Hydrodynamics of CCSNe at the Transition to Explosion

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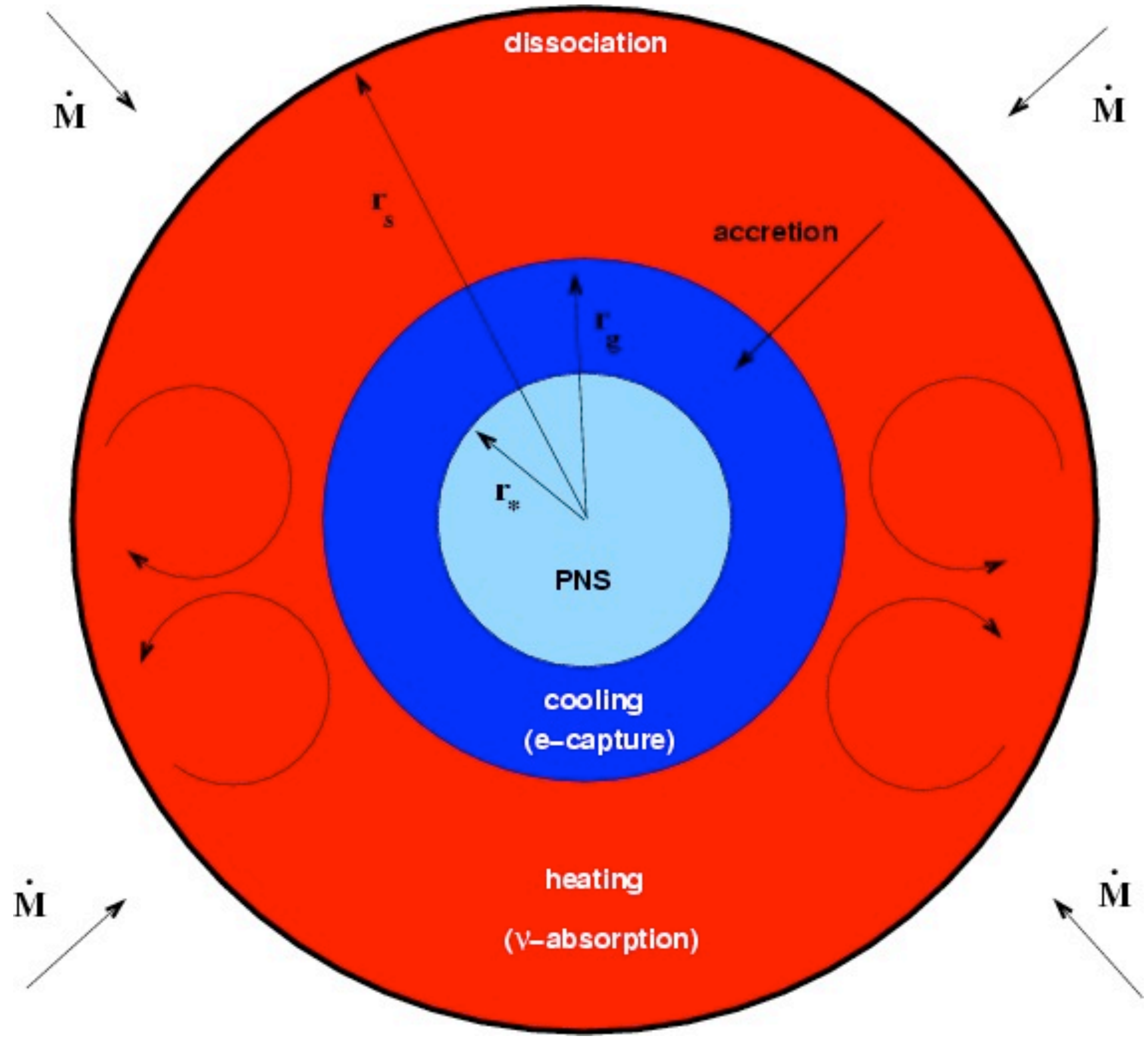
Stalled Shock

Result of Core-Collapse and Bounce

Neutrino Mechanism: heating due to neutrino absorption by nucleons

(Bethe & Wilson 1985)



No Explosion in Spherical Symmetry

For stars that form iron cores

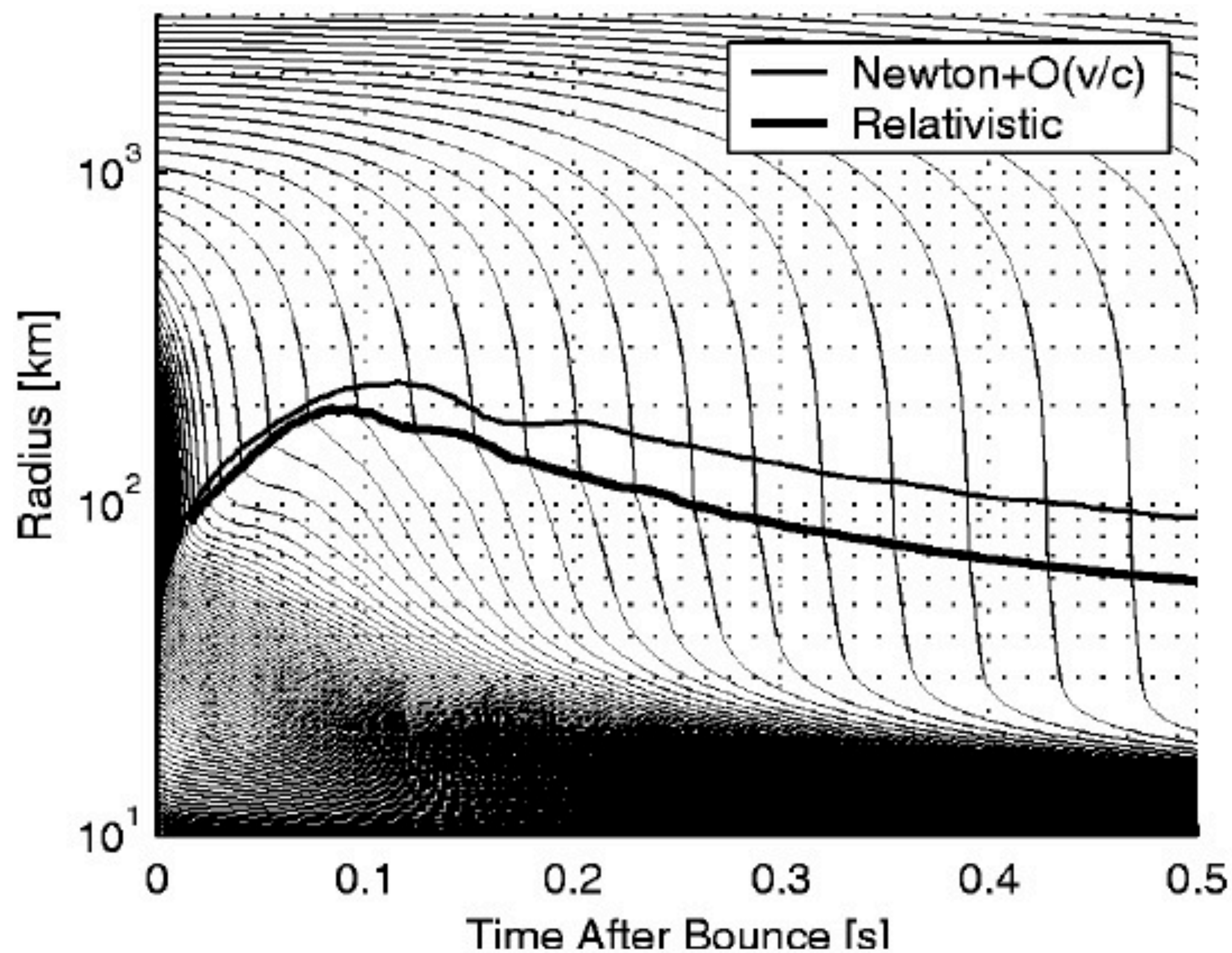
$$M \gtrsim 10 M_\odot$$

Agreement in
Supernova Community

(Liebendörfer et al. 2001,
T.Thompson et al. 2003,
Rampp & Janka 2002,
Sumiyoshi et al. 2005)

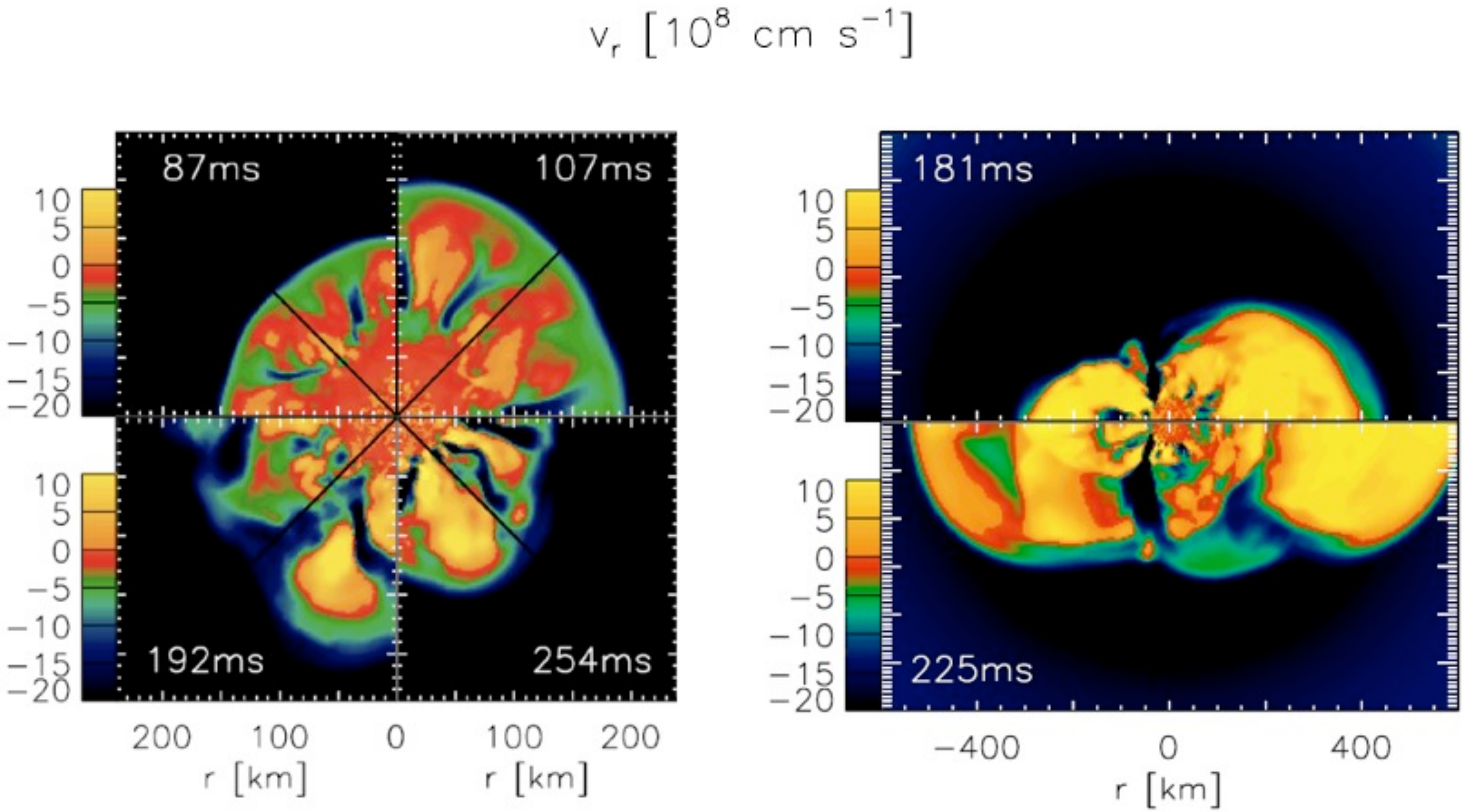
O-Ne-Mg cores do
explode in 1D

(Kitaura et al. 2006, Burrows et al. 2007)



