Neutrino Reactions on the Deuteron in Core-Collapse Supernovae

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Neutrino reactions on the deuteron

$$\nu_e + d \to e^- + p + p \qquad \nu + d \to \nu + p + n$$

$$e^- + d \to \nu_e + n + n \qquad e^+ + d \to \bar{\nu}_e + p + p$$

$$p + p \to d + e^+ + \nu_e \qquad p + n \to d + \nu + \bar{\nu} \qquad n + n \to d + e^- + \bar{\nu}_e$$

Important relevance to neutrino physics, astrophysics

- Supernova (ν–heating, ν–emission)
- ν–oscillation experiment @ SNO
- Solar fusion (*pp*-chain)

Calculational method

Well-established method for electroweak processes in few-nucleon systems

$$\boxed{\langle\psi_f|H_{\mathrm{ew}}|\psi_i\rangle}$$

$|\psi\rangle$: solution of Schröding eq. with high-precision NN (+ NNN) potential

AV18, Nijmegen, Bonn, etc.

H_{ew} : impulse + meson exchange currents

review : Carlson and Schiavilla, Rev. Mod. Phys. 70, 743841 (1998)

cf. chiral effective field theory

Contents

- Model for H_{ew}
- ν−heating in supernova

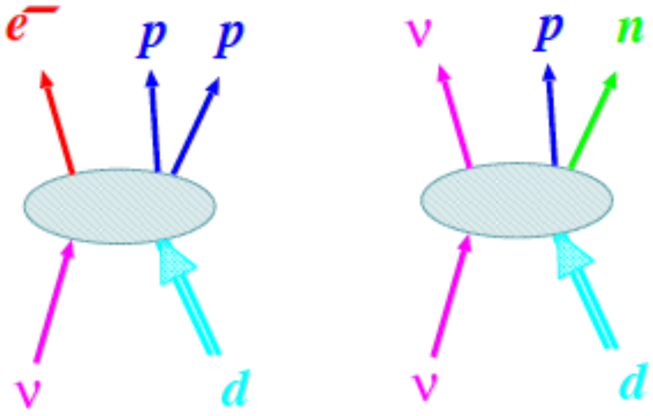
$$\nu_e + d \rightarrow e^- + p + p \qquad \nu \ + d \rightarrow \nu + p + n$$

- ν−emission in supernova

$$e^{\pm} + d \rightarrow \nu_e(\bar{\nu}_e) + N + N \qquad N + N \rightarrow d + l + \bar{l}$$

MODEL

Interaction Hamiltonian



Charged-Current **Neutral-Current**

$$H_W^{CC} = \frac{G'_F V_{ud}}{\sqrt{2}} \int d\boldsymbol{x}[J_\lambda^{CC}(\boldsymbol{x}) L^\lambda(\boldsymbol{x}) + \text{h. c.}] \qquad \text{for CC}$$

$$H_W^{NC} = \frac{G'_F}{\sqrt{2}} \int d\boldsymbol{x}[J_\lambda^{NC}(\boldsymbol{x}) L^\lambda(\boldsymbol{x}) + \text{h. c.}] \qquad \text{for NC}$$

$$L^\lambda(\boldsymbol{x}) = \bar{\psi}_l(\boldsymbol{x})\, \gamma^\lambda\, (1 - \gamma^5)\, \psi_\nu(\boldsymbol{x})$$

Nuclear Current

$$
\begin{aligned}
J_\lambda^{CC}(\boldsymbol{x}) &= V_\lambda^{\pm}(\boldsymbol{x}) + A_\lambda^{\pm}(\boldsymbol{x}) \\
J_\lambda^{NC}(\boldsymbol{x}) &= V_\lambda^{3} - 2\sin^2\theta_W (V_\lambda^3 + V_\lambda^s) + A_\lambda^3
\end{aligned}
$$

$V(A)$: Vector (Axial) current

V^s : Isoscalar vector current

θ_W : Weinberg Angle $\quad \sin^2\theta_W = 0.23$

J_λ = (one-body current) + (two-body exchange current)

Impulse Approximation (IA) Current

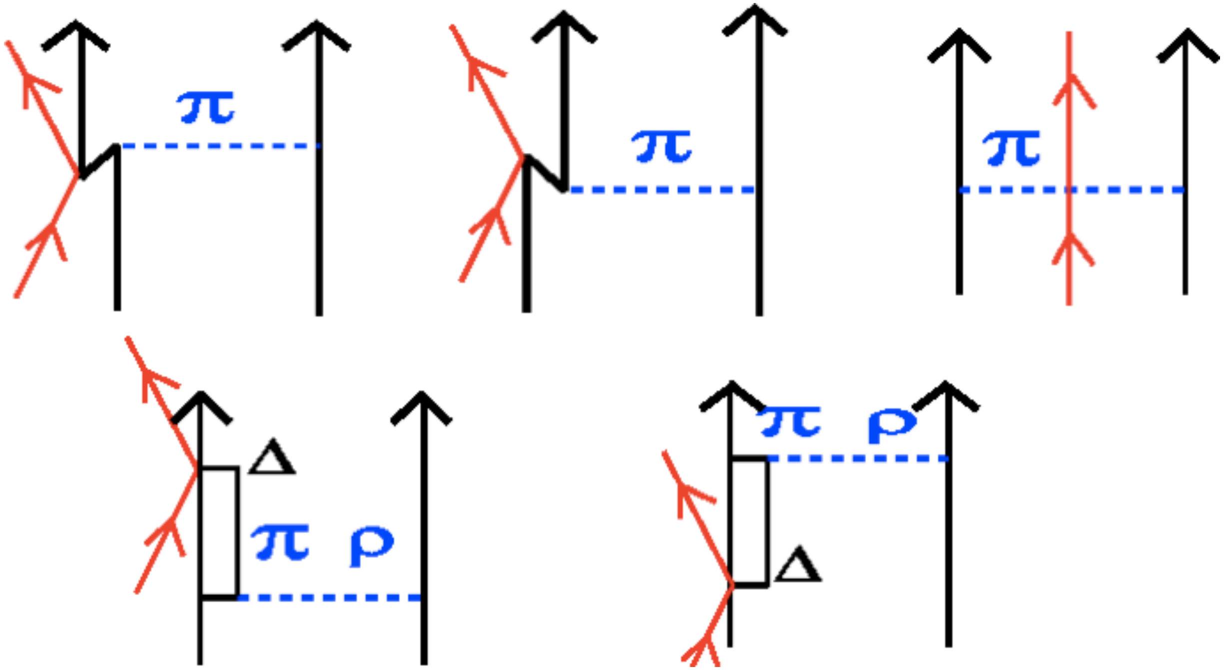
$$
\begin{aligned}
< p' \,|V_\lambda(0)|\, p > &= \bar{u}(p') \left[f_V \gamma_\lambda + i \frac{f_M}{2M_N} \sigma_{\lambda\rho} q^\rho \right] u(p) \\
< p' \,|A_\lambda(0)|\, p > &= \bar{u}(p') \left[f_A \gamma_\lambda \gamma^5 + f_P \gamma^5 q_\lambda \right] u(p)
\end{aligned}
$$

