New approaches to multi-dimensional radiation transport

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Caltech

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Monte Carlo

E. Abdikamalov, A. Burrows, C. D. Ott, F. Löffler, E. O'Connor, J. Dolence, E. Schnetter, 2012, ApJ

Filtered Spherical Harmonics (FP_N)

D. Radice, E. Abdikamalov, L. Rezzolla, and C. D. Ott, *in prep*

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Transport equation

$$\frac{1}{c}\frac{\partial I}{\partial t} + \nabla I = \eta - \kappa I$$

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flux absorption extinction

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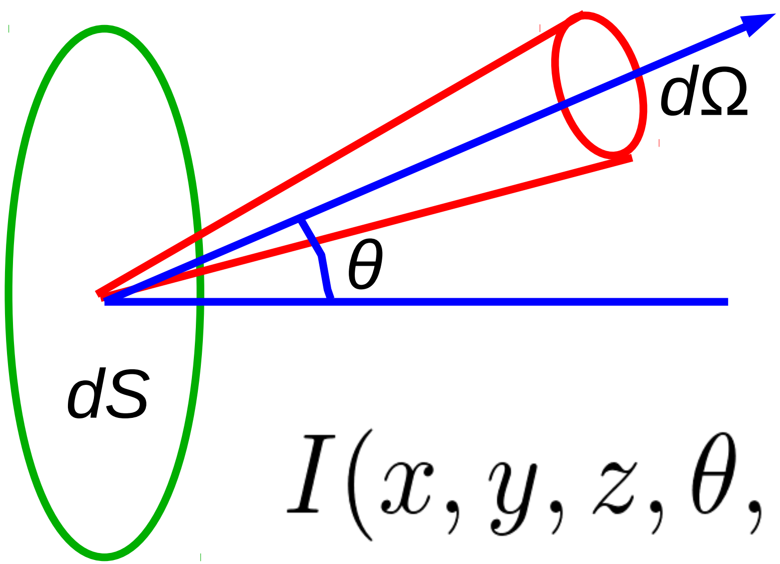
flux absorption extinction



