

Microphysics for SN: Nuclear Equation of State, Neutrino Opacity with Medium Correction

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Outline

- Nuclear EOS
 - Overview
 - Current EOS tables
 - Further development
- Neutrino opacity: unified approach
 - Mean field effect: Charged current reactions
 - RPA correlation
 - Beyond RPA: 2-ph response
- Summary

SN EOS ($T > 0$)

Inputs

- Low density ($< \sim 10^{12}$ g/cm³)
 - Single-nucleus vs **multi-nuclei**
 - Multi-nuclei with nucleons: NSE vs **Virial+NSE**

Mass tables
Medium corrections
Correlations: NN, ...

-
- PNS “crust” ($< \sim 0.5 n_0$)
 - Mean field (MF) models: Skyrme vs **RMF**
 - Liquid drop vs Thomas-Fermi vs **Hartree**
 - efficiency & necessity
 - Uniform matter ($< \sim 2n_0$)
 - MF: efficient/not systematic

Constraints on MF

- Saturation property of symmetric matter
- Stable nuclei (symmetric)
- Isovector observables
 - IVGDR
 - Neutron skin of nuclei
 - Neutron matter

-
- High density $> \sim 2 n_0$
 - Skyrme vs **RMF** vs ??? (meson, quark,...)
 - Best parameterize uncertainties

– Neutron star

Only a few realistic EoS used in astrophysical simulations

- most well known and widely used EoS:
 - [J.M. Lattimer, F.D. Swesty](#) LS 180, 220, 375 - non relativistic Skyrme model
 - [H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi](#) STOS - relativistic MF model + 2nd one with hyperon (Mmax: ~1.7 solar mass)
- more recently developed EoSs:
 - NSE + uniform matter at high density (RMF)
 - [M. Hempel, J. Schaffner-Bielich](#), 2010

Virial EOS (gas) + Hartree nonuniform (solid) + uniform (liquid) (RMF)

- [G. Shen, C.J. Horowitz, S. Teige](#) NL3, 2011
- [G. Shen, C.J. Horowitz, E. O'Connor](#) FSU, 2011

http://cecelia.physics.indiana.edu/gang_shen_eos/

Ensemble of nucleons and nuclei

- Grand partition function in virial expansion

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3} [z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] \quad \triangleright \text{nucleon-nucleon}$$

$$+ \frac{1}{\lambda_\alpha^3} [z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha (z_n + z_p) b_{\alpha n}] \quad \triangleright \text{nucleon-alpha}$$

$$+ \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i \quad \triangleright \text{nuclei}$$

1. Nucleon and alpha: mod. ind.

Horowitz, Schwenk '05

2. Heavy species: 8980 nuclei

FRDM mass table: Moller et al '97. $\Delta_{\text{rms}} \sim 0.6 \text{ MeV}$

Chemical equilibrium -----

$$\mu_i = Z\mu_p + N\mu_n \quad z_i = z_p^Z z_n^N e^{(E_i - E_i^C)/T}$$

Coulomb correction -----

$$E_i^C = \frac{3}{5} \frac{Z_i^2 \alpha}{r_A} \left[-\frac{3}{2} \frac{r_A}{r_i} + \frac{1}{2} \left(\frac{r_A}{r_i} \right)^3 \right]$$

Nuclear partition function Ω_i ----

eg, Fowler, Engelbrecht, Woosley, '78

Nuclei spacing

$$\frac{4}{3} \pi r_i^3 \left(\sum_j Z_j n_j \right) = Z_i$$

3. Solving: z_n, z_p, r_i :

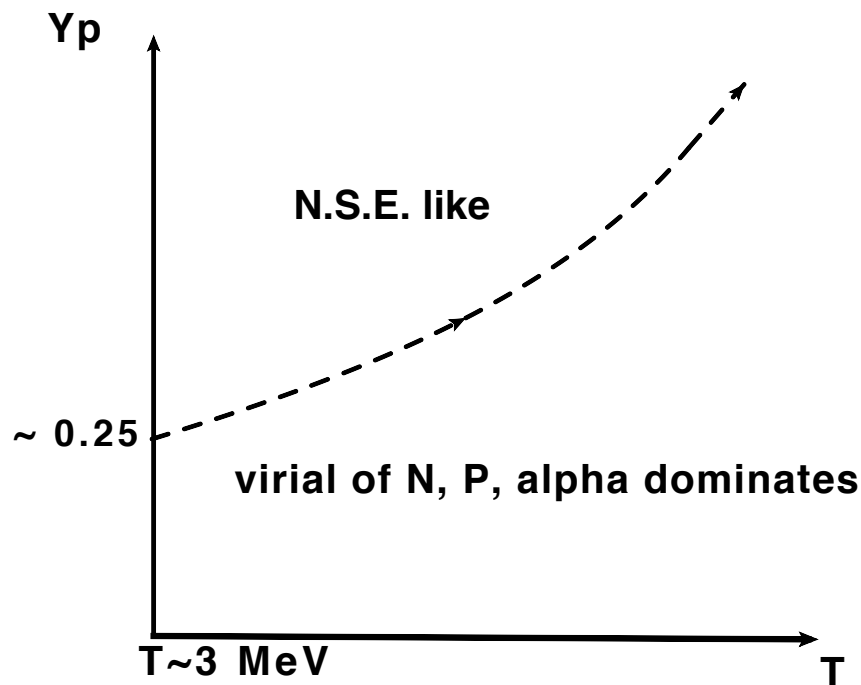
$$n_B = n_n + n_p + 4n_\alpha + \sum_i A_i n_i = \text{Baryon}$$

$$Y_P = (n_p + 2n_\alpha + \sum_i Z_i n_i) / n_B = \text{Charge}$$

4. Mass fraction:

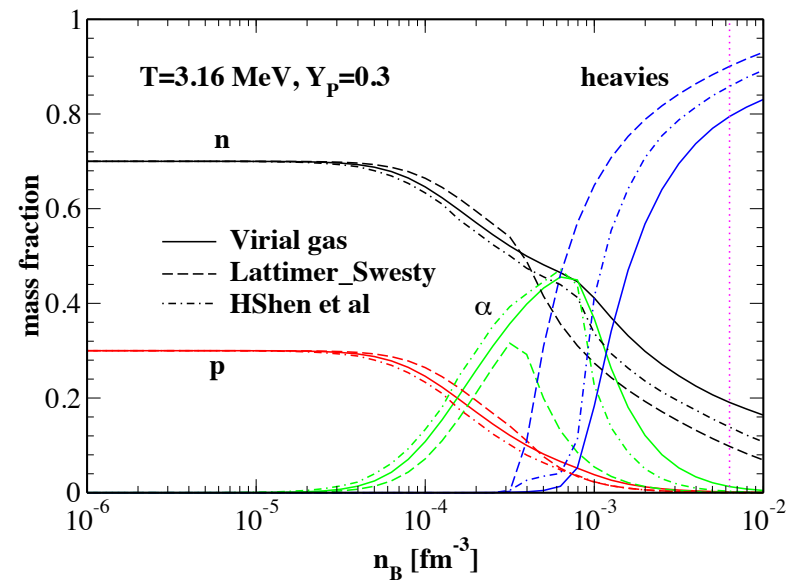
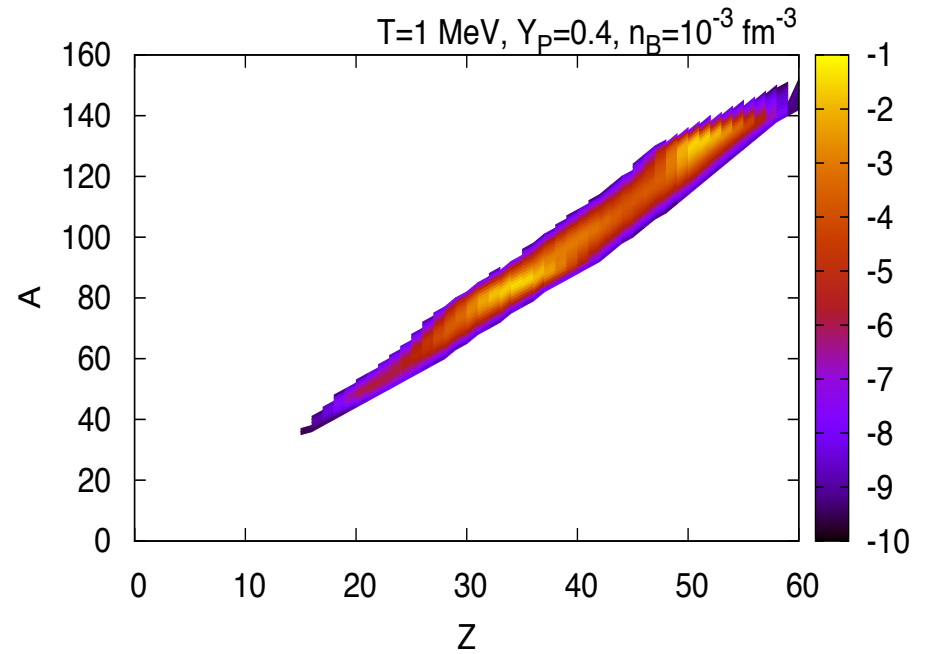
$$X_a = A_a n_a / n_B$$

- $n < 0.1 - 0.01 n_0$
- $T < 12 \text{ MeV}$



- Mass 2, 3 nuclei can be included in virial: O'Connor et al, 2008
- M. Hempel et al, 2012 partially addressed the role of multi-nuclei in 1D SN simulation

