

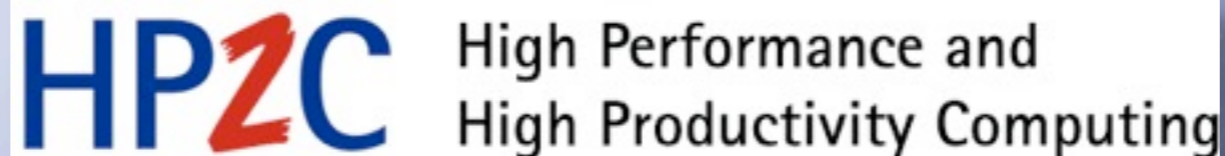
New equations of state in simulations of core-collapse supernovae

INT Seattle, 6.7.2012

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in collaboration with:

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New equations of state in simulations of core-collapse supernovae

Outline:

- 1.) introduction, EOS model, constraints
- 2.) EOS aspects in simulations at
 - a.) sub-saturation densities
 - b.) supra-saturation densities and high temperatures
- 3.) conclusions

Supernova EOS – Introduction

- EOS provides the crucial nuclear physics input for astrophysical simulations: thermodynamic quantities and nuclear composition
- plenty of EOSs for cold neutron stars
- challenge of the supernova EOS:
 - finite temperature: $T = 0 - 100$ MeV
 - wide density range: $\rho = 10^4 - 10^{15}$ g/cm³
 - no weak equilibrium: $Y_e = 0 - 0.6$
 - EOS in tabular form, ~ 1 million grid-points (T, Y_e, ρ)
 - sub-saturation densities: nuclei/non-uniform nuclear matter/crust
- SN EOS: multi-purpose EOS

Supernova EOS – Introduction

- most used SN EOSs:
- Lattimer & Swesty 1991 (LS): non-relativistic liquid drop model
- H. Shen, Toki, Omayatsu and Sumiyoshi 1998 (STOS/Shen): relativistic mean-field (RMF), Thomas-Fermi approximation
- both models:
 - one representative nucleus: “single nucleus approximation”
 - no shell effects
 - only α -particles of light clusters

Available Supernova EOS

Hillebrandt, Wolff, Nomoto (1984)	Skyrme interactions, NSE, Hartree-Fock
Lattimer & Swesty (1991) (LS)	Skyrme interactions, compressible liquid-drop model, three different compressibilities, and tabulated variants
H. Shen et al. (1998) (STOS)	table for TM1, relativistic mean-field (RMF), Thomas-Fermi approximation
MH & Schaffner-Bielich (2010) (HS)	tables for NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx: NSE, RMF, excluded volume
G. Shen, Horowitz, Teige (2010)	tables for NL3 and FSUgold: virial expansion, RMF, Hartree

Nakazato et al. 2008	quark matter with large n_c added to STOS
Ishizuka et al. 2008	hyperons added to STOS
Sagert et al. 2009	quark matter with low n_c added to STOS → explosions in 1D
H. Shen et al. 2010	lambdas added to STOS

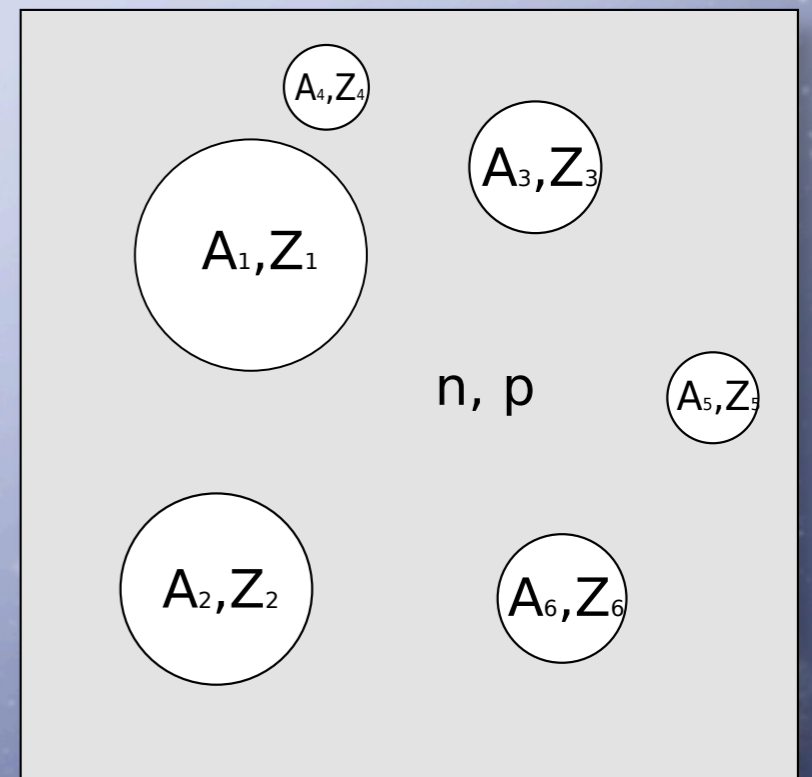
nucleonic EOS

strange physics
at high densities

EOS model: excluded volume NSE with interactions

MH, J. Schaffner-Bieleich; NPA837(2010) (HS)

- chemical mixture of nuclei and interacting nucleons in nuclear statistical equilibrium
- nuclei: theoretical and experimental nuclear mass tables
 - nuclear shell effects
 - ensemble of heavy nuclei and all possible light nuclei
- nucleons: full Fermi-Dirac statistics, various relativistic mean-field (RMF) interactions
- excluded volume effects:
 - interplay nuclei vs. unbound nucleons
 - smooth transition to uniform nuclear matter
- thermodynamic consistent & stable



EOS model: excluded volume NSE with interactions

MH, J. Schaffner-Bieleich; NPA837(2010) (HS)

- seven EOS tables for different RMF interactions:
NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx
- three additional tables for TM1 with selection of light nuclei:
{ α }, { α ,d}, { α ,d,t}
- abundances of all nuclei provided by program

<http://phys-merger.physik.unibas.ch/~hempel/eos.html>

Nucleons – non-linear RMF (TM1, TMA, FSUgold, NL3)

- relativistic mean-field model (RMF)
- interactions mediated via exchange of mesons and meson (self-) interactions

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_\sigma\bar{\psi}\sigma\psi \\ & - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}g_4(\omega_\mu\omega^\mu)^2 - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu \\ & - \frac{1}{4}R^a{}_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} - g_\rho\bar{\psi}\gamma_\mu\tau^a\psi\rho^{\mu a} - \Lambda\omega_\mu\omega^\mu\rho_\nu^a\rho^{a\nu}\end{aligned}$$

- alternative: density-dependent coupling constants (DD2)
- coupling constants fitted to experimental data
- well-established description of finite nuclei and nuclear matter

Sugahara & Toki 1994

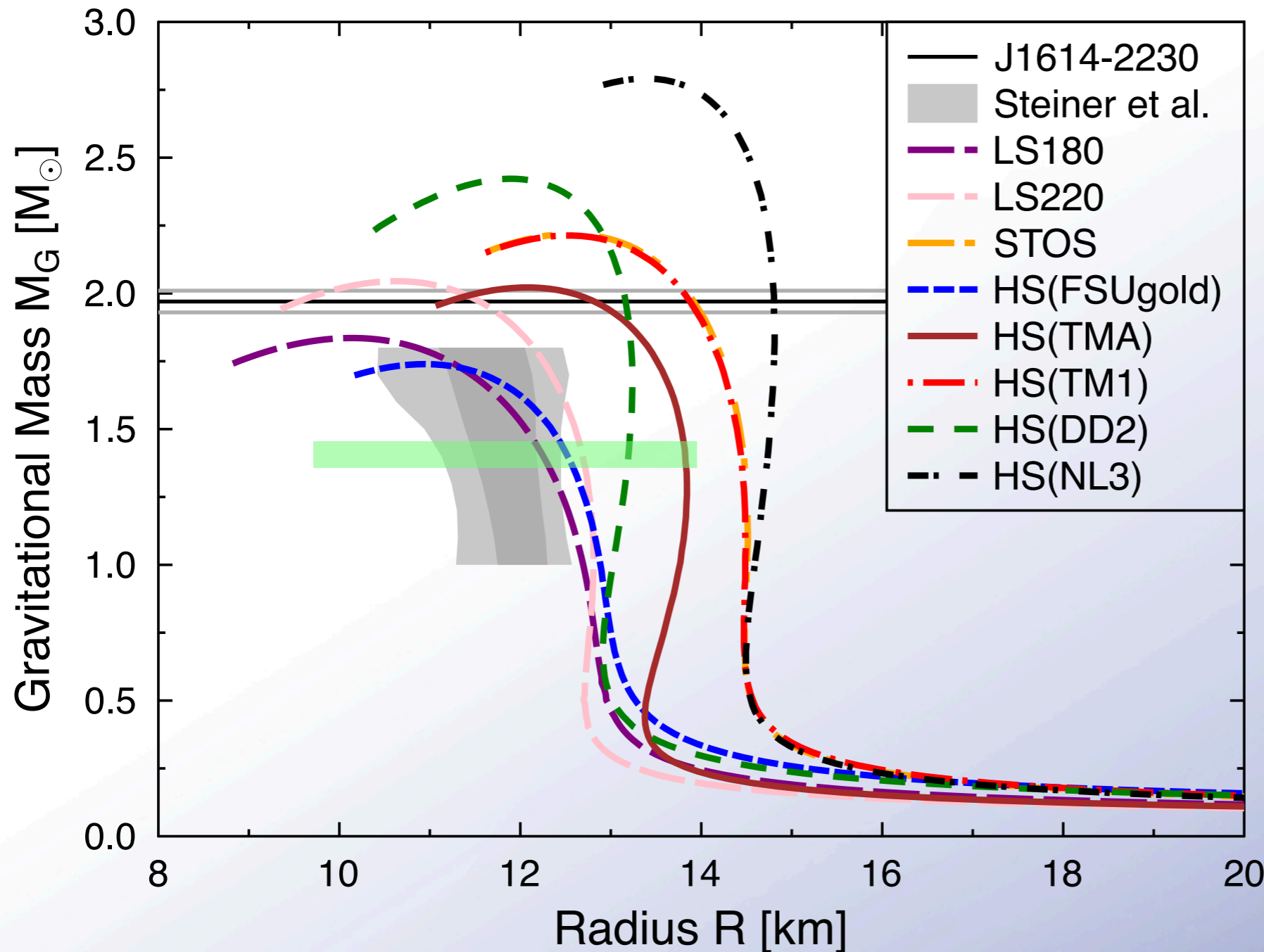
Toki et al. 1995

Todd-Rutel & Piekarewicz 2005

Lalazissis et al. 1997

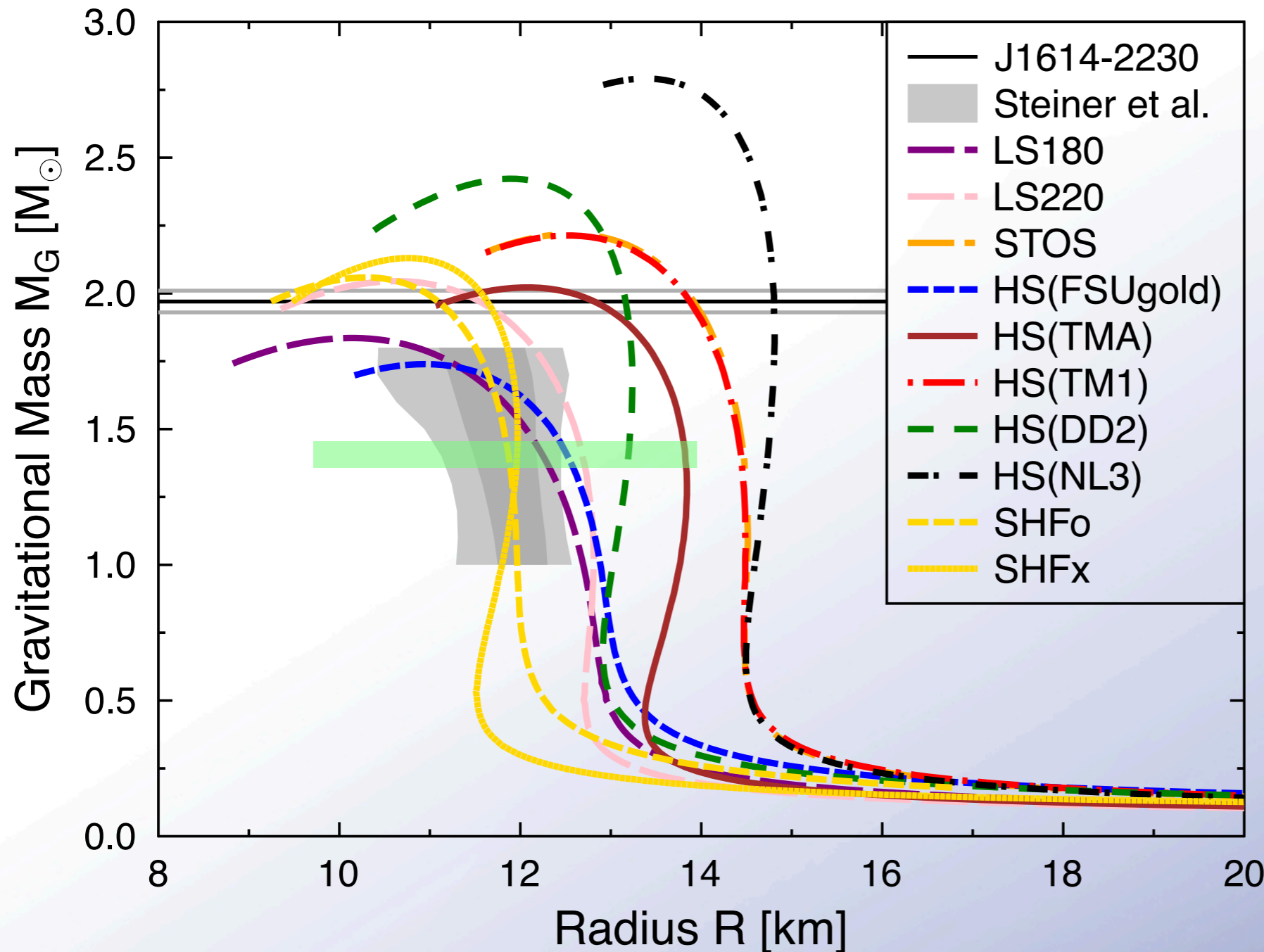
Typel 2005, Typel et al. 2010

EOS constraints – mass and radius measurements



- bayesian analysis of observations of six NS (X-ray burster, low - mass X-ray binaries), Steiner et al. ApJ 2010
- similar results from Chiral EFT (Hebeler et al. 2010)
- new SN EOS fitted to observations: Andrew Steiner (INT), Tobias Fischer (GSI Darmstadt)
- compact PNS \rightarrow more binding energy release in a SN