$$I(x, y, z, \theta, \phi, \nu, t) = \frac{dE}{d\Omega dS d\nu \cos\theta dt}$$

Deterministic and Monte Carlo radiation transport

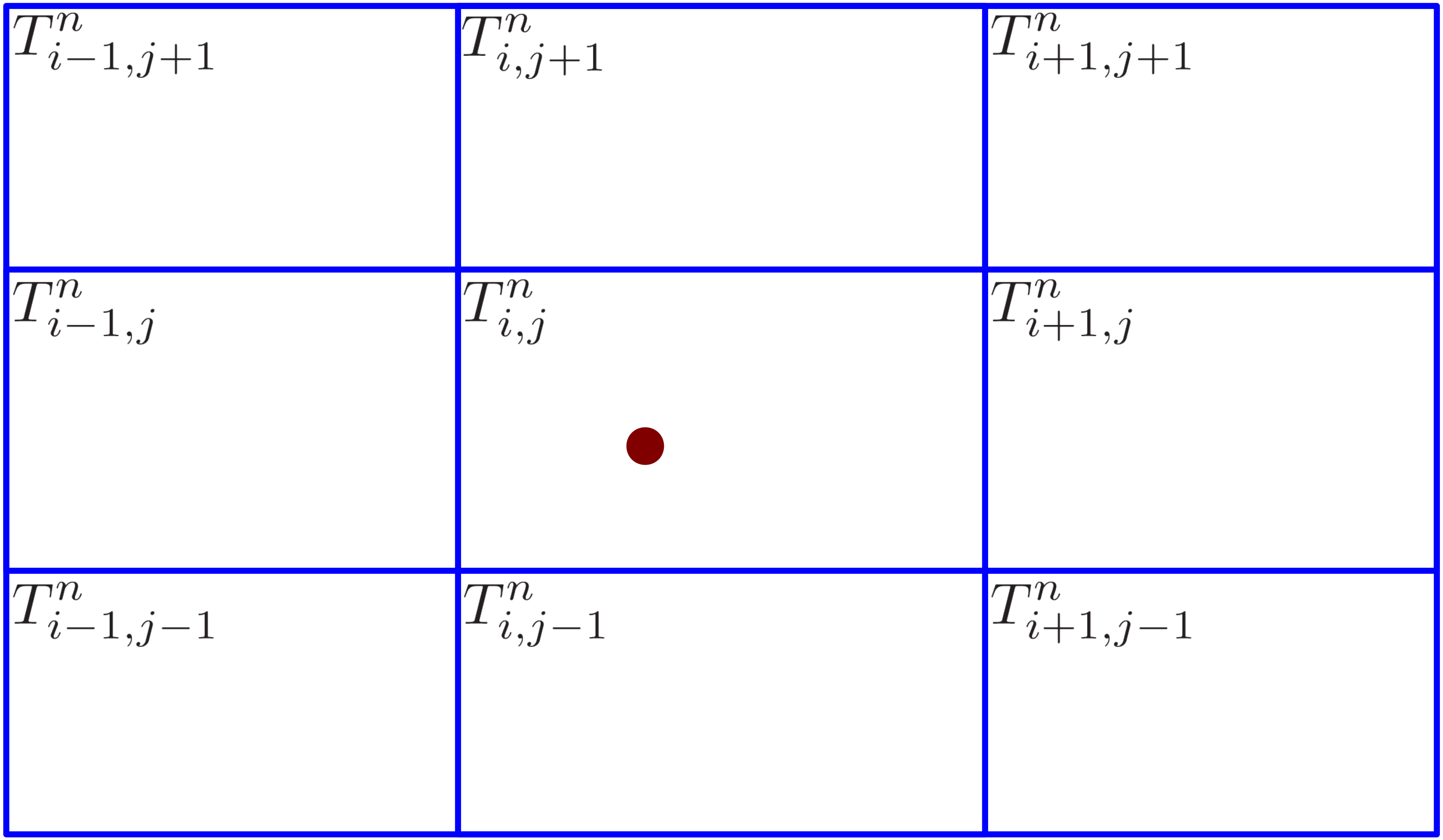
Monte Carlo Transport

Monte Carlo Transport

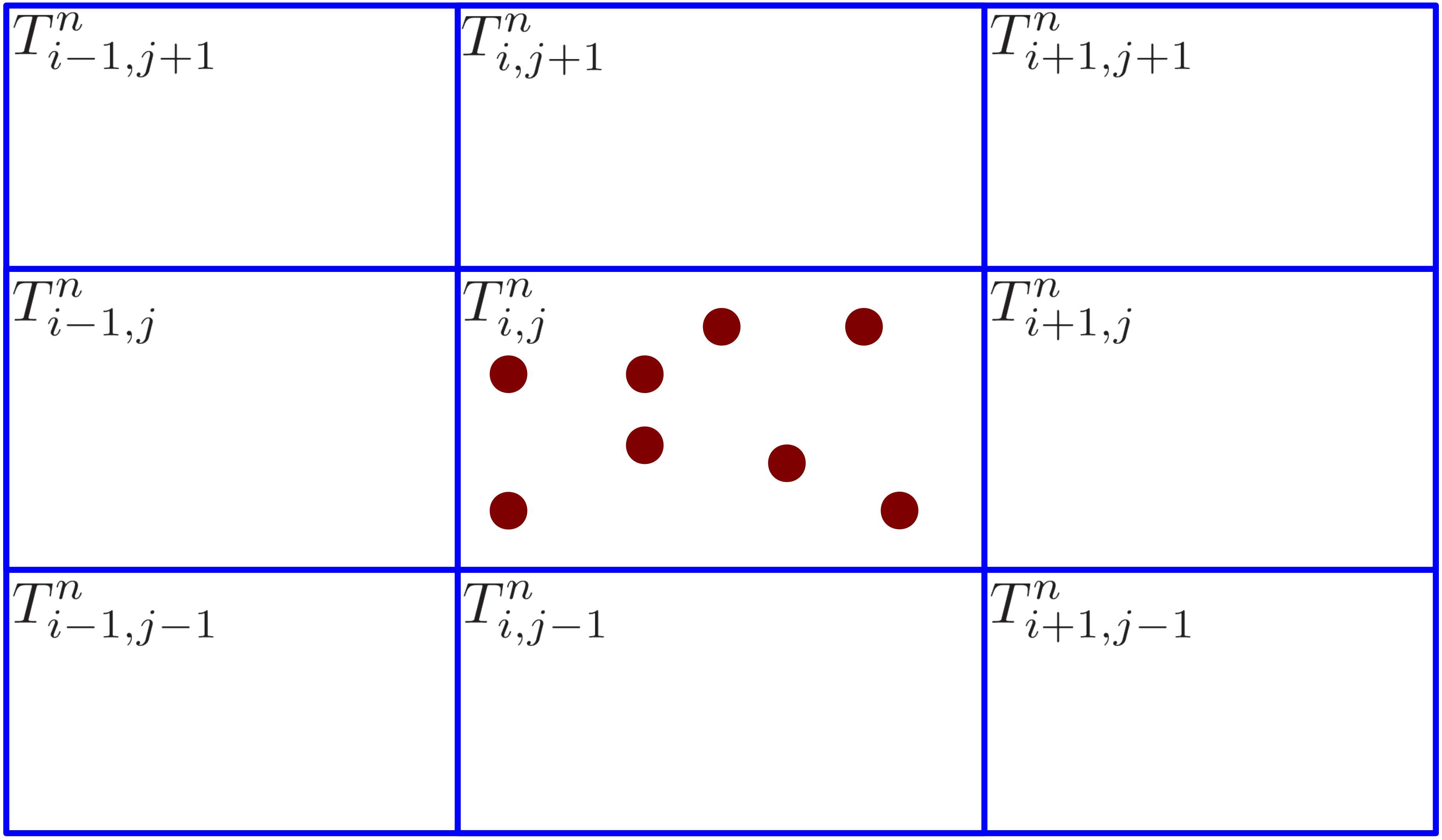
Monte Carlo Transport

$T^n_{i-1,j+1}$	$T^n_{i,j+1}$	$T^n_{i+1,j+1}$
$T^n_{i-1,j}$	$T^n_{i,j}$	$T^n_{i+1,j}$
$T^n_{i-1,j-1}$	$T^n_{i,j-1}$	$T^n_{i+1,j-1}$

