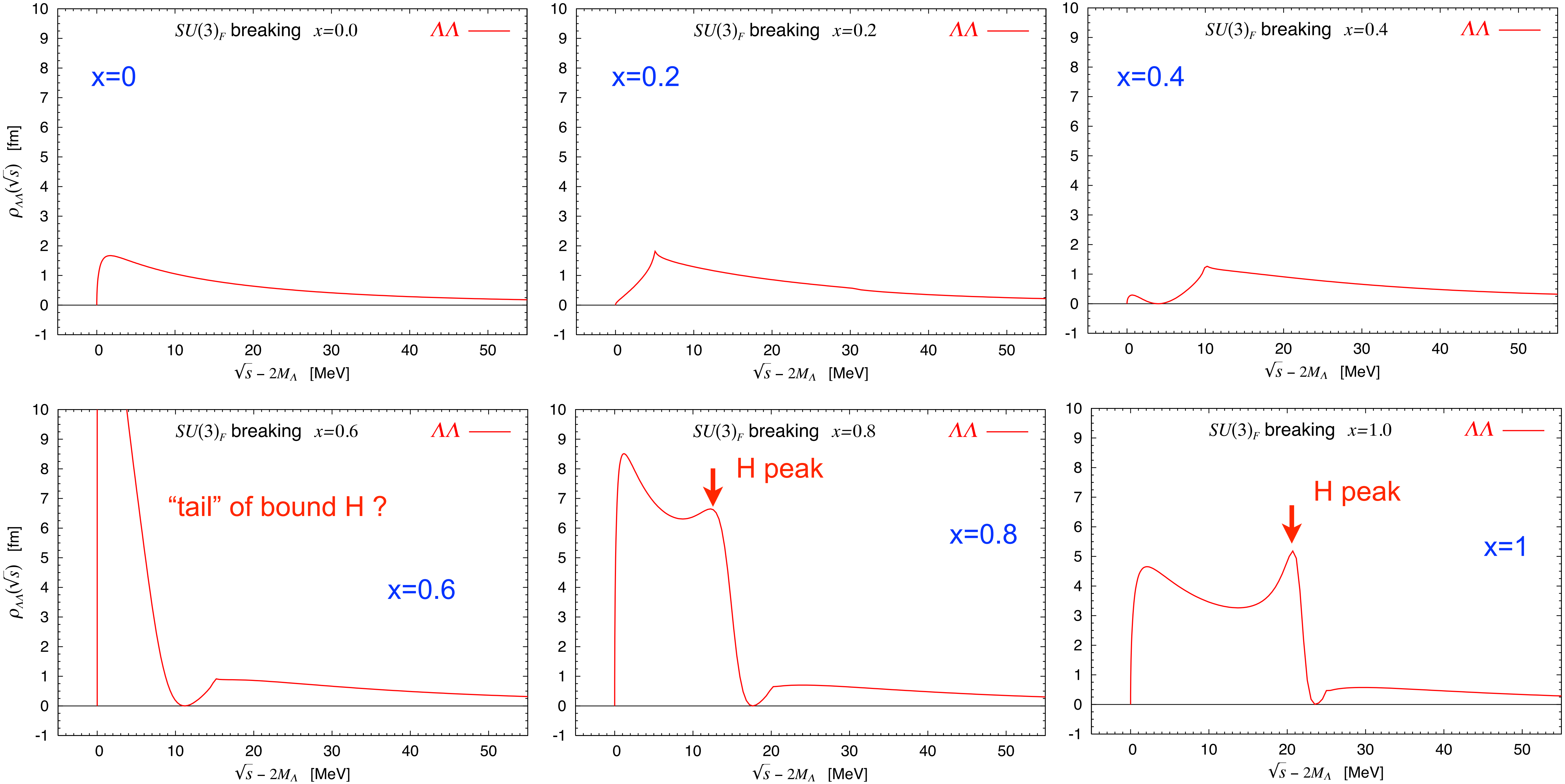
$\Lambda\Lambda$ interaction is attractive.

H has a sizable coupling to $\Lambda\Lambda$ near and above the threshold.

