Amplification of magnetic fields in core collapse

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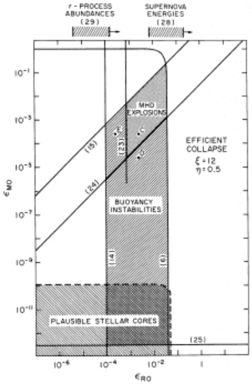
INT Program INT 12-2a
Core-Collapse Supernovae: Models and Observable Signals
Nuclear and Neutrino Physics in Stellar Core Collapse

Contents

Progenitor fields

- magnetic fields need to be strong to have an effect on SNe
- But: stellar evolution theory predicts rather weak fields in the pre-collapse core

→ efficient amplification desired



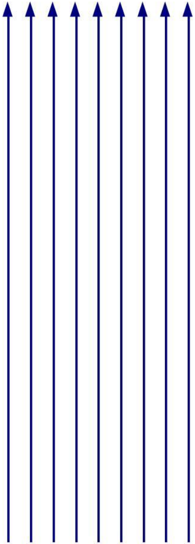
Meier et al., 1976

Magnetic fields and MHD

- magnetic energy $\frac{1}{2}\vec{B}^2$
- ideal MHD: field lines and flux tubes frozen into the fluid
- Lorentz force (Maxwell stress) consists of
  - isotropic pressure $\frac{1}{2}\vec{B}^2$
  - anisotropic tension $B^i B^j$
- increase the energy by working against the forces
  - compressing the field
  - stretching and folding field lines

$\rightarrow$ estimate for the maximum field energy: $\sim$ kinetic energy

- actual amplification may be less



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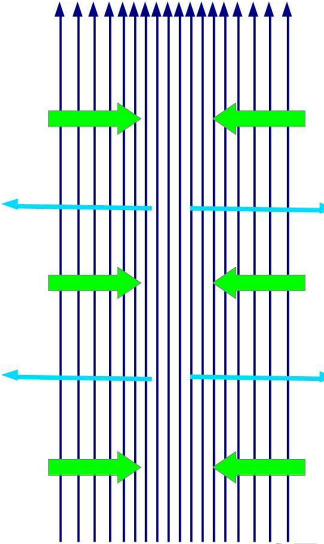

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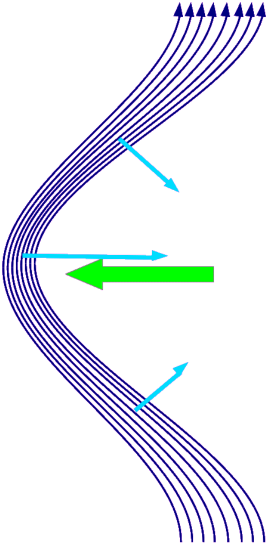


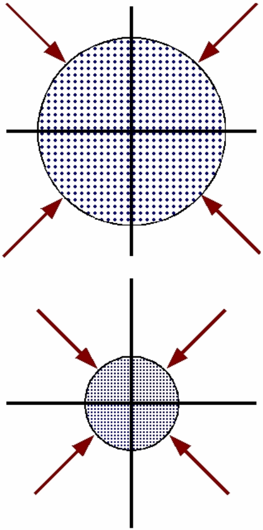
Contents

Compression

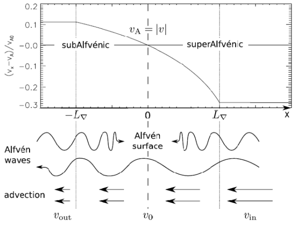
- (radial) collapse and accretion compress the field
- magnetic flux through a surface is conserved

→ $B \propto \rho^{2/3}$ for a fluid element; energy grows faster than gravitational

- no change of field topology
- core collapse: factor of 10^3 in field strength
- possible saturation: $e_{\mathrm{mag}} \sim e_{\mathrm{kin,r}}$ is unrealistic in collapse
- occurs in every collapse
- no difficulties in modelling



Amplification of Alfvén waves



- requires an accretion flow decelerated above the PNS and a (radial) guide field

→ accretion is sub-/super-Alfvénic inside/outside the Alfvén surface

- Alfvén waves propagating along the field are amplified at the Alfvén point
- waves are finally dissipated there → additional heating
- in core collapse: efficient for a limited parameter range (strong guide field); strong time variability of the Alfvén surface may be a problem
- modelling issues: high resolution, uncertainties in the dissipation