EOS constraints – mass and radius measurements

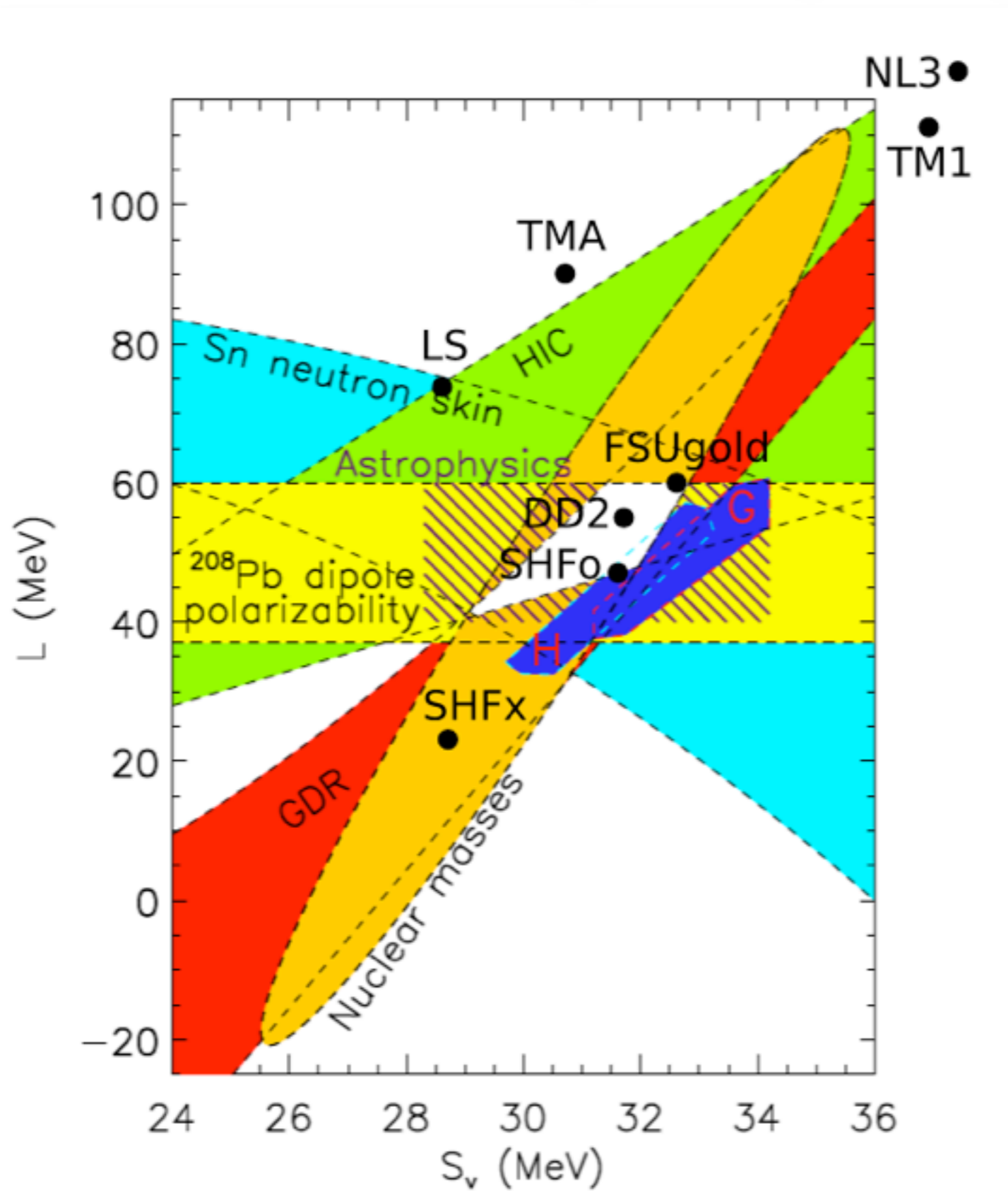


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[A. Steiner, MH & T. Fischer, in preparation]

EOS constraints – symmetry energy

[Lattimer & Lim, arXiv:1203.4286]



- convergence of observational, experimental and theoretical constraints
- standard non-linear RMF in disagreement (TM1, NL3, TMA)
- two new RMF models/SN EOS: SHFo (optimal) SHFx (extreme)

- G: Gandolfi et al. 2012: quantum Monte-Carlo
- H: Hebeler et al. 2010: Chiral EFT, neutron matter

EOS constraints – saturation properties & maximum mass

	n_B^0 [fm ⁻³]	E_0 [MeV]	K [MeV]	J [MeV]	L [MeV]	M_{\max} [M_{\odot}]
TM1	0.146	-16.31	282	36.95	110.99	2.213
TMA	0.147	-16.03	318	30.66	90.14	2.022
FSUgold	0.148	-16.27	230	32.56	60.44	1.739
NL3	0.148	-16.24	271	37.39	118.50	2.791
DD2	0.149	-16.02	243	31.67	55.04	2.422
SHFo	0.158	-16.19	245	31.57	47.10	2.059
SHFx	0.160	-16.16	239	28.67	23.18	2.130
LS180	0.155	-16.00	180	28.61	73.82	1.828
LS220	0.155	-16.00	220	28.61	73.82	2.031
Exp.	~ 0.15	~ -16	240 ± 10 [1]	$30 - 34$ [2]	$40 - 110$ [2]	$> 1.97 \pm 0.04$ [3]

- span a broad range of possible RMF models
- provide a “best fit” EOS

	references	type of constraint
[1]	Piekarewicz JPG 2010	compilation of measurements of isoscalar giant monopole resonances
[2]	Tsang et al. PRL 2009 Carbone et al. PRC 2010	compilation of experiments: isospin diffusion, pygmy dipole resonance, nuclear masses, GDR, isoscaling, antiprotonic atoms
[3]	Demorest et al. Nature 2010	measurement of Shapiro delay

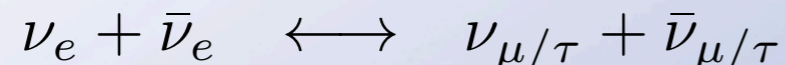
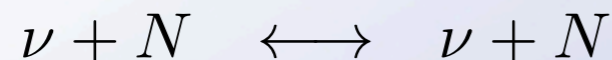
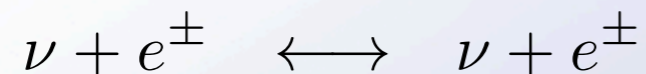
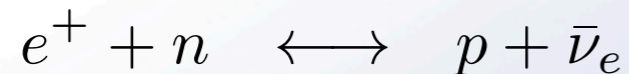
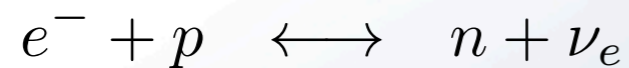
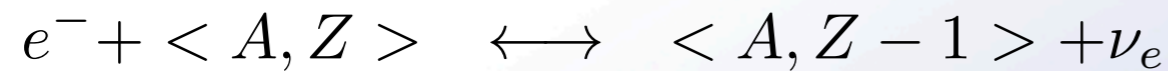
EOS aspects at subsaturation densities

- heavy nuclei

Supernova simulations

MH, T. Fischer, J. Schaffner-Bielich, M. Liebendörfer, ApJ 748, 70 (2012)

- simulations by Tobias Fischer, GSI Darmstadt
 - general relativistic radiation hydrodynamics in spherical symmetry
 - three flavor Boltzmann neutrino transport
- weak reactions:
 - all light clusters treated as alpha-particles
 - only average heavy nucleus

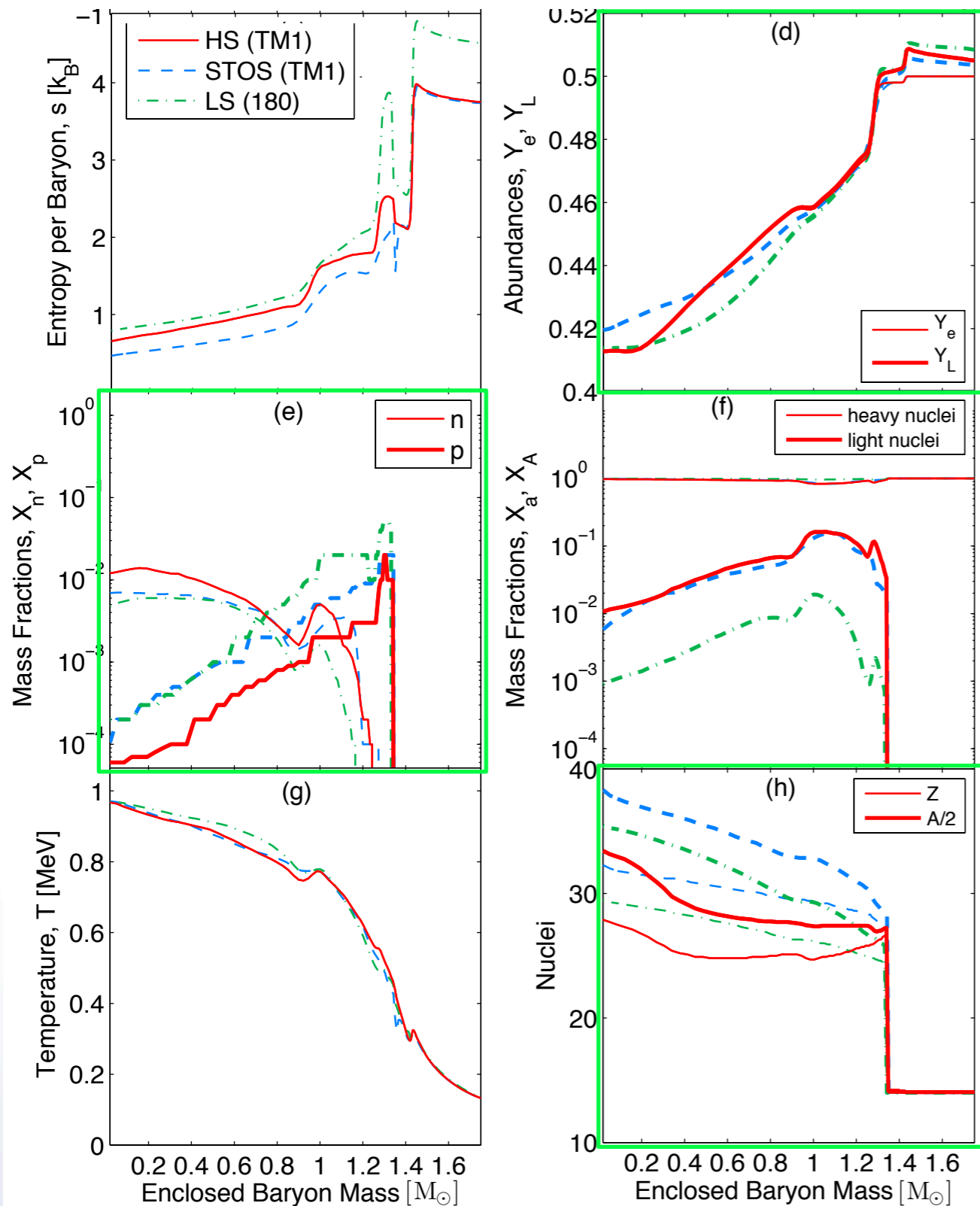


→ mainly sensitive to differences in thermodynamic quantities

- 15 M_{sun} progenitor of Woosley & Weaver ApJS 101 (1995)

