Thermal Effects in Supernova Matter

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Outline

- Equation of State constraints
- Non-Relativistic Potential models
- Mean Field Theoretical model
- Summary

Focus

- Bulk matter

- Strong part of the NN interaction
  - Nucleons and electrons are in weak interaction equilibrium
  - Electromagnetic corrections, mainly from the exchange interaction, are negligibly small.
    For electrons, $\frac{P_{exc}}{P_{FG}} = \frac{-3\alpha_{em}}{2\pi} \simeq -3.5 \times 10^{-3}$

EOS Laboratory Constraints

At saturation density ($n_0 = 0.16 \pm 0.01$ fm^{-3}):

- Compression modulus, $K = 225 \pm 30$ MeV
  (giant monopole resonances)
- Energy per particle, $E/A = -16 \pm 1$ MeV
  (fits to masses of atomic nuclei)
- Symmetry energy, $S_2 = 30 \pm 5$ MeV
  (fits to masses of atomic nuclei)
- Effective mass, $M^*/M = 0.8 \pm 1$
  (neutron evaporation spectra)

EOS Constraints from Neutron Stars

- Largest observed mass, $M = 1.97 M_\odot$
  (binaries)

- Largest observed frequency, $\Omega = 114$ rad/s
  (pulsars)

- Inferred radius range, 10 km $\leq R \leq$ 12.5 km
  (photospheric emission, thermal spectra)

- ...

Solution of Tolman-Oppenheimer-Volkoff (TOV) equations and EOS predicts M_{max}, R_{max}, I_{max}, Ω_{max}, etc.

NR Potential Model:APR

- Due to Akmal & Pandharipande (Phys. Rev. C. 56, 2261 (1997)):

$$v_{NN} = v_{18,ij} + V_{IX,ijk} + \delta v(\mathbf{P}_{ij})$$

where

$$\begin{aligned} v_{18,ij} &= \sum_{p=1,18} v^p(r_{ij}) O^p_{ij} + v_{em} \qquad \text{(Argonne)} \\ V_{IX,ijk} &= V^{2\pi}_{ijk} + V^R_{ijk} \qquad \text{(Urbana)} \\ \delta v(\mathbf{P}) &= -\frac{P^2}{8m^2} u + \frac{1}{8m^2}[\mathbf{P.r}\ \mathbf{P.}\nabla, u] + \frac{1}{8m^2}[(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) \times \mathbf{P.}\nabla, u] \end{aligned}$$

(relativistic boost correction)

NR Potential Model:APR

- Ground state expectation value

$$
\begin{aligned}
E &= \frac{< \Psi|\hat{H}|\Psi >}{< \Psi|\Psi >} \\
\Psi_v &= \left(S \prod_{i<j} F_{ij} \right) \Phi
\end{aligned}
$$

- Features
  - Excellent fit of NN scattering data and of binding energies of light nuclei
  - Phase transition to π^o-condensate at $\sim 1.25\ n_0$
  - Supports neutron star masses up to $2.2 M_\odot$.

NR Potential Model:APR

- Phenomenological fit by Akmal, Pandharipande, and Ravenhall (Phys. Rev. C. 58, 1804 (1998)).
- Hamiltonian density:

$$\begin{aligned}\mathcal{H}_{APR} &= \left[\frac{\hbar^2}{2m} + (p_3 + (1-x)p_5)ne^{-p_4 n}\right]\tau_n \\ &+ \left[\frac{\hbar^2}{2m} + (p_3 + (1-x)p_5)ne^{-p_4 n}\right]\tau_n \\ &+ g_1(n)(1-(1-2x)^2) + g_2(n)(1-2x)^2\end{aligned}$$

$\tau_{n(p)}$ - neutron(proton) kinetic energy density
p_i - fit parameters

- Skyrme-like $\Rightarrow$ Landau effective mass:

$$m_i^* = \left(\left.\frac{\partial \varepsilon_{k_i}}{\partial k_i}\right|_{k_{Fi}}\right)^{-1} k_{Fi} = \left[\frac{1}{m} + \frac{2}{\hbar^2}(p_3 + Y_i p_5)ne^{-p_4 n}\right]^{-1}$$

T=0

- Note that $\tau_i(T=0) = \frac{(3\pi^2)^{5/3}}{5\pi^2}(nY_i)^{5/3}$,

  where $Y_i = n_i/n$

  $\Rightarrow \mathcal{H}(T=0) = \mathcal{H}(n,x)$

  with $x = n_p/n$ (proton fraction)

T=0

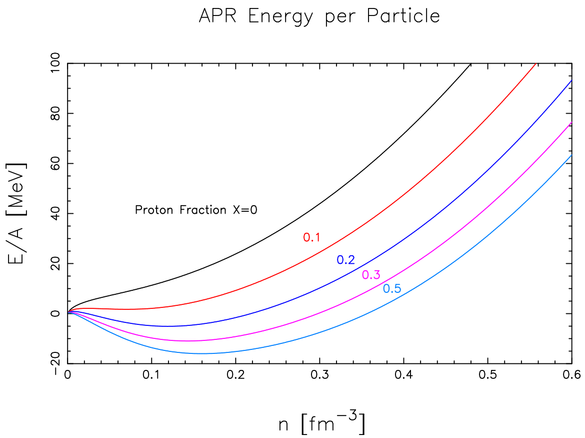
Rest of thermodynamics:

- Energy per particle $E/A = \mathcal{H}/n$
- Pressure $P = n^2 \frac{\partial E}{\partial n}$
- Neutron chem. pot. $\mu_n = E + n \left.\frac{\partial E}{\partial n}\right|_x - x \left.\frac{\partial E}{\partial x}\right|_n$
- Proton chem. pot. $\mu_p = E + n \left.\frac{\partial E}{\partial n}\right|_x + (1-x) \left.\frac{\partial E}{\partial x}\right|_n$
- Compression modulus $K = 9 n_0 \left.\frac{\partial^2 E}{\partial n^2}\right|_{n_0}$
- Symmetry energy $S_2 = \frac{1}{8} \left.\frac{\partial^2 E}{\partial x^2}\right|_{x=1/2}$
- Susceptibilities $\chi_{ij} = \left(\frac{\partial \mu_i}{\partial n_j}\right)^{-1}$

At saturation density, $n_0 = 0.16\ \mathrm{fm}^{-3}$:

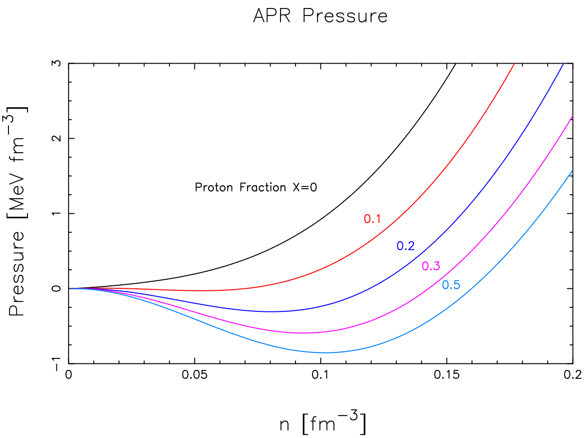
- $K = 266$ MeV
- $E/A = -16$ MeV
- $S_2 = 32.6$ MeV
- $M^*/M = 0.7$

Energy per Particle



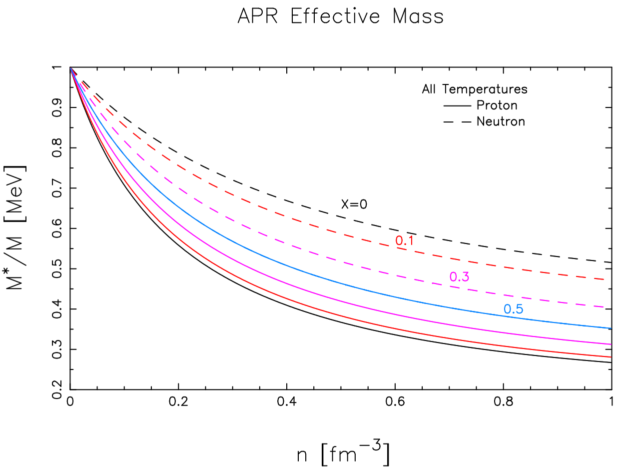
- E/A minimum shifts to lower densities as $x \to 0$.
- For $n < n_o$ need to consider nuclei, clusters, etc.

Pressure



- For $\frac{dp}{dn} < 0$, matter is spinodally unstable.
- Range of n for which instability occurs, shifts to lower densities as $x \to 0$.

Effective Mass



Finite T

- Single-particle energy spectrum:

$$\varepsilon_i = k_i^2 \frac{\partial \mathcal{H}}{\partial \tau_i} + \frac{\partial \mathcal{H}}{\partial n_i} \equiv \varepsilon_{k_i} + V_i$$

- $$n_i = \frac{1}{2\pi^2}\left(\frac{2m_i^* T}{\hbar^2}\right)^{3/2} F_{1/2i}$$

$$\tau_i = \frac{1}{2\pi^2}\left(\frac{2m_i^* T}{\hbar^2}\right)^{5/2} F_{3/2i}$$

$$F_{\alpha i} = \int_0^\infty \frac{x_i^\alpha}{e^{-\psi_i} e^{x_i} + 1} dx_i$$

$$x_i = \frac{\varepsilon_{ki}}{T}, \quad \psi_i = \frac{\mu_i - V_i}{T} = \frac{\nu_i}{T}$$

Finite T Issues

- Analytical expressions for arbitrary degeneracy not possible.
- Numerical simulations need closely-gridded EOS
  as a function of n, T, Y_e.
  Therefore, we need a thermodynamically consistent and efficient method for evaluating FD integrals.

Finite T

- Non-Relativistic Johns-Ellis-Lattimer's (JEL) Scheme (ApJ, 473 (1020),1996)

  Fermi-Dirac integrals as algebraic functions of the degeneracy parameter ψ only:

$$
\begin{aligned}
F_{3/2} &= \frac{3f(1+f)^{1/4-M}}{2\sqrt{2}} \sum_{m=0}^{M} p_m f^m \\
F_{\alpha-1} &= \frac{1}{\alpha}\frac{\partial F_\alpha}{\partial \psi} \\
\psi &= 2\left(1+\frac{f}{a}\right)^{1/2} + \ln\left[\frac{(1+f/a)^{1/2}-1}{(1+f/a)^{1/2}+1}\right]
\end{aligned}
$$

Finite T

Rest of thermodynamics :

- Energy density $$\varepsilon = \frac{\hbar^2}{2m_n^*}\tau_n + \frac{\hbar^2}{2m_p^*}\tau_p + g_1(n)\left[1-(1-2x)^2\right] + g_2(n)(1-2x)^2$$
- Chemical potentials $$\mu_i = T\psi_i + V_i$$
- Entropy density $$s_i = \frac{1}{T}\left(\frac{5}{3}\frac{\hbar^2}{2m_i^*}\tau_+ n_i(V_i - \mu_i)\right)$$
- Pressure $$P = T(s_n + s_p) + \mu_n n_n + \mu_p n_p - \varepsilon$$
- Free energy density $$\mathcal{F} = \varepsilon - Ts$$
- Inverse susceptibilities $$\chi_{ij}^{-1} = T\frac{\partial\psi_i}{\partial n_j} + \frac{\partial V_i}{\partial n_j}$$

