Neutrino-Driven Convection and Neutrino-Driven Explosions

by

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1D simulations (Rad-hydro)

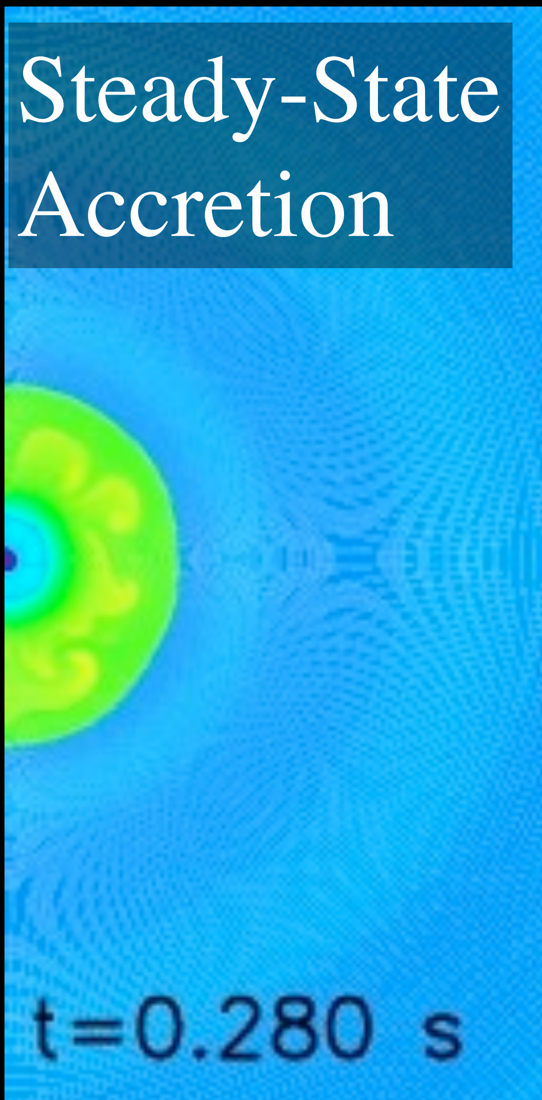
Wilson ‘85
Bethe & Wilson ‘85
Liebendoerfer et al. ‘01
Rampp & Janka ‘02
Buras et al. ‘03
Thompson et al. ‘03
Liebendoer et al. ‘05
Kitaura et al. ‘06
Burrows et al. ‘07

Neutrino mechanism suggested

No Explosions
(Except lowest masses)

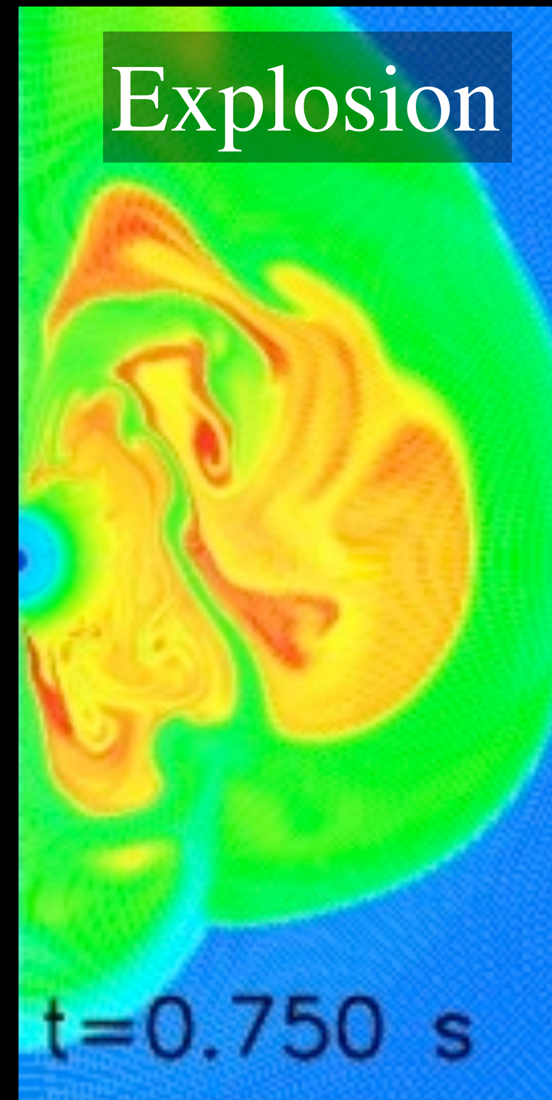
Spherical symmetry!
No GW emission?

Fundamental Question of Core-Collapse Theory

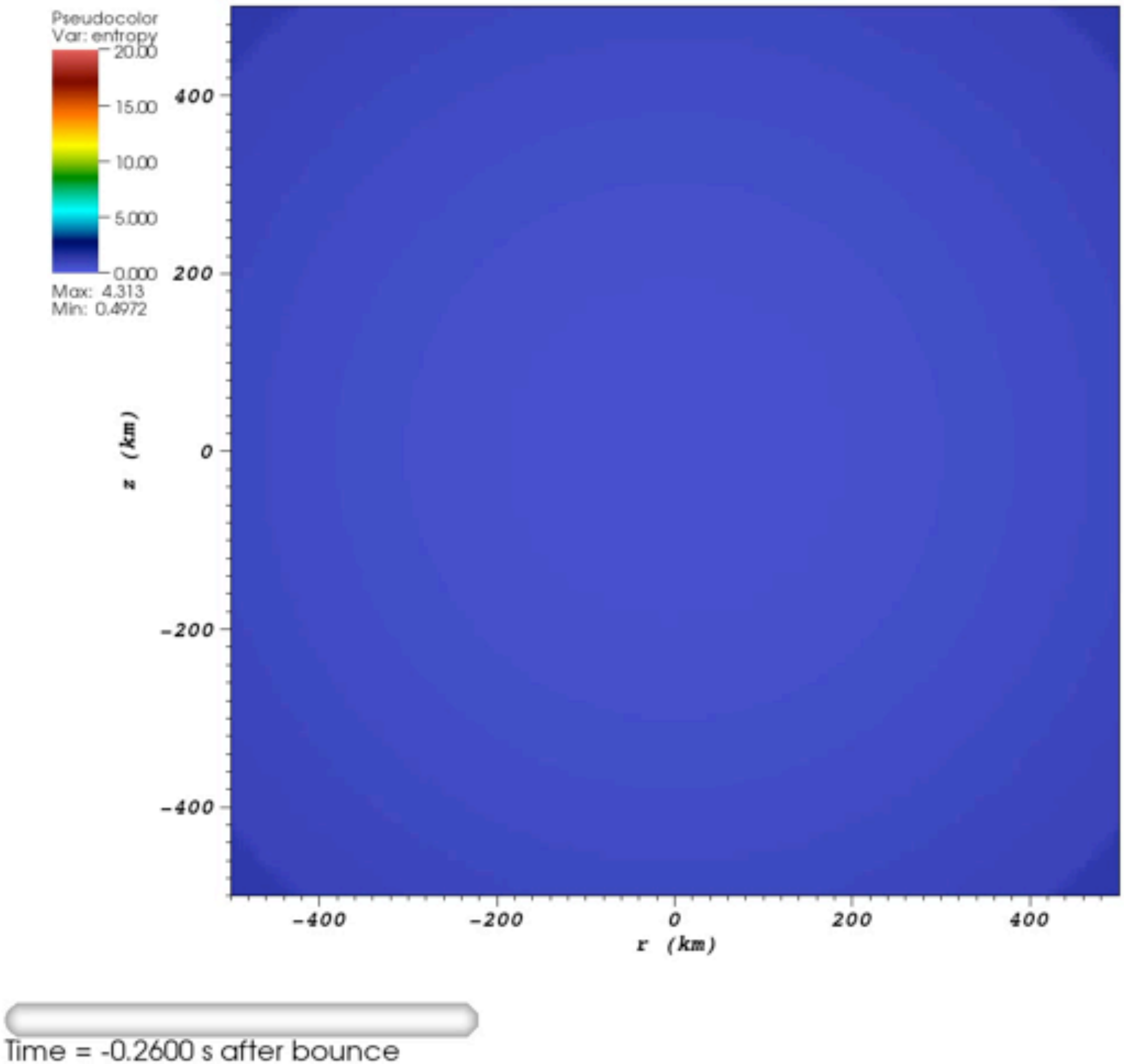


?

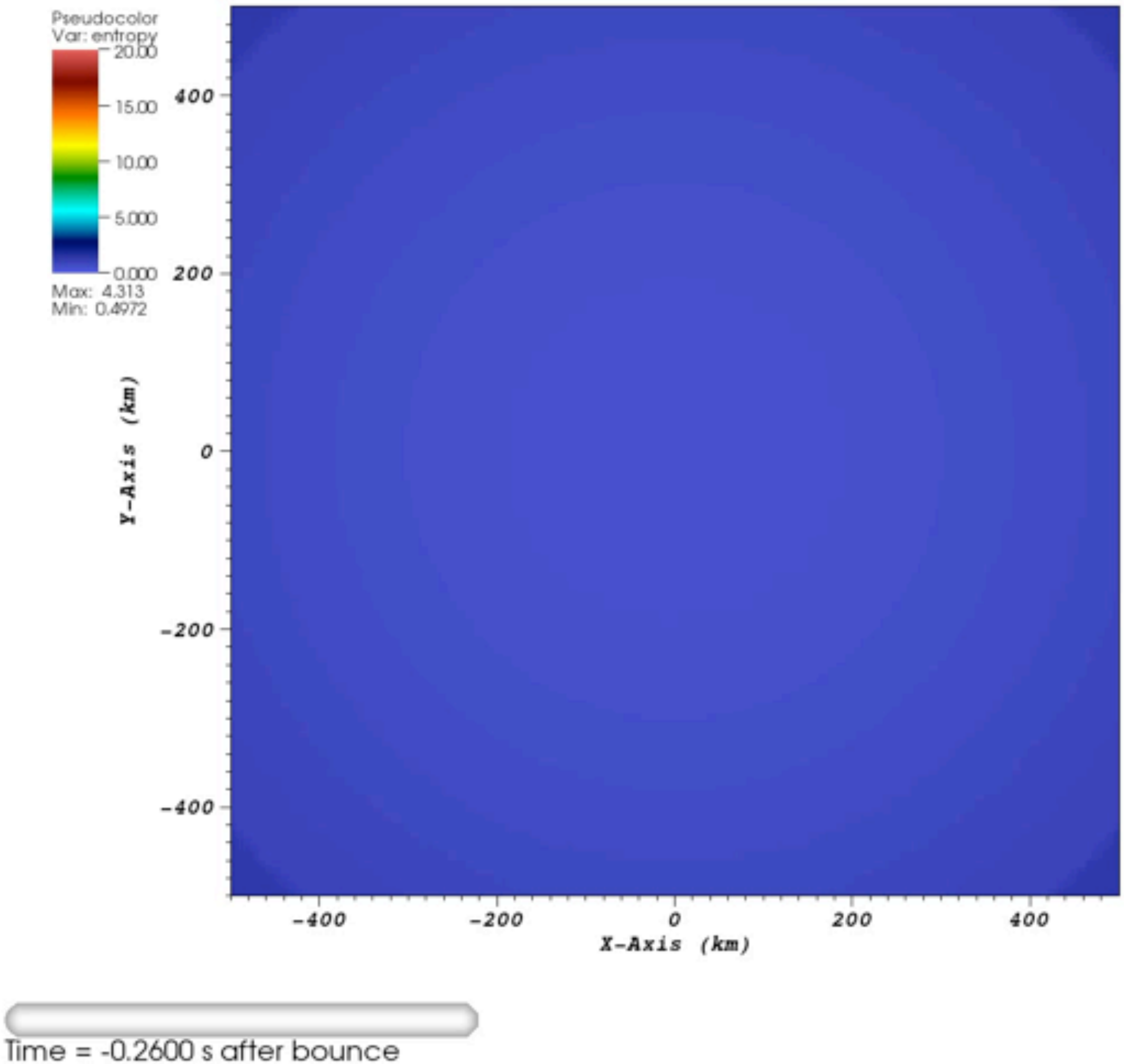
→



Relax 1D assumption?



Pseudocolor
Var: entropy
20.00
15.00
10.00
5.000
0.000
Max: 4.313
Min: 0.4972
400
200
0
-200
-400
z (km)
-400
-200
0
200
400
r (km)
Time = -0.2600 s after bounce


Pseudocolor
Var: entropy
20.00
15.00
10.00
5.000
0.000
Max: 4.313
Min: 0.4972
400
200
0
-200
-400
Y-Axis (km)
-400
-200
0
200
400
X-Axis (km)
Time = -0.2600 s after bounce

Neutrino Mechanism:

- Neutrino-heated convection
- Standing Accretion Shock Instability (SASI)
- Explosions? Maybe

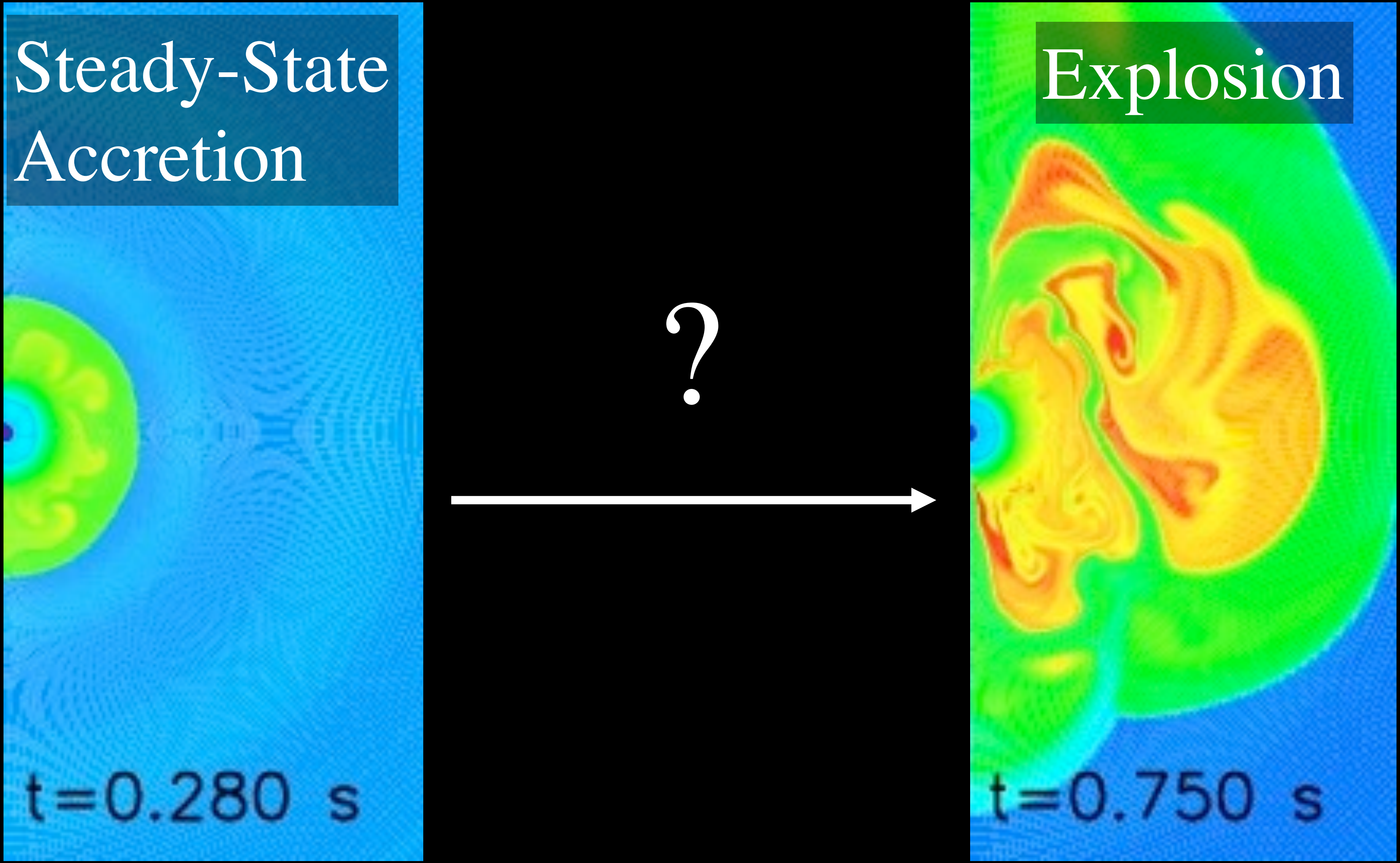
Acoustic Mechanism:

- Explosions but caveats.