Heavy nuclei

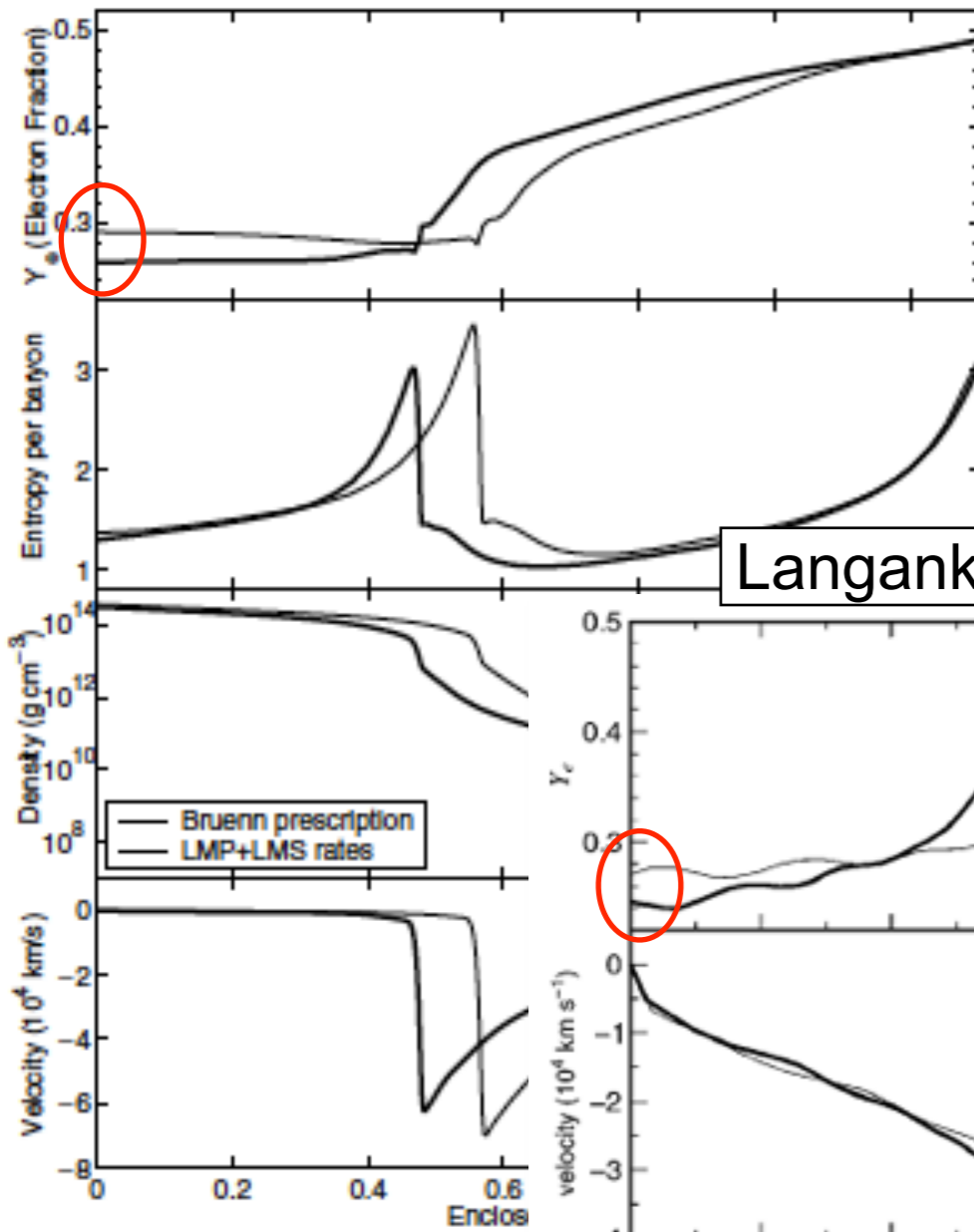
$t_{pb} = -40$ ms



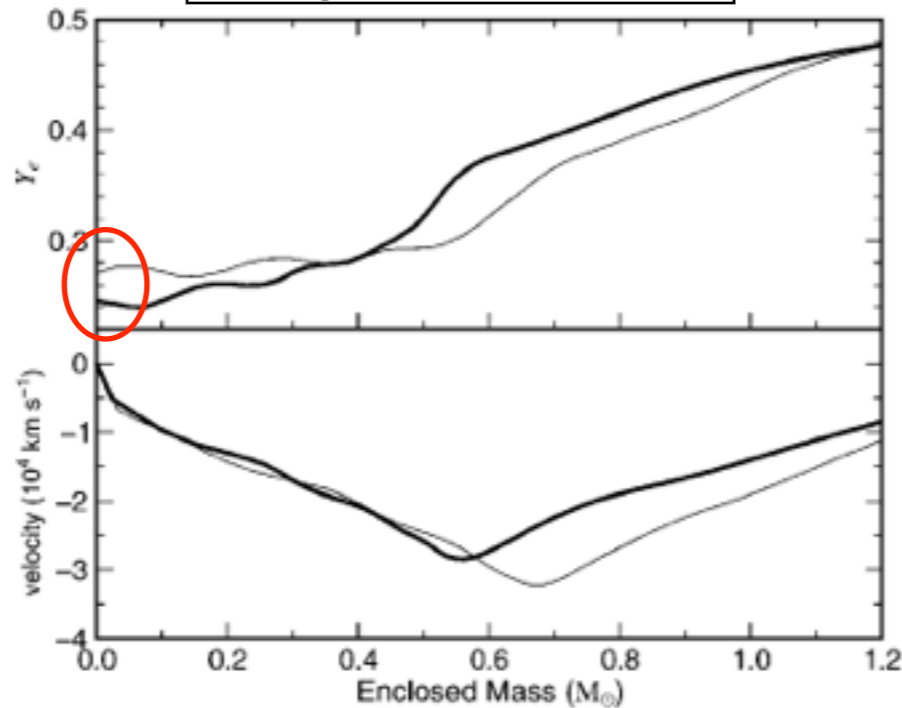
- systematic overprediction of A in STOS \rightarrow underprediction of e-captures (Bruenn '85 rates)
- indirect effect on $X_p \rightarrow$ higher Y_e in outer layers
- correct description of nuclei as important as nuclear interactions

