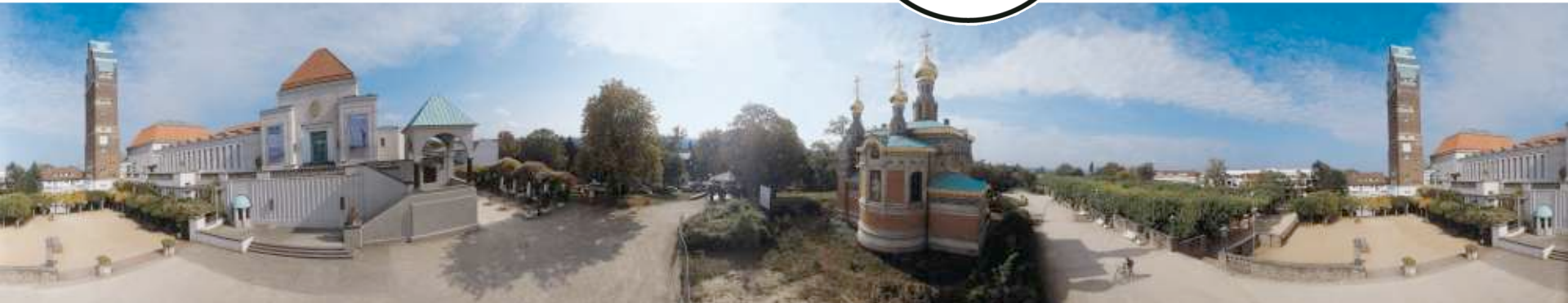


Chiral effective field theory constraints for the equation of state and for supernova neutrino rates

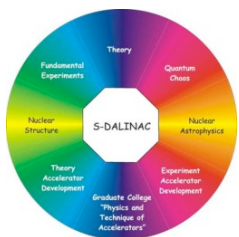
Achim Schwenk



TECHNISCHE
UNIVERSITÄT
DARMSTADT



INT program on core-collapse supernovae
Seattle, July 3, 2012



DFG



Minerva
Stiftung



Bundesministerium
für Bildung
und Forschung

Main points

Advances in nuclear forces and nuclear matter theory

Impact on neutron stars:

provides strong constraints for the equation of state

neutron star radius 9.9-13.8 km for $M=1.4 M_{\text{sun}}$ ($\pm 15\%$)

K. Hebeler, J.M. Lattimer, C.J. Pethick, AS, PRL **105**, 161102 (2010) and in prep.
I. Tews, T. Krüger, K. Hebeler, AS, arXiv:1206.0025.

Impact on neutrino-matter interactions

S. Bacca, K. Hally, C.J. Pethick, AS, PRC **80**, 032802(R) (2009) and
S. Bacca, K. Hally, M. Liebendörfer, A. Perego, C.J. Pethick, AS, arXiv:1112.5185.

Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~ 500 MeV

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

limited resolution at low energies,
can expand in powers $(Q/\Lambda_b)^n$

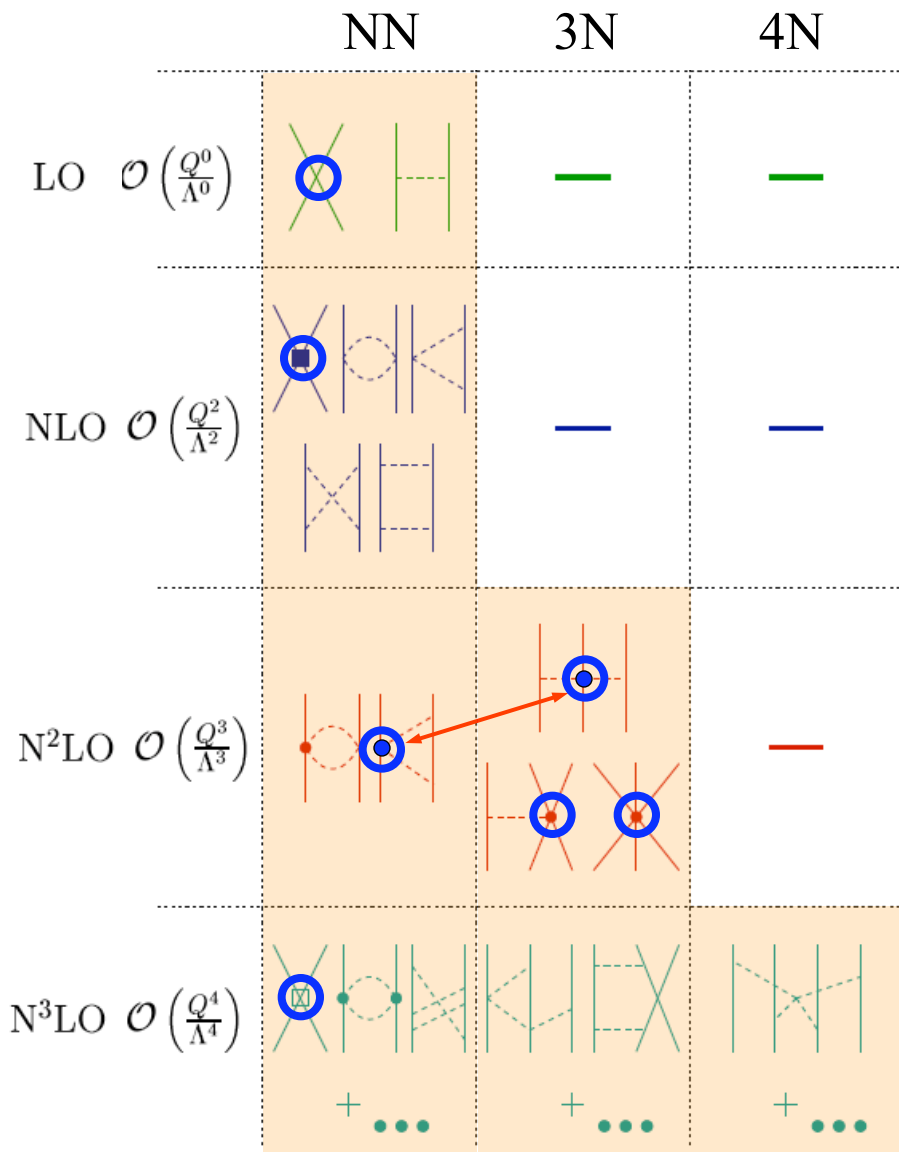
LO, $n=0$ - leading order,
NLO, $n=2$ - next-to-leading order,...

expansion parameter $\sim 1/3$

(compare to multipole expansion
for a charge distribution)

Chiral Effective Field Theory for nuclear forces

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include long-range pion physics

few short-range couplings,
fit to experiment once

systematic: can work to desired
accuracy and obtain **error estimates**

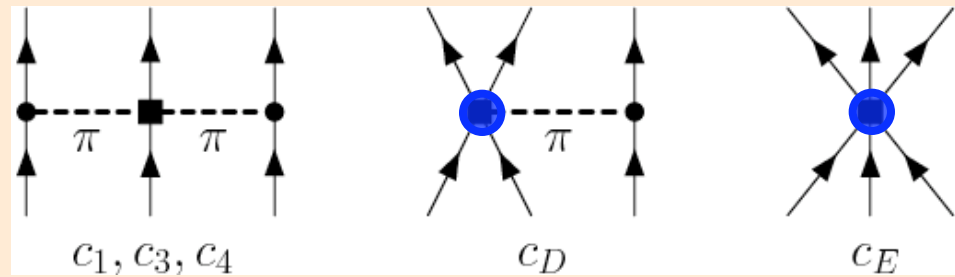
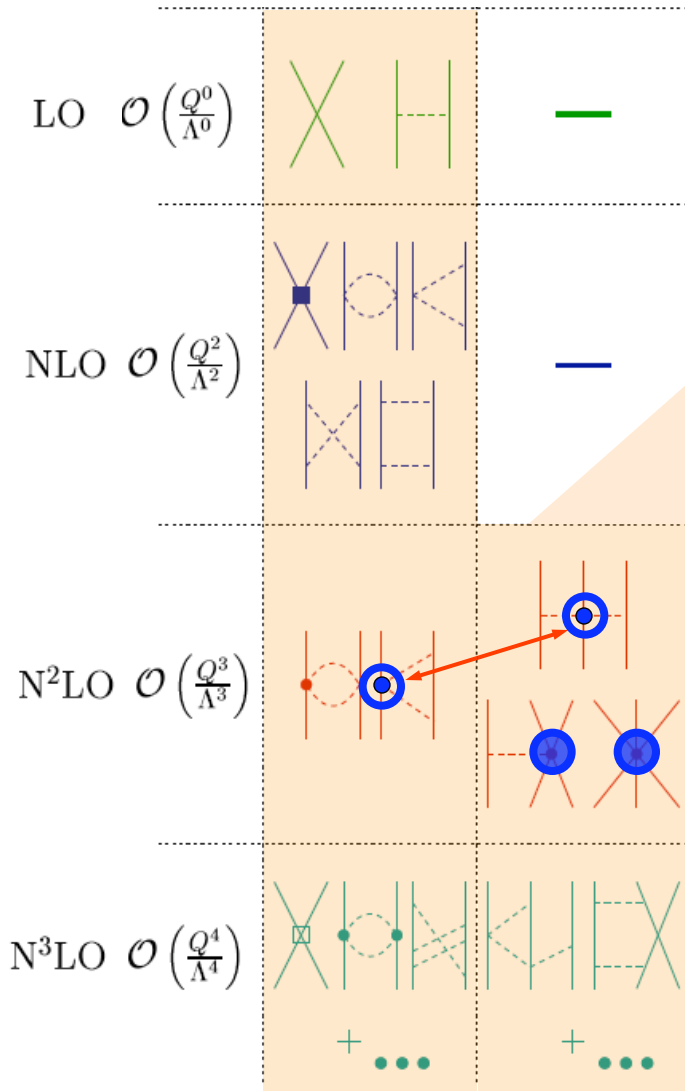
Chiral Effective Field Theory and many-body forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~ 500 MeV

NN 3N

consistent NN-3N interactions

3N,4N: only 2 new couplings to N³LO



long-range 3N: c_i from π N and NN

$$c_1 = -0.9_{-0.5}^{+0.2}, \quad c_3 = -4.7_{-1.0}^{+1.2}, \quad c_4 = 3.5_{-0.2}^{+0.5}$$

3- and 4-neutron forces are predicted to N³LO ($c_{D,E}$ don't contribute)

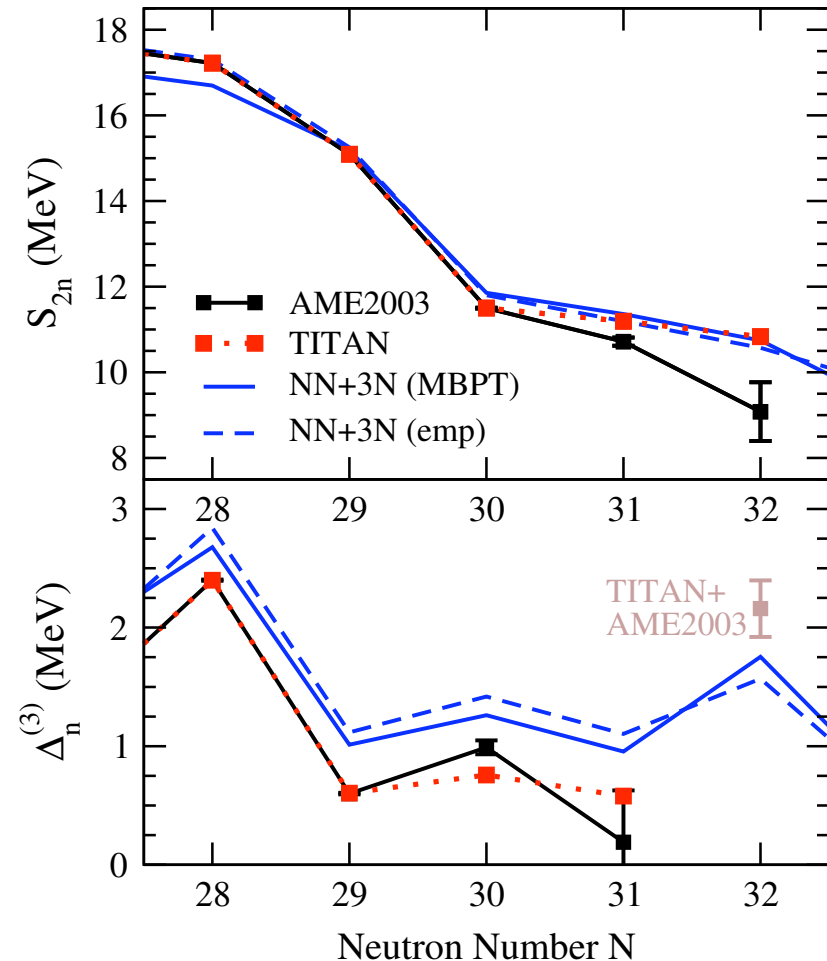
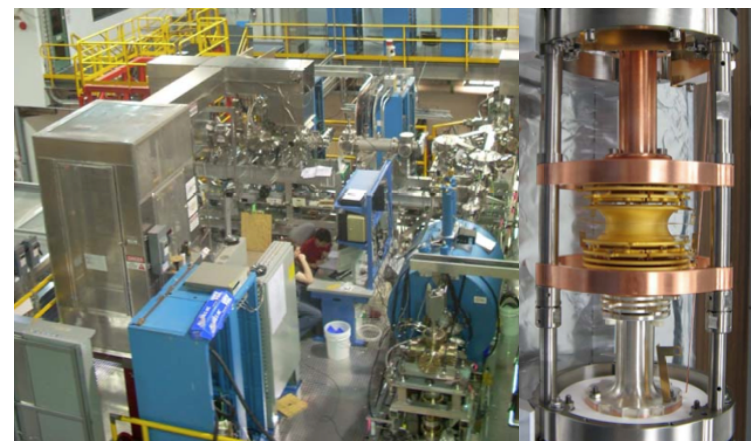
Hebeler, AS (2010)