Mass distributions in Virial gas - multi-nuclei distribution



Relativistic Mean Field Theory

attractive

repulsive

Iso-vector

$$\mathcal{L}_{\text{int}} = \bar{\psi} \left[g_s \phi - \left(g_v V_\mu + \frac{g_\rho}{2} \boldsymbol{\tau} \cdot \mathbf{b}_\mu + \frac{e}{2} (1 + \tau_3) A_\mu \right) \gamma^\mu \right] \psi$$

$$- \frac{\kappa}{3!} (g_s \phi)^3 - \frac{\lambda}{4!} (g_s \phi)^4 + \frac{\zeta}{4!} g_v^4 (V_\mu V^\mu)^2 + \Lambda_v g_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu g_v^2 V_\nu V^\nu$$

- Ψ : Nucleon fields
- Meson/photon fields: classical expectation value \rightarrow mean field
- 7 adjustable parameters

EoMs in real space:

Nucleon:

$$\left[\boldsymbol{\alpha} \cdot \mathbf{p} + V(\mathbf{r}) + \beta (M + S(\mathbf{r})) \right] \psi_i = \varepsilon_i \psi_i$$

Hartree Mean fields:

$$V(\mathbf{r}) = \beta \left\{ g_\omega \rho_0 + g_\rho \vec{\tau} \cdot \vec{\rho}_\mu + e \frac{(1 + \tau_3)}{2} A_\mu + \Sigma^R \right\},$$

$$S(\mathbf{r}) = \Gamma_\sigma \sigma,$$

Mesons and Photons:

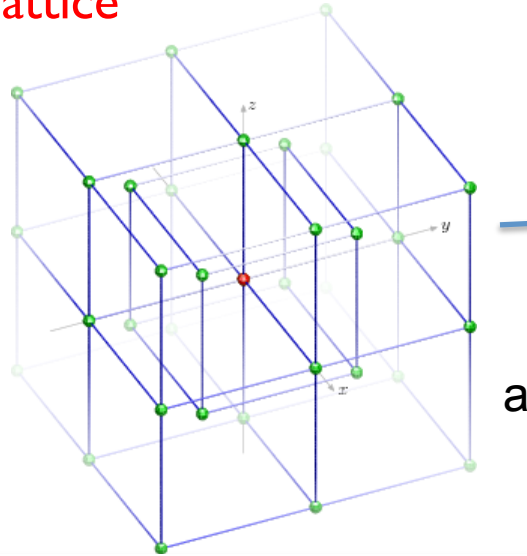
$$\left\{ \begin{array}{l} (-\Delta + \partial_\sigma U(\sigma)) \sigma(\mathbf{r}) = -g_\sigma \rho_s(\mathbf{r}) \\ (-\Delta + m_\omega^2) \omega_0(\mathbf{r}) = g_\omega \rho_v(\mathbf{r}) \\ (-\Delta + m_\rho^2) \rho_0(\mathbf{r}) = g_\rho \rho_3(\mathbf{r}) \\ -\Delta A_0(\mathbf{r}) = e(\rho_c(\mathbf{r}) - \rho_e) \end{array} \right.$$

Finite temperature n_i : Fermi-Dirac stat.

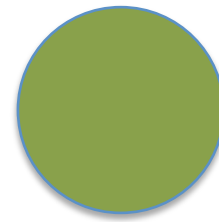
$$\left\{ \begin{array}{l} \rho_s(\mathbf{r}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{r}) \psi_i(\mathbf{r}) n_i, \rho_3(\mathbf{r}) = \sum_{i=1}^A \psi_i^+(\mathbf{r}) \tau_3 \psi_i(\mathbf{r}) n_i \\ \rho_v(\mathbf{r}) = \sum_{i=1}^A \psi_i^+(\mathbf{r}) \psi_i(\mathbf{r}) n_i, \rho_c(\mathbf{r}) = \sum_{i=1}^A \psi_i^+(\mathbf{r}) \frac{1 - \tau_3}{2} \psi_i(\mathbf{r}) n_i \end{array} \right.$$

proto-neutron star crust ($\Gamma \gg 1$)

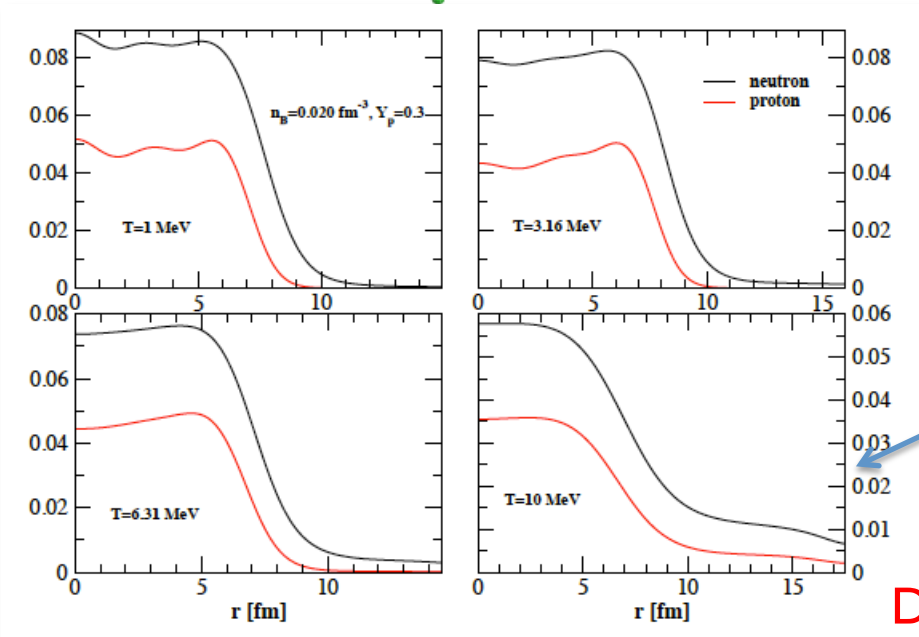
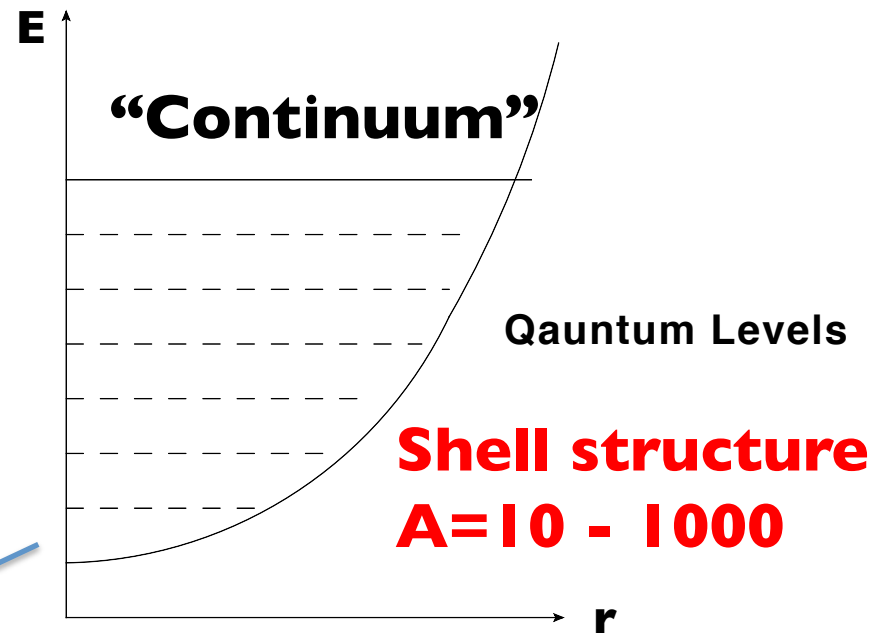
BCC lattice