Consistency Checks

- To infer thermal contributions, eliminate terms that depend only on density :

$$X_{th} = X(n, x, T) - X(n, x, 0)$$

- Compare graphically the exact X_{th} with its degenerate and non-degenerate limits

Degenerate Limit

Landau Fermi Liquid Theory

- Interaction switched-on adiabatically
- Entropy density and number density maintain their free Fermi-gas forms:

$$s_i = \frac{1}{V}\sum_{k_i}[f_{k_i}\ln f_{k_i} + (1-f_{k_i})\ln(1-f_{k_i})]$$

$$n_i = \frac{1}{V}\sum_{k} f_{k_i}(T)$$

- $\int d\varepsilon \frac{\delta s}{\delta T} \quad \Rightarrow \quad s_i = 2a_i n_i T$

$$a_i = \frac{\pi^2}{2k_{f_i}u_{F_i}} \qquad \text{level density parameter}$$

$$u_F = \left.\frac{\partial \varepsilon_{k_i}}{\partial k_i}\right|_{k_{Fi}} \qquad \text{Fermi velocity}$$

Degenerate Limit

Other thermodynamics via Maxwell's relations:

- Energy density
  $$\frac{d\varepsilon}{ds} = T$$
  $$\varepsilon(n, T) = \varepsilon(n, 0) + anT^2$$
- Pressure
  $$\frac{dp}{dT} = -n^2 \frac{d(s/n)}{dn}$$
  $$p(n, T) = p(n, 0) + \frac{1}{3}anT^2 \left[1 + \frac{d\ln u_F}{d\ln k_F}\right]$$
- Chemical potentials
  $$\frac{d\mu}{dT} = -\frac{ds}{dn}$$
  $$\mu(n, T) = \mu(n, 0) - \frac{1}{3}aT^2 \left[2 - \frac{d\ln u_F}{d\ln k_F}\right]$$
- Free energy density
  $$\frac{d\mathcal{F}}{dT} = -s$$
  $$\mathcal{F}(n, T) = \mathcal{F}(n, 0) - anT^2$$

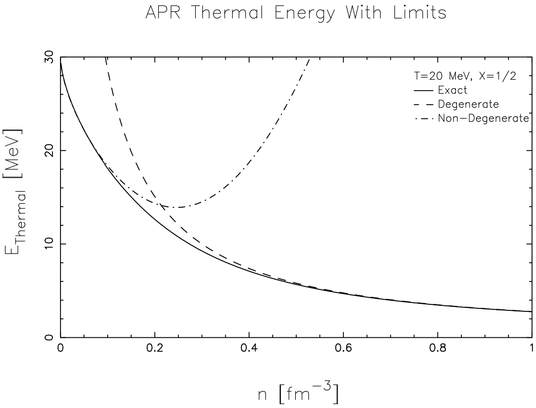
Non-Degenerate Limit

1. $F_\alpha \xrightarrow{z \ll 1} \Gamma(\alpha+1)\left(z - \frac{z^2}{2^{\alpha+1}} + \ldots\right)$
2. Invert $F_{1/2}$ to get z :

   $z = \frac{n\lambda^3}{\gamma} + \frac{1}{2^{3/2}}\left(\frac{n\lambda^3}{\gamma}\right)^2, \quad \lambda = \left(\frac{2\pi\hbar^2}{m^* T}\right)^{1/2}$ (quantum concentration)
3. Plug z in F_α's :

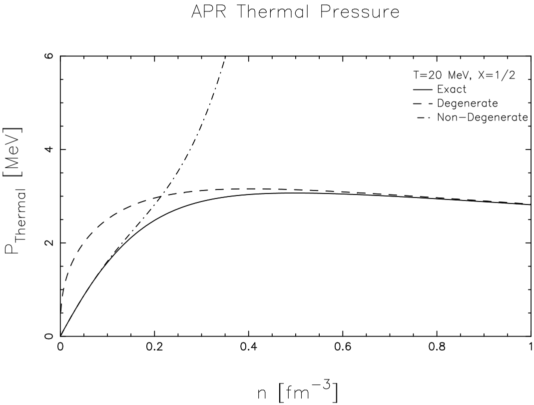
$$
\begin{aligned}
F_{3/2} &= \frac{3\pi^{1/2}}{4}\frac{n\lambda^3}{\gamma}\left[1 + \frac{1}{2^{5/2}}\frac{n\lambda^3}{\gamma}\right] \\
F_{1/2} &= \frac{\pi^{1/2}}{2}\frac{n\lambda^3}{\gamma} \\
F_{-1/2} &= \pi^{1/2}\frac{n\lambda^3}{\gamma}\left[1 - \frac{1}{2^{3/2}}\frac{n\lambda^3}{\gamma}\right]
\end{aligned}
$$

Thermal Energy



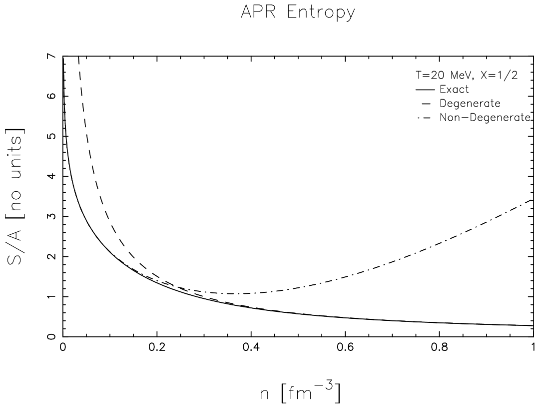
- Exact calculations agree with analytical limits where expected.
- Around nuclear saturation density exact results needed.

