NEUTRINO OPACITIES IN DENSE MATTER:

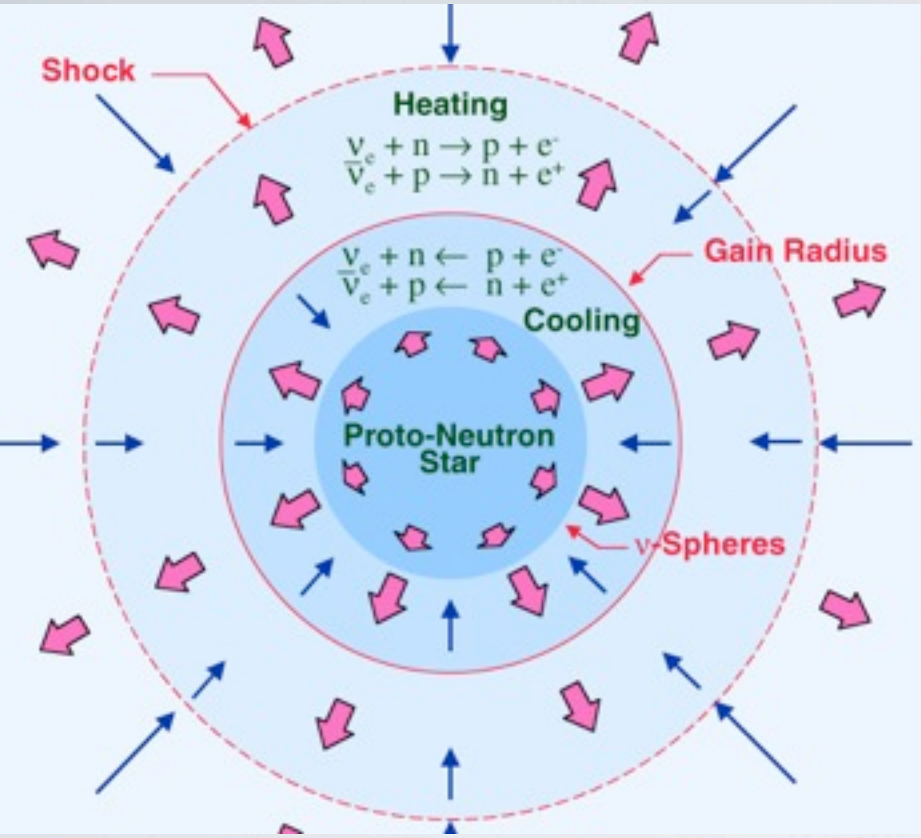
AN APPRAISAL OF PAST AND RECENT RESULTS

- Introduction and motivation.
- Cross-sections and correlation functions.
- Mean field theory, multi-particle excitations.
- Charged current rates.
- Benchmarks and sum rules.
- Summary and outlook.

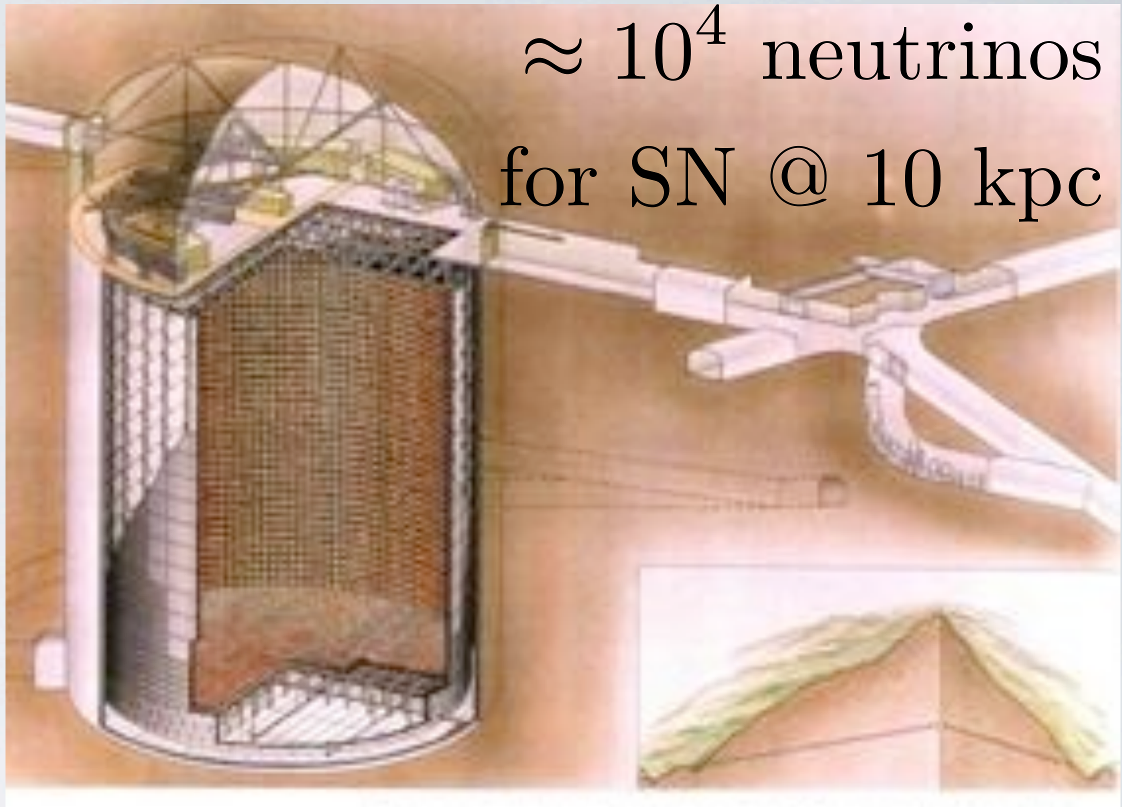
Sanjay Reddy
INT, Univ. of Washington, Seattle

Core-Collapse Supernova: Models and Observable Signals (2012)

MOTIVATION



$\nu_e, \bar{\nu}_e, \nu_X, \bar{\nu}_X$

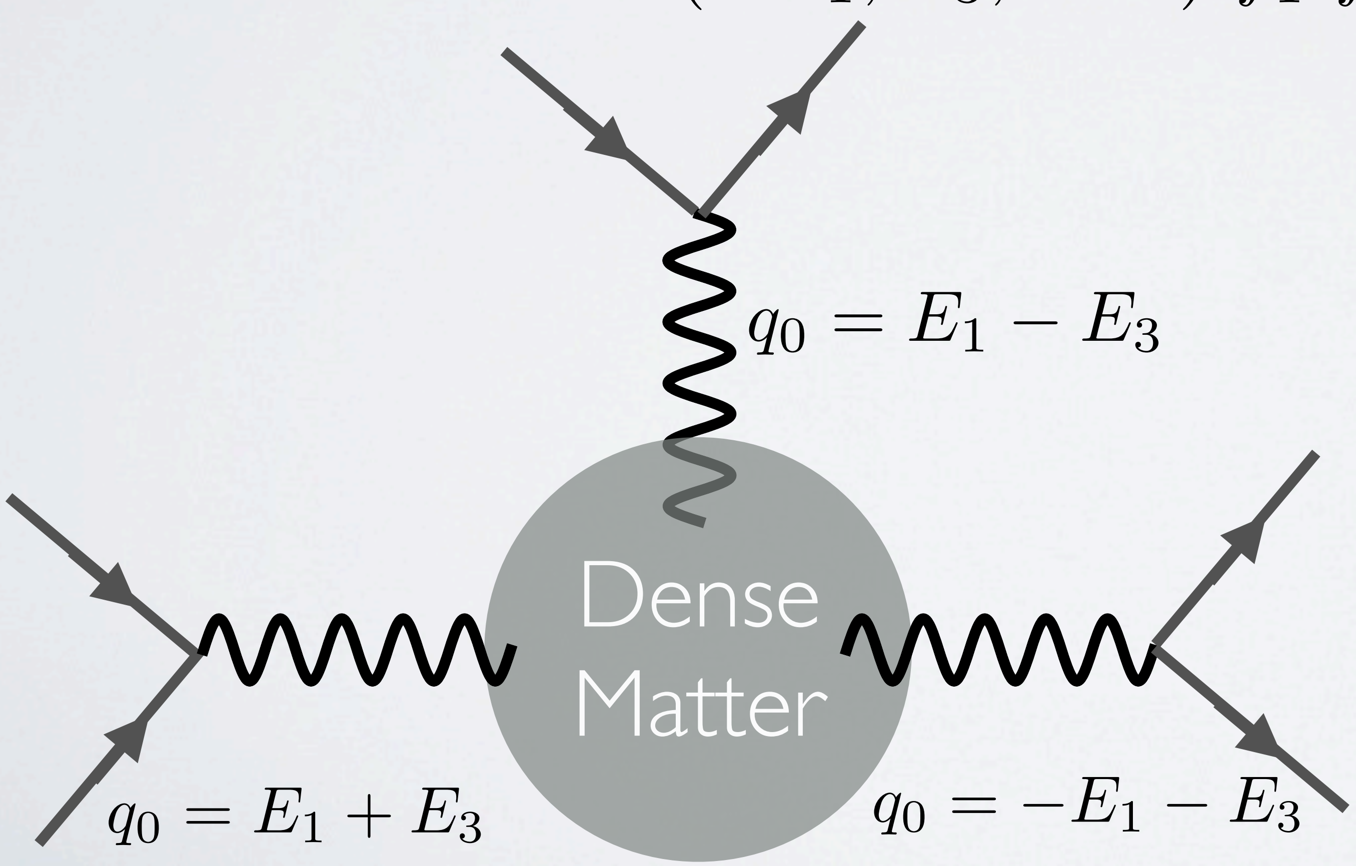


- Supernova neutrinos are detectable.
- Neutrino luminosity and spectra influence the explosion mechanism and nucleosynthesis.
- Temporal and spectral features of the supernova neutrino signal is an unique probe of physics/astrophysics under extreme conditions, neutrino properties and exotic light weakly interacting particles.

NEUTRINO TRANSPORT

- RHS of the Boltzmann Equation.

$$
\begin{aligned}
\frac{\partial f(E_1)}{\partial t} = \int \frac{d^3k_3}{(2\pi)^3}\ & R(E_1, E_3, \cos\theta)\ f_3(1-f_1) \\
& -R(E_3, E_1, \cos\theta)\ f_1(1-f_3) \\
& +R(E_1, -E_3, \cos\theta)\ (1-f_1)(1-f_3) \\
& -R(-E_1, E_3, \cos\theta)\ f_1\ f_3
\end{aligned}
$$