Invariant mass spectrum

$\Lambda\Lambda \to \Lambda\Lambda$

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

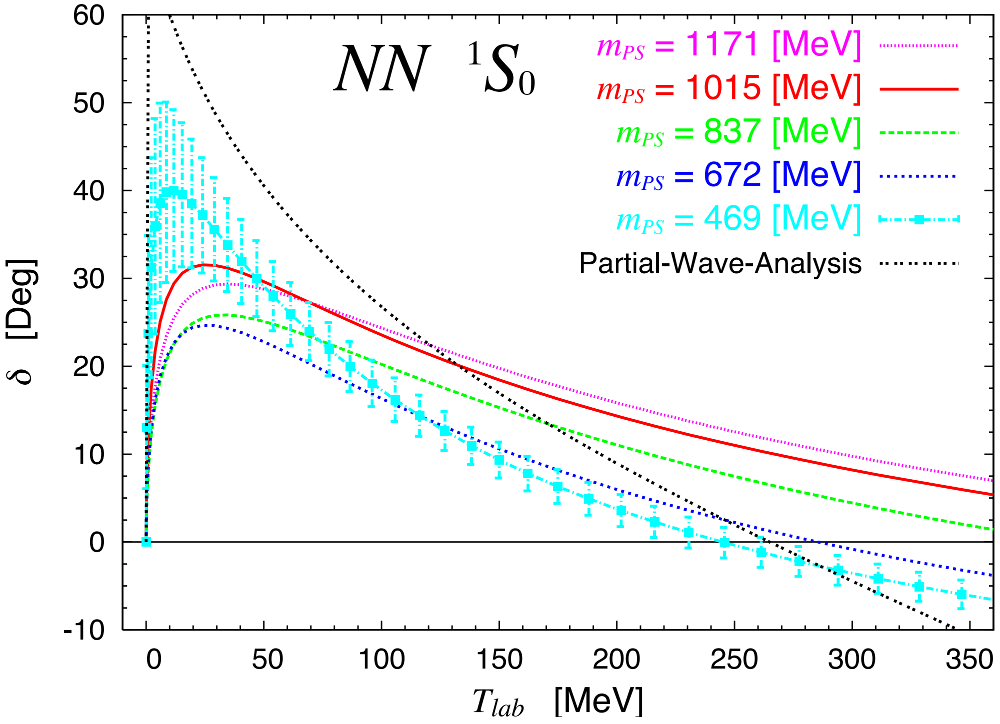


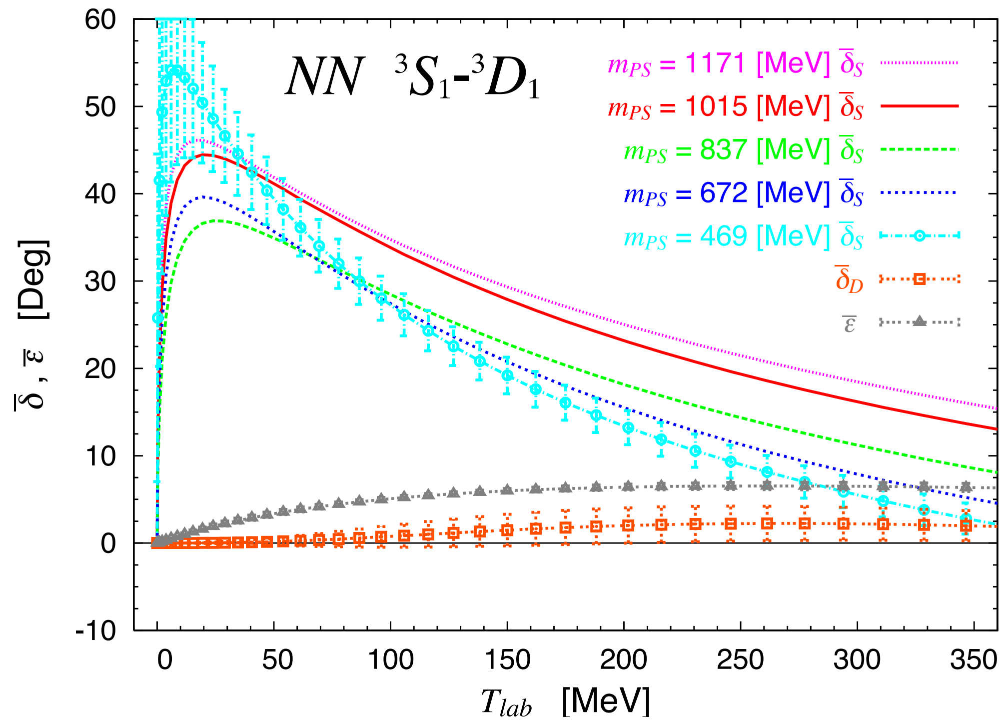
A peak of the resonance H might be observed in experiments !?

Other observables in the flavor SU(3) limit

NN Phase shift, deuteron and 4N state

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

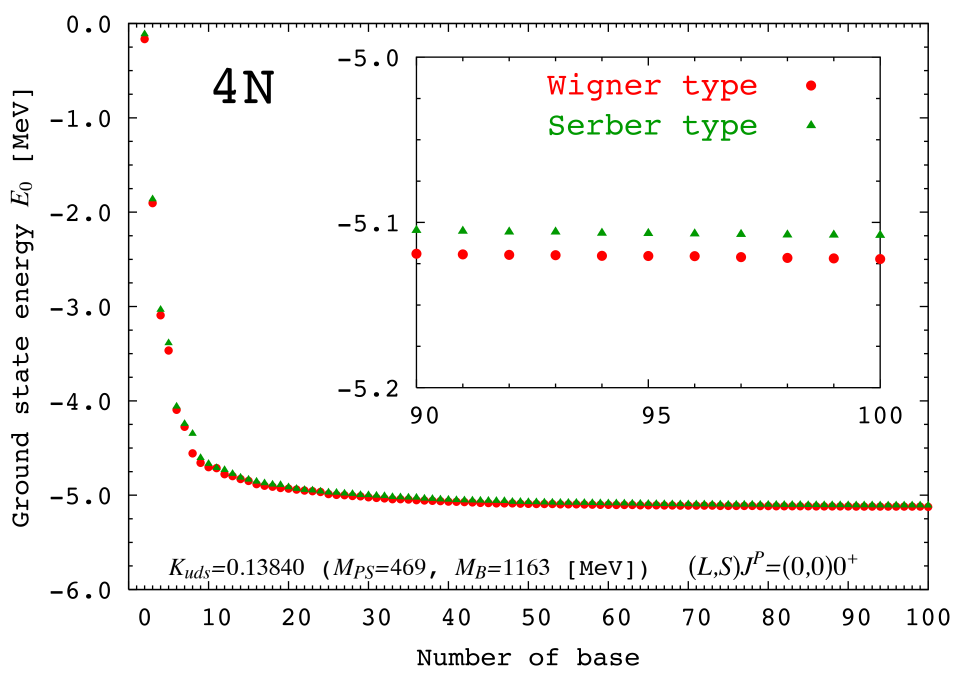




Attraction is stronger in triplet, but no deuteron so far.