$$\Gamma = (Ze)^2 / ak_B T$$

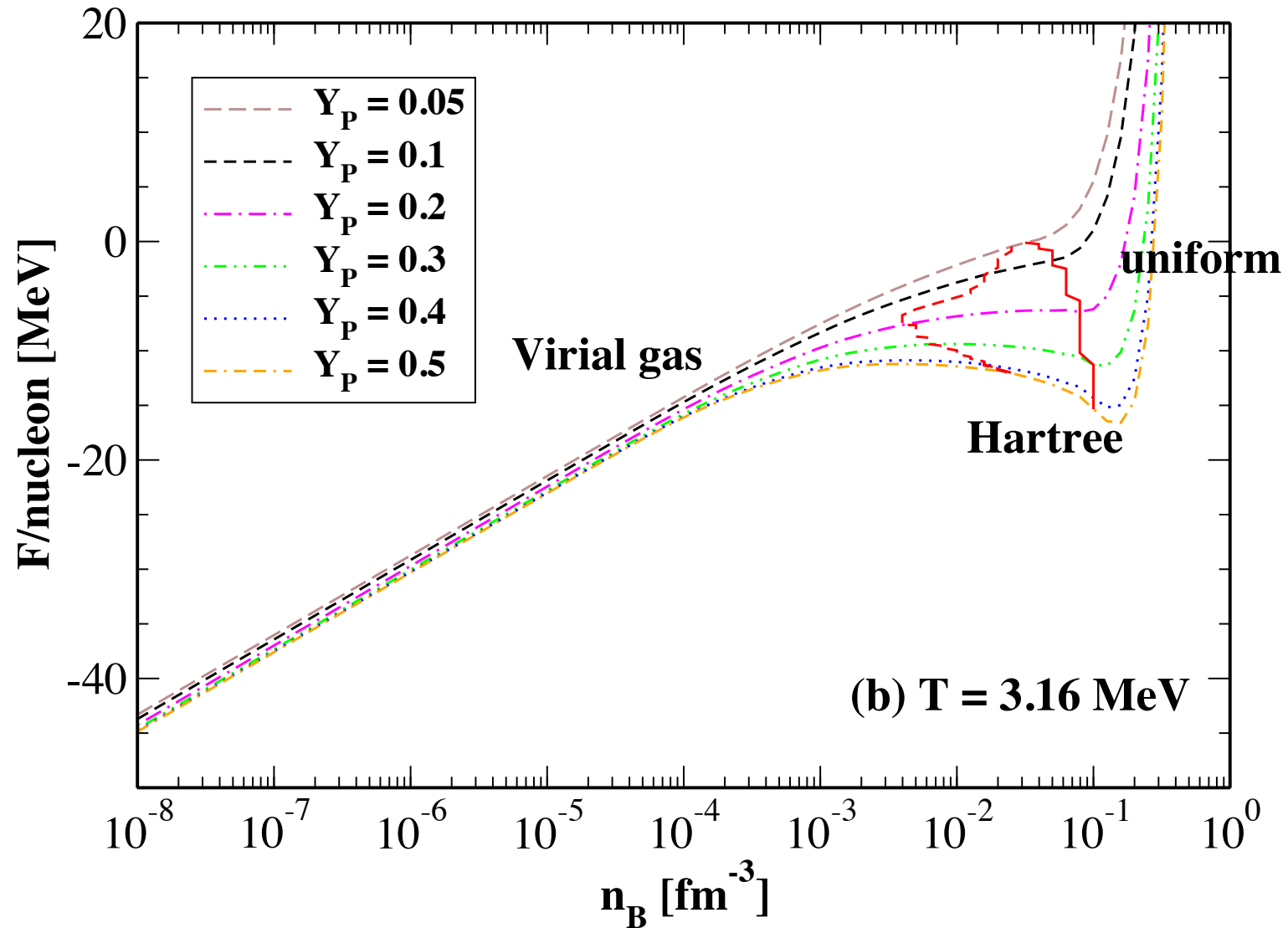


Wigner-Seitz spherical cell



Density distribution in the unit lattice

Matching EOS from low to high densities



3-D Parameter spaces (T, ρ, Y_p) for gas-solid-liquid

	Virial Gas	Hartree	Uniform matter
Temperature [MeV]	0.1~20	0.1~12	0.1~80
Density [fm ⁻³]	1E-8~1E-1	1E-4~1E-1	1E-8~1.6
Proton fraction	0.05~0.56	0.05~0.56	0~0.56
# points in phase space	73,840	17,021	90,478
CPU time (hr)	10,000	100,000	100

- Matching them and interpolate to get thermodynamically consistent Table.

Constraints on mean fields

- Saturation of symmetric nuclear matter

B/A, K, J

- Doubly magic nuclei: B/A, R_{ch}

- Giant/Collective resonances

Isospin ind. observables

- Isoscalar Giant Monopole Resonance

-
- Isovector Giant Dipole Resonance

Isospin dep. observables

- Neutron skin of nuclei

- Benchmark neutron matter: Chiral EFT, QMC

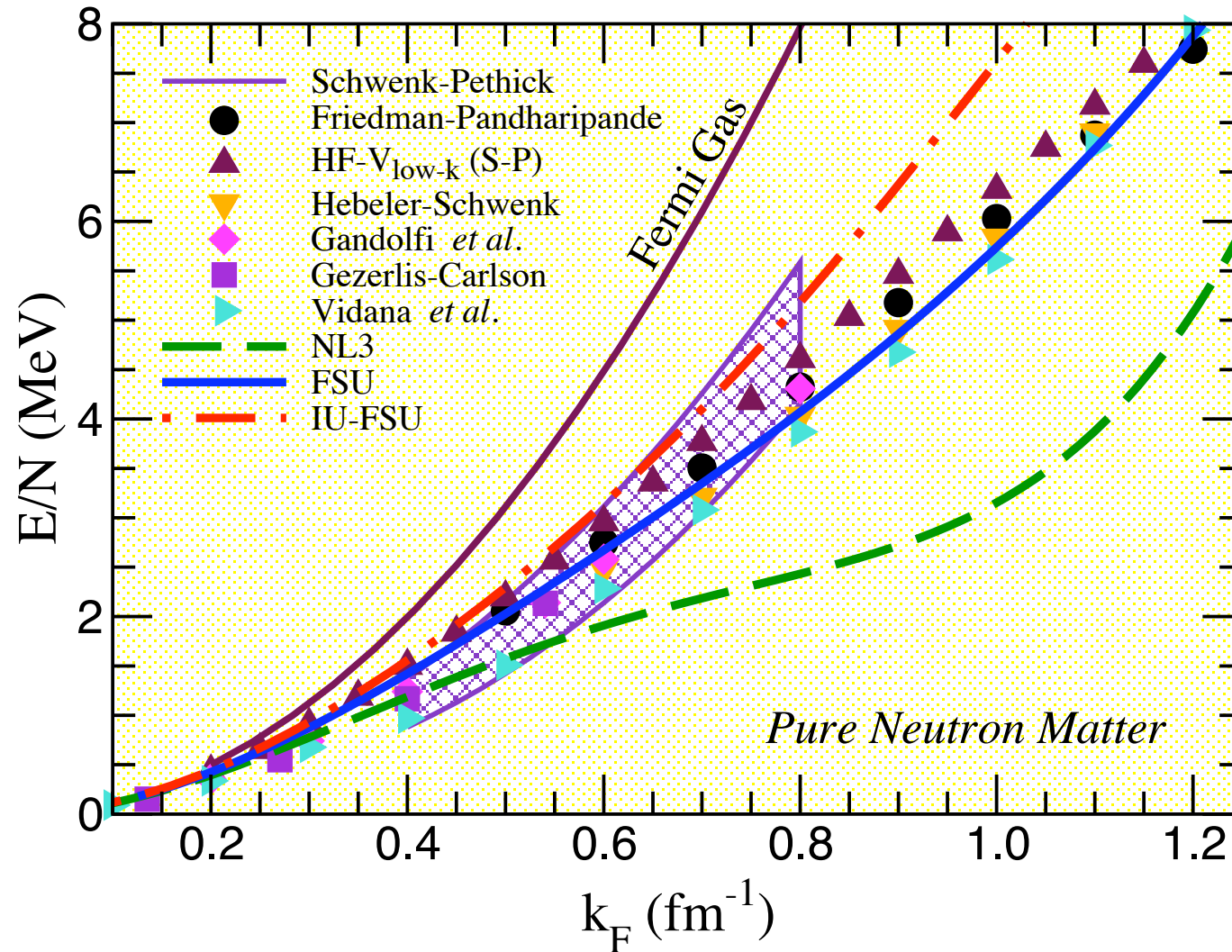
- NS observations

GMR, GDR in Pb208, Zr90

Nucleus	Observable	Experiment	NL3	FSU	IU-FSU
^{208}Pb	GMR (MeV)	14.17 ± 0.28	14.32	14.04	14.17
^{90}Zr	GMR (MeV)	17.89 ± 0.20	18.62	17.98	17.87
^{208}Pb	IVGDR (MeV)	13.30 ± 0.10	12.70	13.07	13.24

Fattoyev, Horowitz, Piekarewicz, Shen (2010)

Energy of Neutron matter



New developments:

Hebeler, Schwenk, 2010

Gandolfi, Carlson, Reddy 2011

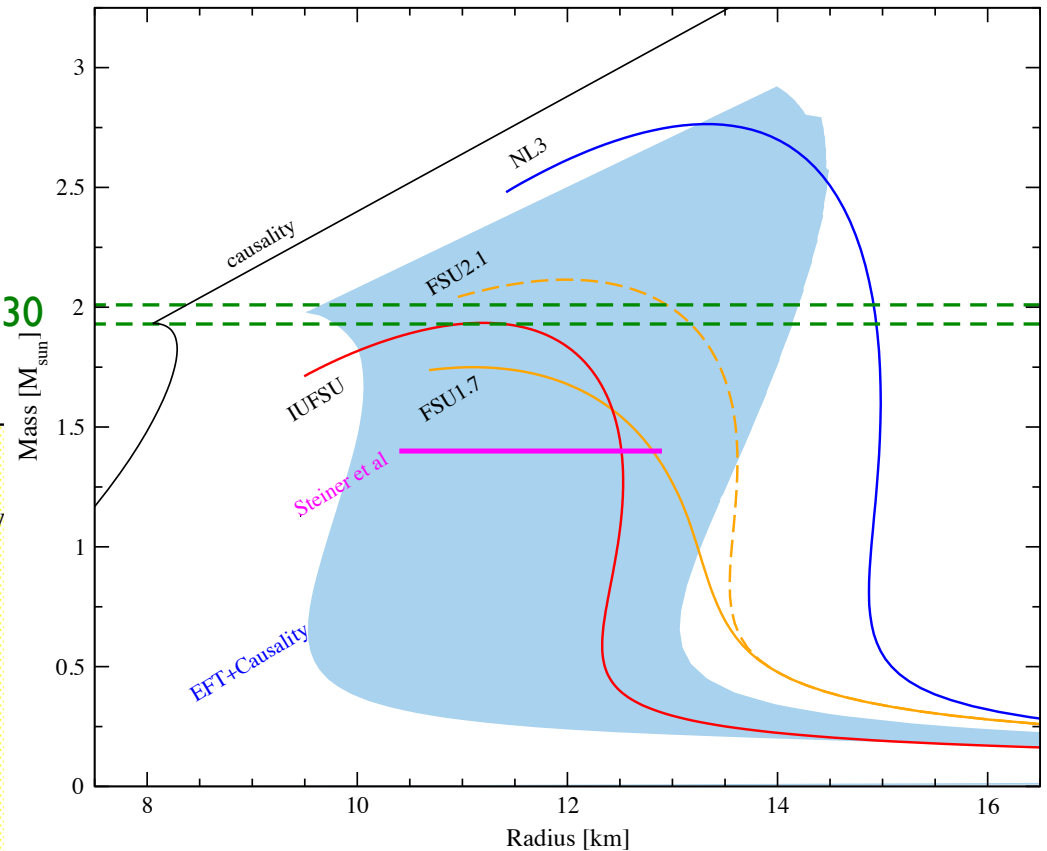
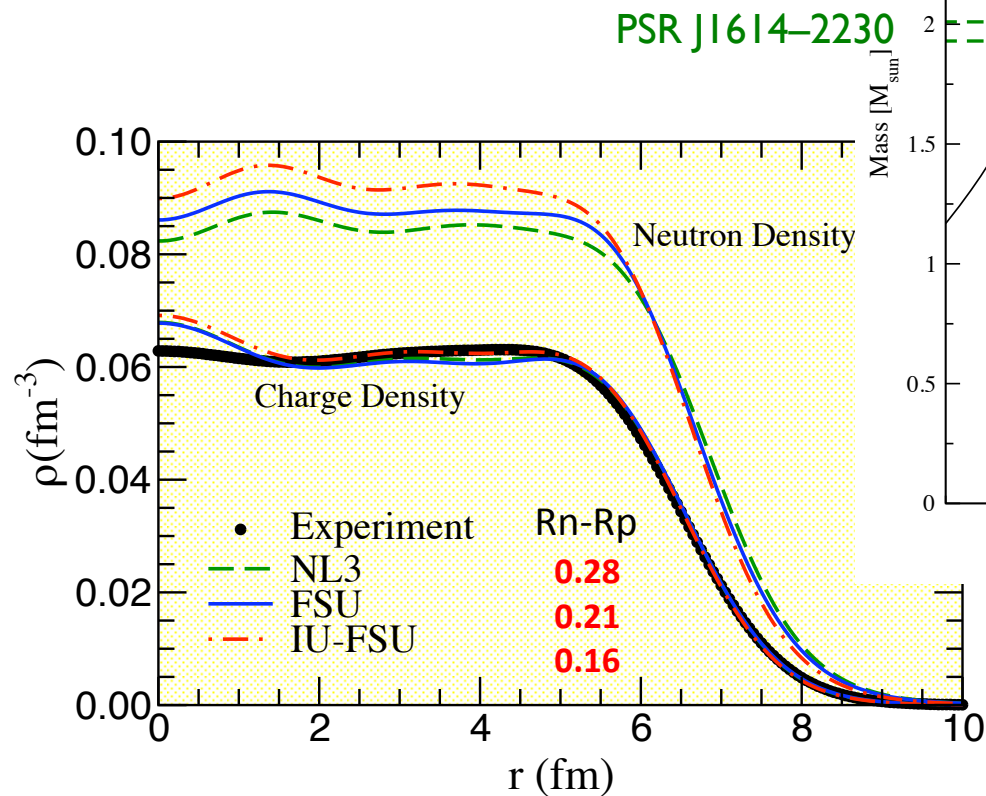
Fattoyev, Horowitz, Piekarewicz, Shen (2010)

Neutron skin and NS radius

A larger $E'_{\text{sym}}(n_0)$ indicates a bigger radius for 1.4 solar mass neutron star and a bigger neutron radius in 208Pb.

$$(L=3n_0E'_{\text{sym}}(n_0))$$

Models	NL3	FSUGold	IU-FSU
$n_0E'_{\text{sym}}(n_0)$ [MeV]	39.4	20.2	15.7



- Neutrino opacity: unified approach
 - Mean field effect: Charged current reactions
 - RPA correlation
 - Beyond RPA: 2-ph response

Neutrino opacity

Corrections to ν Opacities

- | | | | | |
|-------|-----|--|--------|---|
| ✓ | 1. | Phase space | } | Model Independent |
| | 2. | Matrix element | | |
| ✓ | a. | recoil | | |
| ✓ | b. | weak magnetism | | |
| | c. | form factors | | |
| | d. | strange quarks | } ~ 1% | |
| ✓ | 3. | Pauli blocking | | |
| ✓ | 4. | Fermi/thermal motion of initial nucleons | | |
| ✓ | 5. | Coulomb interactions | | |
| ✓ | 6. | Mean field effects | } | Consistent with EOS |
| ✓ | 7. | NN Correlations in RPA | | |
| ✓ | 8. | NN Correlations beyond RPA | | |
| | 9. | Meson exchange currents | ~ 1% | |
| | 10. | Other components such as hyperons | } | Uncertain,
Constrained by
2 Msun NS |
| | 11. | Other phases such as meson condensates or quark matter | | |
| | 12. | Corrections from superfluid/ superconductor pairing | | |
| ????? | 13. | Nonuniform matter | | |
| ????? | 14. | Magnetic field effects | | |

Neutrino cross section via response function

Neutral and charged current reactions contribute.

$$\begin{aligned}
 l_\mu &= \bar{\psi}_l \gamma_\mu (1 - \gamma_5) \psi_\nu, & j_W^\mu &= \bar{\psi}_4 \gamma^\mu (g_V - g_A \gamma_5) \psi_2 & g_V &= 1, g_A = 1.26 \\
 \mathcal{L}_{int}^{cc} &= \frac{G_F}{\sqrt{2}} l_\mu j_W^\mu & \text{for } \nu_l + B_2 &\rightarrow l + B_4 \\
 \mathcal{L}_{int}^{nc} &= \frac{G_F}{\sqrt{2}} l_\mu^\nu j_Z^\mu & \text{for } \nu_l + B_2 &\rightarrow \nu_l + B_4 \\
 l_\mu^\nu &= \bar{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu, & j_Z^\mu &= \bar{\psi}_4 \gamma^\mu (c_V - c_A \gamma_5) \psi_2
 \end{aligned}$$

Weak magnetism

$$F_2 \frac{i\sigma_{\mu\nu} q^\nu}{2M} \vec{\tau}$$

$$\begin{aligned}
 c_V^n &= -1.0, & c_A^n &= -1.26(-1.1) \\
 c_V^p &= 0.07, & c_A^p &= 1.26(1.4) \\
 c_V^e &= 1.92, & c_A^e &= 1. [\nu_e] \\
 c_V^x &= -0.08, & c_A^x &= -1. [\nu_x]
 \end{aligned}$$

$$\frac{d^3\sigma}{d^2\Omega_3 dE_3} = -\frac{G_F^2}{32\pi^2} \frac{E_3}{E_1} \left[1 - \exp\left(-\frac{(q_0 + \hat{\mu})}{T}\right) \right]^{-1} (1 - f_3(E_3)) \text{Im} (L^{\alpha\beta} \Pi_{\alpha\beta}^R),$$

[Reddy, Prakash, Lattimer 1998, Horowitz, Perez-Garcia 2003]

Nucleon

Retarded polarization

$$\text{Im} \Pi_{\alpha\beta}^R = \tanh((q_0 + \hat{\mu})/2T) \text{Im} \Pi_{\alpha\beta}$$

Causal polarization

$$\Pi_{\alpha\beta} = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr}\{T[G_2(p) J_\alpha G_4(p+q) J_\beta]\}$$

Lepton tensor

$$L^{\alpha\beta} = 8[2k^\alpha k^\beta + (k \cdot q)g^{\alpha\beta} - (k^\alpha q^\beta + q^\alpha k^\beta) \mp i\epsilon^{\alpha\beta\mu\nu} k_\mu q_\nu]$$

Non-relativistic limit (simplification for discussion)

Neutrinos couple to (isospin) density
and (isospin) spin density

[Reddy, Prakash, Lattimer 1998]

$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu(C_V - C_A\gamma_5)(\tau_j)\psi(x)$$

$$\text{N.R.} \rightarrow C_V\psi^\dagger(\tau_j)\psi\delta^{\mu 0} - C_A\psi^\dagger\sigma^i(\tau_j)\psi\delta^{\mu i}$$

$$\frac{1}{V} \frac{d^2\sigma}{d\cos\theta dE_3} = \frac{G_F^2}{4\pi^2} E_3^2 (1 - f_3(E_3))$$

$$\times [C_V^2(1 + \cos\theta)S_\rho(q_0, q) + C_A^2(3 - \cos\theta)S_\sigma(q_0, q)]$$

Density and spin density (NC)