NEUTRINO TRANSPORT

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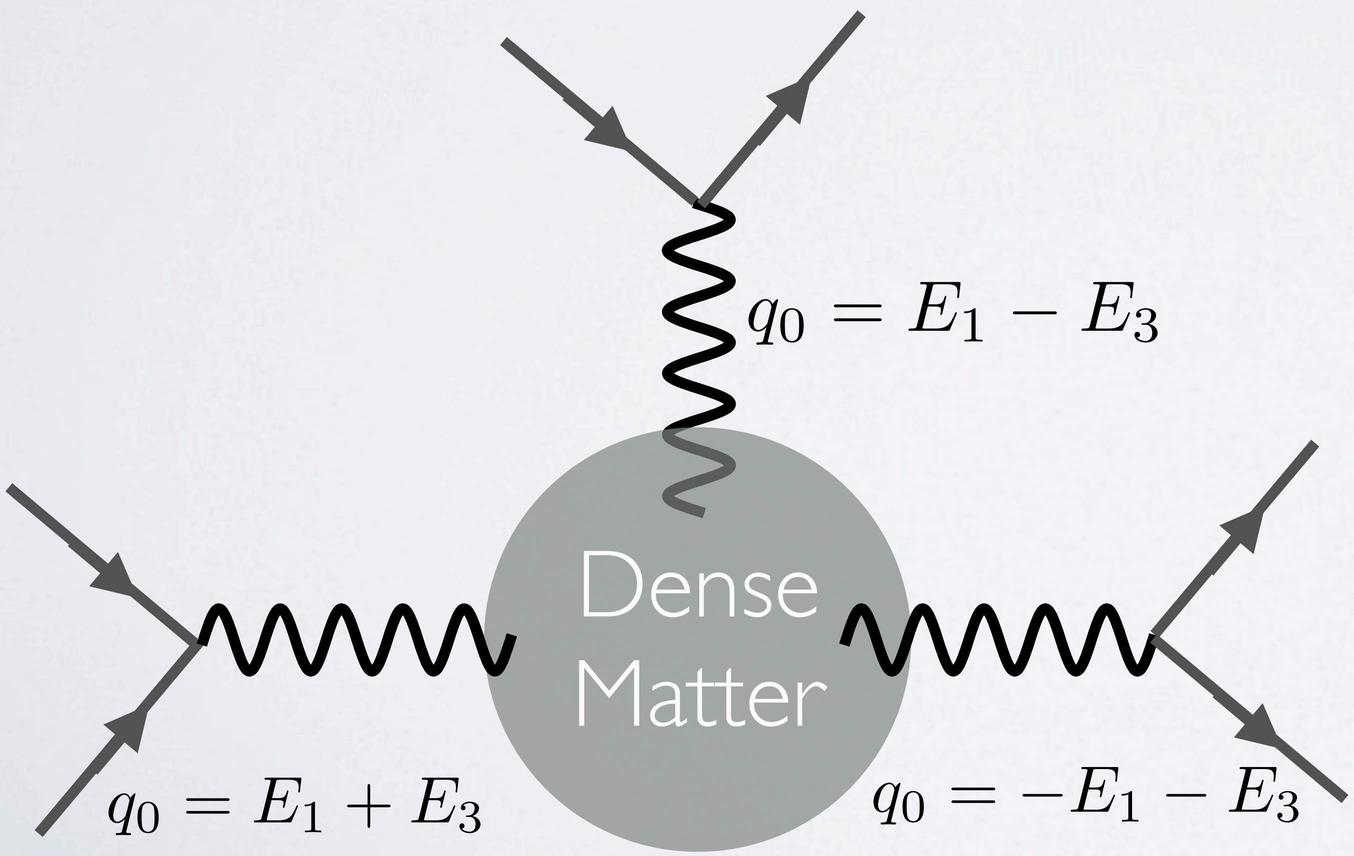
$$\frac{\partial f(E_1)}{\partial t} = \int \frac{d^3k_3}{(2\pi)^3} \; R(E_1, E_3, \cos\theta)\; f_3(1-f_1) \longrightarrow \text{scattering-in}$$

$$-R(E_3, E_1, \cos\theta)\; f_1(1-f_3) \longrightarrow \text{scattering-out}$$

$$+R(E_1, -E_3, \cos\theta)\; (1-f_1)(1-f_3) \longrightarrow \text{pair-production}$$

$$-R(-E_1, E_3, \cos\theta)\; f_1\; f_3 \longrightarrow \text{pair-annihilation}$$



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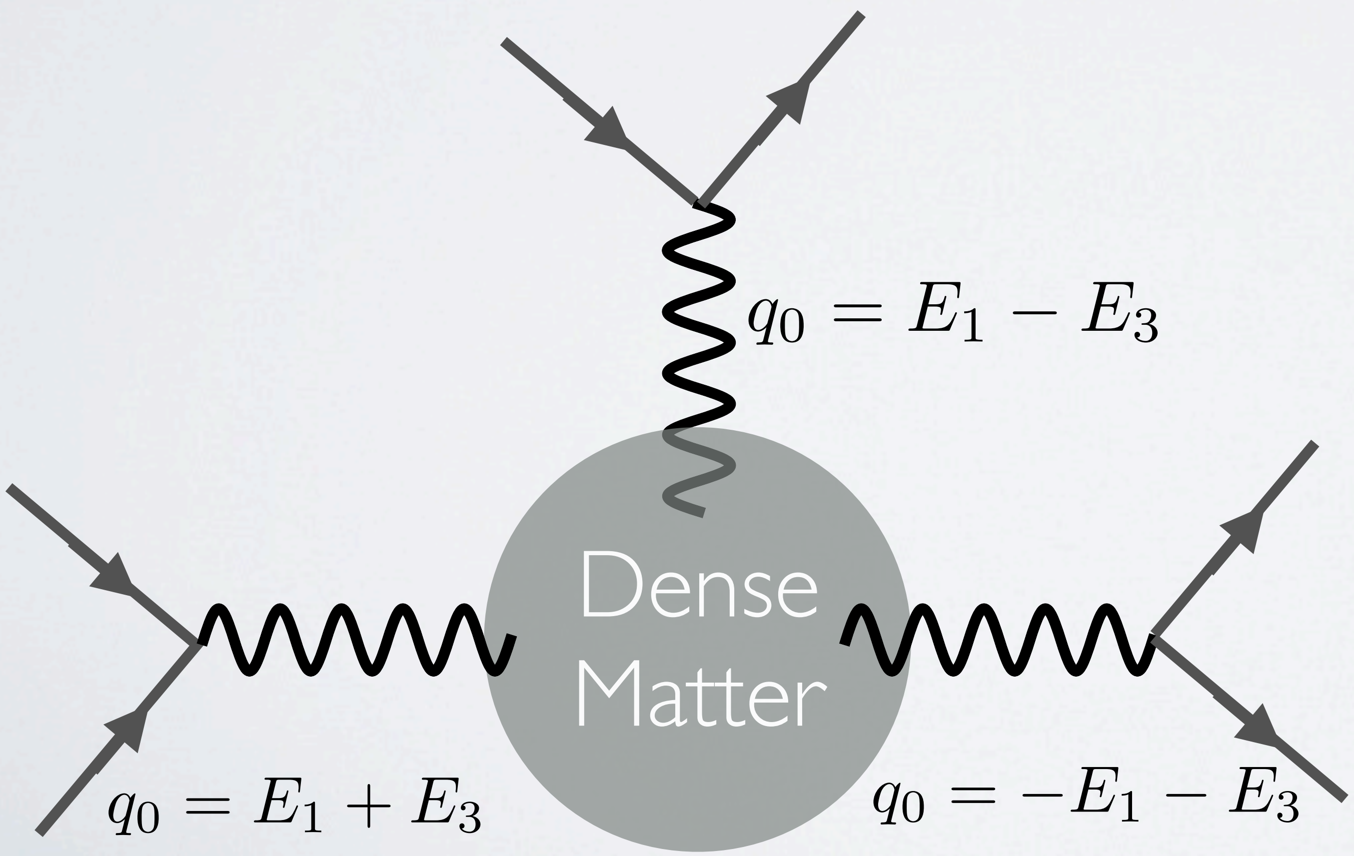

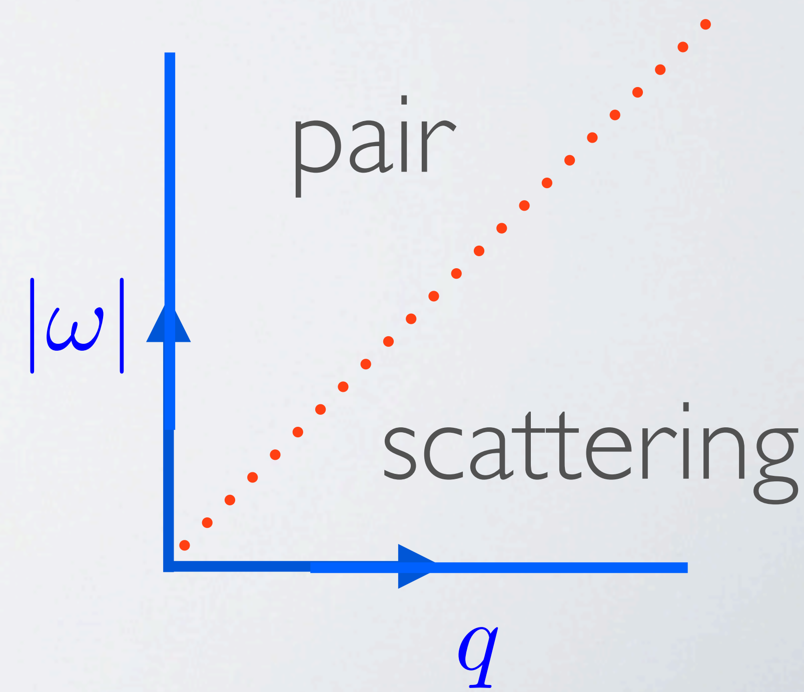

NEUTRINO CROSS SECTIONS

Differential Scattering/Absorption Rate:

response function of the medium

$$R(E_1, E_3, \cos\theta) = G_F^2 \; L(E_1, E_3, \cos\theta) \;\times\; S_{[\rho, Y_e, T]}(q_0, q)$$

neutrino/lepton kinematic factor

- Neutral and charged current reactions contribute.

$$\mathcal{L}_{int}^{cc} = \frac{G_F}{\sqrt{2}} \; l_\mu j_W^\mu \qquad \text{for} \qquad \nu_l + B_2 \rightarrow l + B_4$$

$$\mathcal{L}_{int}^{nc} = \frac{G_F}{\sqrt{2}} \; l_\mu^\nu j_Z^\mu \qquad \text{for} \qquad \nu_l + B_2 \rightarrow \nu_l + B_4$$

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$$l_\mu = \overline{\psi}_l \gamma_\mu (1-\gamma_5)\,\psi_\nu\,, \quad j_W^\mu = \overline{\psi}_4 \gamma^\mu (g_V - g_A\gamma_5)\,\psi_2$$

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$$l_\mu = \overline{\psi}_l \gamma_\mu (1-\gamma_5)\, \psi_\nu \,, \quad j_W^\mu = \overline{\psi}_4 \gamma^\mu (g_V - g_A \gamma_5)\, \psi_2 \qquad g_V = 1, g_A = 1.26$$

$$\mathcal{L}_{int}^{cc} = \frac{G_F}{\sqrt{2}} \; l_\mu j_W^\mu \qquad \text{for} \qquad \nu_l + B_2 \to l + B_4$$

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$c_V^n = -1.0,\; c_A^n = -1.26(-1.1)$

$c_V^p = 0.07,\; c_A^p = 1.26(1.4)$

$c_V^e = 1.92,\; c_A^e = 1.\; [\nu_e]$

$c_V^e = -0.08,\; c_A^e = -1.\; [\nu_X]$

RESPONSE FUNCTIONS

Sawyer (1975), Iwamoto & Pethick (1982)

Neutrinos couple to density and spin

$$j^{\mu}(x) = \bar{\psi}(x)\,\gamma^{\mu}(c_V - c_A\gamma_5)\,\psi(x)$$

$$\xrightarrow{NR} c_V\,\psi^{+}\psi\,\delta^{\mu 0} - c_A\ \psi^{+}\,\sigma^{i}\,\psi\,\delta^{\mu i}$$

$$\frac{d\Gamma(E_1)}{d\cos\theta\ dq_0} \simeq \frac{G_F^2}{4\pi^2}\,(E_1 - q_0)^2\,\left(1 - f_\nu(E_1 - q_0)\right)$$
$$\times\left[C_V^2(1+\cos\theta)S_\rho(q_0,q) + C_A^2(3-\cos\theta)S_\sigma(q_0,q)\right]$$

$$S_\rho(q_0,q) = \int dt\ e^{iq_0 t}\ \langle\rho(q,t)\ \rho(-q,0)\rangle \Rightarrow \rho = \psi^\dagger\psi = \sum_{i=1,N} e^{i\vec{q}\cdot\vec{r}_i}$$

$$= \sum_f\ \langle 0|\rho(q)|f\rangle\langle f|\rho(-q)|0\rangle\ \delta(q_0 - (E_f - E_0))$$

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$$= \sum_f\;\langle 0|\rho(q)|f\rangle\langle f|\rho(-q)|0\rangle\;\delta(q_0-(E_f-E_0))$$

$$S_\sigma(\omega,\mathbf{q}) \;=\; \frac{4}{3n}\frac{1}{2\pi}\int_{-\infty}^{\infty} dte^{i\omega t}\big\langle\mathbf{s}(t,\mathbf{q})\cdot\mathbf{s}(0,-\mathbf{q})\big\rangle \qquad \mathbf{s}(t,q) = V^{-1}\sum_{i=1}^N e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_i(t)}\boldsymbol{\sigma}_i$$

$$=\; \frac{4}{3n}\sum_f\langle 0|s(\mathbf{q})|f\rangle\cdot\langle f|s(-\mathbf{q})|0\rangle\delta(\omega-(E_f-E_0))$$

RESPONSE OF AN IDEAL GAS

- Process involves excitation of single (uncorrelated) particles. Total response is the (incoherent) sum over individual species.
- For nucleons and electrons final state blocking is important. Matter is partially degenerate for typical supernova conditions.
- Nucleons are heavy and recoil energy is small. Response lies at small $|\omega| < q\, v$. Where $v \sim p_F/M$ or $\sqrt{T/M}$.