Thermal Pressure



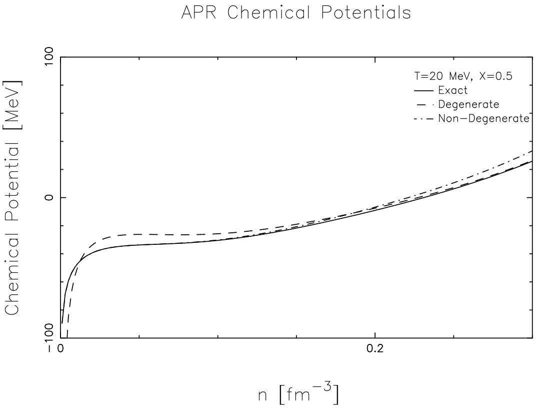
- Exact calculations agree with analytical limits where expected.
- Around nuclear saturation density exact results needed.

Entropy



- Agreement is better for S/A than for E_{th} and P_{th}.

Chemical Potential



- Agreement is better for μ than for E_{th} and P_{th}.

Deficiencies of the APR EOS

- Compression modulus, K, somewhat larger than $K_{empirical}$
- Single particle potential, $U(n,p) = U(n) + const. \times p^2$; inconsistent with optical model fits to nucleon-nucleus reaction data.
- Microscopic calculations (RBHF, variational calculations, etc.) show distinctly different behaviors in their momentum dependence, consistent with optical model fits (results shown later).
- The above features were found necessary to account for heavy-ion data on transverse momentum and energy flow (in conjuction with $K \sim 220$ MeV).

NR Potential Model:MDYI

- Hamiltonian density (that mimics more microscopic calculations)

$$\mathcal{H}_{MDYI} = \frac{\hbar^2}{2m}\tau + \frac{A}{2}\left(\frac{\rho^2}{\rho_o}\right) + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_o^\sigma} + \frac{C}{\rho_o}\left(\frac{4}{h^3}\right)^2 \int\int d^3p d^3p' \frac{f_p(T) f_{p'}(T)}{1+\left(\frac{\vec{p}-\vec{p}'}{\Lambda}\right)^2}$$

- Energy spectrum

$$\epsilon(p) = \frac{p^2}{2m} + R(\rho,p) + A\left(\frac{\rho}{\rho_o}\right) + B\left(\frac{\rho}{\rho_o}\right)^\sigma$$

$$R(\rho,p) = \frac{2C}{\rho_o}\frac{4}{h^3}\int d^3p' \frac{1}{e^{[\epsilon(p')-\mu]/T}+1}\frac{1}{\left[1+\left(\frac{\vec{p}-\vec{p}'}{\Lambda}\right)^2\right]}$$

Calibration

Use known properties of nuclear matter to fix parameters:

$$\left.\begin{array}{l} \rho_o = 0.16\ \text{fm}^{-3} \\ K = 215\ \text{MeV} \\ E/A = -16\ \text{MeV} \\ U(\rho_o, p = 0) = -75\ \text{MeV} \\ U(\rho_o, p = 300\ \text{MeV}) = 0 \\ U(\rho_o, p_{asymptotic}) = 30.5\ \text{MeV} \end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{l} A = -110.44\ \text{MeV} \\ B = 140.9\ \text{MeV} \\ C = -64.95\ \text{MeV} \\ \sigma = 1.24 \\ \Lambda = 1.58 p_{F_o} \end{array}\right.$$

where $$U(\rho, p) = R(\rho, p) + A\left(\frac{\rho}{\rho_o}\right) + B\left(\frac{\rho}{\rho_o}\right)^{\sigma}$$

$U(\rho, p)$ in microscopic and schematic models

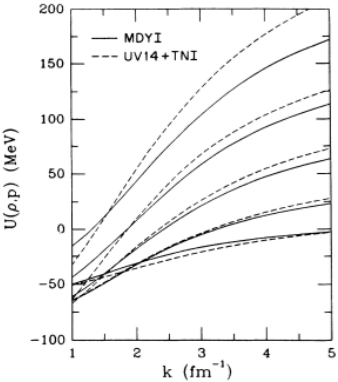


FIG. 1. A comparison of the single-particle potential from MDYI [Eq. (2.4)] with the microscopic calculations of Wiringa (Ref. 10) using the UV14+TNI interaction. The abscissa shows wave numbers. Starting from the bottom at right, the different curves are for densities of 0.1, 0.2, 0.3, 0.4, and 0.5 fm^{-3}.

- Welke et al. PRC 38, 2101 (1988).
- Illustrations with isospin symmetric matter.

Finite Temperature

- Exact

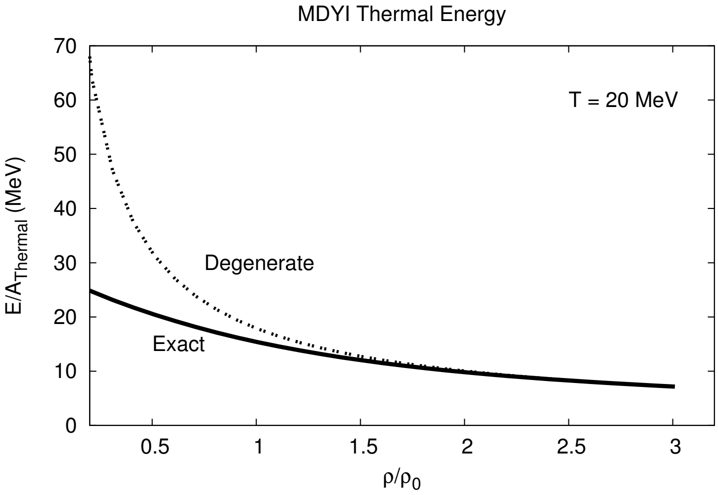
  Iterative numerical procedure adopted to calculate $\epsilon(p)$ just as in Hartree-Fock theory.

