Some Thoughts on the Equation of State of Dense Matter Using Nuclear Experiments, Neutron Matter Theory and Astronomical Observations

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Important Questions

- What is the Nature of the Nucleon-Nucleon Interaction?
- Does Exotic Matter (Hyperons, Kaons/Pions, Deconfined Quarks) Exist in Neutron Star Interiors?
- How Do Nuclear Experiments Constrain Nuclear Parameters?
- How Can We Make the Interpretations of Astronomical Observations Less Model-Dependent?
- How Reliable Are Theoretical Neutron Matter Calculations?
- What is the Best Way to Model the Equation of State at Subnuclear Densities?
  - Single-Nucleus Approximation vs. Ensemble Approaches
  - Liquid-Droplet vs. Thomas-Fermi or Hartree Nuclear Models
  - Equation of State Tables and Interpolation
  - Connections to NSE Networks
- What Constraints Exist for the EOS at Supernuclear Densities?
  - Nuclear Mass Measurements
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Isolated and Quiescent Binary Sources
  - Pulse Modeling of X-ray Bursts, QPOs, etc.

Outline For This Morning

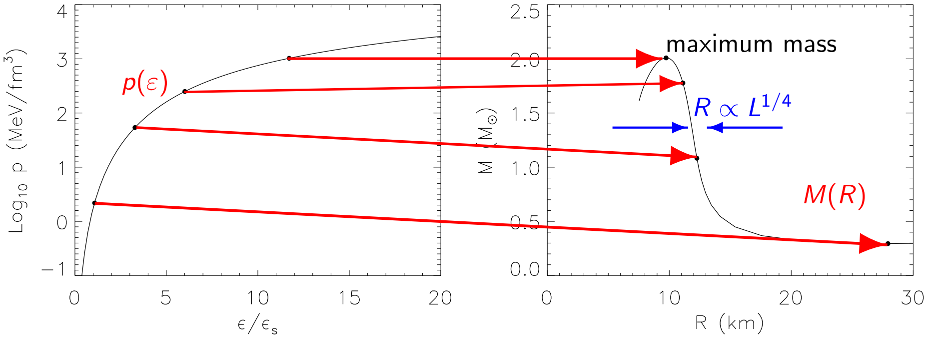
- Neutron Stars and the Symmetry Energy
- Constraints on Symmetry Parameters from Nuclear Experiments
  - Binding Energies
  - Heavy ion Collisions
  - Neutron Skin Thicknesses
  - Dipole Polarizabilities
  - Giant (and Pygmy) Dipole Resonances
- Theoretical Calculations of Pure Neutron Matter
- Astrophysical Observations
  - Mass Measurements of Binary Neutron Stars
  - Simultaneous Mass and Radius Measurements
    - Thermal Emission from Cooling Neutron Stars
    - Photospheric Radius Expansion X-Ray Bursters
  - The Inferred Universal Mass-Radius Relation and the Neutron Star EOS

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ⟵ Observations

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

$M < M_{max}$

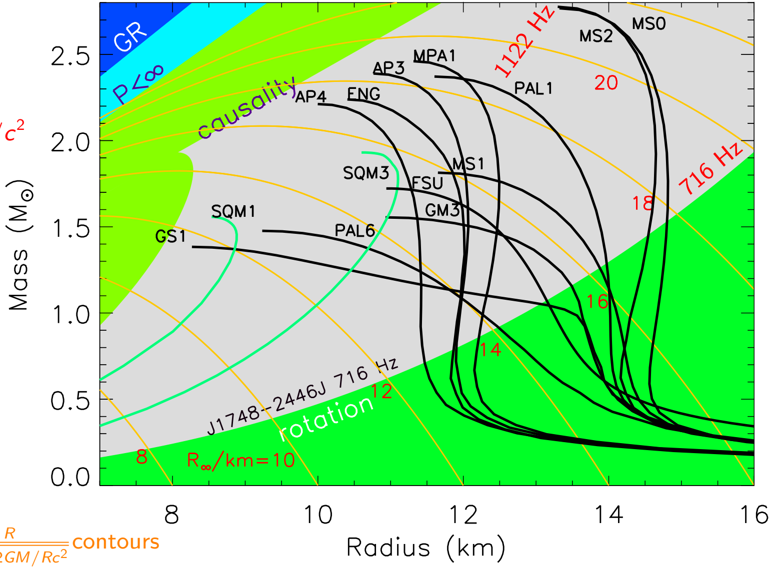
causality:

$R \gtrsim 2.9GM/c^2$

— normal NS

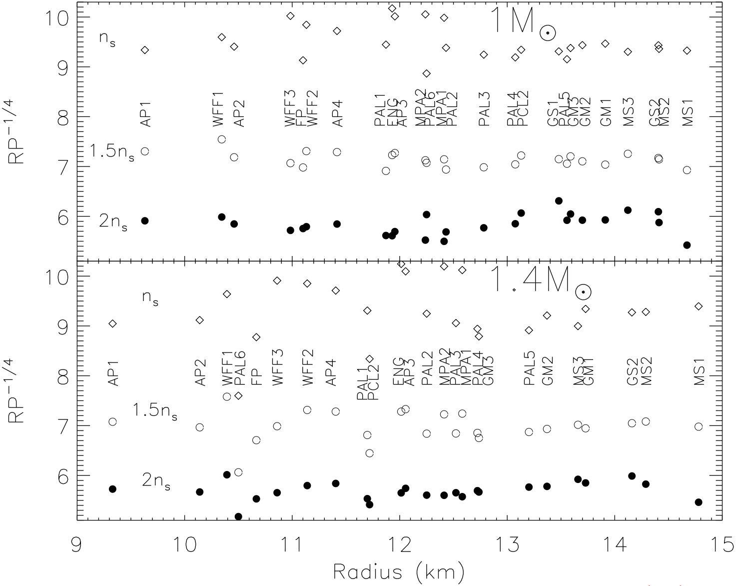
— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

Neutron Star Structure

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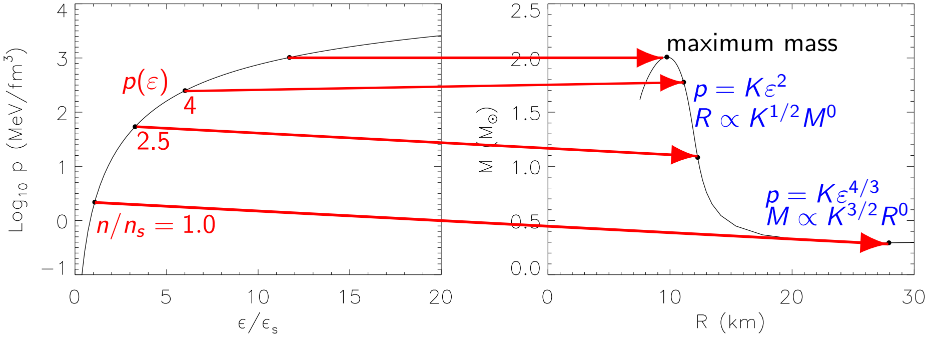
Newtonian Polytropes: $p = K\varepsilon^\gamma$; $\qquad M \propto K^{1/(2-\gamma)}R^{(4-3\gamma)/(2-\gamma)}$



Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter

$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around saturation density (ρ_s) and symmetric matter

$$E(\rho, x) = E(\rho, x = 1/2) + (1 - 2x)^2 E_{sym}(\rho) + \dots$$

$$E_{sym}(\rho) = S_v + \frac{L}{3}\frac{\rho - \rho_s}{\rho_s} + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_s}{\rho_s}\right)^2 + \dots$$

$$S_v = \frac{1}{8}\frac{\partial^2 E}{\partial x^2}\bigg|_{\rho_s, 1/2}, \quad L = \frac{3}{8}\frac{\partial^3 E}{\partial \rho \partial x^2}\bigg|_{\rho_s, 1/2}, \quad K_{sym} = \frac{9}{8}\frac{\partial^4 E}{\partial \rho^2 \partial x^2}\bigg|_{\rho_x, 1/2}$$

Thus, $E_{sym}(\rho) \simeq S(\rho)$ if higher-than-quadratic terms are small. Can be connected to neutron matter:

$$S(\rho_s) = E(\rho_s, x = 0) + B \approx S_v, \qquad p(\rho_s, x = 0) \approx L\rho_s/3$$