Magnetic Jets:

- Only for very rapid rotations
- Collapsar?

Fundamental Question of Core-Collapse Theory



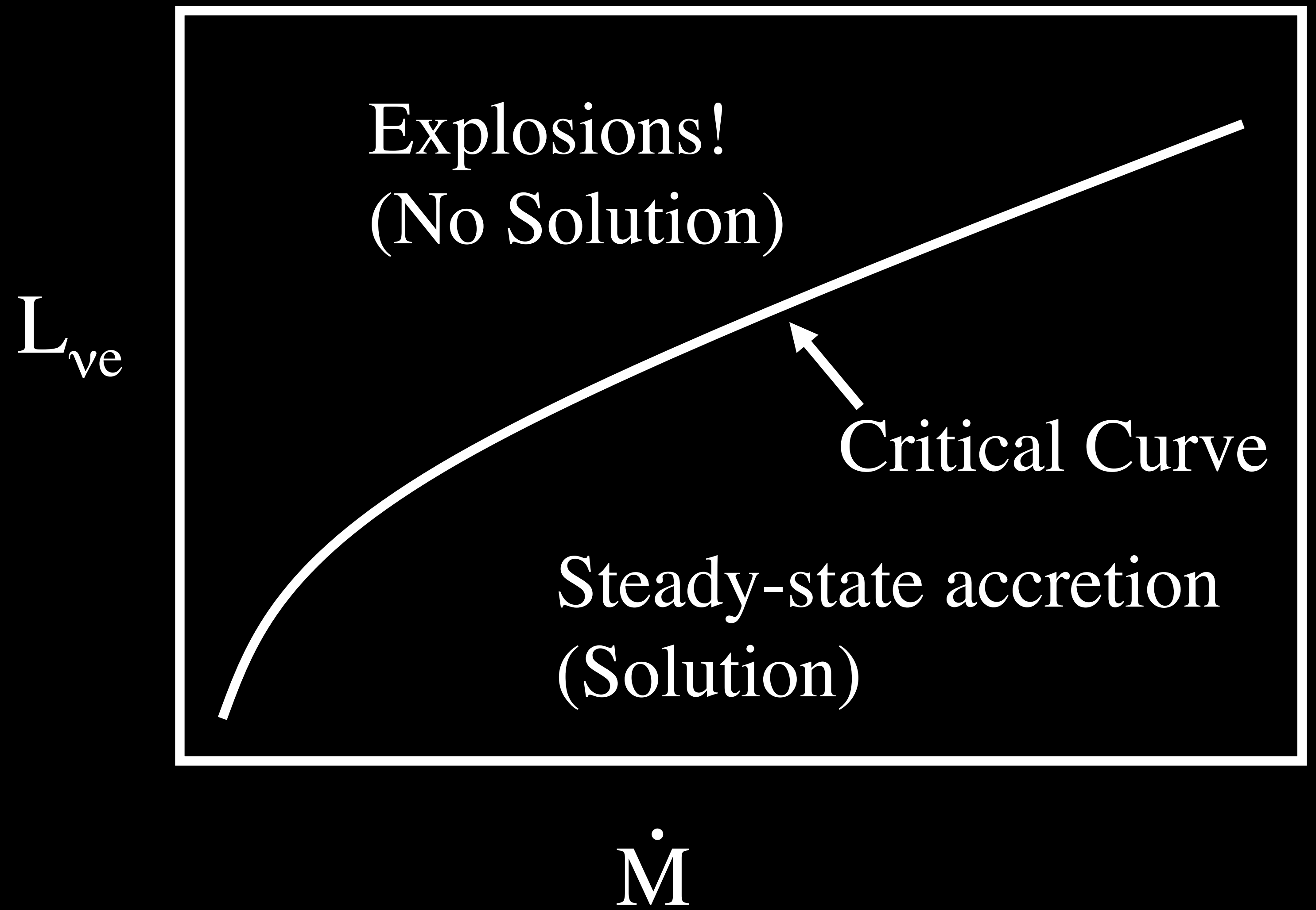
Why is it easier to explode in 2D compared to 1D?

Two Paths to the Solution

- Detailed 3D radiation-hydrodynamic simulations
  ("Accurate" energies, NS masses, nucleo., etc.)

- Parameterizations that capture essential physics
  (Tease out fundamental mechanisms)

Burrows & Goshy ‘93
Steady-state solution (ODE)



Explosion is a global, boundary-value problem

Explosion is a global, boundary-value problem

In other words:
What neutrino forcing is required to change the global structure between the NS and shock such that explosions occur?

Explosion is a global, boundary-value problem

This also means that one can’t easily and cleanly pick out one simple diagnostic

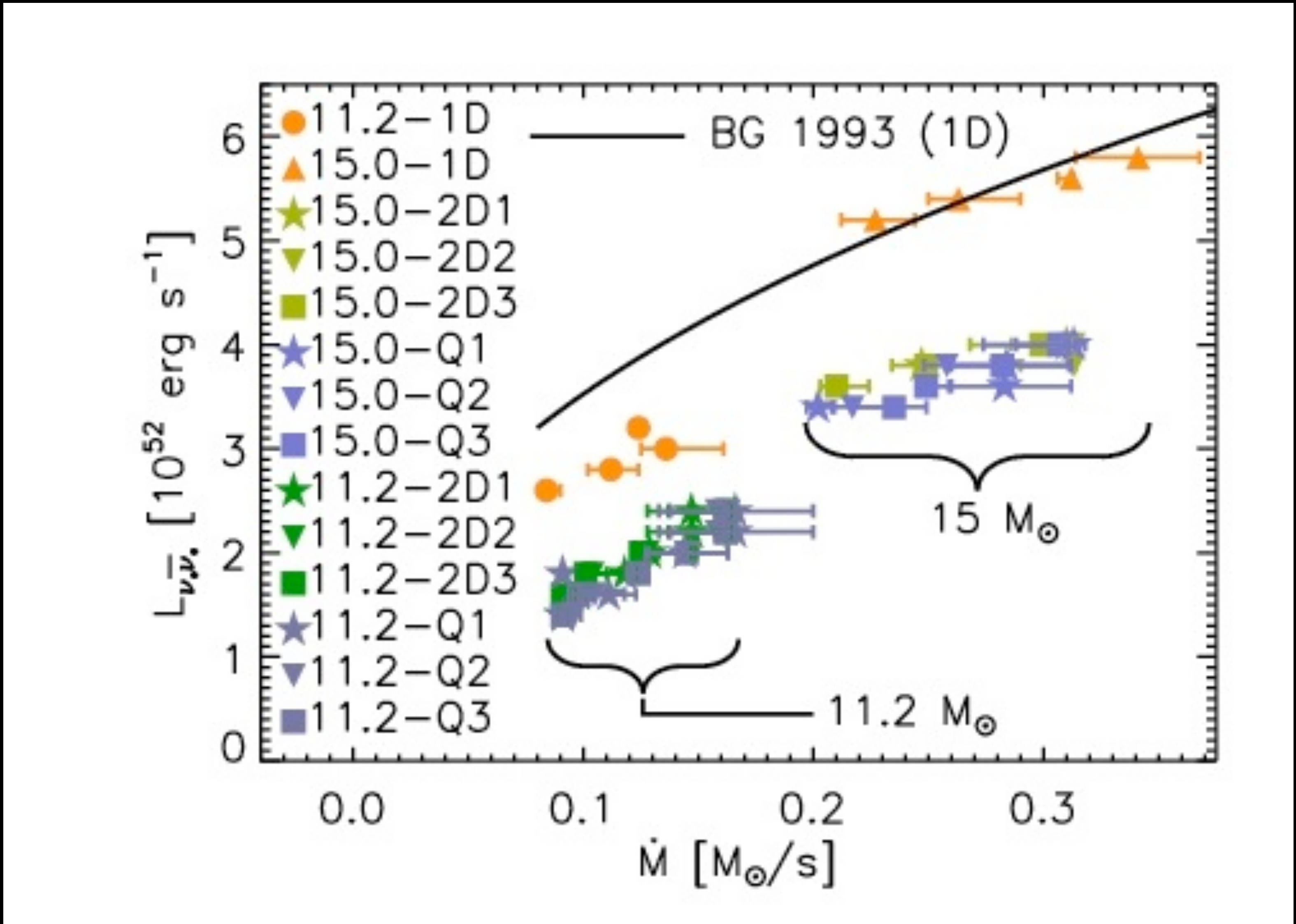
Hence... “Mazurek’s Law”

Is a critical luminosity relevant in hydrodynamic simulations?

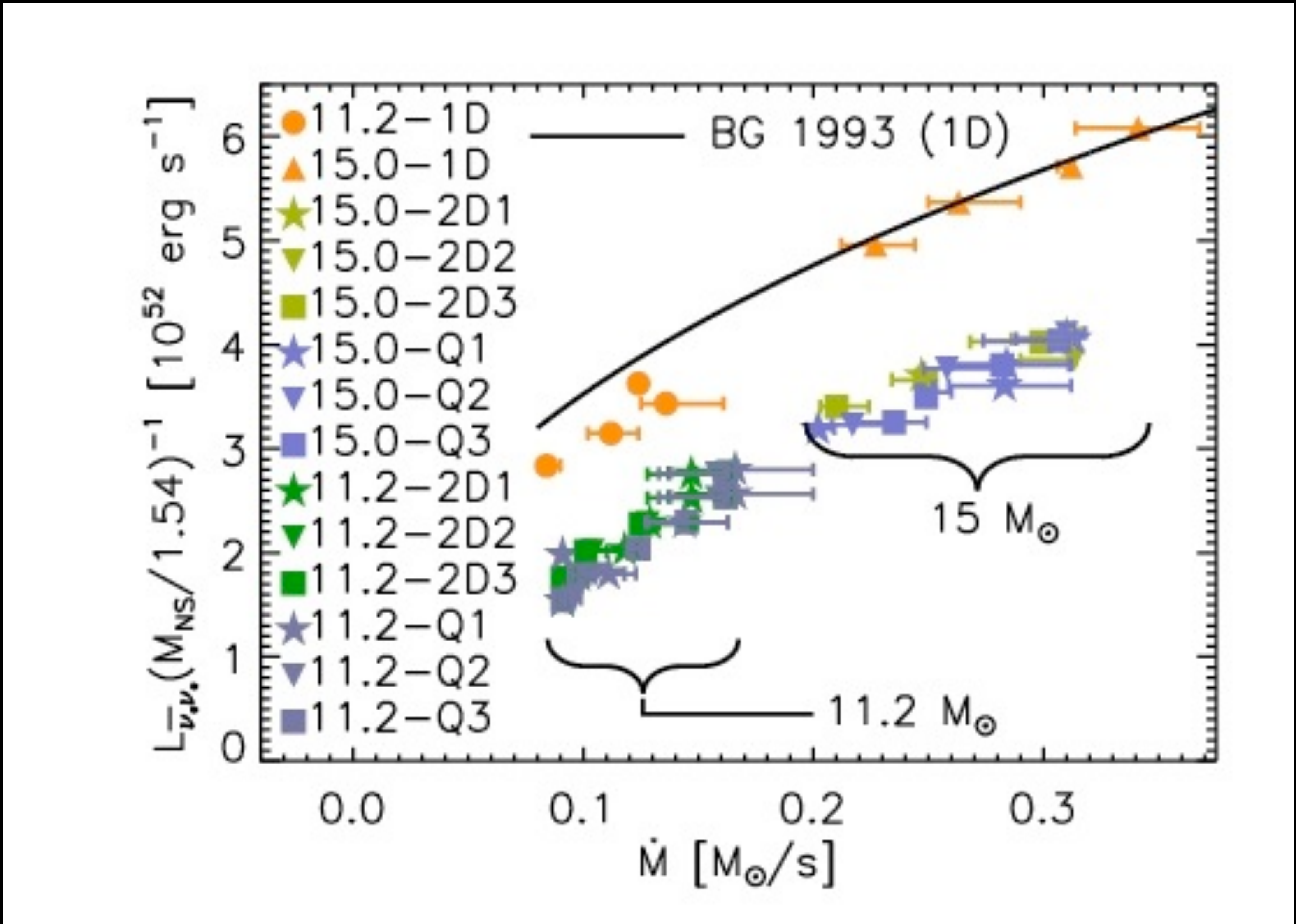
- 1D
- 2D Convection and SASI?

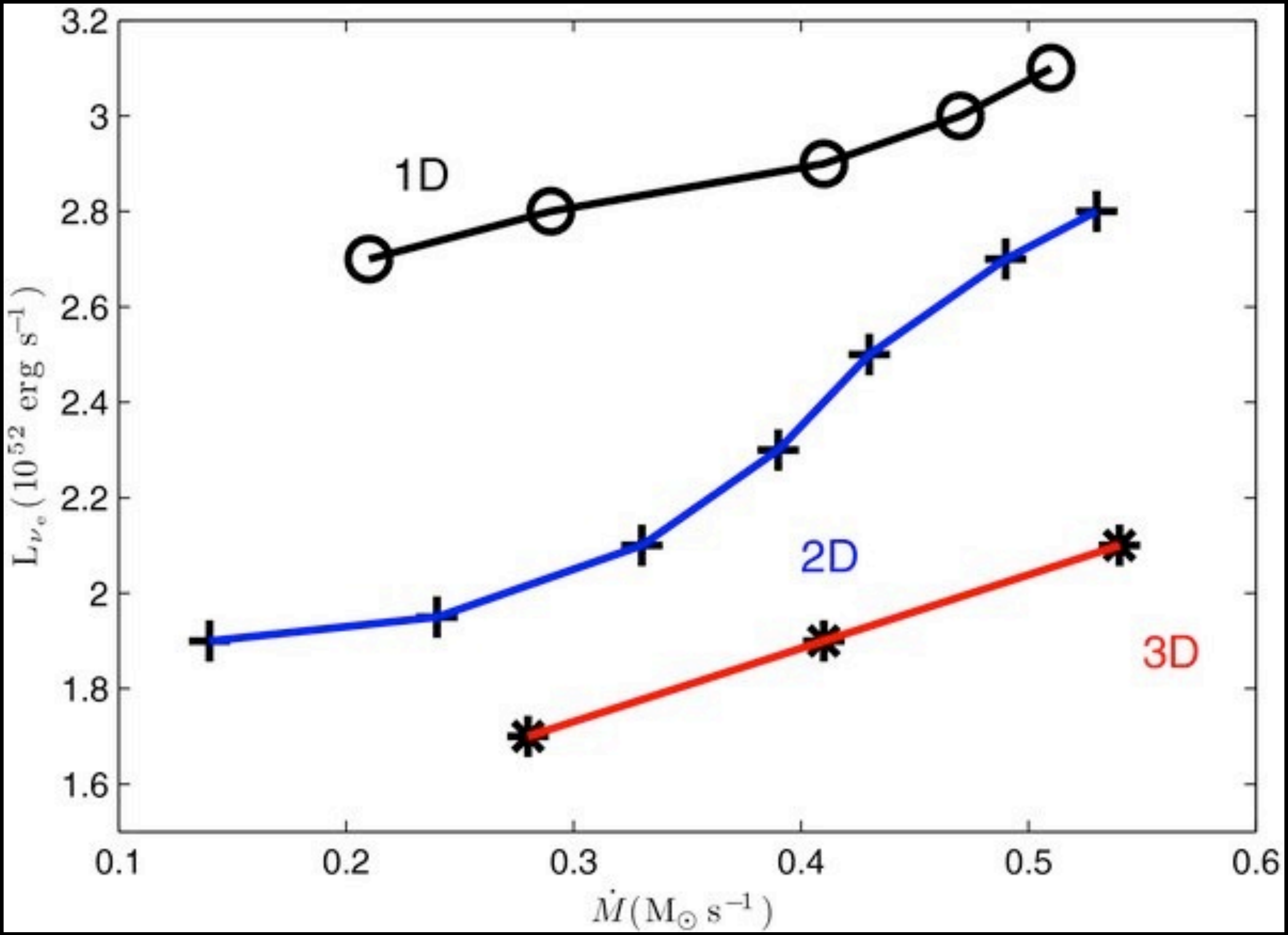
How do the critical luminosities differ between 1D and 2D?