new $^{51,52}\text{Ca}$ TITAN measurements

^{52}Ca is 1.75 MeV more bound compared to atomic mass evaluation

A. Gallant et al., PRL in press, arXiv:1204.1987.

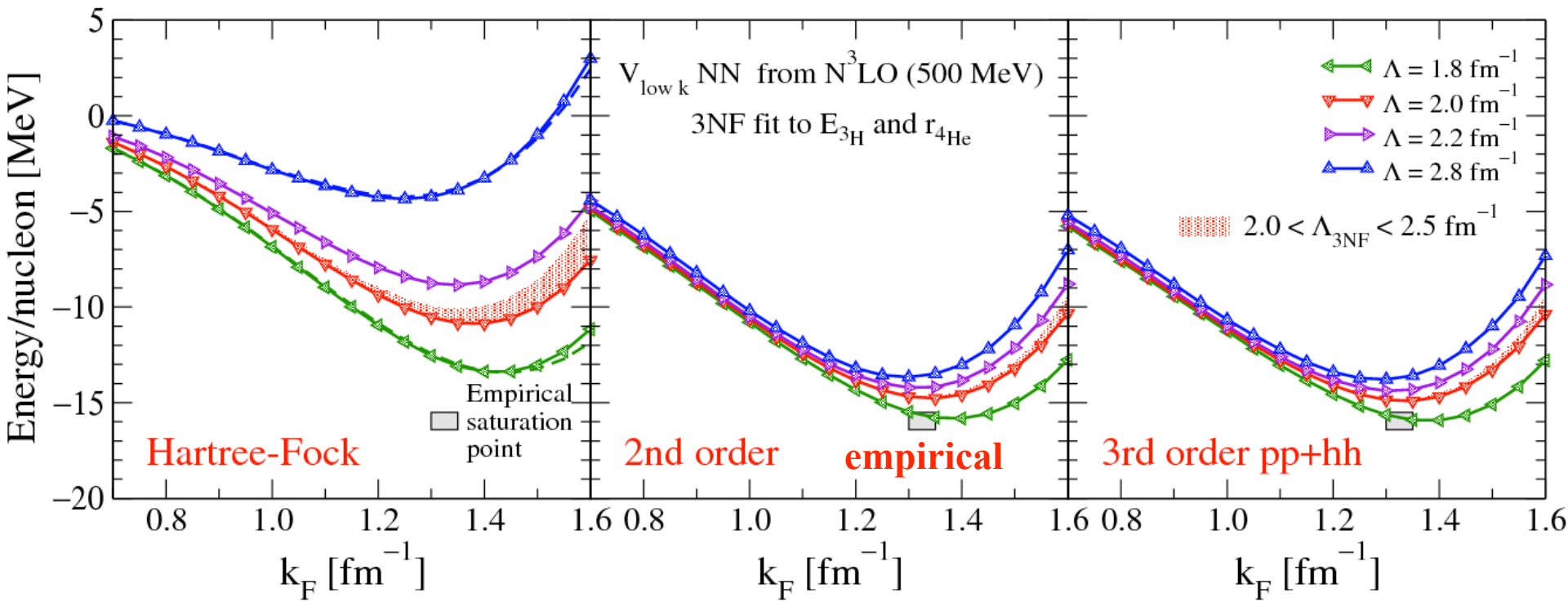
behavior of two-neutron separation energy S_{2n} and odd-even staggering $\Delta_n^{(3)}$ agrees with NN+3N predictions



Impact of 3N forces on nuclear matter

chiral 3N forces fit to light nuclei
predict nuclear matter saturation
with theoretical uncertainties

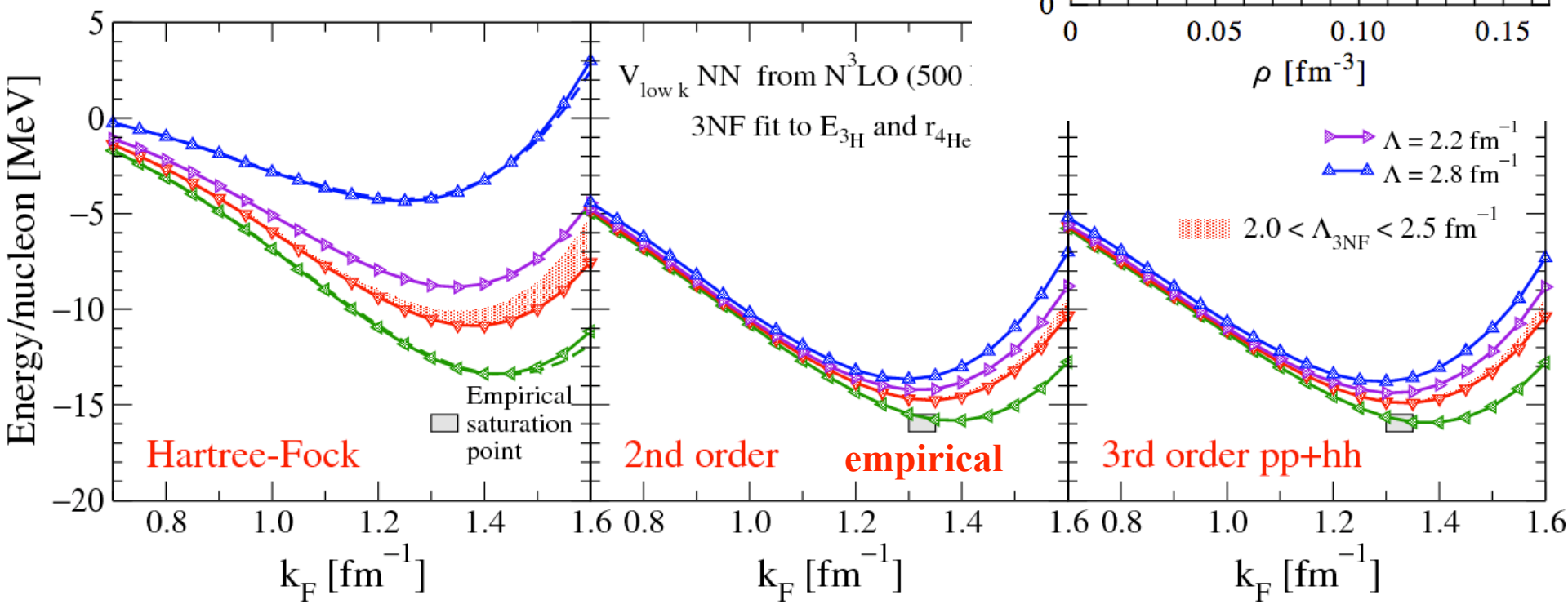
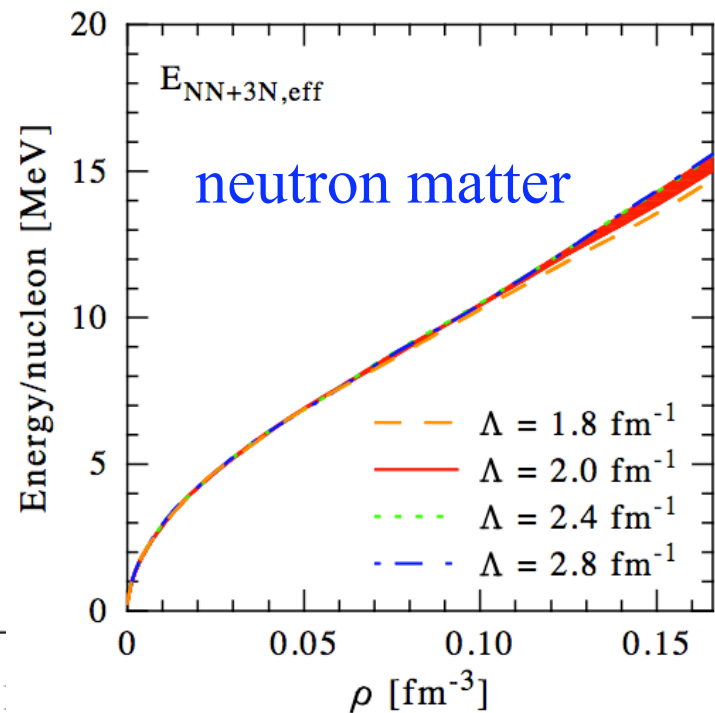
Hebeler et al. (2011), Bogner et al. (2005)



Impact of 3N forces on neutron matter

neutron matter is simpler system,
only long-range parts of 3N forces
contribute (c_1 and c_3)

Hebeler, AS (2010)



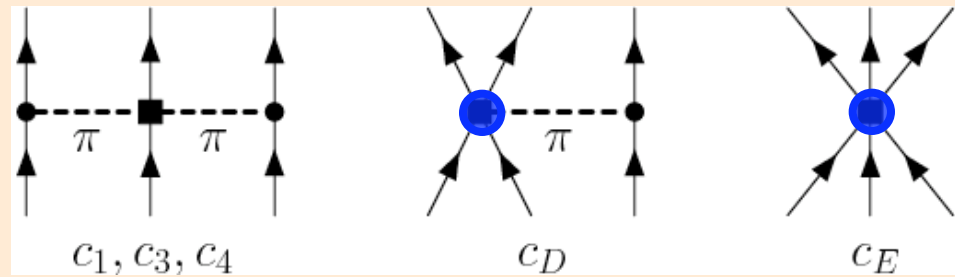
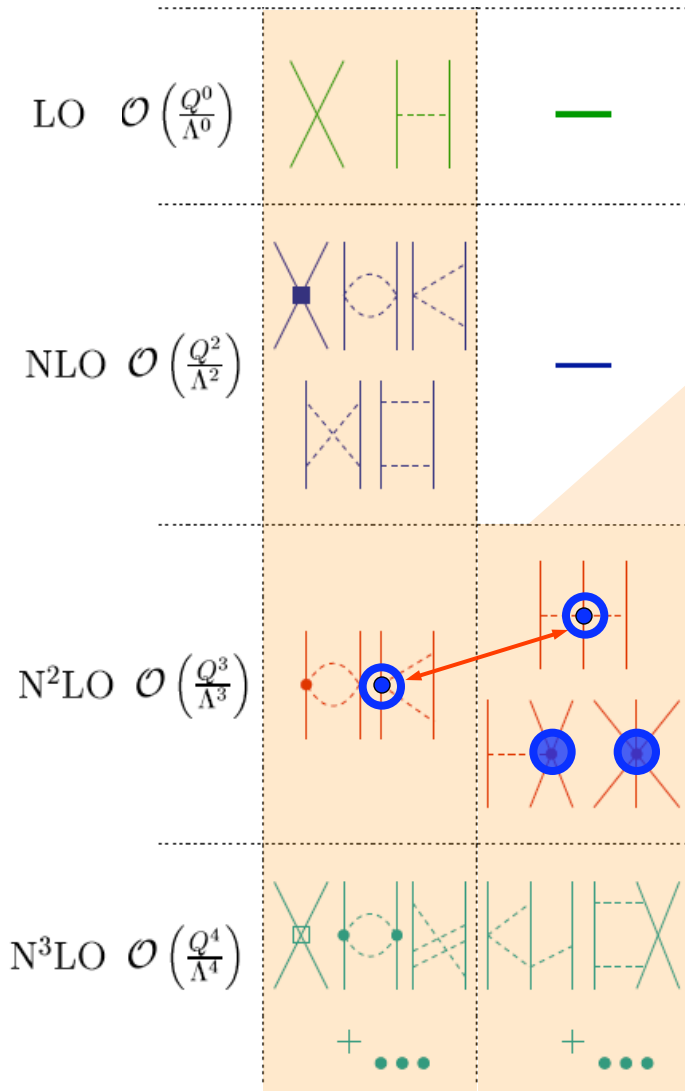
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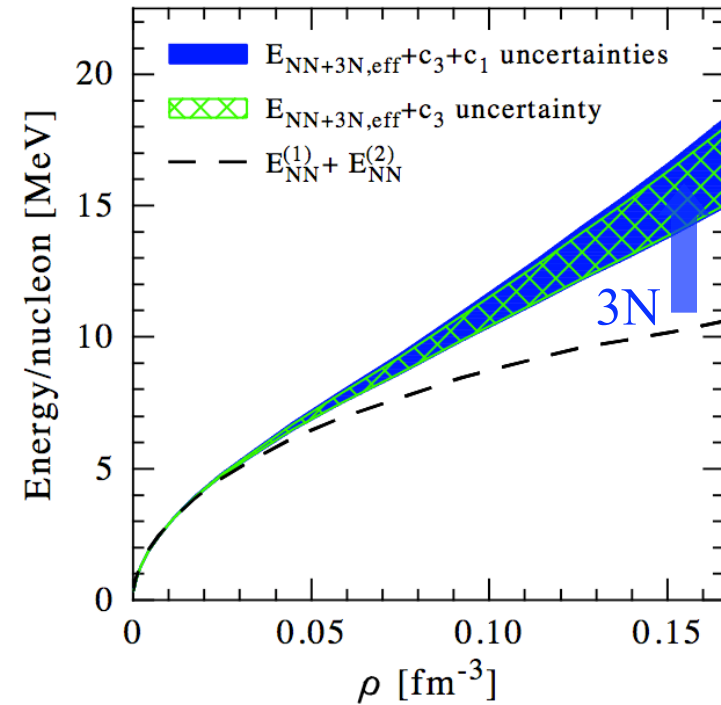
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Hebeler, AS (2010)

Impact of 3N forces on neutron matter

neutron matter uncertainties
dominated by 3N forces (c_3 coupling)

Hebeler, AS (2010)



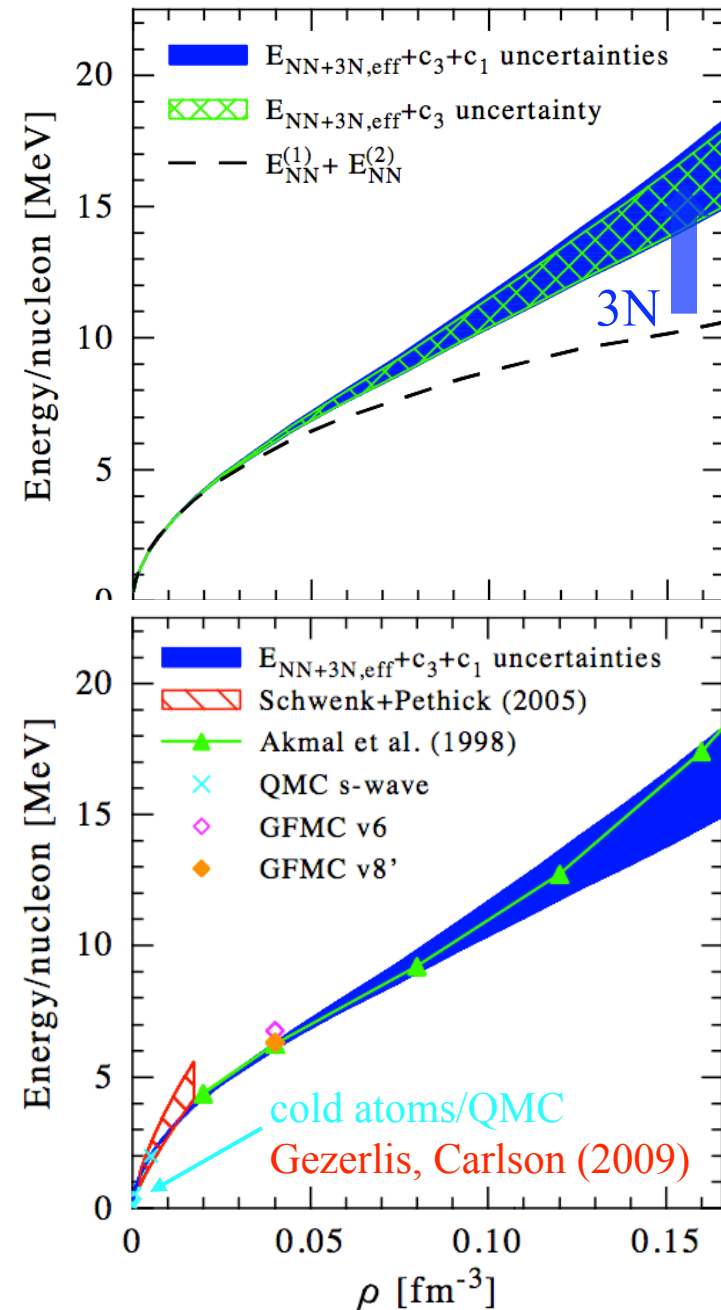
Impact of 3N forces on neutron matter

neutron matter uncertainties

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Hebeler, AS (2010)

other microscopic calculations within band
(but without uncertainties)