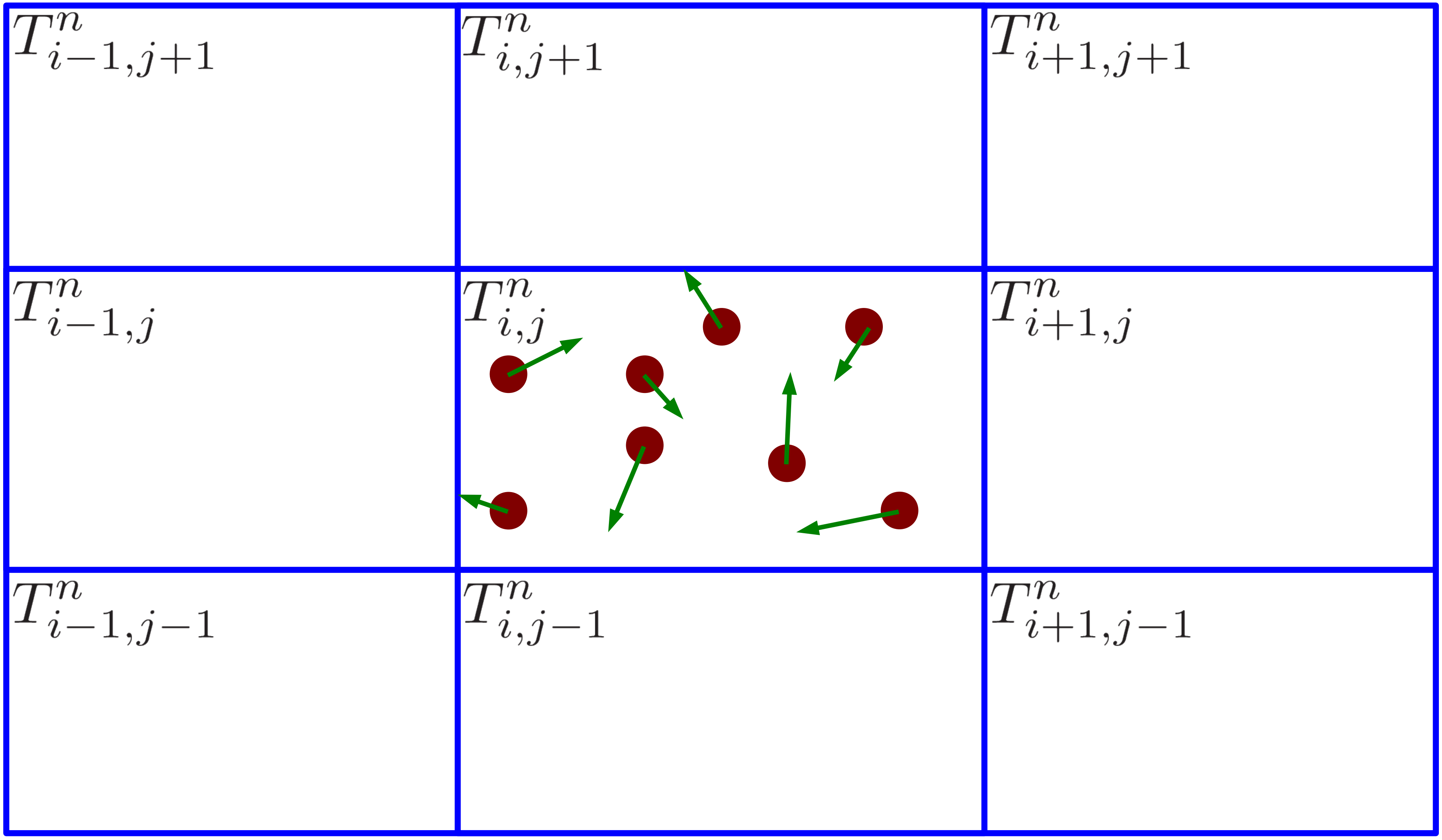
Monte Carlo Transport



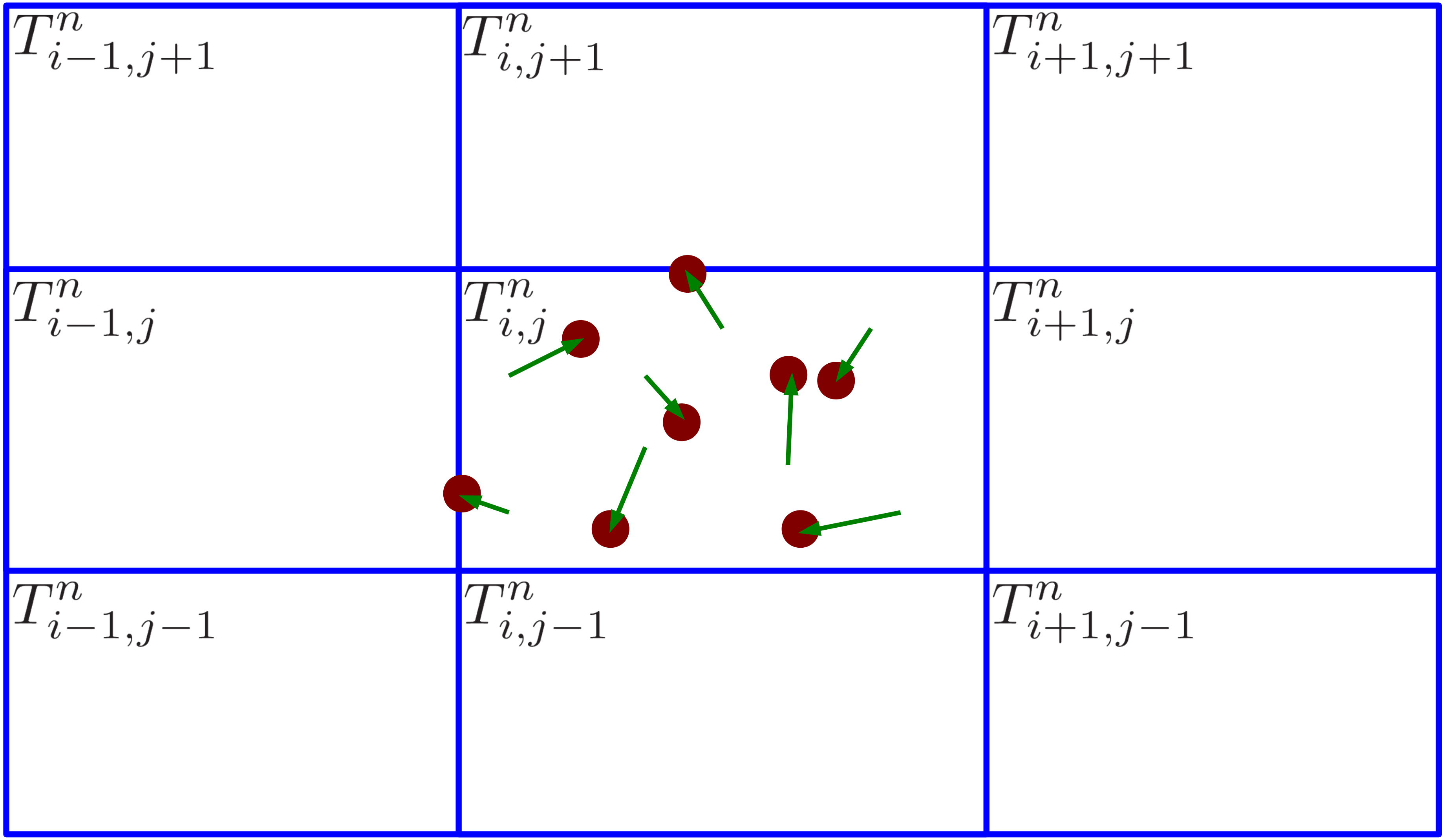
Monte Carlo Transport



Monte Carlo Transport



Monte Carlo Transport



Monte Carlo Transport

- Simple
- Easy extension to multi-D
- Parallelization
- Random noise
- Expensive

Monte Carlo transport scheme goal:

- Time-dependence

  (coupling to matter energy and lepton number)

- Efficient treatment of high optical depth.

