New Developments in Collective Neutrino Oscillations in Supernovae

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Three-Flavor Neutrino Parameters

Three mixing angles θ_{12} , θ_{13} , θ_{23} (Euler angles for 3D rotation), $c_{ij} = \cos \theta_{ij}$, a CP-violating "Dirac phase" δ , and two "Majorana phases" α_2 and α_3 $\begin{pmatrix} v_e \\ v_{\mu} \\ v_{\tau} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $39^{\circ} < \theta_{23} < 53^{\circ} \qquad 7^{\circ} < \theta_{13} < 11^{\circ} \qquad 33^{\circ} < \theta_{12} < 37^{\circ} \qquad \text{Relevant for}$ Atmospheric/LBL-Beams Reactor Solar/KamLAND $0 v_2\beta$ decay



Neutrino oscillations in matter

L. Wolfenstein

3400 citations Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 (Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein



Flavor Oscillations in Core-Collapse Supernovae



Three-Flavor Eigenvalue Diagram



Inverted mass hierarchy



Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

Signature of Flavor Oscillations

	1-3-mixing scenarios			
	A	В	С	
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)	
$\sin^2 \theta_{13}$	$\gtrsim 10^{-3}$		$\lesssim 10^{-5}$	
MSW conversion	adiabatic		non-adiabatic	
v_e survival prob.	0	$\sin^2 \theta_{12} \approx 0.3$	$\sin^2 \theta_{12} \approx 0.3$	
$\overline{ u}_e$ survival prob.	$\cos^2 \theta_{12} \approx 0.7$	0	$\cos^2 \theta_{12} \approx 0.7$	
$\overline{\nu}_e$ Earth effects	Yes	No	Yes	
	May distinguish mass ordering			

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

Signature of Flavor Oscillations

	1-3-mixing scenarios		
	A	В	С
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/V4)
$\sin^2 \theta_{13}$	$\Theta_{13} \approx 9^{\circ}$		≲ 107
MSW conversion	adiabatic		non-adiabatic
v_e survival prob.	0	$\sin^2 \theta_{12} \approx 0.3$	$\sin^2 \theta_{12} \approx 0.3$
$\overline{\nu}_e$ survival prob.	$\cos^2 \theta_{12} \approx 0.7$	0	$\cos^2 \theta_{12} \approx 0.7$
$\overline{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

Flavor Oscillations in Core-Collapse Supernovae



Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$i\frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Effective mixing Hamiltonian

$$i \frac{1}{\partial t} \begin{pmatrix} v_{\mu} \end{pmatrix} = H \begin{pmatrix} v_{\mu} \end{pmatrix}$$

Effective mixing Hamiltonian

$$H = \frac{M^{2}}{2E} + \sqrt{2}G_{F} \begin{pmatrix} N_{e} - \frac{N_{n}}{2} & 0 \\ 0 & -\frac{N_{n}}{2} \end{pmatrix} + \sqrt{2}G_{F} \begin{pmatrix} N_{v_{e}} & N_{\langle v_{e} | v_{\mu} \rangle} \\ N_{\langle v_{\mu} | v_{e} \rangle} & N_{v_{\mu}} \end{pmatrix}$$

flavor basis: causes vacuum oscillations

Mass term in Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe [0710.3112]. Balantekin & Pehlivan [astro-ph/0607527]. Blennow, Mirizzi & Serpico [0810.2297]. Cherry, Fuller, Carlson, Duan & Qian [1006.2175, 1108.4064]. Cherry, Wu, Fuller, Carlson, Duan & Qian [1109.5195]. Cherry, Carlson, Friedland, Fuller & Vlasenko [1203.1607]. Chakraborty, Choubey, Dasgupta & Kar [0805.3131]. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs [1104.4031, 1105.1130]. Choubey, Dasgupta, Dighe & Mirizzi [1008.0308]. Dasgupta & Dighe [0712.3798]. Dasgupta, Dighe & Mirizzi [0802.1481]. Dasgupta, Dighe, Raffelt & Smirnov [0904.3542]. Dasgupta, Dighe, Mirizzi & Raffelt [0801.1660, 0805.3300]. Dasgupta, Mirizzi, Tamborra & Tomàs [1002.2943]. Dasgupta, Raffelt & Tamborra [1001.5396]. Dasgupta, O'Connor & Ott [1106.1167]. Duan, Fuller, Carlson & Qian [astroph/0608050, 0703776, 0707.0290, 0710.1271]. Duan, Fuller & Qian [0706.4293, 0801.1363, 0808.2046, 1001.2799]. Duan, Fuller & Carlson [0803.3650]. Duan & Kneller [0904.0974]. Duan & Friedland [1006.2359]. Duan, Friedland, McLaughlin & Surman [1012.0532]. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl [0807.0659]. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl [0706.2498, 0712.1137]. Fogli, Lisi, Marrone & Mirizzi [0707.1998]. Fogli, Lisi, Marrone & Tamborra [0812.3031]. Friedland [1001.0996]. Gava & Jean-Louis [0907.3947]. Gava & Volpe [0807.3418]. Galais, Kneller & Volpe [1102.1471]. Galais & Volpe [1103.5302]. Gava, Kneller, Volpe & McLaughlin [0902.0317]. Hannestad, Raffelt, Sigl & Wong [astro-ph/0608695]. Wei Liao [0904.0075, 0904.2855]. Lunardini, Müller & Janka [0712.3000]. Mirizzi, Pozzorini, Raffelt & Serpico [0907.3674]. Mirizzi & Serpico [1111.4483]. Mirizzi & Tomàs [1012.1339]. Pehlivan, Balantekin, Kajino & Yoshida [1105.1182]. Pejcha, Dasgupta & Thompson [1106.5718]. Raffelt [0810.1407, 1103.2891]. Raffelt & Sigl [hep-ph/0701182]. Raffelt & Smirnov [0705.1830, 0709.4641]. Raffelt & Tamborra [1006.0002]. Sawyer [hep-ph/0408265, 0503013, 0803.4319, 1011.4585]. Sarikas, Raffelt, Hüdepohl & Janka [1109.3601]. Sarikas, Tamborra, Raffelt, Hüdepohl & Janka [1204.0971]. Saviano, Chakraborty, Fischer, Mirizzi [1203.1484]. Wu & Qian [1105.2068].