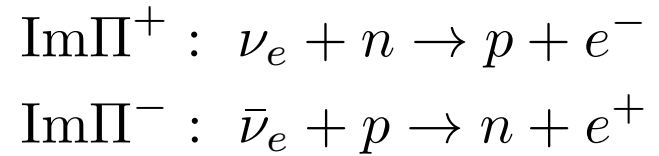
$$S_\rho(q_0, q) = \int dt e^{iq_0 t} \langle \rho(q, t) \rho(-q, 0) \rangle \Rightarrow \rho = \psi^\dagger \psi = \sum_{i=1, N} e^{i\vec{q} \cdot \vec{r}_i}$$

$$= \sum_f \langle 0 | \rho(q) | f \rangle \langle f | \rho(-q) | 0 \rangle \delta(q_0 - (E_f - E_0))$$

$$S_\sigma(\omega, \mathbf{q}) = \frac{4}{3n} \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \mathbf{s}(t, \mathbf{q}) \cdot \mathbf{s}(0, -\mathbf{q}) \rangle \quad \mathbf{s}(t, \mathbf{q}) = V^{-1} \sum_{i=1}^N e^{-i\mathbf{q} \cdot \mathbf{r}_i(t)} \boldsymbol{\sigma}_i$$

$$= \frac{4}{3n} \sum_f \langle 0 | \mathbf{s}(\mathbf{q}) | f \rangle \cdot \langle f | \mathbf{s}(-\mathbf{q}) | 0 \rangle \delta(\omega - (E_f - E_0))$$

Mean field shifts in charged current reactions

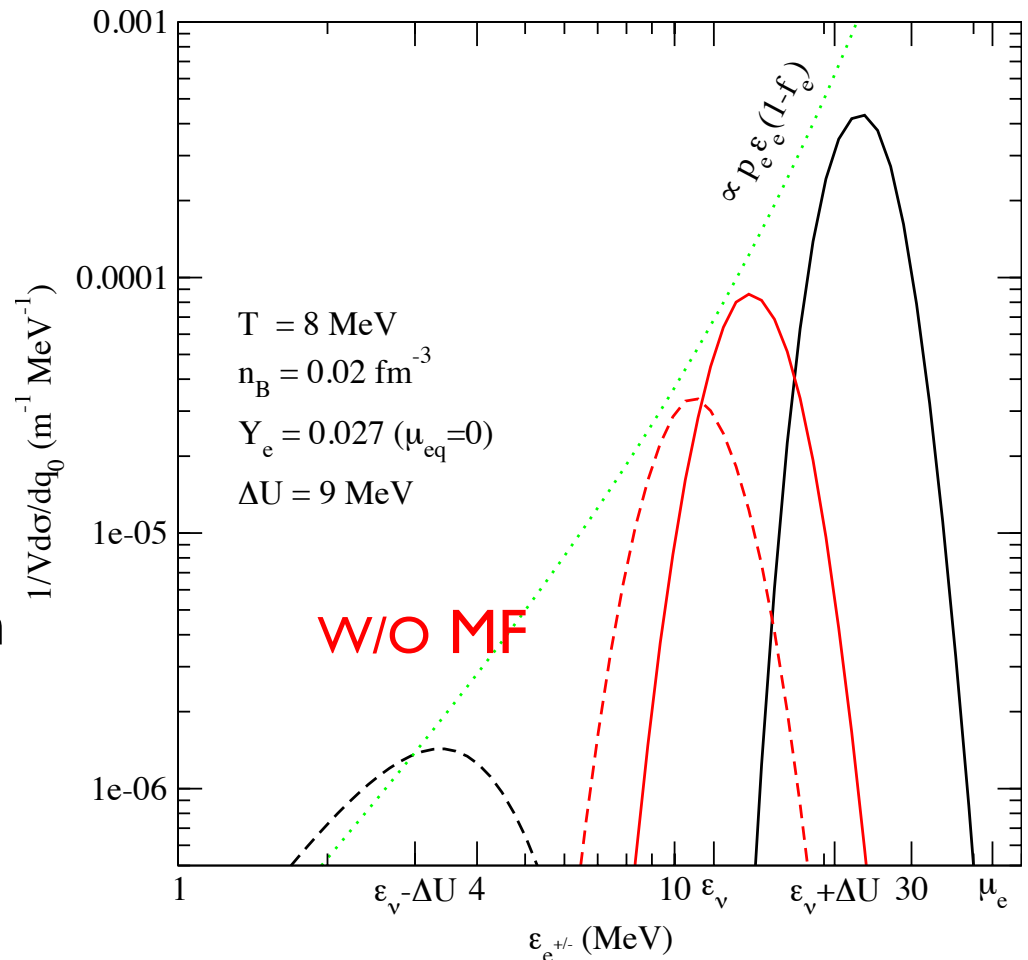


- Dispersion relation of quasi-particle from EOS:

$$E_i(k) = \sqrt{k^2 + M^{*2}} + U_i,$$

$$U_n - U_p = 40 \frac{n_n - n_p}{n_0} \text{MeV}$$

- Response function is peaked around $q_0 \approx U_n - U_p$
- Around neutrino sphere, mean field shift is comparable to temperature.



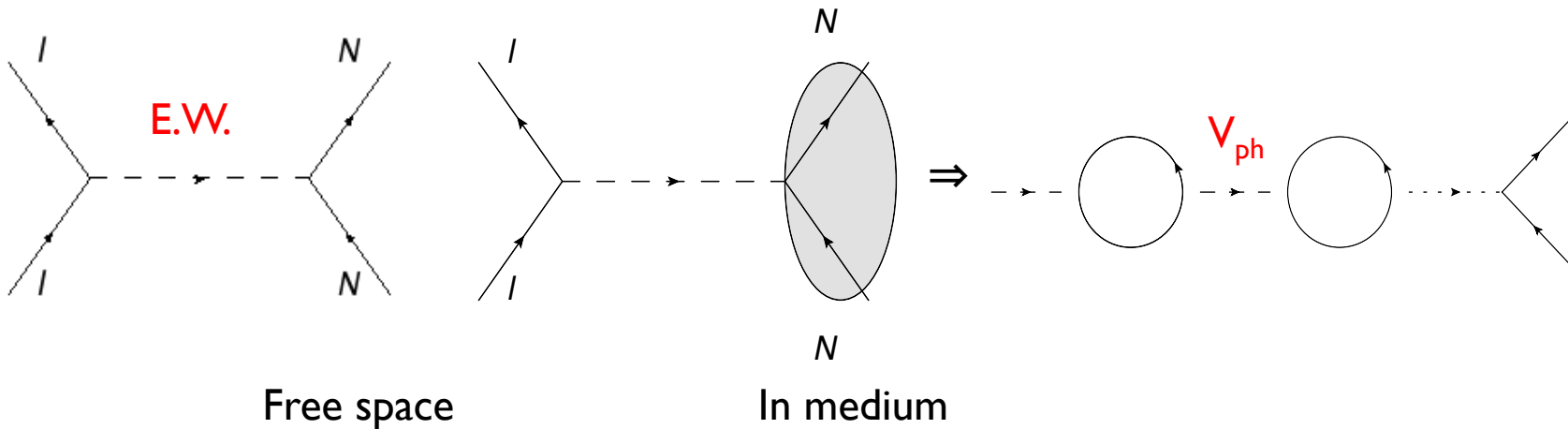
Roberts (2012)

Martinez-Pinedo et al. (2012)

[Roberts, Reddy (2012)]

Random Phase Approximation (RPA)

- An approximate method to include nuclear medium correlations in the response function: I-ph response.



\Rightarrow Dyson Equation RPA response

$$\Pi^{RPA} = \Pi^0 + \Pi^{RPA} V_{ph} \Pi^0$$

- Provides a fair qualitative description of response in nuclei. Mean field models with consistent residual p-h interactions.
- RPA preserves static sum rule and energy-weight sum rules.

Residual p-h interaction in RPA

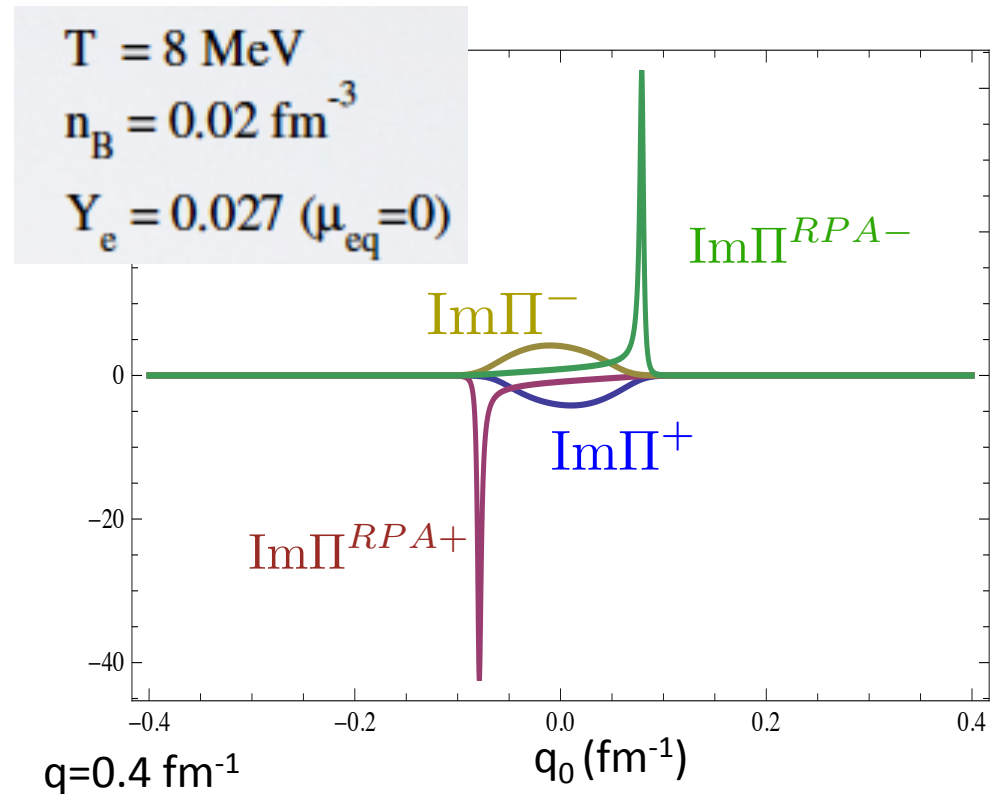
- Relativistic field models [Reddy, Prakash, Lattimer, Pons 1999]
 - V_{ph} is consistent with underlying nuclear mean field EOS.
 - Exception: axial vector V_{ph} is not constrained by EOS: from study of response in finite nuclei.
- Non-relativistic Skyrme models
 - V_{ph} from EOS, or Fermi liquid parameter calculated in a microscopic theory.
 - Residual interactions in density and isospin density fluctuations are consistent with EOS.
 - Exception: spin-flip residual interactions are obtained from study of response in finite nuclei.
- Both axial vector or spin-flip residual interactions are more important in each model.
- Multi-component (n,p,e) matter: RPA mix various polarizations – nn, pp, ee, ...