- Energy dependence

- Velocity dependence

A simple explicit discretization does not work!

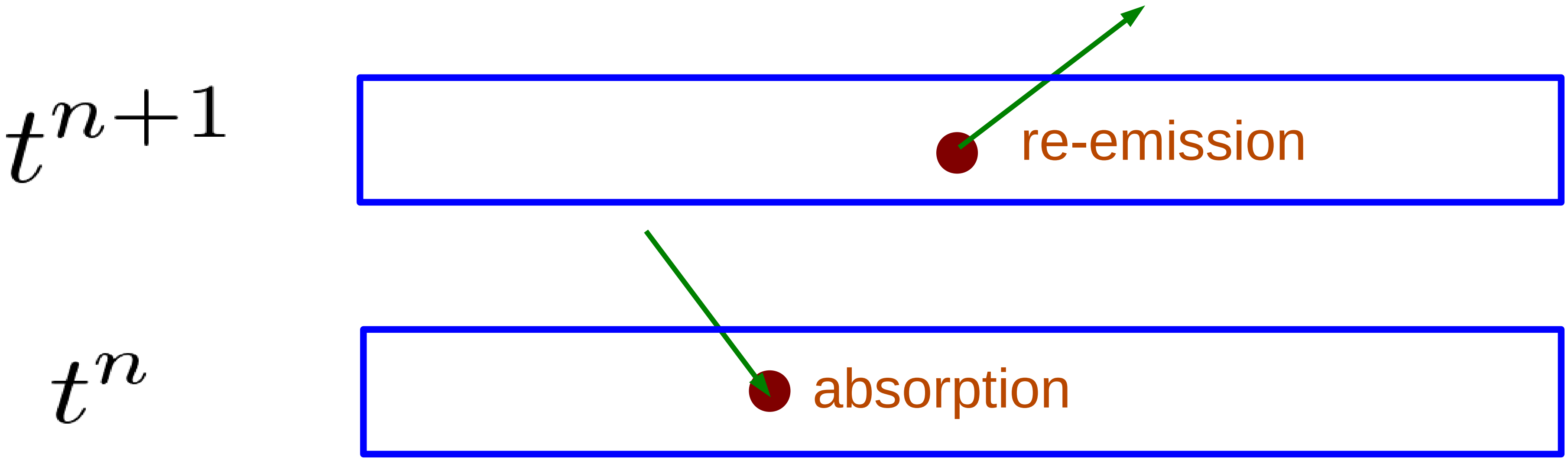
$$\text{Timestep} \lesssim \frac{\rho c_v}{a T^3 c \kappa}$$

A popular solution:

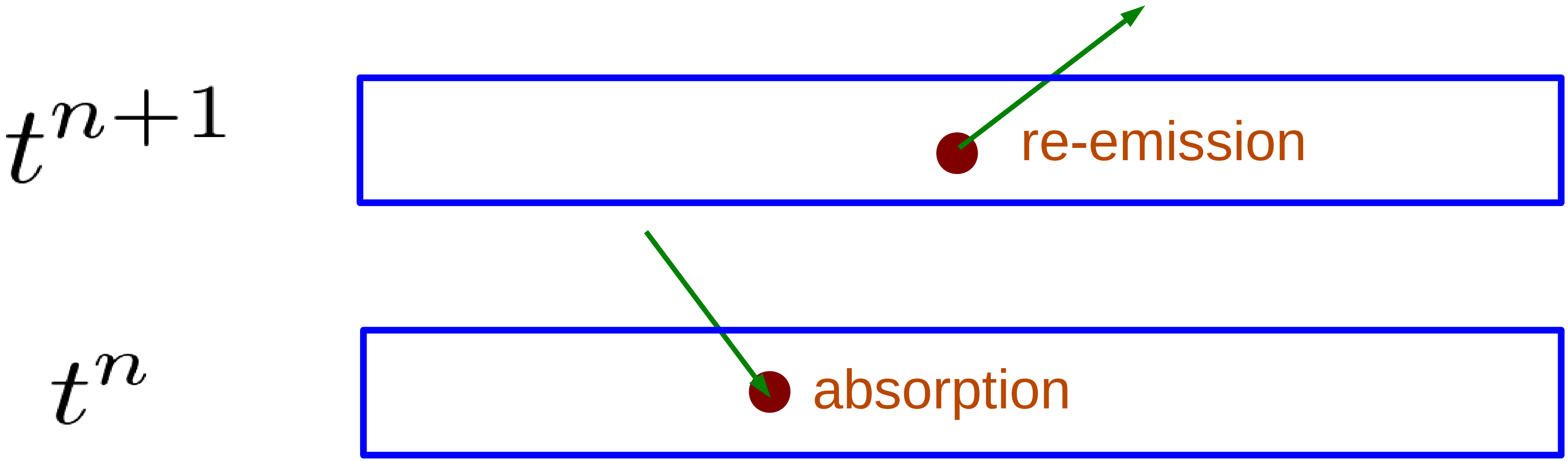
Implicit Monte Carlo method

(Fleck & Cummings 1971)