Heavy nuclei & e-captures

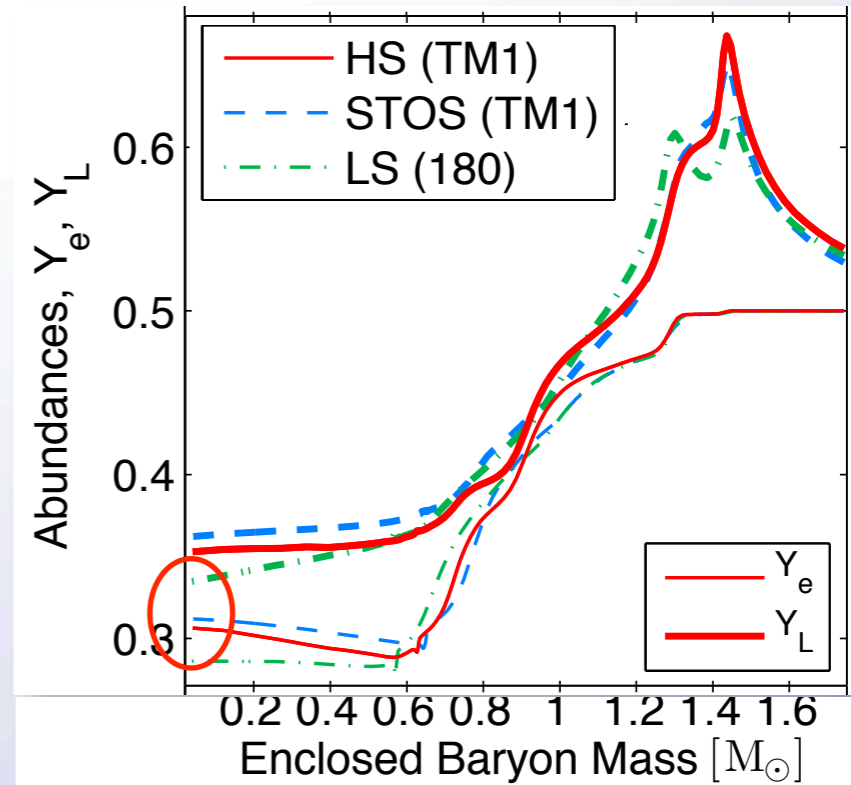
Hix et al. 03



Langanke et al. 03

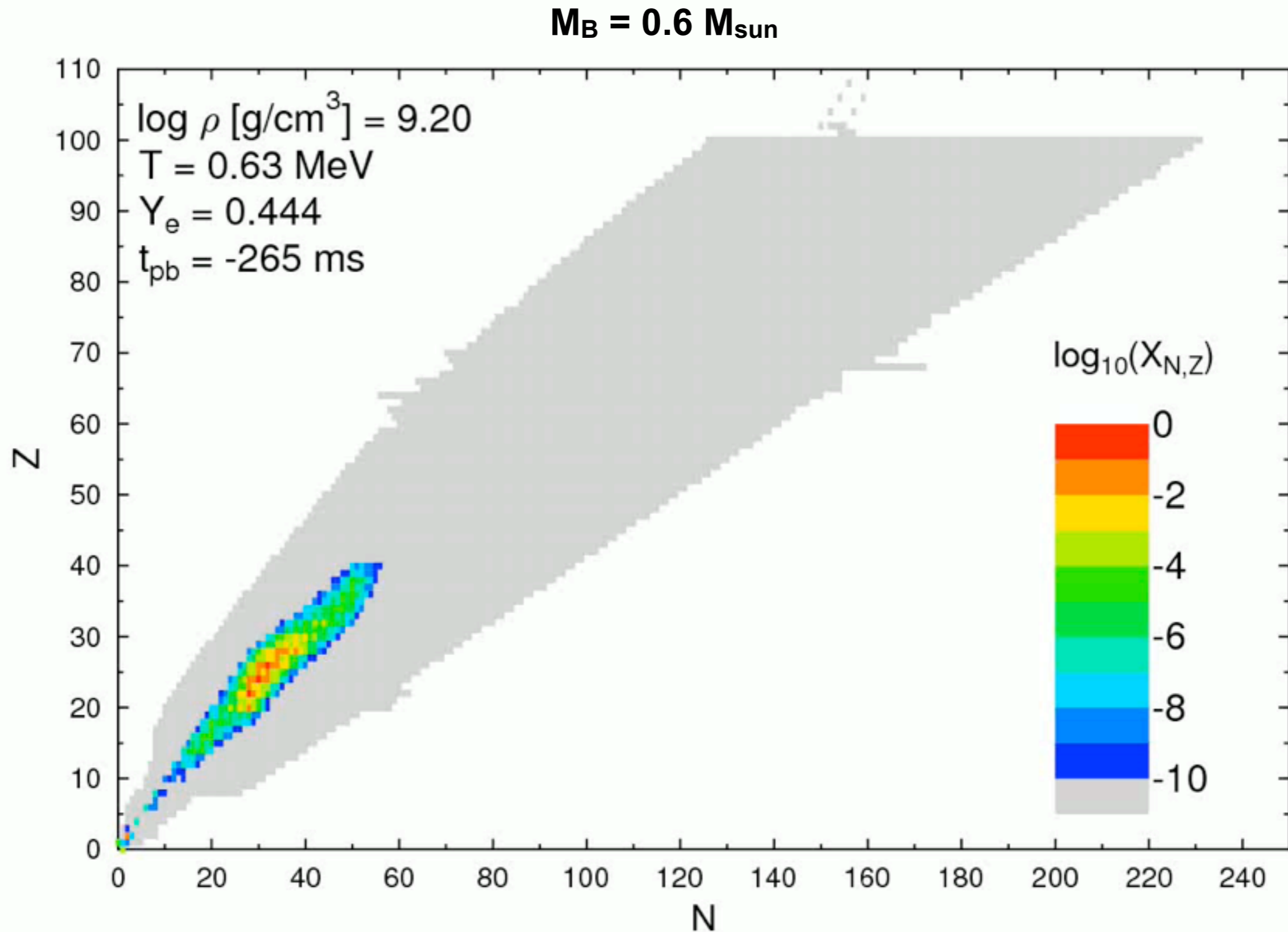


bounce



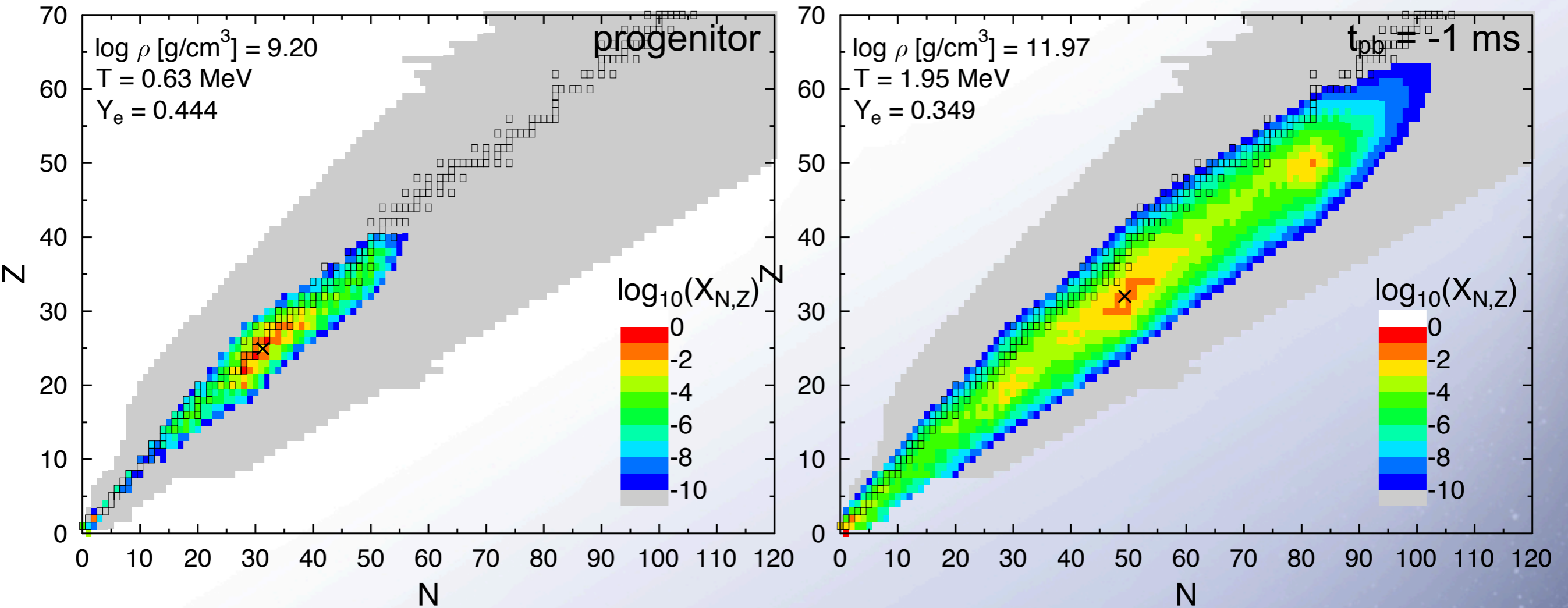
- similar trends as in LMP rates, just due to the EOS
- wiggles in Y_e due to shell effects

Nuclear composition in the SN



Nuclear composition in the SN

$M_B = 0.6 M_{\text{sun}}$



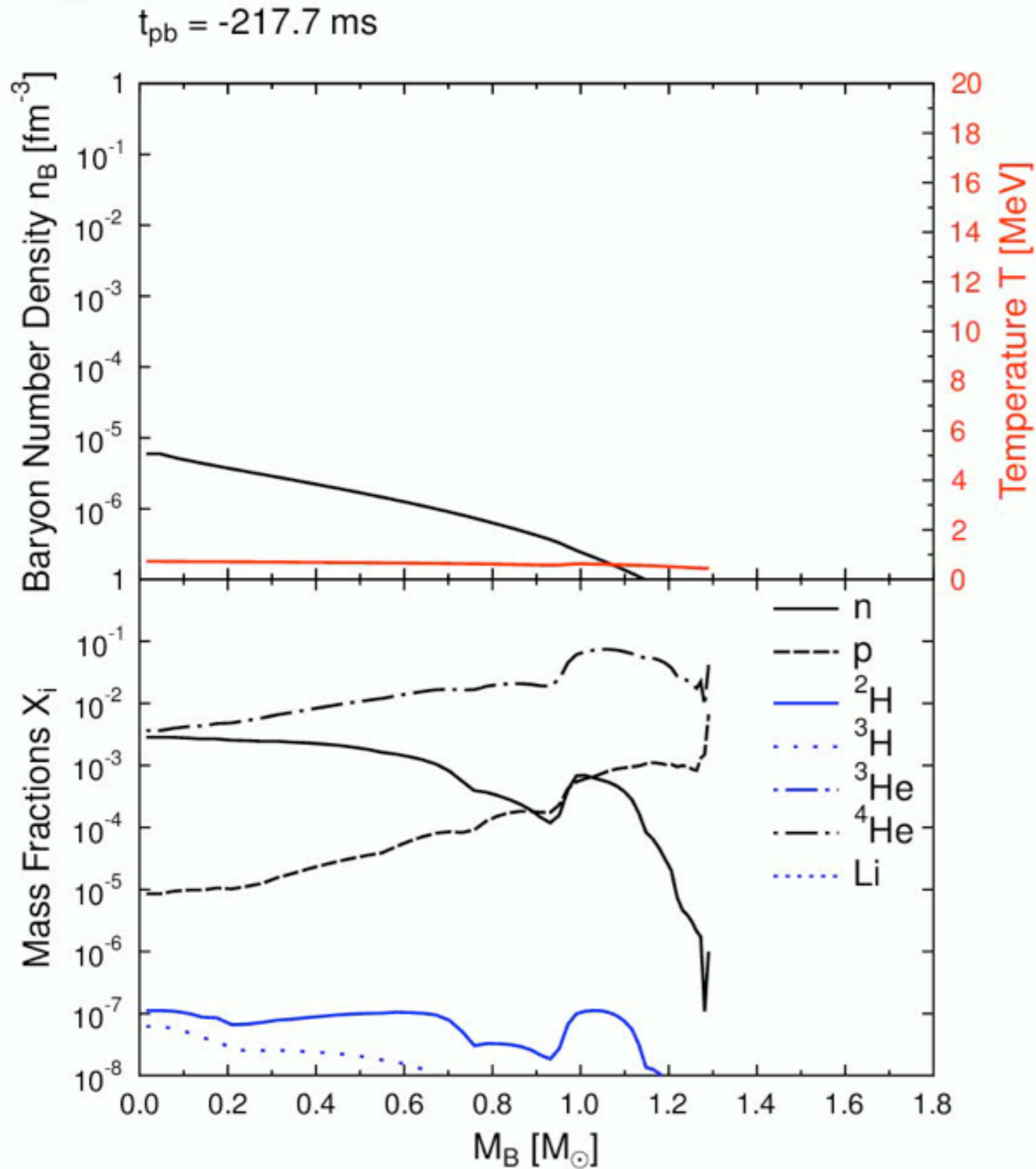
- detailed knowledge of nuclear composition
- based on nuclear structure calculations

→ allows more consistent treatment of e-captures

EOS aspects at subsaturation densities

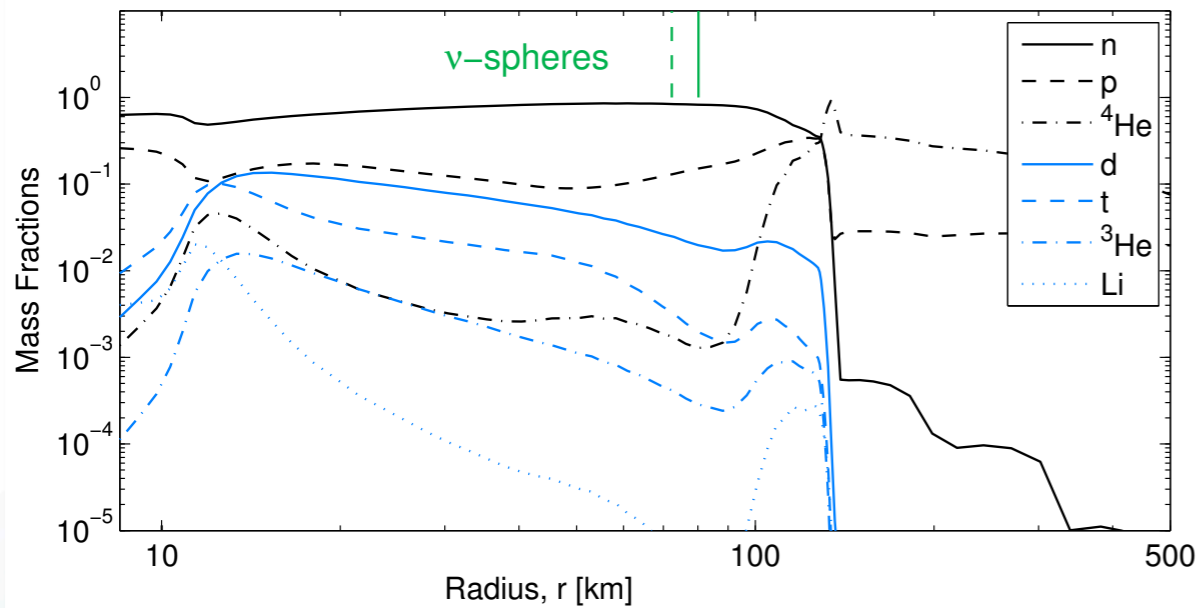
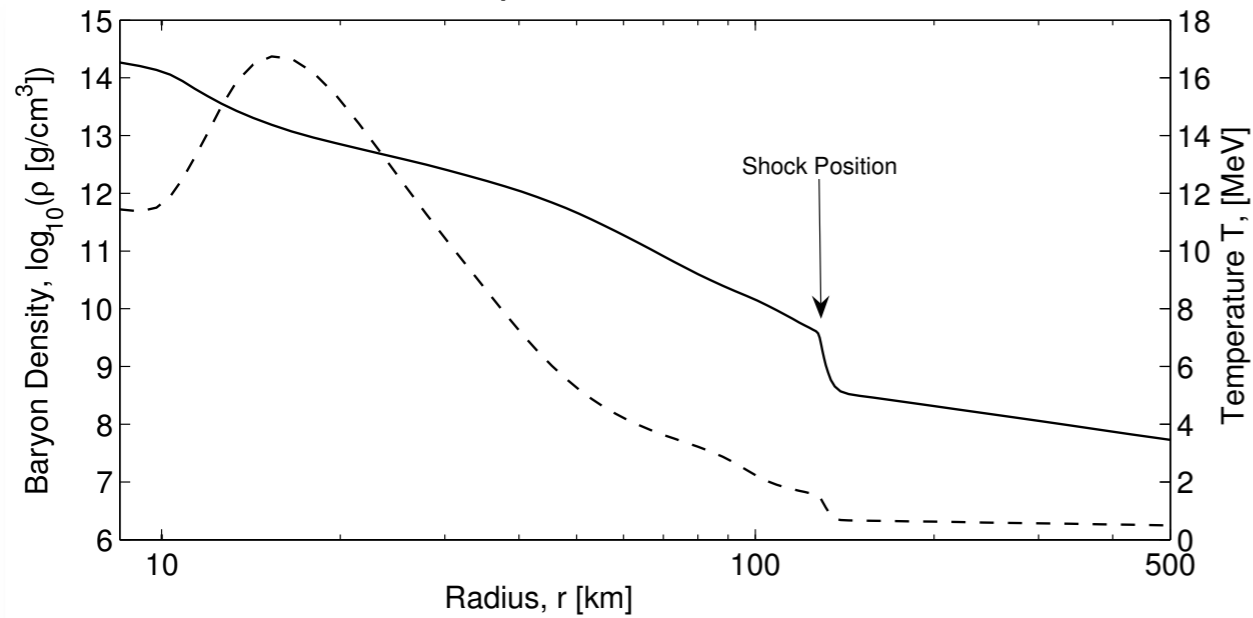
- light nuclei