Charged current reactions in RPA

$$\text{Im}\Pi^+ : \nu_e + n \rightarrow p + e^-$$

$$\text{Im}\Pi^- : \bar{\nu}_e + p \rightarrow n + e^+$$

- Repulsive residual p-h interaction: favor two imbalanced fermi sea and act as restoring force.
- Collective modes (peaks): usually outside I-ph kinematic regime.



- Ikeda sum rule $\int d\omega (\text{Im}\Pi^+ - \text{Im}\Pi^-) = -\pi(n_2 - n_4)$
Ikeda 1964
- f-sum rule $\int d\omega \omega (\text{Im}\Pi^+ + \text{Im}\Pi^-) = \langle \psi_0 | [\hat{O}^+, [H, \hat{O}]] | \psi_0 \rangle + h.c.$
Thouless 1961

Fermi/GT operators

2-ph response: beyond RPA

[Roberts, Shen, Reddy , 2012 (in prep)]

- Ansatz for spin-isospin charge-exchange response function

$$S_{\sigma\tau^-}(q_0, q) = \frac{1}{1 - \exp(-\beta(q_0 + \mu_n - \mu_p))} \text{Im} \left[\frac{\tilde{\Pi}(q_0, q)}{1 - V_{\sigma\tau} \tilde{\Pi}(q_0, q)} \right]$$

- 2PH response is included via relaxation time approximation:

$$\Gamma = \tau_{\sigma}^{-1}, \text{ calculated in Chiral EFT}$$

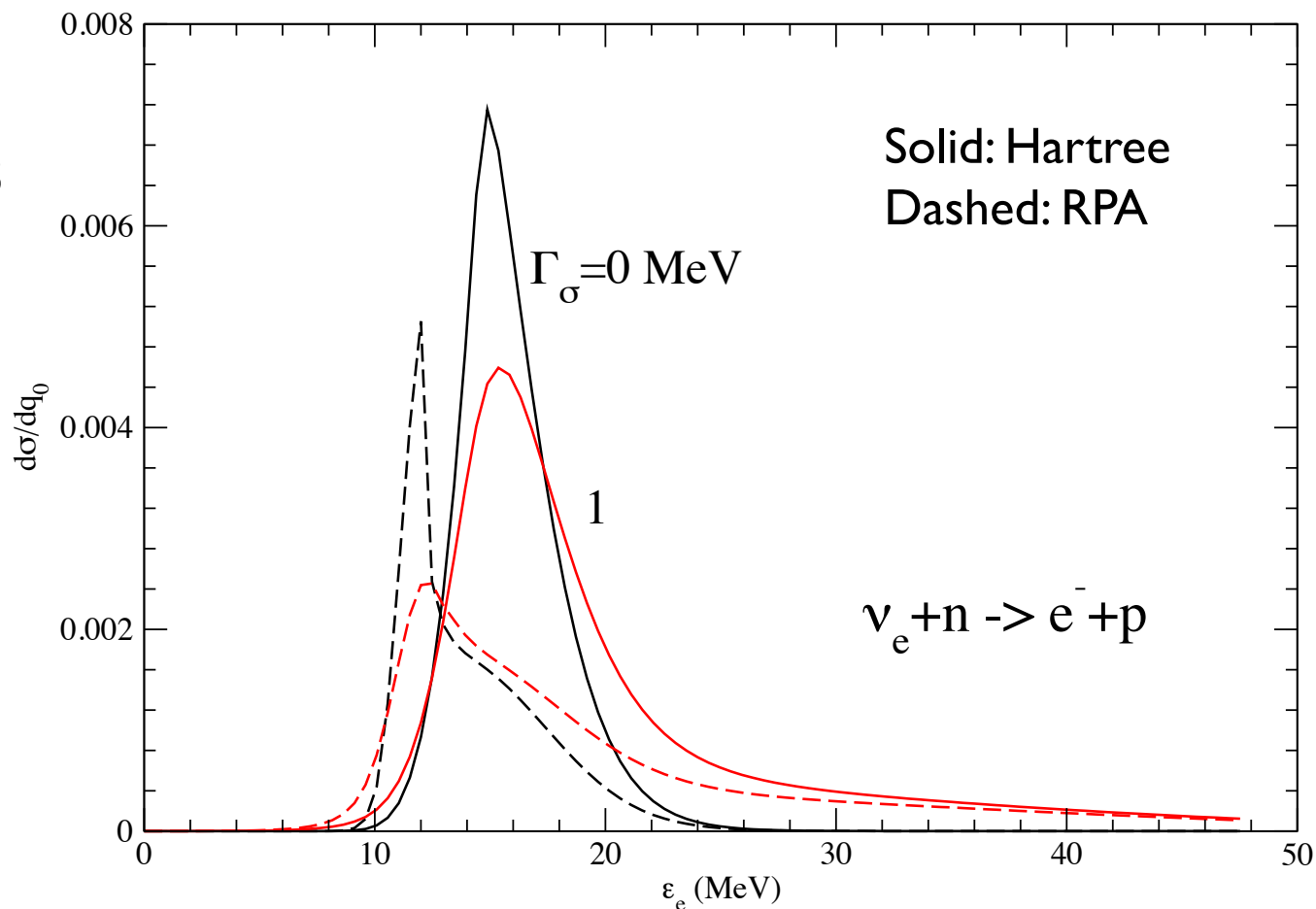
[Bacca et al, 2009, 2011]

$$\text{Im}\tilde{\Pi}(q_0, q) = \frac{1}{\pi} \int \frac{d^3p}{(2\pi)^3} \frac{f_p(\epsilon_{p+q}) - f_n(\epsilon_p)}{\epsilon_{p+q} - \epsilon_p + \hat{\mu}} \mathcal{I}(\Gamma)$$
$$\mathcal{I}(\Gamma) = \frac{\Gamma}{(q_0 + \Delta U - (\epsilon_{p+q} - \epsilon_p))^2 + \Gamma^2}$$

- P-h interaction $V_{\sigma\tau}$ from GT transition of finite nuclei: 220 MeVfm^3 .

Charged current reactions in RPA and beyond

$T=8$ MeV
 $N=0.006$ fm⁻³
 $Y_e=0.03$
 $\mu_e=30$ MeV
 $E_\nu=12$ MeV



- RPA suppresses response and shifts its strength via collective mode. Multi-particle dynamics enhances it.
- Net effect mild suppression. Follow SN trajectory to study details.

Sum rules and multi-particle response

- $q=0$ limit, spin response comes solely from multi-particle dynamics. Density response vanishes.

$$S_{\sigma}(\omega, \mathbf{q}) = \frac{4}{3n} \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \mathbf{s}(t, \mathbf{q}) \cdot \mathbf{s}(0, -\mathbf{q}) \rangle$$

- Response function is constrained by sum rules

Spin susceptibility

$$S_{-1} = \int_0^{\infty} \frac{S_A(\omega, 0)}{\omega} d\omega, \quad \chi = 2nS_{-1} \quad \text{Fermi gas} \quad \chi_{FG} = mk_F/\pi^2$$

Pair correlation function

$$S_0 = \int_{-\infty}^{\infty} S_A(\omega, \mathbf{q}) d\omega = 1 + \frac{4}{3N} \sum_{i \neq j}^N \langle e^{-i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \rangle,$$