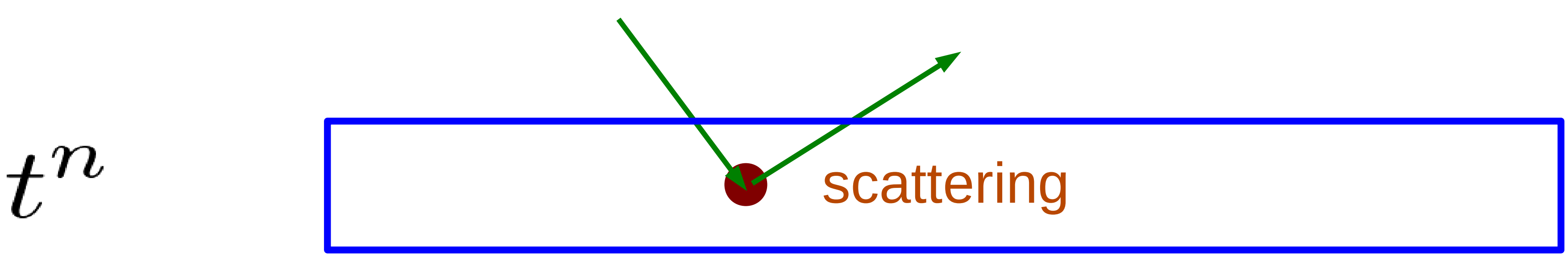
A simple explicit scheme:



A simple explicit scheme:



Implicit Monte Carlo scheme:

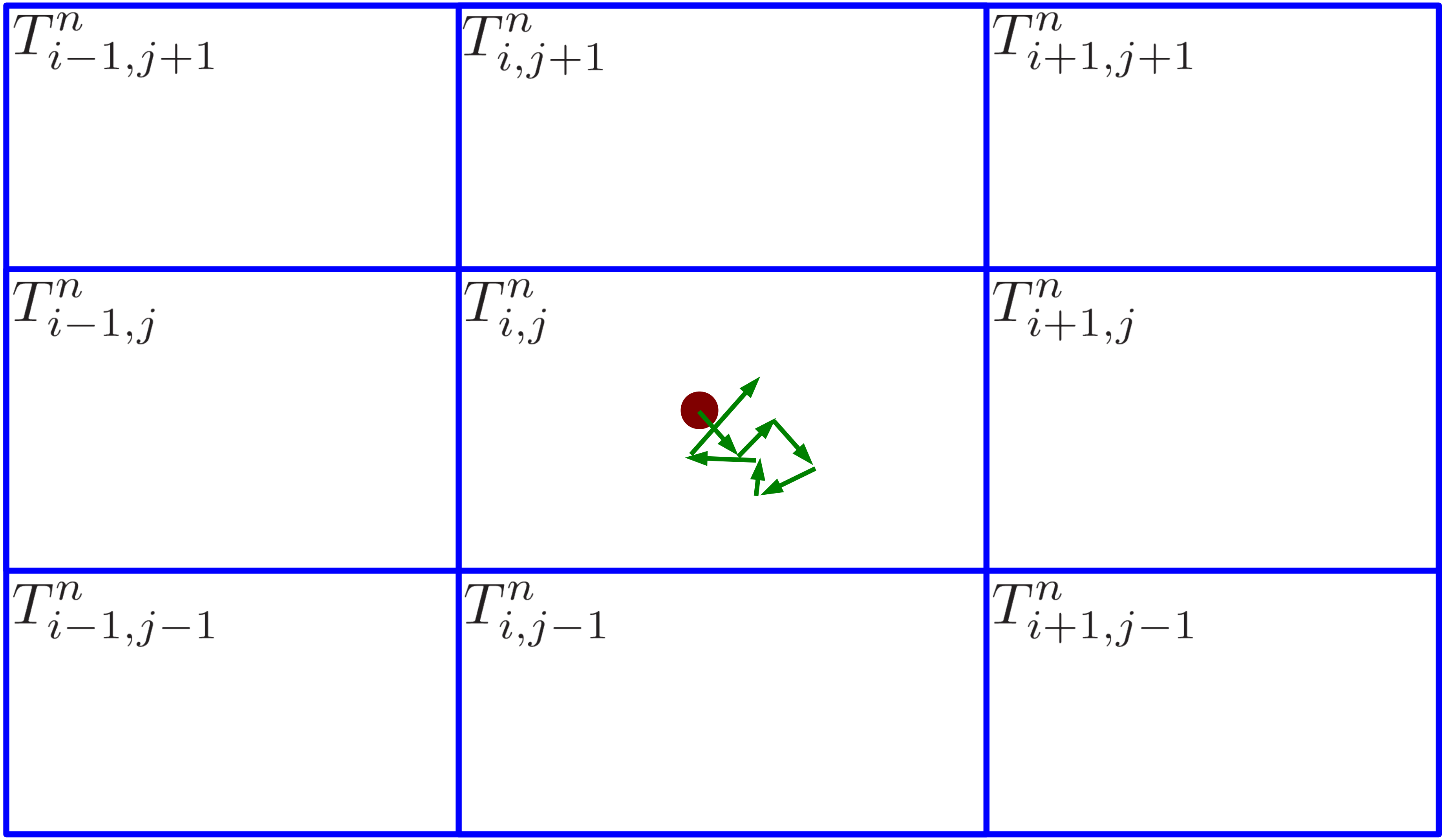


Implicit Monte Carlo for Neutrinos

- Energy and lepton number coupling
- The same benefits as for photons
  (large timestep, unconditional stability, accuracy)
- Both energy and lepton number conservation