Light nuclei – animation from simulation

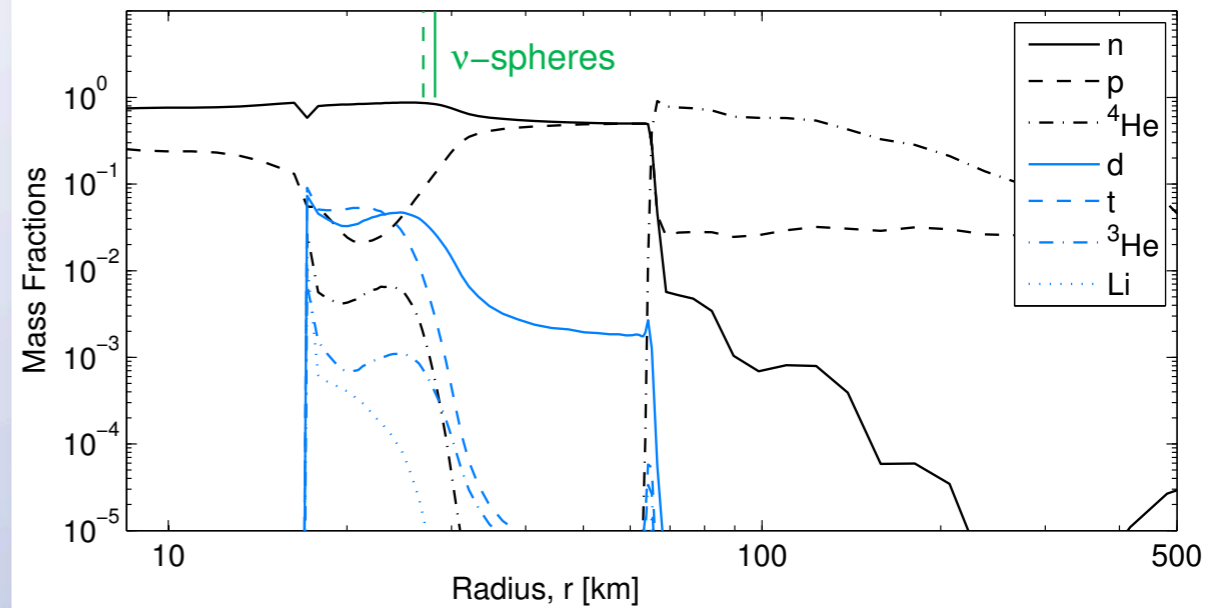
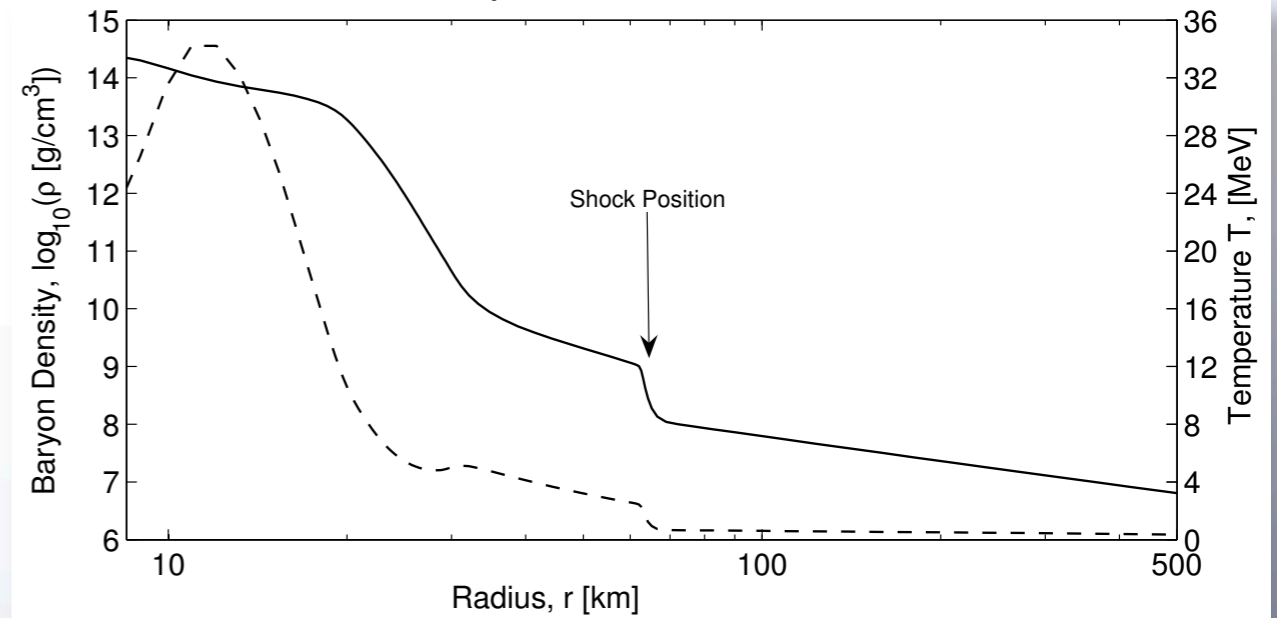


Light nuclei – postbounce evolution

$t_{pb} = 50$ ms



$t_{pb} = 500$ ms



- PNS core: nucleons
- PNS envelope & shock heated matter: light nuclei
- infalling matter: heavies, alphas, nucleons

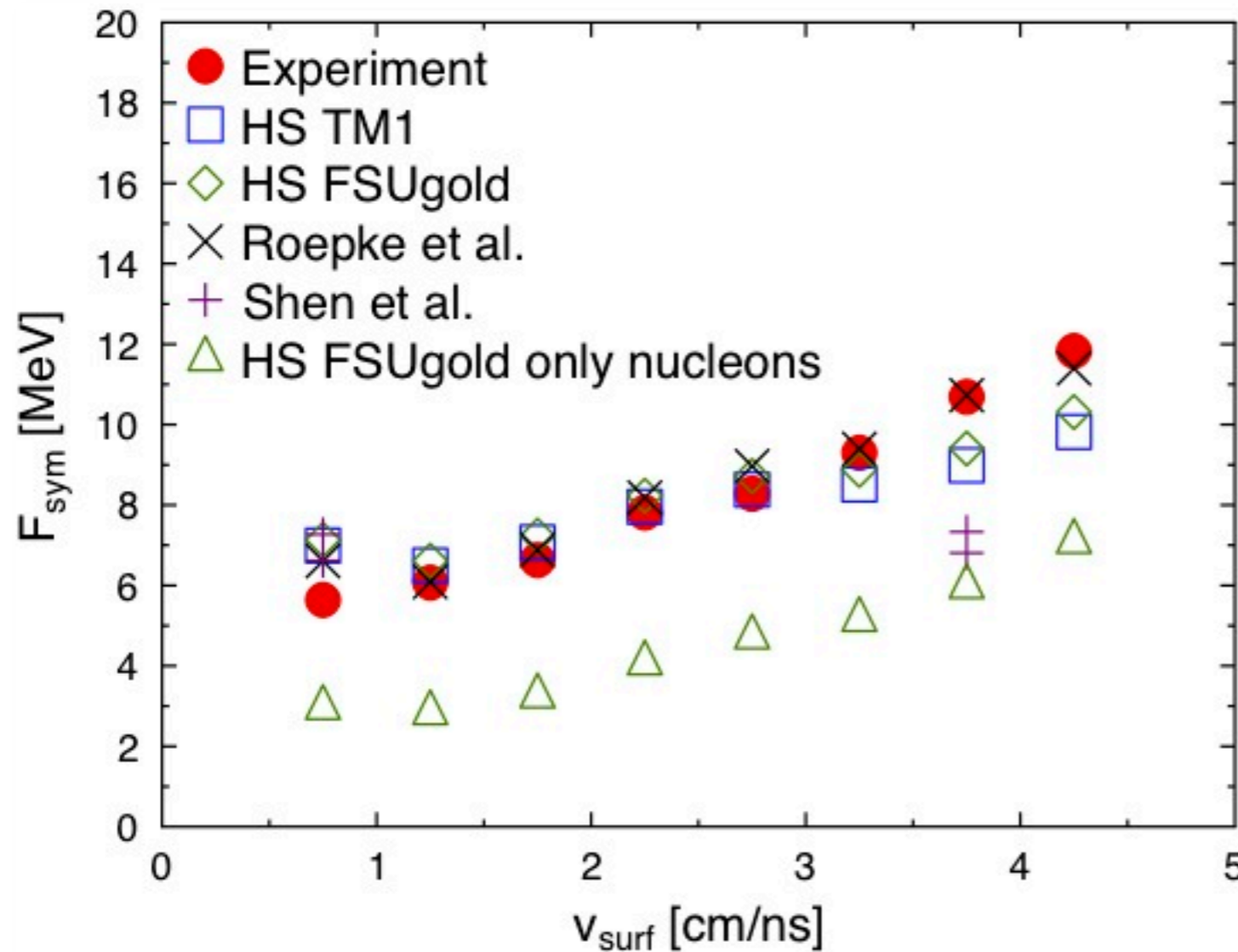
compare also:
Sumiyoshi & Röpke PRC77 2008
Typel et al. PRC81 2010

Effect of light clusters on heating/cooling?

- S. Nakamura's talk
- Arcones et al. 2008: significant change of anti-neutrino energies ~ 1 MeV
- O'Connor et al. 2007: $A=3$ breakup important for ν -energy loss
- difficult! more light nuclei \leftrightarrow less unbound protons

Light nuclei - measurement of symmetry energy

based on: [Natowitz et al.; 2010PRL104]



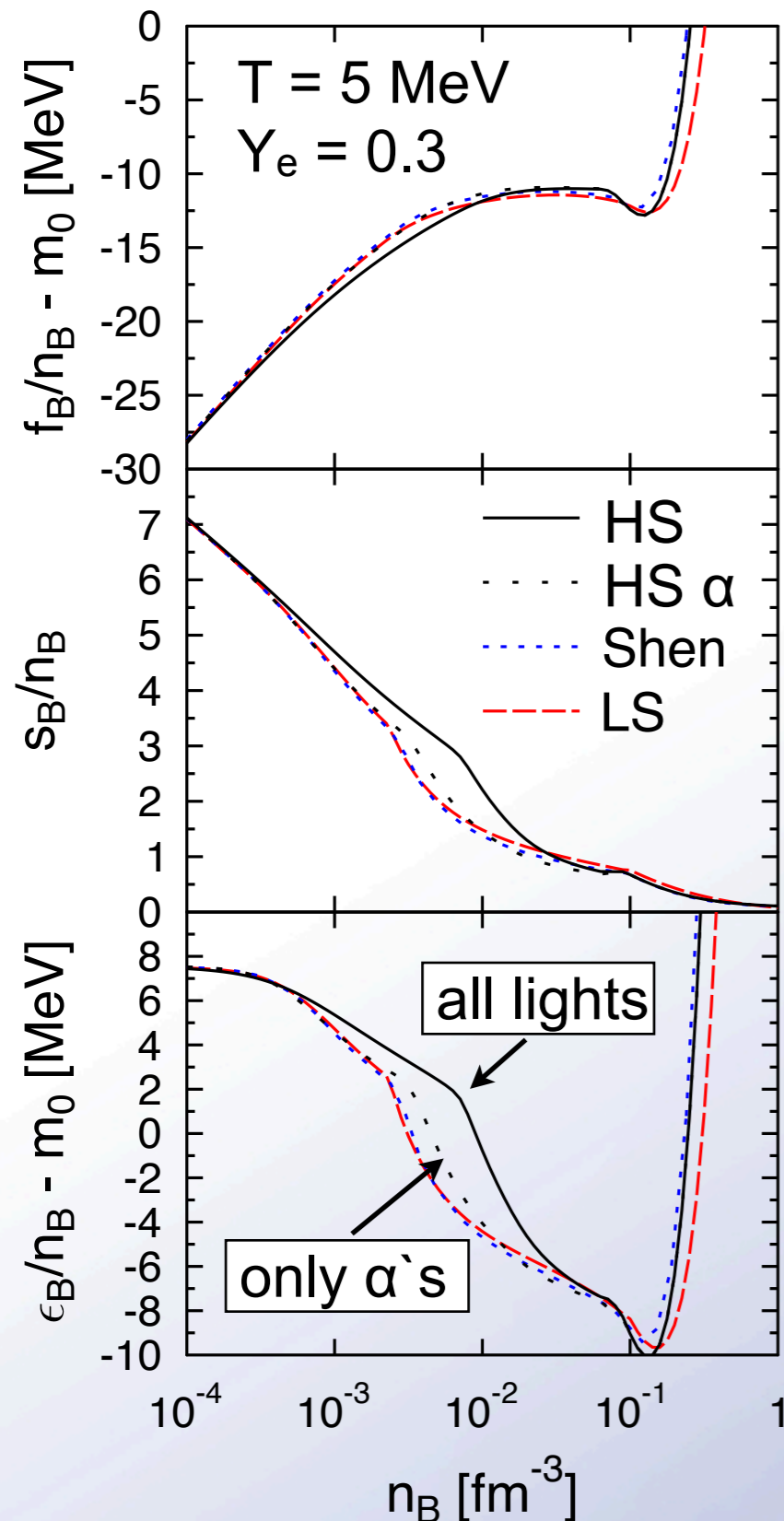
- free symmetry energy extracted from low-energy heavy ion collisions
 - thermodynamic conditions: $T = 3 - 7$ MeV and $n_B = 1/100 - 1/20 n_B^0$
 - cluster formation leads to increased symmetry energy
- experimental evidence for appearance of light clusters in SN

T [MeV]	3.3	3.3	3.6	4.2	4.7	5.3	6.2	7.5
$n_B [10^{-3} \text{ fm}^{-3}]$	2.1	1.7	2.3	3.8	4.7	4.9	5.5	6.4

$$E_{\text{sym}}(n_B, T) = 1/2 (E_{\text{sym}}(n_B, T, Y_p = 1) + E_{\text{sym}}(n_B, T, Y_p = 0)) - E_{\text{sym}}(n_B, T, Y_p = 0.5)$$

[Typel et al.; 2010PRC81]
[Kowalski et al.; 2007PRC75]