$R \propto p(\rho_s - 2\rho_s, x = 0)^{1/4}$ **(Lattimer & Prakash 2001)**

Nuclear Binding Energies

$E_{sym}(N,Z) = I^2(S_v A - S_s A^{2/3})$

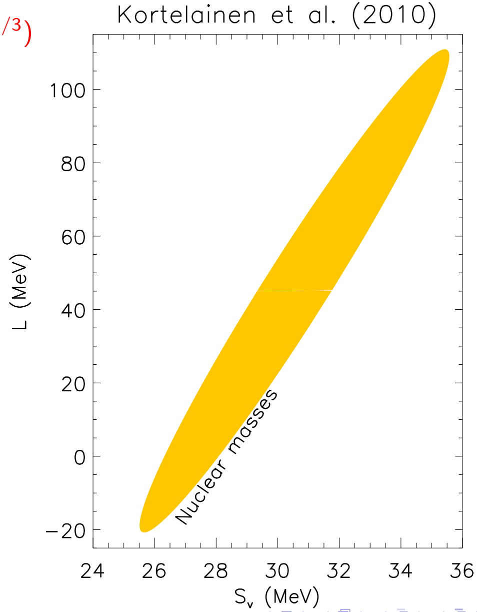
$\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2/\mathcal{N}$
$\chi_{vv} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^2$
$\chi_{ss} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{4/3}$
$\chi_{vs} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{5/3}$

$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$
$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$

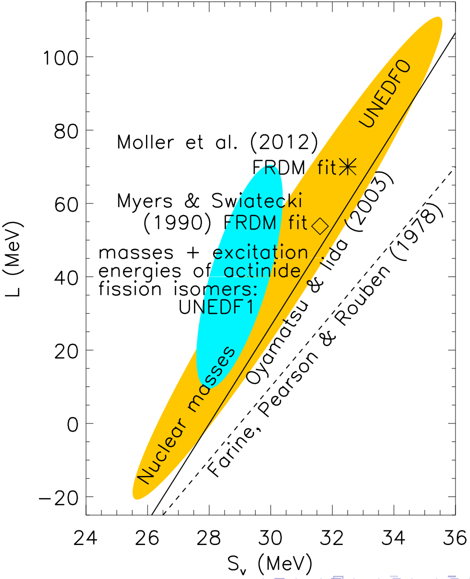
$\alpha = \frac{1}{2} \tan^{-1} \frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}}$
$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}}$

$S_s \approx 0.95 S_v + 0.65 L$

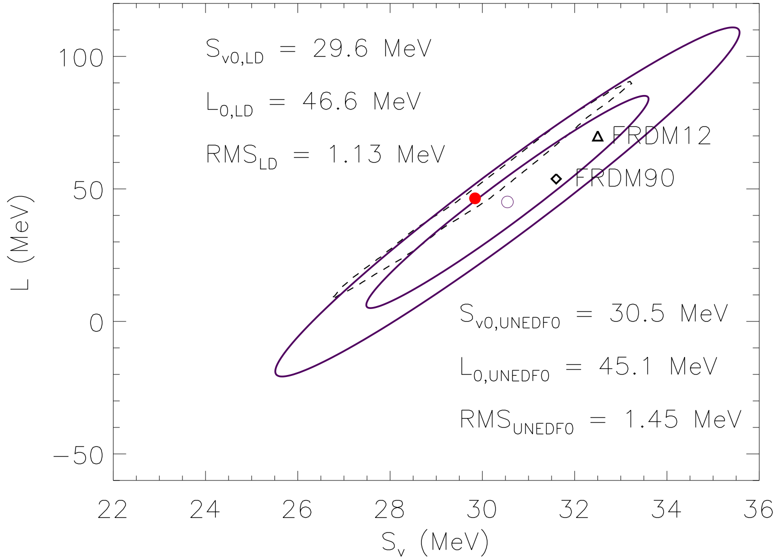
$E_{sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v) A^{-1/3}}$



Nuclear Binding Energies



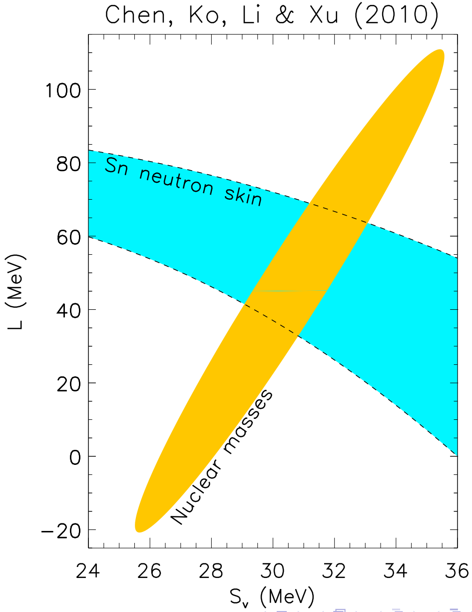
Comparison of Microscopoic and Liquid Droplet Mass Fits