Transition rate ($\Gamma = c/\lambda$) in a Fermi Gas.

$$\Gamma(E_1) = \int \frac{d^3k_3}{(2\pi)^3}\; R(E_1, E_3, \cos\theta)(1 - f_3(E_3))$$

$$\approx G_F^2 \int \frac{d^3k_3}{(2\pi)^3}\; [C_V^2(1+\cos\theta) + C_A^2(3-\cos\theta)]\; S_{\rm FG}(q_0, q)(1 - f_3(E_3))$$

$$S(q_0, q) = 2\int \frac{d^3p_2}{(2\pi)^3}\int \frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^4(P_1 + P_2 - P_3 - P_4)\; \times f_2(E_2)(1 - f_4(E_4))$$

ANALYTIC FORMULA EXIST

- Closed form expressions for the scattering and absorption rates including effects of relativistic kinematics and weak magnetism exist in the literature.
[Reddy, Prakash, Lattimer 1998, Horowitz, Perez-Garcia 2003]

$$S_{\mathrm{FG}}(q_0,q)=\frac{M^2T}{\pi q\left(1-e^{-z}\right)}\ln\left[\frac{\exp\left[\left(e_{\min}-\mu_2\right)/T\right]+1}{\exp\left[\left(e_{\min}-\mu_2\right)/T\right]+\exp\left[-z\right]}\right]$$

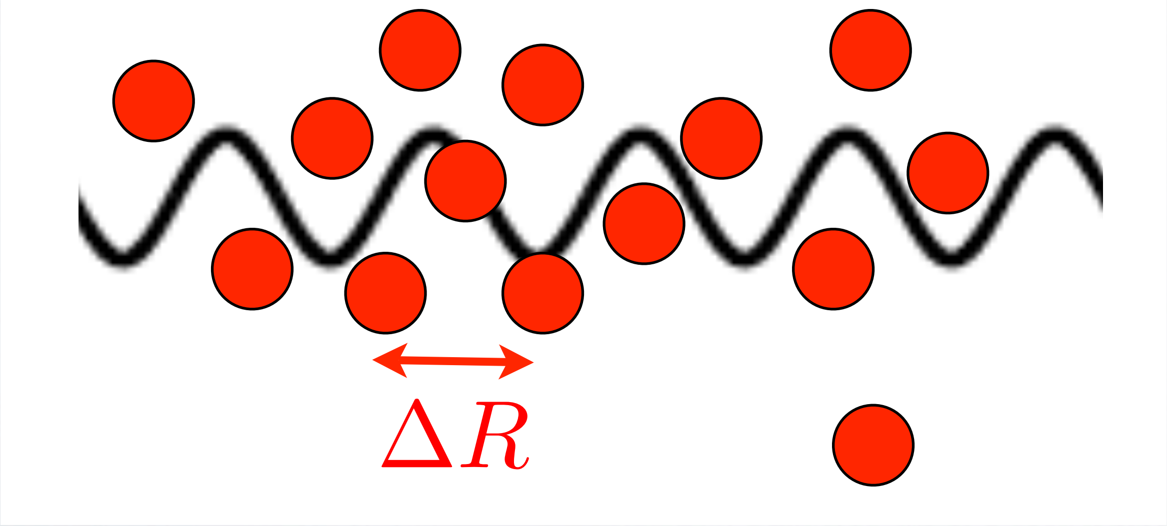
where

$$z=\frac{(q_0+\mu_2-\mu_4)}{T}\qquad e_{\min}=\frac{M}{2q^2}\left(q_0-\frac{q^2}{2M}\right)^2$$

- It would be desirable to use these in supernova simulations to establish baseline results.

MANY-PARTICLE DYNAMICS

- Neutrinos "see" more than one particle in the medium.
- Nature of spatial and temporal correlations between nuclei, nucleons and electrons affect the scattering rate.
- Nucleon dispersion relation is altered. Mean field energy shifts are important. $E_i(k) = \sqrt{k^2 + M^{*2}} + U_i \equiv K(k) + U_i$



At small q_0 and q the neutrino cannot resolve a single nucleon.

Sawyer (1975, 1989)
Iwamoto & Pethick (1982)
Horowitz & Wherberger (1991)
Raffelt & Seckel (1995)

NEUTRINO SCATTERING OFF NUCLEI

- Nuclei are strongly correlated by the (screened) Coulomb force. At 10^{12} g/cm^3 and T > 1 MeV behavior is classical - many particles dynamics can be predicted exactly using F = m a (Molecular Dynamics).

Neutrinos couple coherently to the weak charge in the nucleus.

$$L_{\rm NC} = \frac{G_{\rm F}}{\sqrt{2}}\, Q_W\; l_\mu\; j^\mu \qquad j^\mu = \psi^\dagger\psi\;\delta^\mu_0 \qquad Q_W = (2Z - A - 4Z\sin^2\theta_W)/2$$

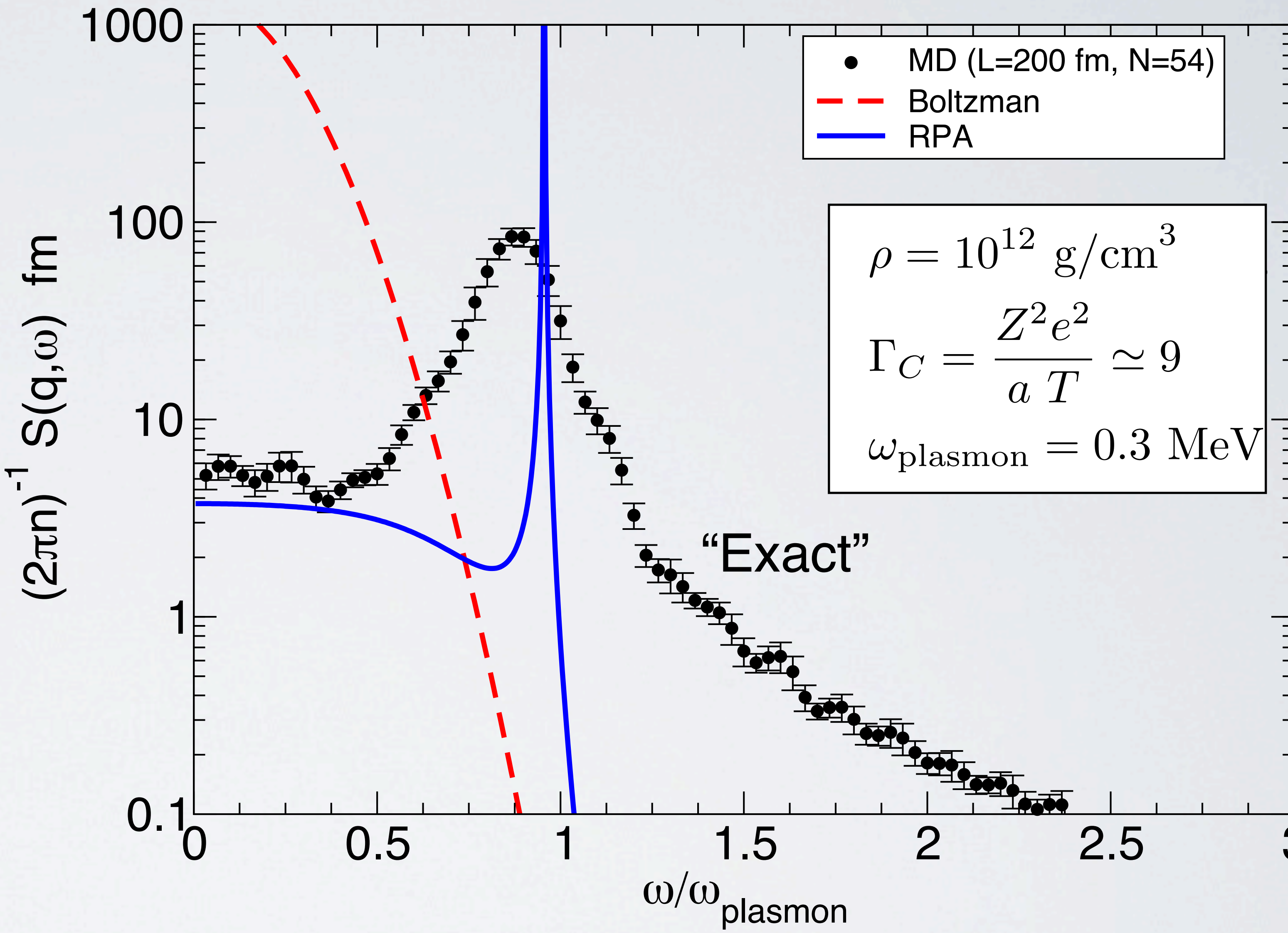
Scattering rate is related to the density-density correlation function

$$\frac{d\Gamma}{d\cos\theta dE'_\nu} = \frac{G_{\rm F}^2}{4\pi^2}\, Q_W^2\; (1+\cos\theta)\; {E'_\nu}^2\; S(|\vec{q}|,\omega)$$

$$S(|\vec{q}|,\omega) = \int_{-\infty}^{\infty} dt\;\exp(i\omega t)\;\langle \rho(\vec{q},t)\rho(-\vec{q},0)\rangle \qquad \rho(\vec{q},t) = \psi^\dagger\psi = \sum_{i=1\cdots N} \exp(i\vec{q}\cdot\vec{r}_i(t))$$

SCREENING, DAMPING & COLLECTIVE MODES

- Strong repulsive Coulomb forces affect the spatial distribution.
- A collective mode exists in the system.
- Response is pushed to high energy.
- Multi-particle excitations smears the response.



Smaller cross-section & larger energy transfer.

Burrows, Reddy & Thompson (2006)

RESPONSE OF A CLASSICAL LIQUID

The density-density correlation for N particles is

$$\langle \rho(\mathbf{q},\mathbf{0})\rho(\mathbf{q},\mathbf{t})\rangle = \langle \mathbf{\Sigma_i \ e^{-iq.r_i} \Sigma_j \ e^{-iq.r_j(t)}} \rangle$$

Ensemble average

Positions at t

Positions at t=0

Need to specify equations of motion ie $\mathbf{r}_j$(t).
Classical limit:

$$\mathbf{r}_j(\Delta t) = \mathbf{r}_j(0) + \mathbf{v}_j \ \Delta t + \frac{1}{2\ m}\Sigma_{i\neq j}\mathbf{F}_{ij} \ t^2$$

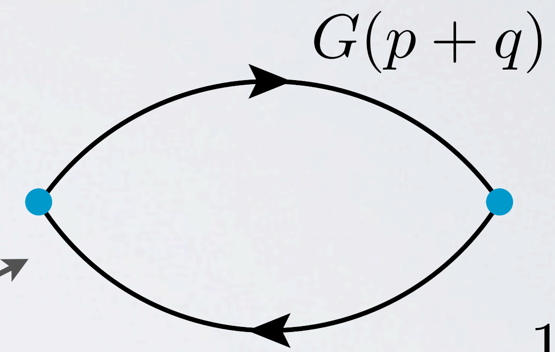
Tractable and could be used under non-degenerate conditions.