Murphy & Burrows '08

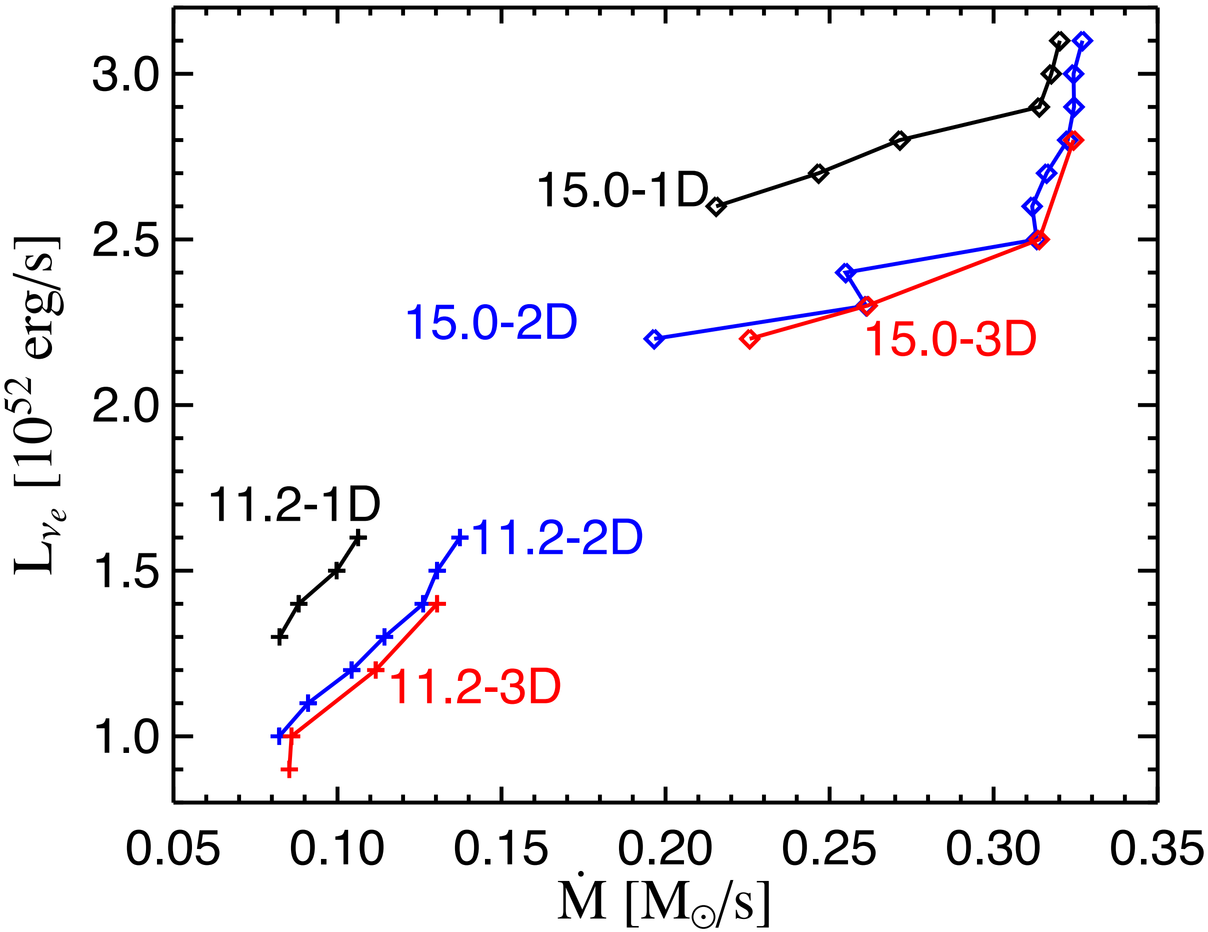


Murphy & Burrows '08

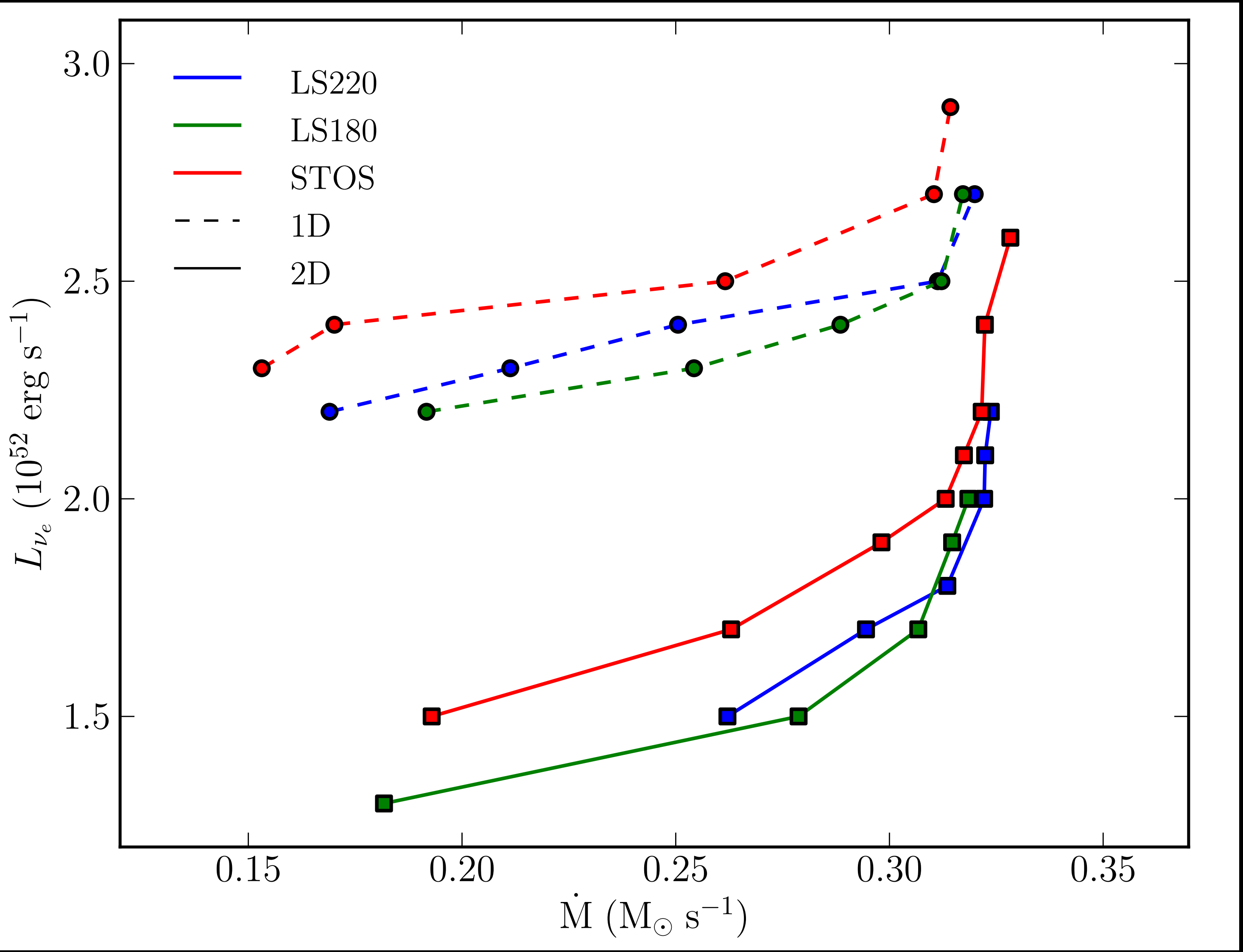




Nordhaus et al. 2010



Hanke et al 2011

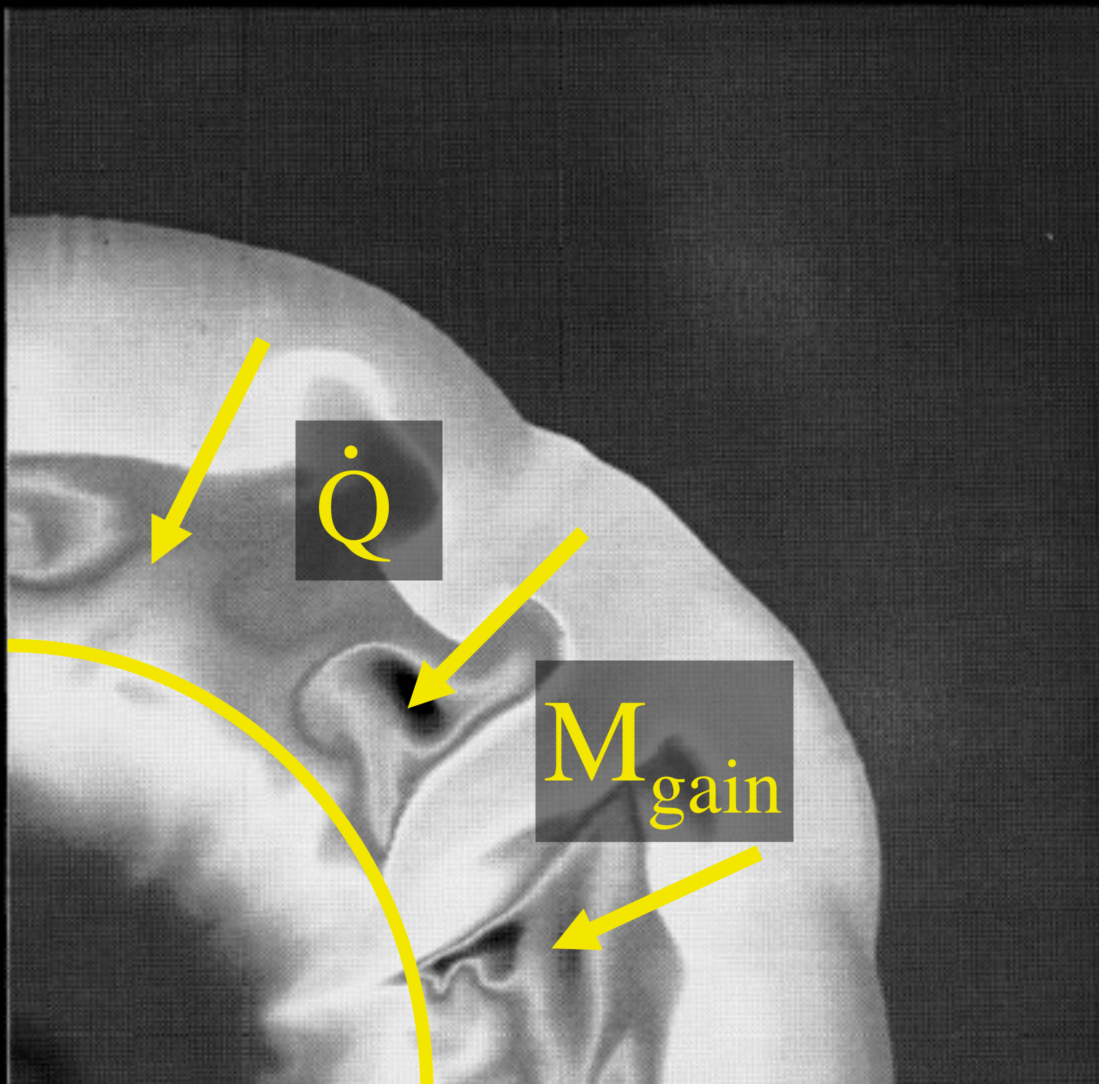

LS220
LS180
STOS
1D
2D
L_{ν_e} (10^{52} erg s^{-1})
3.0
2.5
2.0
1.5
0.15
0.20
0.25
0.30
0.35
$\dot{M}$ ($M_\odot$ s^{-1})

Couch 2012

Why is critical luminosity of multi-D simulations ~70% of 1D?

Comparison of Timescales

(Thompson et al. ‘00, Janka ’01, Thompson et al. ‘03, Murphy et al. ’08, Pejcha ’11, Fernandez ’12)



$$\tau_{adv} = \frac{\Delta r_{gain}}{v_r}$$

$$\tau_q = \frac{E}{\dot{Q}}$$

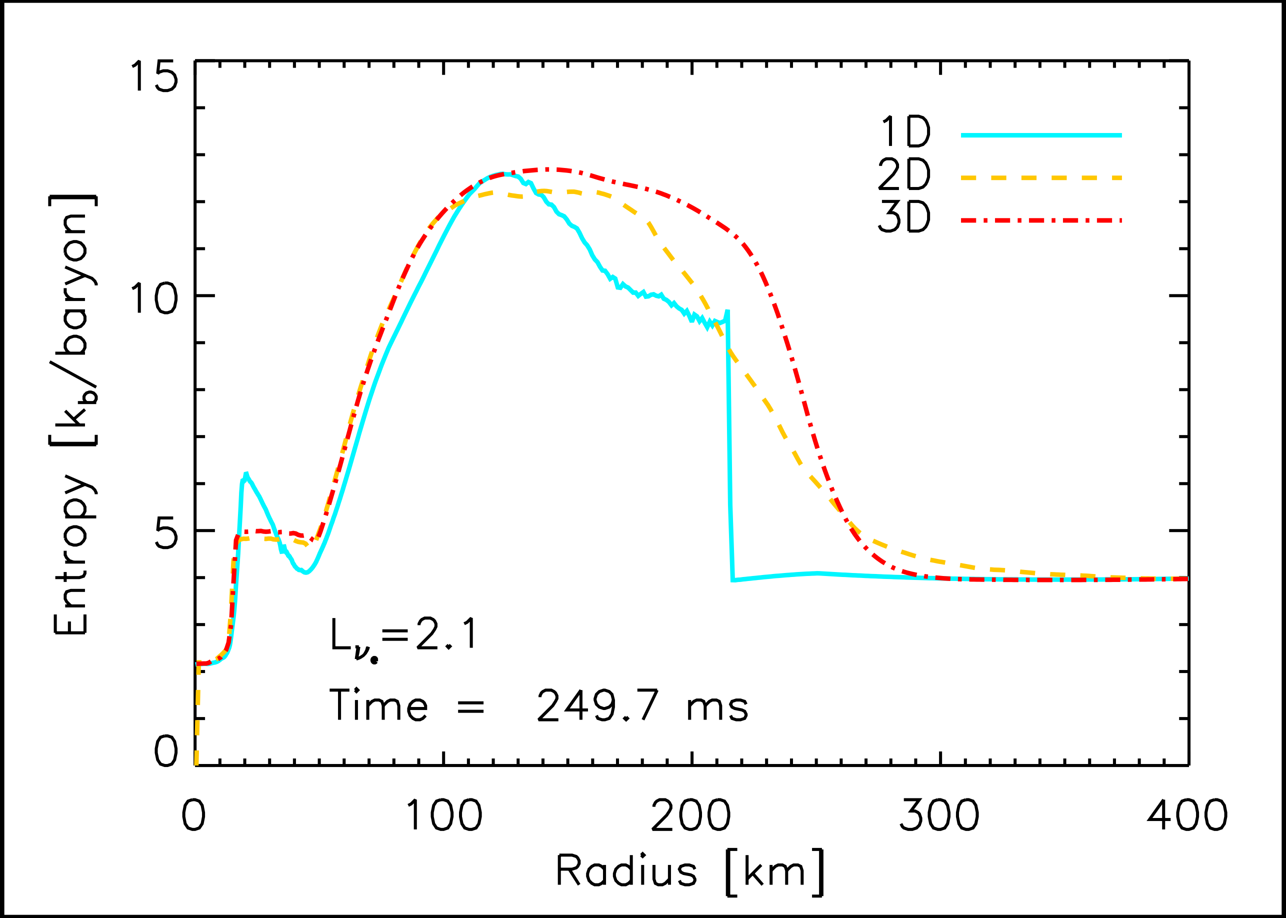
$$\frac{\tau_{adv}}{\tau_q} \gtrsim 1$$

1D → one time

mulit-D → distribution of times

More heating?

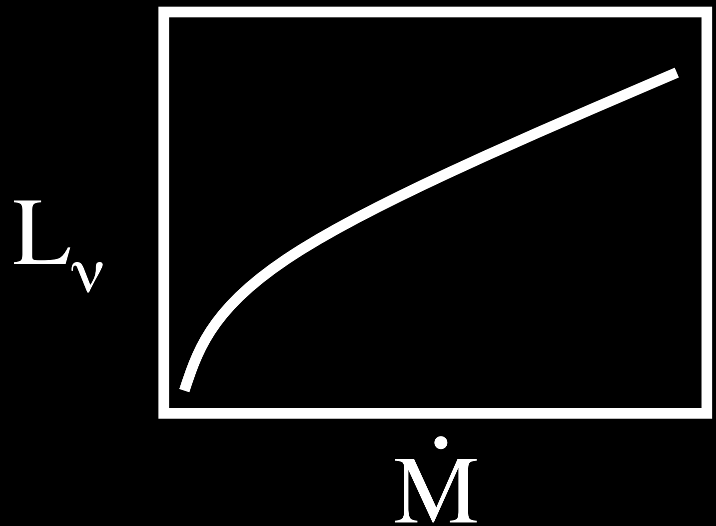
$$\Delta S \propto \frac{\dot{Q}}{T}$$



2D & 3D critical luminosity lower than 1D

Turbulence plays an important role!