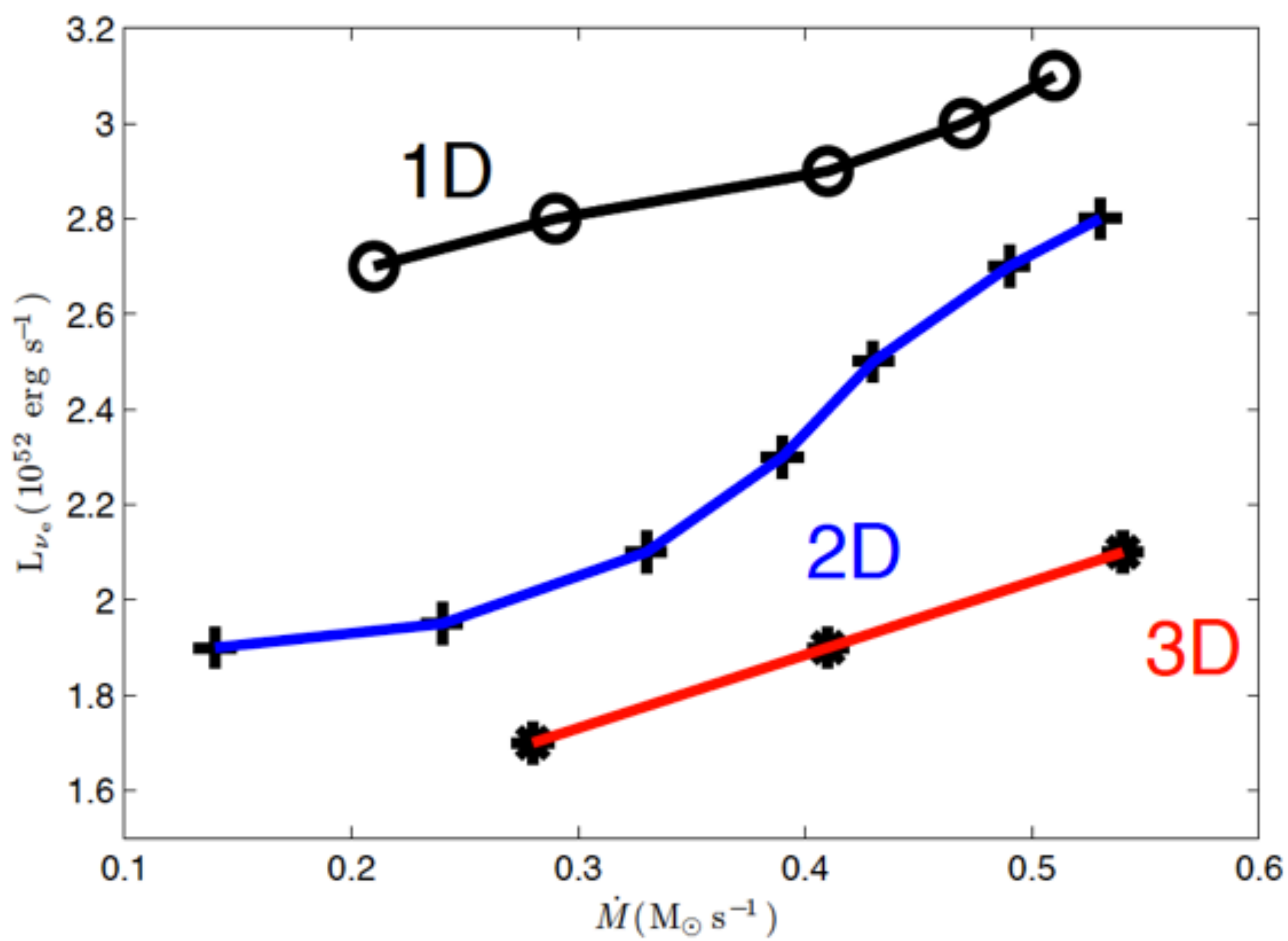
Liebendörfer et al. (2001)

Multidimensional Effects



Buras et al. (2006)

More Efficient with Increasing Dimensionality?



Nordhaus et al. (2010)

In disagreement with Hanke et al. (2011)

Parametric Hydrodynamic Study

- Identify hydrodynamic processes responsible for converting accretion flow into explosion

- Understand their dependence on system parameters and dimensionality

- Assess the effect of hydrodynamic instabilities on explosion mechanism (magnitude and robustness)

Method

- Time-dependent Hydrodynamic Simulations (FLASH3.2, modified grid)
- Steady-state initial and boundary conditions (stalled shock)
- H. Shen (1998) EOS via O'Connor & Ott (2010) implementation
- Bruenn (1985) weak interaction rates, lightbulb heating
- Point-mass gravity (time-independent)
- Begin with spherical symmetry (no turbulence)
- Equations solved:

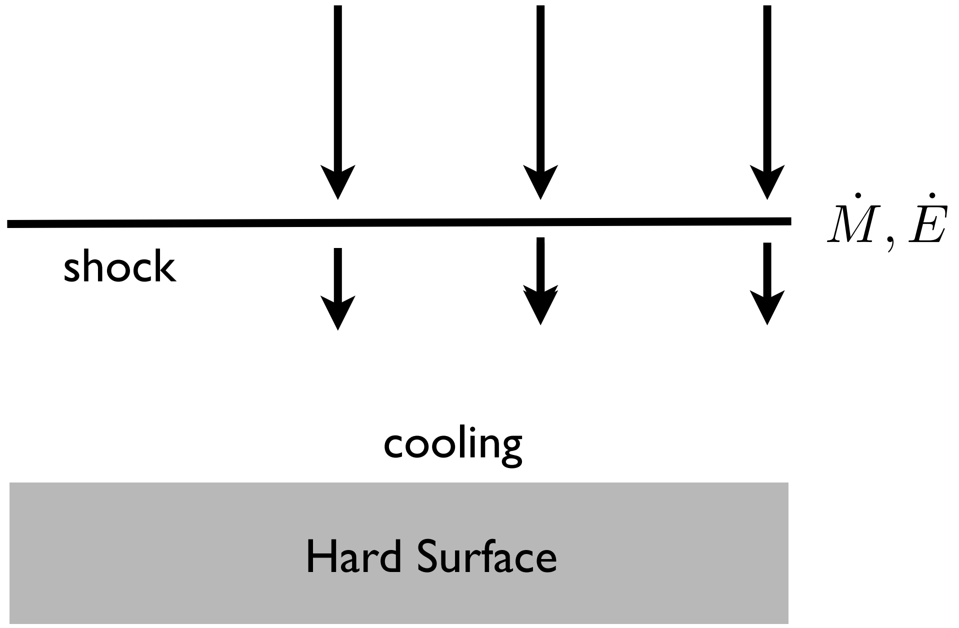
$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) = 0 \qquad \frac{\partial(\rho e)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(v_r[\rho e + p]) + \rho v_r \frac{GM}{r^2} = \mathscr{L}_{\rm net}$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{GM}{r^2} = 0 \qquad \frac{\partial Y_e}{\partial t} + v_r \frac{\partial Y_e}{\partial r} = \Gamma_{\rm net}.$$

RF (2012)

Limiting Cases (1D)

1) No Heating: Accretion Shock



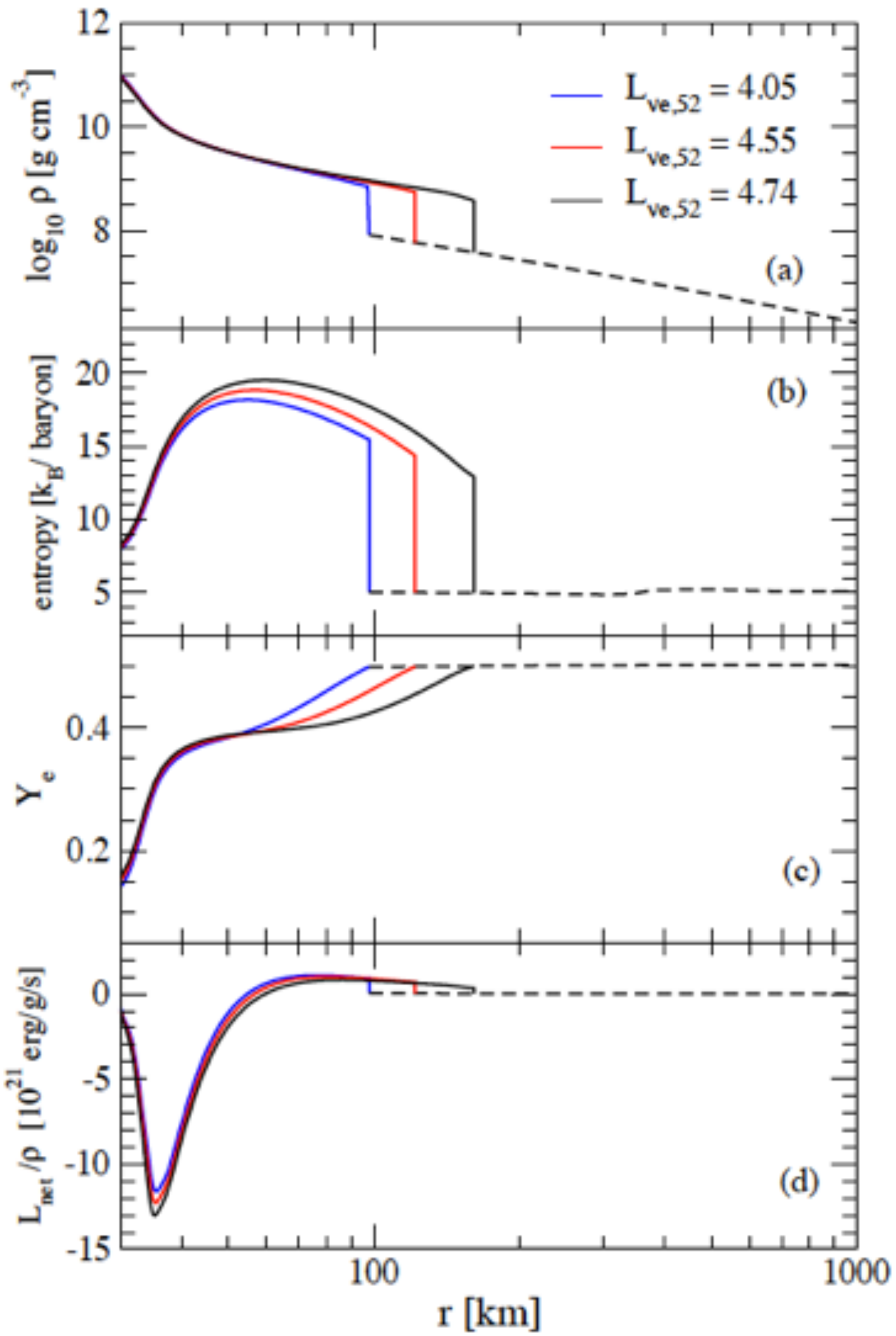
2) Strong Heating: Sedov-like explosion

$$L_\nu \left(\frac{\sigma_\nu}{r_g^2}\right)\left(\frac{M_\mathrm{g}}{m_n}\right) t_\mathrm{dyn} \gg E_\mathrm{g}$$

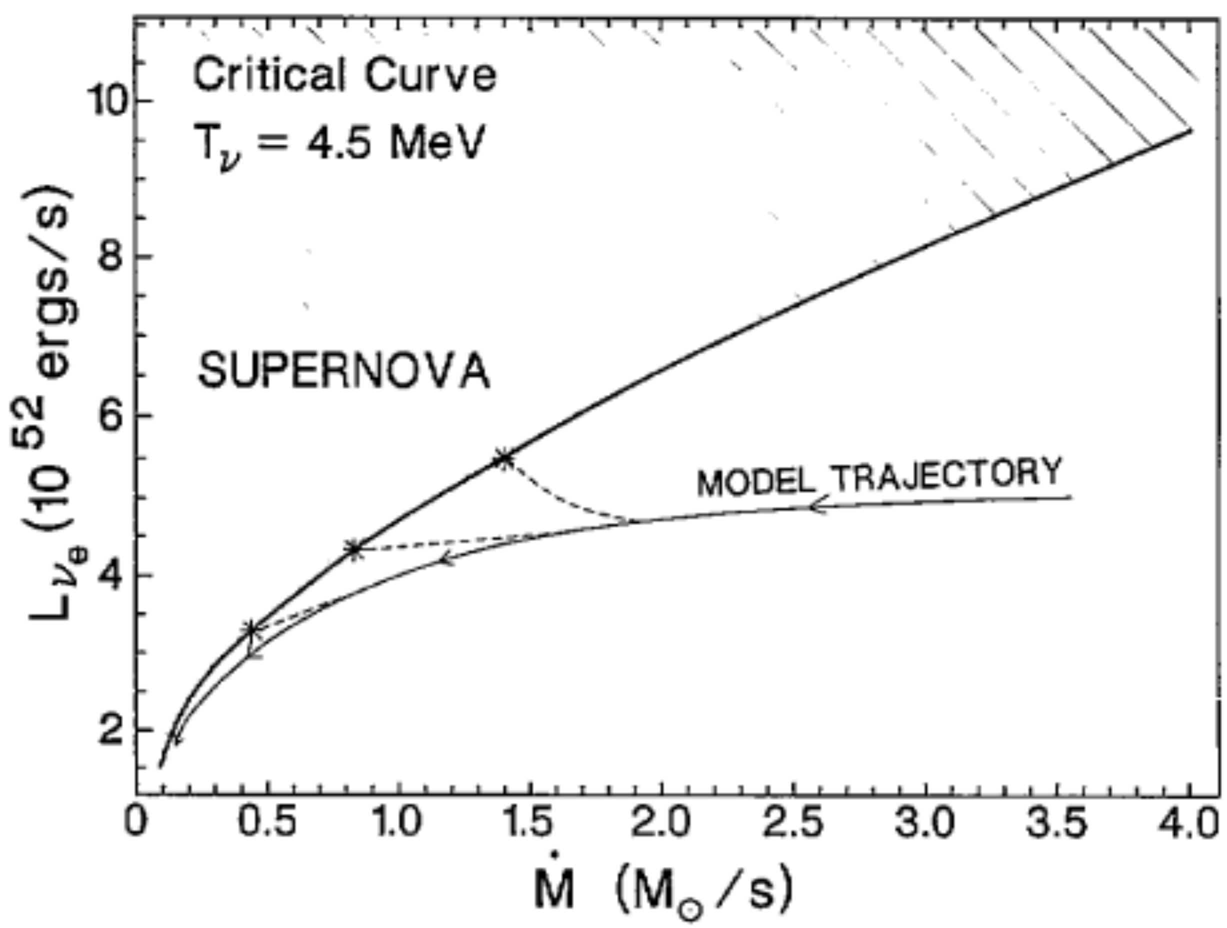
$$t_\mathrm{dyn} \gg t_\mathrm{heat}$$

Limiting Luminosity in Steady-State

(Burrows & Goshy 1993)