RANDOM PHASE APPROXIMATION (RPA)

- An approximate method to include correlations in the response function. Required for consistency with the mean field equation of state.

$$S_{\mathrm{RPA}}(q_0, q) = \frac{1}{1 - \exp\left(-\beta\omega\right)} \; \mathrm{Im}[\Pi^{\mathrm{RPA}}]$$

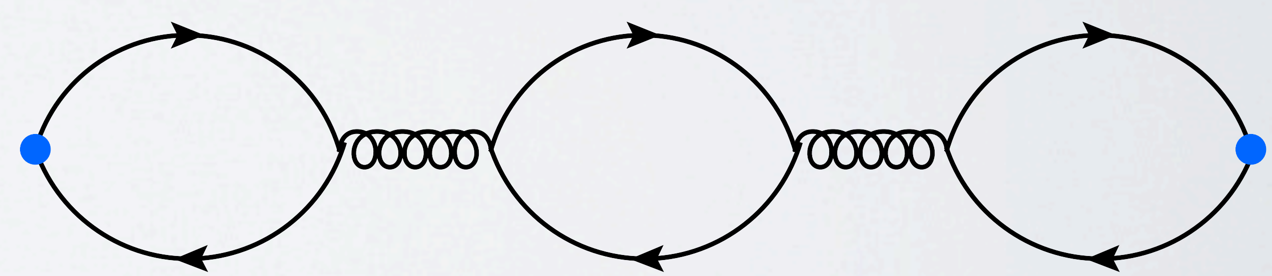
$$\Pi^{\mathrm{RPA}} = \left[\frac{\Pi^0(q_0, q)}{1 - V_c(q)\; \Pi^0(q_0, q)}\right]$$



$$G(p) = \frac{1}{p_0 - \mu - (p^2/2M)}$$

$$\Pi^0(q_0, q) = i \int \frac{d^4p}{(2\pi)^2}\; G(p)\; G(p+q)$$

$$\Pi^{\mathrm{RPA}} = \Pi^0 + \Pi^{\mathrm{RPA}}\; V_c\; \Pi^0$$



- Provides a fair qualitative description of response in nuclei. Mean field models with consistent residual p-h interactions.

SIMPLE RPA IN A NUCLEAR LIQUID

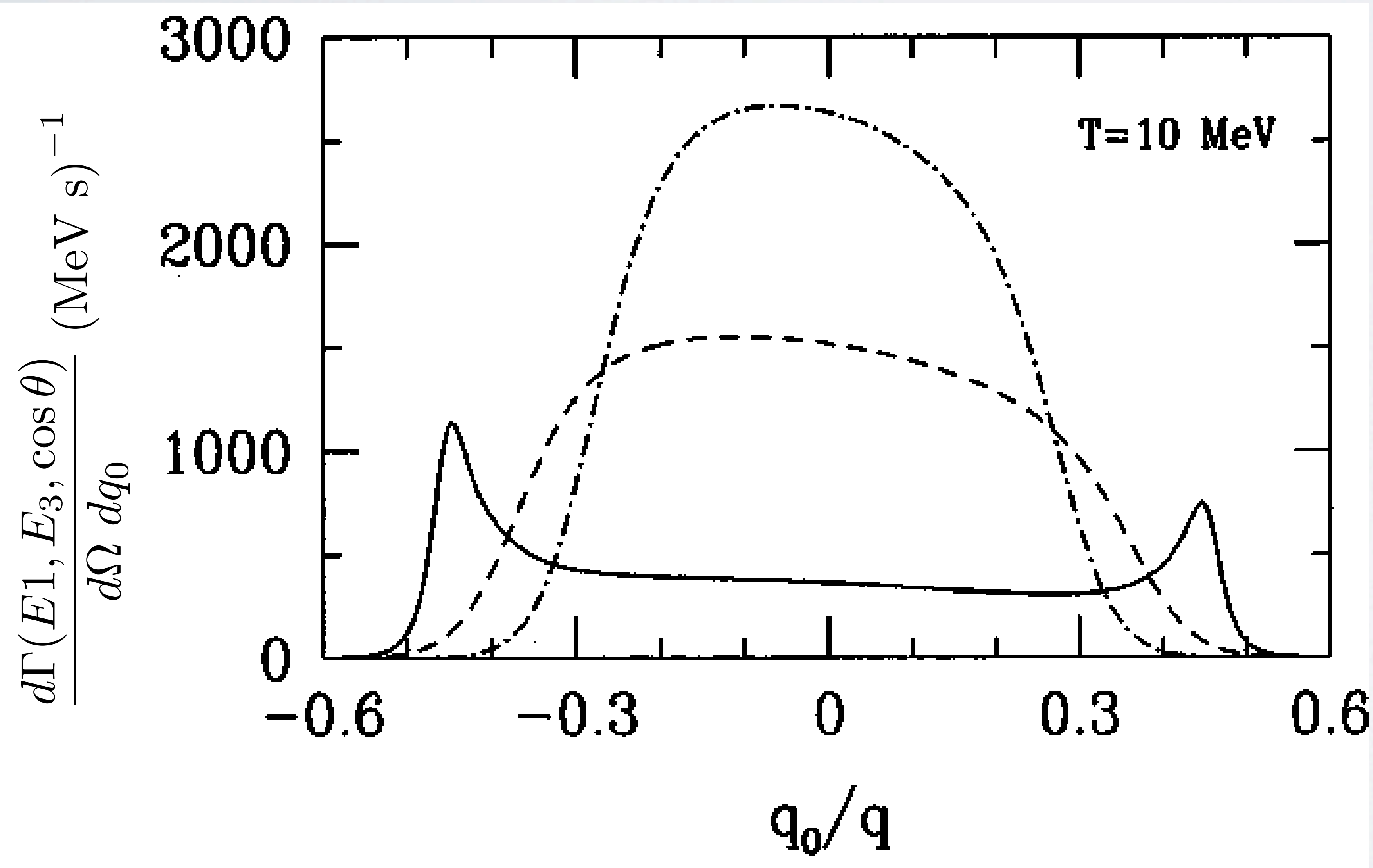
$$\frac{d\Gamma(E_1)}{d\cos\theta\ dq_0} = \frac{G_F^2}{4\pi^2}\ (E_1 - q_0)^2\ \left[(1+\cos\theta)\ S_{\rm V}^{\rm RPA}(q_0,q) + (3-\cos\theta)\ S_{\rm A}^{\rm RPA}(q_0,q)\right]$$

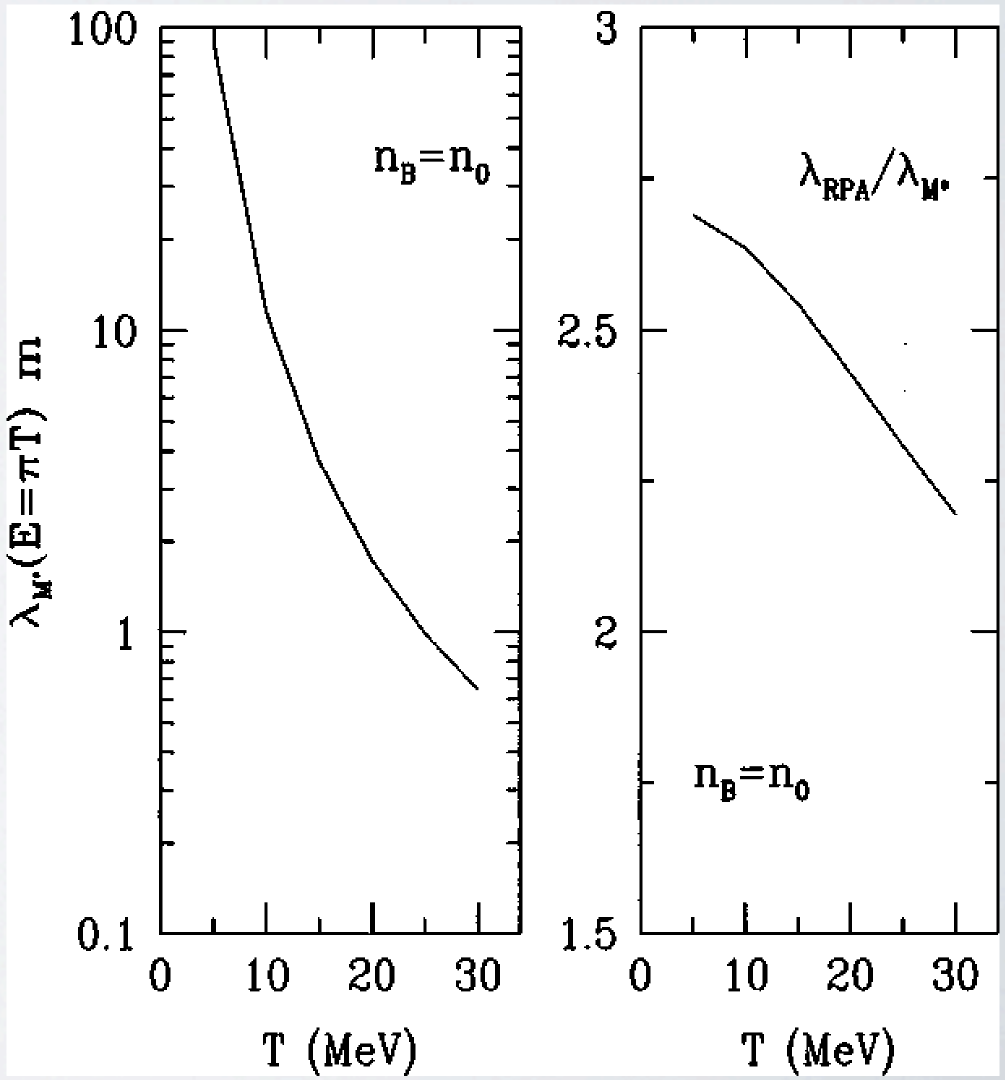
$$S_V(q_0,q) = \frac{1}{1-\exp(-q_0/T)}\,{\rm Im}\,\Pi_V^{\rm RPA}, \qquad S_A(q_0,q) = \frac{1}{1-\exp-(q_0/T)}\,{\rm Im}\,\Pi_A^{\rm RPA},$$

$$\Pi_V^{\rm RPA} = \left[\frac{(c_V^p)^2(1-f_{nn}\Pi_n^0)\Pi_p^0 + (c_V^n)^2(1-f_{pp}\Pi_p^0)\Pi_n^0 + 2c_V^p c_V^n f_{np}\Pi_n^0\Pi_p^0}{\Delta_V}\right] \qquad \Pi_A^{\rm RPA} = \left[\frac{(c_A^p)^2(1-g_{nn}\Pi_n^0)\Pi_p^0 + (c_A^n)^2(1-g_{pp}\Pi_p^0)\Pi_n^0 + 2c_A^p c_A^n g_{np}\Pi_n^0\Pi_p^0}{\Delta_A}\right]$$

$$\Delta_V = [1 - f_{nn}\,\Pi_n^0 - f_{pp}\,\Pi_p^0 + f_{pp}\,\Pi_p^0 f_{nn}\,\Pi_n^0 - f_{np}{}^2\Pi_n^0\,\Pi_p^0] \qquad \Delta_A = [1 - g_{nn}\,\Pi_n^0 - g_{pp}\,\Pi_p^0 + g_{pp}\,\Pi_p^0 g_{nn}\,\Pi_n^0 - g_{np}{}^2\Pi_n^0\,\Pi_p^0]$$

Reddy, Prakash, Lattimer, Pons (1999)





THE RESIDUAL INTERACTION IN RPA

Very simple s-wave interaction is used

p-h interaction obtained from the equation of state.

$$\langle k_1 k_3^{-1} | V_{\rm ph} | k_4 k_2^{-1} \rangle = \frac{\delta^2 \langle V \rangle}{\delta n_{k_3 k_1} \delta n_{k_4 k_2}}$$

$$f_{nn} = \frac{\delta U_n}{\delta n_n}, \quad f_{pp} = \frac{\delta U_p}{\delta n_p}, \quad f_{np} = \frac{\delta U_n}{\delta n_p} = \frac{\delta U_p}{\delta n_n}$$

Or from Fermi Liquid parameters calculated in a microscopic theories.