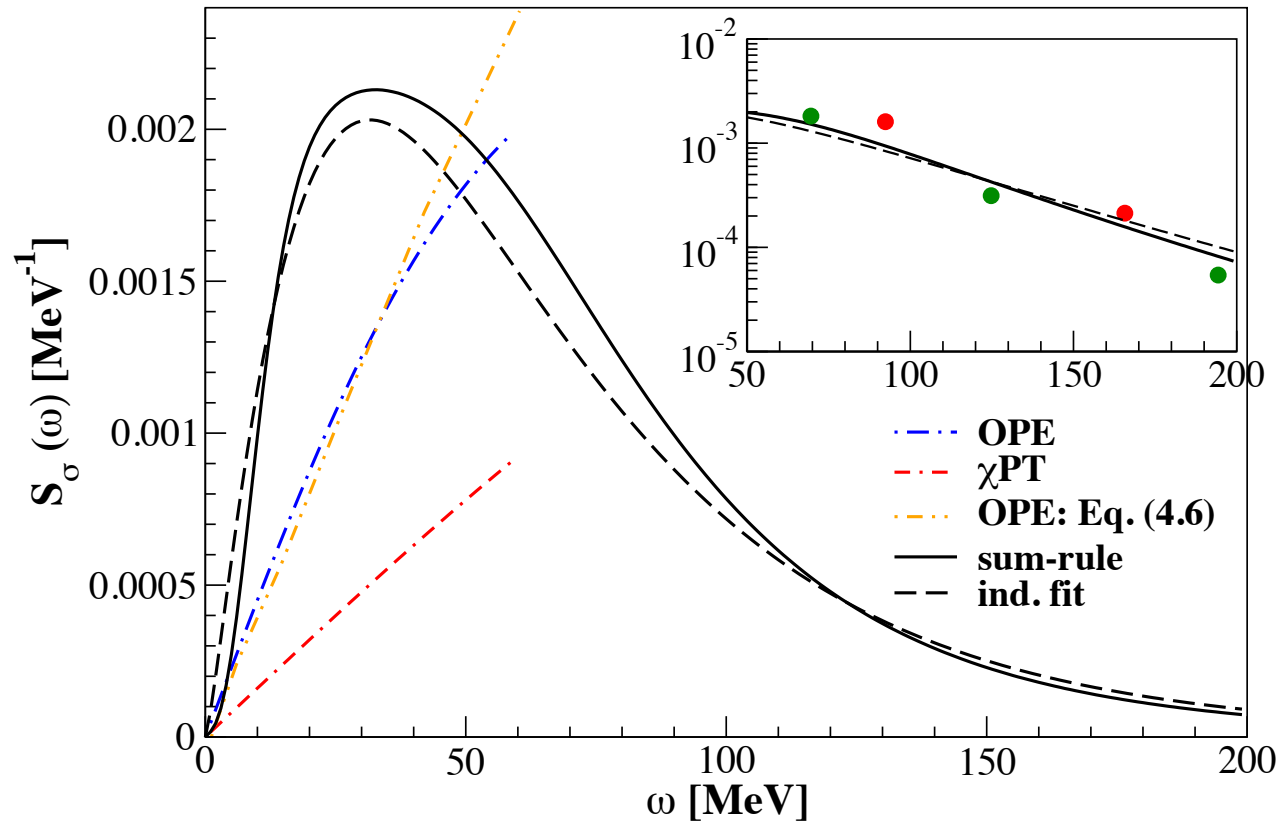
Energy-weighted sum rule

$$S_{+1} = \int_{-\infty}^{\infty} S_A(\omega, \mathbf{q}) \omega d\omega = -\frac{4}{3N} \langle [H, \mathbf{s}(\mathbf{q})] \cdot \mathbf{s}(-\mathbf{q}) \rangle,$$

[Shen, Gandolfi, Reddy, Carlson, 2012]

AFDMC results for the sum-rules

Density (fm^{-3})	S_{σ}^{-1} (MeV^{-1})	S_{σ}^0	S_{σ}^{+1} (MeV)	$\bar{\omega}_0$ (MeV)	$\bar{\omega}_1$ (MeV)
$n = 0.12$	0.0057(9)	0.20(1)	8(1)	35(9)	40(8)
$n = 0.16$	0.0044(7)	0.20(1)	11(1)	46(11)	55(8)
$n = 0.20$	0.0038(6)	0.18(1)	14(1)	47(12)	78(10)



Enhanced response
at 10-50 MeV

- Low ω : from Landau Fermi liquid theory
- Corrections at intermediate ω
- High ω : reduced to 2-particle response – solved exactly from two-body Schrödinger eq.

Neutrino pair production

- . $\nu_1 + \bar{\nu}_1 \rightleftharpoons \nu_2 + \bar{\nu}_2$ Buras et al, 2003
- . $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$ Dicus '72; Bruenn '85
- . $\gamma^* \rightleftharpoons \nu\bar{\nu}$ Braaten & Seckel '91, Ratkovic et al '03
- . $\gamma^* + e \rightleftharpoons e + \nu\bar{\nu}$ Schinder et al '87, Dutta et al '04
- . $NN \rightleftharpoons NN\nu\bar{\nu}$

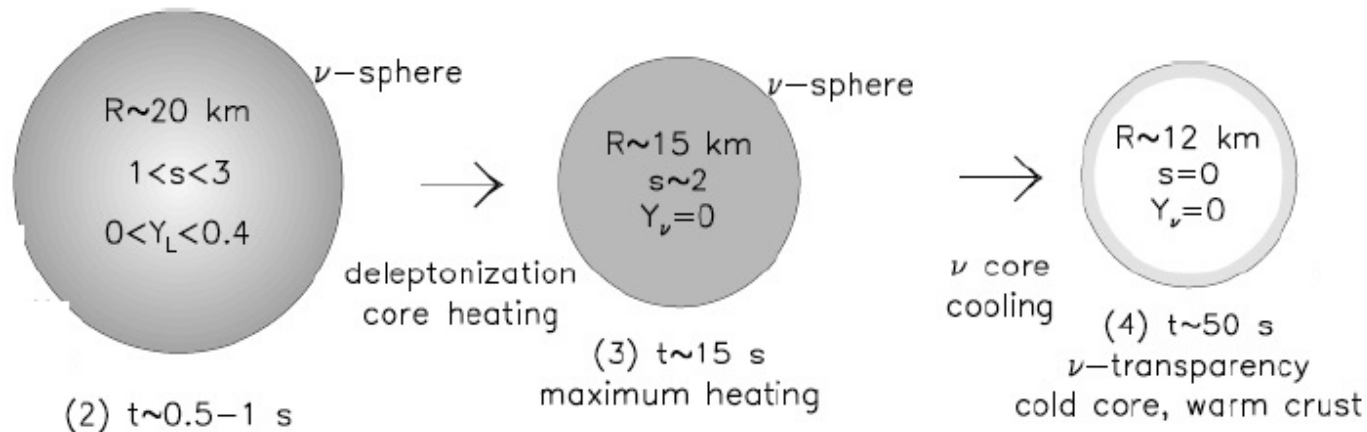
- Friman & Maxwell 1979, OPE Born
- Hannestad & Raffelt 1998, OPE Born – fitting formula
- Hanhart, Philips, Reddy 1999, T-matrix – reduction compared to OPE
- Lykasov et al 2008, Bacca et al 2009, 2011: 2-body response in chiral EFT – reduction of response
- Shen, Gandolfi, Reddy, Carlson (2012): QMC sum rules – low energy enhancement

Application

- Late time supernova neutrino signal
 - Essential physics suite
 - Nuclear symmetry energy and ν convection
 - Observables

Late time supernova ν signal (PNS)

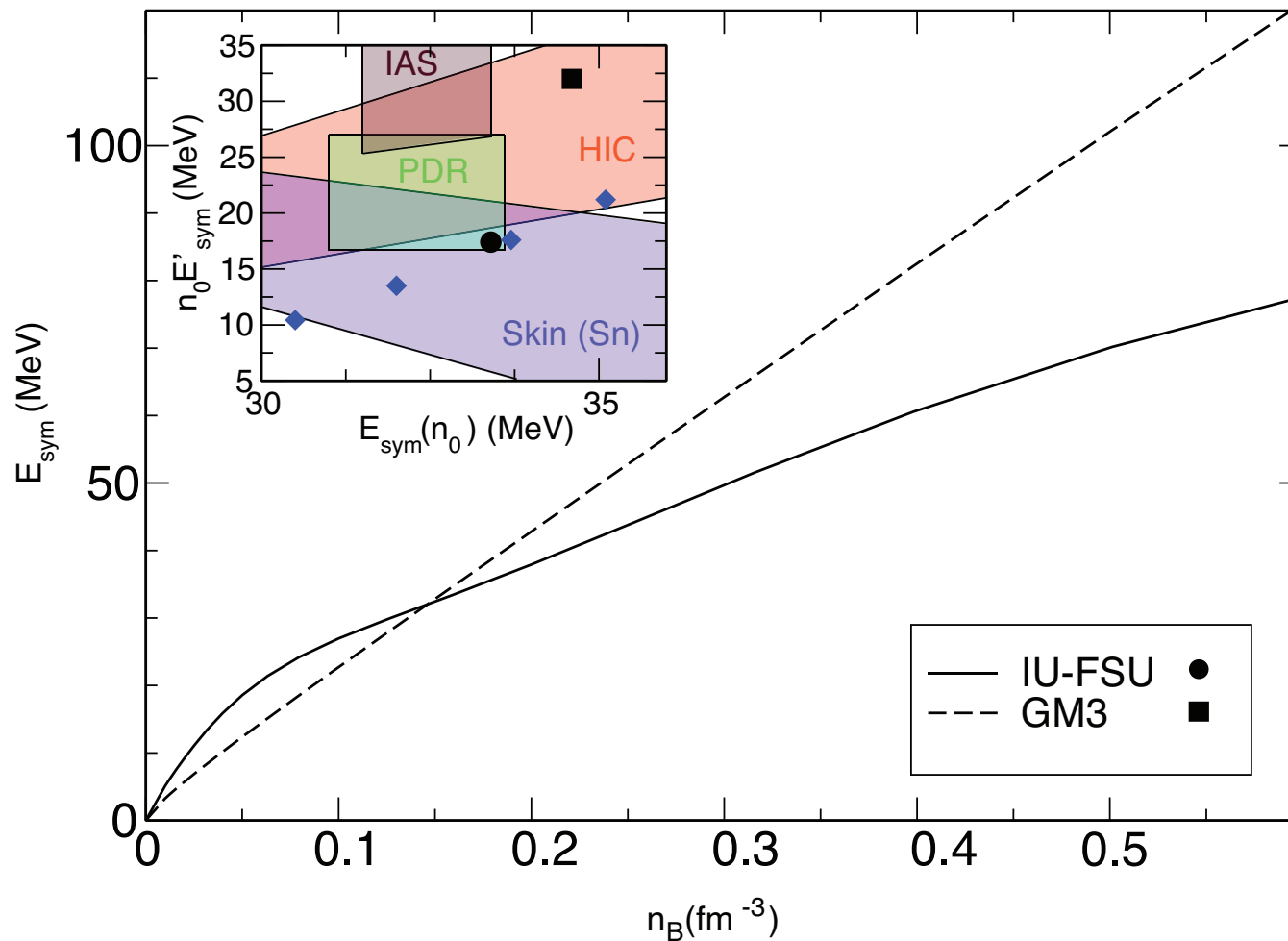
- Neutrino transport drives the evolution of a PNS (hot, lepton rich, SN remnant) \rightarrow NS (cold, deleptonized, compact)