Implicit Monte Carlo

at high optical depth



Implicit Monte Carlo

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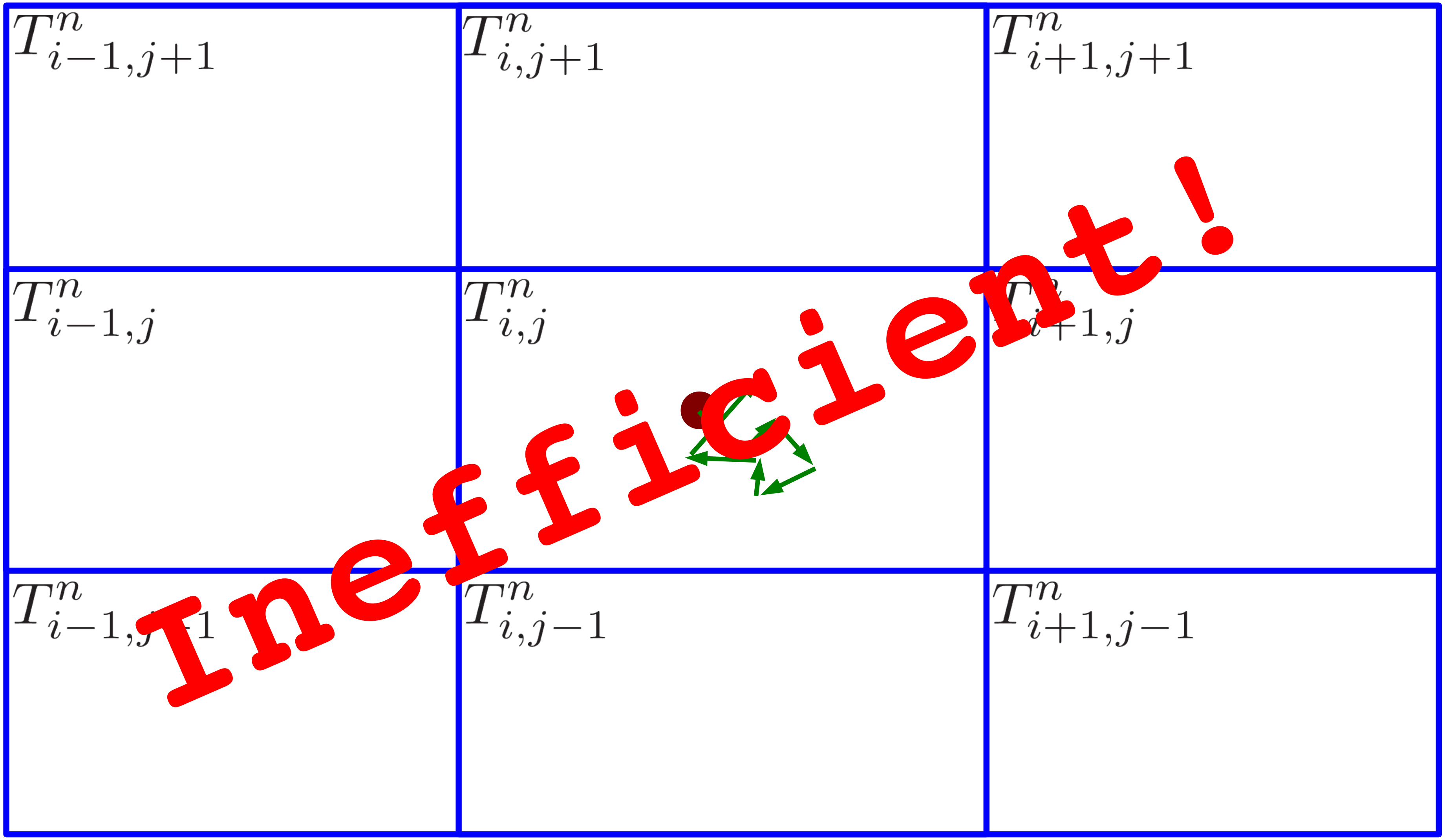
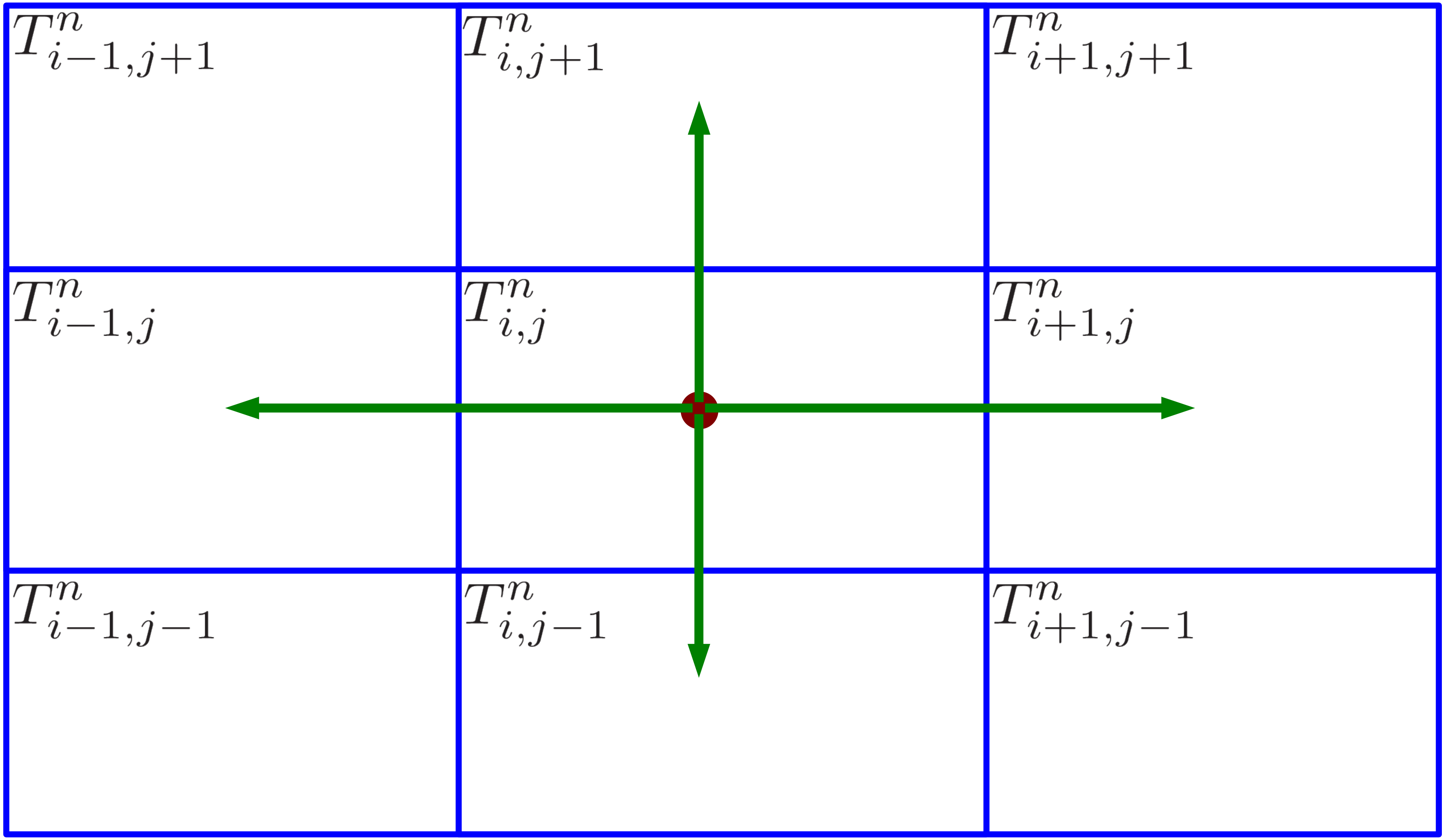