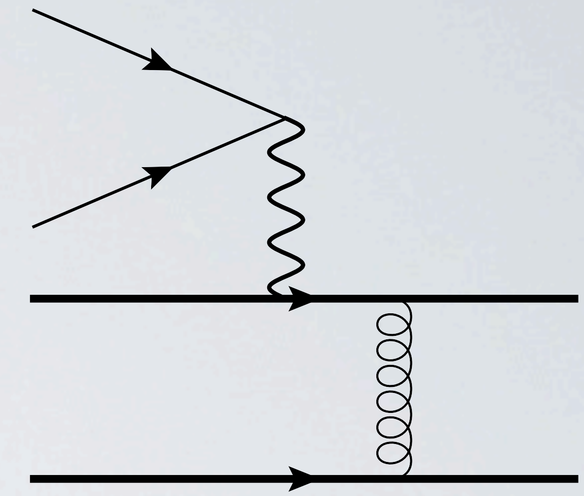
$$f_{nn} = \frac{F_0 + F_0'}{N_0}, \quad f_{np} = \frac{F_0 - F_0'}{N_0}$$

$$g_{nn} = \frac{G_0 + G_0'}{N_0}, \quad g_{np} = \frac{G_0 - G_0'}{N_0}$$

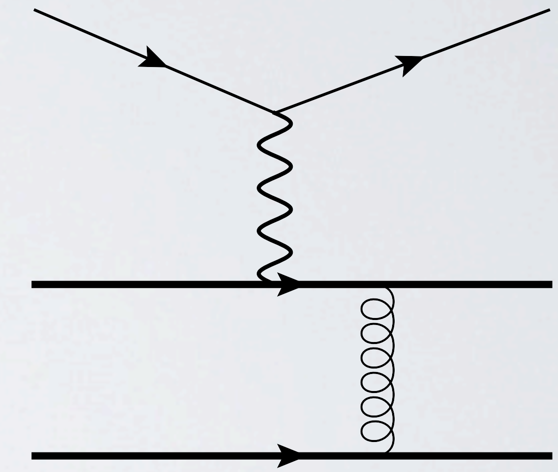
- The residual interaction for density and isospin density fluctuations obtained from the EoS is consistent.
- Important feedback may exist in SN simulations.
- The more important spin-flip interaction strength is chosen from phenomenology of response in nuclei.

MULTI-PARTICLE EXCITATIONS

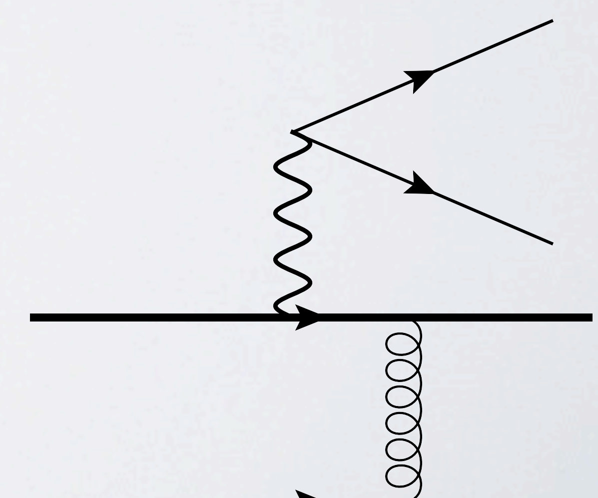
- Excitation of 2 particle-2 hole states enables pair-processes and larger energy transfer during scattering.
- In strongly coupled systems leads to significant smearing of the single particle and collective strength.
- Especially important for the spin response because spin is not conserved in nuclear interactions.
- Can enhance the charged current rate at small Y_e.



$$\nu + n + n \to n + n + \nu$$



$$\nu + \bar{\nu} + n + n \to n + n$$



$$n + n \to n + n + \nu + \bar{\nu}$$

Raffelt & Seckel (1995)

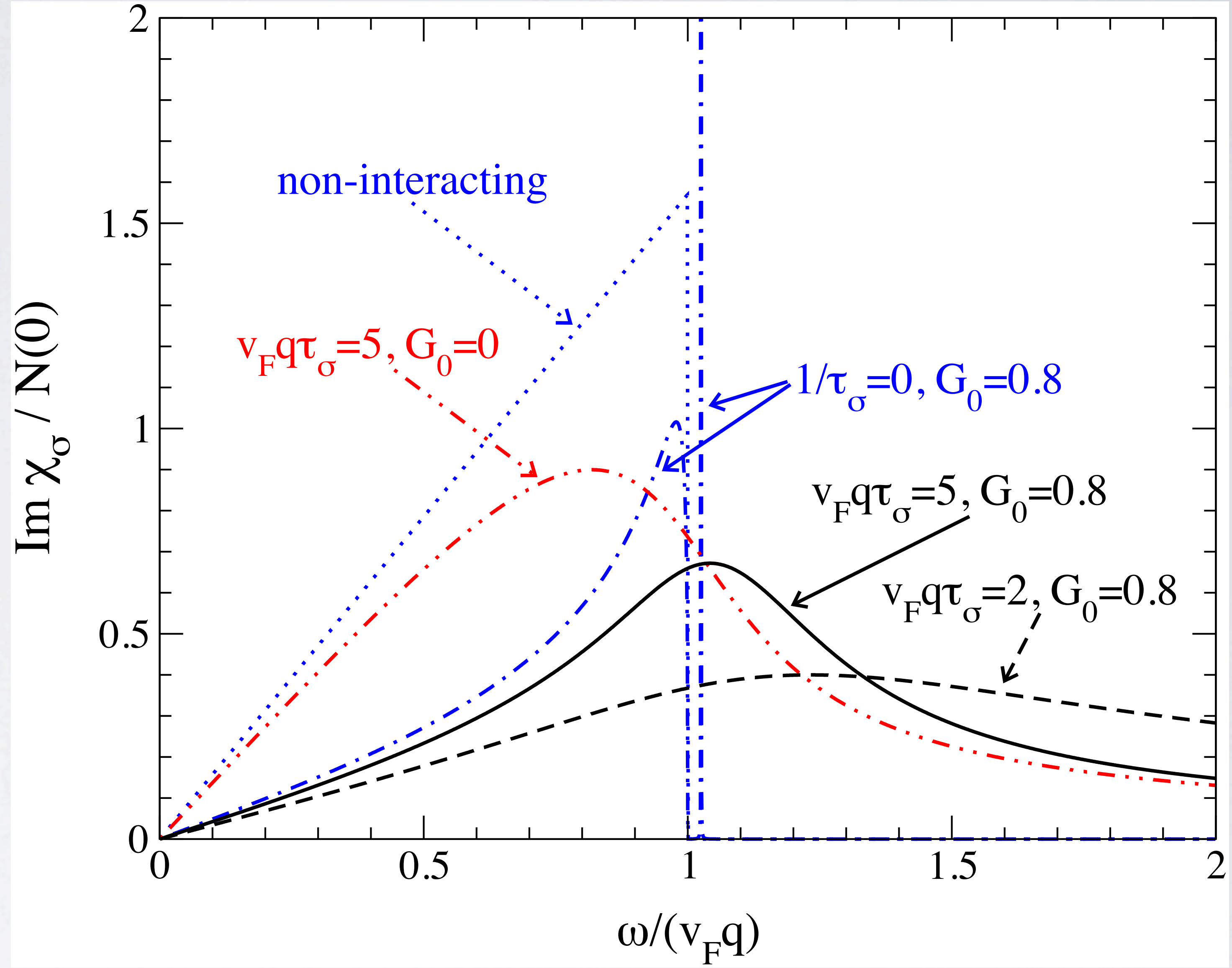
UNIFIED TREATMENT OF SPIN RESPONSE

Lykasov, Olsson, Pethick (2005)
Lykasov, Pethick, Schwenk (2006)

- 2p-2h response is incorporated through a finite quasi-particle lifetime correction in RPA. Combines single-pair and multi-pair excitations and RPA correlations.

- Captures key aspects of the response (screening, damping and collectivity).

- Quasi-particle life-times have been calculated using realistic and modern nucleon-nucleon interactions.

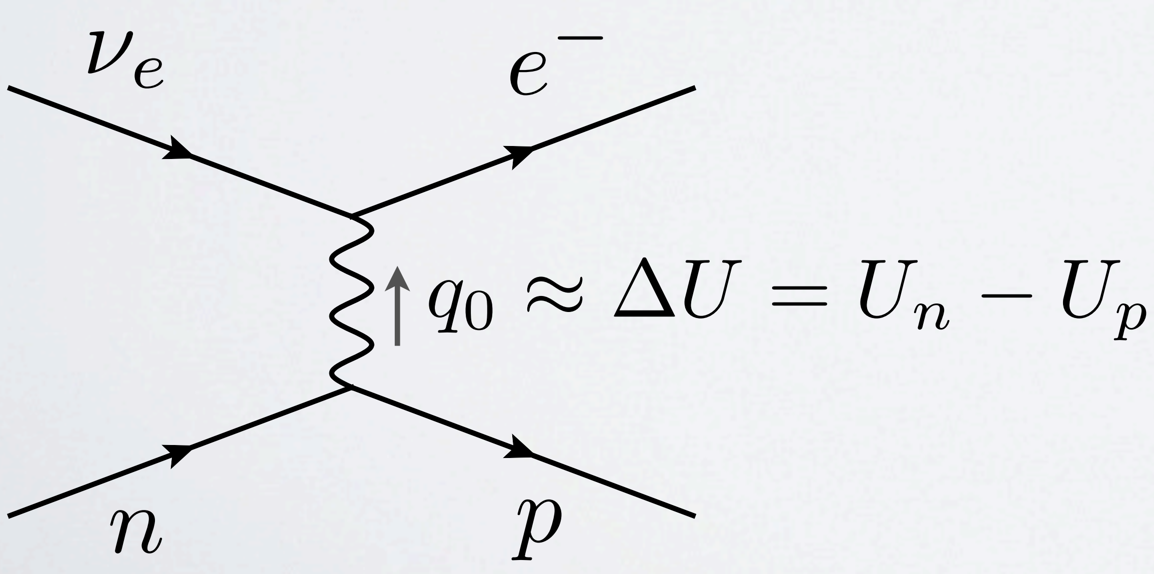


$$\mathrm{Im}\widetilde{\chi}_\sigma(\omega, q \to 0) = \frac{\omega\tau_\sigma}{(1+G_0)^2 + (\omega\tau_\sigma)^2}$$

$$S_\sigma(q \to 0, \omega) = \frac{\mathrm{Im}\tilde{\chi}_\sigma(\omega)}{1 - \exp(-\beta\omega)}$$