RF (2012)



Burrows & Goshy (1993)

BG 93 conjecture I: explosion involves global instability of accretion flow

BG93 conjecture II: instability threshold lies at the limiting luminosity

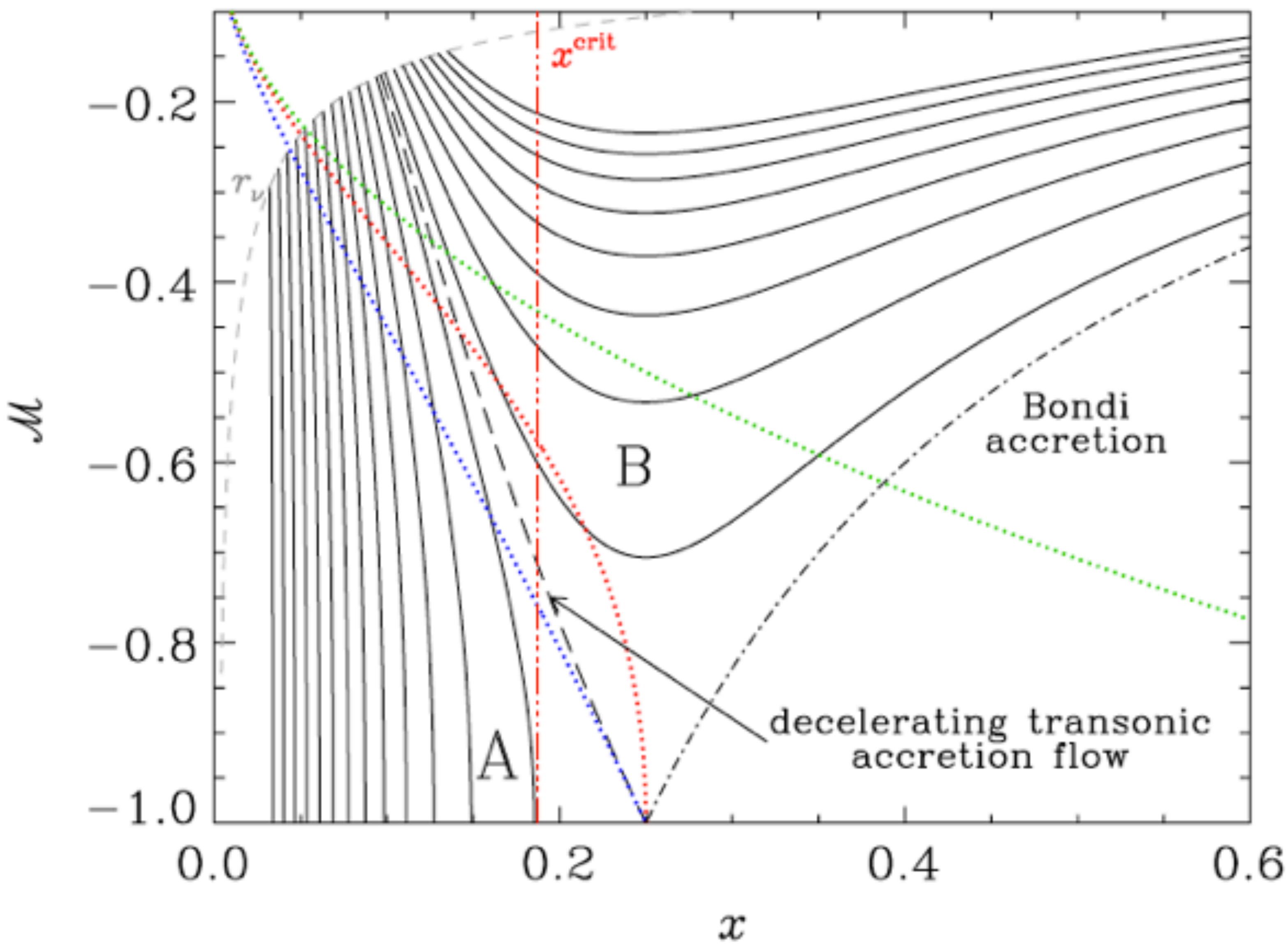
Limiting Luminosity in Steady-State

Antesonic condition:

$$\frac{c_s^2}{v_{\rm esc}^2} \simeq 0.2$$

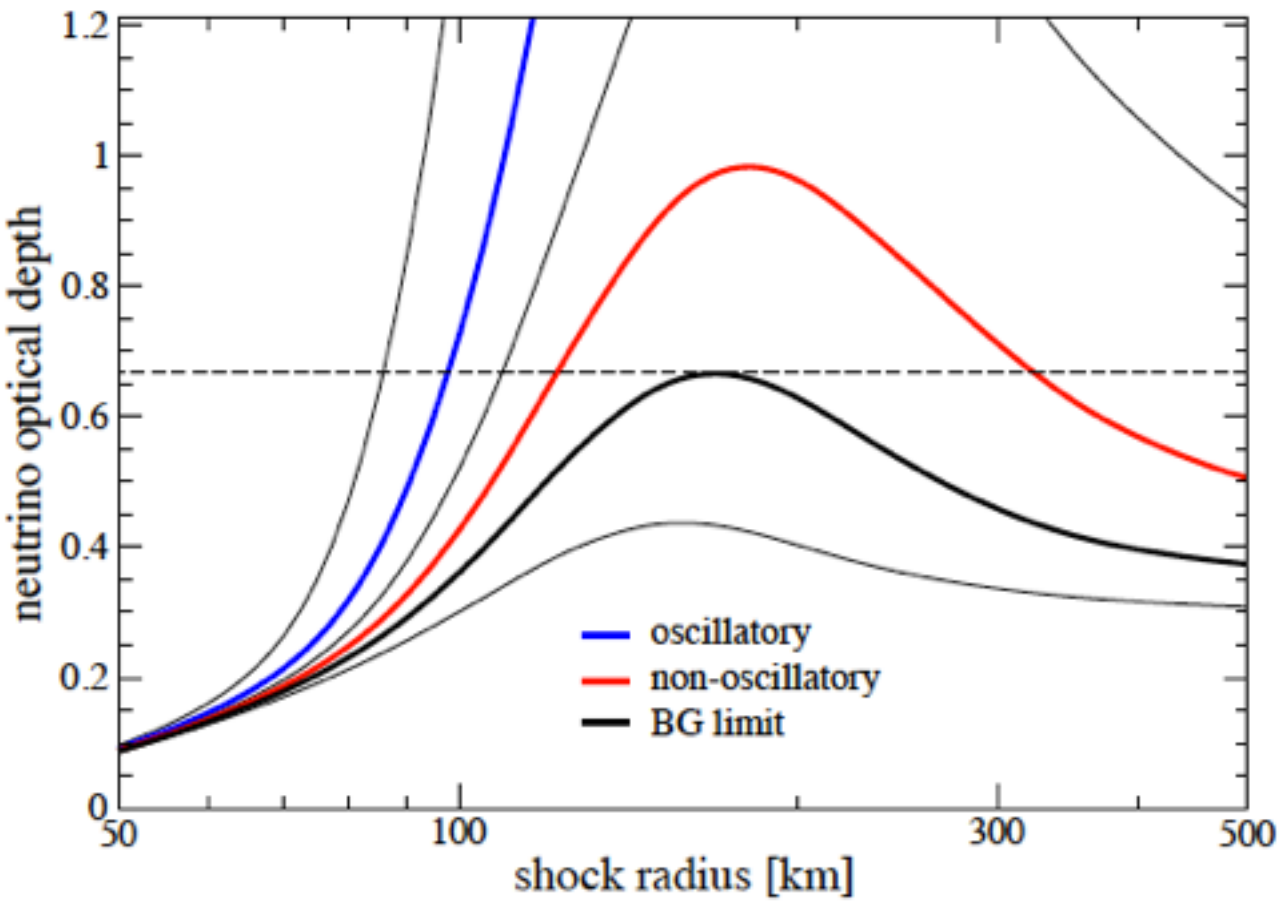
Assumes fixed boundary conditions at neutrinosphere and upstream flow

$$x = \frac{r c_T^2}{2GM} \qquad \mathcal{M} = \frac{v_r}{c_T}$$



(Pejcha & T.Thompson 2012)