$$
q_\lambda \equiv {p'}_\lambda - p_\lambda
$$

$$
f_M \;:\; \mathrm{CVC} \qquad f_P : \mathrm{PCAC}
$$

$$
f_A(q_\mu^2) = -g_A \left(1 - \frac{q_\mu^2}{1.04\,[\mathrm{GeV}^2]} \right)^{-2}, \quad g_A = 1.2670 \pm 0.0030 \;\; (\mathrm{PDG})
$$

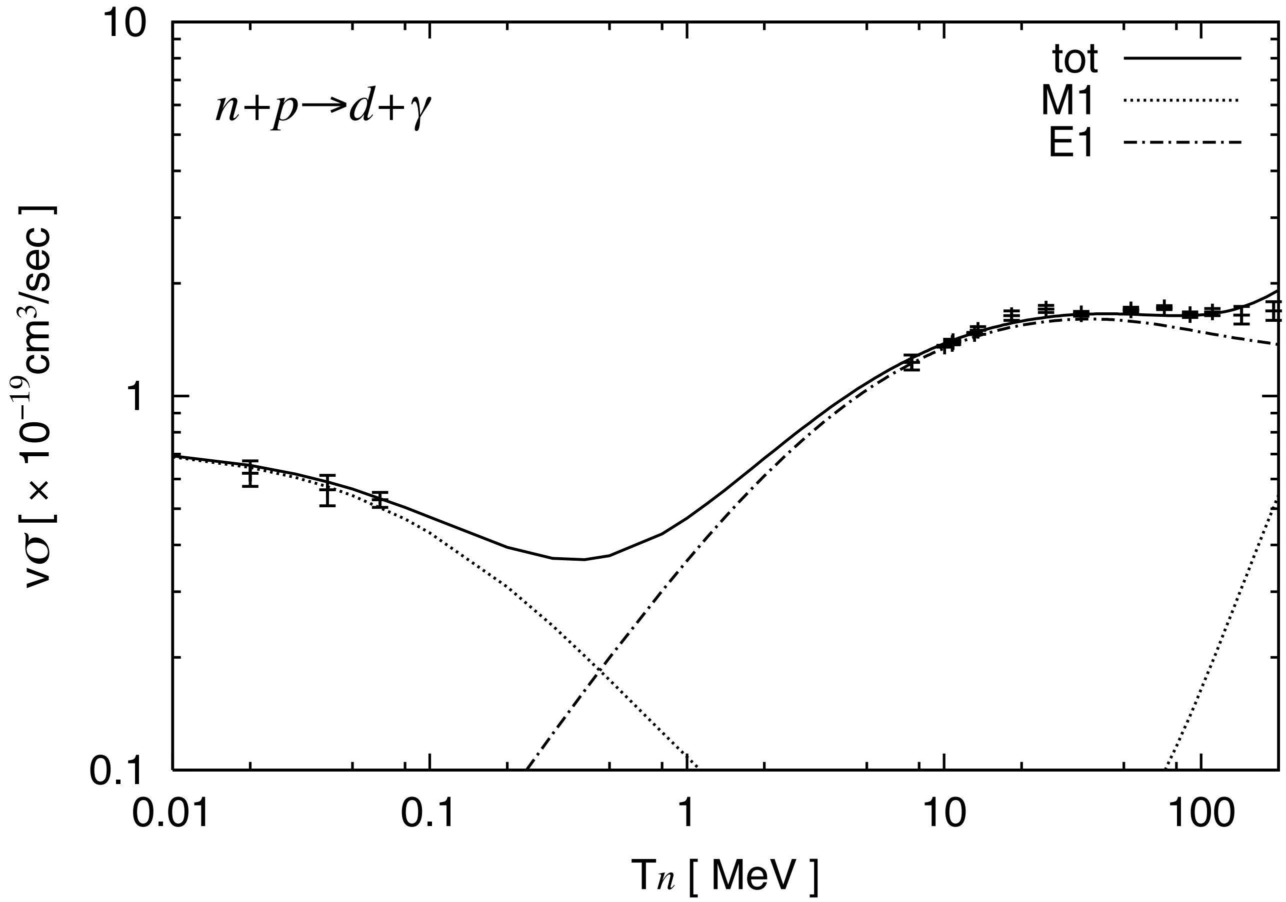
Exchange vector current



★ Current conservation for one-pion-exchange potential

★ $VN\Delta$ coupling is fitted to $np \rightarrow d\gamma$ data

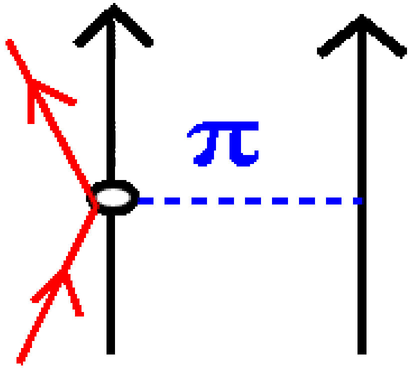
Comparison with *np* ➔ *dγ* data



Exchange currents contribute about 10 %

Exchange axial charge

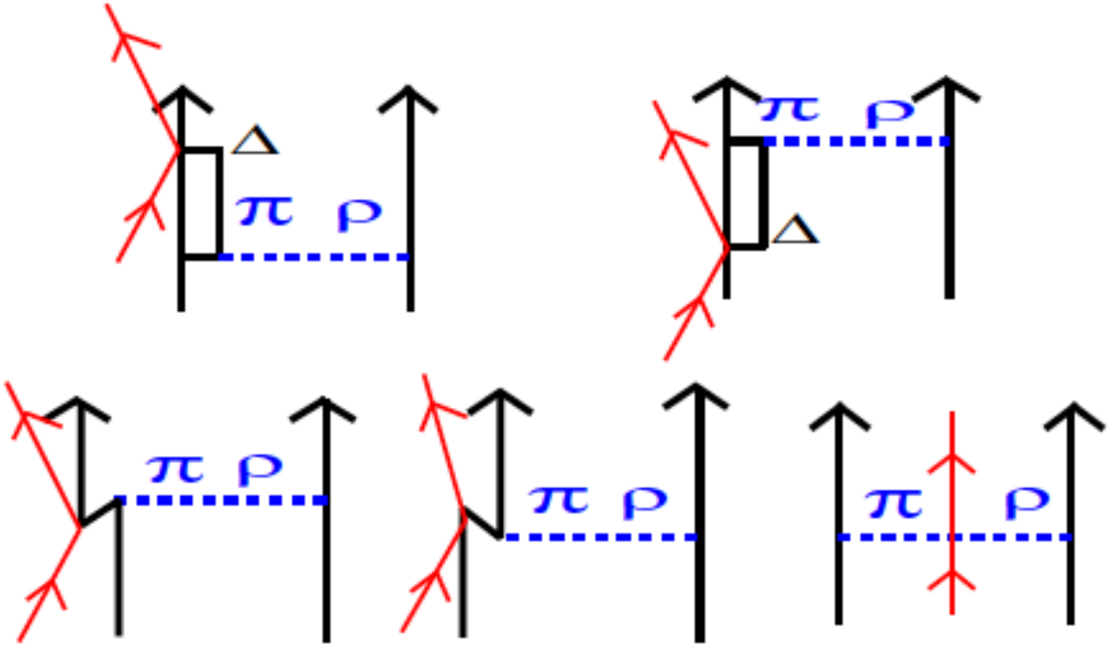
Kubodera, Delorme, Rho, PRL 40 (1978)



Soft pion theorem + PCAC

Exchange axial-vector current

R. Schiavilla et al. PRC **58**, 1263 (1998)



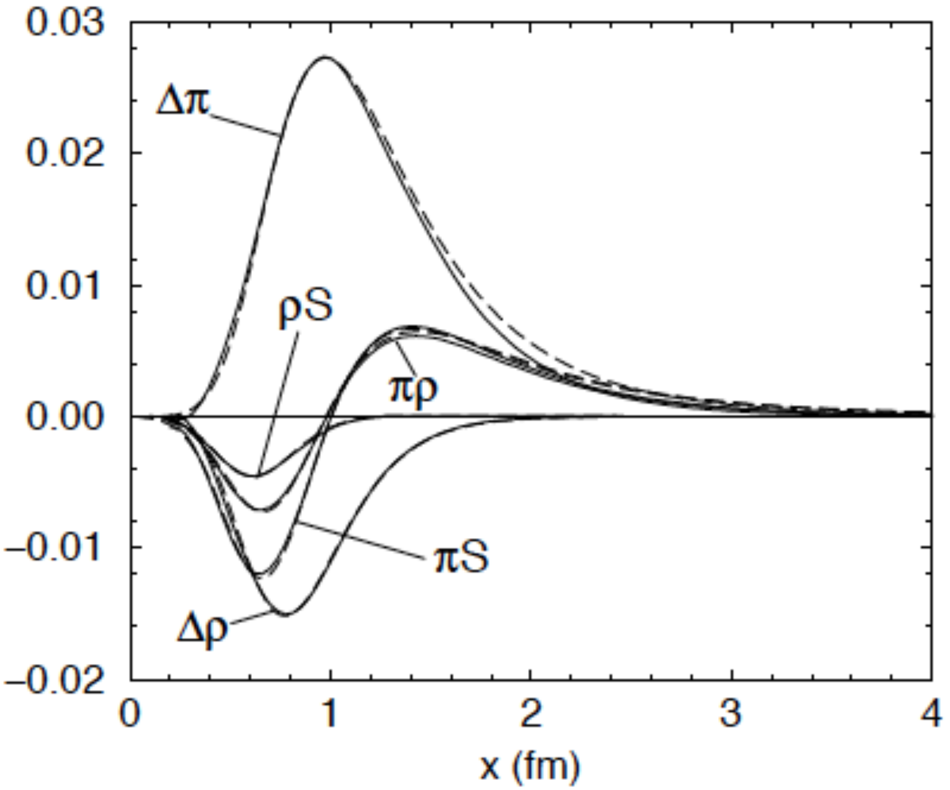
- Fit $AN\Delta$ coupling to tritium β-decay rate
- Rigorous three-body calculation

Why tritium β decay?

νd : Gamow-Teller $({}^3S_1 \rightarrow {}^1S_0) \Rightarrow \mathbf{A}_{EXC}$ is main correction

^{3}H : Fermi $({}^1S_0 \rightarrow {}^1S_0)$ & Gamow-Teller