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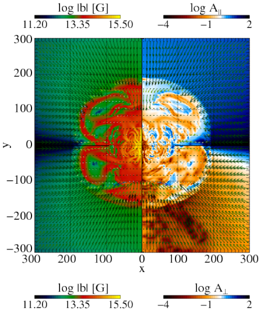

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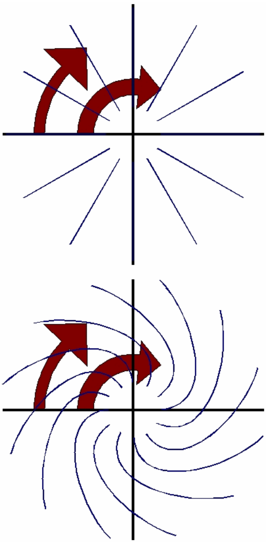
Linear winding



- works if $\vec{\nabla}\Omega \cdot \vec{B} \neq 0$, e.g., differential rotation and poloidal field
- creates toroidal field $B^\phi \propto \Omega t$
- feedback: slows down rotation

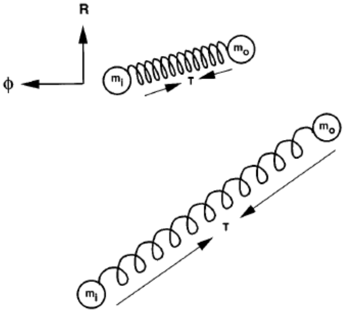
$\rightarrow$ saturation: complete conversion of differential to rigid rotation

- core collapse: slow compared to dynamic times except for rapid rotation and strong seed field
- should be present in all rotating cores
- no difficulties in modelling

The magneto-rotational instability

- instability of differentially rotating fluids with weak magnetic field
- (simplest) instability criterion $\nabla_{\varpi}\Omega < 0$
- exponential growth $\propto \exp \Omega t$
- starts with coherent channel modes, but leads to turbulence
- feedback: redistribution of angular momentum

→ maximum saturation: conversion of differential to rigid rotation



Balbus & Hawley (1998)

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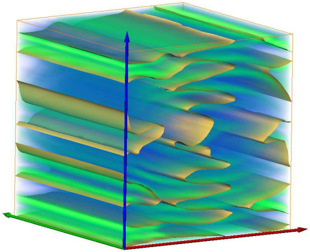

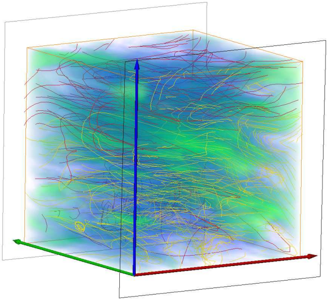

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$\rightarrow$ maximum saturation: conversion of differential to rigid rotation

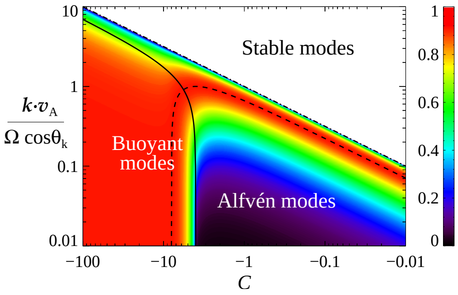
- main physical issue: saturation level
- numerical difficulties: high resolution, low numerical diffusion

The MRI: no amplification without representation

- dispersion relation of the MRI: only short modes, $\lambda \propto |B|$, grow rapidly
- in core collapse, this can be $\sim 1\ \mathrm{m}$
- grid width $< \lambda$ computationally not feasible

→ high-resolution shearing-box simulations to determine fundamental properties of the MRI

- use these results to build models that can be coupled to global simulations



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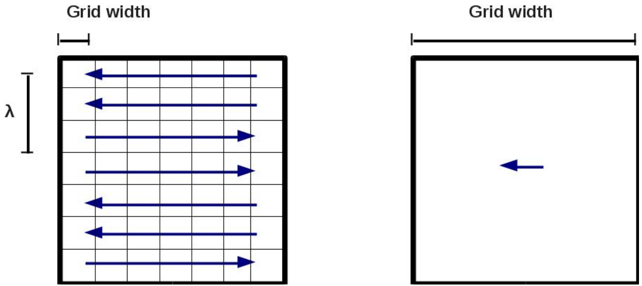