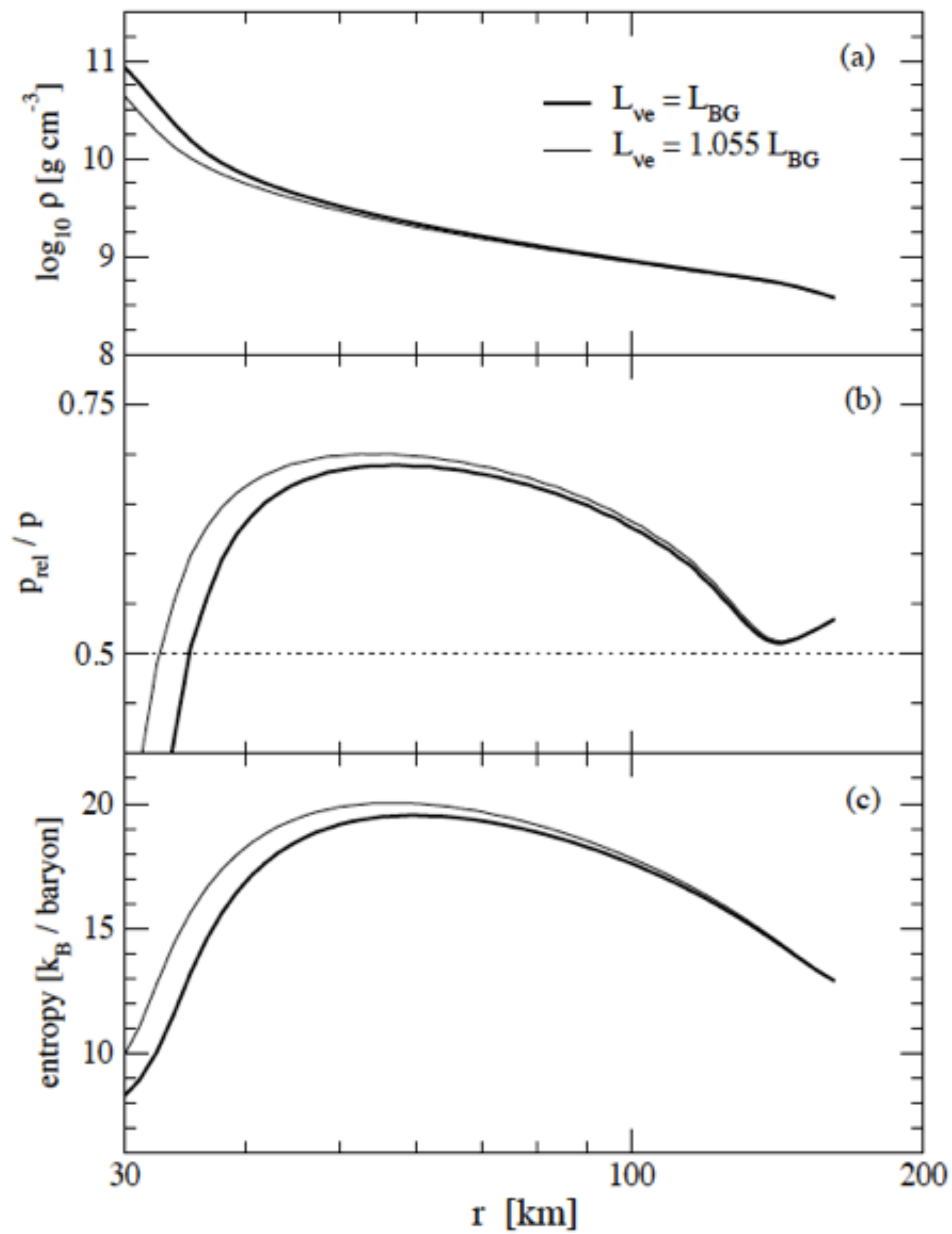
Limiting Luminosity in Steady-State



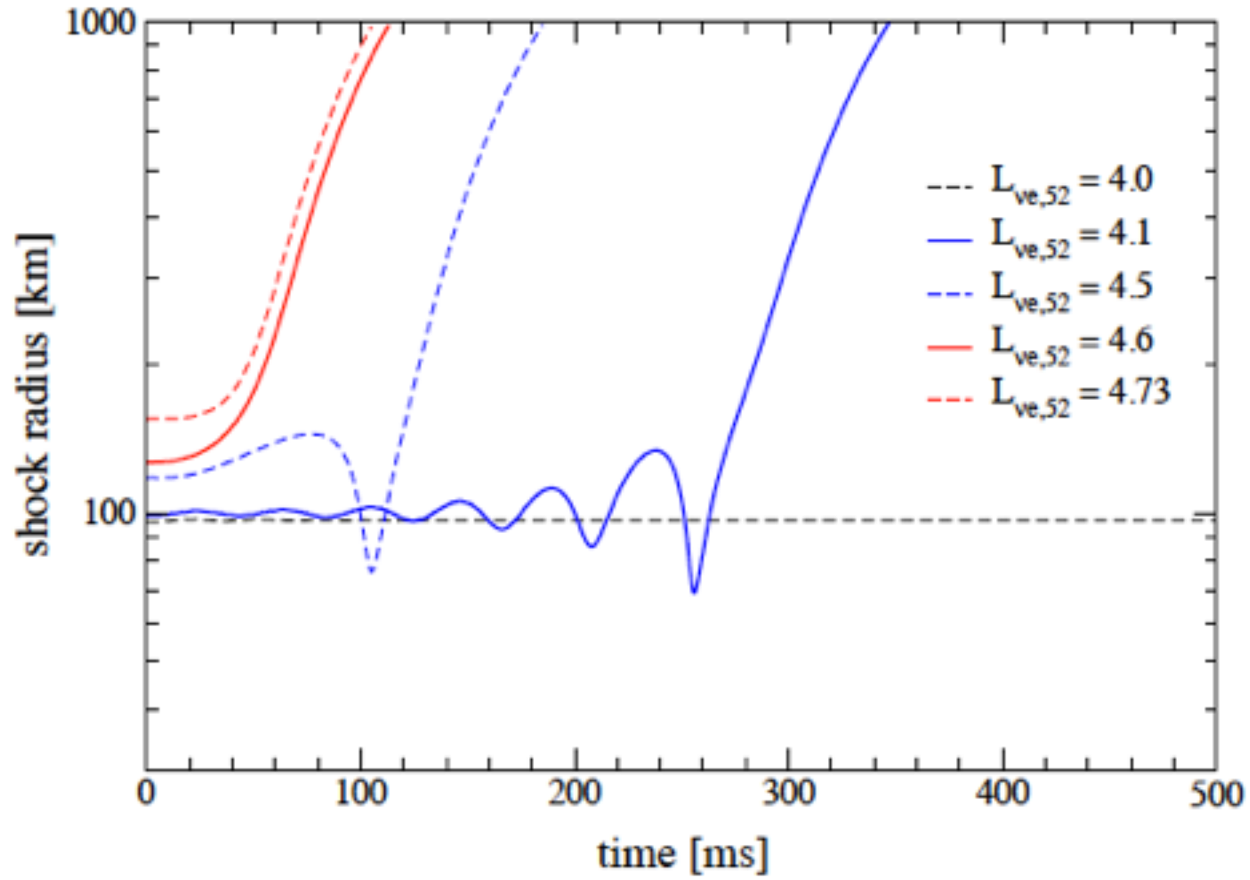
RF (2012)

$$\tau_\nu = \int_{R_\nu}^{R_s} (\rho\kappa_\nu)\, dr$$

$$\frac{\partial \tau_\nu}{\partial R_s} = (\rho\kappa_\nu)\big|_{R_s} + \int_{R_\nu}^{R_s} \frac{\partial(\rho\kappa_\nu)}{\partial R_s} dr$$

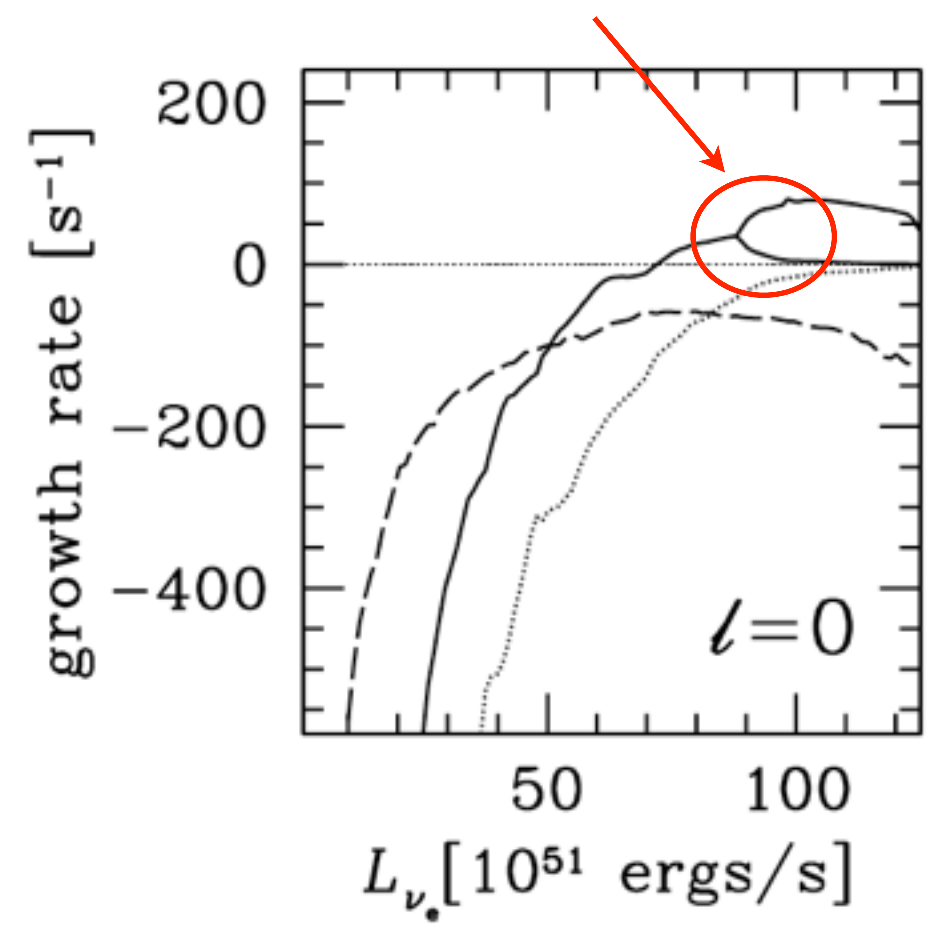


Transition to Explosion



RF (2012)

Linear instability bifurcation predicted by Yamasaki & Yamada (2007)



Yamasaki & Yamada (2007)

Fiducial model: $\dot{M} = 0.3\, M_\odot \ \mathrm{s}^{-1}$

$R_\nu = 30 \ \mathrm{km}$

$t_{\rm dyn} \sim 2 \ \mathrm{ms}$ $\quad t_{\rm heat} \sim 10 \ \mathrm{ms}$

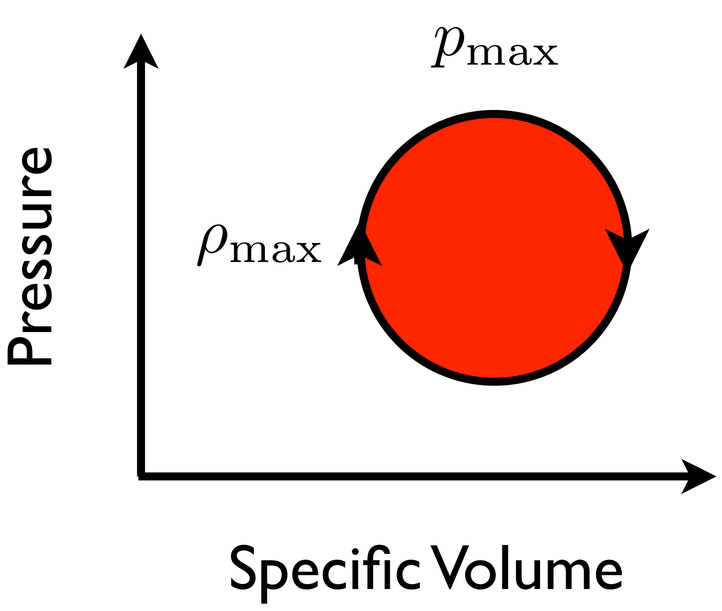
Work Integral

From stellar pulsation theory (Eddington 1926):

$$W = \oint \frac{dE}{dt} dt$$

If W > 0, driving

If W < 0, damping

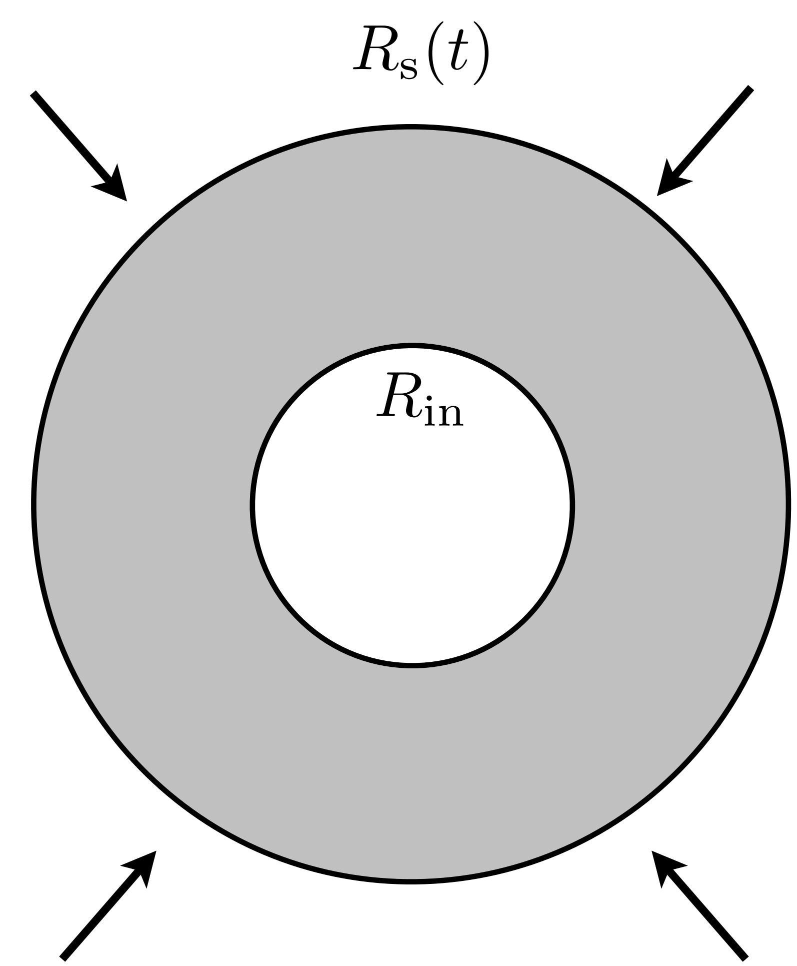


Positive work leads to increase in pulsation kinetic energy

(e.g., Cox 1974)

Work Integral in CCSNe

- Include region in sonic contact
- Neither mass nor volume are constant



$$\frac{\partial E}{\partial t} = \int d^3x \frac{\partial(\rho e_{tot})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{tot})\big|_{R_s} \equiv \dot{E}_{tot}$$

$$e_{tot} = \frac{1}{2}v^2 + e_{int} - \frac{GM}{r}$$

Energy equation

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho v_r^2 + \rho e_{\text{int}}) + \nabla \cdot \left(\rho \mathbf{v} \left[\frac{1}{2}v_r^2 + e_{\text{int}} + \frac{p}{\rho}\right]\right) + \rho v_r \frac{GM}{r^2} = \mathscr{L}_{\text{net}}$$

- Eulerian Rate of Change
- Energy Flux
- Work by Gravity
- Neutrino Source Terms