Spectral Split



Figures from Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in Raffelt & Smirnov arXiv:0705.1830 and 0709.4641 Duan, Fuller,

Carlson & Qian arXiv:0706.4293 and 0707.0290

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$i\partial_t \binom{\nu_e}{\nu_{\mu}} = H \binom{\nu_e}{\nu_{\mu}} = \frac{\Delta m^2}{2E} \binom{\cos 2\theta}{\sin 2\theta} - \frac{\sin 2\theta}{\cos 2\theta} \binom{\nu_e}{\nu_{\mu}}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

 $i\partial_t \rho = [H, \rho]$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

$$\rho = \frac{1}{2} [\operatorname{Tr}(\rho) + \mathbf{P} \cdot \boldsymbol{\sigma}]$$
 and $H = \frac{\Delta m^2}{2E} \mathbf{B} \cdot \boldsymbol{\sigma}$ with $\mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$

Equivalent spin-precession form of equation of motion

$$\dot{\mathbf{P}} = \boldsymbol{\omega} \mathbf{B} \times \mathbf{P}$$
 with $\boldsymbol{\omega} = \frac{\Delta m^2}{2E}$

P is "polarization vector" or "Bloch vector" or "flavor isospin vector"

Flavor Oscillation as Spin Precession



Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2/2E$

Collective Nu Oscillations as a Many-Body Problem



"Spin-pairing H" for isotropic system (or single angle), ignoring matter effect $H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \mu \mathbf{P}_{tot}^2$

BCS theory (using Anderson's pseudo-spin), nuclear physics, ... Integrable system (as many "Gaudin invariants" as spins) → Pehlivan, Balantekin, Kajino & Yoshida [arxiv:1105.1182] for introduction

N-mode coherent solutions ("Normal and anomalous solitons")

- Emil Yuzbashian, Phys. Rev. **B** 78, 184507 (2008) Super-conductivity (BCS)
- Georg Raffelt, Phys. Rev. D 83, 105022 (2011) Collective Nus

Synchronized Flavor Oscillations

Precession equation for each ν mode with energy E, i.e. $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_{\omega} = \underbrace{(\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{P})}_{\mathbf{H}_{\text{eff}}} \times \mathbf{P}_{\omega} \text{ with } \lambda = \sqrt{2}G_{\text{F}}N_{e} \text{ and } \mu = \sqrt{2}G_{\text{F}}N_{\nu}$$

Total flavor spin of entire ensemble

$$\mathbf{P} = \sum_{\omega} \mathbf{P}_{\omega}$$
 normalize $|\mathbf{P}_{t=0}| = 1$

Individual spins do not remain aligned – feel "internal" field $\mathbf{H}_{\nu\nu} = \mu \mathbf{P}$



Instability in Flavor Space

Two-mode example in co-rotating frame, initially $P_1 = \downarrow$, $P_2 = \uparrow$ (flavor basis)



- Initially aligned in flavor direction and $\mathbf{P} = \mathbf{0}$
- Free precession $\pm \omega$

After a short time, transverse **P** develops by free precession

Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$) Free precession in opposite directions Even for very small mixing angle, large-amplitude flavor oscillations