- Study the **sensitivity** of cooling time-scale and neutrino signal to **EoS, opacities, convection**
- 1d PNS model: TOV equations + diffusion transport + convection

Updates in neutrino transport: Roberts (2012)

- Use two different high density EoS with associated opacities (without and with RPA correlations)
- EoS differ in the behavior of the symmetry energy



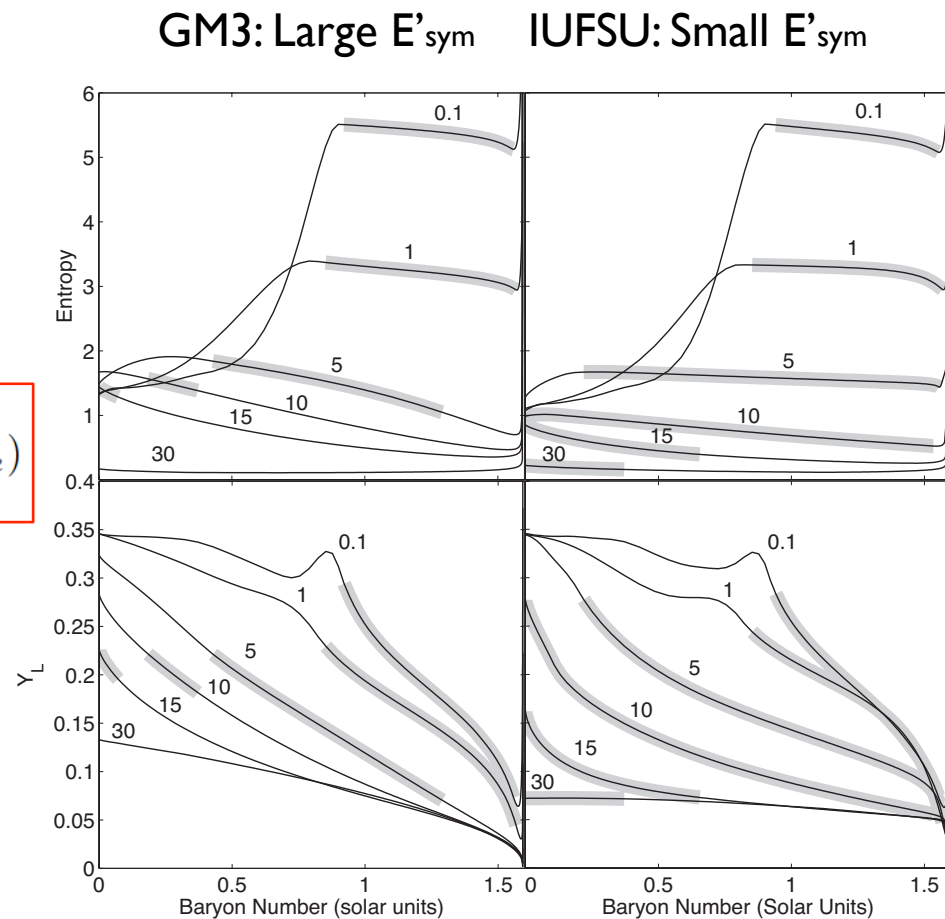
- Large regions are convectively unstable, according to Ledoux criterion (competition of leptonic and entropy gradients)

$$-\left(\frac{\partial P}{\partial s}\right)_{n, Y_L} \frac{ds}{dr} - \left(\frac{\partial P}{\partial Y_L}\right)_{n, s} \frac{dY_L}{dr} > 0$$

- Density dependence of nuclear symmetry is key to understand composition driven convective instability:

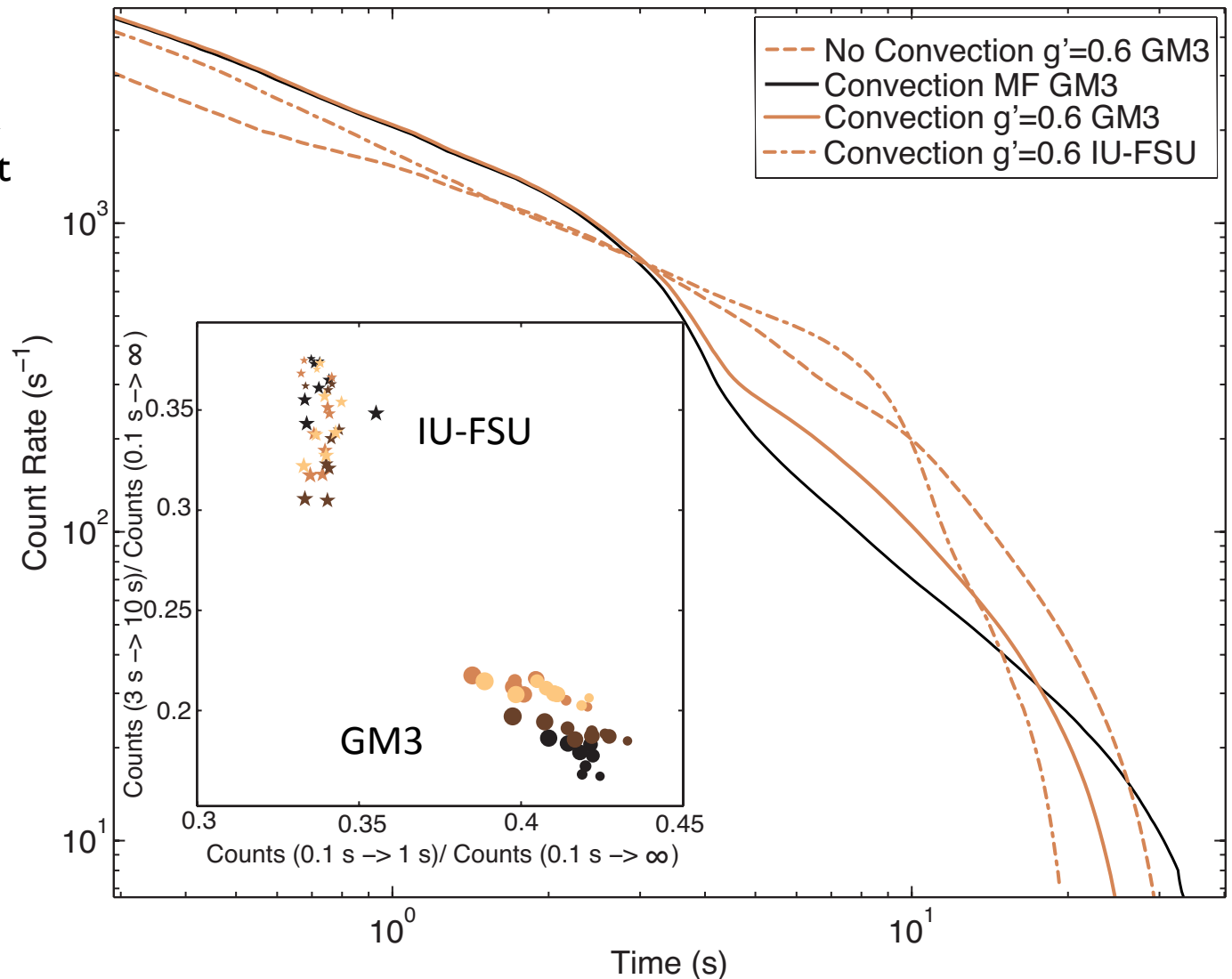
$$\left(\frac{\partial P}{\partial Y_L}\right)_{n_B} \simeq n_B^{4/3} Y_e^{1/3} - 4n_B^2 E'_{\text{sym}} (1 - 2Y_e)$$

- Larger E'_{sym} stabilizes/shuts off convection at earlier time



Observable signatures of convective transport

- Count rate in Super-K for galactic supernova at 10 kpc.
- Convection produces most dramatic features (larger luminosity and shorter cooling time scale).
- RPA opacities imply larger luminosity between 3-10 secs.



Summary

- Constraints on nuclear EOS
 - Symmetric matter: saturation, stable nuclei
 - Neutron rich matter: isovector response, neutron skin, benchmark from EFT, QMC
 - Above $2n_0$: NS observations
- Neutrino opacity
 - Kinematics
 - Mean field effects
 - NN correlations in RPA
 - NN correlations beyond RPA: 2PH
- Neutrino pair production
- Applications

Thank you !