A Theoretical Framework for Successful Explosions

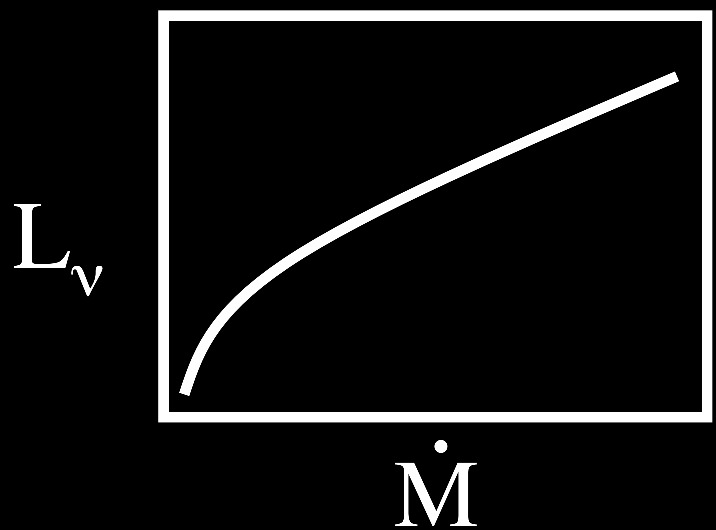


+

Turbulence
Model

Murphy & Meakin 2011

A Theoretical Framework for Successful Explosions



+

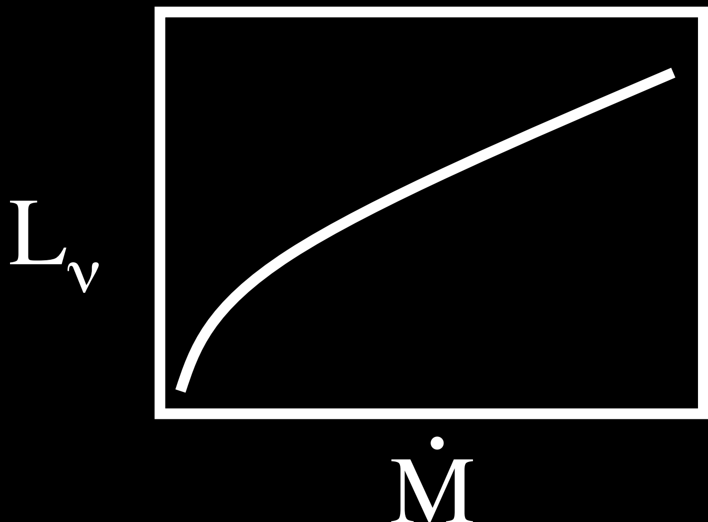
Turbulence Model

↕

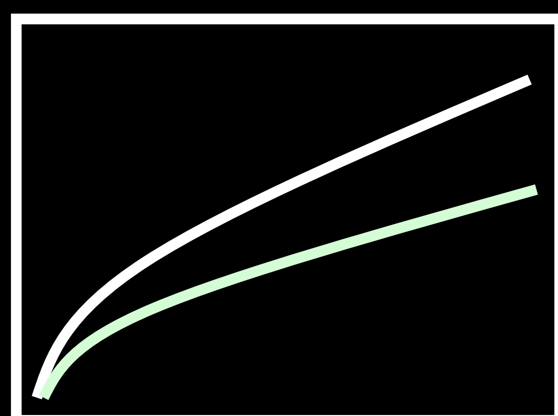
Calibrate with 3D Simulations

Murphy et al. 2012, in prep

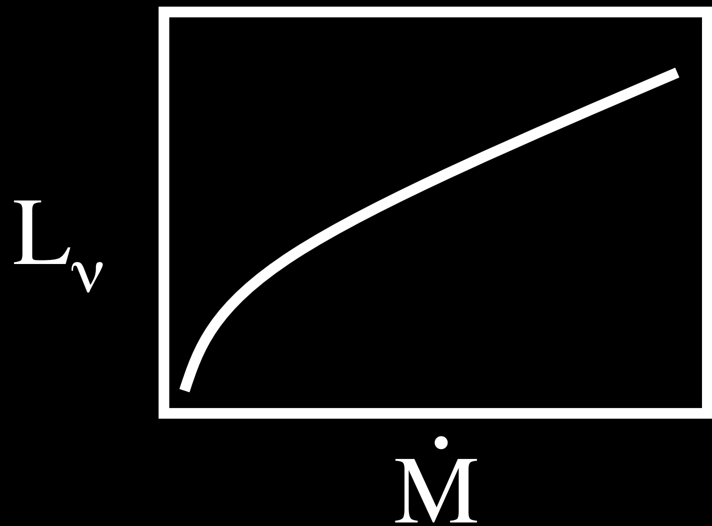
A Theoretical Framework for Successful Explosions