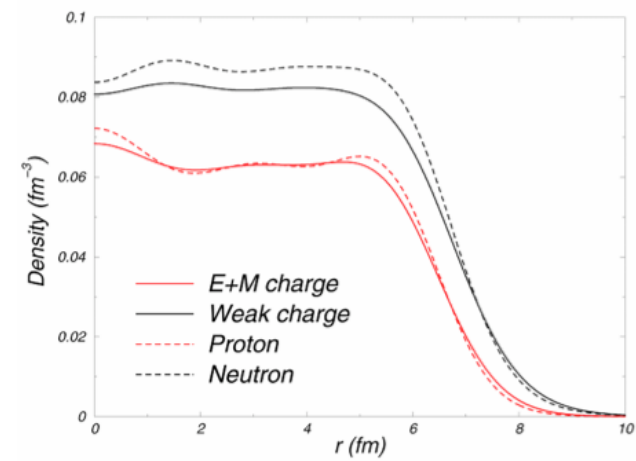


Neutron skin of ^{208}Pb

probes neutron matter energy/pressure,
neutron matter band predicts

neutron skin of ^{208}Pb : 0.17 ± 0.03 fm

Hebeler et al. (2010)

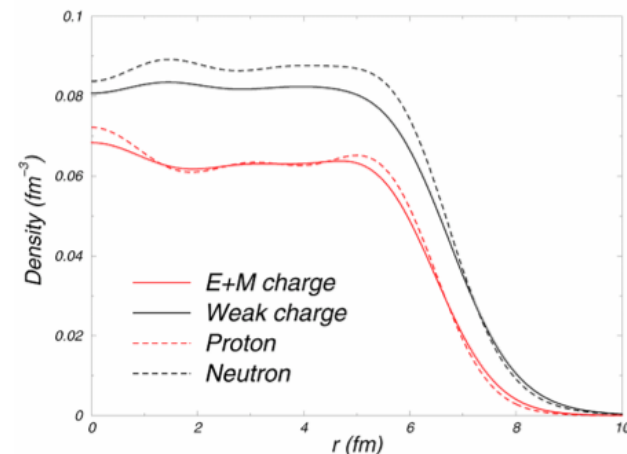


Neutron skin of ^{208}Pb

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Hebeler et al. (2010)



in excellent agreement with extraction from complete E1 response

$0.156 + 0.025 - 0.021$ fm

PRL 107, 062502 (2011)

PHYSICAL REVIEW LETTERS

week ending
5 AUGUST 2011

Complete Electric Dipole Response and the Neutron Skin in ^{208}Pb

A benchmark experiment on ^{208}Pb shows that polarized proton inelastic scattering at very forward angles including 0° is a powerful tool for high-resolution studies of electric dipole (E1) and spin magnetic dipole (M1) modes in nuclei over a broad excitation energy range to test up-to-date nuclear models. The extracted E1 polarizability leads to a neutron skin thickness $r_{\text{skin}} = 0.156^{+0.025}_{-0.021}$ fm in ^{208}Pb derived within

PREX: neutron skin from parity-violating electron-scattering at JLAB

electron exchanges Z-boson, couples preferentially to neutrons

goal II: ± 0.06 fm

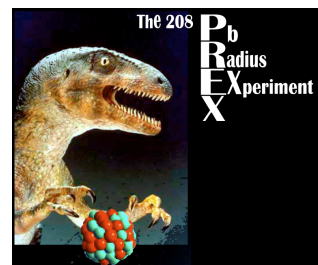
PRL 108, 112502 (2012)

PHYSICAL REVIEW LETTERS

week ending
16 MARCH 2012

Measurement of the Neutron Radius of ^{208}Pb through Parity Violation in Electron Scattering

We report the first measurement of the parity-violating asymmetry A_{PV} in the elastic scattering of polarized electrons from ^{208}Pb . A_{PV} is sensitive to the radius of the neutron distribution (R_n). The result $A_{\text{PV}} = 0.656 \pm 0.060(\text{stat}) \pm 0.014(\text{syst})$ ppm corresponds to a difference between the radii of the neutron and proton distributions $R_n - R_p = 0.33^{+0.16}_{-0.18}$ fm and provides the first electroweak observation of the neutron skin which is expected in a heavy, neutron-rich nucleus.



Symmetry energy and pressure of neutron matter

neutron matter band predicts
symmetry energy S_v and
its density dependence L

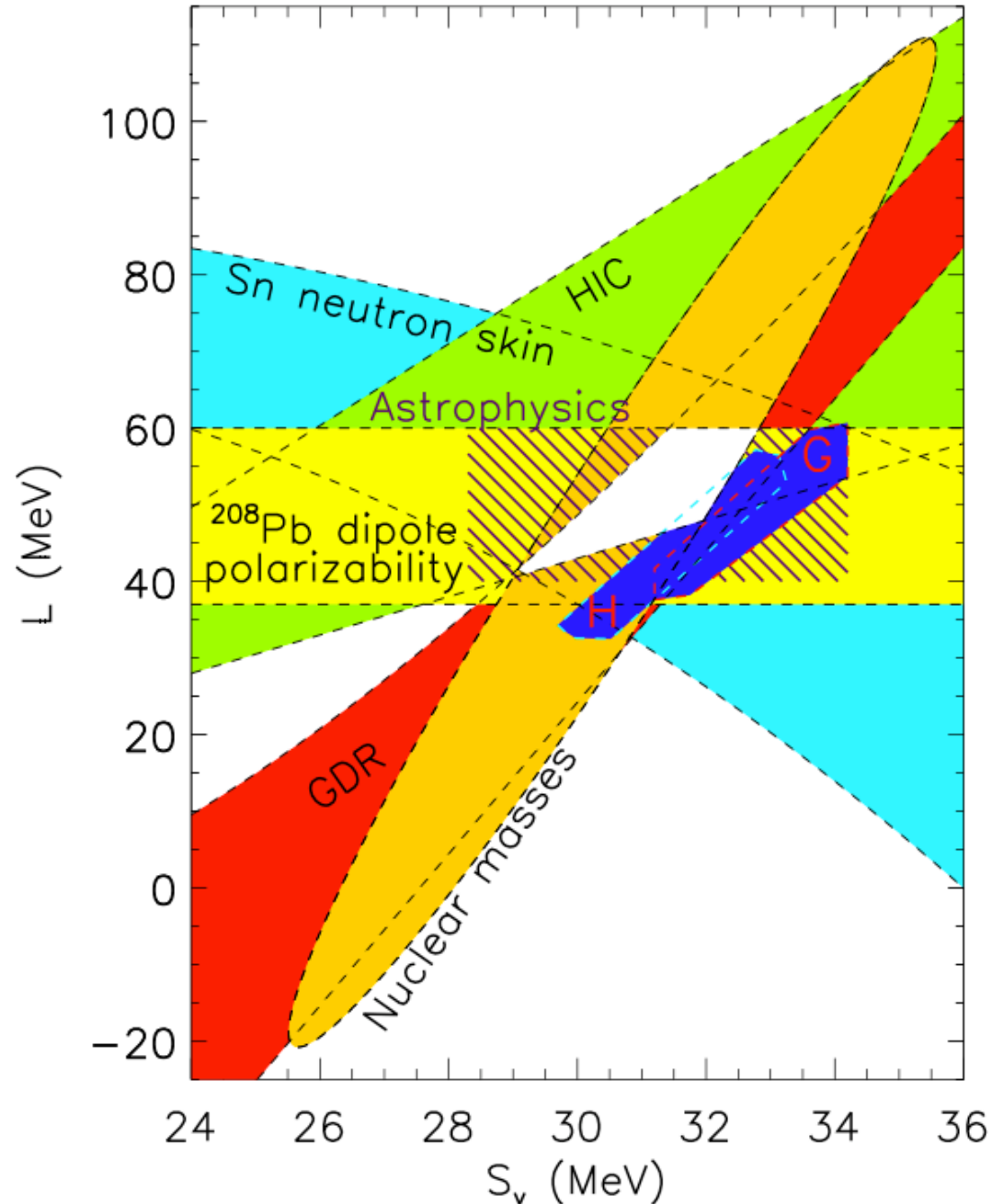
comparison to experimental
and observational constraints
Lattimer, Lim (2012)

neutron matter constraints

H: Hebeler et al. (2010) and in prep.

G: Gandolfi et al. (2011)

predicts correlation
but not range of S_v and L



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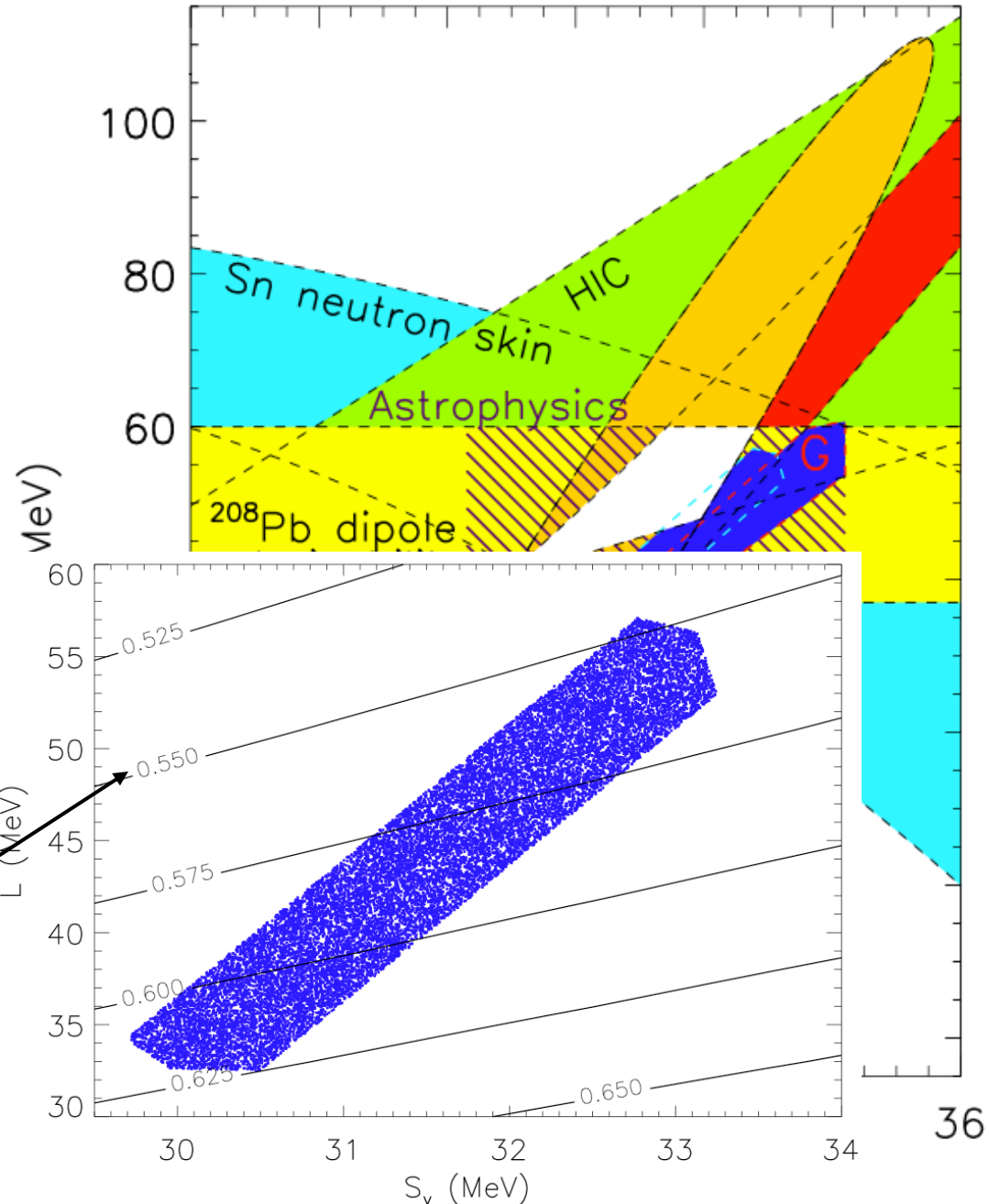
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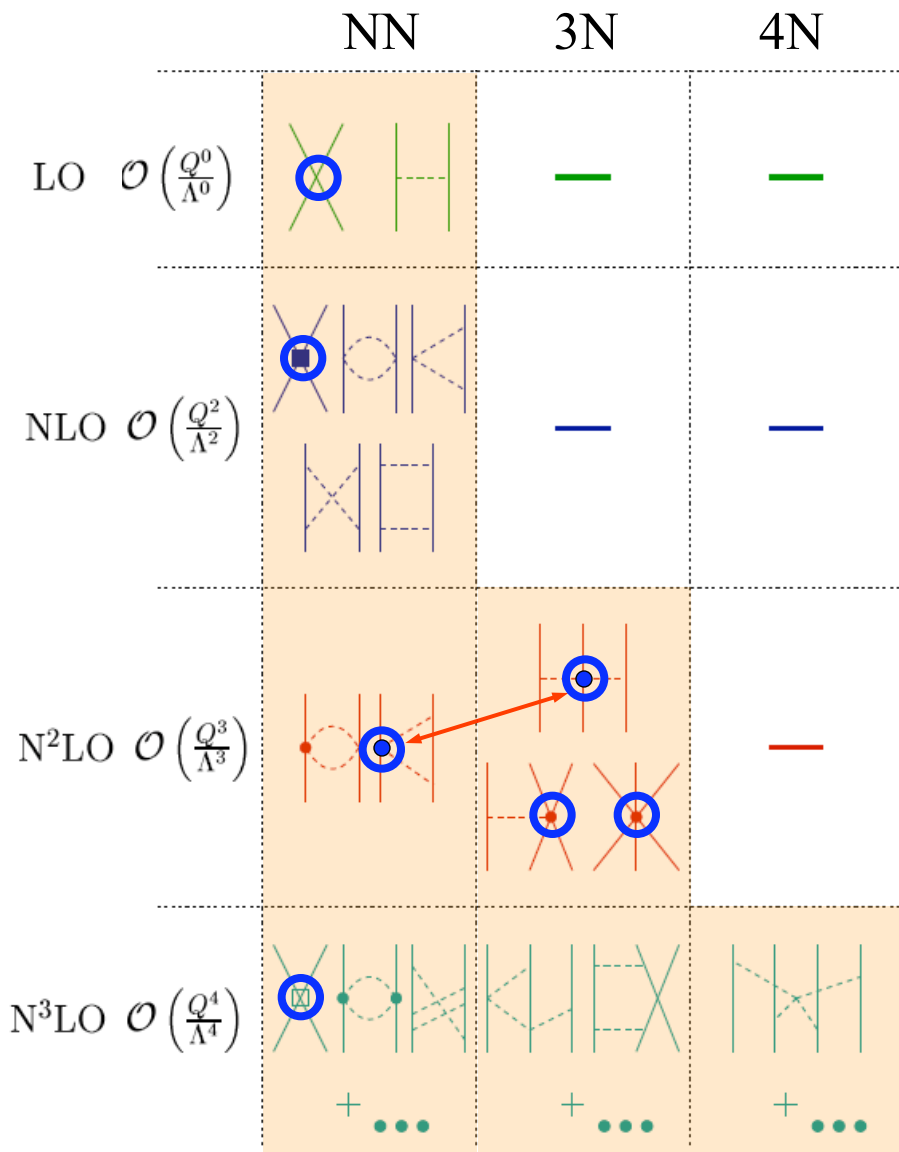
but not range of S_v and L