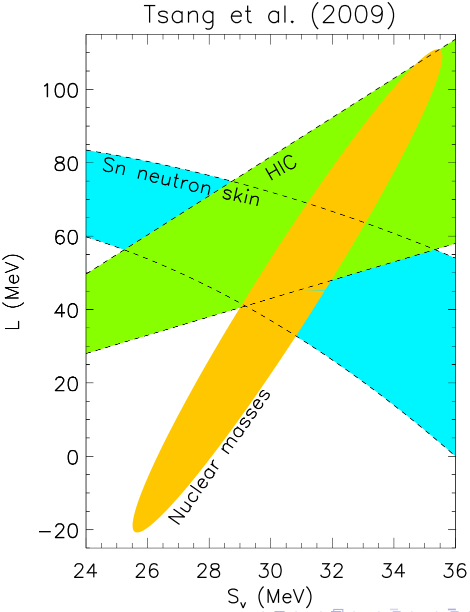
Neutron Skin Thickness

$R_n - R_p \simeq \sqrt{3/5}\,t_{np}$

$t_{np} = \frac{2r_o}{3}\frac{S_s I}{S_v + S_s A^{-1/3}}$



Heavy Ion Collisions

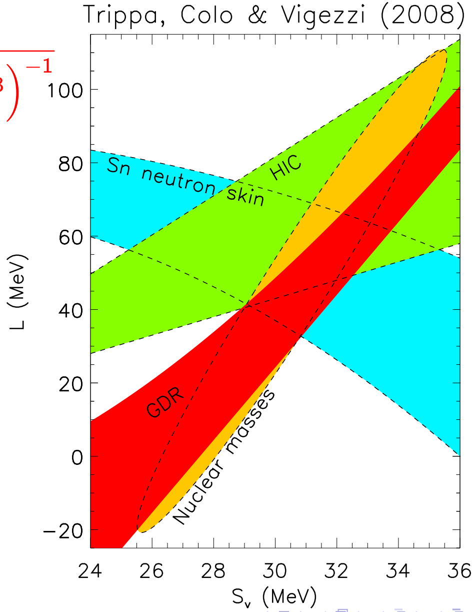


Giant Dipole Resonances

$$E_{-1} \propto \sqrt{S_v \left(1 + \frac{5S_s}{3S_v} A^{-1/3}\right)^{-1}}$$

Correlation between E_{-1} and E_{sym} maximized when

$$E_{sym,208}/AI^2 = \frac{S_v}{1+(S_s/S_v)A^{-1/3}} \simeq S(\rho = 0.1)$$

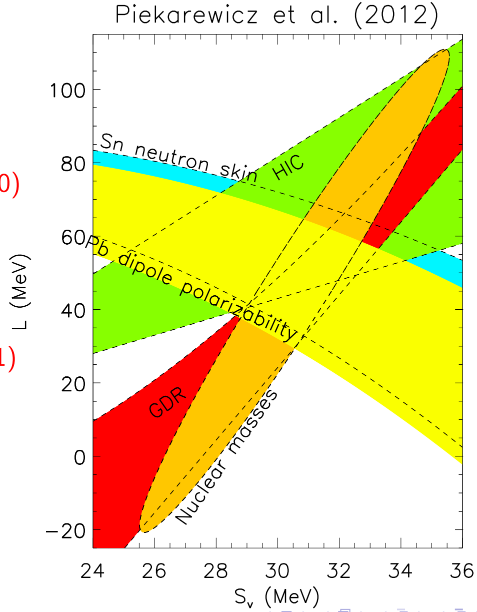


Dipole Polarizability

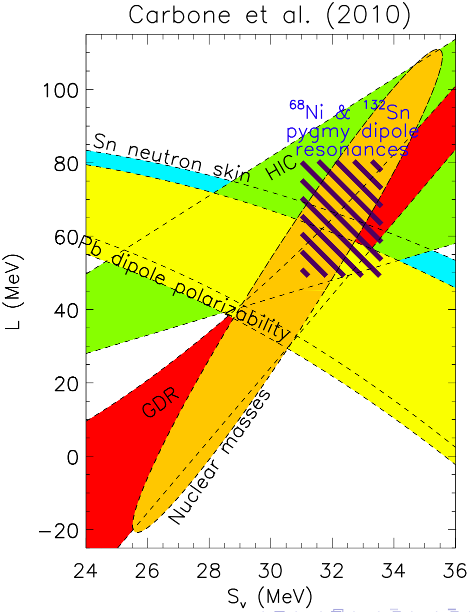
α_D and $R_n - R_p$ in ^{208}Pb are 98% correlated