+ Turbulence Model



A Theoretical Framework for Successful Explosions



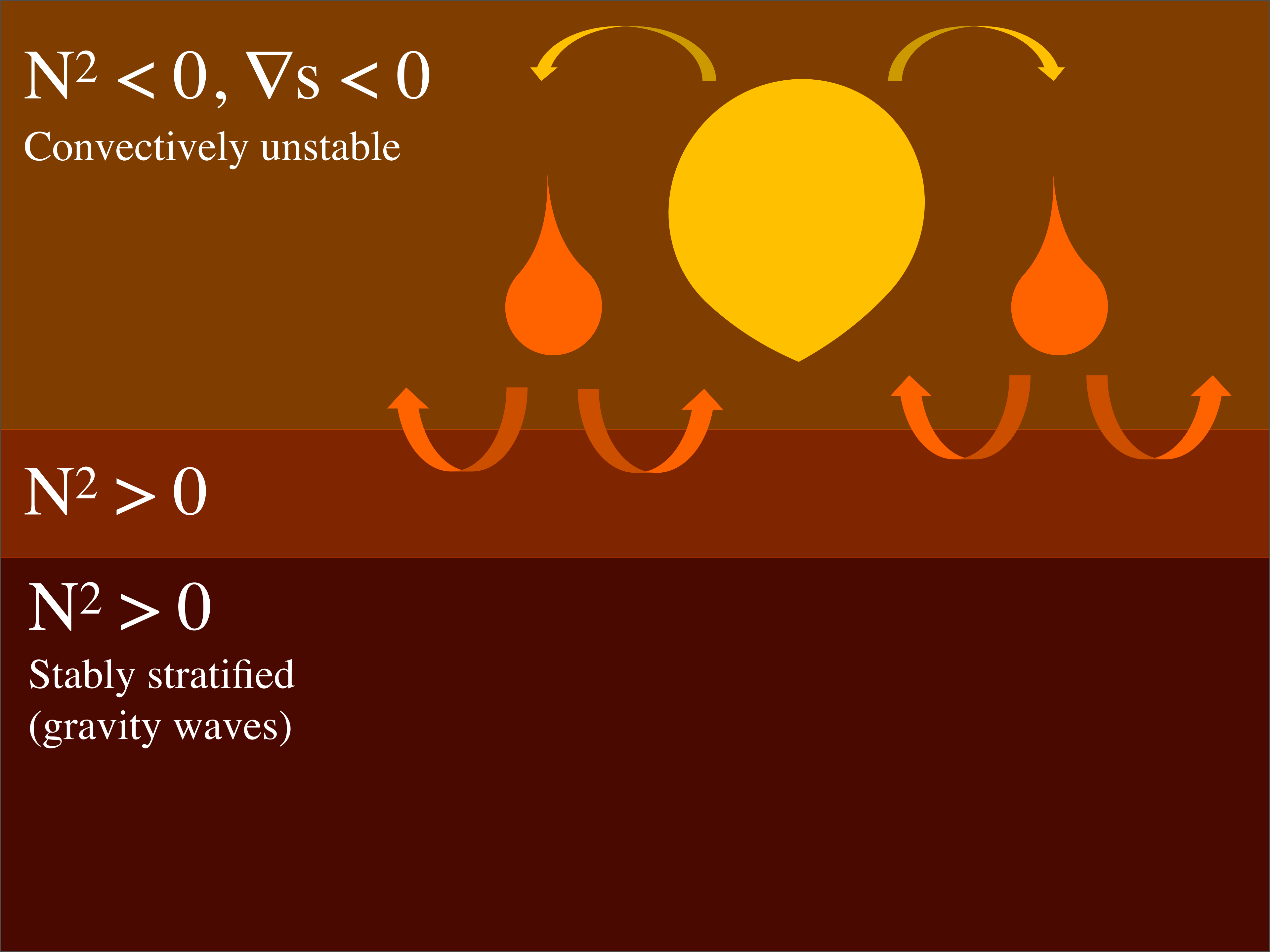
\+ Turbulence Model

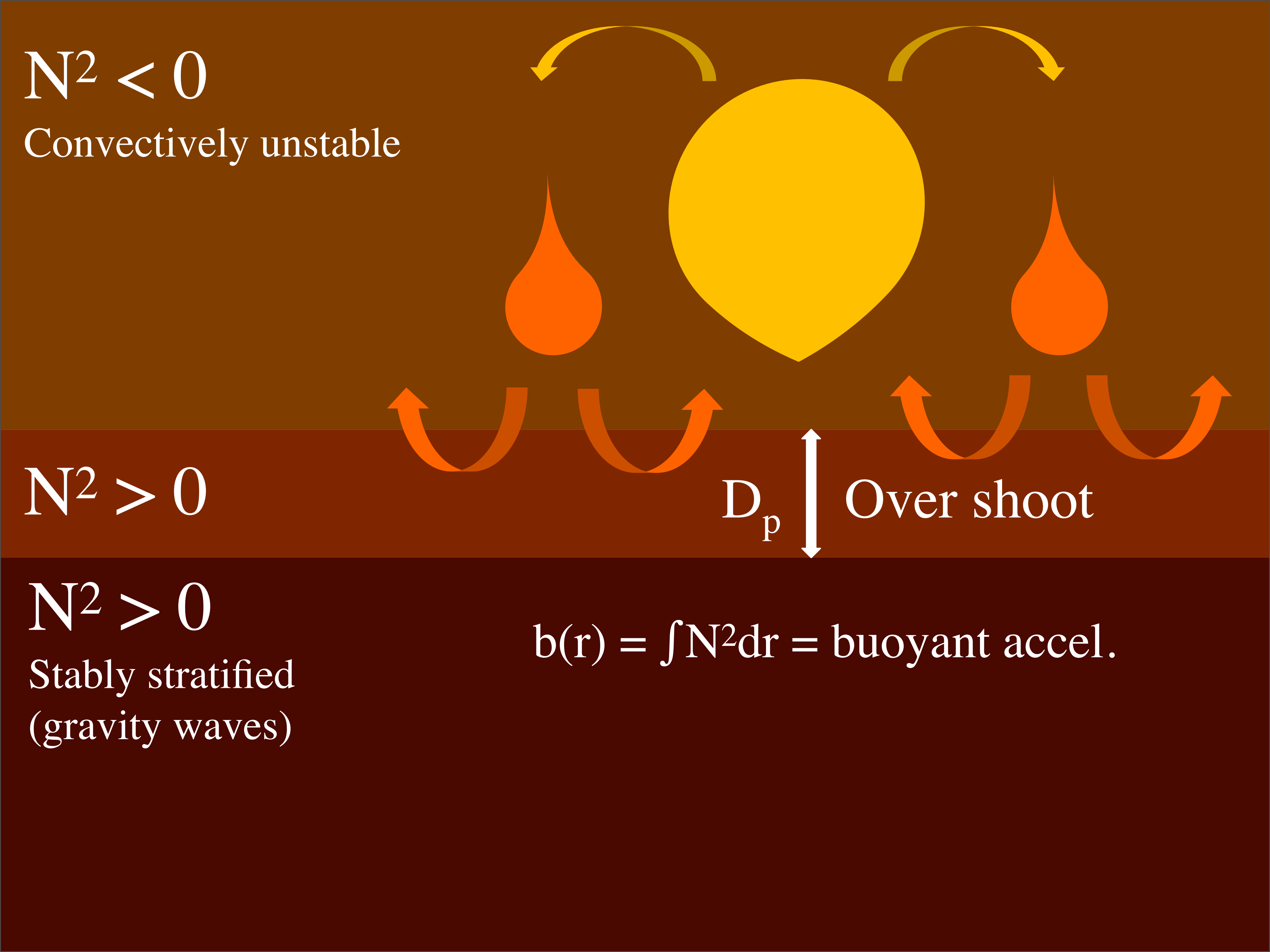
↓

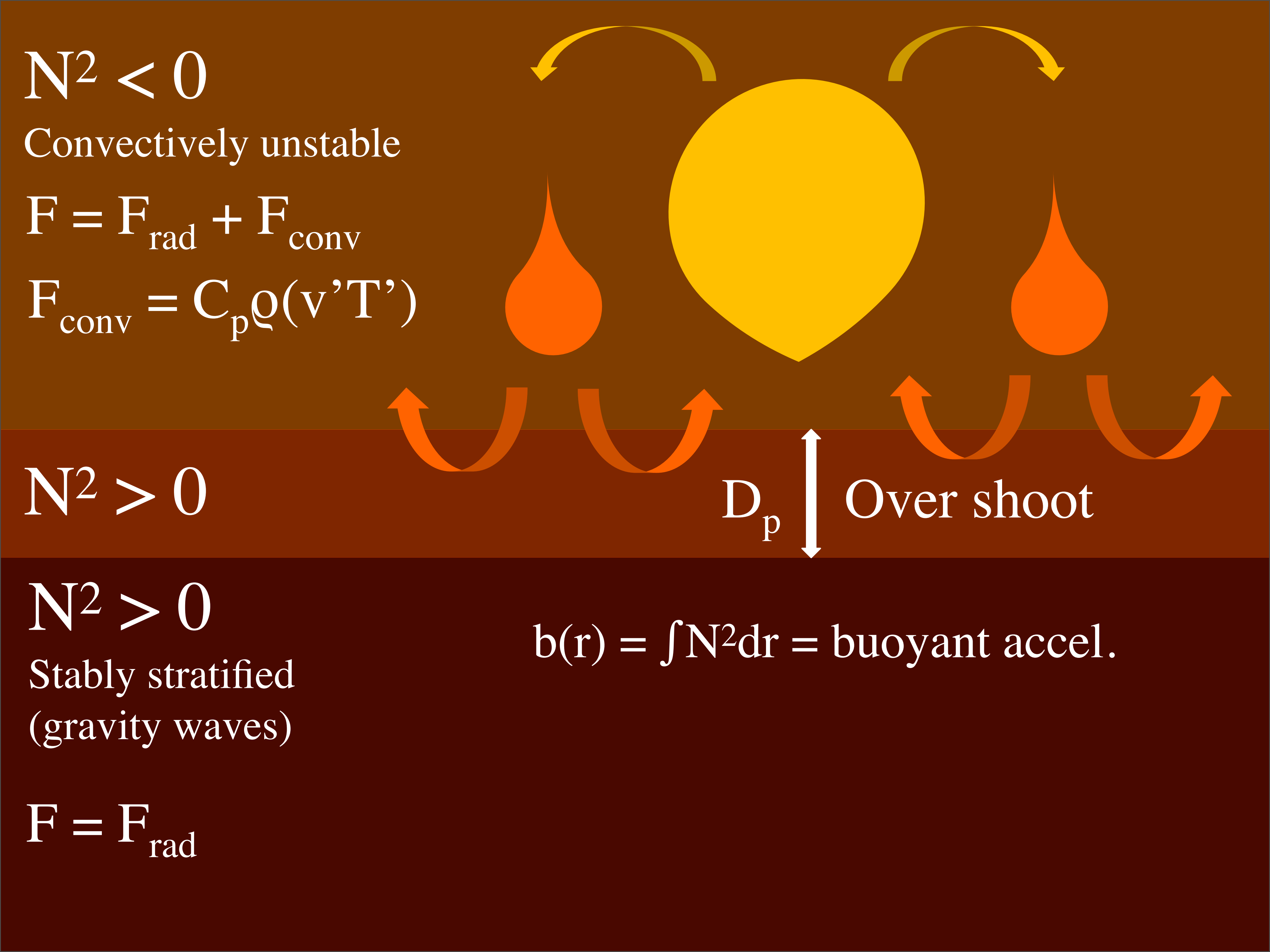
1D Rad-hydro simulations

Realistic and quantitative explosions

Systematic exploration

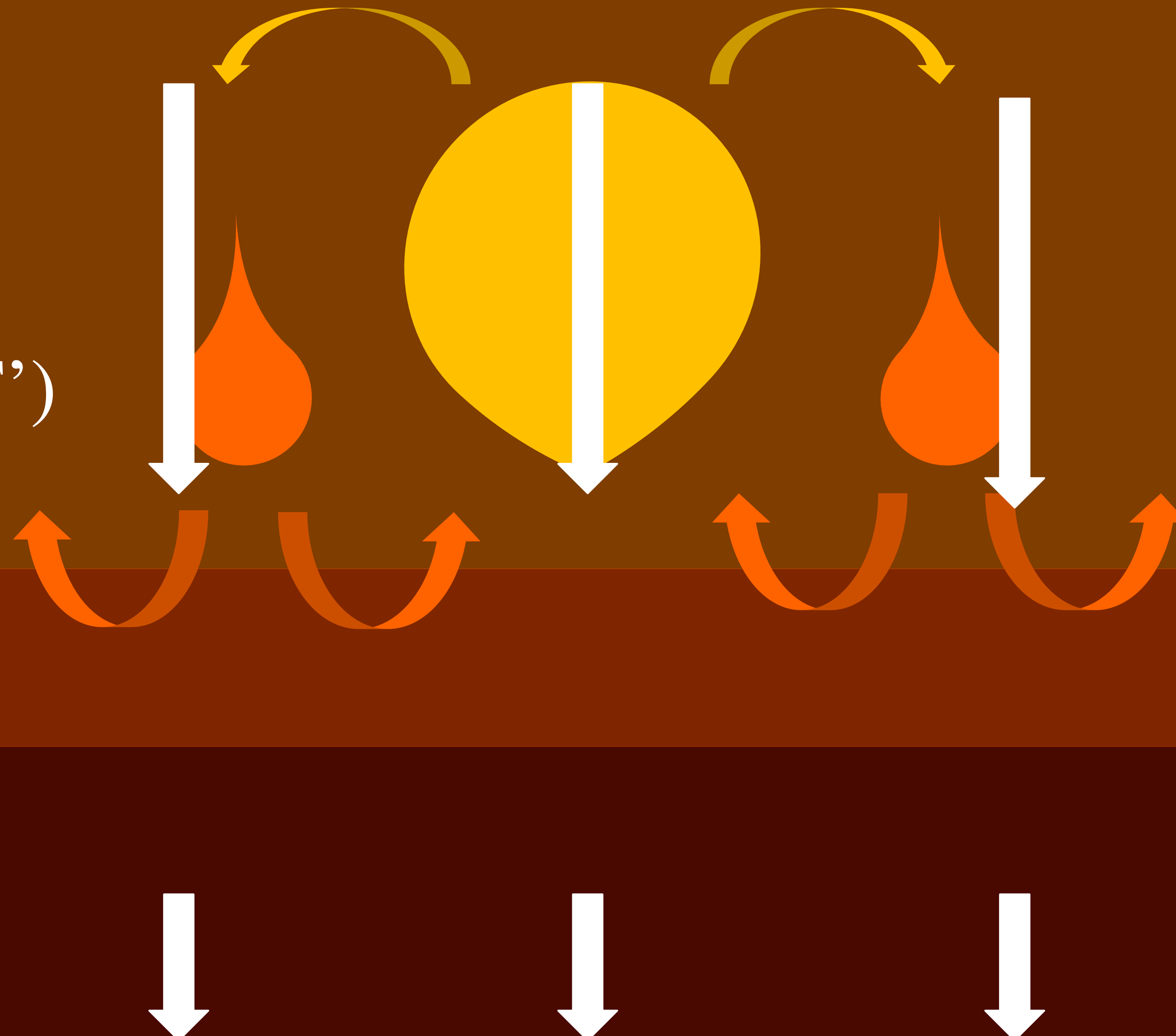

N² < 0, ∇s < 0
Convectively unstable
N² > 0
N² > 0
Stably stratified
(gravity waves)


$N^2 < 0$
Convectively unstable
$N^2 > 0$
D_p
Over shoot
$N^2 > 0$
Stably stratified
(gravity waves)
$b(r) = \int N^2 dr$ = buoyant accel.


$N^2 < 0$
Convectively unstable
$F = F_{rad} + F_{conv}$
$F_{conv} = C_p \varrho (v'T')$
$N^2 > 0$
D_p
Over shoot
$N^2 > 0$
Stably stratified
(gravity waves)
$F = F_{rad}$
$b(r) = \int N^2 dr$ = buoyant accel.

$F = F_{rad} + F_{conv}$

$F_{conv} = C_p \rho (v'T')$



Need a More General Turbulence Model (Reynolds Decomposition)

Back to the Beginning

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\partial_t (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla P + \rho \mathbf{g} + \nabla \cdot \sigma$$

$$\partial_t (\rho s) + \nabla \cdot (\rho \mathbf{v} s) = \frac{\dot{Q} + \varepsilon}{T}$$

$$P = P(\rho, s, X_i)$$

Reynolds Decomposition

$\phi = \phi_0 + \phi'$ $\qquad$ $\phi_0 = \langle \phi \rangle$

↓

Hydro Equations

↓

Mean-Field Equations

↓

New steady-state solutions & Critical Curve

Reynolds-Averaged Equations

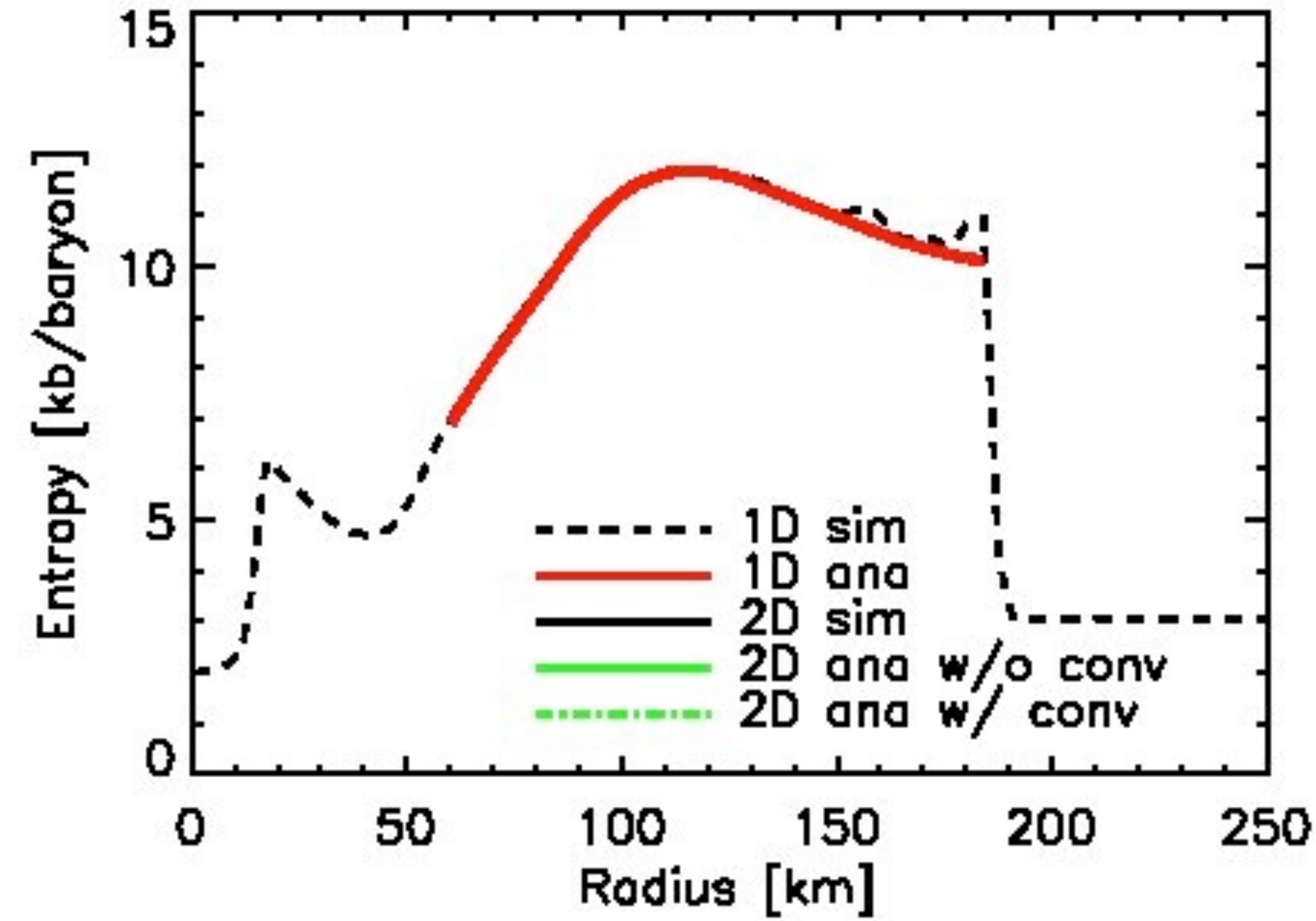
$$\dot{M} = 4\pi r^2 \left(\rho_0 v_0 + \langle \rho' v' \rangle\right)$$

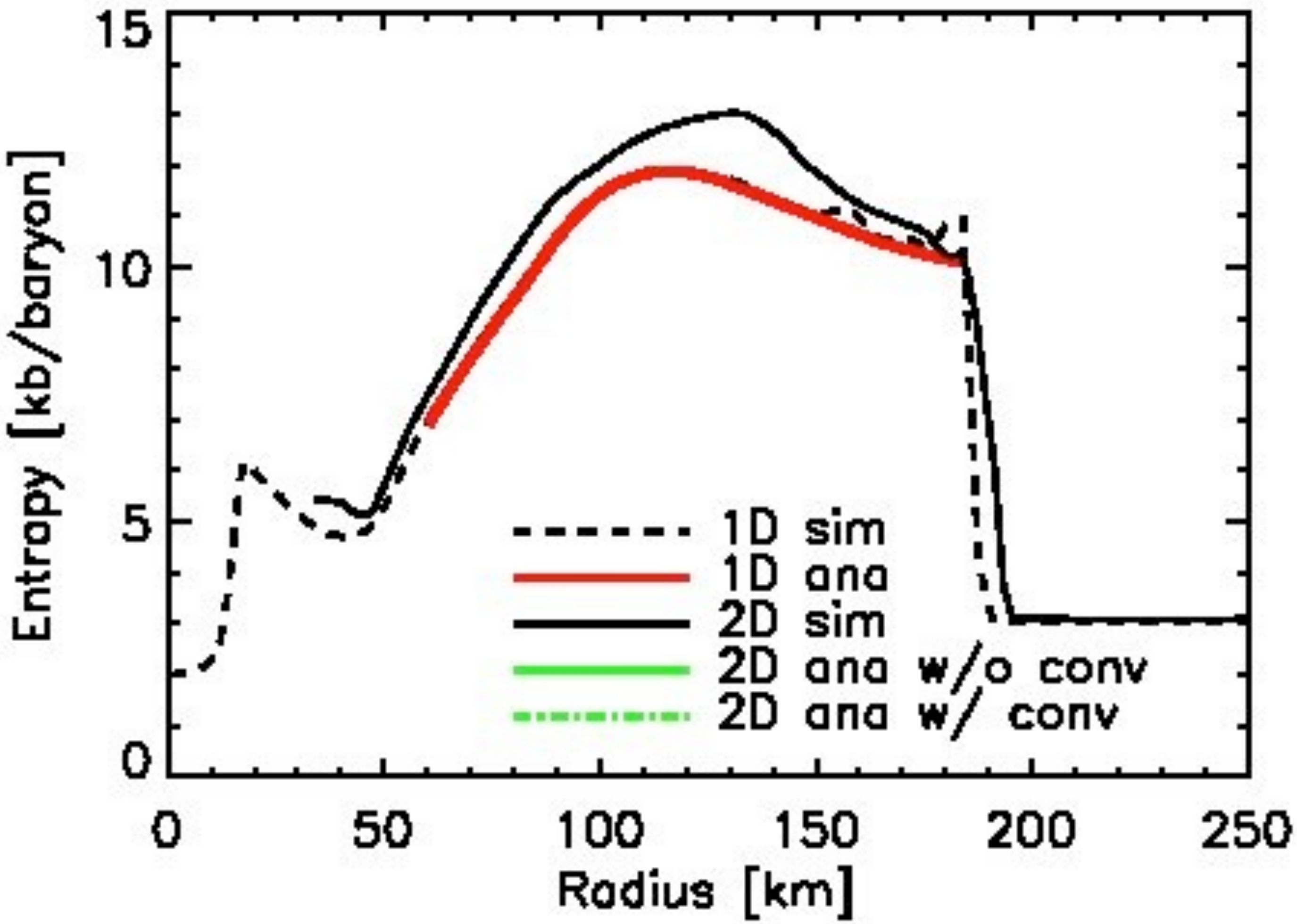
$$\langle \rho \mathbf{v} \rangle \cdot \nabla \mathbf{v}_0 = -\nabla P_0 + \rho_0 g - \nabla \cdot \langle R \rangle$$

$$\langle \rho \mathbf{v} \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle \mathbf{F}_s \rangle$$

Murphy & Meakin 2011

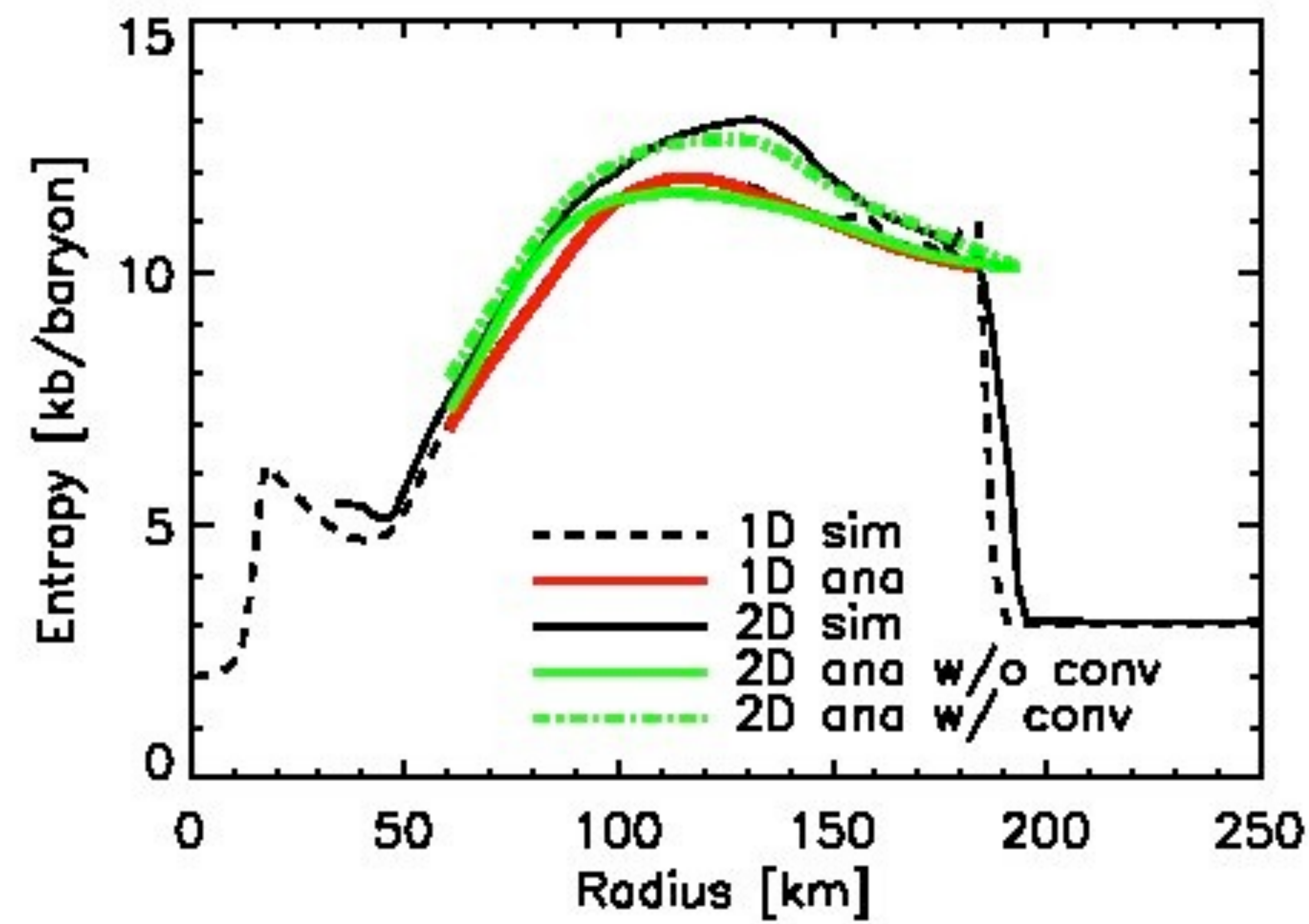
$$\rho \mathbf{v} \cdot \nabla s = \frac{\dot{Q}}{T}$$





Entropy [kb/baryon]
Radius [km]
0
50
100
150
200
250
0
5
10
15
1D sim
1D ana
2D sim
2D ana w/o conv
2D ana w/ conv

$$\langle \rho \mathbf{v} \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle \mathbf{F}_s \rangle$$



Reynolds-Averaged Equations

$$\dot{M} = 4\pi r^2 \left(\rho_0 v_0 + \langle \rho' v' \rangle\right)$$

$$\langle \rho \mathbf{v} \rangle \cdot \nabla \mathbf{v}_0 = -\nabla P_0 + \rho_0 g - \nabla \cdot \langle R \rangle$$

$$\langle \rho \mathbf{v} \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle \mathbf{F}_s \rangle$$

Murphy & Meakin 2011

Turbulent Moment Equations

Equations for 2nd order moments

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \left\langle \rho' v' \right\rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

and more ...