Also, no 3N state.

binding energy by variational method



^{4}He $\quad (L,S)J^P = (0,0)0^+$

A 4N bound state exists at lightest pion mass.

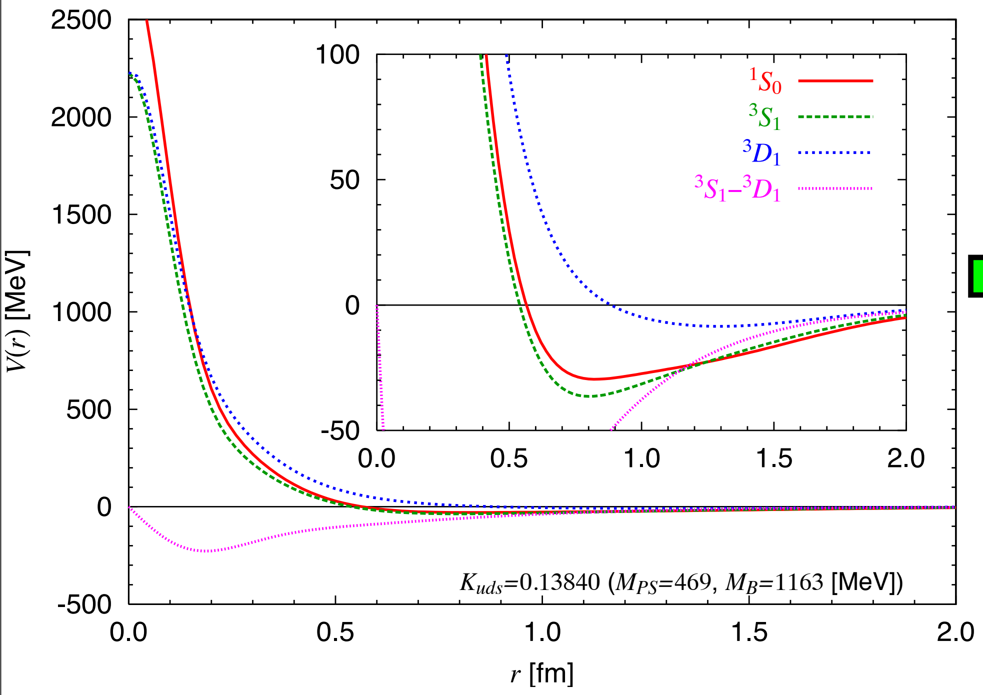
$$m_\pi = 470 \text{ MeV}$$

$$E_{4N} = -5.1 \text{ MeV}$$

EoS of nuclear matter

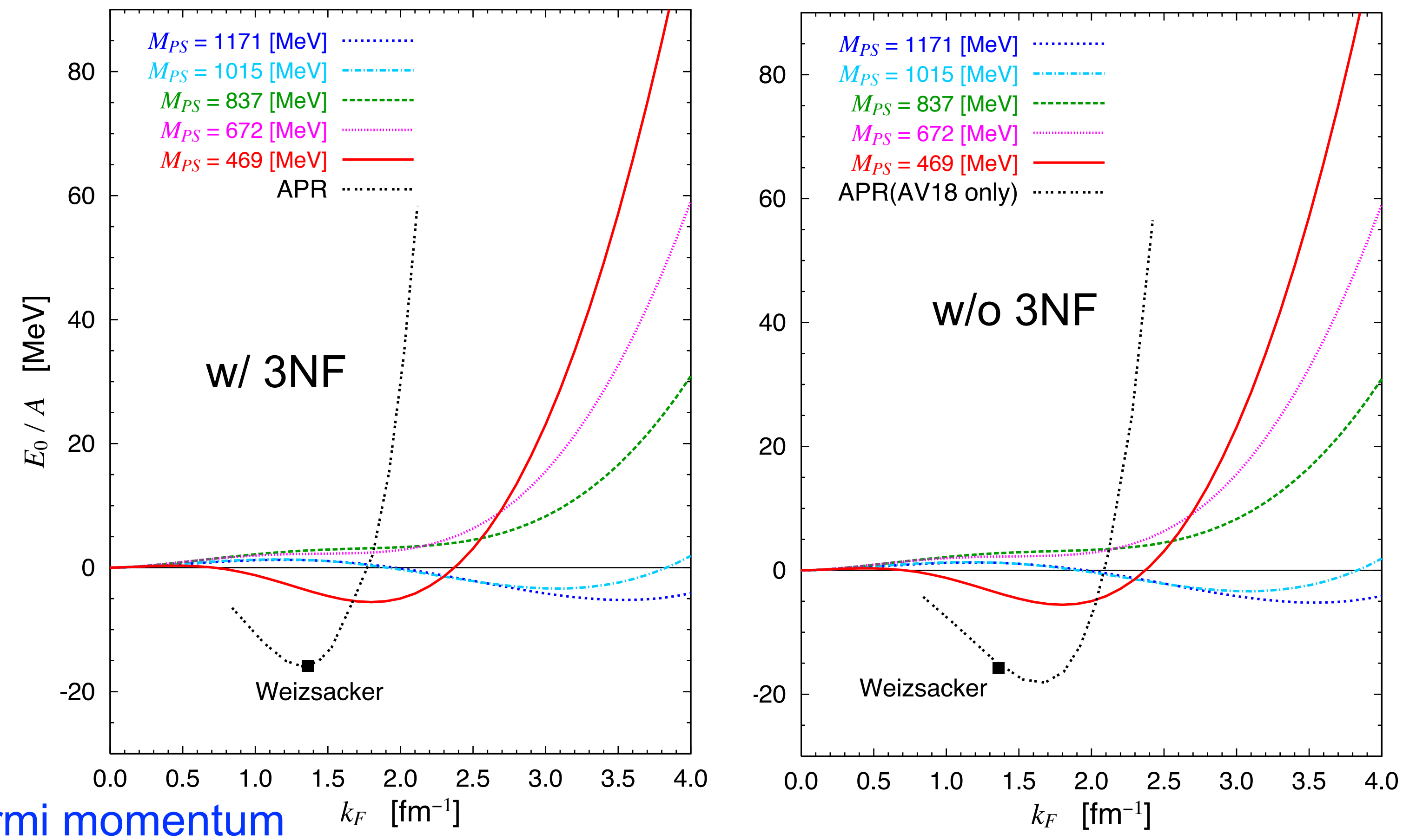
Inoue *et al.* (HAL QCD Coll.), in preparation