- Degenerate Limit

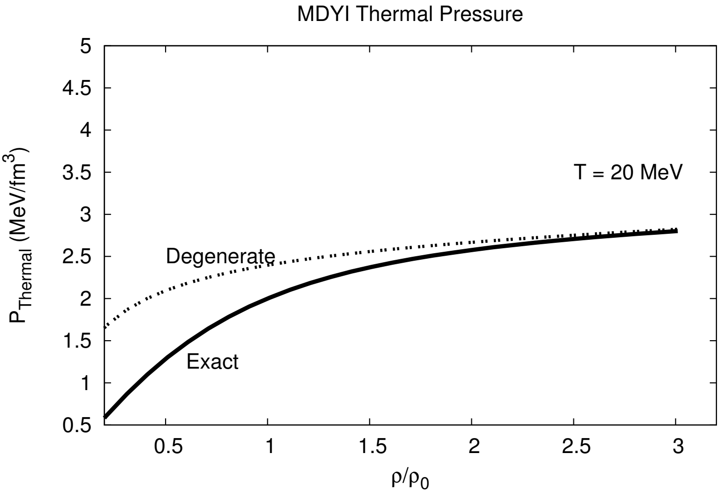
  Use Fermi Liquid results with level density parameter:

$$
\begin{aligned}
a &= \frac{\pi^2}{2p_F u_F} \\
u_F &= \frac{p_F}{m} + \frac{2C}{\rho_o}\frac{4}{h^3}2\pi\Lambda^2\left[1 - \frac{1}{2}\left(1 + \frac{\Lambda^2}{2p_F^2}\right)\ln\left(1 + \frac{4p_F^2}{\Lambda^2}\right)\right]
\end{aligned}
$$

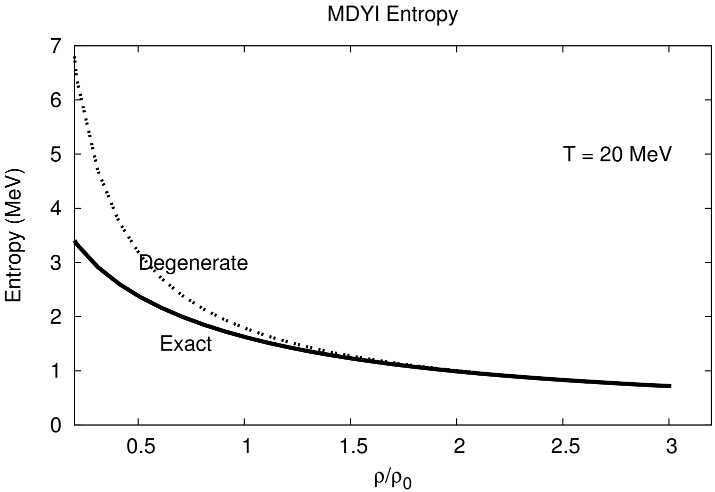
Thermal Energy



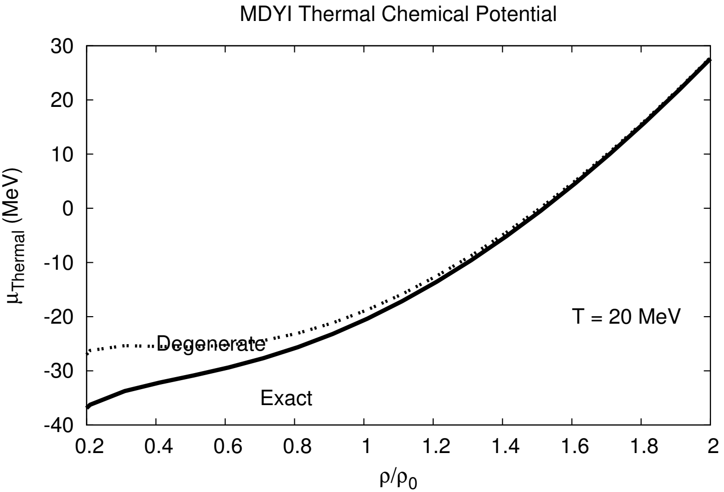
Thermal Pressure



Entropy


MDYI Entropy
Entropy (MeV)
0
1
2
3
4
5
6
7
0.5
1
1.5
2
2.5
3
ρ/ρ0
T = 20 MeV
Degenerate
Exact

Chemical Potential



Ongoing Work

- Comparison of thermal properties when Skyrme-like models and finite-range models are calibrated similarly at nuclear saturation density.
- Development of JEL-like scheme to calculate thermal properties for finite-range models.
- Address isospin asymmetric matter.

Relativistic Mean Field Theory

- Nucleons, Ψ, coupled to σ, ω, and $\vec{\rho}$ mesons:

$$
\begin{aligned}
\mathcal{L} \;=\;& \bar{\Psi}\left[\gamma_\mu\left(i\partial^\mu - g_\omega\omega^\mu - \frac{g_\rho}{2}\vec{\rho}^\mu.\vec{\tau}\right) - (M - g_\sigma\sigma)\right]\Psi \\
&+\frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2 - \frac{\kappa}{3}(g_\sigma\sigma)^3 - \frac{\lambda}{12}(g_\sigma\sigma)^4\right] \\
&+\frac{1}{2}\left[-\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + m_\omega^2\omega^\mu\omega_\mu\right] \\
&+\frac{1}{2}\left[-\frac{1}{2}\vec{B}_{\mu\nu}\vec{B}^{\mu\nu} + m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu\right]
\end{aligned}
$$

- Exclusions: Higgs, electromagnetic interactions, pions.

