New Developments in Collective Neutrino Oscillations in Supernovae

Crab Nebula

Max-Planck-Institut für Physik, München, Germany Georg G. Raffelt

Three-Flavor Neutrino Parameters

Three mixing angles θ_{12} , θ_{13} , θ_{23} (Euler angles for 3D rotation), $c_{ij} = \cos \theta_{ij}$, a CP-violating "Dirac phase" δ , and two "Majorana phases" α_2 and α_3 v Relevant for Atmospheric/LBL-Beams Reactor Solar/KamLAND 0 ν 2 β decay

Georg Raffelt, MPI Physics, Munich Core Collapse Supernovae, INT, Seattle, July 2012

Neutrino oscillations in matter

L. Wolfenstein

3400 citations Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 (Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

Lincoln Wolfenstein

Flavor Oscillations in Core-Collapse Supernovae

Three-Flavor Eigenvalue Diagram

Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

Signature of Flavor Oscillations

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

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Flavor Oscillations in Core-Collapse Supernovae

Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$
i\frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
$$

Effective mixing Hamiltonian

$$
i\frac{\partial}{\partial t}(v_{\mu}) = H(v_{\mu})
$$

\n
$$
H = \frac{M^2}{2E} + \sqrt{2}G_F \begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2} \end{pmatrix} + \sqrt{2}G_F \begin{pmatrix} N_{\nu_e} & N_{\langle \nu_e | \nu_\mu \rangle} \\ N_{\langle \nu_\mu | \nu_e \rangle} & N_{\nu_\mu} \end{pmatrix}
$$

flavor basis: causes vacuumoscillations

Mass term in Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe [0710.3112]. Balantekin & Pehlivan [astro-ph/0607527]. Blennow, Mirizzi & Serpico [0810.2297]. Cherry, Fuller, Carlson, Duan & Qian [1006.2175, 1108.4064]. Cherry, Wu, Fuller, Carlson, Duan & Qian [1109.5195]. Cherry, Carlson, Friedland, Fuller & Vlasenko [1203.1607]. Chakraborty, Choubey, Dasgupta & Kar [0805.3131]. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs [1104.4031, 1105.1130]. Choubey, Dasgupta, Dighe & Mirizzi [1008.0308]. Dasgupta & Dighe [0712.3798]. Dasgupta, Dighe & Mirizzi [0802.1481]. Dasgupta, Dighe, Raffelt & Smirnov [0904.3542]. Dasgupta, Dighe, Mirizzi & Raffelt [0801.1660, 0805.3300]. Dasgupta, Mirizzi, Tamborra & Tomàs [1002.2943]. Dasgupta, Raffelt & Tamborra [1001.5396]. Dasgupta, O'Connor & Ott [1106.1167]. Duan, Fuller, Carlson & Qian [astroph/0608050, 0703776, 0707.0290, 0710.1271]. Duan, Fuller & Qian [0706.4293, 0801.1363, 0808.2046, 1001.2799]. Duan, Fuller & Carlson [0803.3650]. Duan & Kneller [0904.0974]. Duan & Friedland [1006.2359]. Duan, Friedland, McLaughlin & Surman [1012.0532]. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl [0807.0659]. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl [0706.2498, 0712.1137]. Fogli, Lisi, Marrone & Mirizzi [0707.1998]. Fogli, Lisi, Marrone & Tamborra [0812.3031]. Friedland [1001.0996]. Gava & Jean-Louis [0907.3947]. Gava & Volpe [0807.3418]. Galais, Kneller & Volpe [1102.1471]. Galais & Volpe [1103.5302]. Gava, Kneller, Volpe & McLaughlin [0902.0317]. Hannestad, Raffelt, Sigl & Wong [astro-ph/0608695]. Wei Liao [0904.0075, 0904.2855]. Lunardini, Müller & Janka [0712.3000]. Mirizzi, Pozzorini, Raffelt & Serpico [0907.3674]. Mirizzi & Serpico [1111.4483]. Mirizzi & Tomàs [1012.1339]. Pehlivan, Balantekin, Kajino & Yoshida [1105.1182]. Pejcha, Dasgupta & Thompson [1106.5718]. Raffelt [0810.1407, 1103.2891]. Raffelt & Sigl [hep-ph/0701182]. Raffelt & Smirnov [0705.1830, 0709.4641]. Raffelt & Tamborra [1006.0002]. Sawyer [hep-ph/0408265, 0503013, 0803.4319, 1011.4585]. Sarikas, Raffelt, Hüdepohl & Janka [1109.3601]. Sarikas, Tamborra, Raffelt, Hüdepohl & Janka [1204.0971]. Saviano, Chakraborty, Fischer, Mirizzi [1203.1484]. Wu & Qian [1105.2068].