NN potentials $m_\pi = 470$ MeV



BHF

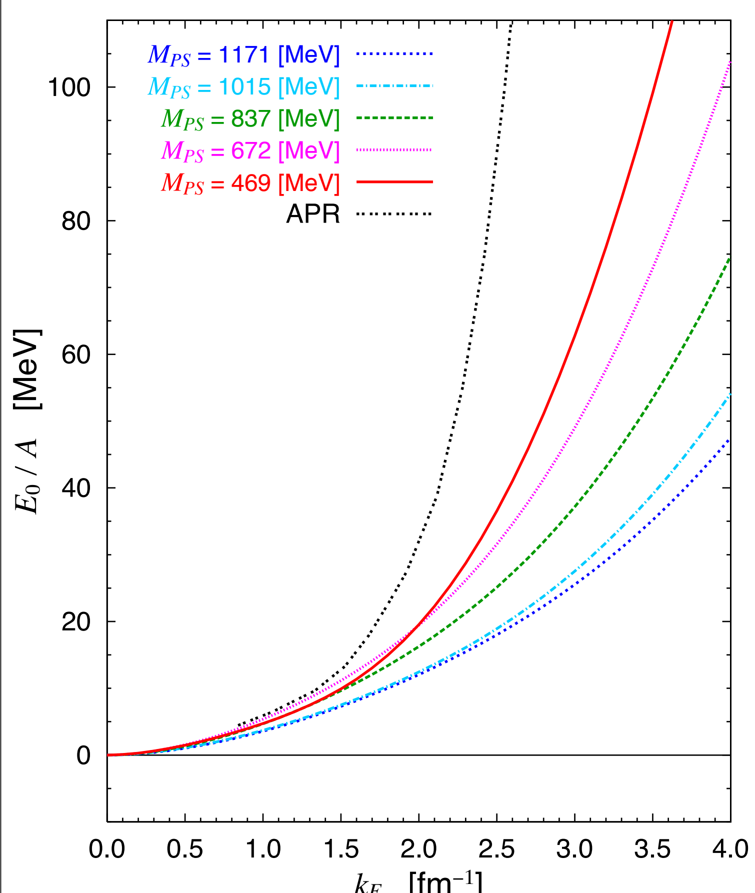
Energy density of Nuclear matter



Fermi momentum

A. Akmal, V.R. Pandharipande, G.G. Ravenhall, Phys. Rev. C58 1804 (1998)

Neutron matter