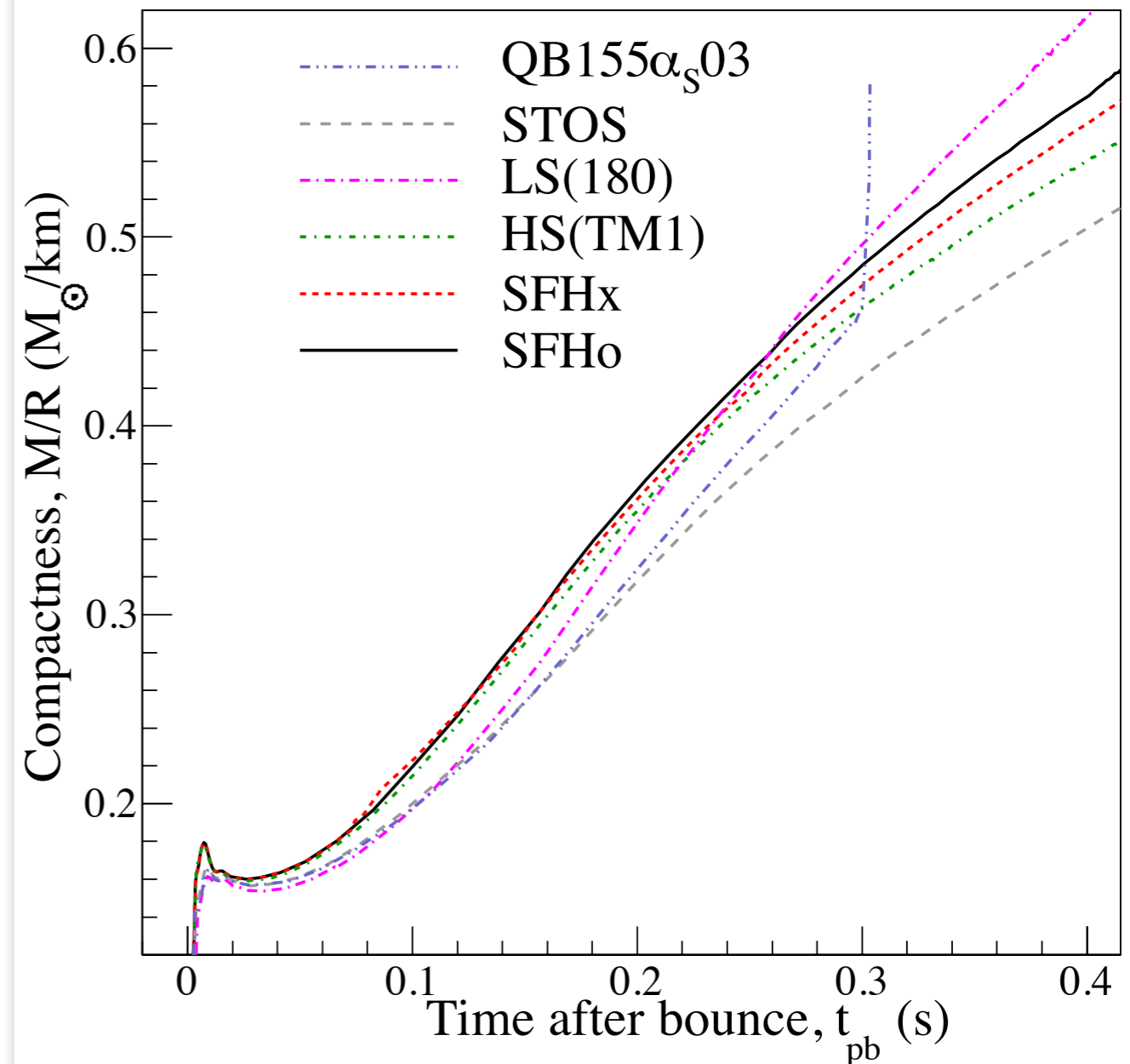
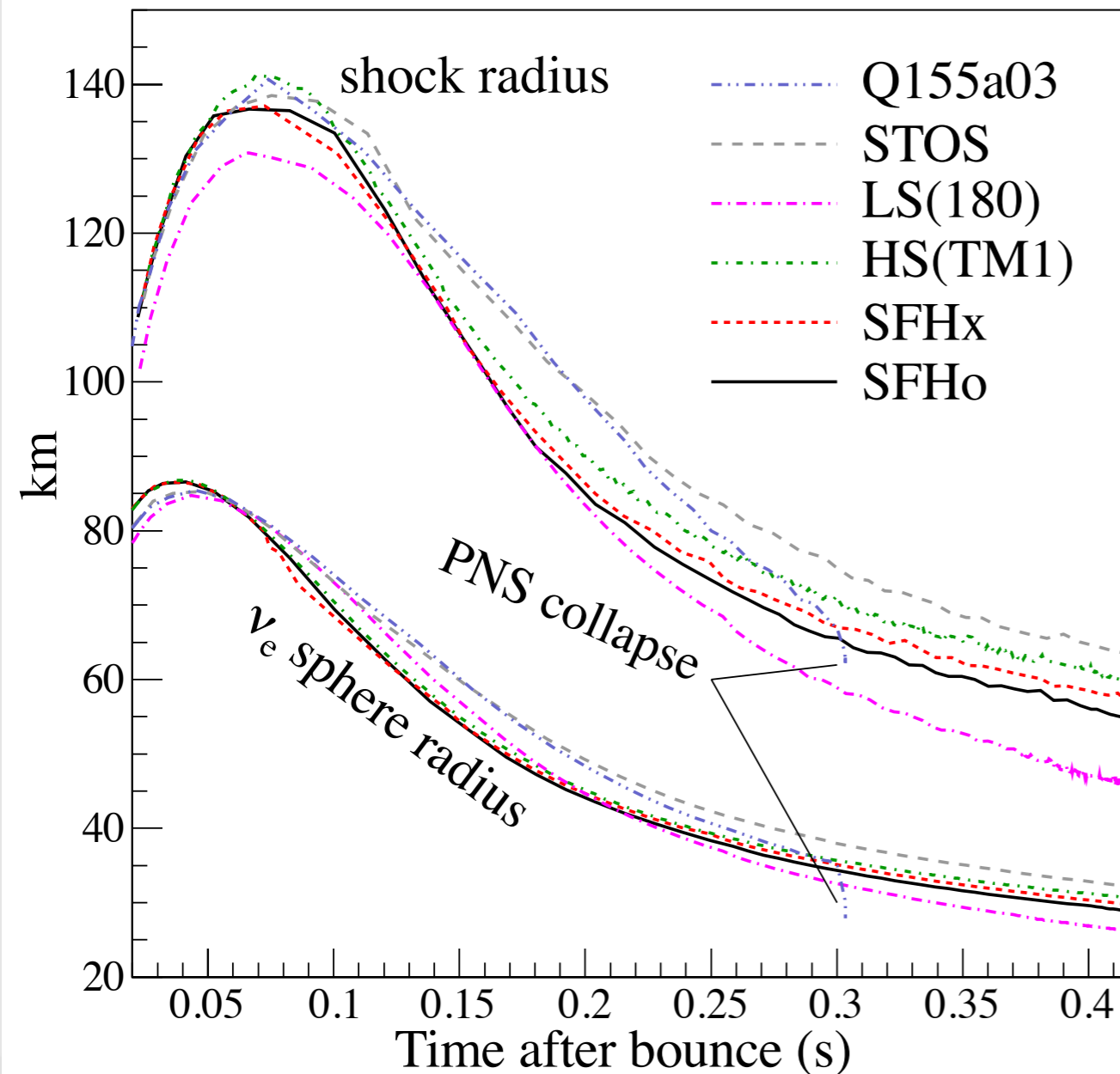
Effect of light clusters on the EOS



- significant increase of entropy and internal binding energy
 - light nuclei contribute to the symmetry energy \rightarrow increase of Y_e in beta-equilibrium
- \rightarrow non-trivial effect of LC on the EOS
 \rightarrow change of the structure of the proto-neutron star envelope

EOS effect on SN dynamics

[A. Steiner, MH & T. Fischer, in preparation]



- new EOS give most compact PNS up to 300 ms post-bounce
- complicated role of J, K, L
- low density EOS as important as nuclear interactions

EOS aspects at supersaturation densities

- time until black hole formation for a $40 M_{\text{sun}}$ progenitor

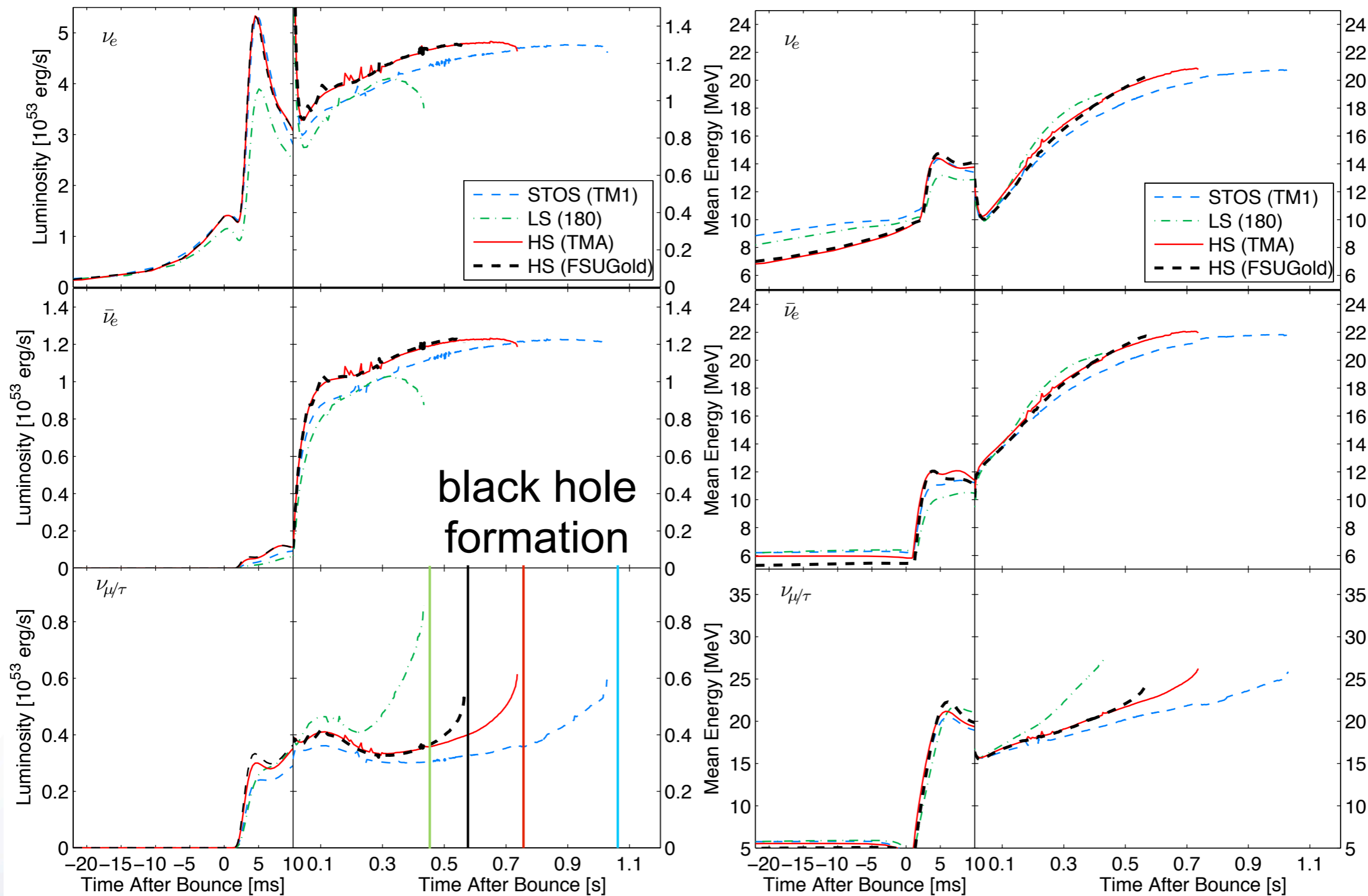
Supernova simulations – EOS signal from black hole formation

MH, T. Fischer, J. Schaffner-Bielich, M. Liebendörfer, ApJ 748, 70 (2012)

- simulations by Tobias Fischer, GSI Darmstadt
- 40 M_{sun} progenitor of Woosley & Weaver ApJS 101 (1995)
- setup as for the 15 M_{sun} progenitor