If point mass is time-independent:

$$\rho v_r \frac{GM}{r^2} = \rho \mathbf{v} \cdot \nabla \left(-\frac{GM}{r}\right) = \nabla \cdot \left(\rho \mathbf{v} \left[-\frac{GM}{r}\right]\right) - \nabla \cdot (\rho \mathbf{v}) \left[-\frac{GM}{r}\right]$$

to energy flux

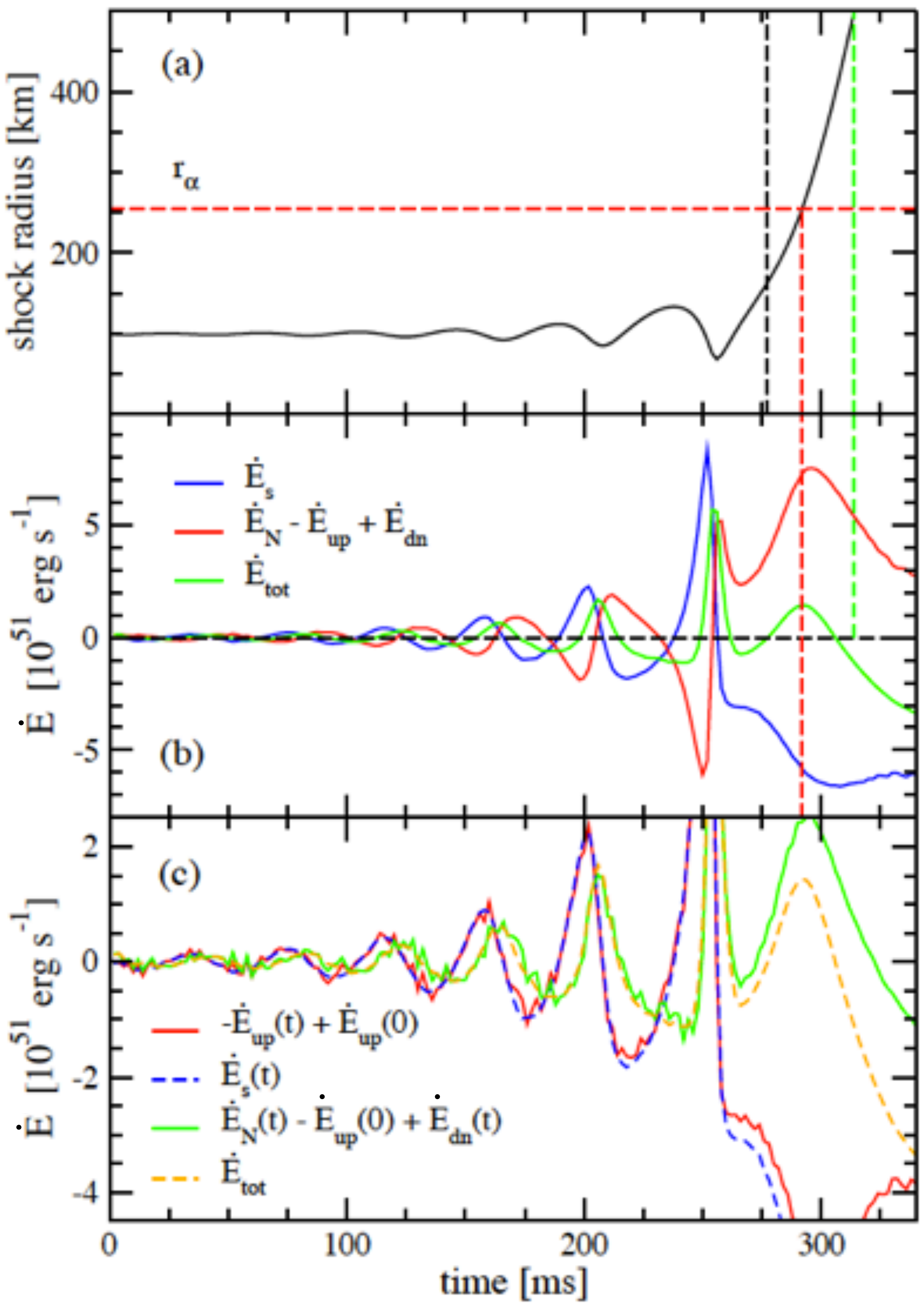
$$+\frac{\partial \rho}{\partial t}\left[-\frac{GM}{r}\right] = \frac{\partial}{\partial t}\left(-\rho \frac{GM}{r}\right)$$

to rate of change

$$\longrightarrow \boxed{\frac{\partial}{\partial t}(\rho e_{\text{tot}}) + \nabla \cdot \left(\rho \mathbf{v} \left[e_{\text{tot}} + \frac{p}{\rho}\right]\right) = \mathscr{L}_{\text{net}}}$$

$$e_{\text{tot}} = \frac{1}{2}v^2 + e_{\text{int}} - \frac{GM}{r}$$

Driving and Damping: Oscillatory Mode



Change of Total Energy (Work Integral)

$$e_{\rm tot} = \frac{1}{2}v^2 + e_{\rm int} - \frac{GM}{r}$$

$$\frac{\partial E}{\partial t} = \int d^3x \frac{\partial(\rho e_{\rm tot})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{\rm tot})\big|_{R_s} \equiv \dot{E}_{\rm tot}$$

$$= \dot{E}_{\rm N} - \dot{E}_{\rm up} + \dot{E}_{\rm dn} + \dot{E}_{\rm s},$$

$$\dot{E}_{\rm N} = \int_{R_{\rm in}}^{R_s} 4\pi r^2 dr\, \mathscr{L}_{\rm net}$$

$$\dot{E}_{\rm up} = 4\pi R_s^2 [v_r(\rho e_{\rm tot} + p)]\big|_{R_s}$$

$$\dot{E}_{\rm dn} = 4\pi R_{\rm in}^2 [v_r(\rho e_{\rm tot} + p)]\big|_{R_{\rm in}}$$

Cancel out in steady-state

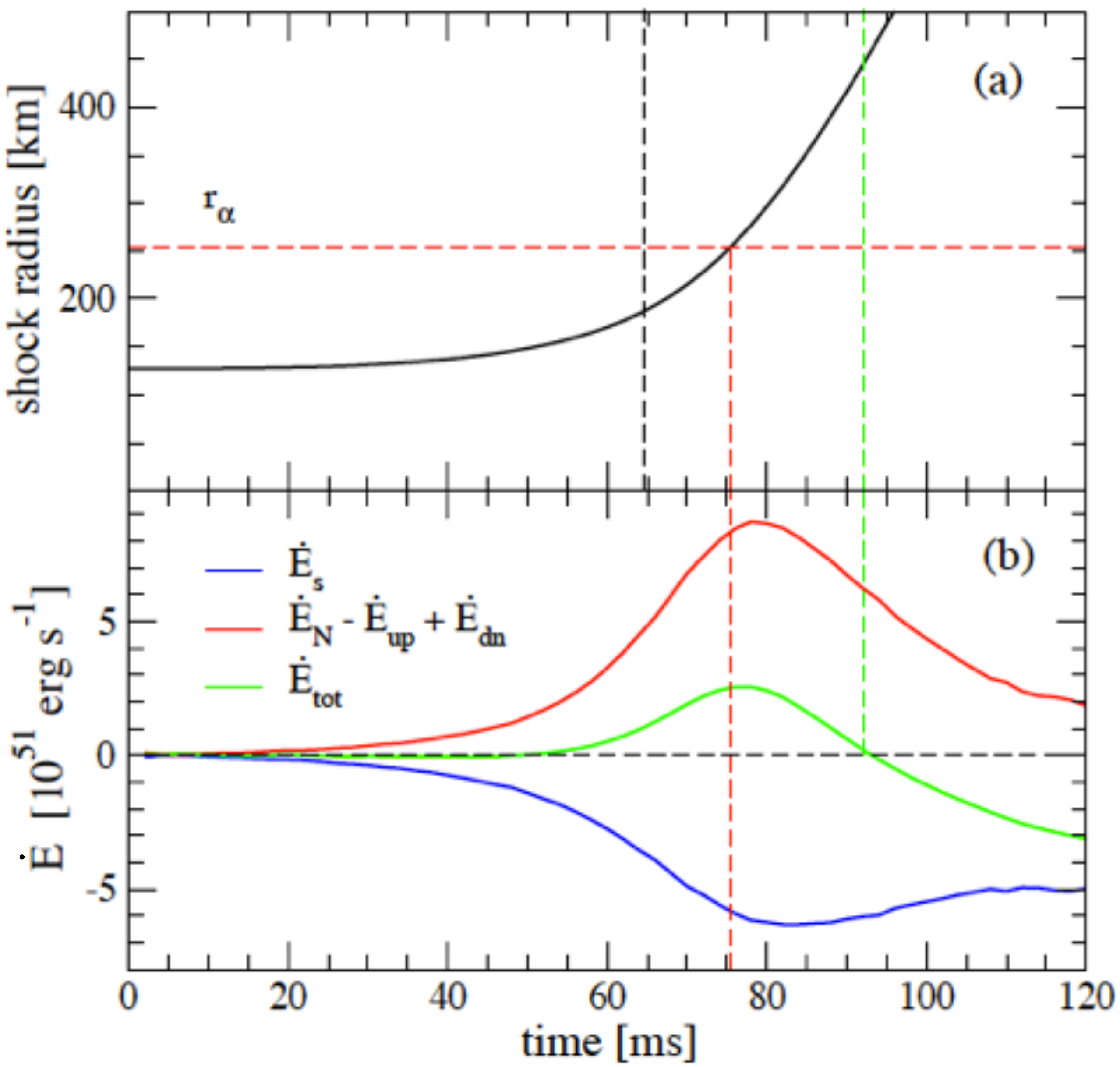
$$\dot{E}_{\rm s} = 4\pi R_s^2 \dot{R}_s (\rho e_{\rm tot})\big|_{R_s},$$

← Damps on expansion

Driving: positive energy generation

Damping: negative energy generation

Driving and Damping: Non-Oscillatory Mode



Change of Total Energy (Work Integral)

$$e_{tot} = \frac{1}{2}v^2 + e_{int} - \frac{GM}{r}$$

$$\frac{\partial E}{\partial t} = \int d^3x \frac{\partial(\rho e_{tot})}{\partial t} + 4\pi R_s^2 \dot{R}_s (\rho e_{tot})\big|_{R_s} \equiv \dot{E}_{tot}$$

$$= \dot{E}_N - \dot{E}_{up} + \dot{E}_{dn} + \dot{E}_s,$$

$$\dot{E}_N = \int_{R_{in}}^{R_s} 4\pi r^2 dr \mathscr{L}_{net}$$

$$\dot{E}_{up} = 4\pi R_s^2 [v_r(\rho e_{tot} + p)]\big|_{R_s}$$

$$\dot{E}_{dn} = 4\pi R_{in}^2 [v_r(\rho e_{tot} + p)]\big|_{R_{in}}$$

Cancel out in steady-state

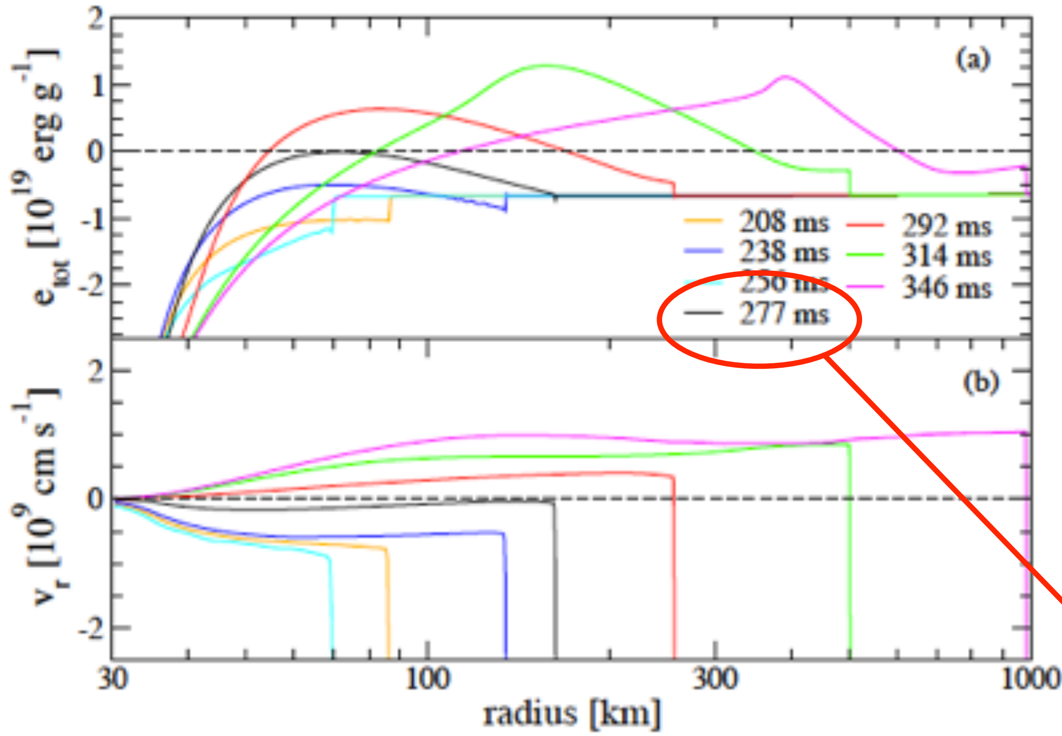
$$\dot{E}_s = 4\pi R_s^2 \dot{R}_s (\rho e_{tot})\big|_{R_s},$$