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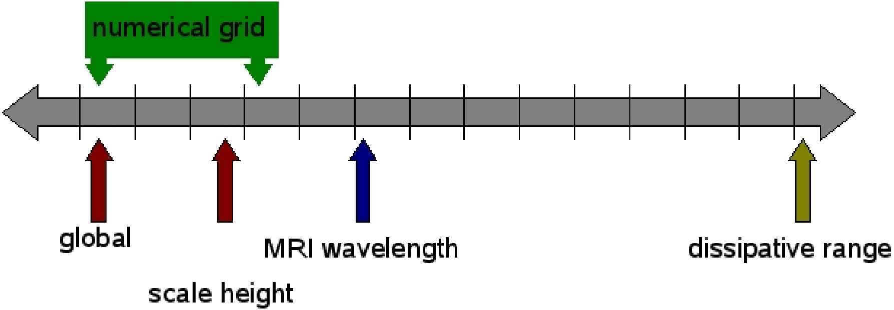

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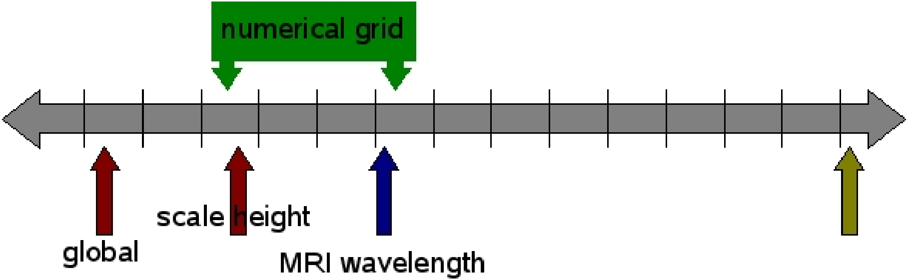

The MRI: saturation mechanism

- amplification until $e_{\rm mag} \sim e_{\rm diffrot}$?
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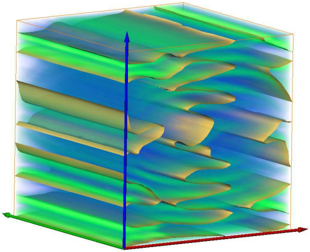
The MRI: saturation mechanism

- MRI channel modes are separated by current sheets and shear layers $\rightarrow$ unstable against parasitic instabilities: Kelvin-Helmholtz and tearing modes
- parasites grow at rates $\propto B_{\mathrm{MRI}}/\lambda \propto \exp \sigma_{\mathrm{MRI}} t / B_0$, i.e., faster as the MRI proceeds
- at some point, they overtake the MRI and break the channels down into turbulence
- weaker field $\rightarrow$ thinner channels $\rightarrow$ lower termination amplitude



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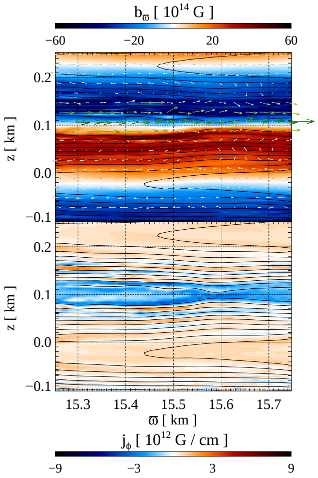

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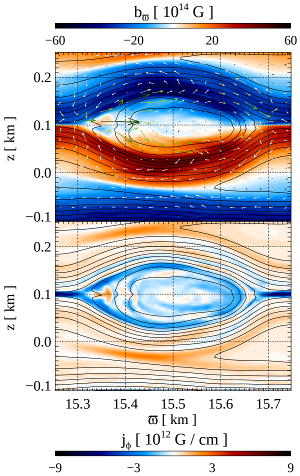

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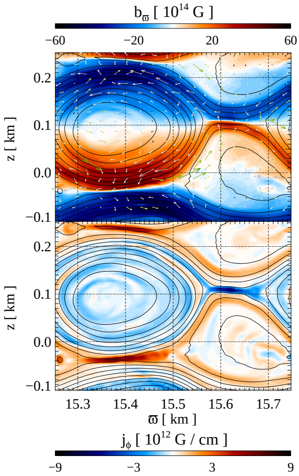