lower limit on crust-core
transition density



Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~ 500 MeV

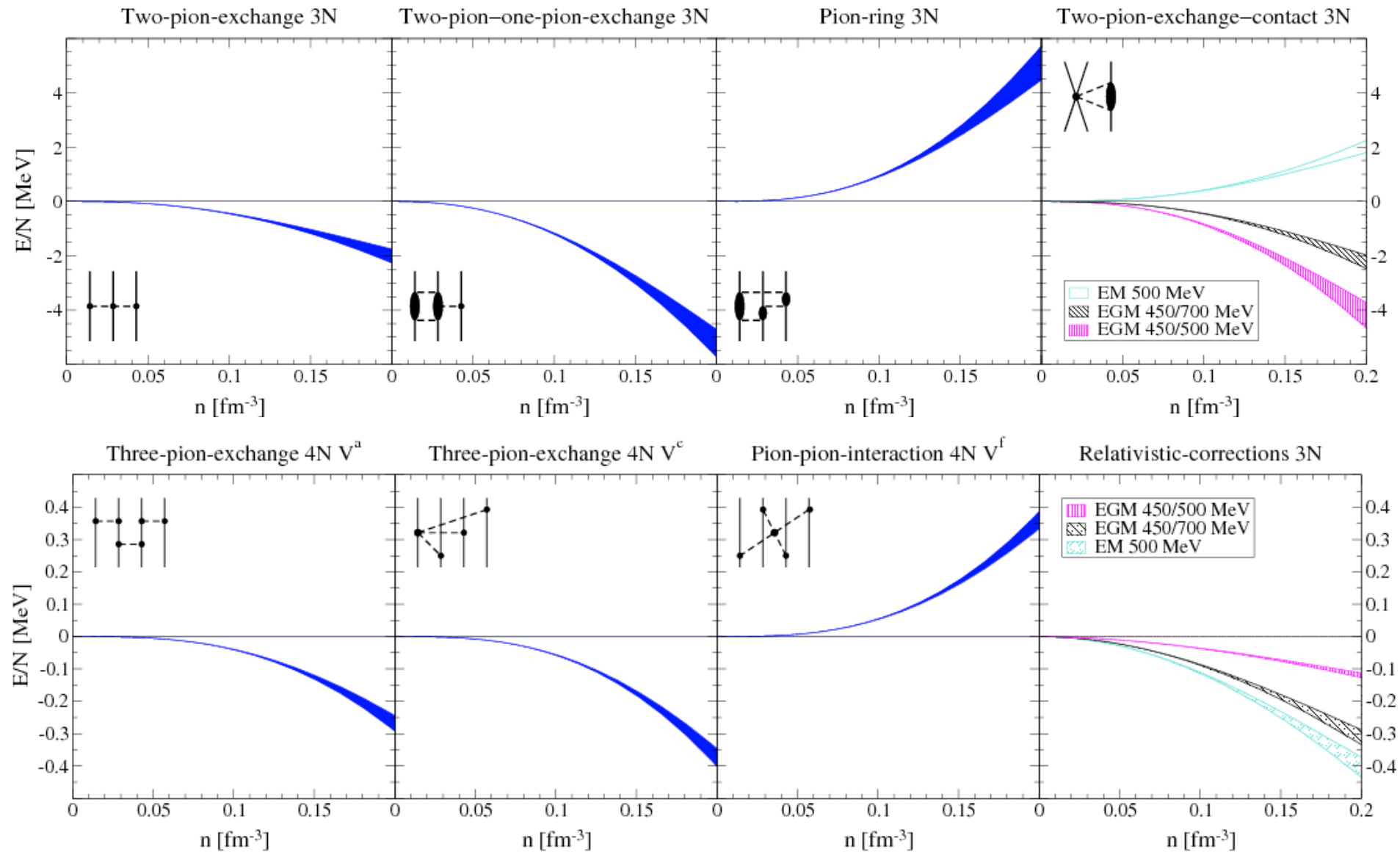


include long-range pion physics

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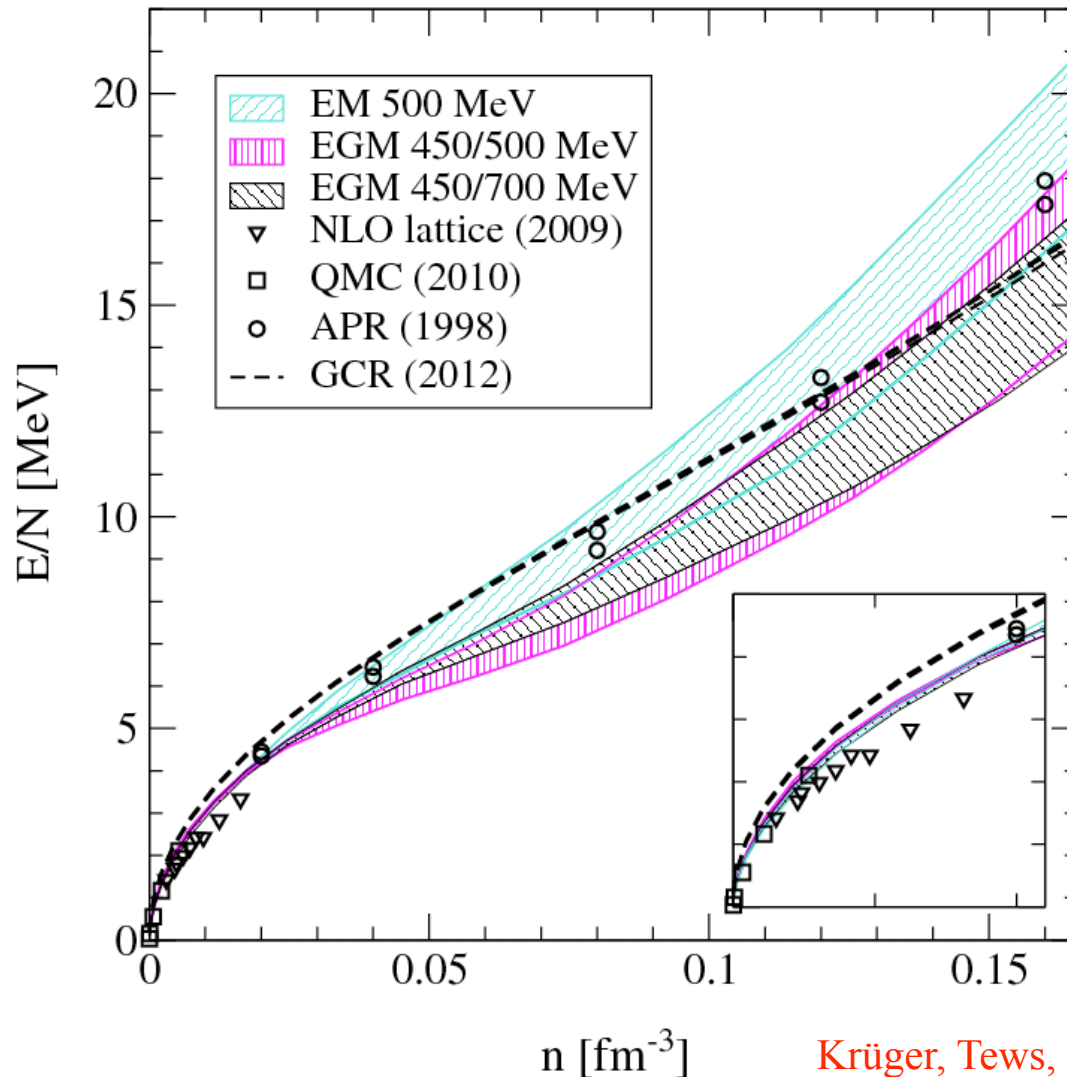
Complete N^3LO calculation of neutron matter



Complete N^3 LO calculation of neutron matter

first complete N^3 LO result

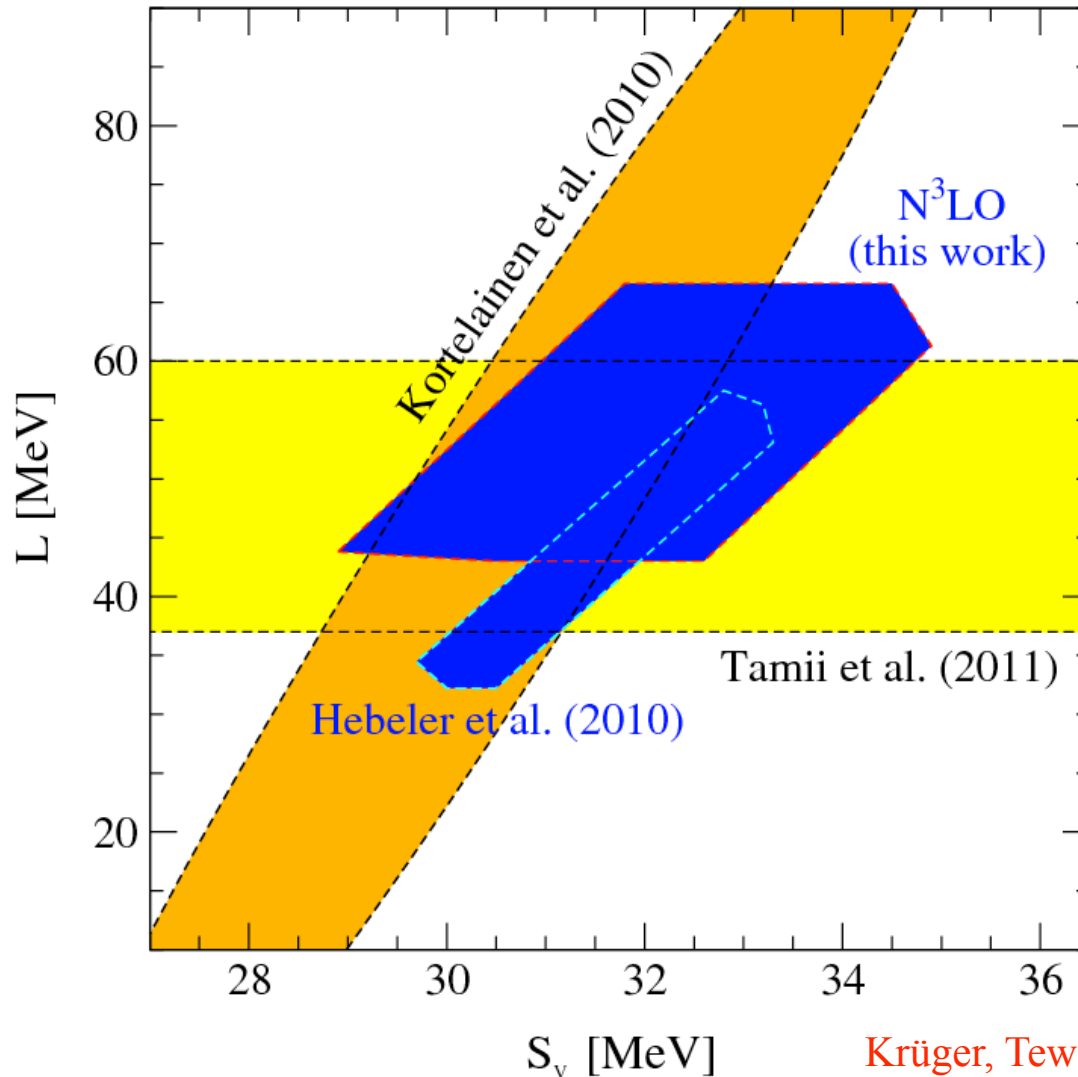
no RG evolution necessary, includes uncertainties from bare NN, 3N, 4N