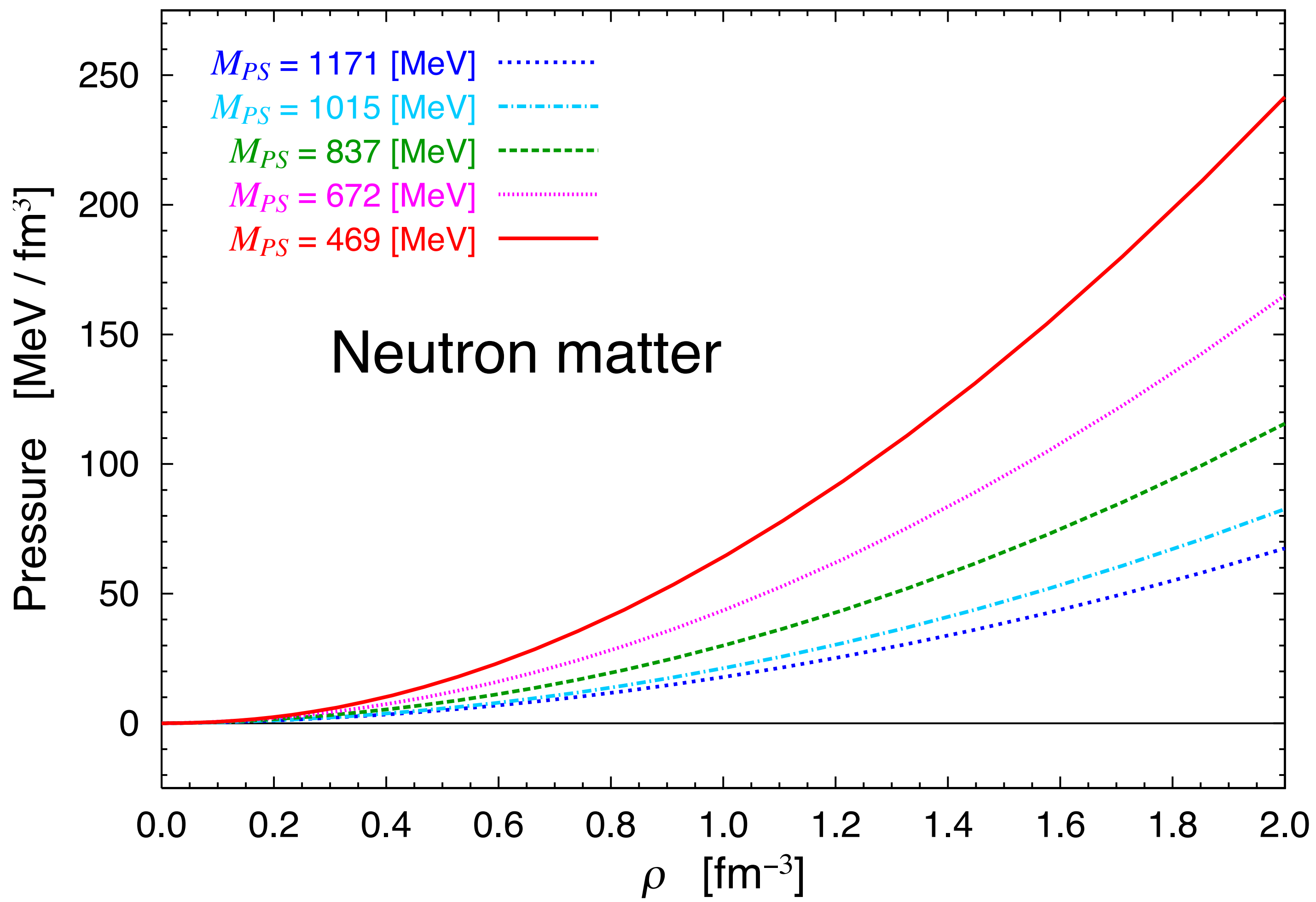


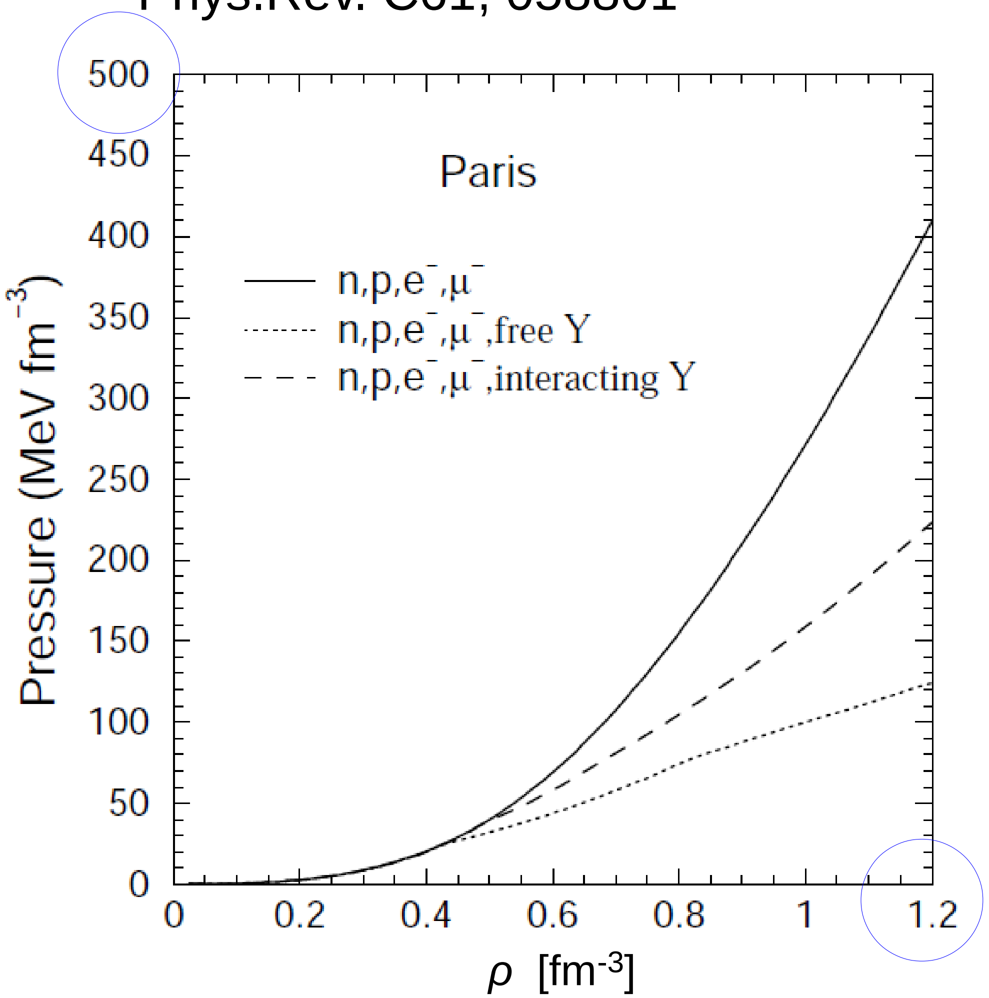
Nuclear matter shows the saturation at the lightest pion mass, but the saturation point deviates from the empirical one obtained by Weizsacker mass formula.