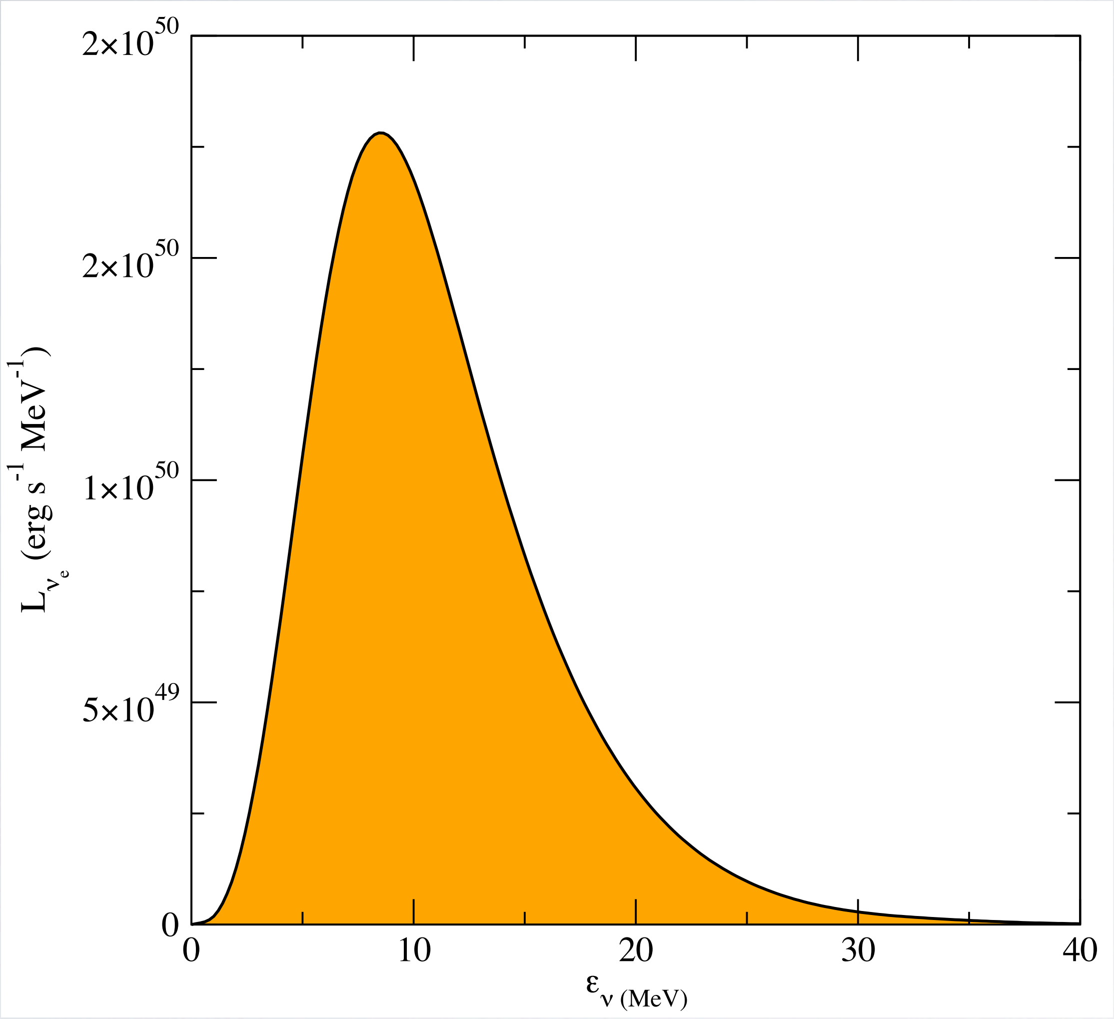
CHARGED CURRENT REACTIONS $\begin{cases} \nu_e + n \to p + e^- \\ \bar{\nu}_e + p \to n + e^+ \end{cases}$

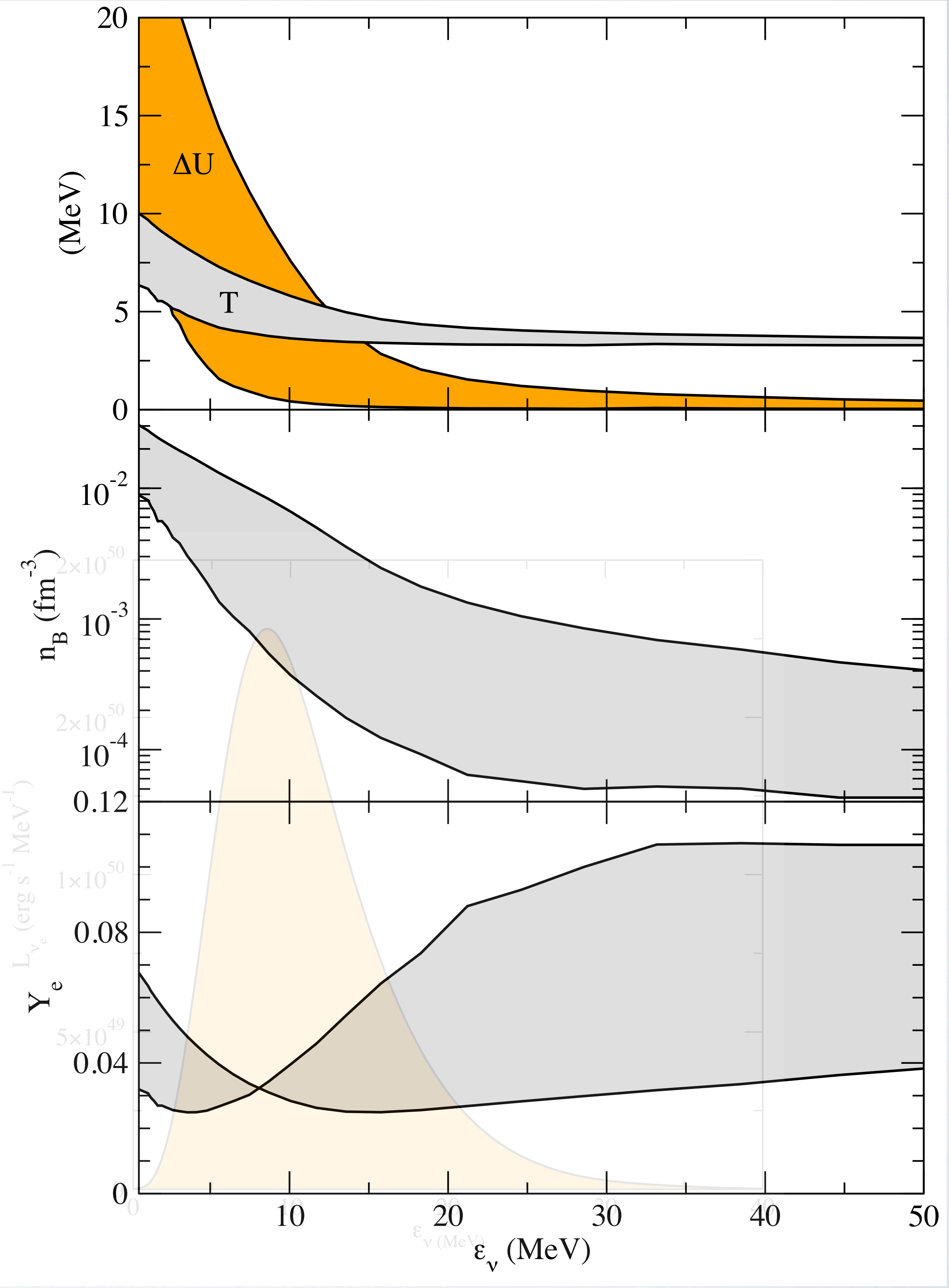
- Determine the electron neutrino spectra and deleptonization times.
- Final state electron blocking is strong for electron neutrino absorption reaction.
- Asymmetry between mean field energy between neutrons and protons alters the kinematics.



Reddy, Prakash & Lattimer (1998)
Roberts (2012)
Martinez-Pinedo et al. (2012)
Roberts & Reddy (2012)

SPECTRA AT LATE TIMES





- Decoupling occurs at relatively high density.
- Spectra influenced by nuclear correlations.

Figures from PNS simulations by Roberts (2012)

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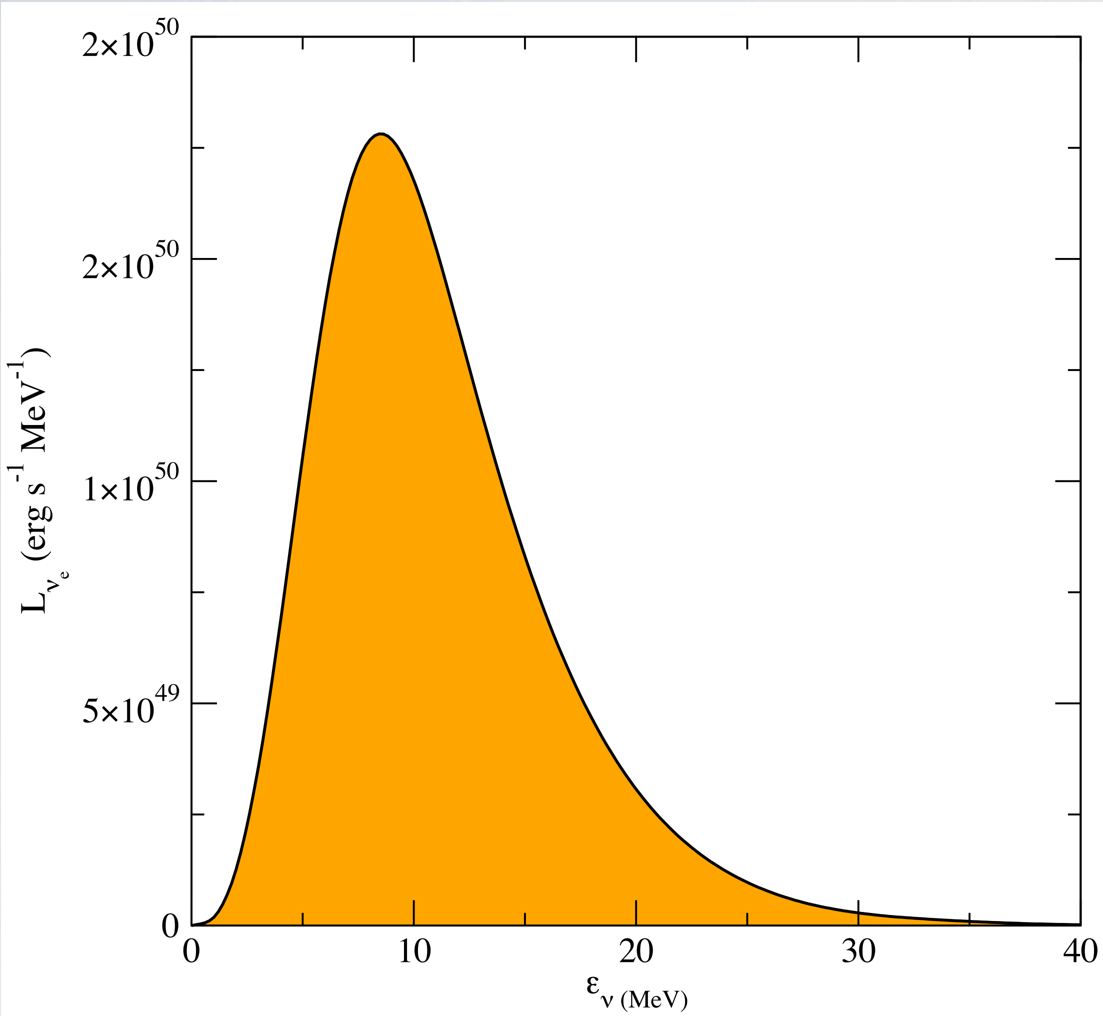