Spectral Split

Figures from Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in Raffelt & SmirnovarXiv:0705.1830and 0709.4641Duan, Fuller, Carlson & Qian arXiv:0706.4293

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$
i\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
$$

Neutrino flavor density matrix

$$
\rho = \begin{pmatrix} \langle v_e | v_e \rangle & \langle v_e | v_\mu \rangle \\ \langle v_\mu | v_e \rangle & \langle v_\mu | v_\mu \rangle \end{pmatrix}
$$

Equivalent commutator form of Schrödinger equation

 $i\partial_t \rho = [H, \rho]$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

$$
\rho = \frac{1}{2} [\text{Tr}(\rho) + \mathbf{P} \cdot \mathbf{\sigma}] \quad \text{and} \quad H = \frac{\Delta m^2}{2E} \mathbf{B} \cdot \mathbf{\sigma} \quad \text{with} \quad \mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)
$$

Equivalent spin-precession form of equation of motion

$$
\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}
$$
 with $\omega = \frac{\Delta m^2}{2E}$

P is "polarization vector" or "Bloch vector" or "flavor isospin vector"

Flavor Oscillation as Spin Precession

Collective Nu Oscillations as a Many-Body Problem

"Spin-pairing H" for isotropic system (or single angle), ignoring matter effect $H = \sum \omega_i \mathbf{B} \cdot \mathbf{P}_i + \mu \mathbf{P}_{\text{tot}}^2$

BCS theory (using Anderson's pseudo-spin), nuclear physics, ... Integrable system (as many "Gaudin invariants" as spins) \rightarrow Pehlivan, Balantekin, Kajino & Yoshida [arxiv:1105.1182] for introduction

N-mode coherent solutions ("Normal and anomalous solitons")

- Emil Yuzbashian, Phys. Rev. **B** 78, 184507 (2008) Super-conductivity (BCS)
- Georg Raffelt, Phys. Rev. **D** 83, 105022 (2011) Collective Nus

Synchronized Flavor Oscillations

Precession equation for each v mode with energy E, i.e. $\omega = \Delta m^2 / 2E$

$$
\dot{\mathbf{P}}_{\omega} = \underbrace{(\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{P})}_{\mathbf{H}_{\text{eff}}} \times \mathbf{P}_{\omega} \quad \text{with} \quad \lambda = \sqrt{2} G_{\text{F}} N_e \quad \text{and} \quad \mu = \sqrt{2} G_{\text{F}} N_v
$$

Total flavor spin of entire ensemble

$$
\mathbf{P} = \sum_{\omega} \mathbf{P}_{\omega} \quad \text{normalize} \quad |\mathbf{P}_{t=0}| = 1
$$

Individual spins do not remain aligned – feel "internal" field $H_{\nu\nu} = \mu P$

Instability in Flavor Space

Two-mode example in co-rotating frame, initially $P_1 = \downarrow$, $P_2 = \uparrow$ (flavor basis)

- Initially aligned in flavor direction and $P = 0$
- Free precession $\pm \omega$

After a short time, transverse P develops by free precession

Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$) Free precession in opposite directions

Even for very small mixing angle, large-amplitude flavor oscillations

Inverse-Energy Spectrum

Fermi-Dirac energy spectrum

$$
\frac{dN}{dE} \propto \frac{E^2}{e^{E/T - \eta} + 1}
$$

 η degeneracy parameter, $-\eta$ for $\overline{\nu}$

Spectrum in terms of $\omega = T/E$

- Antineutrinos $E \rightarrow -E$
- and dN/dE negative (flavor isospin convention)

$$
\omega > 0: v_e = \uparrow \text{ and } v_\mu = \downarrow
$$

$$
\omega < 0: \overline{v}_e = \downarrow \text{ and } \overline{v}_\mu = \uparrow
$$