No saturation for Neutron matter.

Pressure of Neutron matter

M. Baldo, F. Burgio, H.-J.Schulze,
Phys.Rev. C61, 058801





pressure $$P = \rho^2 \frac{d(E_0/A)}{d\rho} = \frac{\gamma k_F^4}{18\pi^2} \frac{d(E_0/A)}{dk_F}$$

density $$\rho = \frac{\gamma k_F^3}{6\pi^2}$$

Our Neutron matter becomes harder as the pion mass decreases,
but it is still softer than phenomenological models.

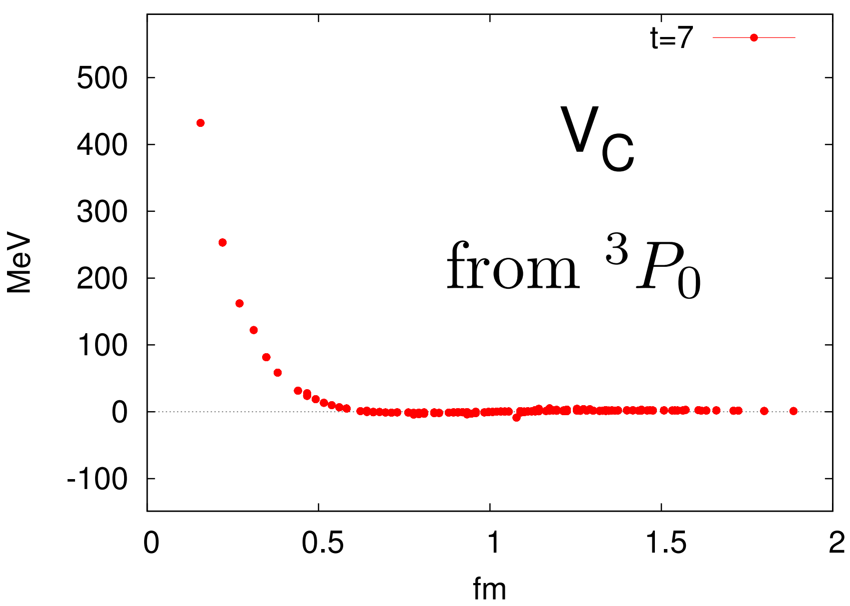
6. Other recent developments