Complete N^3LO calculation of neutron matter

first complete N^3LO result

no RG evolution necessary, includes uncertainties from bare NN, 3N, 4N



N^3LO correlation
broader because more
density dependences

Discovery of the heaviest neutron star

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}

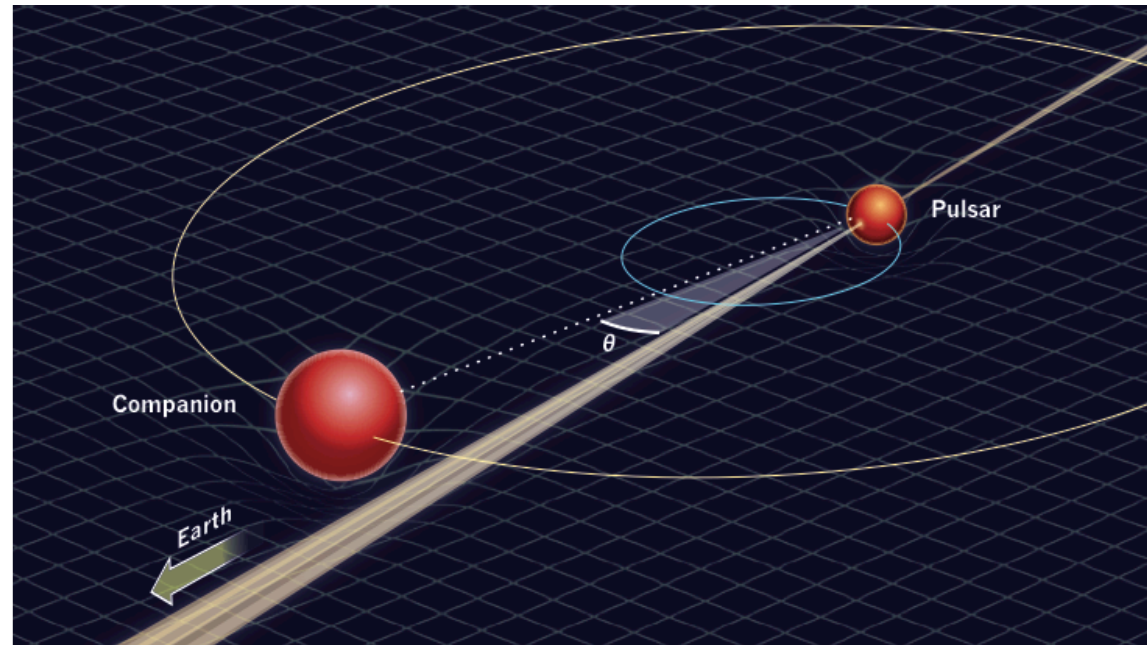
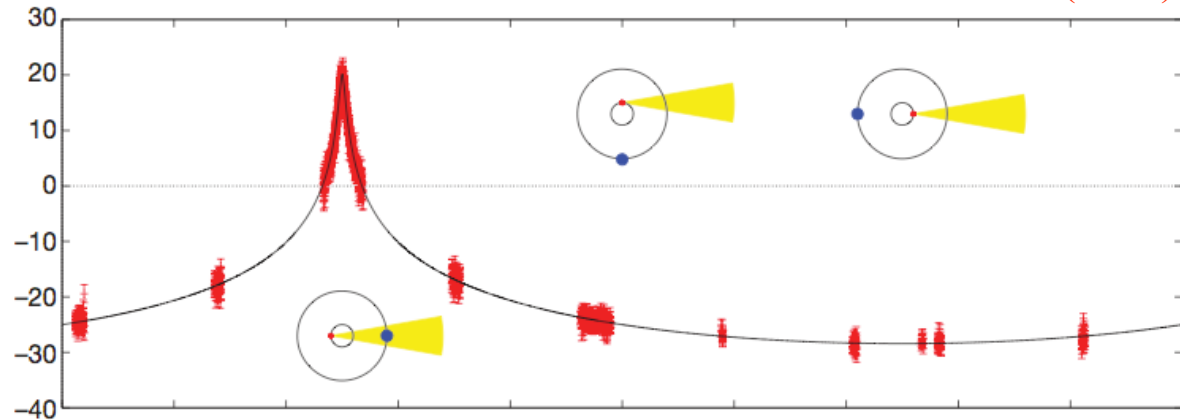
Nature (2010)

direct measurement of
neutron star mass from
increase in signal travel
time near companion

J1614-2230

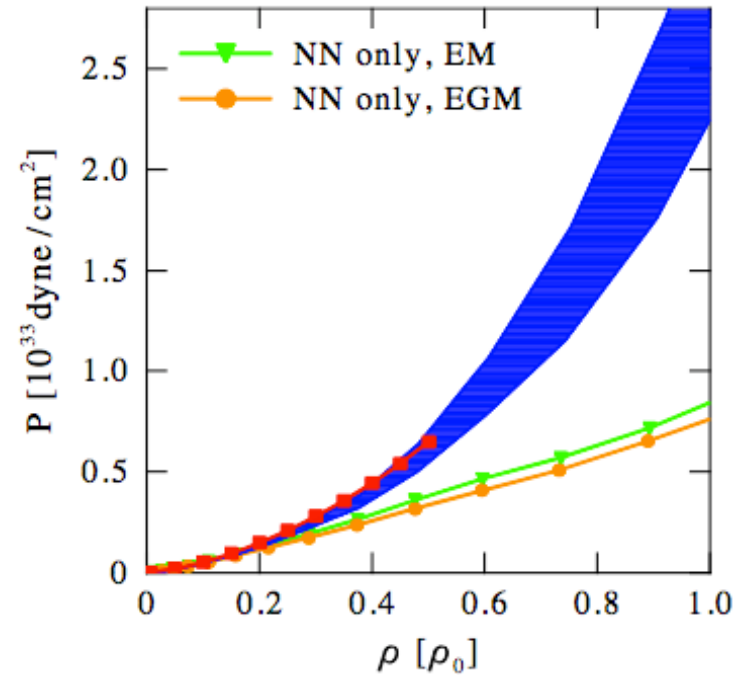
most edge-on binary
pulsar known (89.17°)
+ massive white dwarf
companion ($0.5 M_{\text{sun}}$)

heaviest neutron star
with $1.97 \pm 0.04 M_{\text{sun}}$



Impact on neutron stars Hebeler et al. (2010) and in prep.

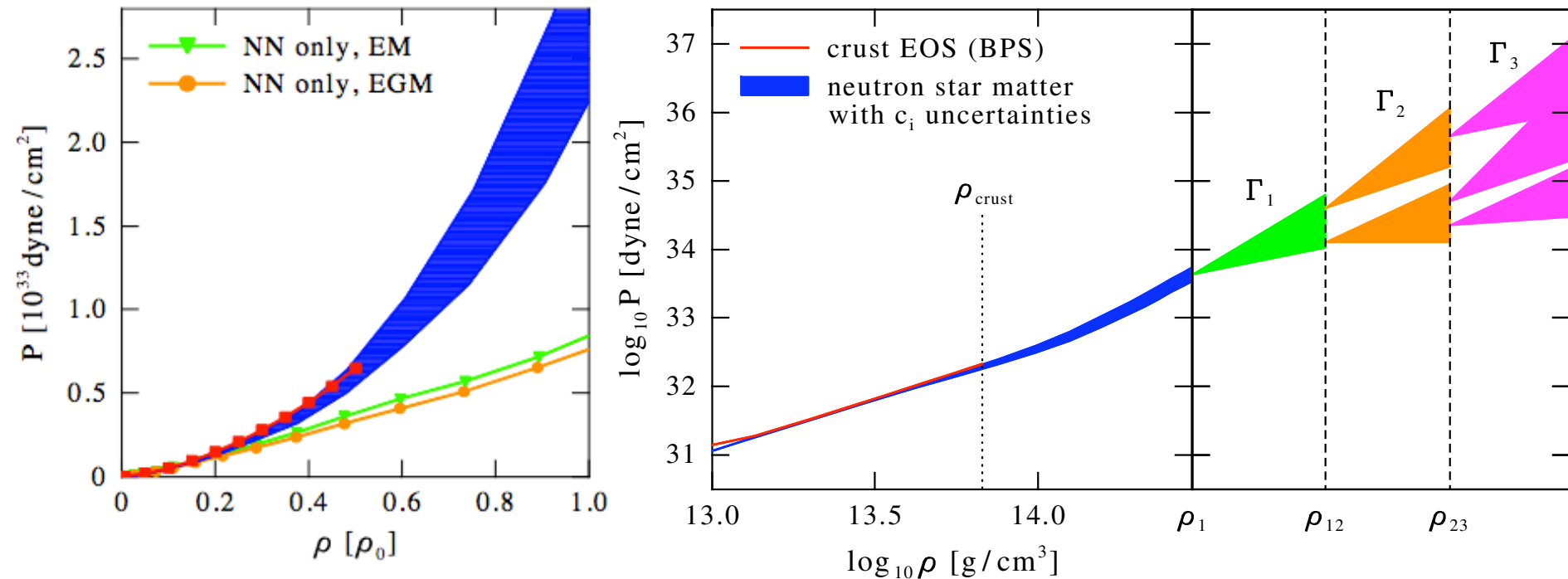
Equation of state/pressure for **neutron-star matter** (includes small $Y_{e,p}$)



pressure below nuclear densities agrees with standard crust equation of state only after 3N forces are included

Impact on neutron stars Hebeler et al. (2010) and in prep.

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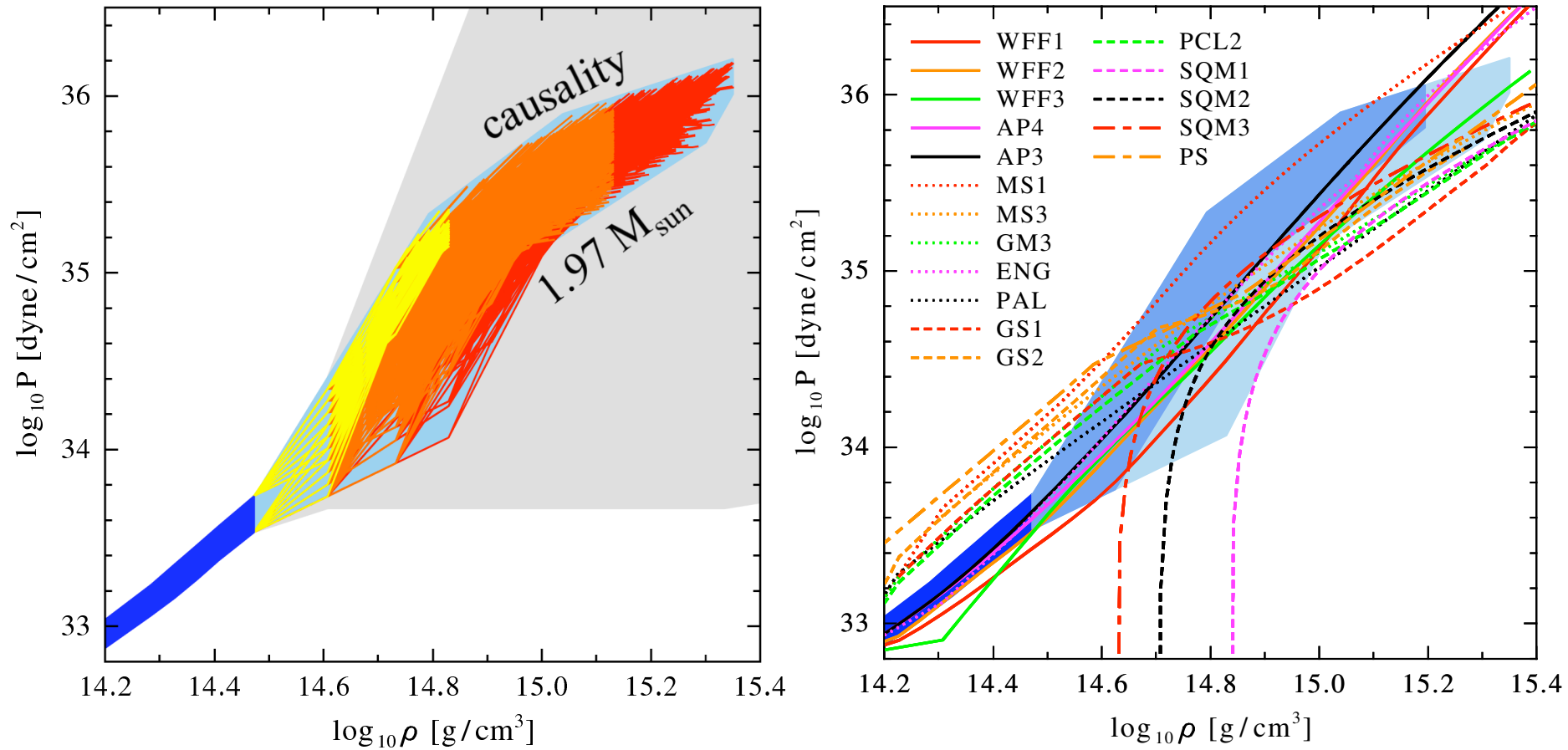


pressure below nuclear densities agrees with standard crust equation of state only after 3N forces are included

extend uncertainty band to higher densities using piecewise polytropes
allow for soft regions

Pressure of neutron star matter

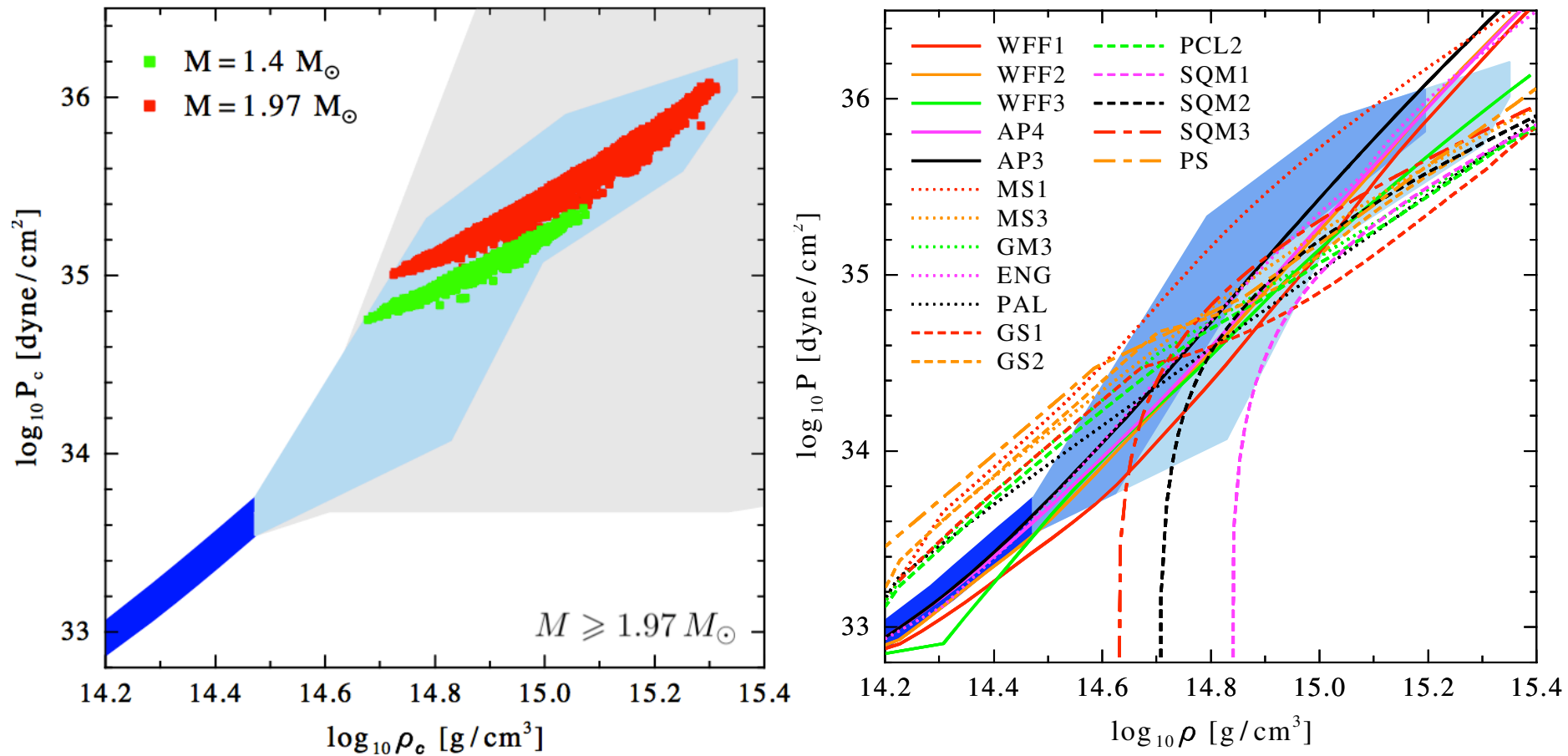
constrain polytropes by causality and require to support $1.97 M_{\text{sun}}$ star



low-density pressure sets scale, chiral EFT interactions provide strong constraints, ruling out many model equations of state