Reinhard & Nazawericz (2010)

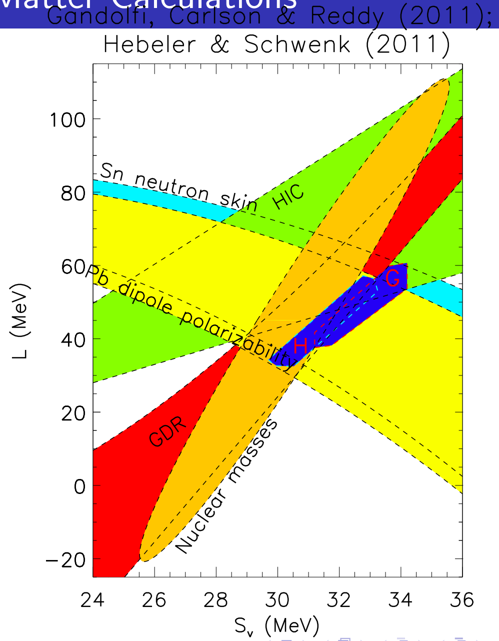
Data from Tamii et al. (2011)



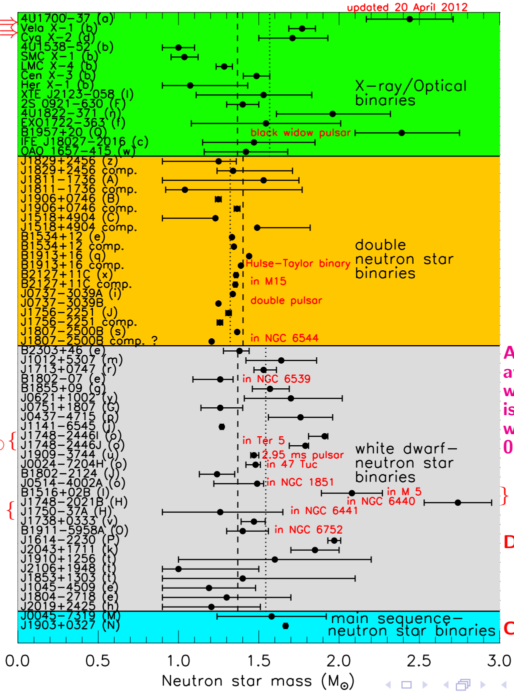
Pygmy Dipole Resonances



Theoretical Neutron Matter Calculations



Black hole?
Firm lower mass limit?



$M > 1.68\ \mathrm{M}_\odot$ {
95% confidence

Freire et al. 2007 {

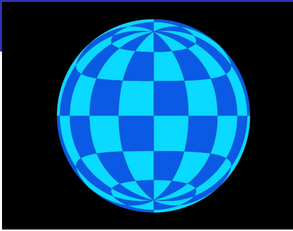
Although simple average mass of w.d. companions is 0.27 $\mathrm{M}_\odot$ larger, weighted average is 0.08 $\mathrm{M}_\odot$ smaller

} **w.d. companion? statistics?**

Demorest et al. 2010

Champion et al. 2008

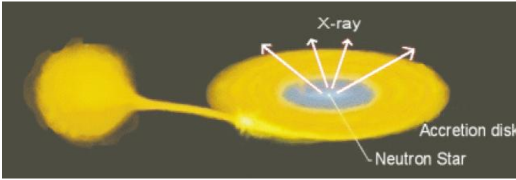
Simultaneous Mass/Radius Measurements



- The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar absorption (UV and X-rays), atmospheric composition