Assumptions

- The fluctuations of the meson fields are negligible
- Uniform, static system $\Rightarrow$

$$
\begin{aligned}
\sigma_0 &= \frac{g_\sigma}{m_\sigma^2} < \bar{\Psi}\Psi > - \frac{1}{m_\sigma^2}\left(\frac{\kappa}{2} g_\sigma^3 \sigma_0^2 + \frac{\lambda}{6} g_\sigma^4 \sigma_0^3\right) \\
&= \frac{g_\sigma}{m_\sigma^2} n_s - \frac{1}{m_\sigma^2}\left(\frac{\kappa}{2} g_\sigma^3 \sigma_0^2 + \frac{\lambda}{6} g_\sigma^4 \sigma_0^3\right) \\
\omega_0 &= \frac{g_\omega}{m_\omega^2} < \bar{\Psi}\gamma^0\Psi >= \frac{g_\omega}{m_\omega^2} n \\
\rho_0 &= \frac{g_\rho}{2m_\rho^2} < \bar{\Psi}\gamma^0\tau_3\Psi >= -\frac{g_\rho}{2m_\rho^2} n(1-2x)
\end{aligned}
$$

Here, $x = \frac{n_p}{n_n+n_p} = \frac{n_p}{n}$ is the proton fraction

T=0

1. Stress-Energy tensor

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu q_i)}\partial_\nu q_i - g_{\mu\nu}\mathcal{L}$$

2. Diagonal elements of $T_{\mu\nu}$ give
   - Energy density, $\varepsilon(n, T=0) = < T_{00} >$
   - Pressure, $P(n, T=0) = \frac{1}{3} < T_{ii} >$

3. Effective mass, M^*, derived from the requirement $\frac{\delta\varepsilon}{\delta\sigma} = 0$

Rest of thermodynamics

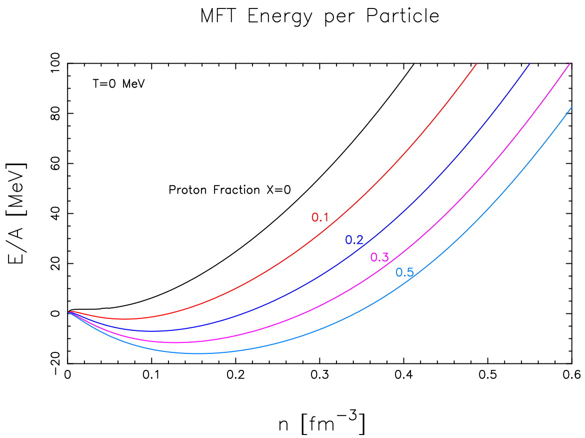
- Chemical potentials $\mu_i = \frac{d\varepsilon}{dn_i}$
- Compression modulus $K = 9n_0 \left.\frac{d\mu}{dn}\right|_{n_0}$
- Symmetry energy $S_2 = \frac{1}{8} \left.\frac{d^2(\varepsilon/n)}{dx^2}\right|_{x=1/2}$
- Susceptibilities $\chi_{ij} = \left(\frac{d\mu_i}{dn_j}\right)^{-1}$

Calibration

Use known properties of nuclear matter to fix couplings:

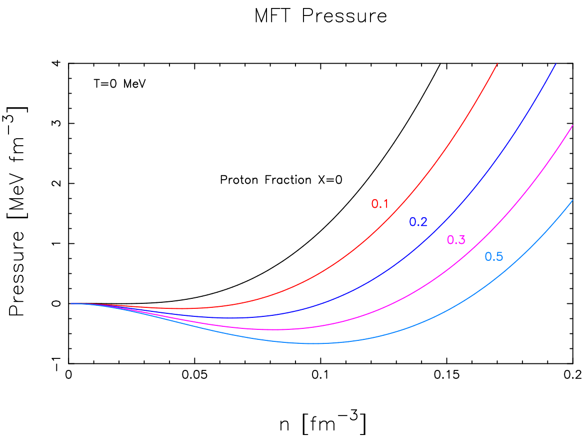
$$\left.\begin{array}{l} n_0 = 0.16\ \mathrm{fm}^{-3} \\ K = 225\ \mathrm{MeV} \\ E/A = -16\ \mathrm{MeV} \\ M^*/M = 0.7 \\ S_2 = 30\ \mathrm{MeV} \end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{l} g_\sigma = 9.061 \\ g_\omega = 10.55 \\ g_\rho = 7.475 \\ \kappa = 9.194\ \mathrm{MeV} \\ \lambda = -3.280 \times 10^{-2} \end{array}\right.$$

Energy per particle



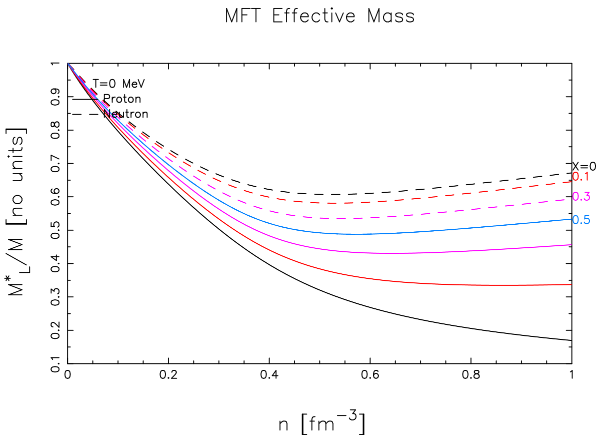
- E/A minimum shifts to lower densities as $x \to 0$.
- For $n < n_o$ need to consider nuclei, clusters, etc.