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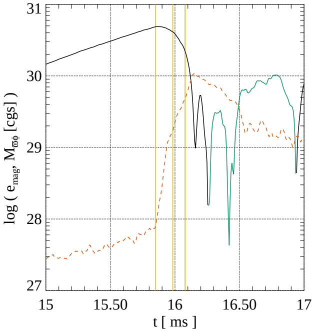
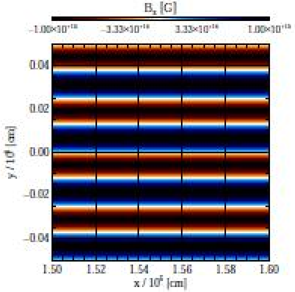
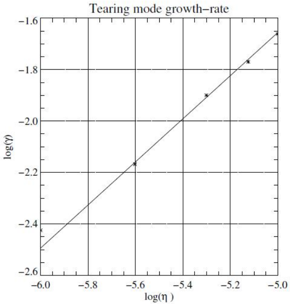


The MRI: saturation mechanism

⇒ MRI growth limited by initial field strength?

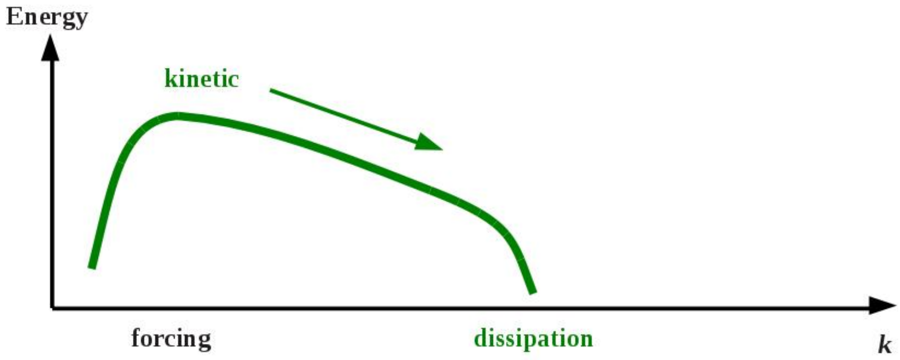
- current simulations (T. Rembiasz) try to test the predictions of Pessah (2010) and focus on how parasites depend on resistivity and viscosity. Prerequisite: careful determination of numerical resistivity and viscosity.





Dynamos driven by hydrodynamic instabilities

- instabilities such as convection and SASI drive turbulence
- energy cascades from the large scale at which the instability operates down to dissipation in a Kolmogorov-like cascade



Dynamos driven by hydrodynamic instabilities

→ dynamo converting turbulent kinetic to magnetic energy by stretching and folding flux tubes

- small-scale dynamo: amplifies the field only on length scales of turbulent velocity fluctuations → Kolmogorov-like spectrum of turbulent magnetic field
- large-scale dynamo adds an inverse cascade of field to larger length scales, i.e., generates an ordered component. Key ingredient: helicity $\vec{v} \times \mathrm{curl}\vec{v}$.

A. Brandenburg, K. Subramanian / Physics Reports 417 (2005) 1–209

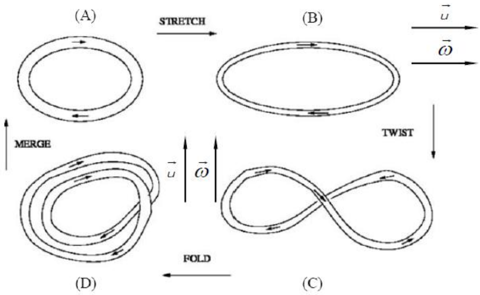


Fig. 4.6. A schematic illustration of the stretch-twist-fold-merge dynamo.

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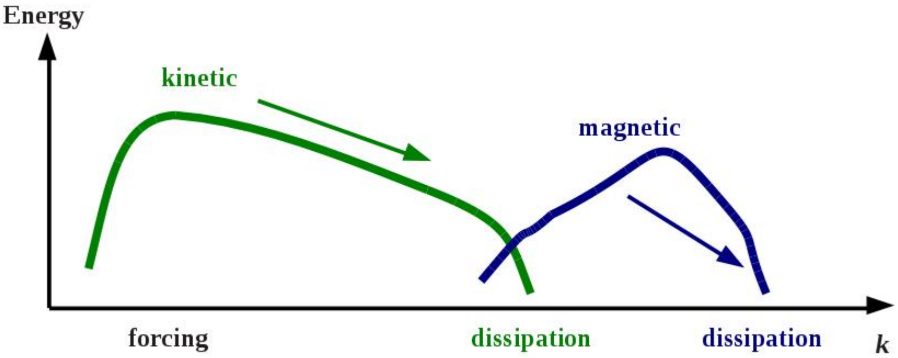