Turbulent Moment Equations

Equations for 2nd order moments

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle)$$

$$\approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

$$\varepsilon \sim \frac{{v'}^3}{L} \qquad F_K = \rho K v'$$

Depends upon higher order moments

Turbulent Moment Equations

Equations for 2nd order moments

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \left\langle \rho' v' \right\rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

A Closure Problem!

Closure Strategies

Local Algebraic | Local Single-Point | Global

Closure Strategies

Local Algebraic | Local Single-Point | Global

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

Closure Strategies

Local Algebraic | Local Single-Point | Global

$$\sout{\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle)}$$

$$\approx \langle \rho' v' \rangle g - \varepsilon - \sout{\nabla \cdot \langle F_K \rangle}$$

$$\langle \rho' v' \rangle g \approx \varepsilon \sim \frac{\rho v'^3}{L}$$

MLT is a classic example

Closure Strategies

Local Algebraic | Local Single-Point | Global

$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

Local models for these

Closure Strategies

Local Algebraic | Local Single-Point | Global

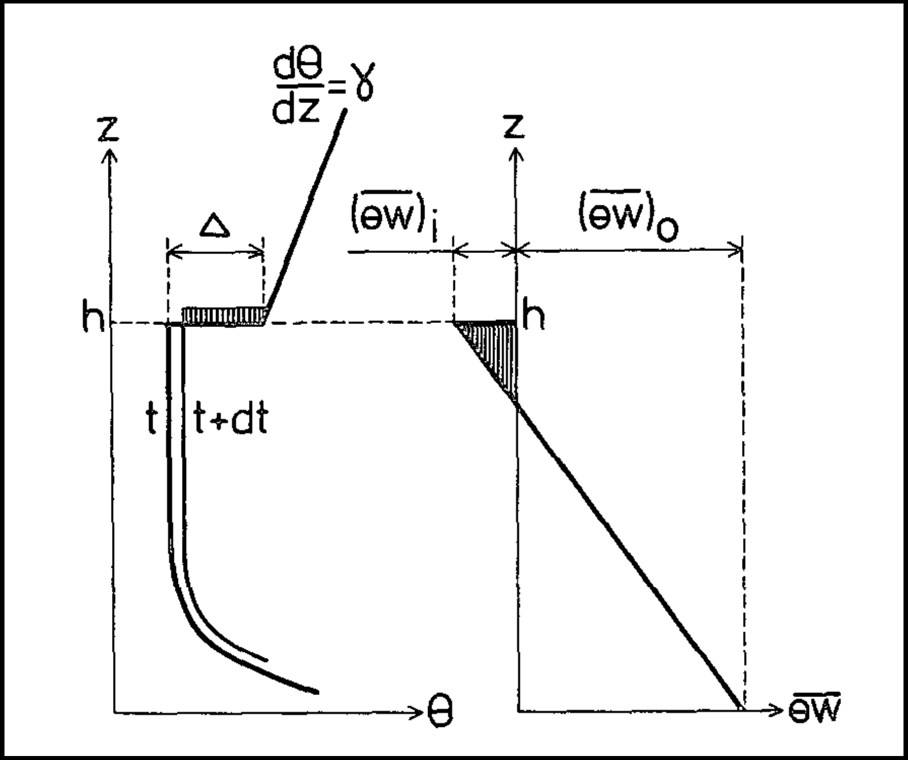
$$\partial_t \langle \rho K \rangle + \nabla \cdot (v_0 \langle \rho K \rangle) \approx \langle \rho' v' \rangle g - \varepsilon - \nabla \cdot \langle F_K \rangle$$

$$\int \langle \rho' v' \rangle g \, dV = \int \varepsilon \, dV$$

Global Closure Examples

Earth's Atmospheric Convective Layer

$$\nabla s_0 = 0 \Longrightarrow \rho_0 \langle \dot{s} \rangle = -\partial_z \langle F_s \rangle$$

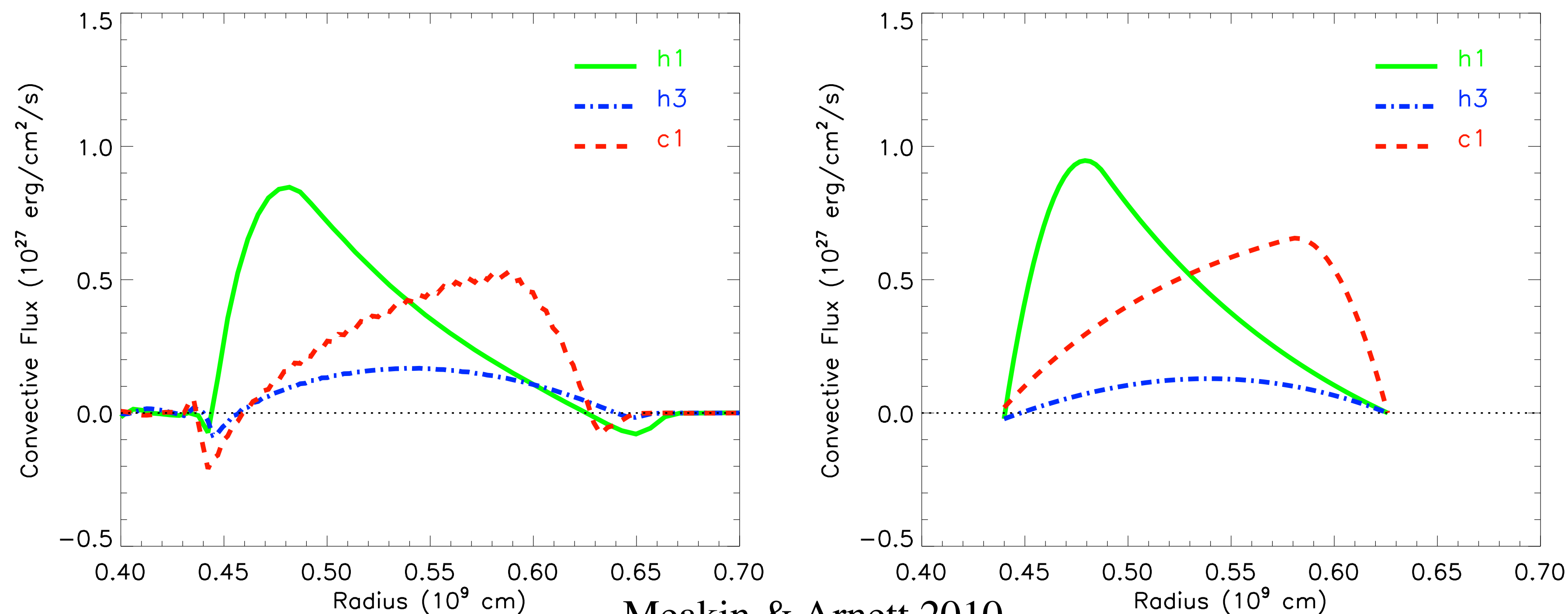


Tennekes 1973

Global Closure Examples

Stellar Convection

$$\nabla s_0 = 0 \Rightarrow \rho_0 \langle \dot{s} \rangle = -\nabla \cdot \langle F_s \rangle + \frac{\dot{Q} + \varepsilon}{T}$$



Meakin & Arnett 2010

Global Closure For CCSNe

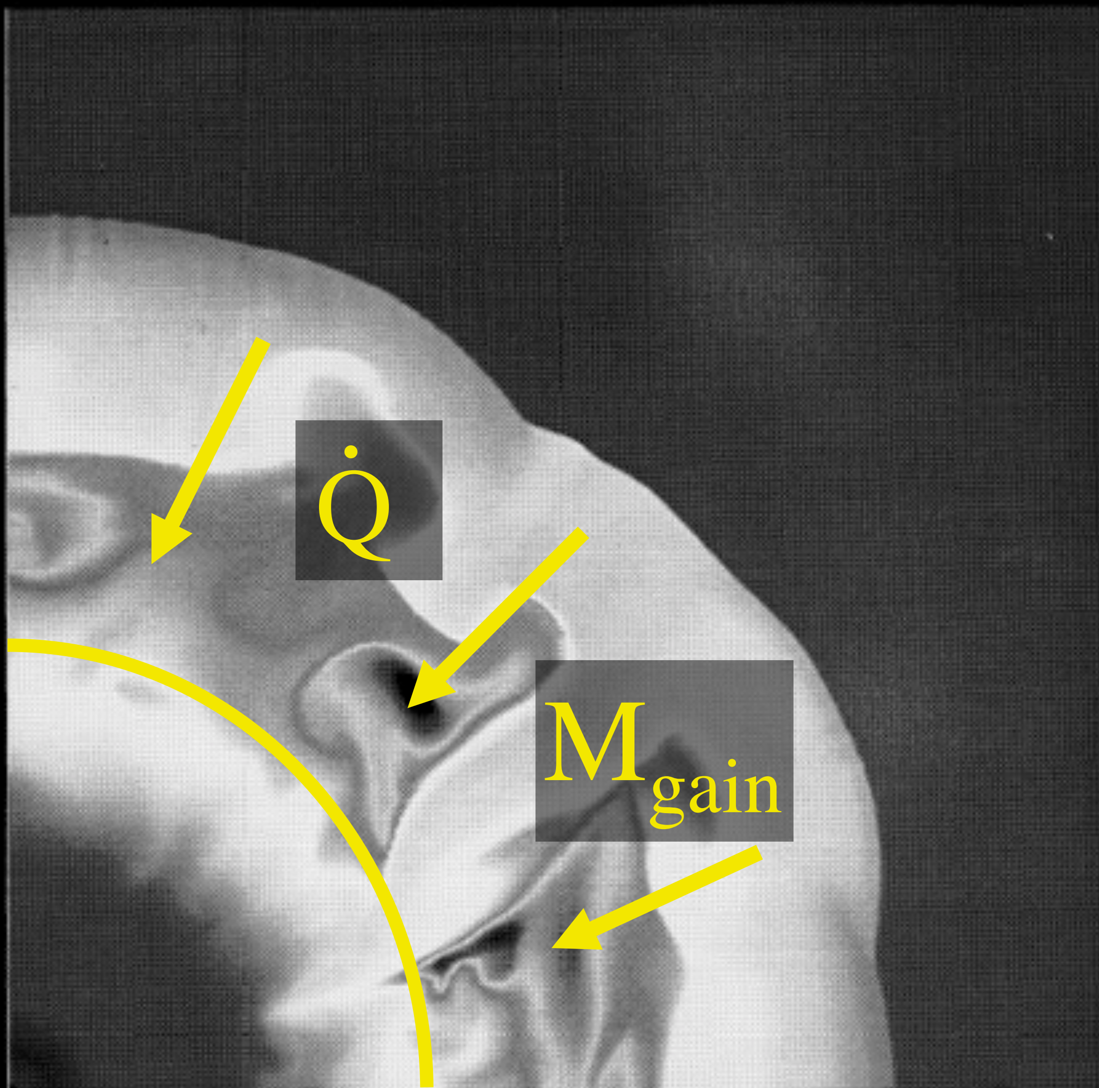
$$\langle \rho \mathbf{v} \rangle \cdot \nabla s_0 = \frac{\dot{Q} + \varepsilon}{T} - \nabla \cdot \langle \mathbf{F}_s \rangle$$

Use

$$\int \langle \rho' v' \rangle \, g \, dV = \int \varepsilon \, dV$$

And simulations to inform assumptions about profiles

Comparison of Timescales



$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \gtrsim 1$$

Heuristic & Emperical

See...

Thompson et al. ’00

Janka ’01

Thompson et al. ’03

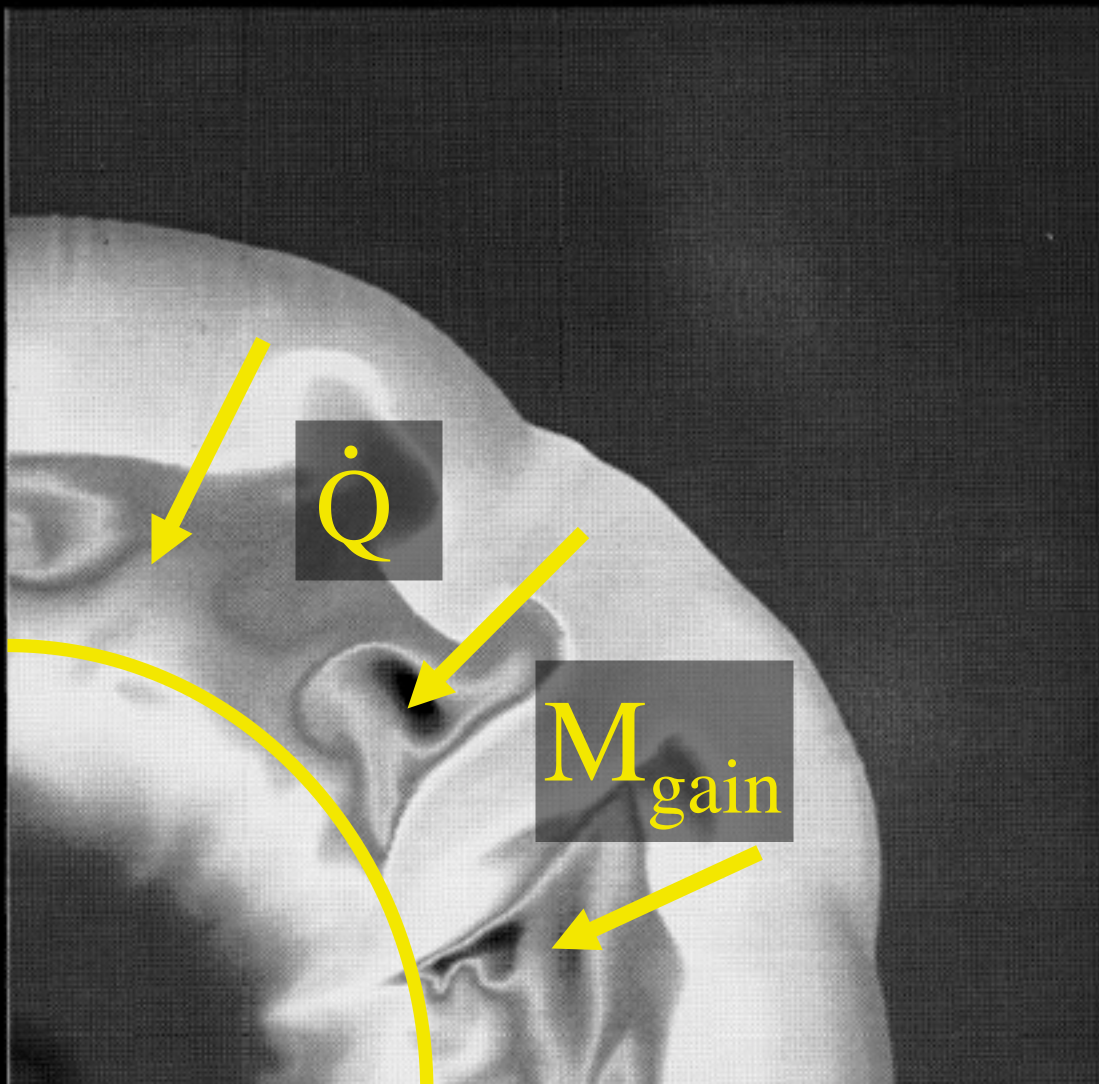
Murphy et al. ’08

Buras ’06

Pejcha ’11

Fernandez ’12

Comparison of Timescales



$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \gtrsim 1$$

$$\rho \mathbf{v} \cdot \nabla s = \frac{\dot{Q}}{T}$$

Comparison of Timescales

$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \sim \frac{1}{\Delta s}\int \frac{\dot{Q}}{T}\frac{dr}{\rho v}$$