Pressure



- For $\frac{dp}{dn} < 0$, matter is spinodally unstable.
- Range of n for which instability occurs, shifts to lower densities as $x \to 0$.

Effective mass



- Landau effective mass, $M_i^* = \left(\left. \frac{\partial \varepsilon_{k_i}}{\partial k_i} \right|_{k_{Fi}} \right)^{-1} k_{Fi}$

Finite T

Fermi-Dirac distributions ($f_{k_i}(T) = 1 + e^{(E^*_{k_i} - \nu)/T}$) in kinetic integrals:

$$
\begin{aligned}
\varepsilon_k(n,T) &= \sum_{i=n,p} \frac{1}{\pi^2} \int_0^\infty dk_i k_i^2 E^*_{k_i} f_{k_i}(T) \\
P_k(n,T) &= \frac{1}{3} \sum_{i=n,p} \frac{1}{\pi^2} \int_0^\infty dk_i \frac{k_i^4}{E^*_{k_i}} f_{k_i}(T) \\
n_i &= \frac{1}{\pi^2} \int_0^\infty dk_i k_i^2 f_{k_i}(T) \\
M^*_k(n,T) &= \frac{g_\sigma^2}{m_\sigma^2} \sum_{i=n,p} \frac{1}{\pi^2} \int_0^\infty dk_i \frac{k_i^2 M^*_T}{E^*_{k_i}} f_{k_i}
\end{aligned}
$$

- At temperatures of interest ($T \leq 50$ MeV) we can ignore antiparticles

P_k, ε_k, and n_i as algebraic functions of M^*, T, and ν (kinetic part of chemical potential):

$$
\begin{aligned}
p_i &= \frac{M^{*4}}{\pi^2}\frac{f_i g_i^{5/2}(1+g_i)^{3/2}}{(1+f_i)^{M+1}(1+g_i)^N}\sum_{m=0}^{M}\sum_{n=0}^{N} p_{mn} f_i^m g_i^n \equiv \frac{M^{*4}}{\pi^2} p_i^* \\
\epsilon_i &= \frac{M^{*4}}{\pi^2}\left[t\left(\frac{\partial p_i^*}{\partial t}\right)_{\psi_i} - p_i^*\right] \\
n_i &= \frac{M^{*3}}{\pi^2}\frac{1}{t}\left(\frac{\partial p_i^*}{\partial \psi_i}\right)_t \\
\psi_i &= \frac{\nu_i - M^*}{T} = 2(1+f_i/a)^{1/2}\ln\left[\frac{(1+f_i/a)^{1/2}-1}{(1+f_i/a)^{1/2}+1}\right] \\
g_i &= \frac{T}{M^*}(1+f_i)^{1/2} = t(1+f_i)^{1/2}
\end{aligned}
$$

Rest of thermodynamics

- Chemical potentials, $\mu_i = \nu_i + \frac{g_\omega^2}{m_\omega^2} n + \frac{g_\rho^2}{4m_\rho^2}(n_i - n_j)$
- Entropy density, $s = \frac{1}{T}\left(\varepsilon + p + \sum_i \mu_i n_i\right)$
- Free energy density, $\mathcal{F} = \varepsilon - Ts$
- Inverse kinetic susceptibilities,

$$\frac{d\nu_n}{dn_n} = T \frac{\left(\frac{dt}{d\psi_p}\right)^{-1} \frac{dn_p}{d\psi_p}\left(1 - \frac{1}{t^2}\frac{dt}{d\psi_n}\right) + \frac{dn_p}{dt}}{\left(\frac{dt}{d\psi_p}\right)^{-1} \frac{dn_p}{d\psi_p}\left[\frac{dn_n}{dt}\frac{dt}{d\psi_n} + \frac{dn_n}{d\psi_n}\right] + \frac{dn_n}{d\psi_n}\frac{dn_p}{dt}}$$

$$\frac{d\nu_n}{dn_p} = -T \frac{\frac{dt}{d\psi_p}\left(\frac{dt}{d\psi_n}\right)^{-1}\left[\frac{1}{t^2}\frac{dn_n}{d\psi_n} + \frac{dn_n}{dt}\right]}{\left(\frac{dt}{d\psi_n}\right)^{-1} \frac{dn_n}{d\psi_n}\left[\frac{dn_p}{dt}\frac{dt}{d\psi_p} + \frac{dn_p}{d\psi_p}\right] + \frac{dn_p}{d\psi_p}\frac{dn_n}{dt}}$$

Consistency Checks

Degenerate Limit

- Earlier Fermi Liquid Theory results for P, ϵ, and μ apply, but with level density parameter:

$$a_i = \frac{\pi^2 E^*_{F_i}}{2k^2_{F_i}}; \qquad E^*_{F_i} = (k^2_{F_i} + M^{*2})^{1/2}$$

and Fermi velocity

$$u_{F_i} = \frac{k_{F_i}}{E^*_{F_i}}$$

Non-Degenerate Limit

Expand in Bessel functions, K_α, and fugacity, $z = e^{-\nu/T}$:

$$\begin{aligned}
n_i &= \frac{M^{*3}}{4\pi^2} \sum_{m=1} (-1)^{m+1} z_i^m \frac{K_2(mx)}{mx} \\
\varepsilon_i &= \frac{M^{*4}}{\pi^2} \sum_{m=1} (-1)^{m+1} z_i^m \left[\frac{K_1(mx)}{mx} + \frac{3K_2(mx)}{m^2x^2}\right] \\
p_i &= \frac{M^{*4}}{4\pi^2} \sum_{m=1} (-1)^{m+1} z_i^m \frac{K_2(mx)}{m^2x^2}
\end{aligned}$$