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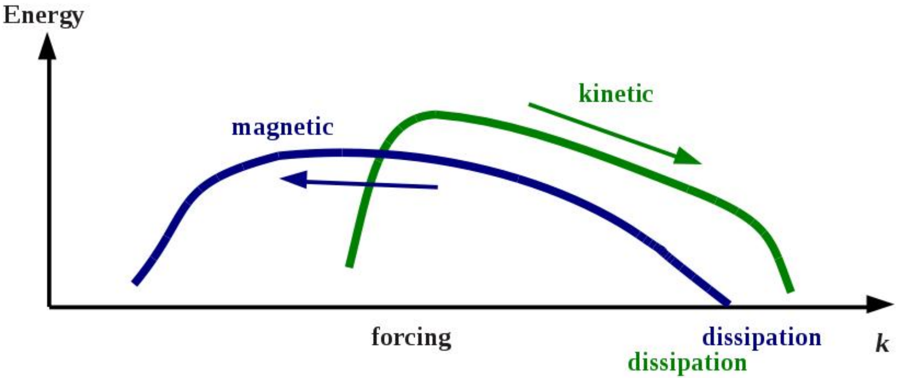
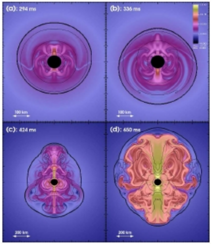
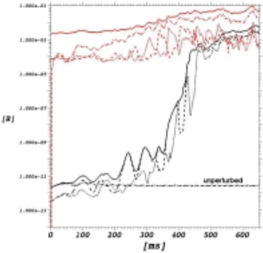
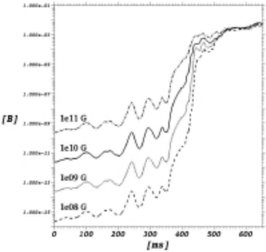


Instabilities: an overview

	SASI	convection	MRI
energy mechanism	accretion flow advective-acoustic cycle	thermal buoyant transport of energy/species	diff. rotation magnetic transport of angular momentum
role of $\vec{b}$	passive; turbulent dynamo	passive; turbulent dynamo	instability driver; turbulent dynamo







Endeve et al., 2008

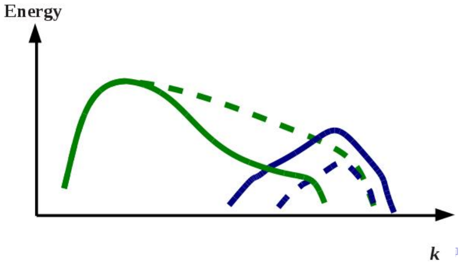
Dynamos

- MHD uncertainties: type of dynamo, saturation mechanism and level
- technical complications: 3d necessary, high resolution and Reynolds numbers
- kinematic dynamo: weak fields, back-reaction negligible $\rightarrow$ mean-field dynamo theory
  solve the induction equation, $\partial_t \vec{B} = \vec{\nabla} \times (\vec{v} \times \vec{B})$, for a fixed turbulent velocity field
  $\rightarrow \alpha$ effect: $\partial_t \vec{B} = \alpha \vec{B}$. α parametrises the unknown physics of helical turbulence.

Saturation mechanisms

Will instabilities and turbulence amplify a seed magnetic field to dynamically relevant strength or will the amplification cease earlier?

- if the properties of the instability depend strongly on the field, amplification might be limited by the initial field (MRI channel disruption)
- quenching of turbulent dynamos: small-scale field resists further stretching ($e_{\mathrm{mag}} \sim e_{\mathrm{kin}}$ locally in k-space), reducing the efficiency of mean-field dynamos by orders of magnitude.
- magnetic helicity: $\vec{A} \cdot \vec{B}$ (where $\vec{B} = \vec{\nabla} \times \vec{A}$), is conserved in ideal MHD. Box simulations indicate that fluxes out of the domain may be important to avoid catastrophic quenching. Inhomogeneity of cores may provide that.



Contents

Summary

mechnism	requires	results & issues
compression	infall	amplifies field robustly
winding	diff. rotation	works well in rapid rotators
Alfvén waves	accretion flow	works for rather strong fields
MRI	diff. rotation	saturation level?
dynamo	instabilities	saturation level?

Saturation level

Better understanding of turbulence and dynamos is required!