Parity-odd potential and LS force

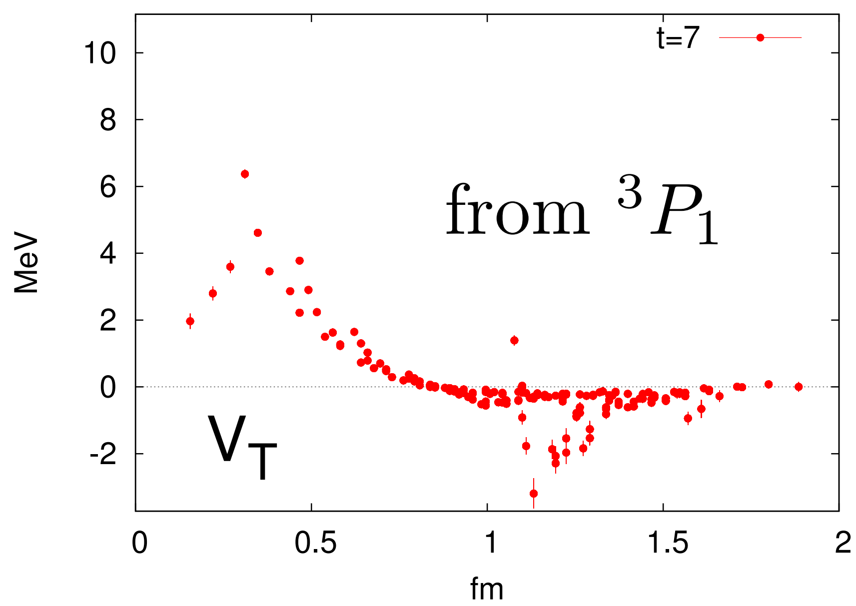
Murano et al. (HAL QCD), lat2012

2-flavor QCD, a=0.16 fm $m_\pi \simeq 1.1$ GeV

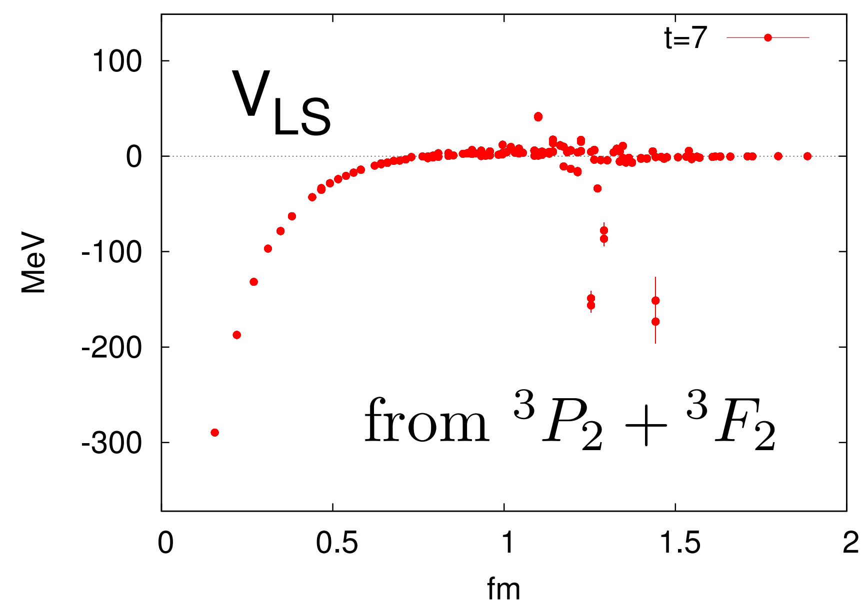
LO



LO

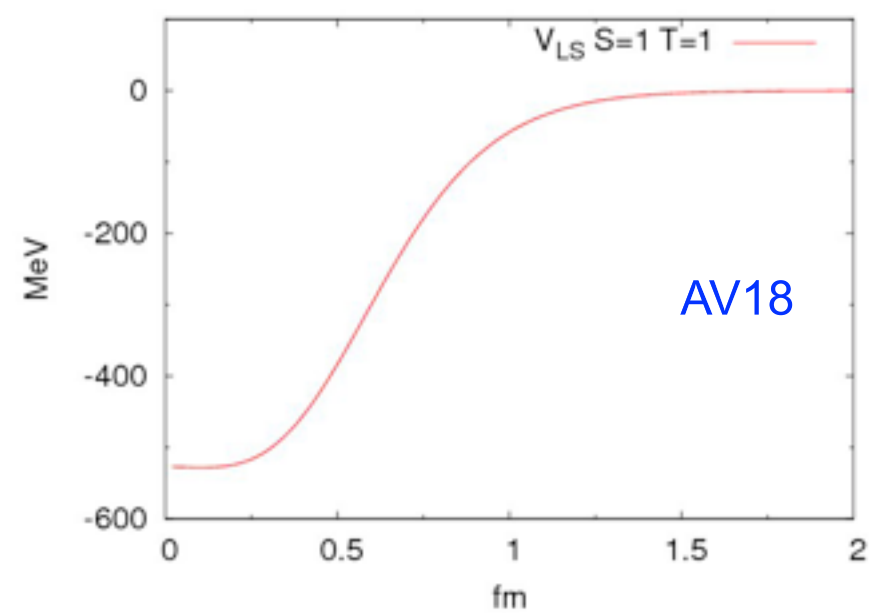


NLO



Very weak !

We now have all potentials at LO.



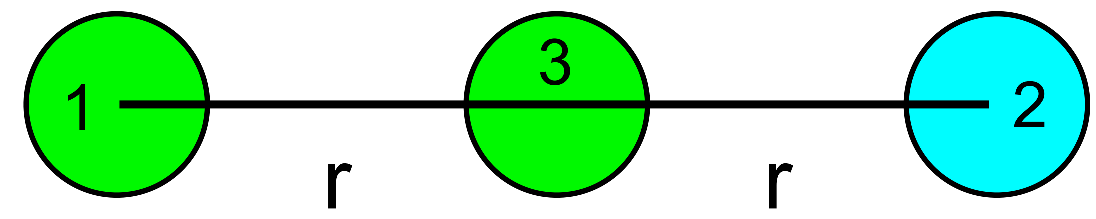
Three nucleon force (TNF)

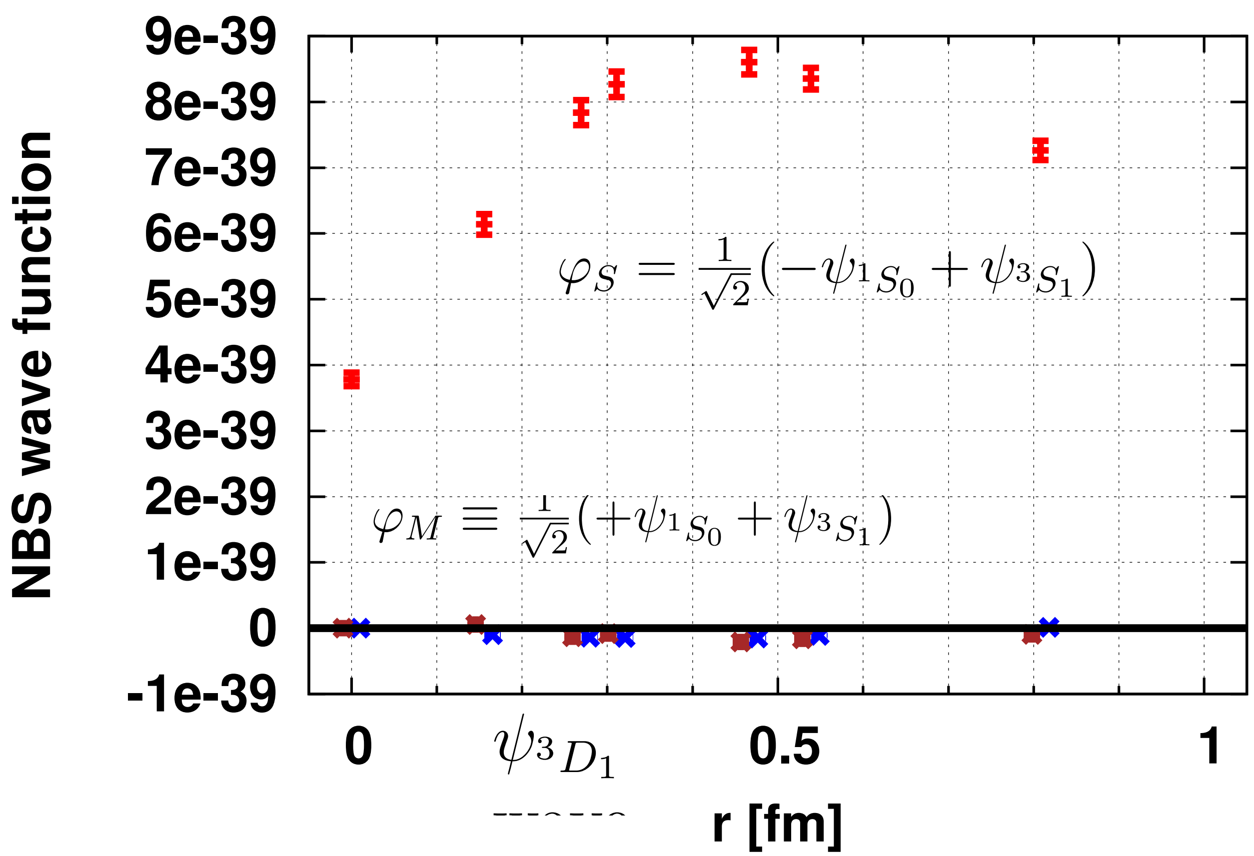
Doi et al. (HAL QCD), PTP 127 (2012) 723