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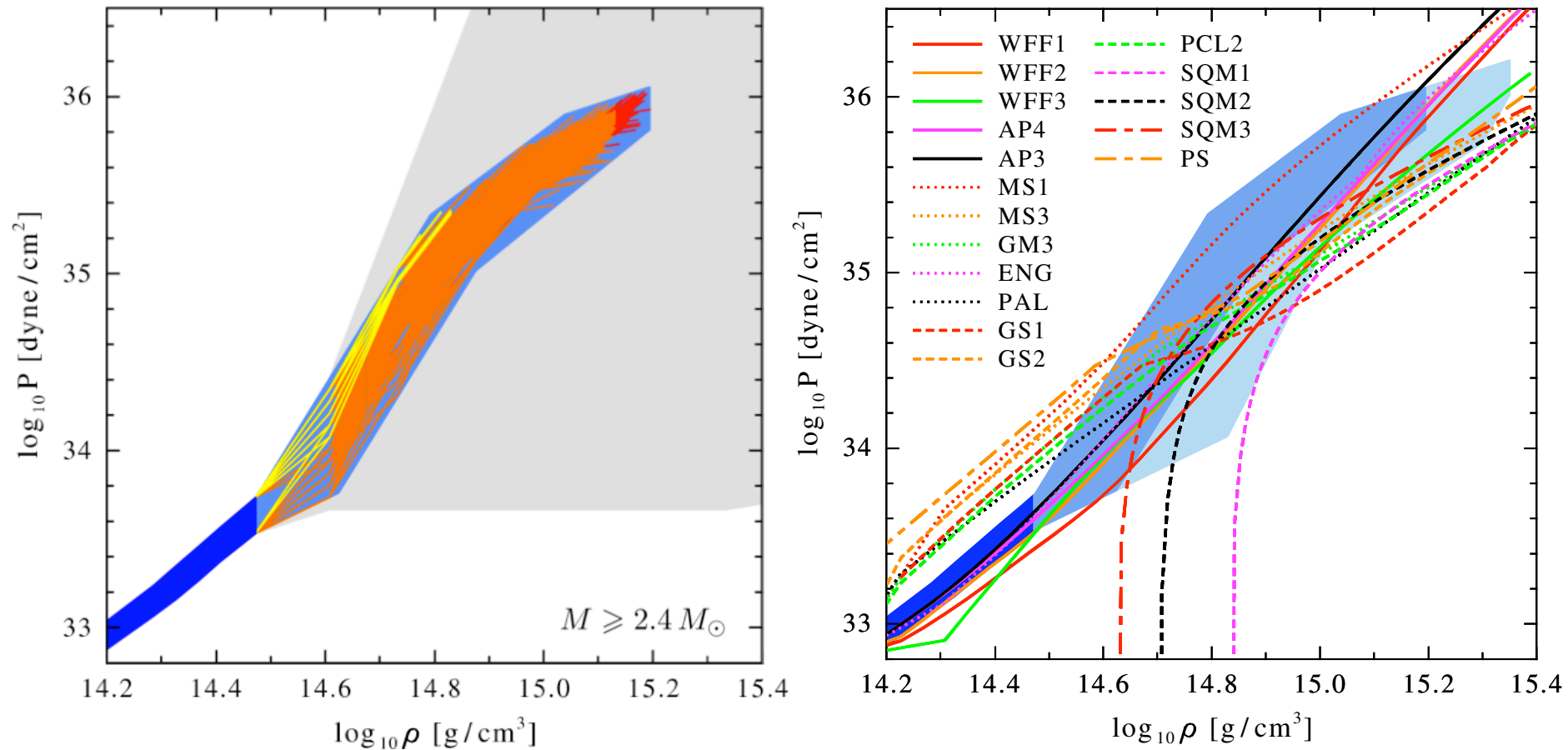


low-density pressure sets scale, chiral EFT interactions provide strong constraints, ruling out many model equations of state

central densities for $1.4 M_{\text{sun}}$ star: $1.7\text{-}4.4 \rho_0$

Pressure of neutron star matter

constrain polytropes by causality and require to support $1.97 M_{\text{sun}}$ star

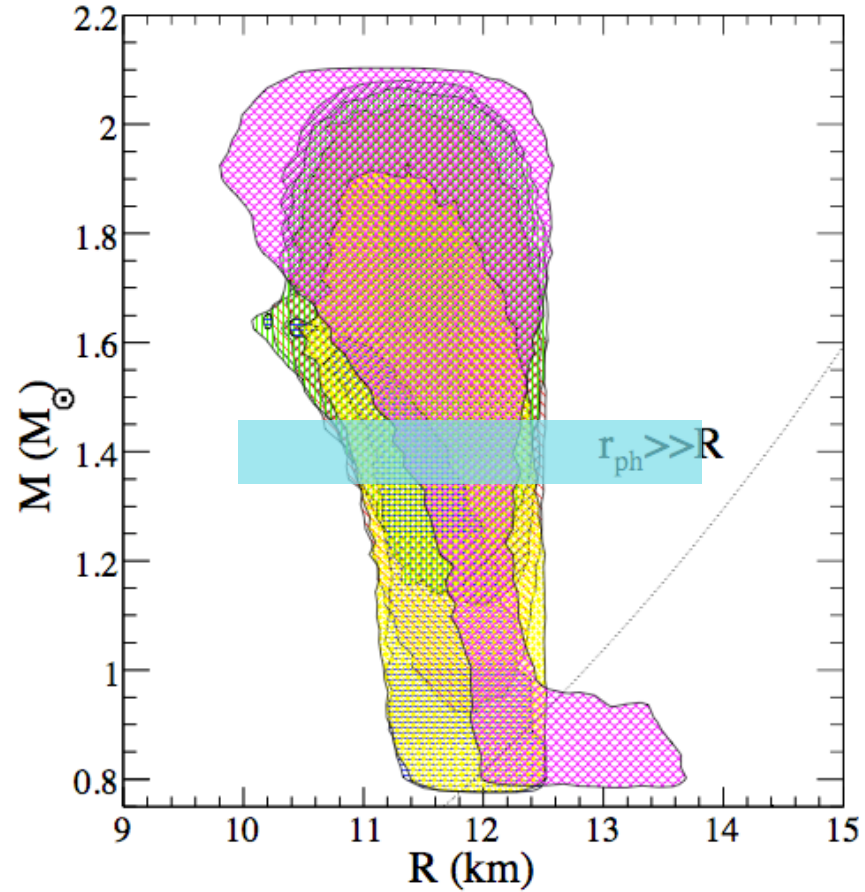
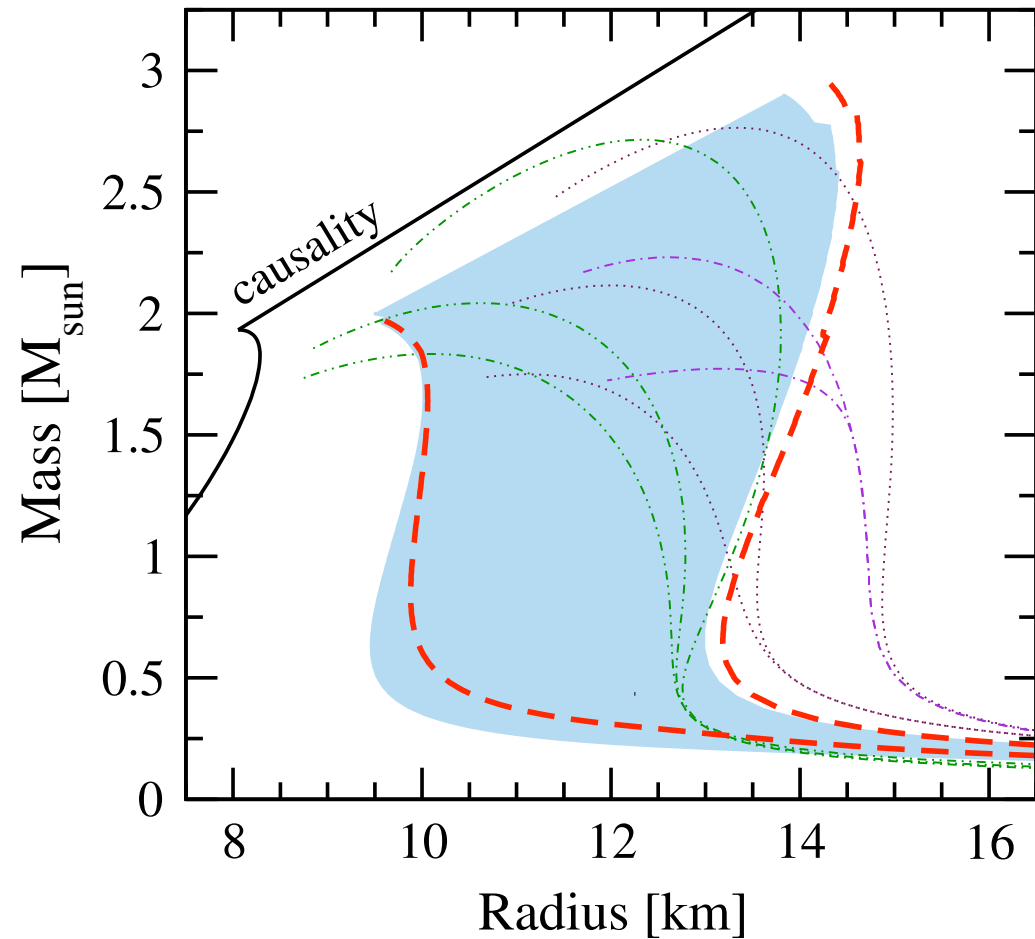


low-density pressure sets scale, chiral EFT interactions provide strong constraints, ruling out many model equations of state

darker blue band for $2.4 M_{\text{sun}}$ star

Neutron star radius constraints

uncertainty from many-body forces and general extrapolation



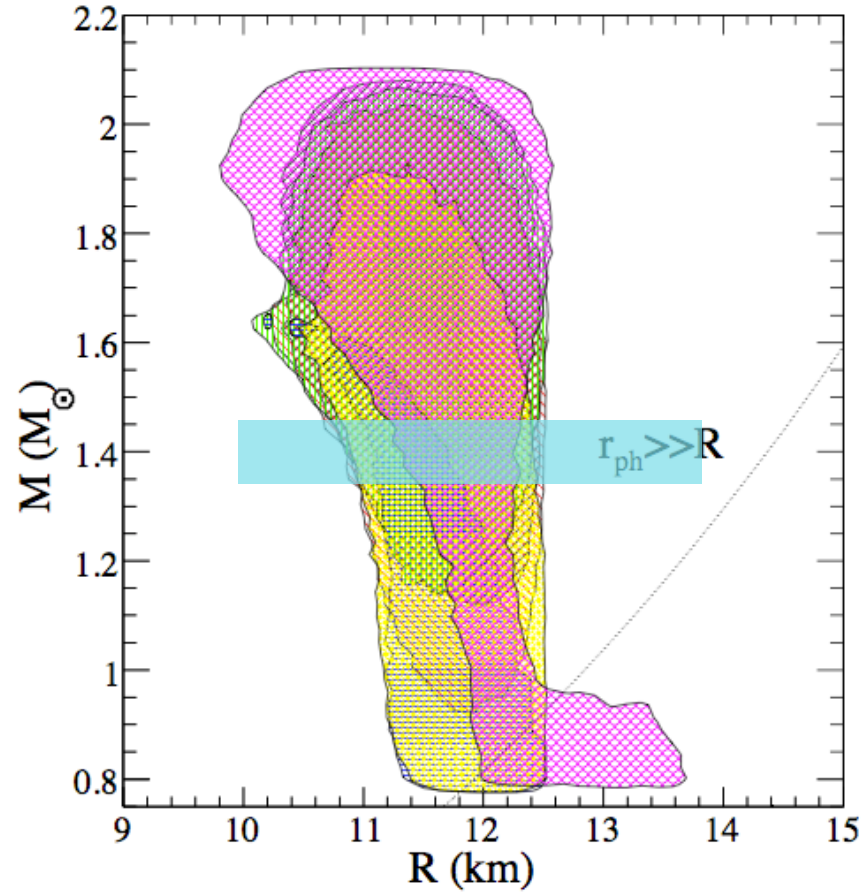
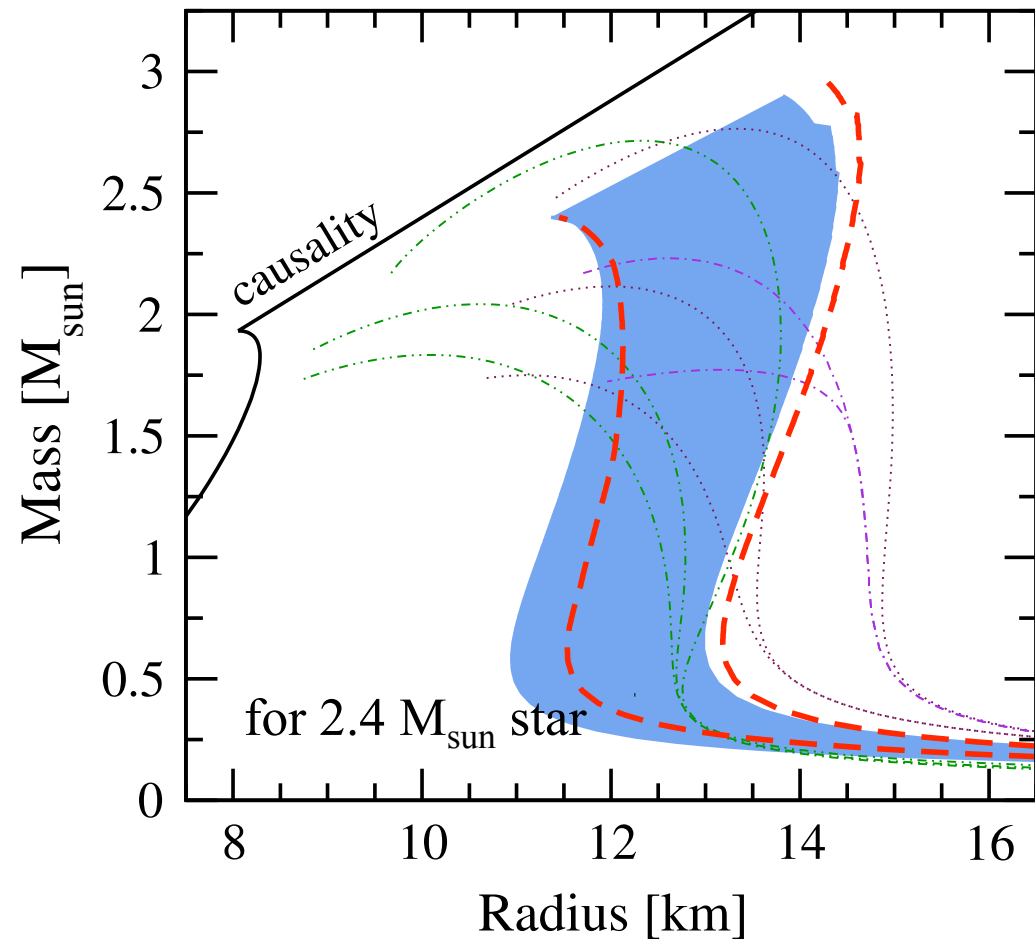
constrains neutron star radius: 9.9-13.8 km for $M=1.4 M_{\text{sun}}$ ($\pm 15\%$!)