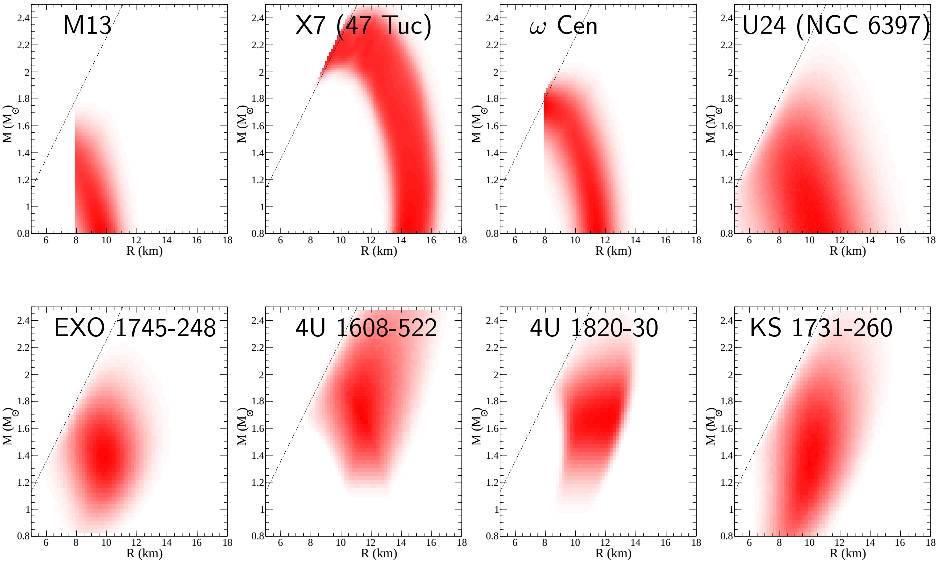


Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
- Bursting sources with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

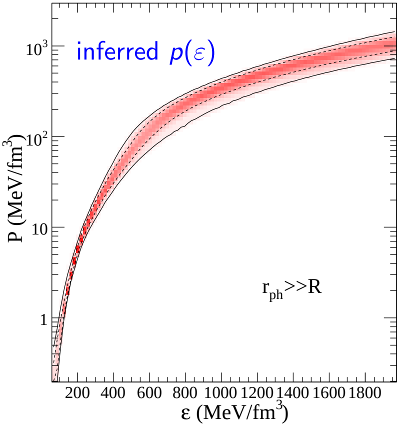
$$F_{Edd} = \frac{cGM}{\kappa D^2}$$

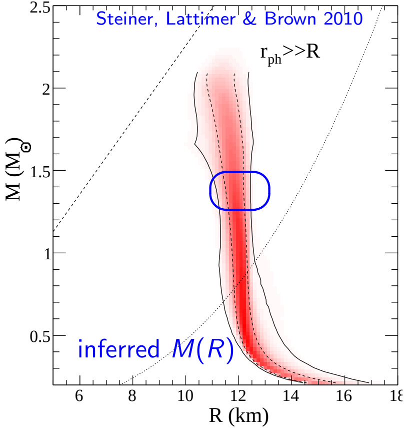
$M - R$ Probability Estimates



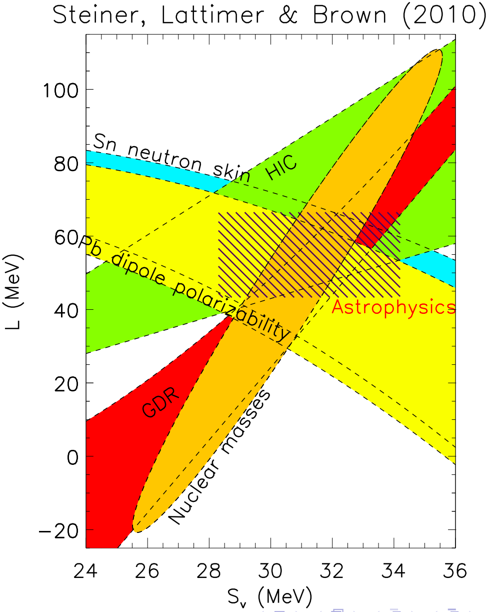
Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{max} \geq 1.97$ M$_\odot$, causality enforced
- All stars equally weighted

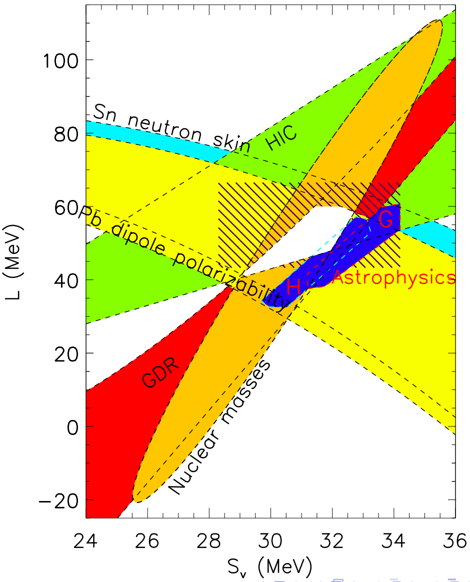




Astronomical Observations



Combined Constraints



Consistency with Neutron Matter and Heavy-Ion Collisions