$\rho(x)$ [fm^{-1}]



$\frac{\rho_{\nu d}}{\rho_{{}^3H}} \approx$ const.

Schiavilla et al., PRC**58**,1263(1998)

Most recent applications of the model to weak processes

★ *pp*-fusion $(p+p\rightarrow d+e^{+}+\nu_{e})$ for solar model , Schiavilla et al. PRC 58 (1998)

★ Muon capture $(\mu^{-}+d\rightarrow n+n+\nu_{\mu}\ ,\ \mu^{-}+{}^{3}He\rightarrow {}^{3}H+\nu_{\mu})$, Marcucci et al., PRC 83 (2011)

	[1]	[2]	Theory	MuSun@PSI
$\Gamma(\mu^{-}+d)[s^{-1}]$	409 ± 40	470 ± 29	393	$???$

	[3]	Theory
$\Gamma(\mu^{-}+{}^{3}He)[s^{-1}]$	1496 ± 4	1496

[1] Cargnelli et al. (1998)
[2] Bardin et al. NPA 453 (1986)
[3] Ackerbauer et al. PLB 417 (1998)

★ *νd*-reactions $(\nu_{e}+d\rightarrow e^{-}+p+p\ ,\ \nu\ +d\rightarrow \nu+p+n)$ for SNO experiment

SN et al. PRC 63 (2000) ; NPA707 (2002)

➔ evidence of *ν*-oscillation, solar *ν* problem resolved

Neutrino-deuteron reaction as heating mechanism in Supernova

SXN, K. Sumiyoshi, T. Sato, PRC 80, 035802 (2009)

In many simulations, supernova doesn't explode !