$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \gtrsim 1$$

$$\rho \mathbf{v} \cdot \nabla s = \frac{\dot{Q}}{T}$$

Comparison of Timescales

$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \sim \frac{1}{\Delta s} \int \frac{\dot{Q}}{T} \frac{dr}{\rho v}$$

$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \gtrsim 1$$

$$\Delta s \gtrsim \Delta s_{\rm crit}$$

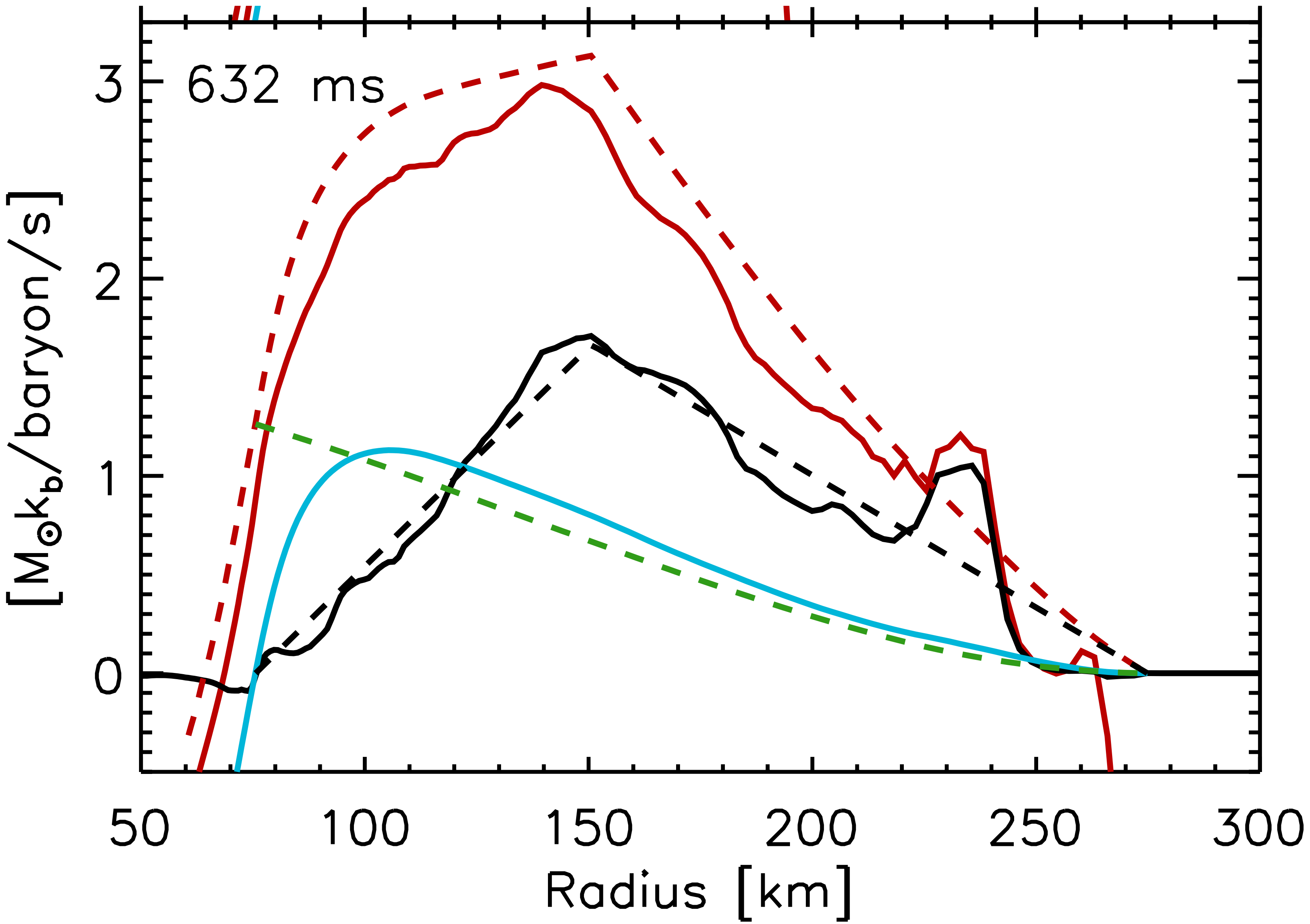
$$\rho \mathbf{v} \cdot \nabla s = \frac{\dot{Q}}{T}$$

Comparison of Timescales

$$\Delta s \gtrsim \Delta s_{\rm crit}$$

$$\frac{\tau_{\rm adv}}{\tau_{\rm heat}} \gtrsim 1$$

$$\dot{M}\Delta s = \int \frac{\dot{Q}}{T} dV + \int \frac{\varepsilon}{T} dV + L_s$$



632 ms
[M⊙k_b/baryon/s]
Radius [km]
50
100
150
200
250
300
0
1
2
3

“What about the SASI?”

What dominates the post shock flow?
Convection, SASI... both?

Compare nonlinear theories for convection and SASI with post shock flow

SASI nonlinear theory

?

Compare nonlinear theories for convection and SASI with post shock flow

Convection nonlinear theory

100+ years

In CCSN...Murphy & Meakin 2012

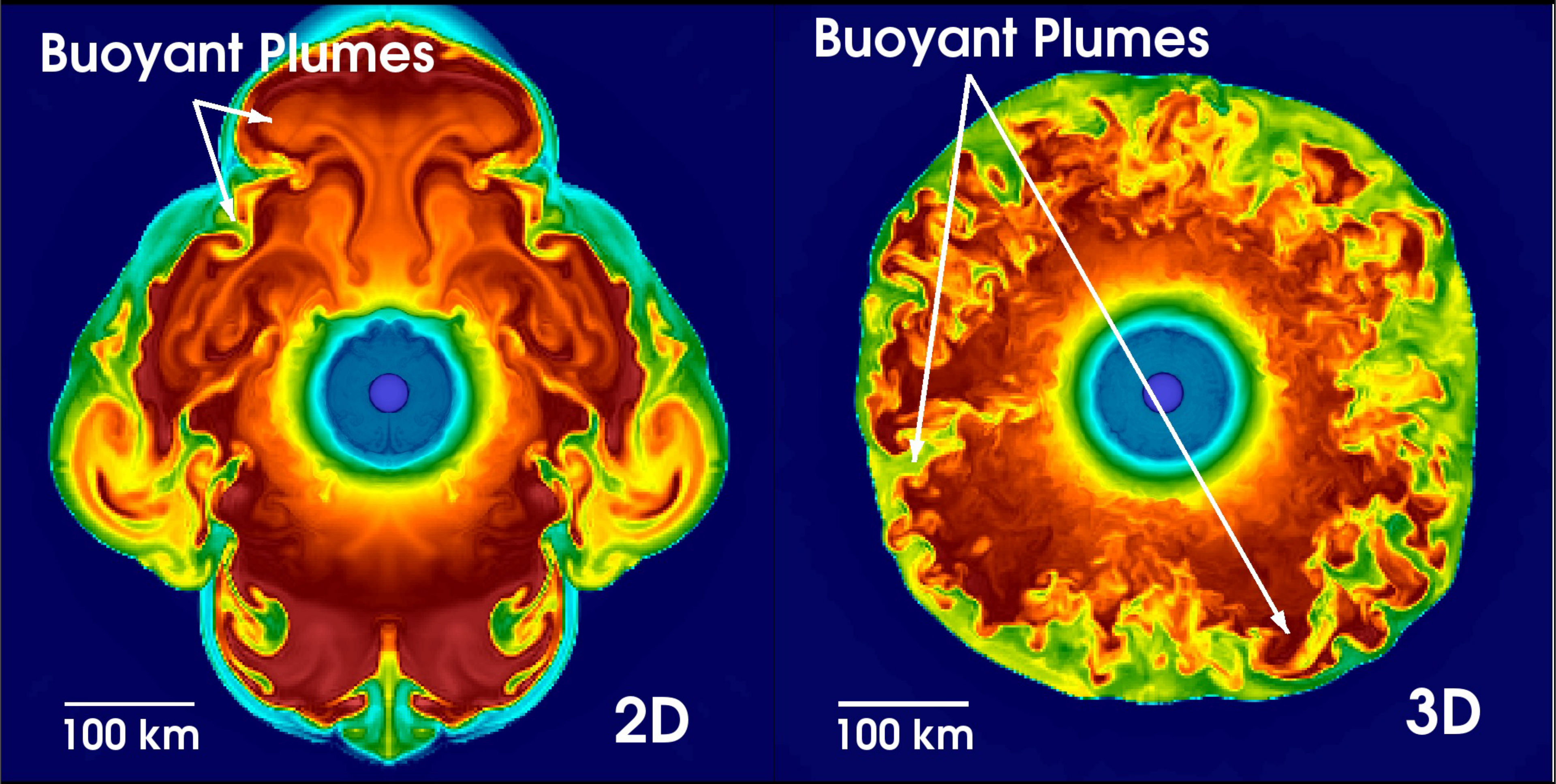
Compare nonlinear theories for convection and SASI with post shock flow

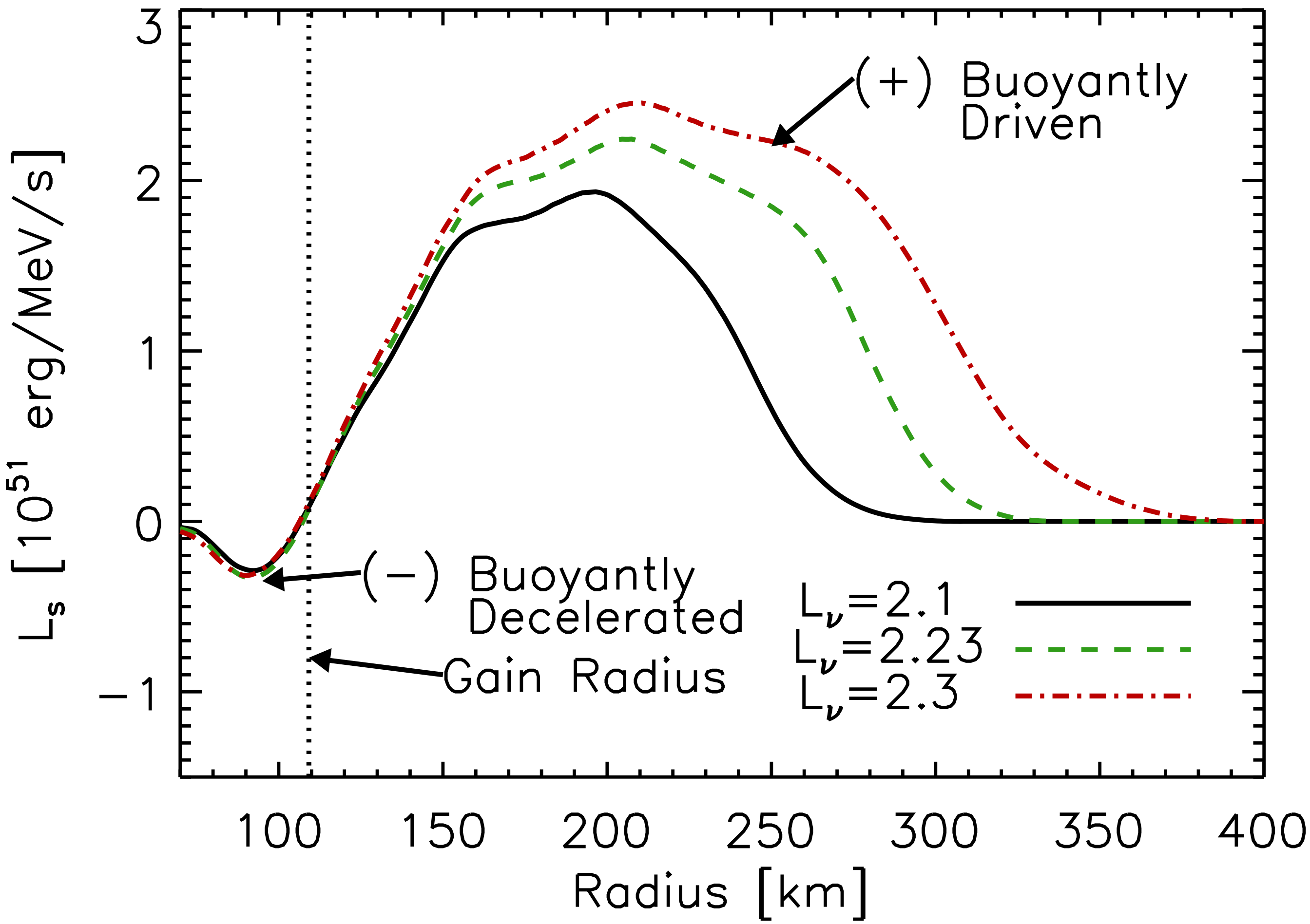
Convection nonlinear theory

100+ years

In CCSN...Murphy & Meakin 2012

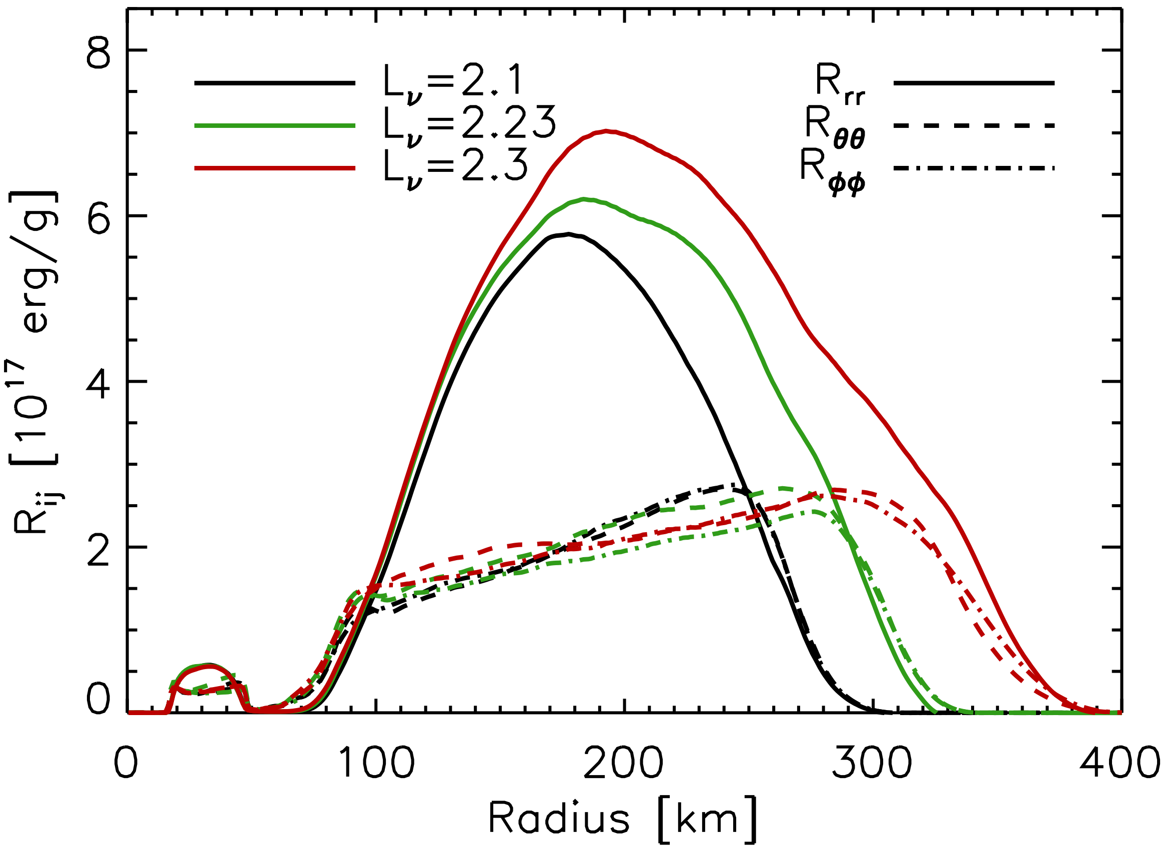
We can test this theory with 3D simulations