Flavor Pendulum

Single "positive" crossing (IH) (potential energy at a maximum)

Single "negative" crossing (NH) (potential energy at a minimum)

Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542 For movies see http://www.mppmu.mpg.de/supernova/multisplits

- **Flavor content exchanged between different momentum modes(or nus and anti-nus changing together)**
- **No net flavor conversion of ensemble**
- **Instability required to get started: Exponential growth of the off-diagonal density matrix parts**
- **→ Linearized Stability Analysis (first stressed by Ray Sawyer)**

Sawyer, arXiv:0803.4319 – Banerjee, Dighe & Raffelt, arXiv:1107.2308

Linearized Stability Analysis

Schrödinger equation for flavor matrices of neutrino fluxes $\Phi_{\omega,u}$ $\omega = \pm \Delta m^2 / 2E$ $u = \sin^2(\text{emission angle})$ $v_u = \text{radial velocity at } r$ $i\partial_r \Phi_{\omega,u} = \left| \frac{\omega + \sqrt{2} G_F N_\ell}{v_u} + \frac{\sqrt{2} G_F}{4\pi r^2} \int d\omega' du' \Phi_{\omega',u'} \frac{1 - v_u v_{u'}}{v_u v_{u'}} , \Phi_{\omega,u} \right|$

Linearize in small off-diagonal flux terms and Fourier transform

$$
\Phi_{\omega,u} = \frac{g_{\omega,u}}{2} \begin{pmatrix} 1 & Q_{\omega,u} \ e^{-i\Omega r} \\ Q_{\omega,u}^* \ e^{i\Omega r} & -1 \end{pmatrix}
$$

Eigenvalue equation for $Q_{\omega,u}$ in terms of eigenfrequency $\Omega = \gamma + i \kappa$, where κ is the exponential growth rate

$$
\left[\omega + u\left(\lambda + \int d\omega' du' g_{\omega',u'}\right) - \Omega\right]Q_{\omega,u} = \mu \int d\omega' du' \left(u + u'\right)g_{\omega',u'}Q_{\omega',u'}
$$

Straightforward to solve for eigenvalue Ω and eigenfunction $Q_{\omega,u}$

Banerjee, Dighe & Raffelt, arXiv:1107.2308

Stability Analysis for Simple SN Example

Normal vs Inverted Hierarchy

Represent neutrino field by discrete energy and angle modes

- Number of energy modes chosen to fit desired precision
- $N_a \gg 1$ of angle modes required N_a too small: Unphysical solutions

Liouville form of oscillation equation

Self-induced conversion suppressed for $N_e \gtrsim N_v$

Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Accretion-Phase Matter Profiles

Dasgupta, O'Connor & Ott, arXiv:1106.1167

Multi-Angle Matter Effect (Basel Model 10.8 M $_{sun}$ **)**

Schematic single-energy, multi-angle simulations with realistic density profile

Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130

Multi-Angle Multi-Energy Stability Analysis

Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601

To investigate neutrino oscillations in SNe need

- Profile of electron density
- Flavor-dependent neutrino flux spectra
- Flavor-dependent neutrino angular distribution

Neutrino Radiation Field

Small "scattering halo" important for nu-nu refraction? (Cherry et al., arXiv:1203.1607)

Picture from Ott, Burrows, Dessart & Livne, arXiv:0804.0239

Scattered Neutrinos as a Source of Refraction

Cherry, Carlson, Friedland, Fuller & Vlasenko, arXiv:1203.1607 Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1204.0971

Multi-Angle Matter Suppression in Realistic Model

Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1109.3601, 1204.0971

Summary

Supernova neutrino flavor evolution remains a complicated subject

- **Axial symmetry was always assumed – too symmetric?**
- **Numerical treatments challenging**
- **Novel role for neutrino "scattering halo"?**
- **Simultaneous space and time dependence important?**

Theoretical developments

- **Analogy to BCS theory**
- **Linearized stability analysis provides many conceptual insights**
- **And practical results**

Working hypothesis for SN neutrinos

- Multi-angle matter effect can prevent instability
- No collective conversion during early accretion phase
- Can test for nu mass hierarchy (because θ_{13} is large)

More theory progress is needed to reliably interpret neutrino signal of next galactic supernova!

Looking forward to the next galactic supernova