- “failed supernova“: collapse to black hole
- high densities and temperatures

Neutrino signal

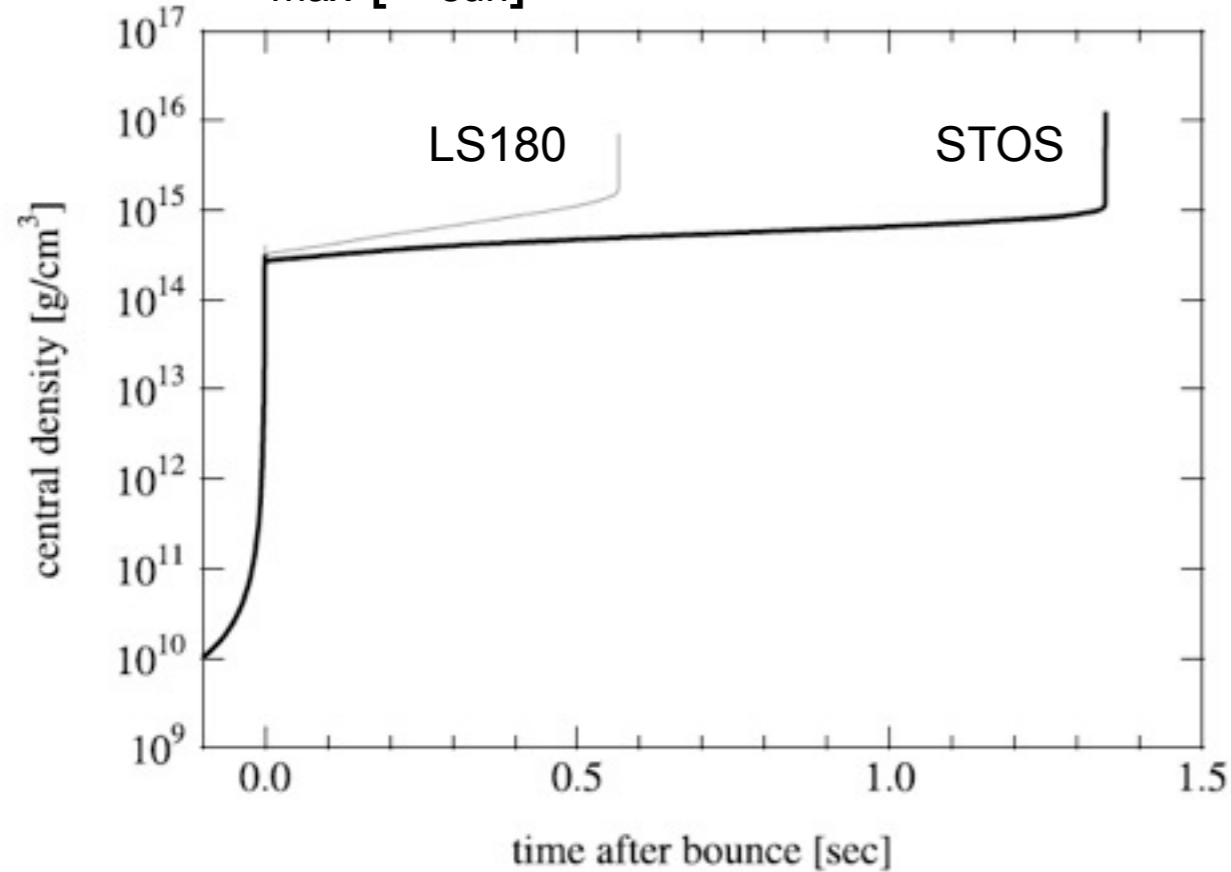


- μ/τ -neutrinos most sensitive to EOS because emitted from deeper layers

Time until black hole formation

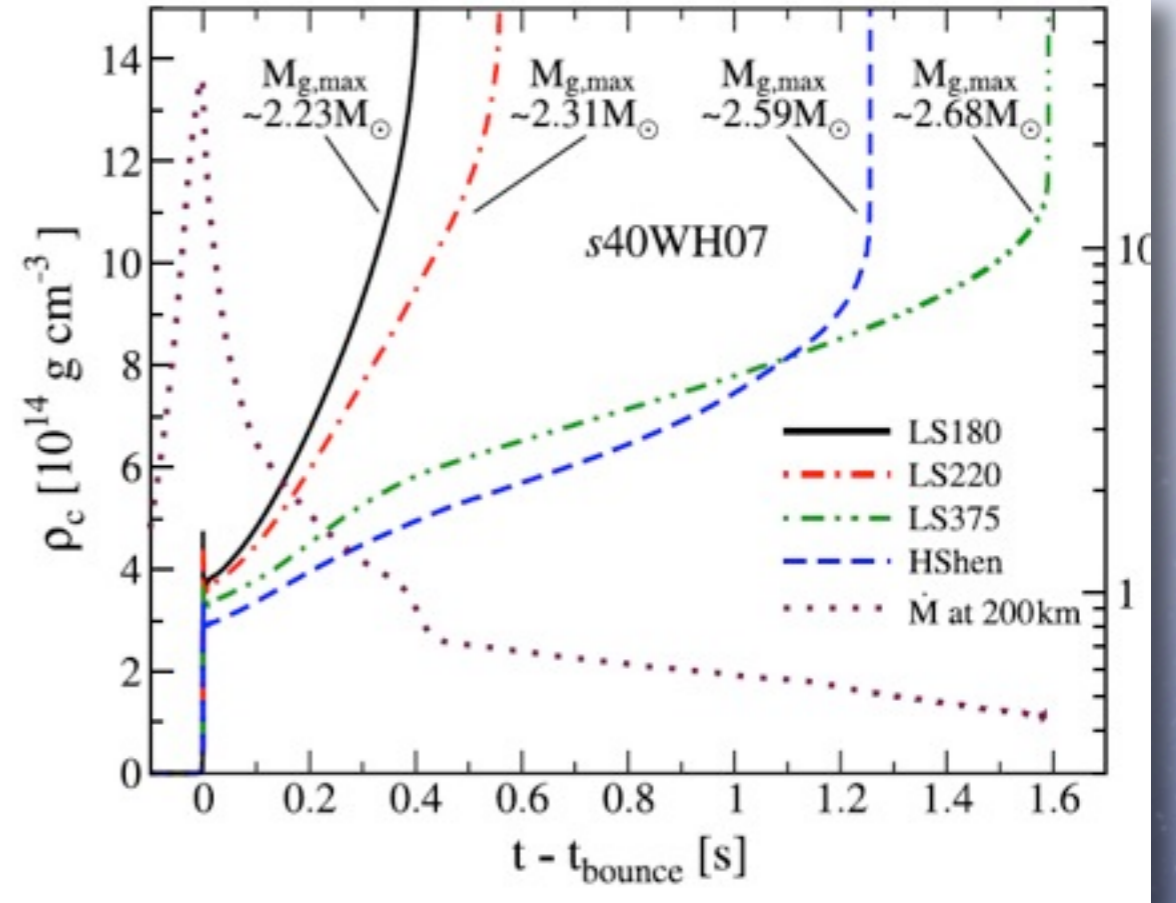
- previous EOS studies for this progenitor

K [MeV]:	180	282
M_{max} [M_{sun}]:	1.83	2.21



[Sumiyoshi et al.; 2007ApJ667]

180	220	282	375
1.83	2.03	2.21	2.72



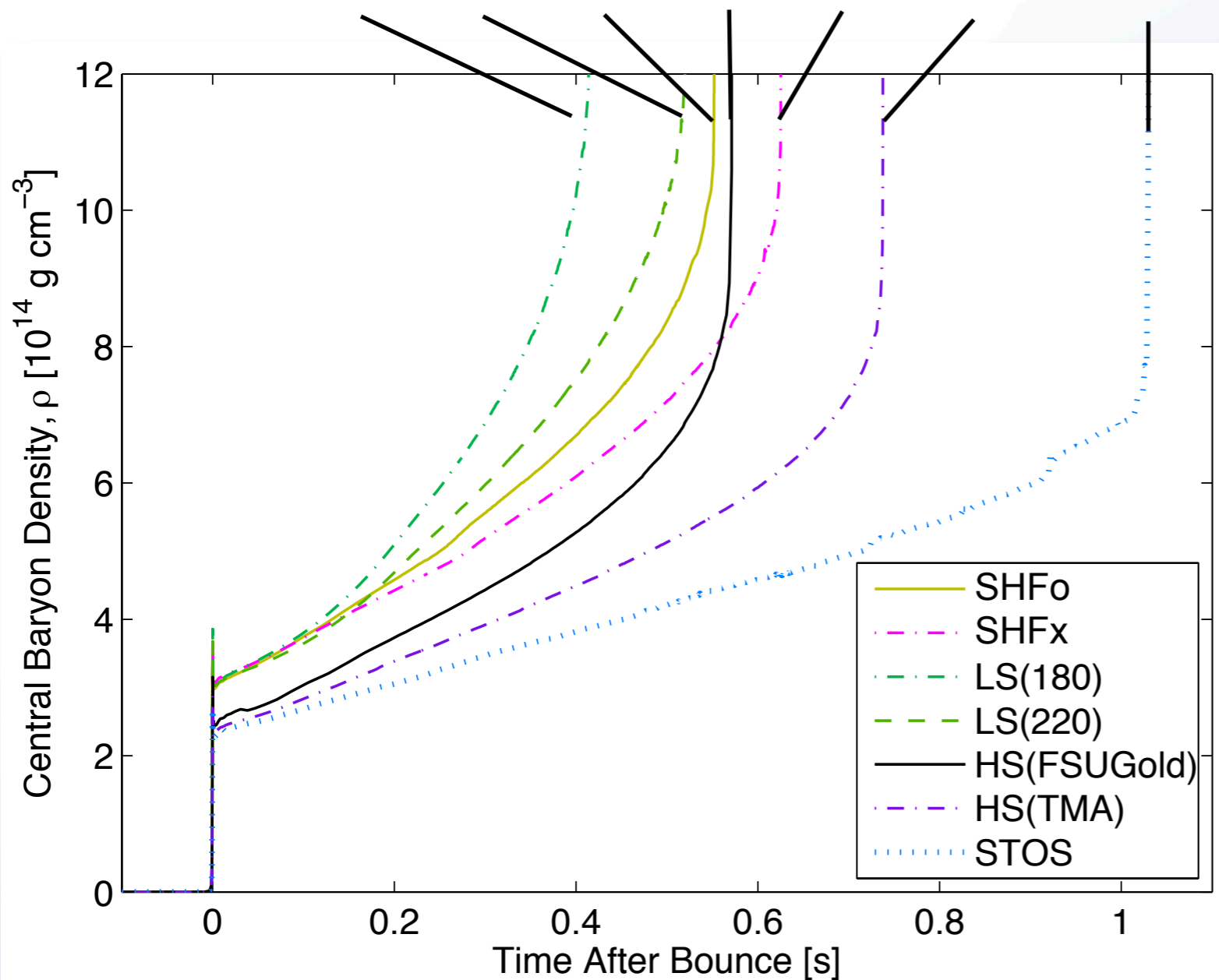
[O'Connor & Ott.; 2011ApJ730]

- possible conclusion: the compressibility K / the maximum mass M_{max} dictates „soft“ or „stiff“ behavior

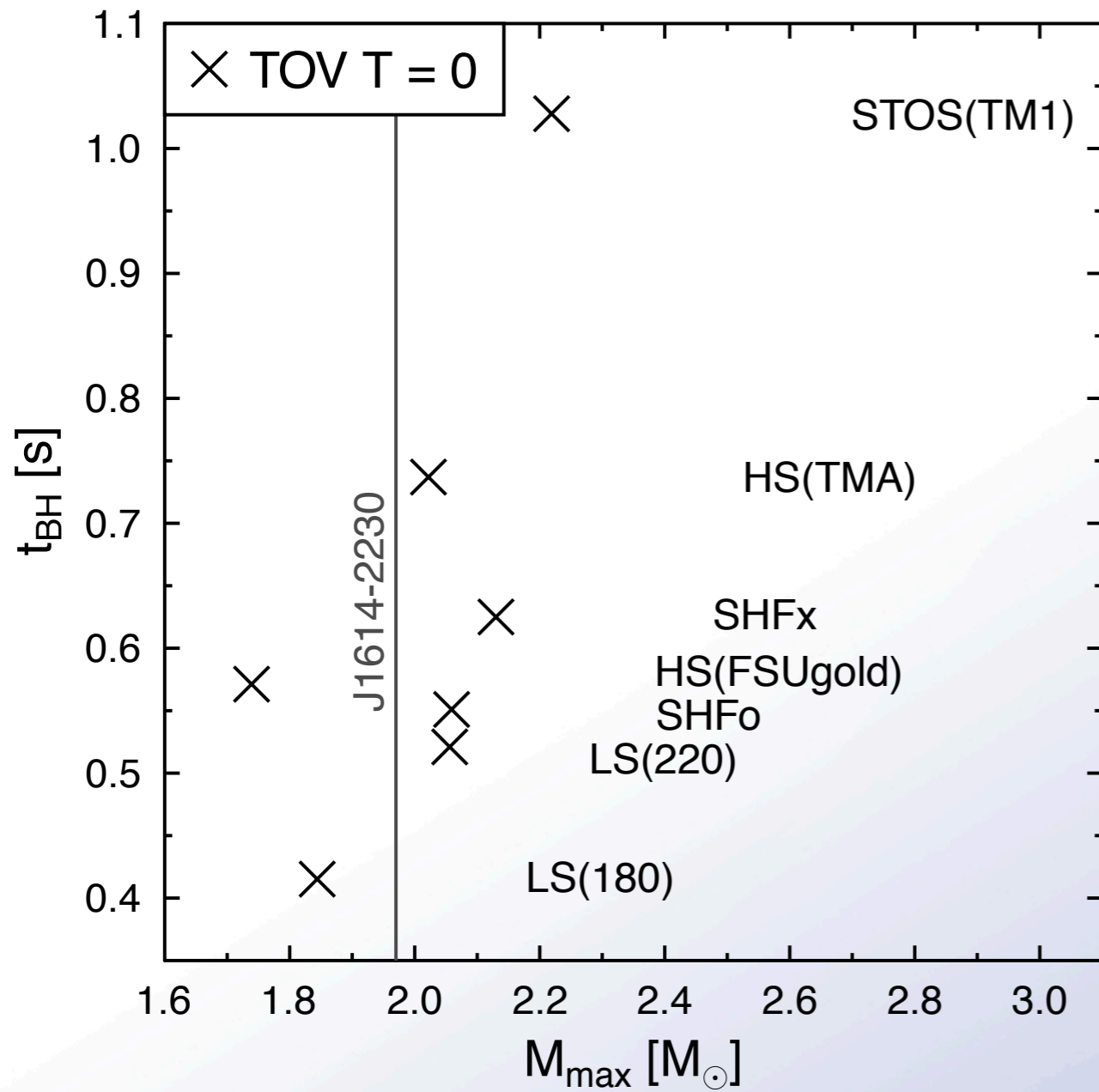
Time until black hole formation

K [MeV]:	180	220	245	230	239	318	282
M_{\max} [M_{sun}]:	1.83	2.03	2.06	1.74	2.13	2.02	2.21
J [MeV]:	28.6	28.6	31.6	32.6	28.7	30.7	37.0
L [MeV]:	73.8	73.8	47.1	60.4	23.2	90.1	111.0