➔ extra assistance needed for re-accelerating shock-wave

★ neutrino absorption on nucleon (main)

★ neutrino scattering or absorption on nuclei (extra agent)

NC can contribute to the heating

Heating through neutrino-nucleus scattering

- ^{4}He, ^{56}Fe Haxton, PRL 60, 1999 (1988)

  ⇒ small effect on supernova dynamics

- ^{3}He, ^{3}H O'Connor et al. PRC 75, 055803 (2007)

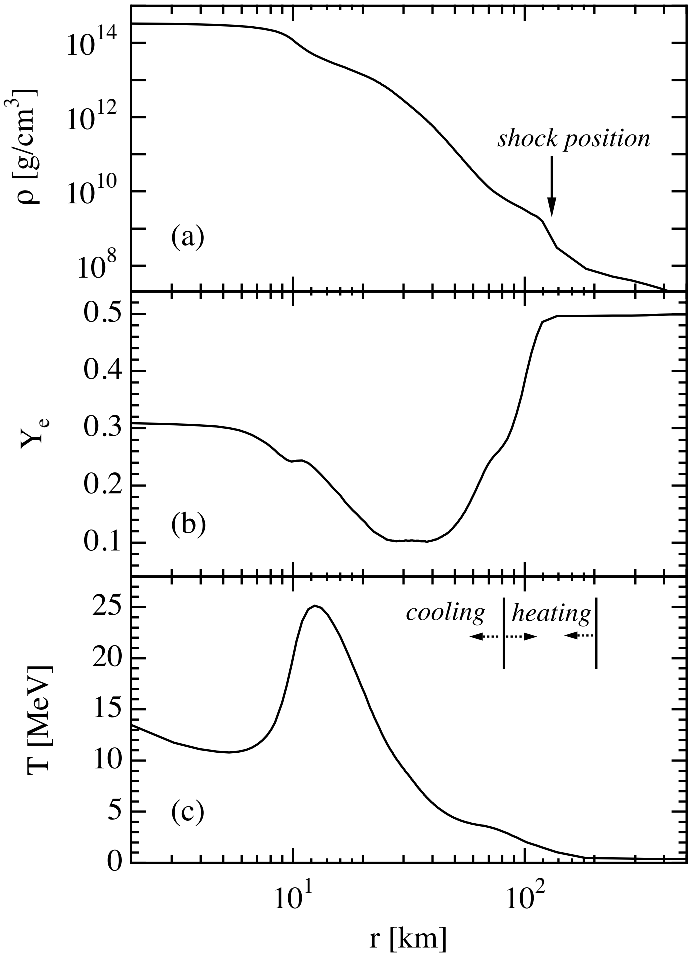
  more effective heating than ^{4}He

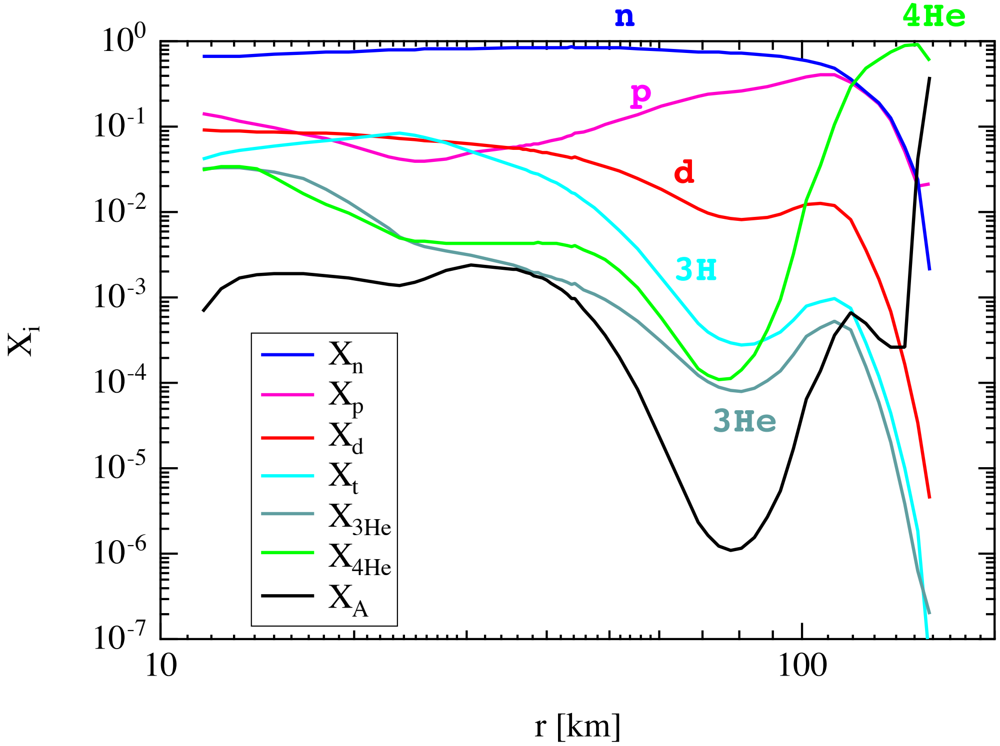
- deuteron ?

  can be abundant in supernova, $\sigma_{\nu d} \gg \sigma_{\nu^3\mathrm{He}}, \sigma_{\nu^3\mathrm{H}}$

Abundance of light elements in supernova

Sumiyoshi, Röpke, PRC 77, 055804 (2008)





15 $M_\odot$, 150 ms after core bounce

Nuclear statistical equilibrium assumed

cf. Arcones et al. PRC 78, 015806 (2008)

Energy transfer cross section

CC (absorption)

$$\sigma\omega(E_\nu) = \int dE'_l \frac{d\sigma}{dE'_l} E_\nu$$

NC (scattering)