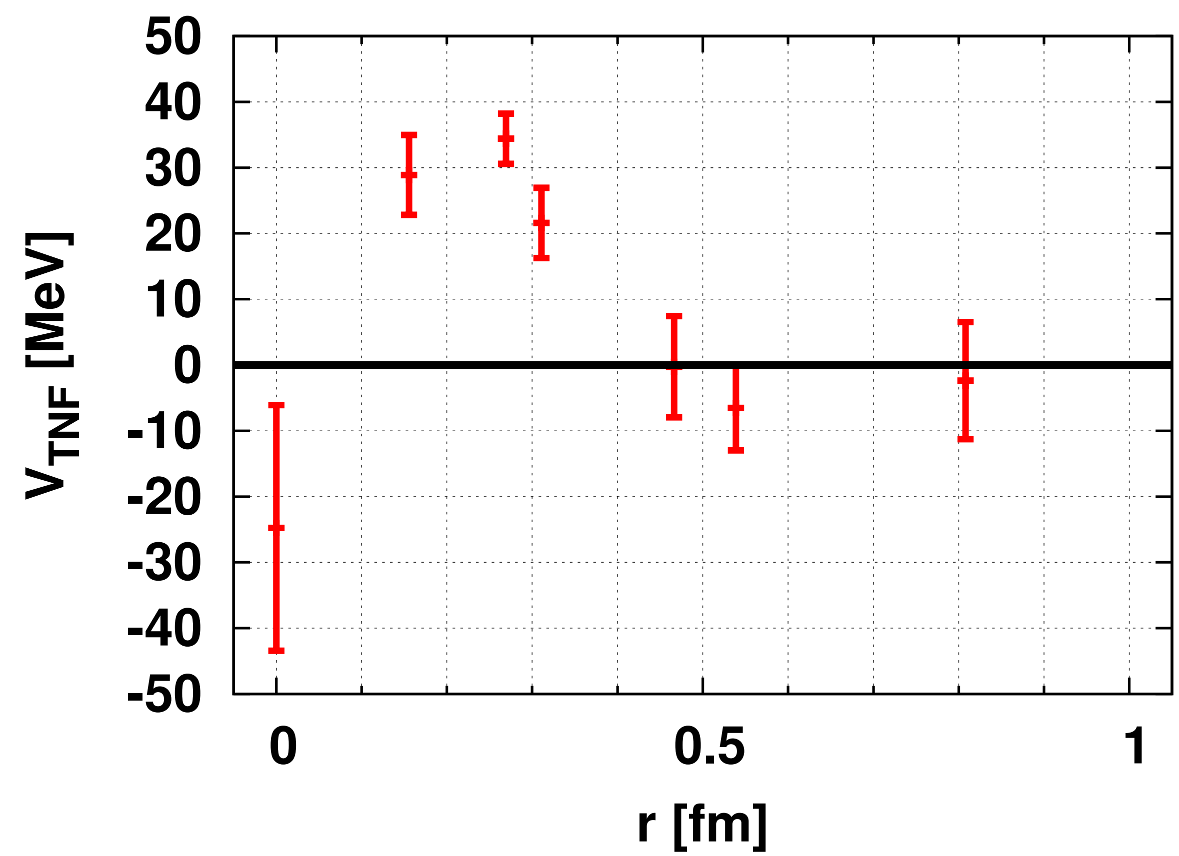
(1,2) pair $\quad {}^1S_0,\ {}^3S_1,\ {}^3D_1 \quad$ S-wave only

$\mathrm{Triton}(I = 1/2,\ J^P = 1/2^+)$

Linear setup







scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

Analysis by OPE (operator product expansion) in QCD predicts
universal short distance repulsions in TNF.

Aoki, Balog and Weisz, NJP14(2012)043046

7. Future prospect

- HAL QCD scheme is shown to be a promising method to extract hadronic interactions in lattice QCD.
  - ground state saturation is not required.
  - Calculate potential (matrix) in lattice QCD on a finite box.
  - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
  - bound/resonance/scattering
- Future directions
  - calculations at the physical pion mass on “K-computer”
  - hyperon interactions with the SU(3) breaking
  - Baryon-Meson, Meson-Meson
  - Exotic other than H such as penta-quark, X, Y etc.
  - 3 Nucleon forces
  - Other applications ? (weak interaction ?)

Please join us if you are interested in.