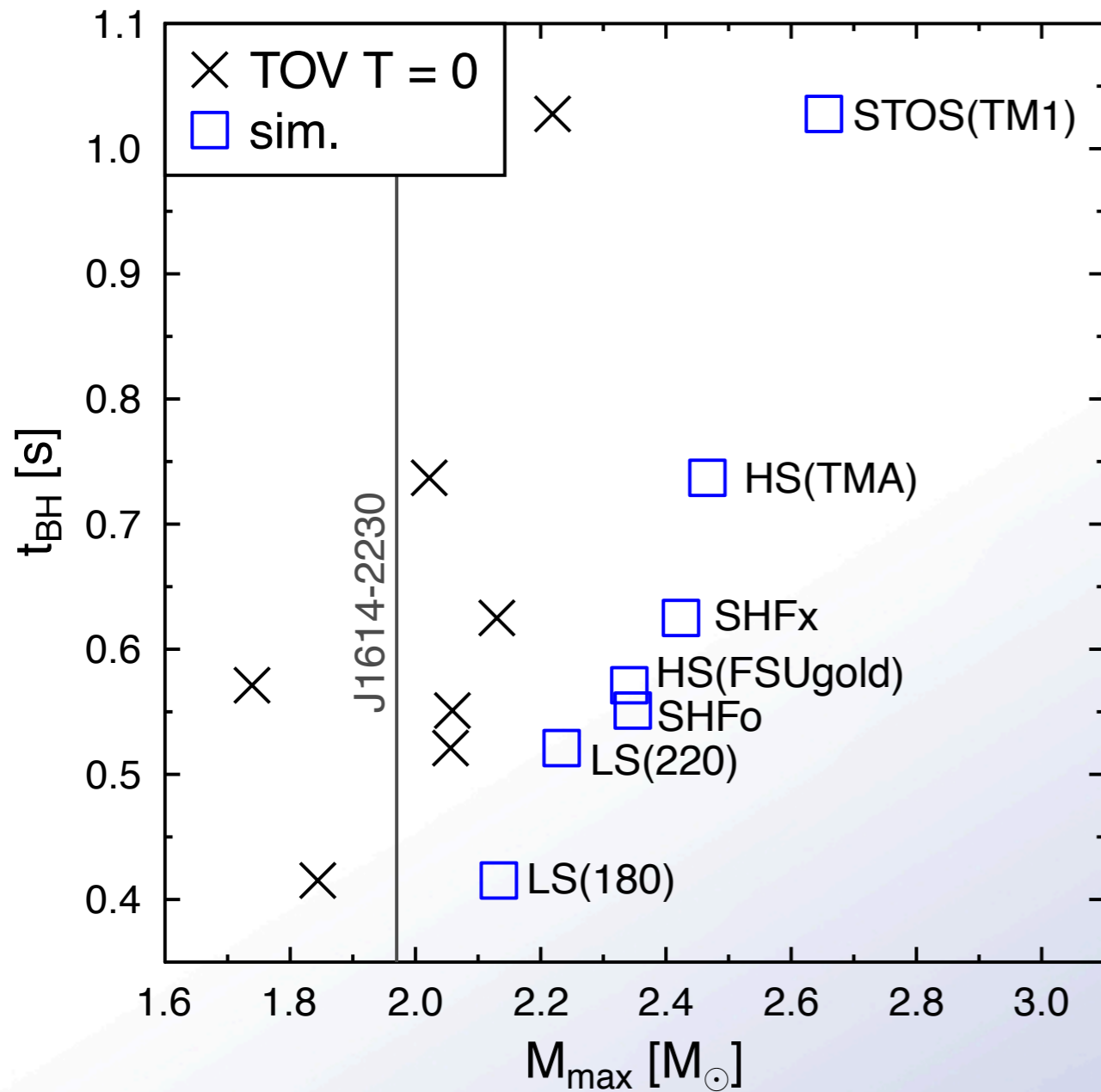
- unexpected: maximum mass of cold neutron stars and K are not directly correlated with t_{BH}
- neither correlated with J or L
- not found before, because only STOS and LS were available



Correlation of t_{BH} with maximum mass

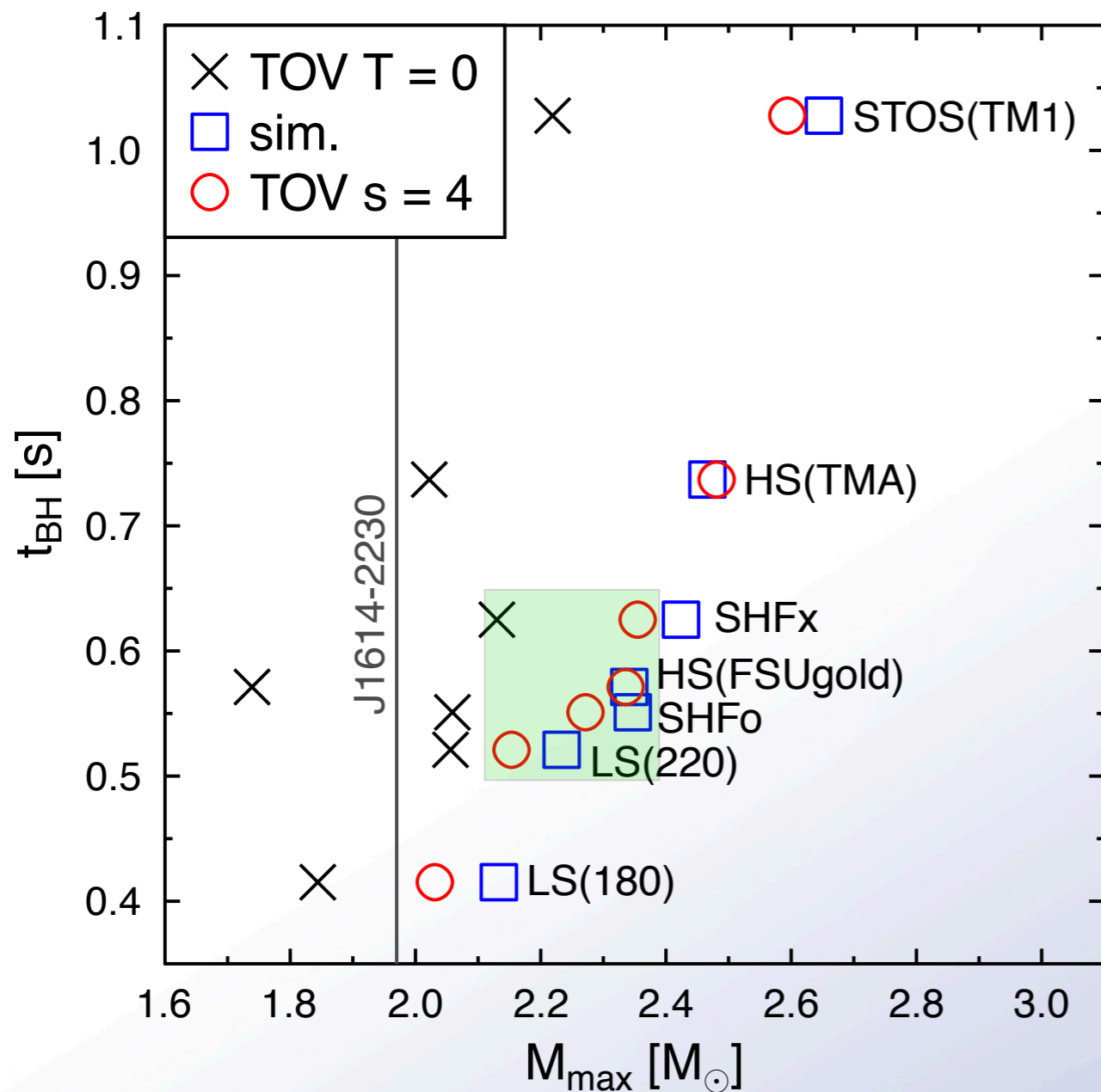


Correlation of t_{BH} with maximum mass



- maximum masses in the simulations are significantly increased (up to $0.6 M_{\text{sun}}$) compared to $T = 0$

Correlation of t_{BH} with maximum mass



- maximum masses in the simulations are significantly increased (up to $0.6 M_{\text{sun}}$) compared to $T = 0$
- static $s = 4$ configurations reproduce the simulations
- new correlation: t_{BH} gives information about the finite entropy EOS
- degeneracy with progenitor star
[O'Connor & Ott.; 2011ApJ730]
- experimental/observational EOS constraints important

- extreme temperature effects
→ is there a simple explanation?

Specific heat

- specific heat in the degenerate limit as a guide-line:

$$c_V = T\alpha_V = s$$

$$\epsilon(T, n_B, Y_e) = \epsilon(T = 0, n_B, Y_e) + \frac{1}{2}T^2 n_B \alpha_V$$

$$\alpha_V = \left(\frac{\pi}{3}\right)^{2/3} n_B^{-2/3} m^* \left[Y_e^{1/3} + (1 - Y_e)^{1/3} \right] \quad \text{“level-density parameter”}$$

[Lattimer & Prakash, *Phys. Rep.* 2000]

- change of pressure:

$$p(T, n_B, Y_e) = p(T = 0, n_B, Y_e) + \Delta\epsilon \left(\frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} \frac{n_B}{Y_e^{2/3}} \frac{\partial Y_e}{\partial n_B} \right)_{T=0}$$

$$\Delta\epsilon = \frac{1}{2}T^2 n_B \alpha_V$$

Finite entropy effect in neutron stars

- beta-equilibrium, (here: without neutrinos):

$$Y_e = Y_e(n_B)$$

$$Y_e \sim \frac{4^3 E_{\text{sym}}^3}{3\pi^2 n_B}$$

- TOV eqs: only $p(\epsilon)$ relevant, not $p(n)$
- change of the pressure due to entropy at given energy density:

$$\begin{aligned} \Delta p_\epsilon &= p(s, \epsilon) - p(T = 0, \epsilon) && \boxed{\text{at } n_0:} && \boxed{\text{J, L}} && \boxed{\text{K, J, L}} \\ &= \frac{1}{2} s^2 n_B \frac{1}{\alpha_V} \left(\frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} \frac{n_B}{Y_e^{2/3}} \frac{\partial Y_e}{\partial n_B} - \frac{n_B}{\mu_B} \frac{\partial \mu_B}{\partial n_B} \right) \\ &&& \boxed{<0} && \boxed{>0} && \boxed{>0} \end{aligned}$$

$$\alpha_V = \left(\frac{\pi}{3}\right)^{2/3} n_B^{-2/3} m^* \left[Y_e^{1/3} + (1 - Y_e)^{1/3} \right]$$



J

Finite entropy effect in neutron stars – effective mass?

	m^*/m at n_0	$M_{\max}(T=0)$	$\Delta M_{\max}(s=4)$
LS(220)	1	2.03	0.09
NL3	0.595	2.79	0.10
DD2	0.563	2.42	0.15
LS(180)	1	1.83	0.19
SHFo	0.761	2.06	0.21
SHFx	0.718	2.13	0.24
TM1	0.634	2.21	0.37
TMA	0.635	2.02	0.46
FSUgold	0.611	1.74	0.62

- despite the same specific heat LS(180) and LS(220) differ
 - larger K in agreement with smaller ΔM for LS(220)
 - note: $\Delta M < 0$ possible, i.e. softening, e.g. for LS(375) (O'Connor & Ott 2011)
 - temperature effects tend to decrease with high $M_{\max}(T=0)$
- it's not only m^{*0} or α_V^0

$$\Delta p_\epsilon = \frac{1}{2} s^2 n_B \frac{1}{\alpha_V} \left(\frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} \frac{n_B}{Y_e^{2/3}} \frac{\partial Y_e}{\partial n_B} - \frac{n_B}{\mu_B} \frac{\partial \mu_B}{\partial n_B} \right)$$

$$\alpha_V = \left(\frac{\pi}{3} \right)^{2/3} n_B^{-2/3} m^* \left[Y_e^{1/3} + (1 - Y_e)^{1/3} \right]$$

• around saturation density: $\left. \frac{\partial \mu_B}{\partial n_B} \right|_{n_B^0} = \frac{1}{n_B^0} \left(\frac{1}{9} (K + K_{\text{sym}}) + \frac{2}{3} L \right)$

Conclusions

- new SHFo and SHFx EOS: fit to NS observations
- EOS tables and nuclear composition available for NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx:
<http://phys-merger.physik.unibas.ch/~hempel/eos.html>
- the model for nuclei can be as important as the nuclear interactions
- could light nuclei influence the PNS cooling and/or the v-driven wind?
- black hole formation gives information about the finite entropy EOS
- temperature effects depend significantly on the EOS model
- complex role of the EOS, not “soft or stiff“
- individual nuclear matter properties are not sufficient to characterize the EOS
- EOS effect on explosions: → 3D simulations

New equations of state in simulations of core-collapse supernovae

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