$$\sigma\omega(E_\nu) = \int dE'_\nu \frac{d\sigma}{dE'_\nu} (E_\nu - E'_\nu)$$

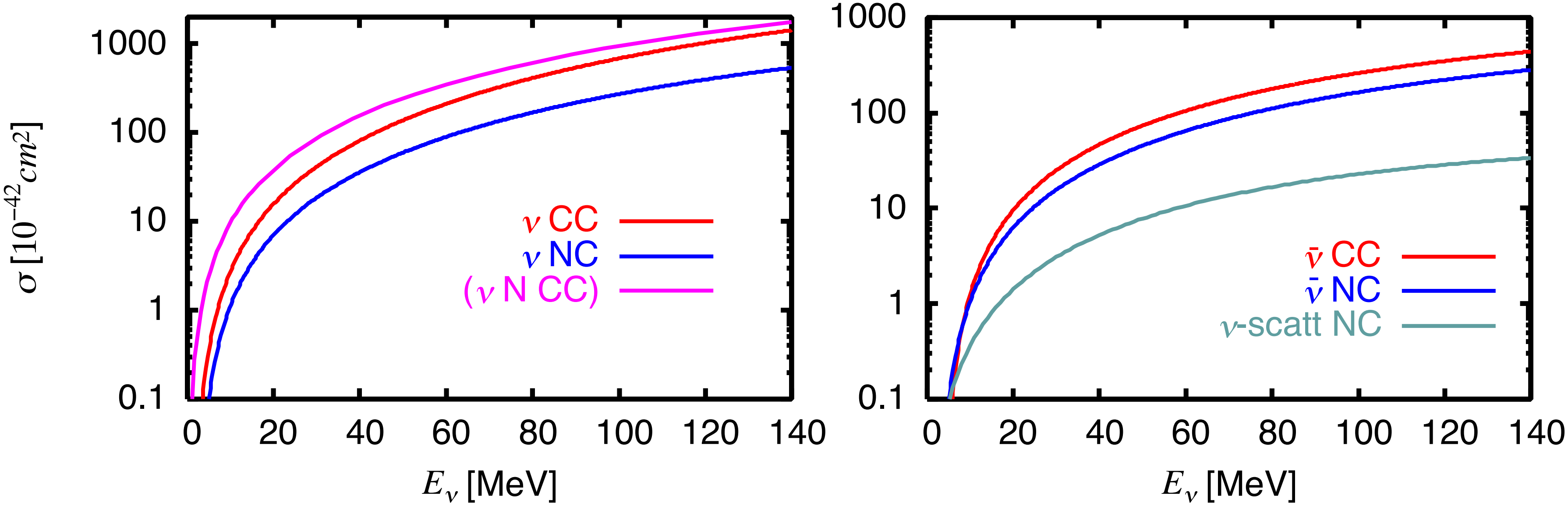
Thermal average

$$< \sigma\omega >_{T_\nu} = \int dE_\nu f(T_\nu, E_\nu) \sigma\omega(E_\nu)$$

$$f(T_\nu, E_\nu) = \frac{N}{T_\nu^3} \frac{E_\nu^2}{e^{E_\nu/T_\nu} + 1}$$

Results

Neutrino-deuteron cross sections

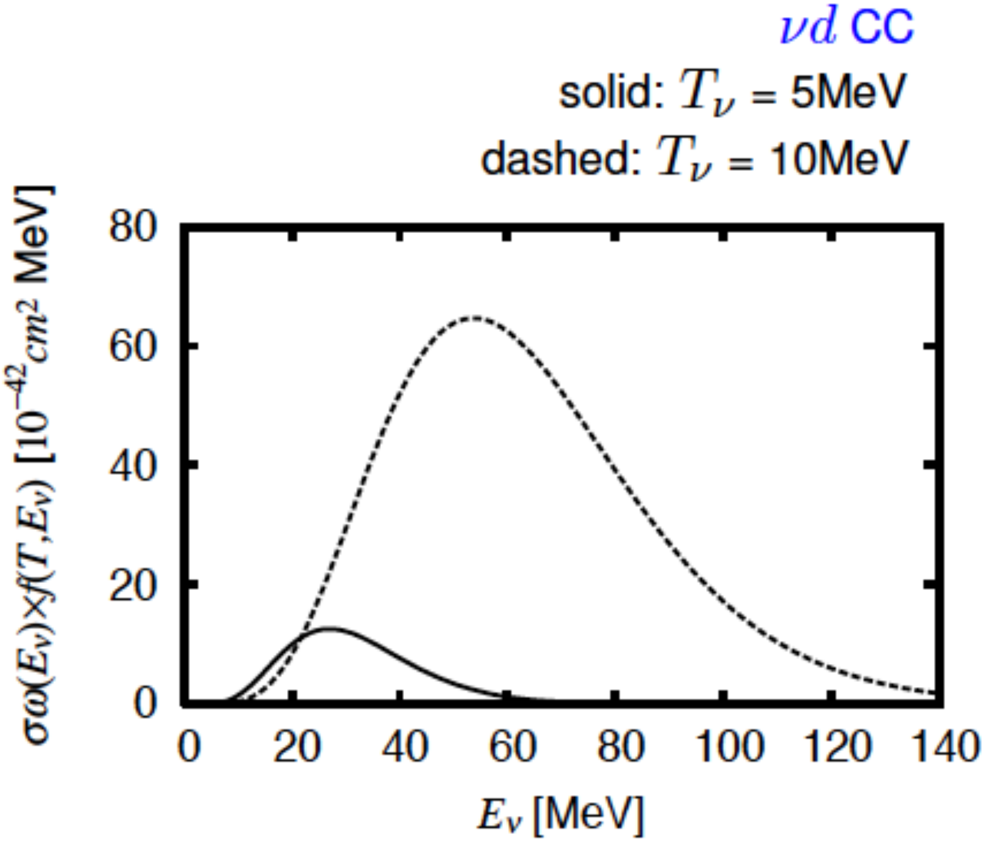


$* \; \sigma(\nu d\, CC) \sim \sigma(\nu N\, CC)/3$ at E_ν = 10 MeV

$* \; \sigma(\nu d\, CC) \sim \sigma(\nu N\, CC)/2$ at E_ν = 50 MeV

$* \; \sigma(\mathrm{elastic}\, \nu d)$ is very small

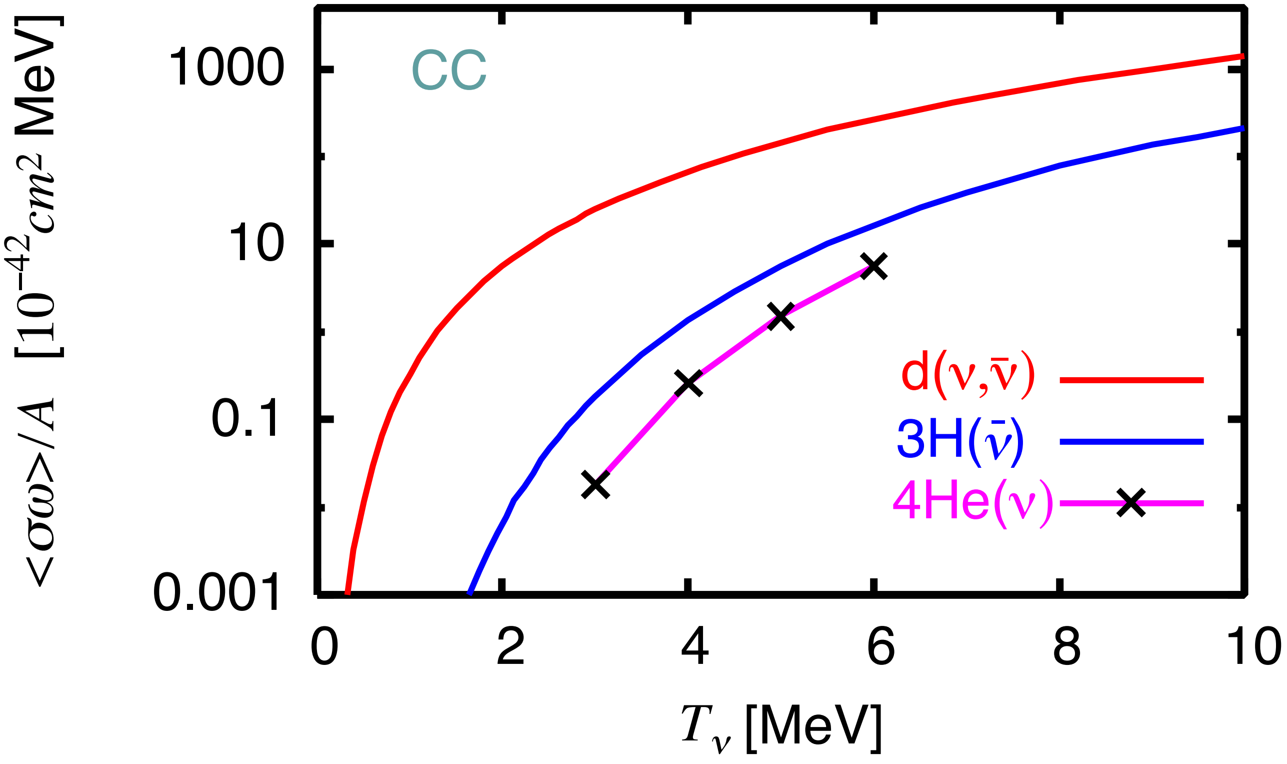
E_ν-dependence of energy transfer cross section