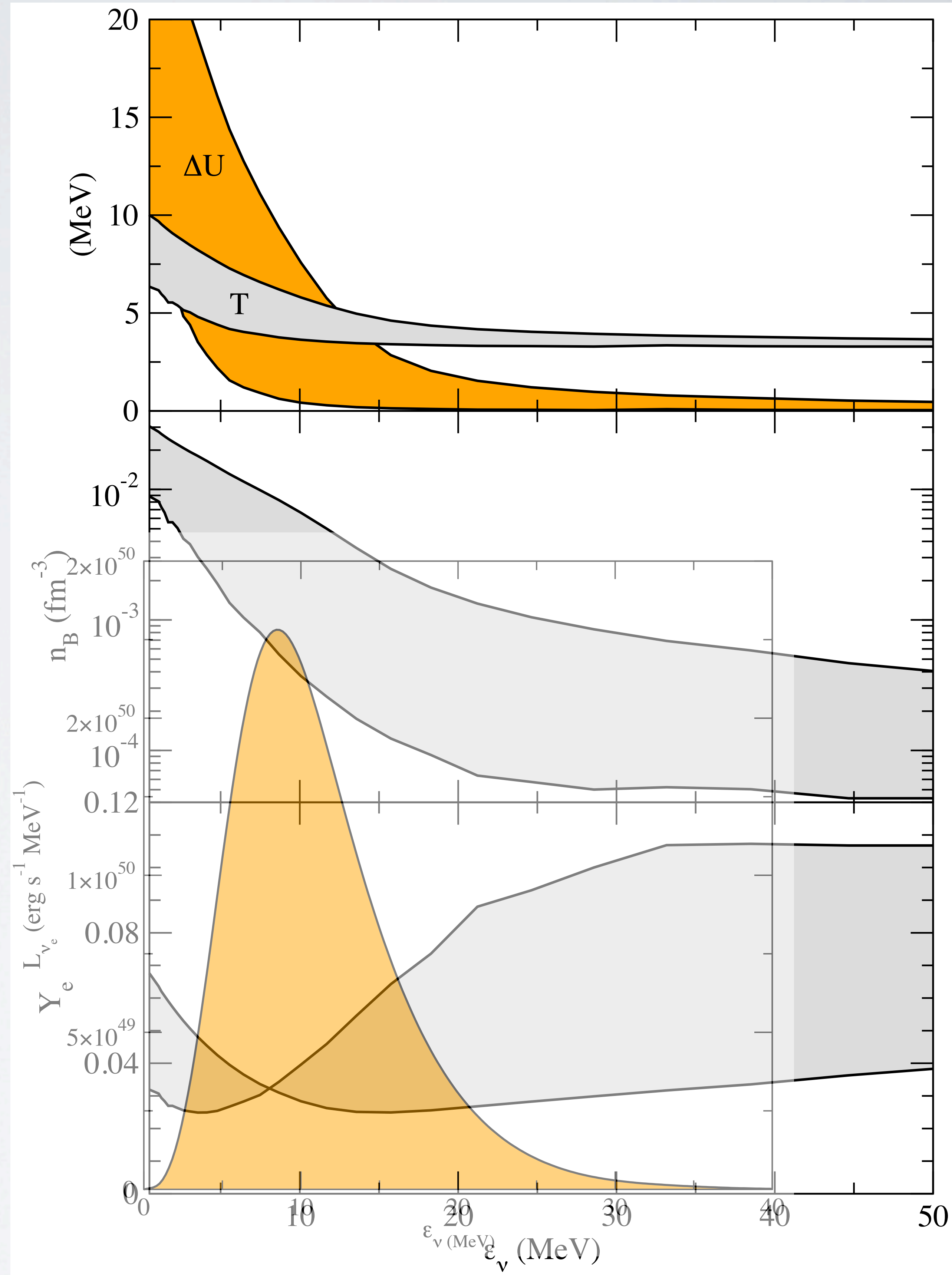

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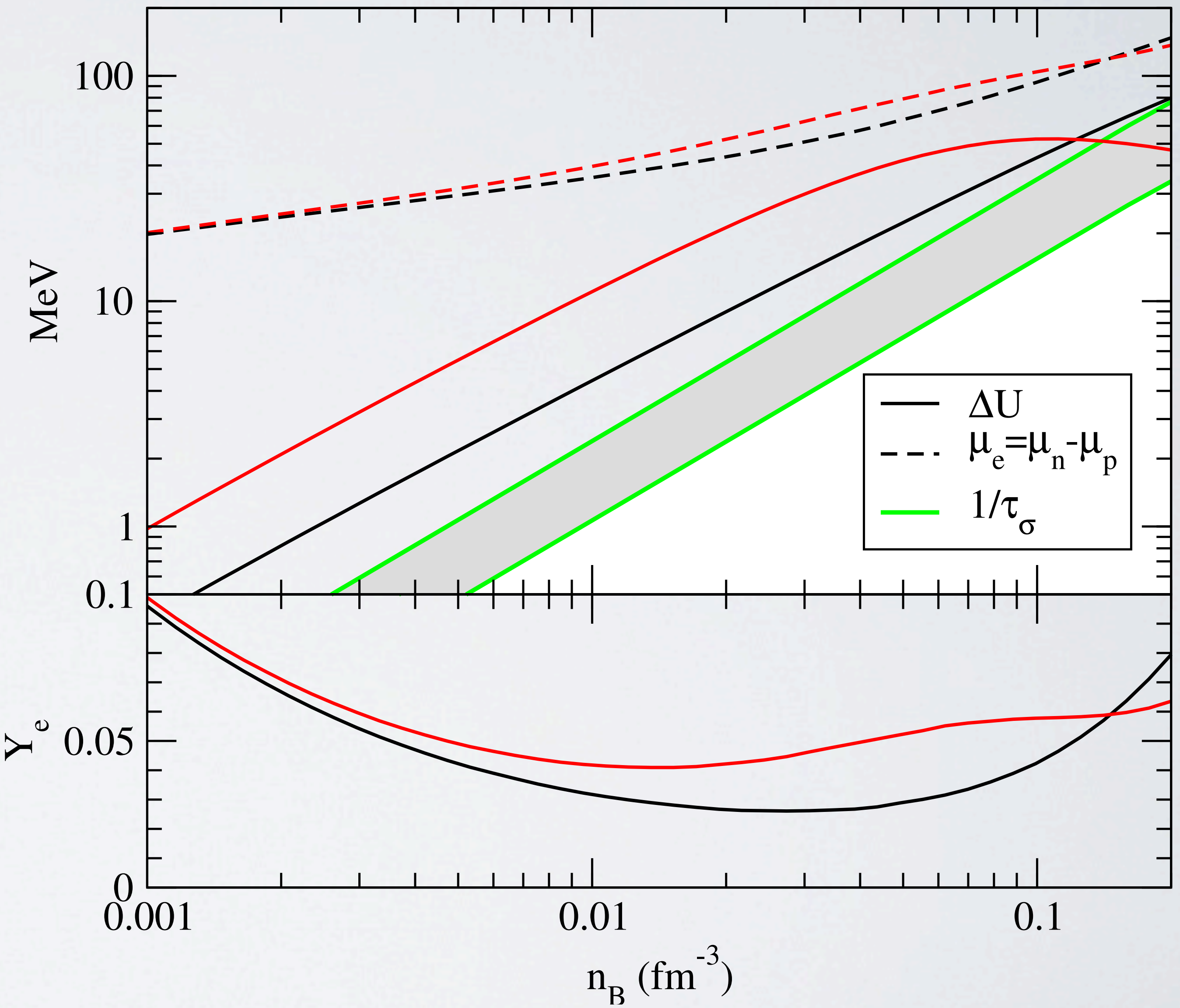
Figures from PNS simulations by Roberts (2012)

MEAN FIELD SHIFT IN THE NEUTRINO SPHERE

$$\Delta U = U_n - U_p \approx 40\ \frac{n_n - n_p}{n_0}\ \mathrm{MeV}$$

- After a few seconds, the density at the neutrino sphere is large. ~$n_0/50$-$n_0/10$.
- Nucleon propagation is affected by mean fields and collisions.
- Sensitive to the low-density behavior of the symmetry energy.

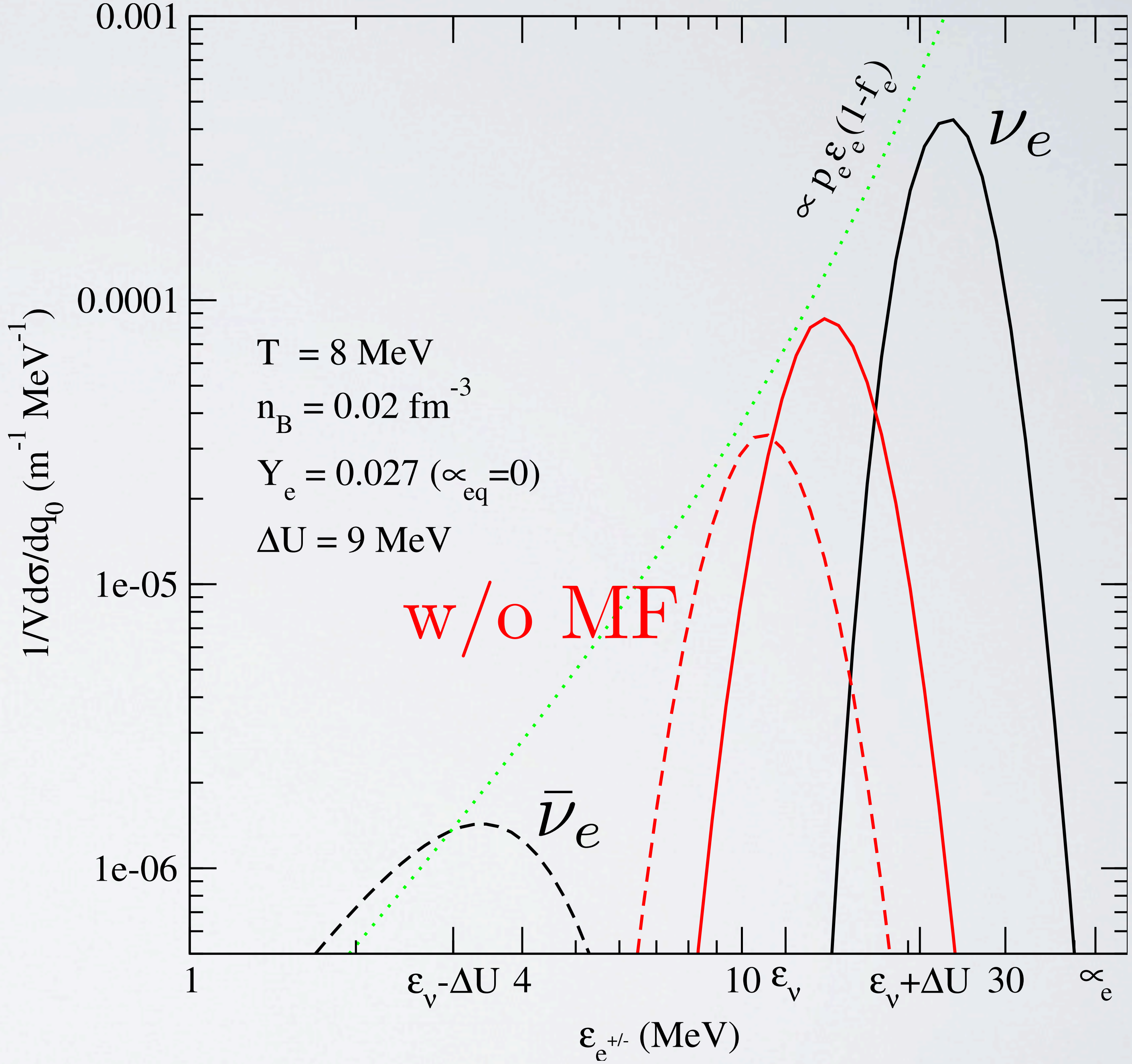


Roberts (2012)
Martinez-Pinedo et al. (2012)

ABSORPTION RATES

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta dE_e} = \frac{G_F^2}{2\pi}\left[(1+\cos\theta)+g_A^2(3-\cos\theta)\right] S(q_0,q)\times\ p_e\ E_e\left[1-f_e(E_e)\right]$$

- Mean field energy shift helps overcome electron final state blocking.
- Enhances ν_e absorption
- Larger energy needed to produce neutrons suppresses anti-ν_e absorption.