Inverse-Energy Spectrum

Fermi-Dirac energy spectrum

$$\frac{dN}{dE} \propto \frac{E^2}{e^{E/T - \eta} + 1}$$

 η degeneracy parameter, $-\eta$ for $\overline{\nu}$

Spectrum in terms of $\omega = T/E$

- Antineutrinos $E \rightarrow -E$
- and dN/dE negative (flavor isospin convention)

$$\omega > 0: \nu_e = \uparrow \text{ and } \nu_\mu = \downarrow$$

$$\omega < 0: \ \overline{
u}_e = \downarrow \ \ ext{and} \ \ \overline{
u}_\mu = \uparrow$$



Flavor Pendulum

Single "positive" crossing (IH) (potential energy at a maximum) Single "negative" crossing (NH) (potential energy at a minimum)



Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542 For movies see http://www.mppmu.mpg.de/supernova/multisplits

- Flavor content exchanged between different momentum modes (or nus and anti-nus changing together)
- No net flavor conversion of ensemble
- Instability required to get started: Exponential growth of the off-diagonal density matrix parts
- → Linearized Stability Analysis (first stressed by Ray Sawyer)

Sawyer, arXiv:0803.4319 – Banerjee, Dighe & Raffelt, arXiv:1107.2308

Linearized Stability Analysis

Schrödinger equation for flavor matrices of neutrino fluxes $\Phi_{\omega,u}$ $\omega = \pm \Delta m^2/2E$ $u = \sin^2(\text{emission angle})$ $v_u = \text{radial velocity at } r$ $i\partial_r \Phi_{\omega,u} = \left[\frac{\omega + \sqrt{2}G_F N_\ell}{v_u} + \frac{\sqrt{2}G_F}{4\pi r^2}\int d\omega' du' \Phi_{\omega',u'} \frac{1 - v_u v_{u'}}{v_u v_{u'}}, \Phi_{\omega,u}\right]$

Linearize in small off-diagonal flux terms and Fourier transform

$$\Phi_{\omega,u} = \frac{g_{\omega,u}}{2} \begin{pmatrix} 1 & Q_{\omega,u} e^{-i\Omega r} \\ Q_{\omega,u}^* e^{i\Omega r} & -1 \end{pmatrix}$$

Eigenvalue equation for $Q_{\omega,u}$ in terms of eigenfrequency $\Omega = \gamma + i \kappa$, where κ is the exponential growth rate

$$\left[\omega + u\left(\lambda + \int d\omega' du' g_{\omega',u'}\right) - \Omega\right] Q_{\omega,u} = \mu \int d\omega' du' (u+u') g_{\omega',u'} Q_{\omega',u'}$$

Straightforward to solve for eigenvalue Ω and eigenfunction $Q_{\omega,u}$

Banerjee, Dighe & Raffelt, arXiv:1107.2308

Georg Raffelt, MPI Physics, Munich

Stability Analysis for Simple SN Example



Normal vs Inverted Hierarchy



Represent neutrino field by discrete energy and angle modes

- Number of energy modes chosen to fit desired precision
- N_a >> 1 of angle modes required
 N_a too small: Unphysical solutions





















Multi-Angle Matter Effect

Liouville form of oscillation equation



Self-induced conversion suppressed for $N_e \gtrsim N_{\nu}$

Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Accretion-Phase Matter Profiles



Dasgupta, O'Connor & Ott, arXiv:1106.1167



Core Collapse Supernovae, INT, Seattle, July 2012













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Multi-Angle Matter Effect (Basel Model 10.8 M_{sun})

Schematic single-energy, multi-angle simulations with realistic density profile



Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130

Multi-Angle Multi-Energy Stability Analysis



Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601

To investigate neutrino oscillations in SNe need

- Profile of electron density
- Flavor-dependent neutrino flux spectra
- Flavor-dependent neutrino angular distribution

Neutrino Radiation Field

Small "scattering halo" important for nu-nu refraction? (Cherry et al., arXiv:1203.1607)



Picture from Ott, Burrows, Dessart & Livne, arXiv:0804.0239

Scattered Neutrinos as a Source of Refraction



Cherry, Carlson, Friedland, Fuller & Vlasenko, arXiv:1203.1607 Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1204.0971

Multi-Angle Matter Suppression in Realistic Model



Sarikas, Tamborra, Raffelt, Hüdepohl & Janka, arXiv:1109.3601, 1204.0971

Summary

Supernova neutrino flavor evolution remains a complicated subject

- Axial symmetry was always assumed too symmetric?
- Numerical treatments challenging
- Novel role for neutrino "scattering halo"?
- Simultaneous space and time dependence important?

Theoretical developments

- Analogy to BCS theory
- Linearized stability analysis provides many conceptual insights
- And practical results

Working hypothesis for SN neutrinos

- Multi-angle matter effect can prevent instability
- No collective conversion during early accretion phase
- Can test for nu mass hierarchy (because θ_{13} is large)

More theory progress is needed to reliably interpret neutrino signal of next galactic supernova!