consistent with extraction from X-ray burst sources [Steiner et al. \(2010\)](#)

provides important constraints for EOS for core-collapse supernovae

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Neutrino rates – Motivation

Neutrino rates important for neutron star crust and core cooling, supernova explosions, neutrino spectra,...

processes involving two nucleons play a special role **Friman,...** **Suzuki, Raffelt,...**

neutrino-pair bremsstrahlung and absorption $NN \leftrightarrow NN\nu\bar{\nu}$

standard cooling of low-mass neutron stars

key for production of muon and tau neutrinos in supernovae

and for equilibrating neutrino number densities

at subnuclear densities $\rho < 10^{14} \text{ g cm}^{-3}$ no systematic calculations

beyond one-pion exchange (OPE) approximation for nuclear interactions

can calculate systematically using

chiral effective field theory, electroweak interactions, many-body theory

Many-body theory: single- and two-nucleon processes

elastic scattering from nucleons (space-like $\omega < q$)

initial and final state interactions, inelastic scattering $\nu nn \leftrightarrow \nu nn$

collisional damping - Landau-Pomeranchuk-Migdal effect

neutrino-pair bremsstrahlung/absorption $nn \leftrightarrow nn\nu\bar{\nu}$ (time-like $\omega > q$)

need collisions between nucleons for the latter processes

noncentral contributions, due to tensor forces from pion exchanges and spin-orbit forces, are essential for the two-neutron response

follows from direct calculations [Friman, Maxwell \(1979\)](#)

and from conservation laws [Olsson, Pethick \(2002\)](#)

developed a unified treatment that consistently includes one- and two-nucleon response in a strongly-interacting many-body system (Boltzmann eqn for collisions, spin-dependent mean-field effects,...)

Neutrino processes and dynamical structure factors

neutrinos interact weakly \rightarrow rates for neutrino scattering, emission and absorption determined by dynamical structure factors of nucleon matter

generally axial/spin response most important, \sim factor 3

neutrino rates $\Gamma(\omega, \mathbf{q}) = 2\pi n G_F^2 C_A^2 (3 - \cos \theta) S_A(\omega, \mathbf{q})$

with spin dynamical structure factor $S_A(\omega, \mathbf{q}) = \frac{1}{\pi n} \frac{1}{1 - e^{-\omega/T}} \text{Im} \chi_\sigma(\omega, \mathbf{q})$

energy, momentum transfers ω, \mathbf{q} small compared with Fermi momentum

for low temperatures \sim Fermi temperature or less

\rightarrow Landau Fermi liquid theory is a reasonable first approximation
problem is to calculate structure factors of nucleon matter

not included: reduction of axial coupling g_a for nucleon quasiparticles
by 5-10% in neutron matter [Cowell, Pandharipande \(2003\)](#)

beyond quasiparticle contributions (incoherent parts)

Unified approach to structure factors

solve linearized quasiparticle transport equation for the spin response

$$(\omega - \varepsilon_{\mathbf{p}+\mathbf{q}/2} + \varepsilon_{\mathbf{p}-\mathbf{q}/2}) \delta \mathbf{s}_{\mathbf{p}} + (n_{\mathbf{p}+\mathbf{q}/2} - n_{\mathbf{p}-\mathbf{q}/2}) \delta \mathbf{h}_{\mathbf{p}} = i I_{\sigma}[\mathbf{s}_{\mathbf{p}'}]$$

includes one-pair states through perturbation of the quasiparticle energy, spin-dependent mean-field

$$\delta \mathbf{h}_{\mathbf{p}} = \mathbf{U}_{\sigma} + 2 \int \frac{d\mathbf{p}'}{(2\pi)^3} g_{\mathbf{p}\mathbf{p}'} \delta \mathbf{s}_{\mathbf{p}'}$$

two-nucleon contributions through collision term $I_{\sigma}[\mathbf{s}_{\mathbf{p}'}] = -\frac{\delta \mathbf{s}_{\mathbf{p}} - \delta \mathbf{s}_{\mathbf{p}}|_{\text{le}}}{\tau_{\sigma}}$
in relaxation time approximation

spin relaxation rate $\frac{1}{\tau_{\sigma}} = C_{\sigma} [T^2 + (\omega/2\pi)^2]$

from quasiparticle scattering amplitude averaged over the Fermi surface

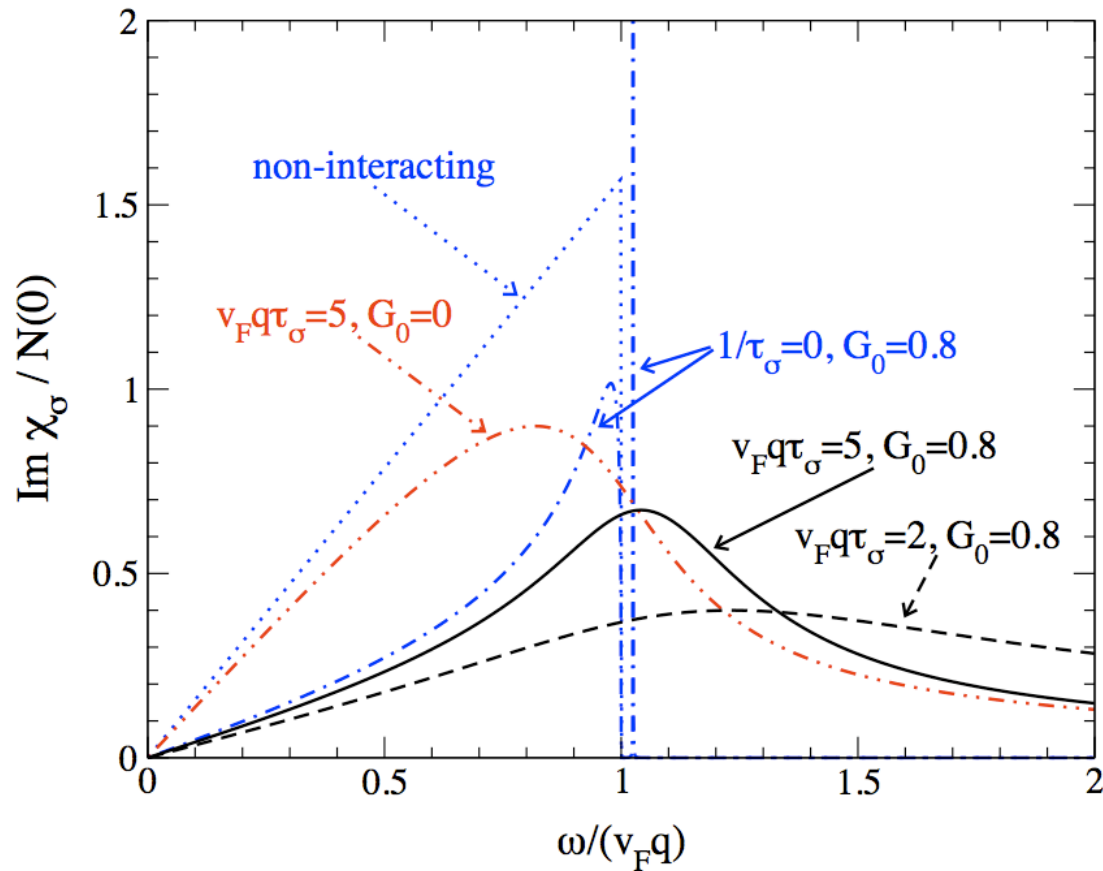
solution to qp transport equation includes multiple-scattering effects, Landau-Pomeranchuk-Migdal effect

Spin dynamical structure factor

solution to quasiparticle transport equation $\text{Im}\chi_\sigma = N(0) \frac{\text{Im}\tilde{X}_\sigma}{|1 + G_0\tilde{X}_\sigma|^2}$

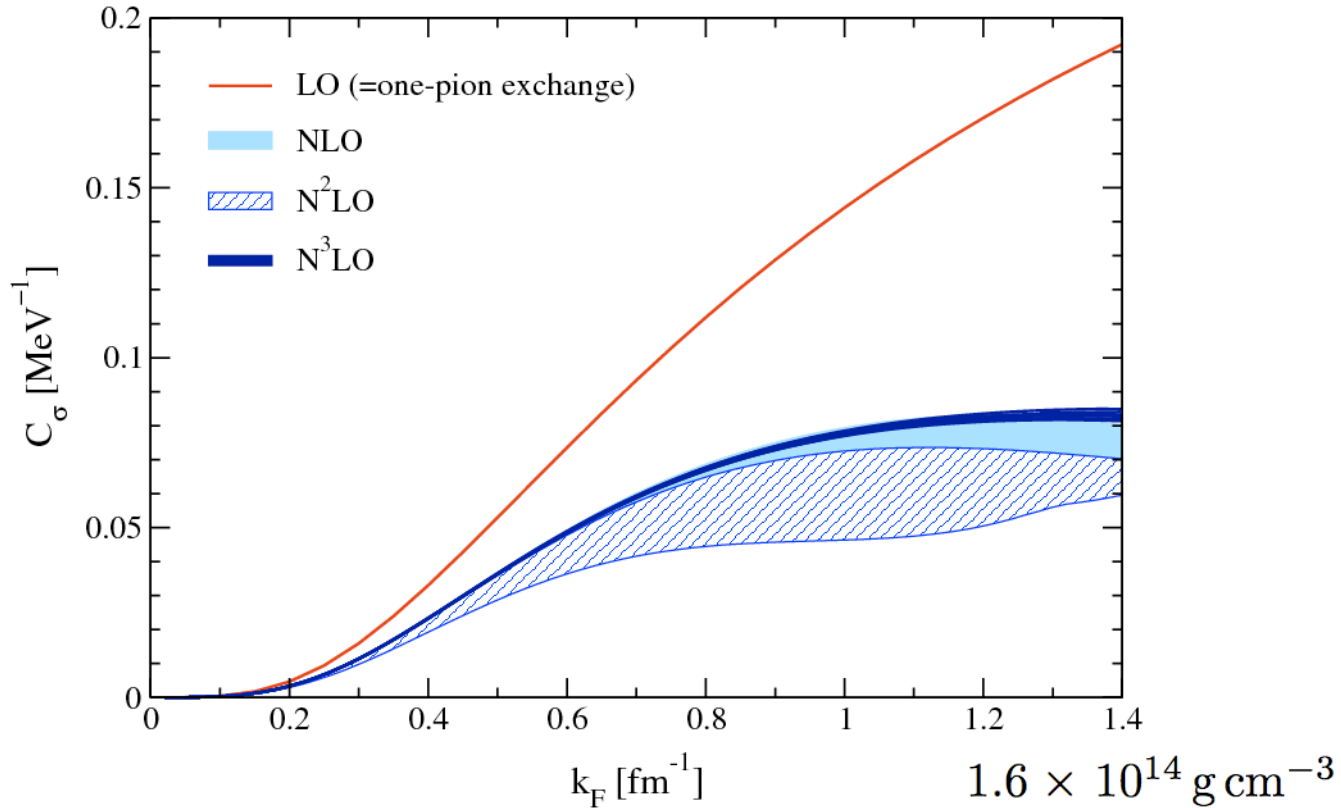
isotropic Landau interaction G_0 dominates in neutron matter

in relaxation time approximation $\tilde{X}_\sigma = 1 - \frac{\omega}{2v_Fq} \ln\left(\frac{\omega + i/\tau_\sigma + v_Fq}{\omega + i/\tau_\sigma - v_Fq}\right)$



Neutrino rates from chiral EFT S. Bacca et al. (2009)

neutrino rates in $2N$ processes determined by spin relaxation time
= rate of change of nucleon spin through collisions with other nucleons