Roberts & Reddy (2012)

MEAN FIELD & COLLISIONAL BROADENING

Ansatz for the spin-isospin charge-exchange response function:

$$S_{\sigma\tau^-}(q_0, q) = \frac{1}{1 - \exp\left(-\beta(q_0 + \mu_n - \mu_p)\right)} \mathrm{Im}\left[\frac{\tilde{\Pi}(q_0, q)}{1 - V_{\sigma\tau}\tilde{\Pi}(q_0, q)}\right]$$

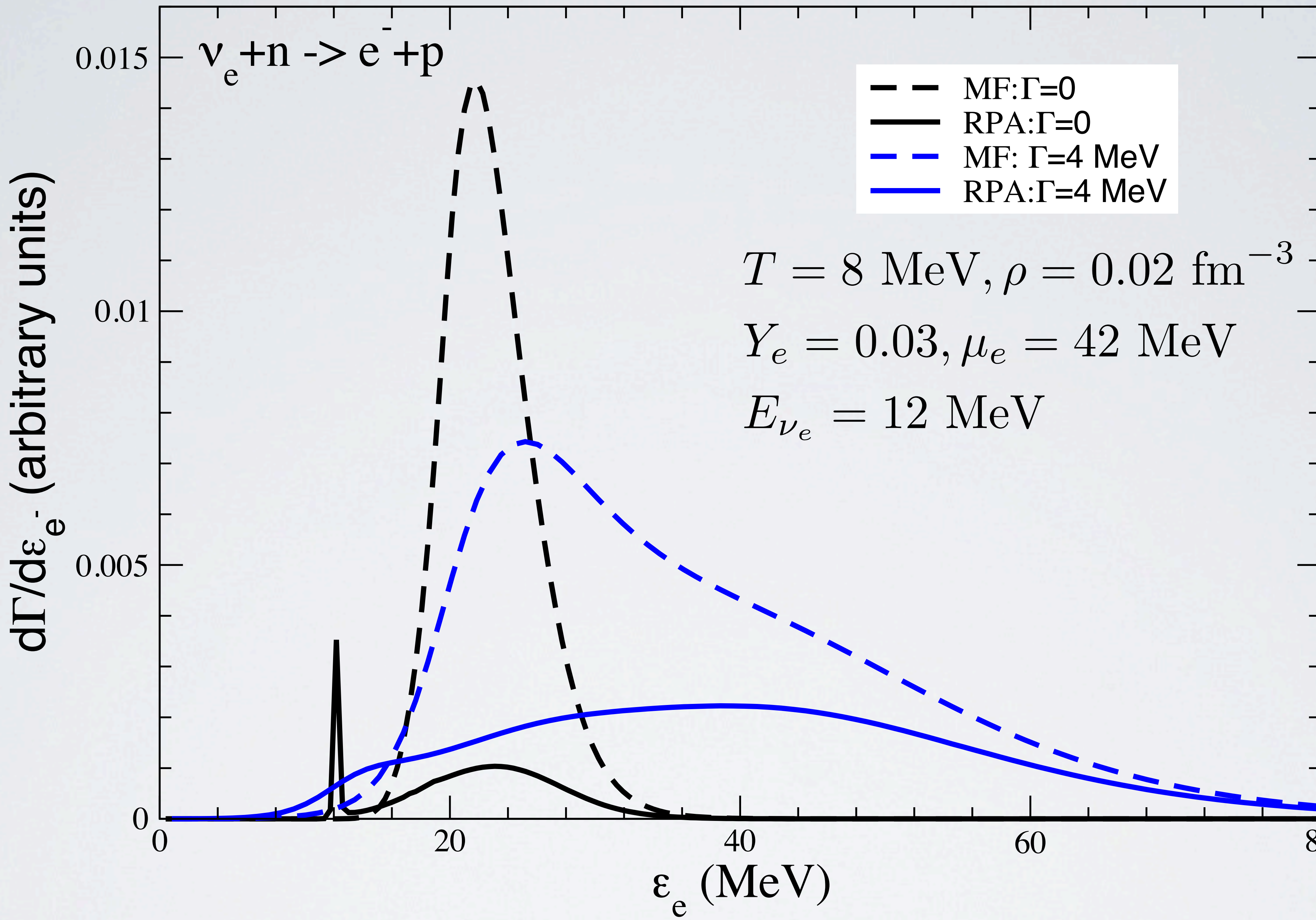
Collisional broadening (finite lifetime) introduced in the relaxation time approximation: $\Gamma = \tau_\sigma^{-1}$

$$\mathrm{Im}\tilde{\Pi}(q_0, q) = \frac{1}{\pi}\int \frac{d^3p}{(2\pi)^3} \frac{f_p(\epsilon_{p+q}) - f_n(\epsilon_p)}{\epsilon_{p+q} - \epsilon_p + \hat{\mu}} \, \mathcal{I}(\Gamma)$$

$$\mathcal{I}(\Gamma) = \frac{\Gamma}{(q_0 + \Delta U - (\epsilon_{p+q} - \epsilon_p))^2 + \Gamma^2}$$

ABSORPTION RATES IN RPA & DAMPING

Roberts, Shen, Reddy (2012) in prep.



- RPA correlations suppress cross-section. Collisional broadening enhances it.
- Net effect mild suppression. Need further investigation.

SUM RULES

Response functions are constrained by sum rules

$$S_\sigma^n = \int_0^\infty S_\sigma(\omega, \boldsymbol{q} = 0)\ \omega^n\ d\omega$$

$$\begin{aligned} S_\sigma(\omega, \mathbf{q}) &= \frac{4}{3n}\frac{1}{2\pi}\int_{-\infty}^{\infty} dt e^{i\omega t}\langle \mathbf{s}(t, \mathbf{q}) \cdot \mathbf{s}(0, -\mathbf{q})\rangle \\ &= \frac{4}{3n}\sum_f \langle 0|s(\mathbf{q})|f\rangle \cdot \langle f|s(-\mathbf{q})|0\rangle \delta(\omega - (E_f - E_0)) \end{aligned}$$

Spin susceptibility $\quad S_\sigma^{-1} = \frac{\chi_\sigma}{2n}$

Static Spin Structure $\quad S_\sigma^0 = 1 + \lim_{q\to 0}\frac{4}{3N}\sum_{i\neq j}^{N}\langle 0|e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j|0\rangle$

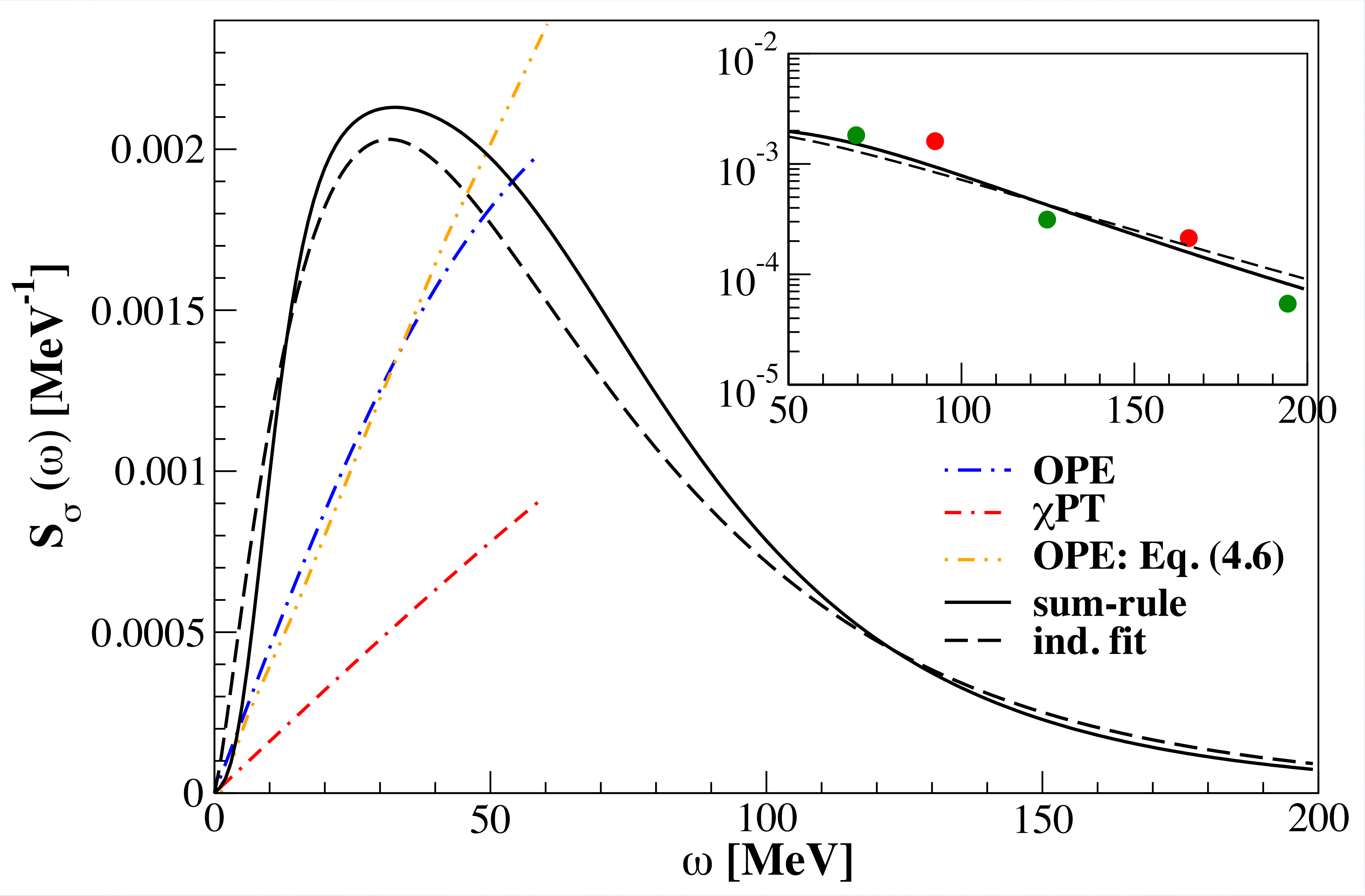
Energy or F-sum Rule $\quad S_\sigma^{+1} = -\frac{4}{3N}\ \lim_{q\to 0}\langle 0|[H_N, \boldsymbol{s}(\mathbf{q})] \cdot \boldsymbol{s}(-\mathbf{q})|0\rangle$

Shen, Gandolfi, Reddy & Carlson (2012)

STRENGTH AT INTERMEDIATE ENERGY

Table I: AFDMC results for the sum-rules

Density (fm^{-3})	S_σ^{-1} (MeV^{-1})	S_σ^0	S_σ^{+1} (MeV)	$\bar{\omega}_0$ (MeV)	$\bar{\omega}_1$ (MeV)
$n = 0.12$	0.0057(9)	0.20(1)	8(1)	35(9)	40(8)
$n = 0.16$	0.0044(7)	0.20(1)	11(1)	46(11)	55(8)
$n = 0.20$	0.0038(6)	0.18(1)	14(1)	47(12)	78(10)



- Response function constructed to satisfy QMC sum-rules at T=0 predict significant strength at 10-50 MeV.

SUMMARY & OUTLOOK

- Formula that incorporate kinematics(recoil), full structure of the weak current(weak magnetism) and Pauli blocking exactly are available.

- Correlations are relevant. Recent progress in including damping effects beyond RPA are important.

- Charged current rates in the neutrino sphere are especially sensitive to many-body effects.

- How do we benchmark calculations of response functions ?

- General trends indicate large suppression at and above nuclear density - Implications ?