$*$ Main contribution is from E_ν = 20 (60) MeV for T_ν = 5 (10) MeV

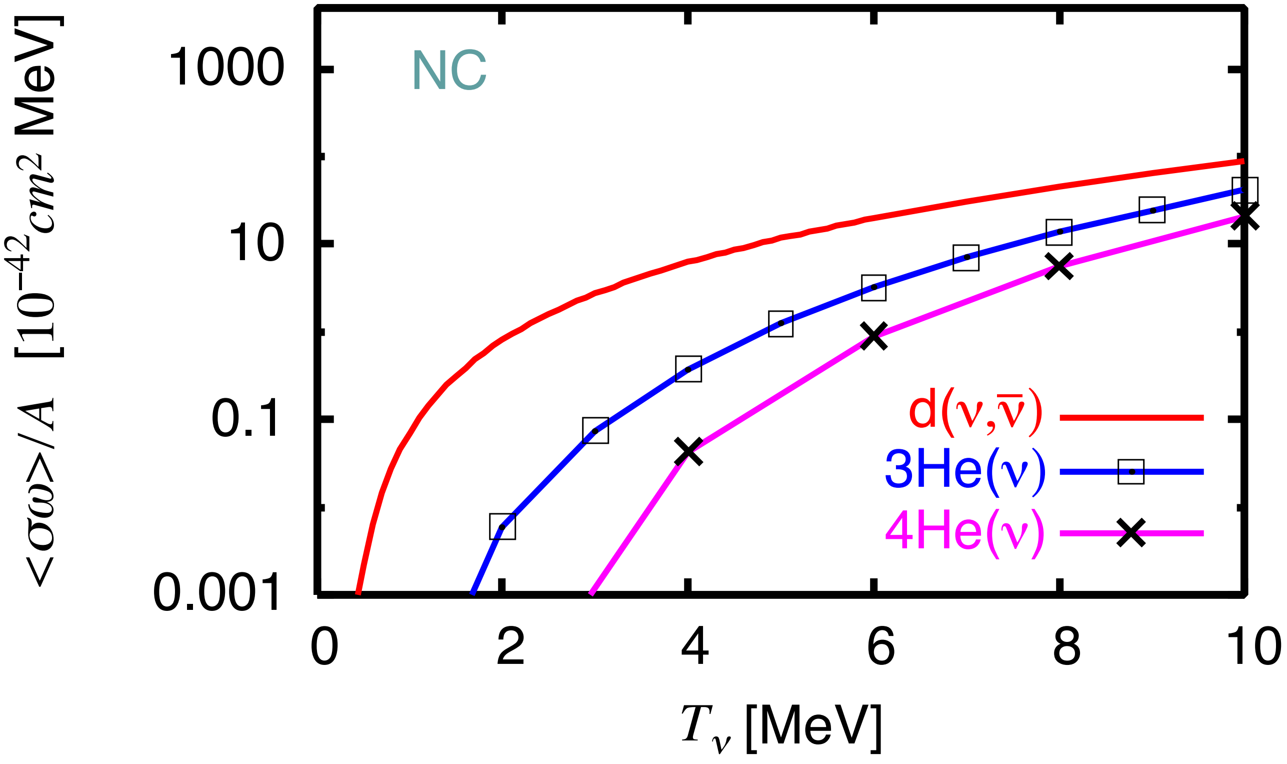
$*$ High energy tail of $\sigma\omega \times f$ is appreciable

Thermal average of energy transfer cross sections



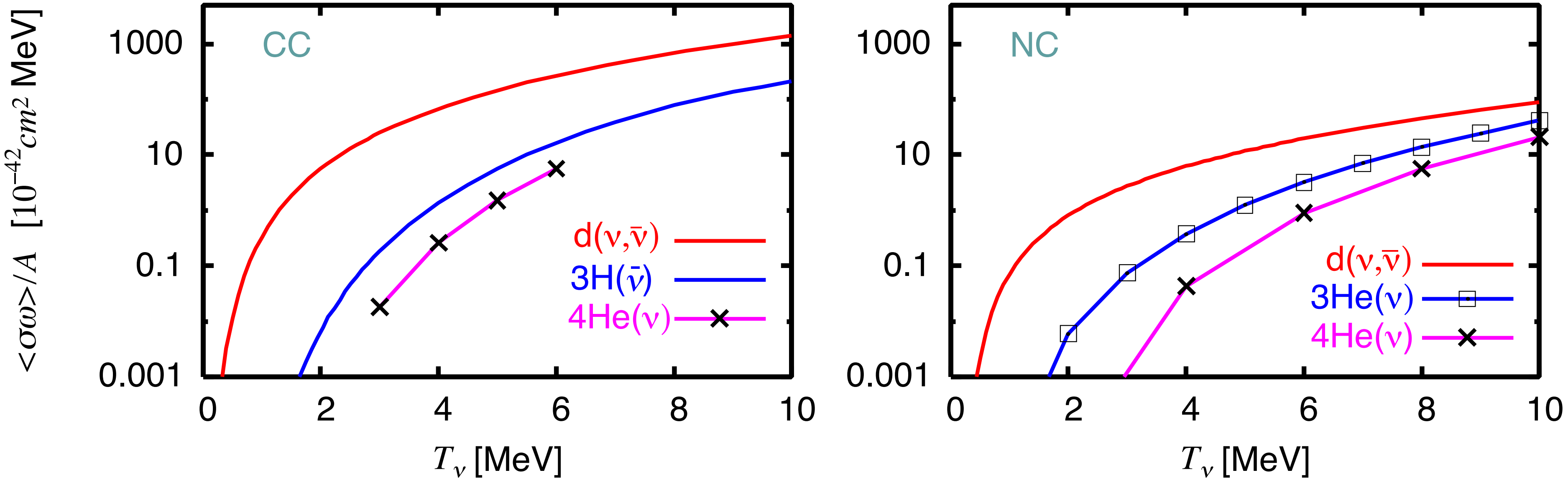
^{3}H (ν̄) : Arcones et al. PRC 78 (2008)
^{4}He(ν) : Haxton PRL 60 (1998)