Buoyant Plumes
100 km
2D
Buoyant Plumes
100 km
3D



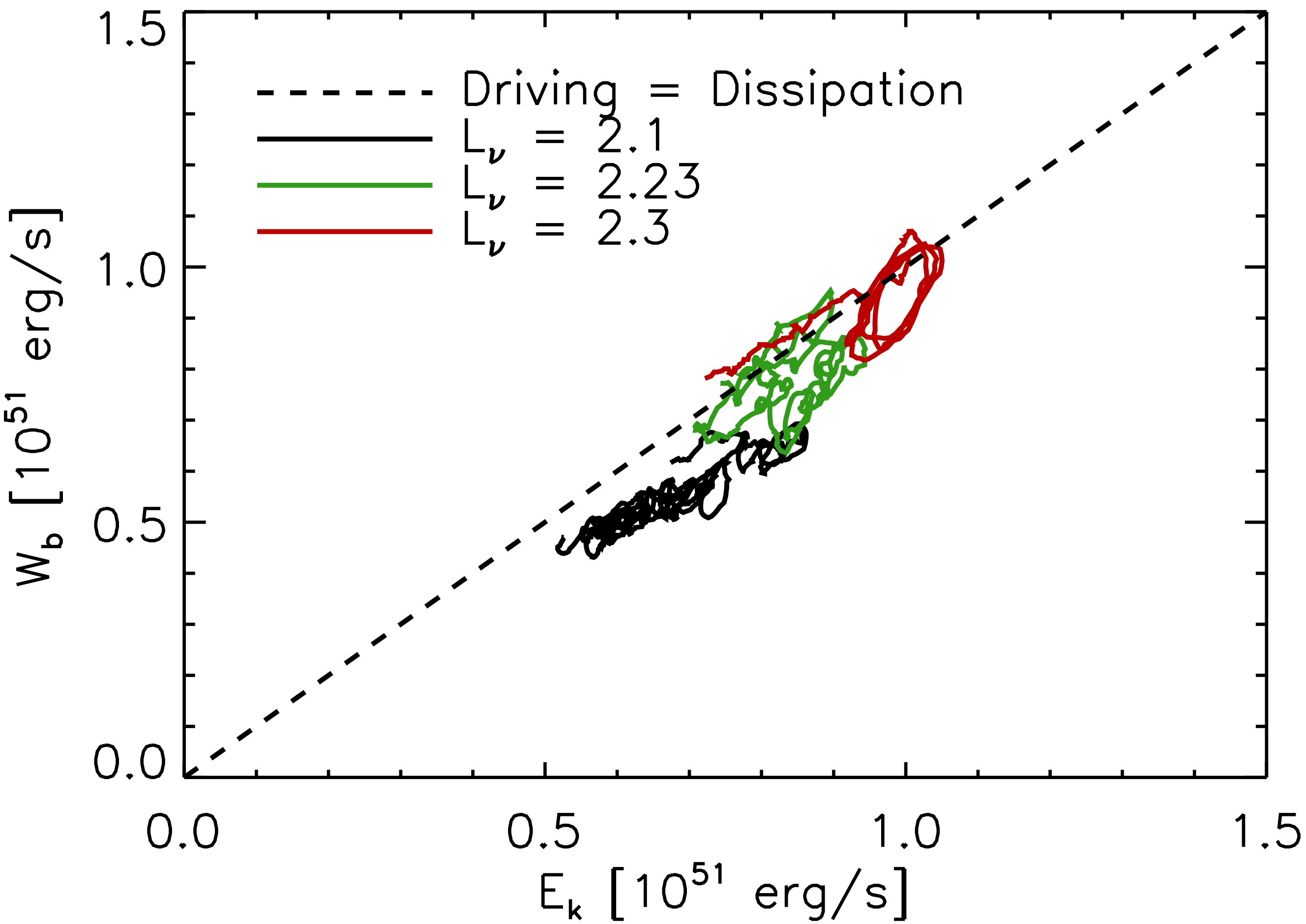
$$L_s = \langle \rho v' s' \rangle 4\pi r^2$$

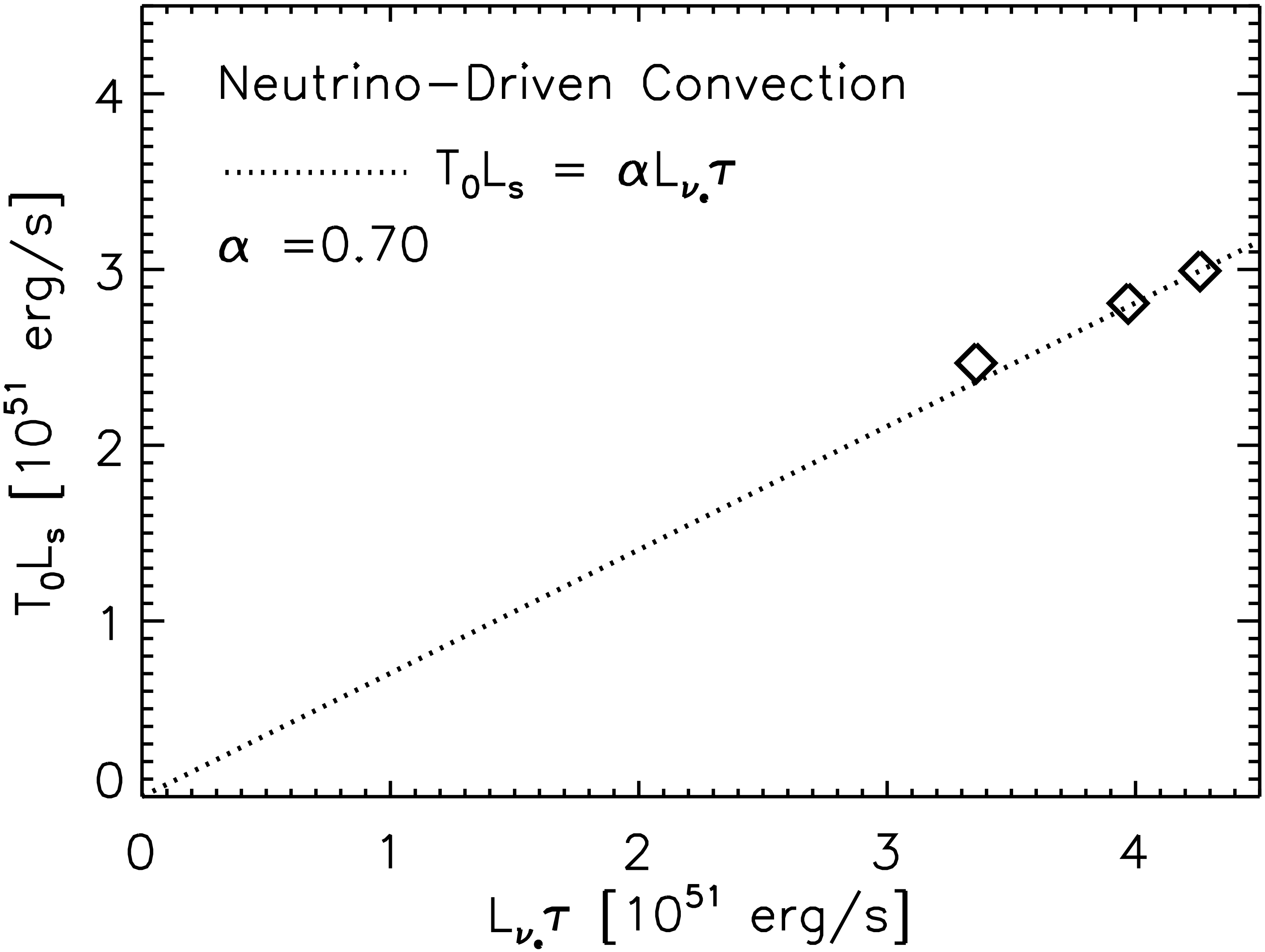
$$R_{ij} = \left\langle v'_i v'_j \right\rangle$$



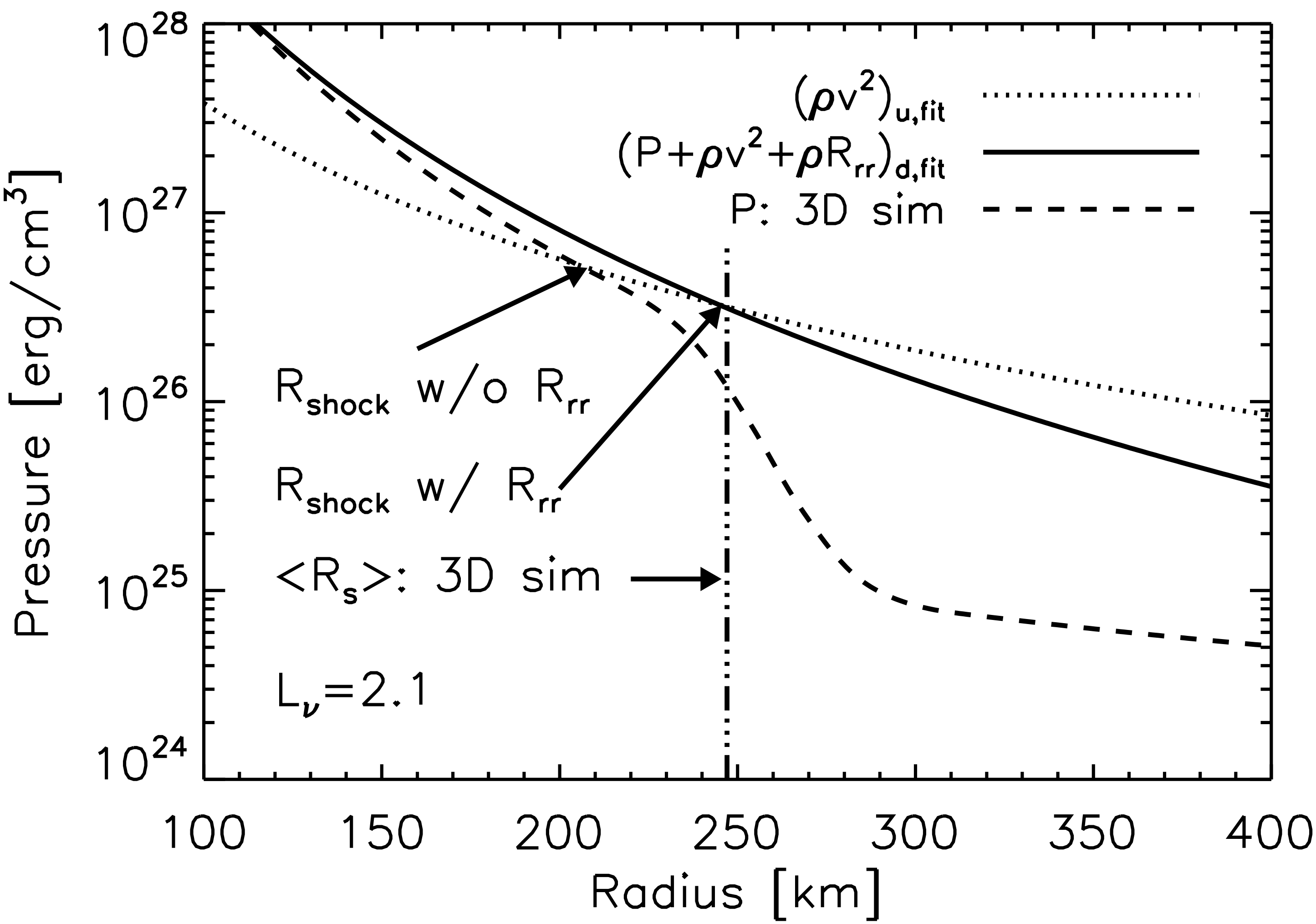
$$R_{rr} \approx R_{\theta\theta} + R_{\phi\phi} \qquad R_{\theta\theta} \approx R_{\phi\phi}$$

$$\int \langle \rho' v' \rangle \, g \, dV = \int \frac{\rho (v_r')^3}{L} dV$$

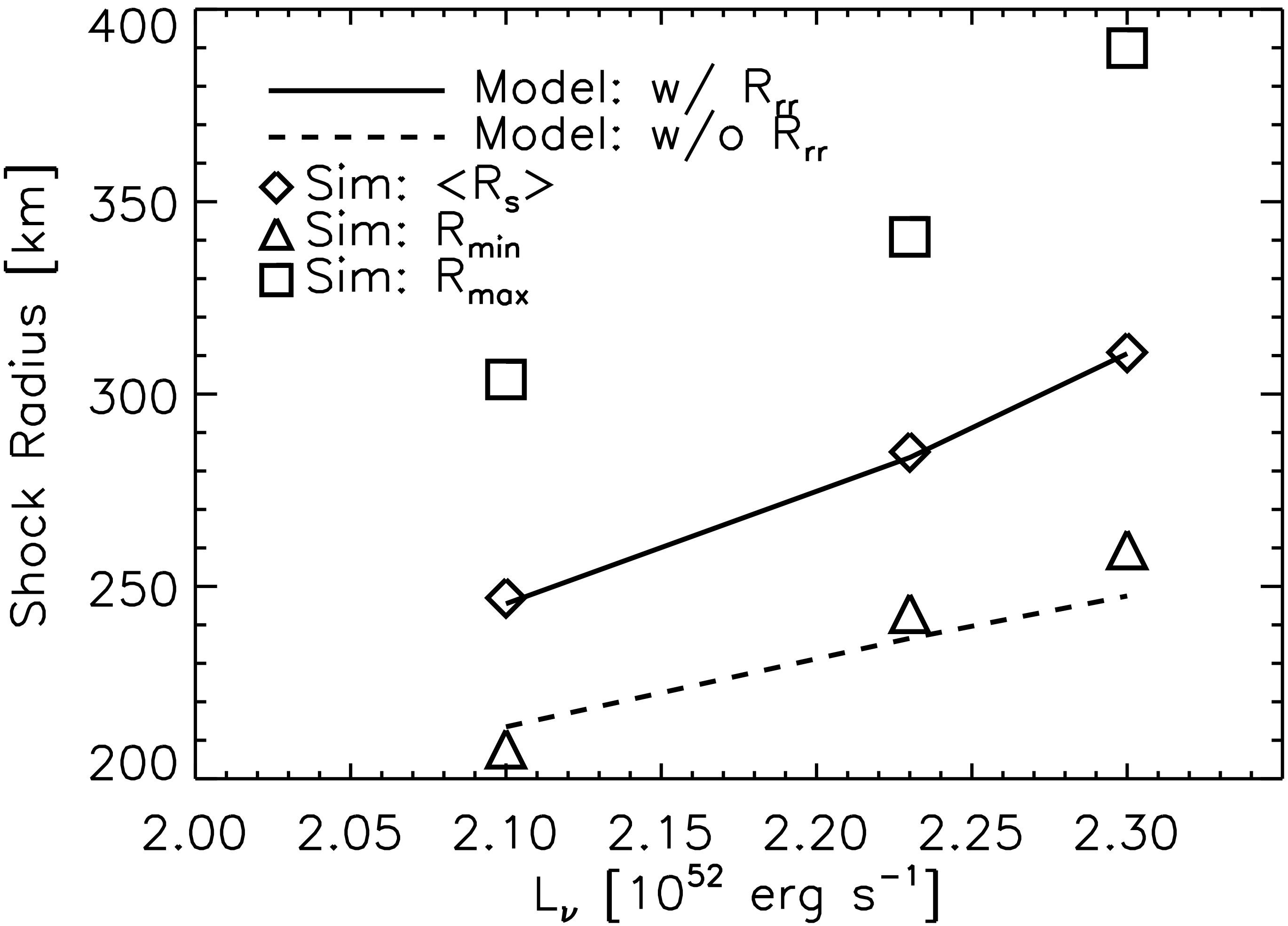




Neutrino−Driven Convection
$T_0 L_s = \alpha L_{\nu_e} \tau$
α =0.70
$T_0 L_s$ [10^{51} erg/s]
$L_{\nu_e} \tau$ [10^{51} erg/s]
0
1
2
3
4



10^{28}
10^{27}
10^{26}
10^{25}
10^{24}
Pressure [erg/cm^3]
100
150
200
250
300
350
400
Radius [km]
$(\rho v^2)_{u,fit}$
$(P+\rho v^2+\rho R_{rr})_{d,fit}$
P: 3D sim
R_{shock} w/o R_{rr}
R_{shock} w/ R_{rr}
<R_s>: 3D sim
L_ν=2.1



Model: w/ R_{rr}
Model: w/o R_{rr}
Sim: $\langle R_s \rangle$
Sim: R_{min}
Sim: R_{max}
Shock Radius [km]
200
250
300
350
400
2.00
2.05
2.10
2.15
2.20
2.25
2.30
L_ν [10^{52} erg s^{-1}]

Nonlinear Convection is Consistent with Post Shock Flow

1. Consistent buoyancy flux profile
2. Consistent Reynolds stresses
3. Buoyant driving balances dissipation
4. Analytic scaling between buoyant flux and neutrino driving
5. Expansion of shock due to turbulent ram pressure

Nonlinear Convection is Consistent with Post Shock Flow

But what about the SASI?

A theory for neutrino-driven explosions

A turbulence model for CCSNe

Post shock flow is consistent with nonlinear convection theory