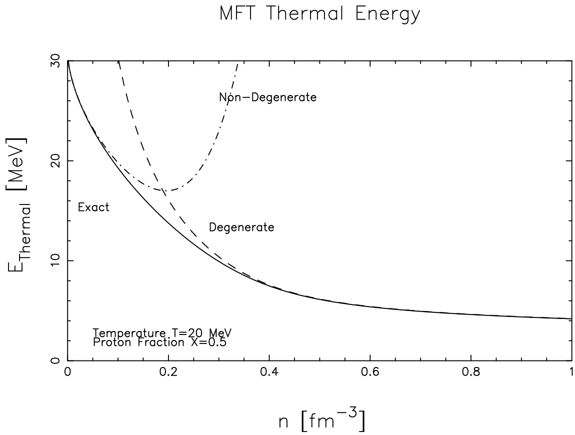
Here, $x \equiv \frac{M^*}{T}$

Non-Degenerate Limit

Neglect $\mathcal{O}(z^3)$, invert n_i for z, expand K_n for large x :

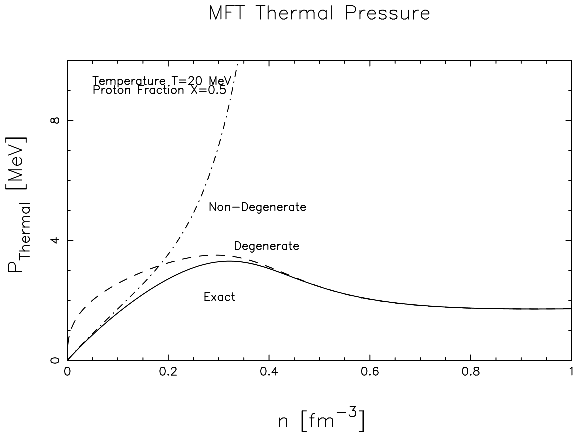
$$\begin{aligned}
\varepsilon_i &= M^* n_i + \frac{3Tn_i}{2}\left[1 + \frac{n_i}{4}\left(\frac{\pi}{M^* T}\right)^{3/2} + \frac{5T}{4M^*}\right] \\
p_i &= Tn_i\left[1 + \frac{n_i}{4}\left(\frac{\pi}{M^* T}\right)^{3/2}\right] \\
\nu_i &= M^* + T\left\{\ln\left[\left(\frac{2\pi}{M^* T}\right)^{3/2}\frac{n_i}{2}\right] + \frac{n_i}{2}\left(\frac{\pi}{M^* T}\right)^{3/2} - \frac{15T}{8M^*}\right\}
\end{aligned}$$

Thermal Energy



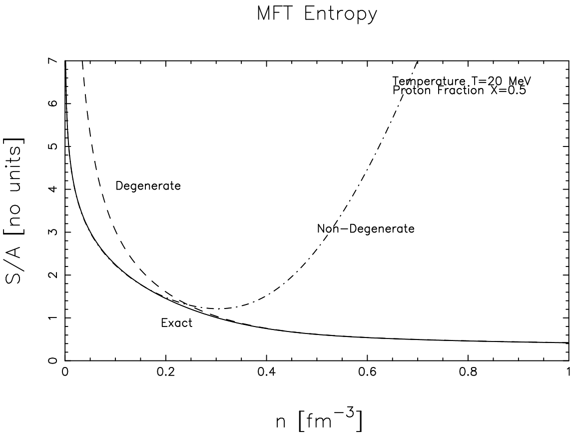
- Exact calculations agree with analytical limits where expected.
- Around nuclear saturation density exact results needed.

Thermal Pressure



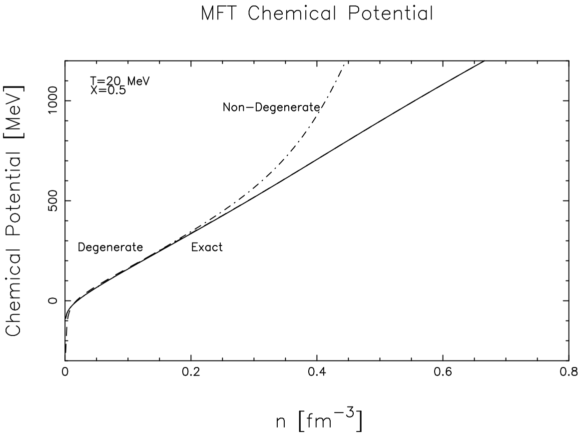
- Exact calculations agree with analytical limits where expected.
- Around nuclear saturation density exact results needed.

Entropy



- Agreement is better for S/A than for E_{th} and P_{th}.

Chemical Potential



- Agreement is better for μ than for E_{th} and P_{th}.

Deficiencies of Relativistic Mean Field Models

- From an analysis of the Dirac equation, the optical potential is linear in energy inconsistent with optical model fits of nucleon-nucleus reaction data.
- Good fits can likely be obtained by extension to include Fock terms; not achieved yet.
- ...

Ongoing Work

- Treatment of inhomogeneous phase with nuclei at subnuclear densities.
- Thermal properties, entropy and specific heat, of nuclei consistent with the treatment of the bulk phase hamiltonian.
- Preparation of tables for use in supernova simulations.
- Collaborators: Lattimer, Prakash, Gang Shen, Steiner, and Muccioli.