Thermal average of energy transfer cross sections



^{3}He (ν) : O’conner et al. PRC 78 (2007)
^{4}He (ν) : Gazit et al. PRL 98 (2007)

Thermal average of energy transfer cross sections



$\ast$ $< \sigma\omega >$ for the deuteron is much larger than those of ^{3}H, ^{3}He, ^{4}He

$\ast$ Small binding energy $\Rightarrow$ rapid increase of $< \sigma\omega >$ at low T_ν

$\ast$ $< \sigma\omega >_{\nu_e d} / < \sigma\omega >_{\nu_e N} \sim$ 0.44 at T_{ν_e} = 5MeV

$\ast$ $< \sigma\omega >_{\nu_\mu d} / < \sigma\omega >_{\nu_e N} \sim$ 0.25 at T_{ν_e} = 5MeV and T_{ν_μ} = 10MeV

Electron capture on deuteron & NN fusion as neutrino emission mechanism

S. Nasu, SXN, T. Sato, K. Sumiyoshi, F. Myrer, K. Kubodera (2012)

ν-emission previously considered (A≤2)

- $p + e^- \to n + \nu_e$
- $n + e^+ \to p + \bar{\nu}_e$
- $n + n \to p + n + e^- + \bar{\nu}_e$
- $p + p \to p + n + e^+ + \nu_e$
- $N + N \to N + N + \nu + \bar{\nu}$

New agents

$*$	$p + e^- \rightarrow n + \nu_e$	$d + e^- \rightarrow n + n + \nu_e$
$*$	$n + e^+ \rightarrow p + \bar{\nu}_e$	$d + e^+ \rightarrow p + p + \bar{\nu}_e$
$*$	$n + n \rightarrow p + n + e^- + \bar{\nu}_e$	$n + n \rightarrow d + e^- + \bar{\nu}_e$
$*$	$p + p \rightarrow p + n + e^+ + \nu_e$	$p + p \rightarrow d + e^+ + \nu_e$
$*$	$N + N \rightarrow N + N + \nu + \bar{\nu}$	$p + n \rightarrow d + \nu + \bar{\nu}$

Emissivity (*Q*)

$$N_1 + N_2 \to N_1' + N_2' + \nu + \bar{\nu}$$

$$\begin{aligned} Q \quad &= \quad \frac{(2\pi)^4}{\mathcal{S}} \int \frac{d\boldsymbol{p}_{N_1}}{(2\pi)^3} \frac{d\boldsymbol{p}_{N_2}}{(2\pi)^3} \frac{d\boldsymbol{p}_{N_1'}}{(2\pi)^3} \frac{d\boldsymbol{p}_{N_2'}}{(2\pi)^3} \frac{d\boldsymbol{p}_{\nu}}{(2\pi)^3} \frac{d\boldsymbol{p}_{\bar{\nu}}}{(2\pi)^3} \delta^{(4)}(p_f - p_i) \\ &\times \quad (E_\nu + E_{\bar{\nu}}) \sum_{spin} |\langle \psi_f | H_{\mathrm{ew}} | \psi_i \rangle|^2 F_{N_1} F_{N_2} (1 - F_{N_1'})(1 - F_{N_2'}) \end{aligned}$$

$$F_N = \frac{1}{1 + \exp[(\varepsilon_N - \mu_N)/kT]}$$

Emissivity (*Q*)

$$N_1 + N_2 \to d + \nu + \bar{\nu}$$

$$\begin{aligned} Q &= \frac{(2\pi)^4}{\mathcal{S}} \int \frac{d\boldsymbol{p}_{N_1}}{(2\pi)^3} \frac{d\boldsymbol{p}_{N_2}}{(2\pi)^3} \frac{d\boldsymbol{p}_{d}}{(2\pi)^3} \frac{d\boldsymbol{p}_{\nu}}{(2\pi)^3} \frac{d\boldsymbol{p}_{\bar{\nu}}}{(2\pi)^3} \delta^{(4)}(p_f - p_i) \\ \times & \quad (E_\nu + E_{\bar{\nu}}) \sum_{spin} |\langle \psi_f | H_{\mathrm{ew}} | \psi_i \rangle|^2 F_{N_1} F_{N_2} \end{aligned}$$

11 dimensional integral !!

Approximation necessary to evaluate *Q*

Emissivity (Q)

$$Q \;\propto\; \int \underbrace{d\boldsymbol{p}_{N_1} d\boldsymbol{p}_{N_2}}_{p^2 \cancel{dp}\, d\Omega_{\boldsymbol{p}}\, d\boldsymbol{P}} \cancel{d\boldsymbol{p}_d}\, d\boldsymbol{p}_\nu d\boldsymbol{p}_{\bar{\nu}}\, \omega\, \underbrace{\delta(E_f - E_i)}_{\cancel{\delta\left[\left(-B + \frac{\cancel{p_d^2}}{\cancel{2}M_d} + \omega\right) - \left(\frac{\cancel{P^2}}{\cancel{4}M_N} + \frac{p^2}{M_N}\right)\right]}} \cancel{\delta^{(3)}(\vec{p}_f - \vec{p}_i)} |M|^2 F_{N_1}(\vec{p}_1) F_{N_2}(\vec{p}_2)$$

Approximation !

$$Q \;\propto\; \int \underbrace{d\boldsymbol{p}_\nu d\boldsymbol{p}_{\bar{\nu}}}_{8\pi^2\, p_\nu^2 p_{\bar{\nu}}^2\, dp_\nu dp_{\bar{\nu}}\, d\cos\theta_{\nu\bar{\nu}}}\, \omega p^2 \left[\int d\Omega_{\boldsymbol{p}} |M|^2\right] \left[\int d\boldsymbol{P}\, F_{N_1}(\vec{P}/2 + \vec{p}) F_{N_2}(\vec{P}/2 - \vec{p})\right]$$