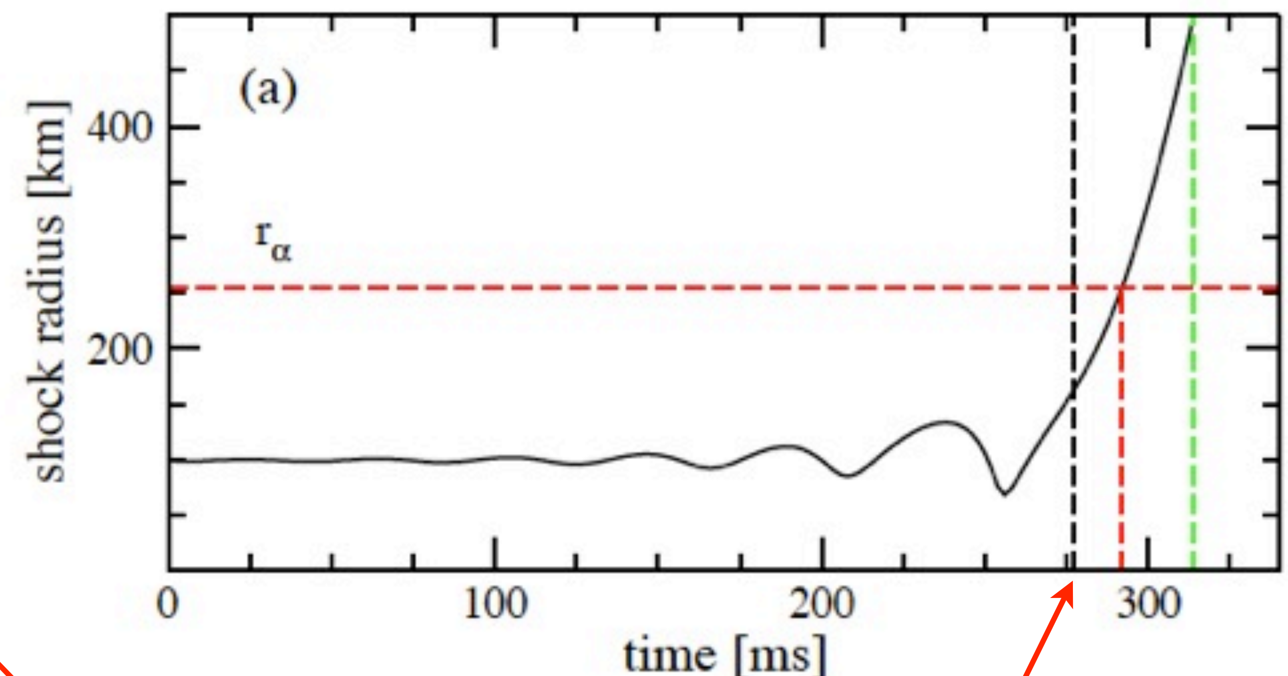
← Damps on expansion

Driving: positive energy generation

Damping: negative energy generation

Radial instability leads to Explosion (1D)





RF (2012)

$$\dot{E}_{\rm N} = \int_{R_{\rm in}}^{R_s} 4\pi r^2 {\rm d}r\, \mathscr{L}_{\rm net}$$

← Does not decrease if heating by accretion neglected and neutrinospheric parameters are constant

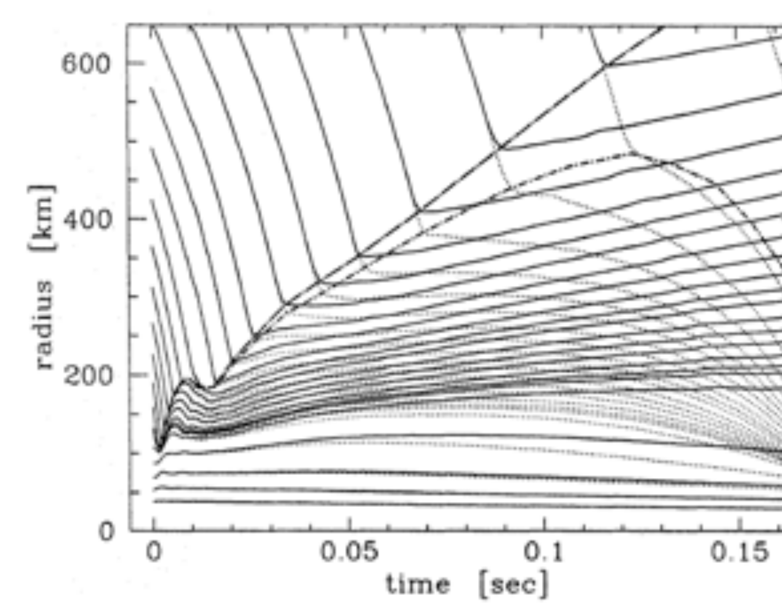
$$\dot{E}_{\rm up} = 4\pi R_s^2 [v_r(\rho e_{\rm tot} + p)]\big|_{R_s}$$

$$\dot{E}_{\rm dn} = 4\pi R_{\rm in}^2 [v_r(\rho e_{\rm tot} + p)]\big|_{R_{\rm in}}$$

← Positive, and largely exceeds $\dot{E}_{\rm up}$

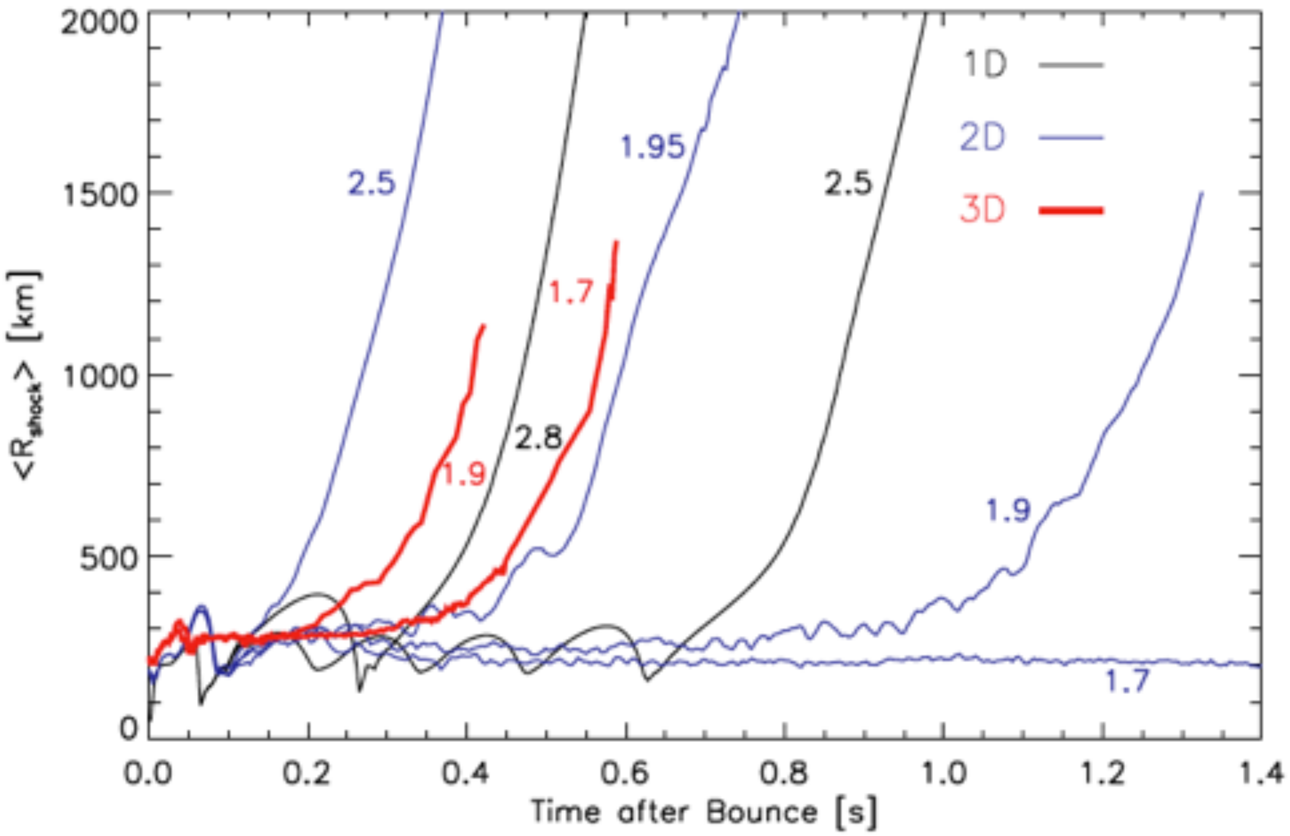
$$\dot{E}_s = 4\pi R_s^2 \dot{R}_s (\rho e_{\rm tot})\big|_{R_s},$$

← Only source of damping, decreases in magnitude due to nuclear recombination

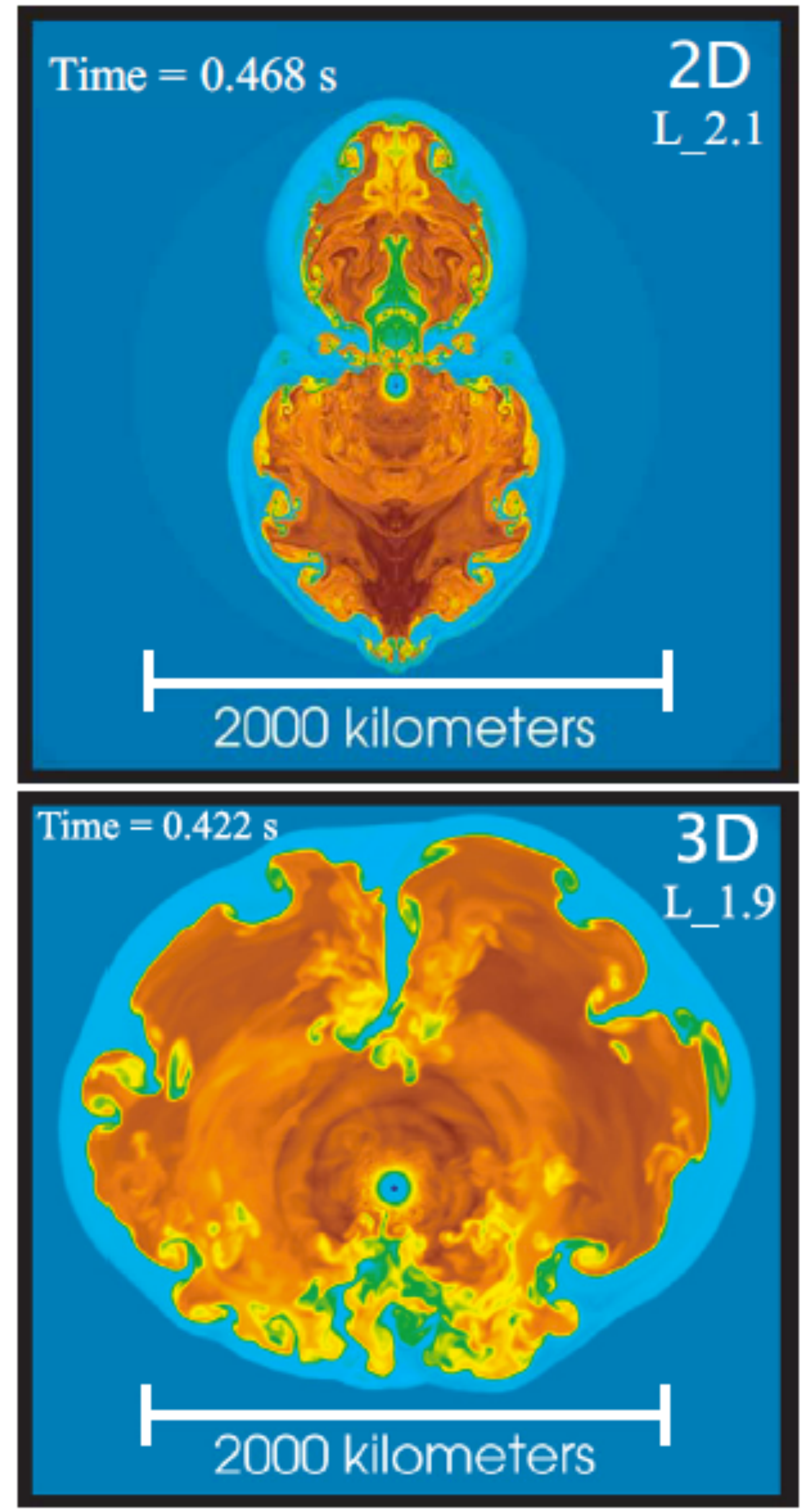


Janka & Müller (1996)