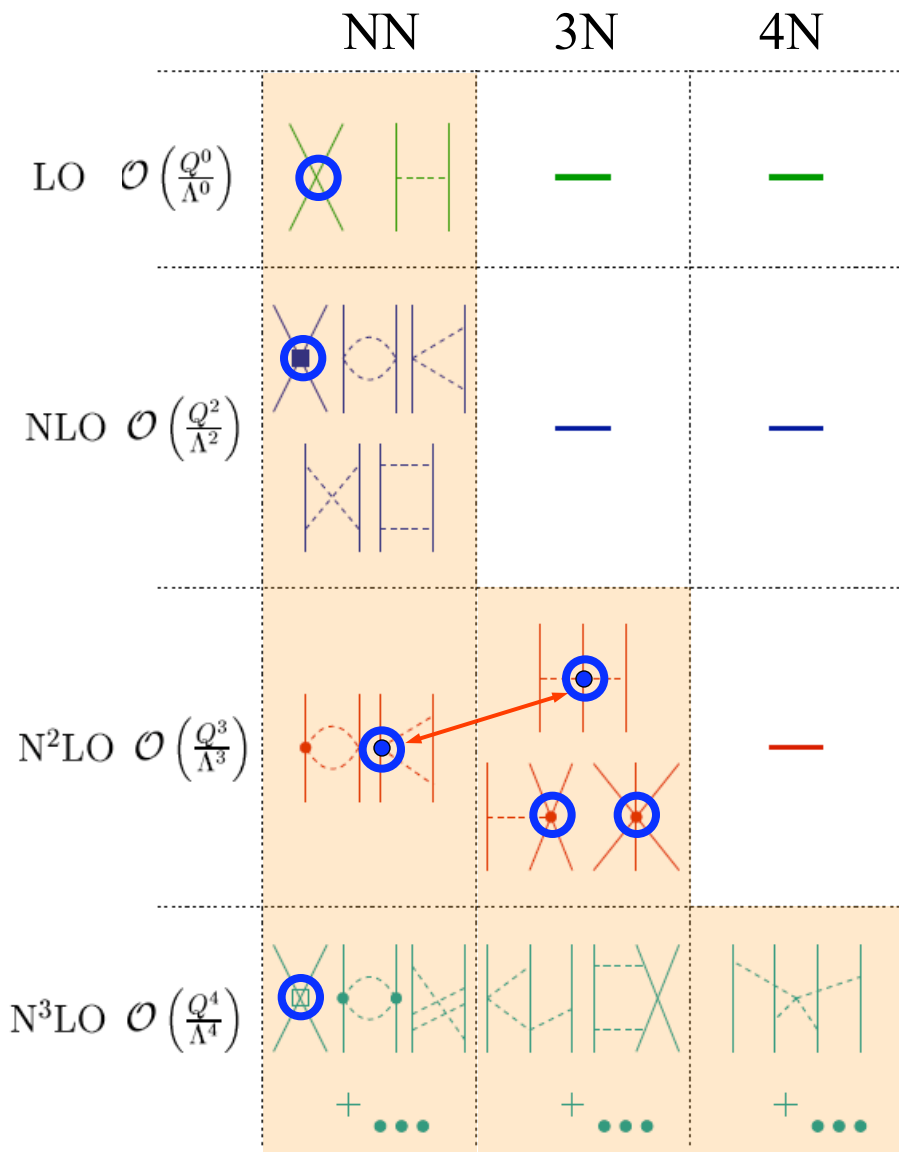


shorter-range interactions significantly reduce neutrino rates (compared to OPE) in neutron matter for all relevant densities

first calculation of neutrino processes in dense matter from chiral EFT

Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~ 500 MeV



include long-range pion physics

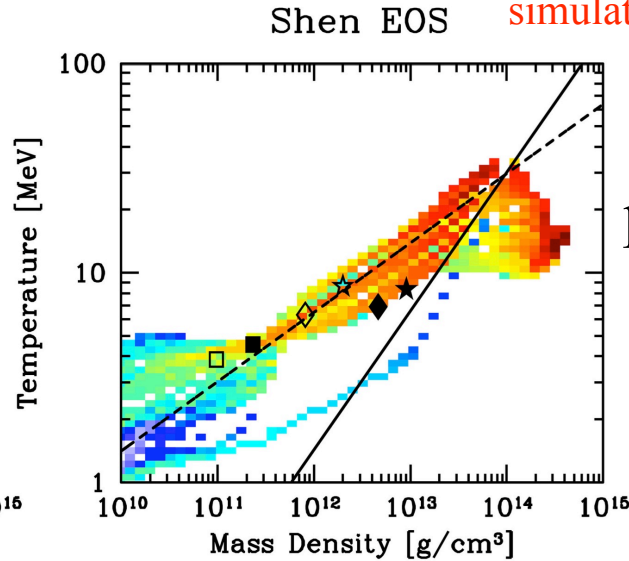
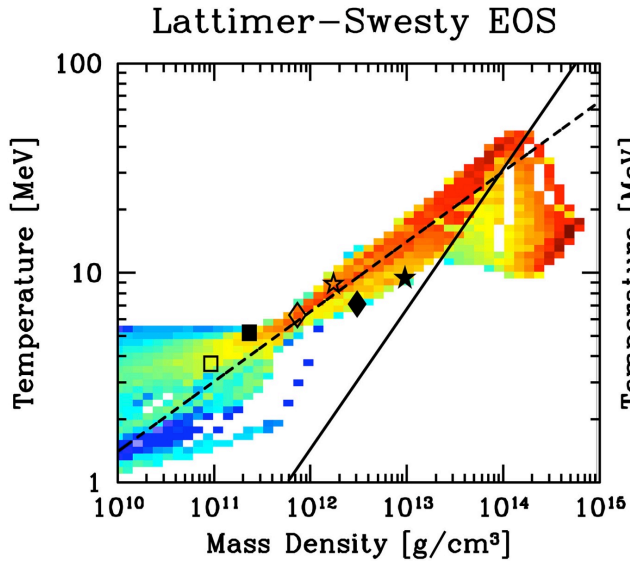
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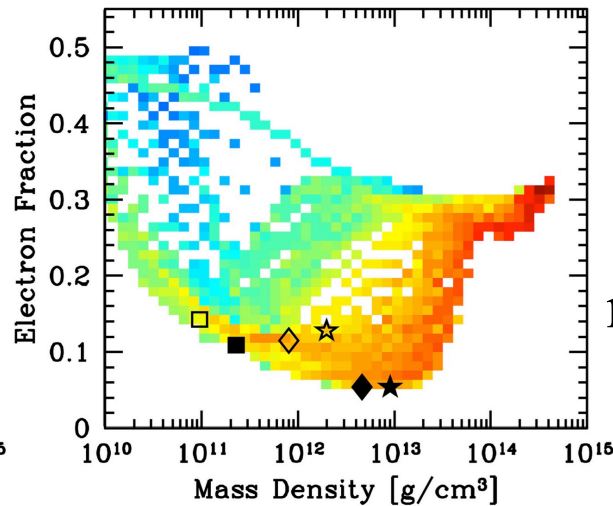
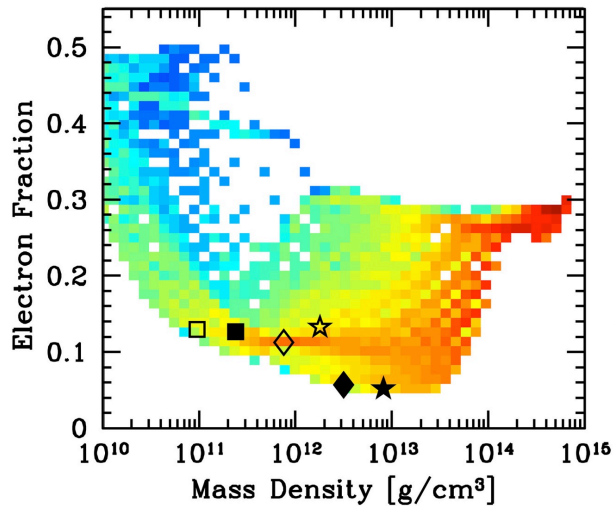
Relevant conditions in core-collapse supernovae

15 M_{\odot} progenitor

S. Bacca et al., arXiv:1112.5185
simulations by M. Liebendörfer et al.



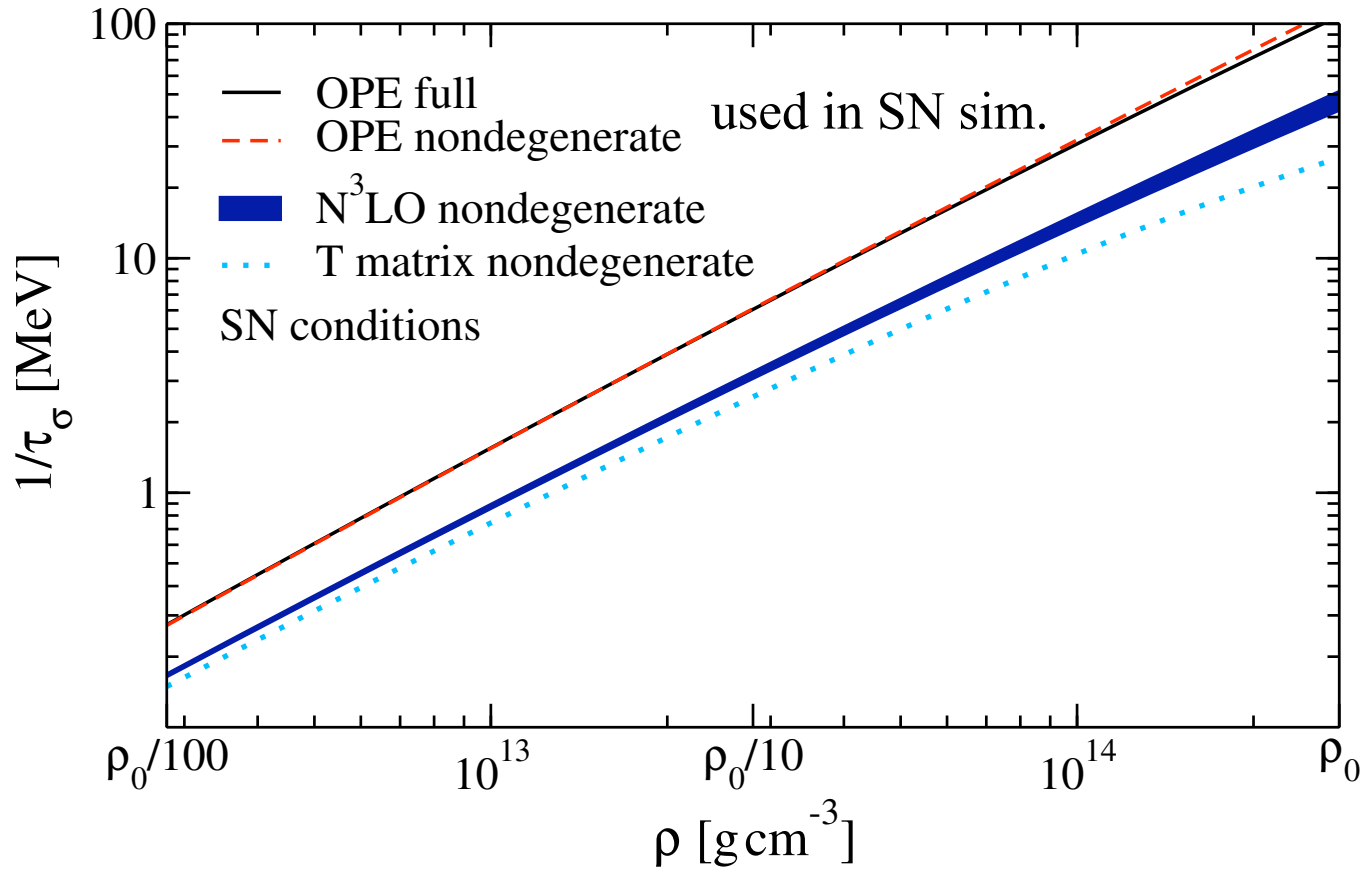
partially degenerate



neutron-rich

crucial densities below nuclear matter density $\sim 10^{13}$ - 10^{14} g/cm^3
(high densities: neutrinos trap; low densities: few interactions)

neutrons only, arbitrary degeneracy



similar reduction along SN conditions

towards chiral EFT rates in supernova simulations with A. Bartl et al.

Energy transfer in neutrino scattering from nucleons

mean-square neutrino energy transfer in $\nu nn \leftrightarrow \nu nn$

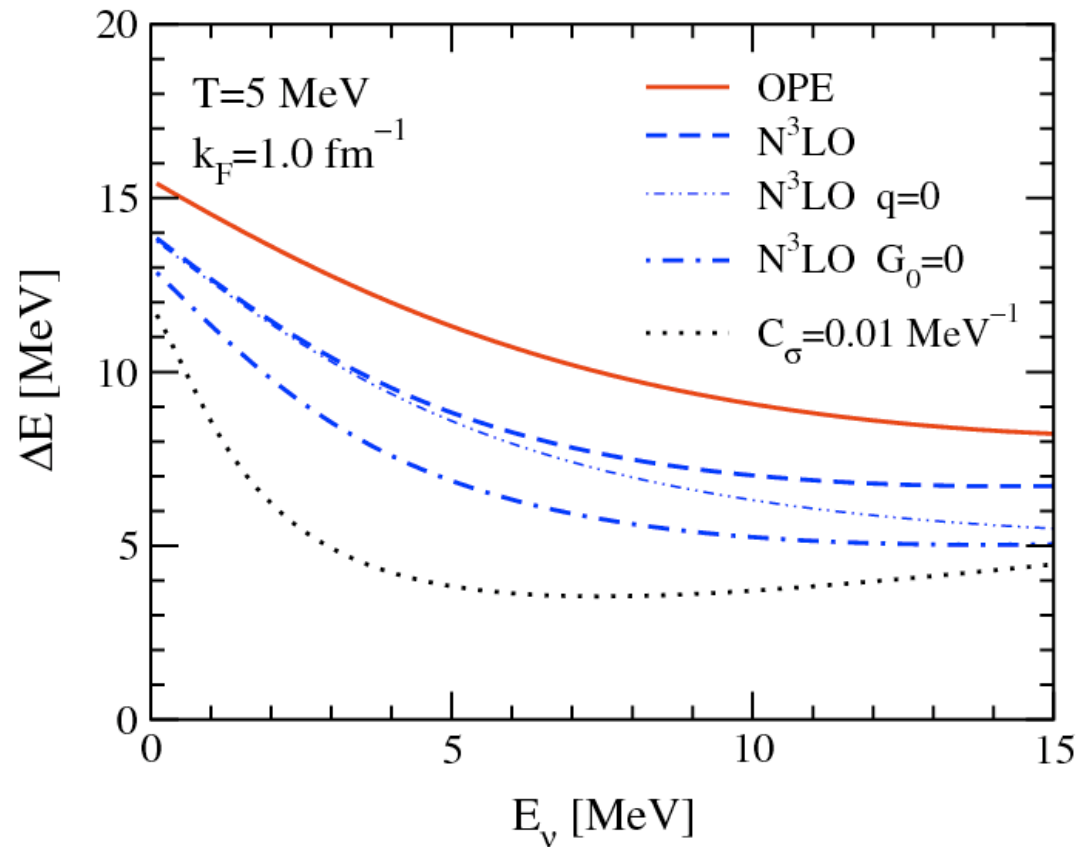
$$(\Delta E)^2 = \frac{\int d\mathbf{p}'_\nu (E_\nu - E'_\nu)^2 \Gamma(E_\nu - E'_\nu, p_\nu - p'_\nu)}{\int d\mathbf{p}'_\nu \Gamma(E_\nu - E'_\nu, p_\nu - p'_\nu)}$$

leads to heating,

NN analogue of inelastic
excitations of nuclei
(but post-collapse)

energy transfer significant,
dominates over recoil effects

not included in simulations



Main points and summary

Chiral EFT interactions provide strong constraints for EOS,
3N forces are a frontier for neutron-rich nuclei/matter

dominant uncertainty of neutron (star) matter below nuclear densities
also key to explain neutron-rich nuclei

neutron star radius 9.9-13.8 km for $M=1.4 M_{\text{sun}}$ ($\pm 15\%$)

towards chiral EFT rates in supernova simulations,
will provide simple structure factors to use