3 dimensional integral

Previous common approximation to evaluate $Q_{\text{NN-brem}}$

- One-pion-exchange potential, Born approximation

- Neglect momentum transfer ($\vec{p}_\nu + \vec{p}_{\bar{\nu}}$)

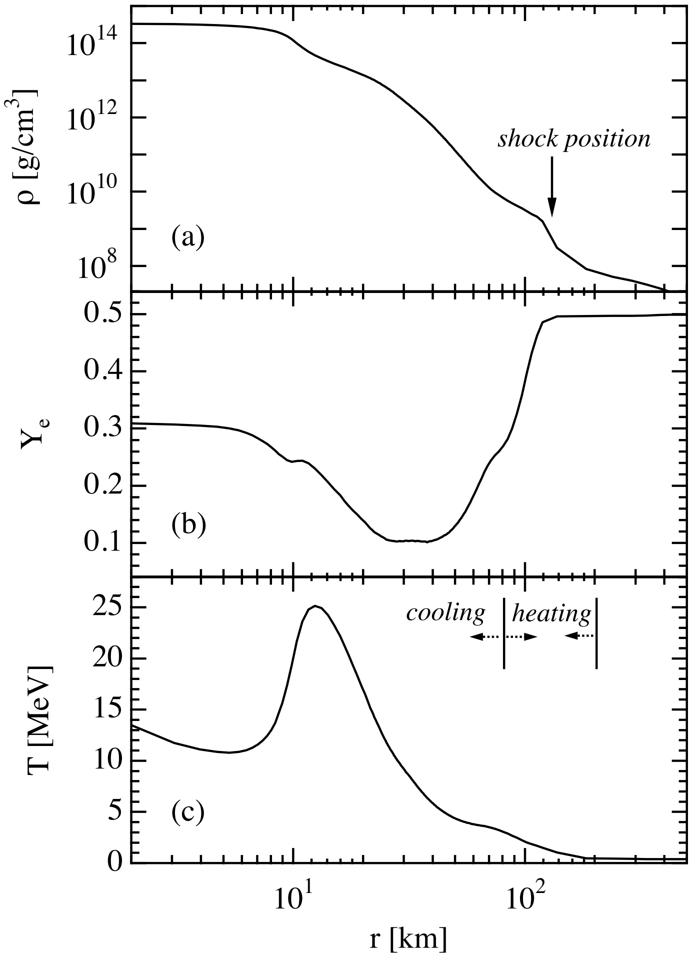
  ➔ also angular correlation between ν and $\bar{\nu}$

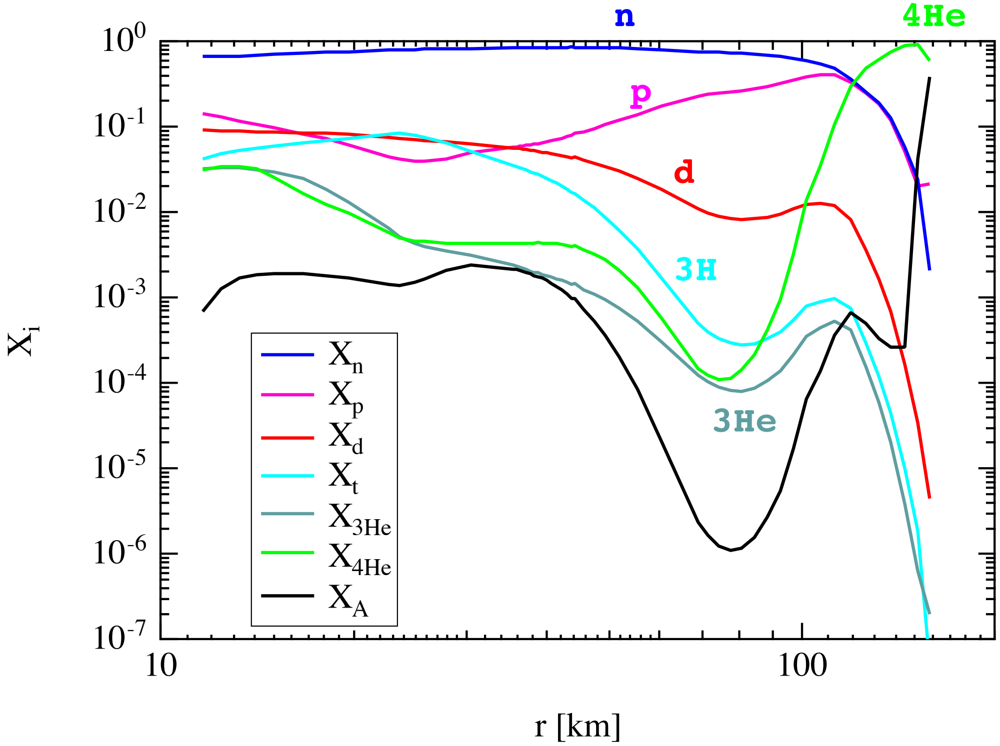
- Nuclear matrix element ➔ long wave length limit

  ➔ constant

Supernova profile

Sumiyoshi, Röpke, PRC 77, 055804 (2008)





150 ms after core bounce

Nuclear statistical equilibrium assumed

Summary

Deuteron breakup (ν-heating) & formation (ν-emission) in SNe and NS

Framework : *NN* wave functions based on high-precision *NN* potential

+ 1 & 2-body nuclear weak currents (tested by data)

ν-heating: $\nu_e + d \to e^- + p + p \qquad \nu + d \to \nu + p + n$

Substantial abundance of light elements (NSE model)

$\langle \sigma \omega \rangle$ for deuteron : much larger than those for ^{3}H, ^{3}He, ^{4}He

25-44% of $\langle \sigma \omega \rangle$ for the nucleon

Summary

ν-emission: $e^{\pm} + d \rightarrow \nu_e(\overline{\nu}_e) + N + N$ $N + N \rightarrow d + l + \overline{l}$

New agents other than direct & modified Urca, *NN* bremsstrahlung

Emissivities ← Rigorous evaluation of nuclear matrix elements

No long wave length limit, no Born approximation

Electron captures → effectively reduced ν_e emissivity

→ Need careful estimate of light element abundance & emissivity

NN fusions → *np* → $d\nu\overline{\nu}$ can be very important for $\overline{\nu}_e$ & ν_μ emissivites

→ play a role comparable to *NN* bremsstrahlung & modified Urca