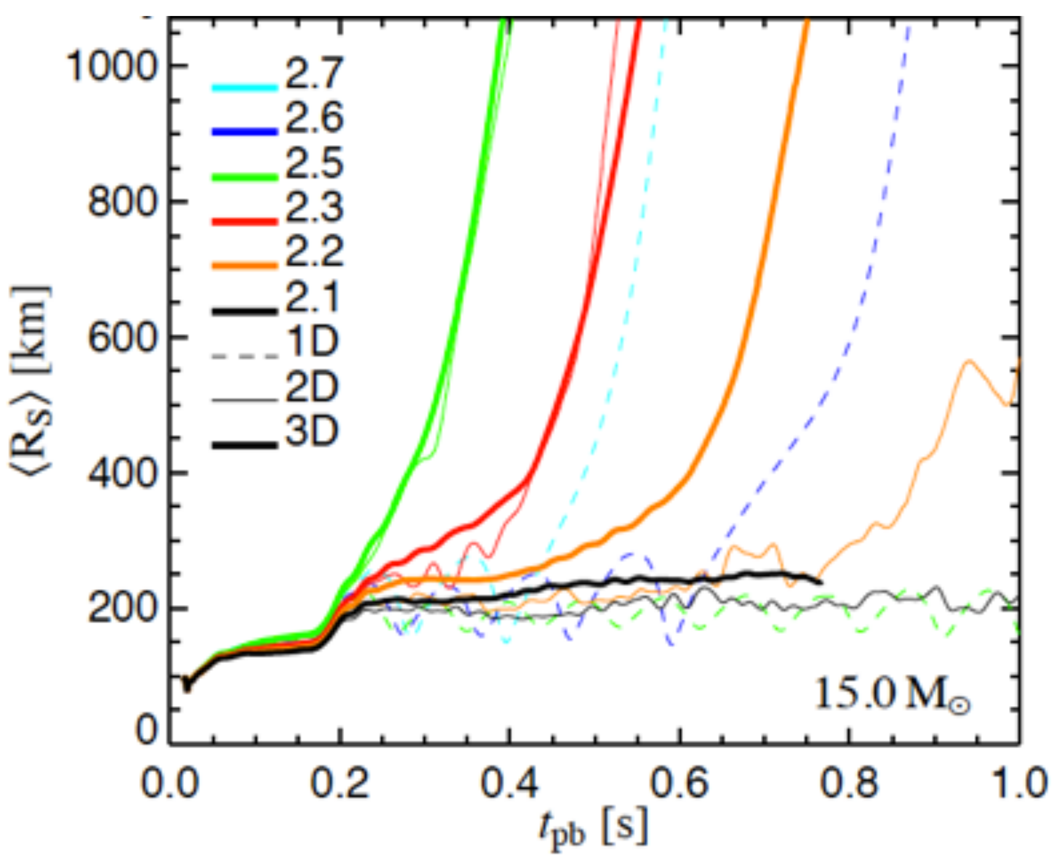
3D Hydrodynamic Studies



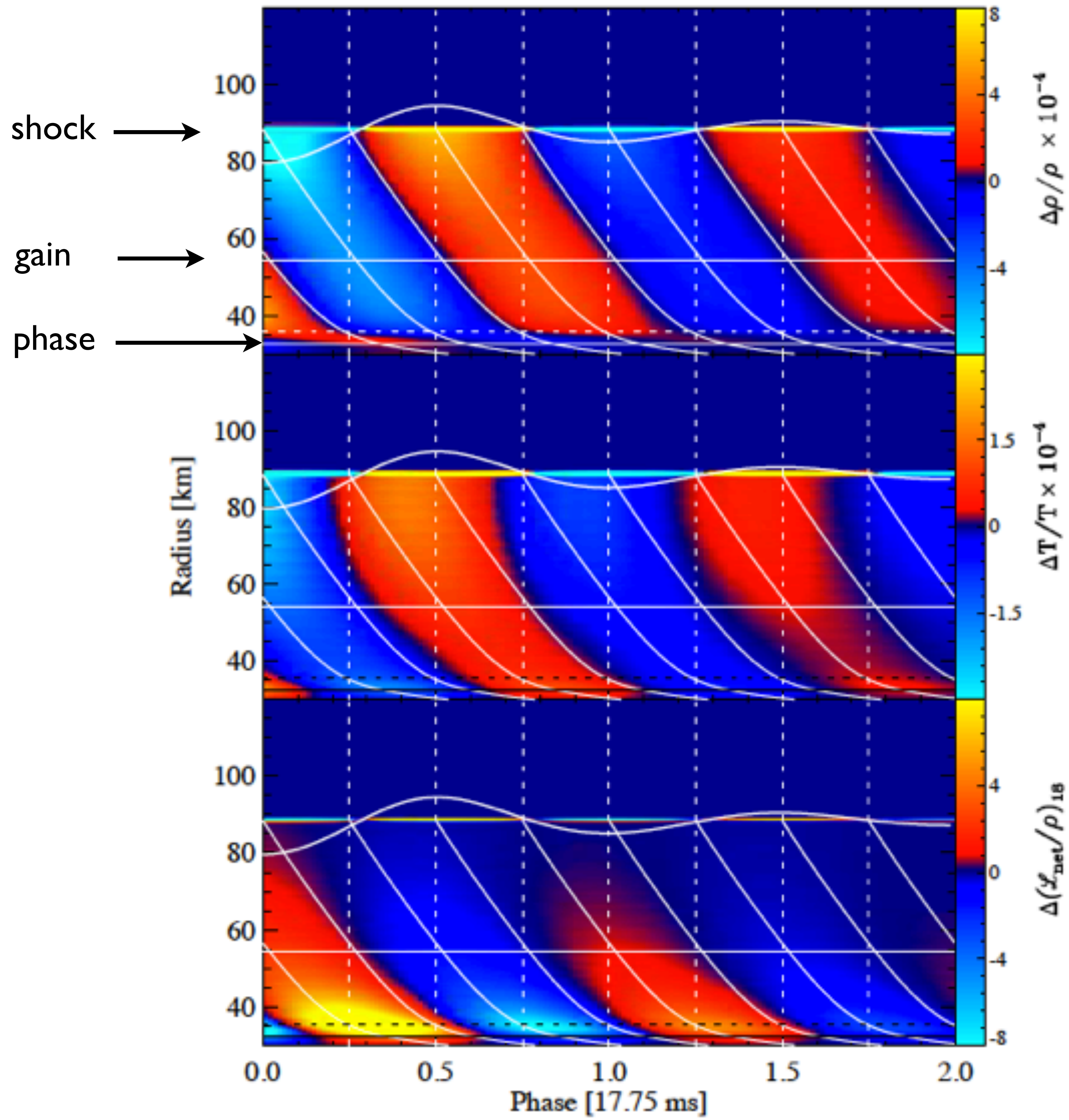
Nordhaus et al. (2010)



Hanke et al. (2011)



Approximate Instability Criteria (1D)



Oscillatory: $t_{\rm adv-g} > t_{\rm adv-e}$

Non-Oscillatory: $t_{\rm adv-g} > t_{\rm heat-e}$

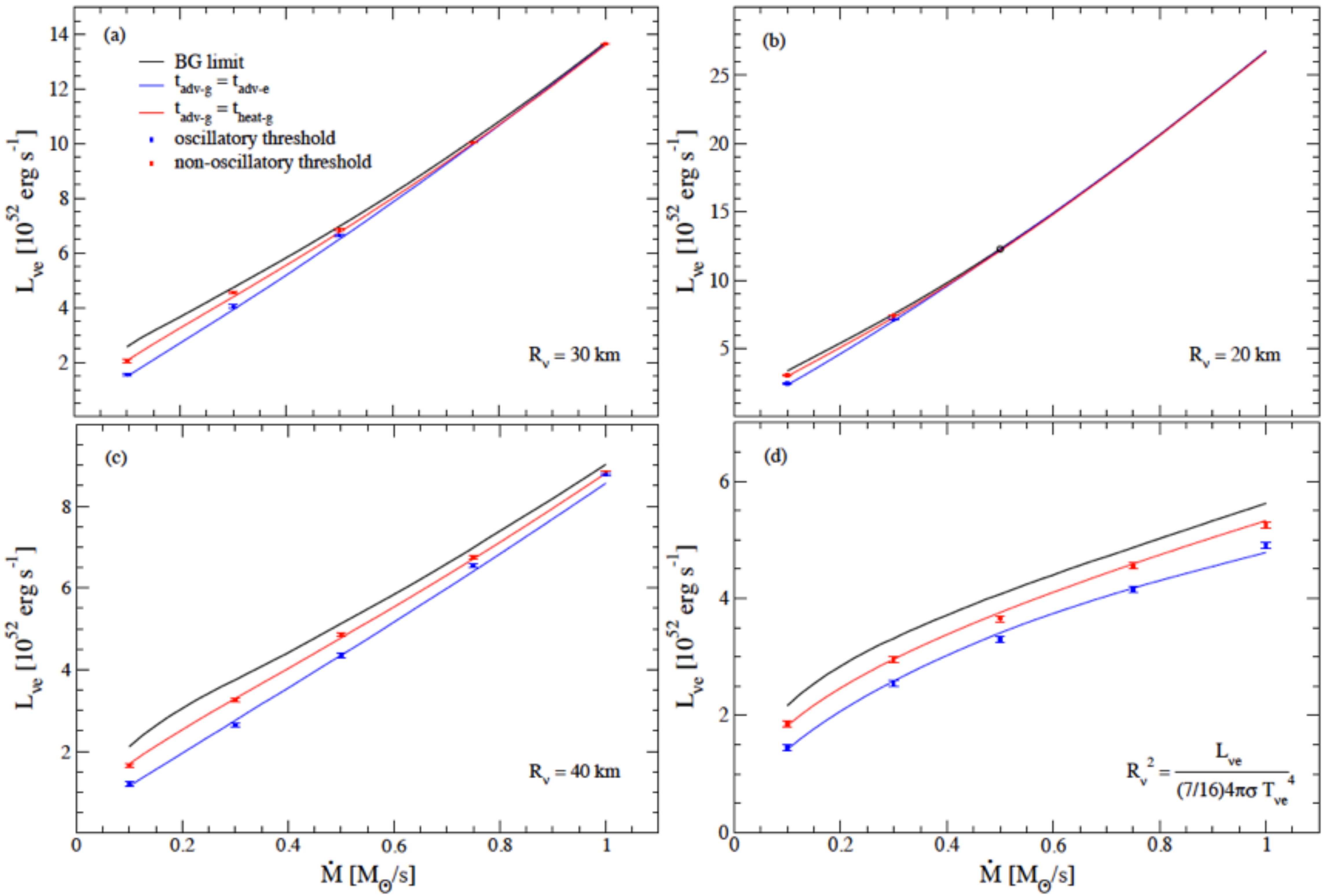
$$t_{\rm adv}(r_1, r_2) = \int_{r_1}^{r_2} \frac{dr}{|v_r|} = \frac{M(r_1, r_2)}{\dot{M}}$$

$$t_{\rm adv-g} = \int_{R_g}^{R_s} \frac{\mathrm{d}r}{|v_r|} \qquad R_g = \text{gain radius}$$

$$t_{\rm adv-e} = \int_{r_e}^{R_g} \frac{\mathrm{d}r}{|v_r|} \qquad r_e = \text{“phase shift” radius}$$

$$t_{\rm heat-g} = \frac{\int_{R_g}^{R_s} \mathrm{d}^3x\,(\rho e_{\rm tot})}{\int_{R_g}^{R_s} \mathrm{d}^3x\,\mathscr{L}_{\rm net}}$$

Threshold Luminosities for Instability (1D)



Implications for Multi-Dimensional Case

1. Unstable oscillatory modes in multi-D (SASI) do not explode the system by themselves.

(e.g., Iwakami et al. 2008)

2. Non-oscillatory modes of high angular degree associated with convection ($\ell \gtrsim 5$)

(Yamasaki & Yamada 2007)

Implications for Multi-Dimensional Case

3. Hence, multi-D explosion mediated by either:

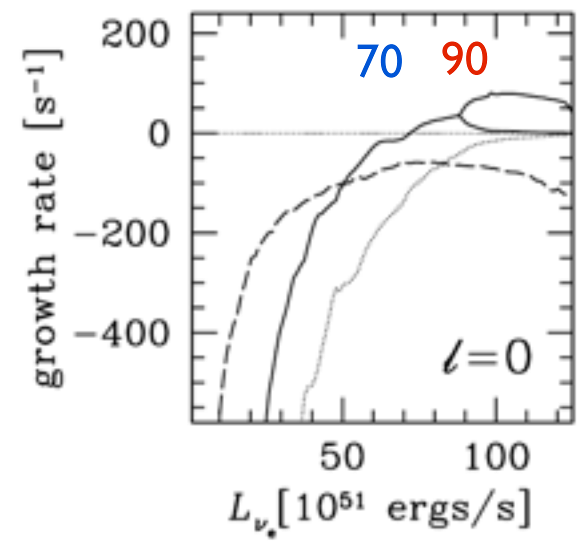
   a) Unstable spherical mode (oscillatory and/or non-oscillatory), and possibly

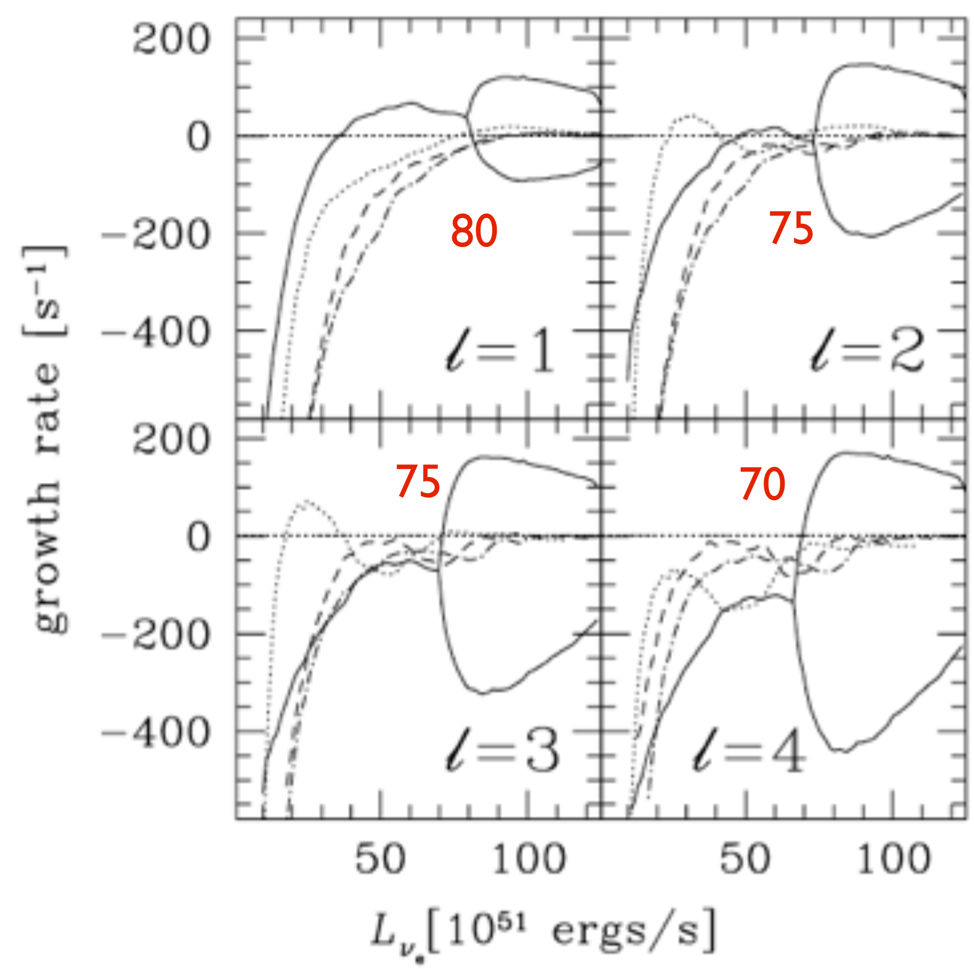
   b) Unstable non-oscillatory mode of low angular degree ($\ell = 1, 2$)

      Will **couple non-linearly** to $\ell = 0$

4. Background flow is turbulent in multi-D, thus instability thresholds will change.

   A semi-analytic study would require perturbing time-averaged quantities.

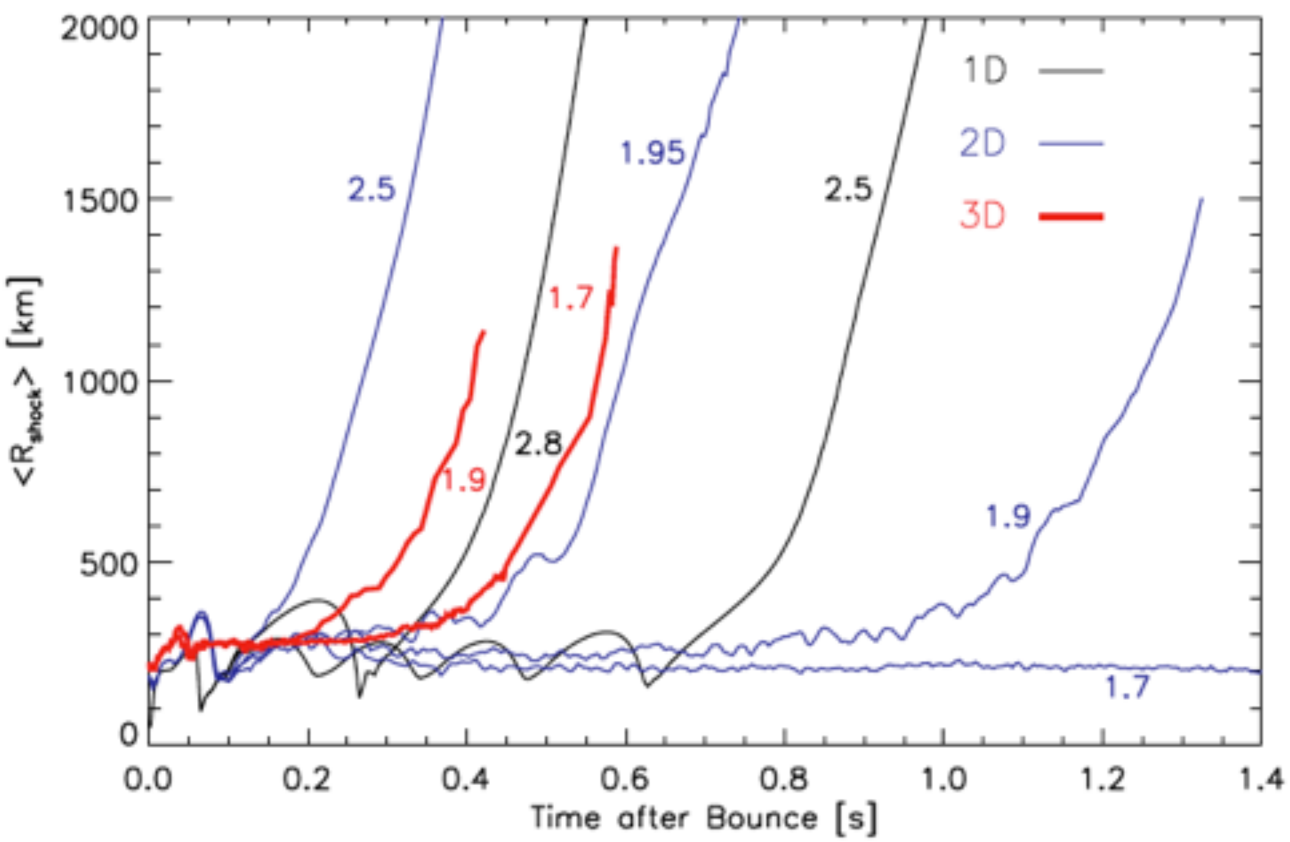


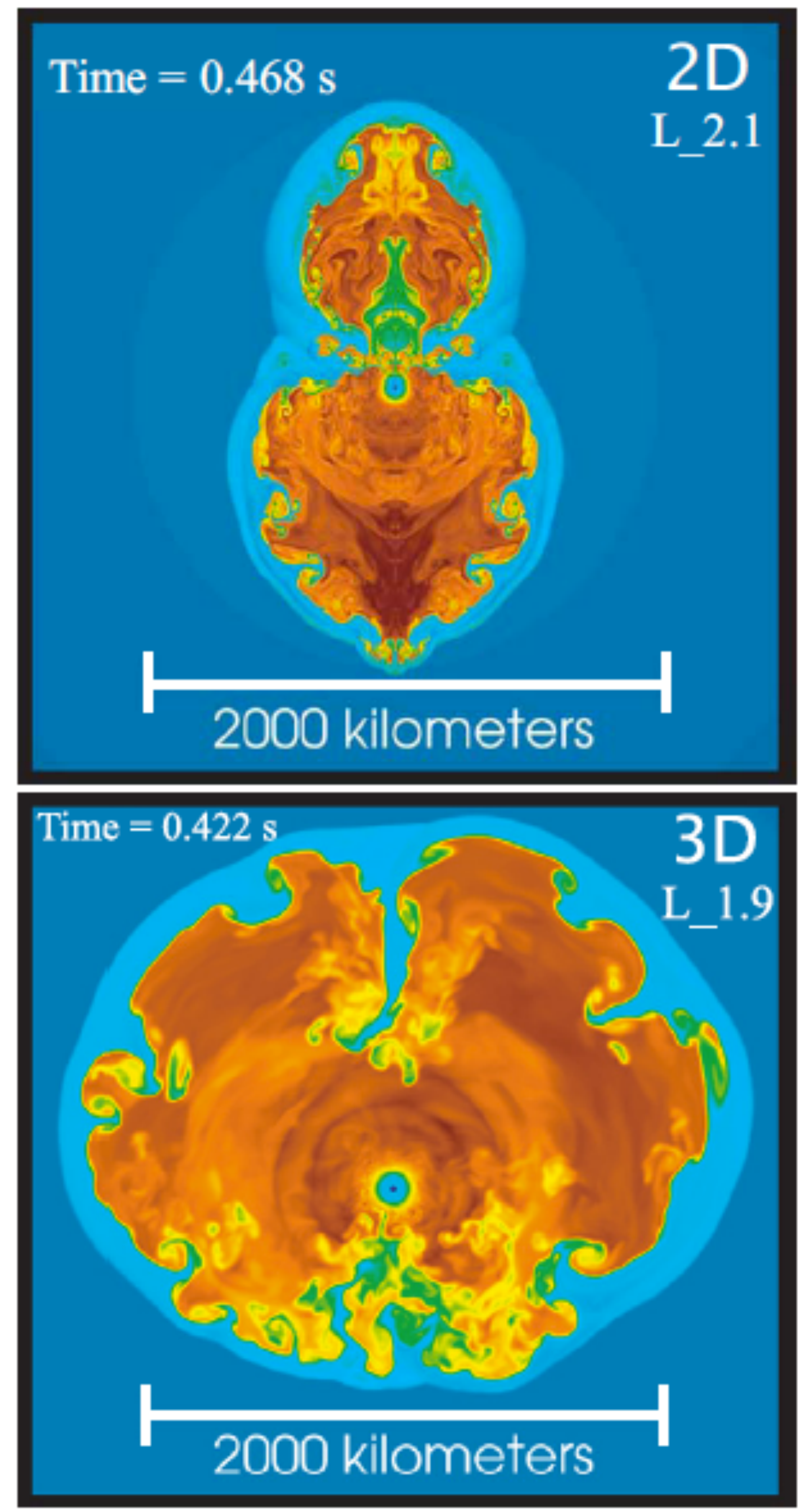


Yamasaki & Yamada (2007)

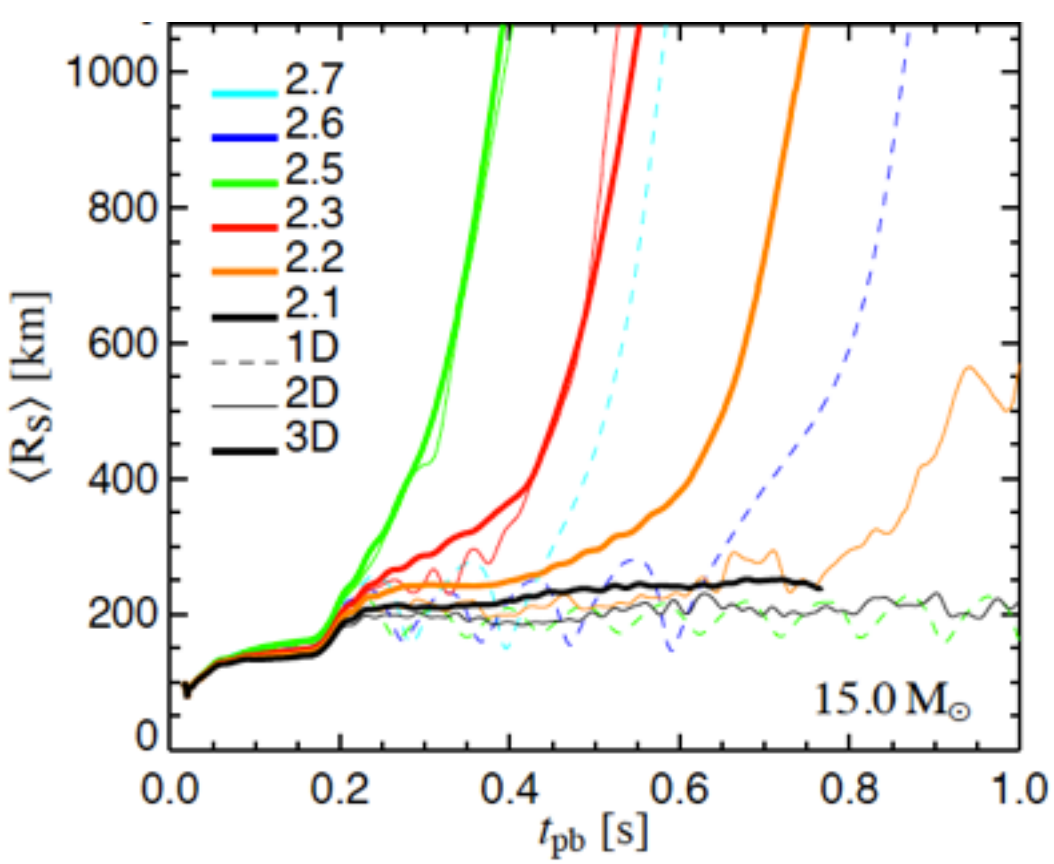
3D Hydrodynamic Studies

Nordhaus et al. (2010)





Hanke et al. (2011)



Summary

1. Radial instability of the shock is a sufficient condition for explosion

   IF

   a) Neutrinospheric parameters are constant in time
   b) Heating from accretion luminosity is neglected
   c) Mass accretion rate is constant or decreases with time

2. Radial instability thresholds can be approximately (~5% in L_ν) described by global properties of the flow

3. Instability thresholds are different from limiting luminosity for a steady-state configuration (BG93 limit)

4. Multi-dimensional explosions involve growth of spherical and/or non-spherical modes in turbulent background flow