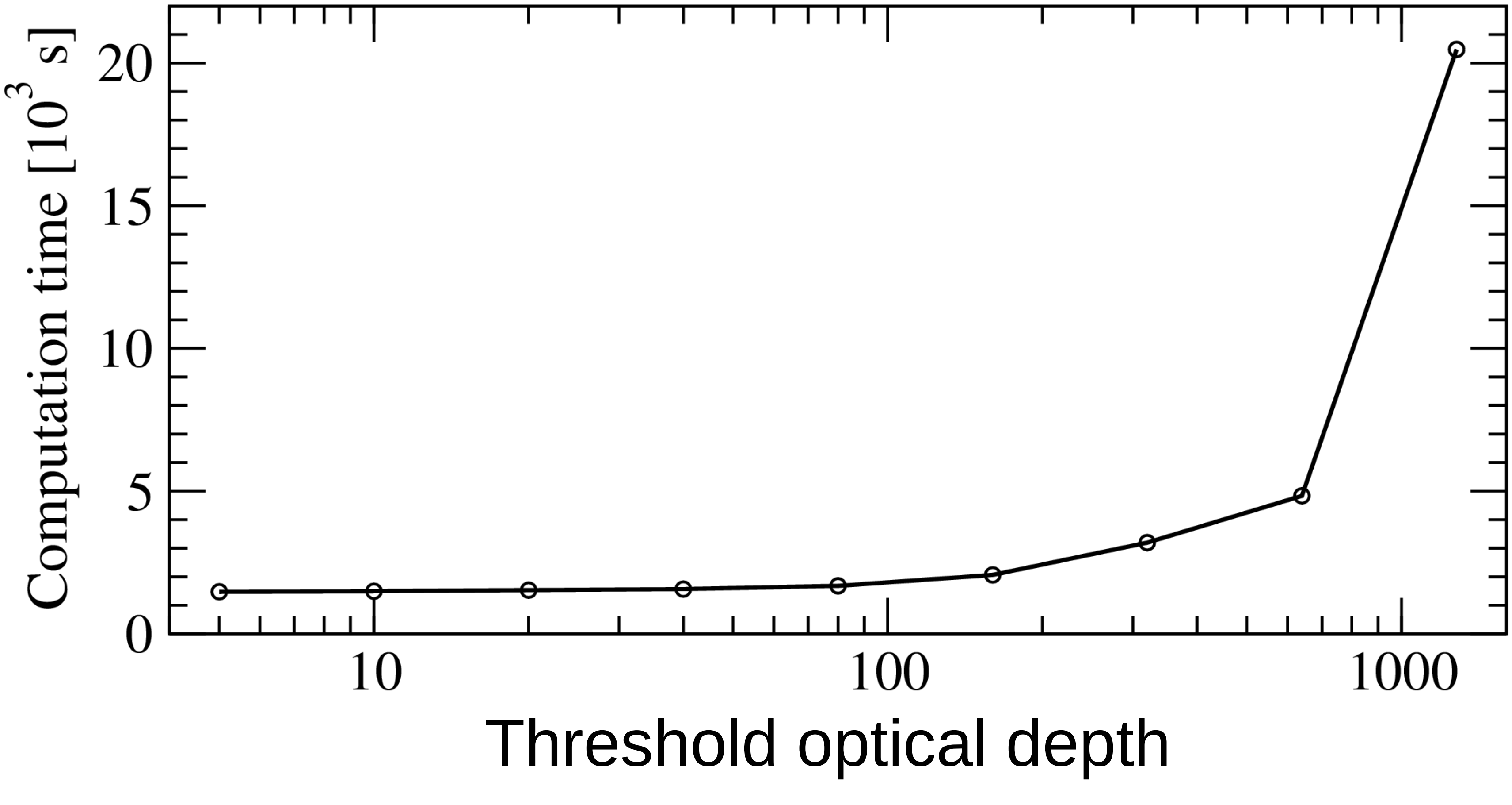
Discrete-Diffusion Monte Carlo

by Densmore+ '07 for gray transport for non-moving matter



Discrete-diffusion speed-up

for proto-neutron star cooling



Velocity dependence

Velocity-dependent Monte Carlo

Mixed frame formalism

Eulerian frame: Transport

Comoving frame: emission, absorption, scattering

Velocity-dependent discrete-diffusion

Transport is performed in comoving frame with O(λ*v*/*Lc*) accuracy:

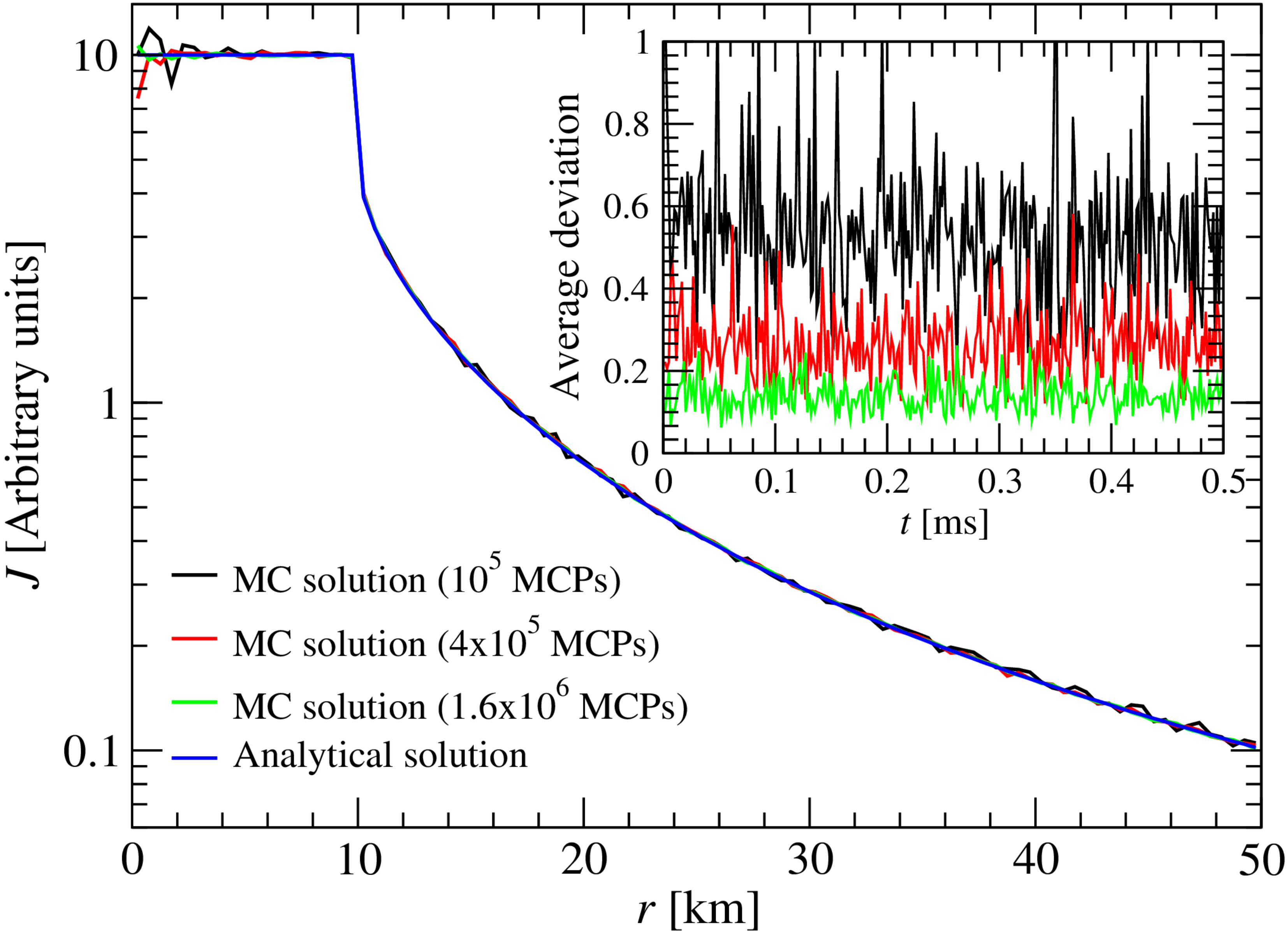
$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{v}{c}\frac{\partial J_0}{\partial r} + \frac{J_0}{c}\frac{\partial v}{\partial r} + \frac{\varepsilon_0}{3c}\frac{\partial J_0}{\partial \varepsilon_0}\frac{D\ln\rho}{Dt} + \frac{\partial H_0}{\partial x} = \kappa_0(B - J_0)$$

Three effects: advection, compression/expansion, and Doppler shift.

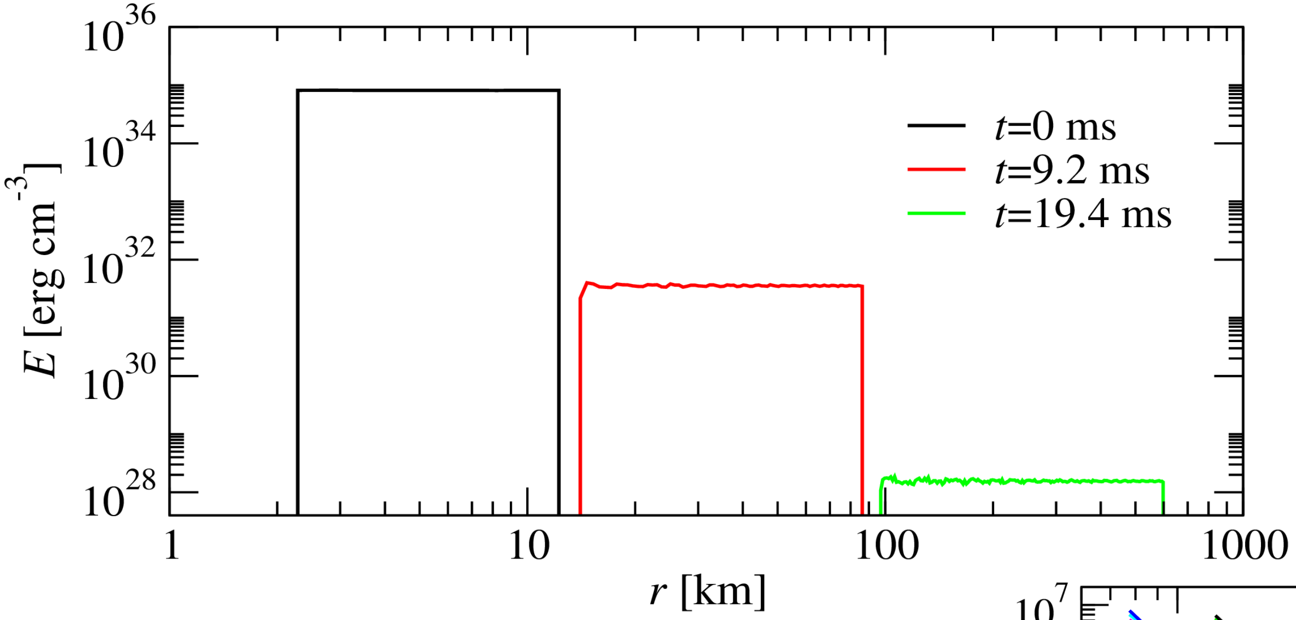
Solution method: Operator-splitting

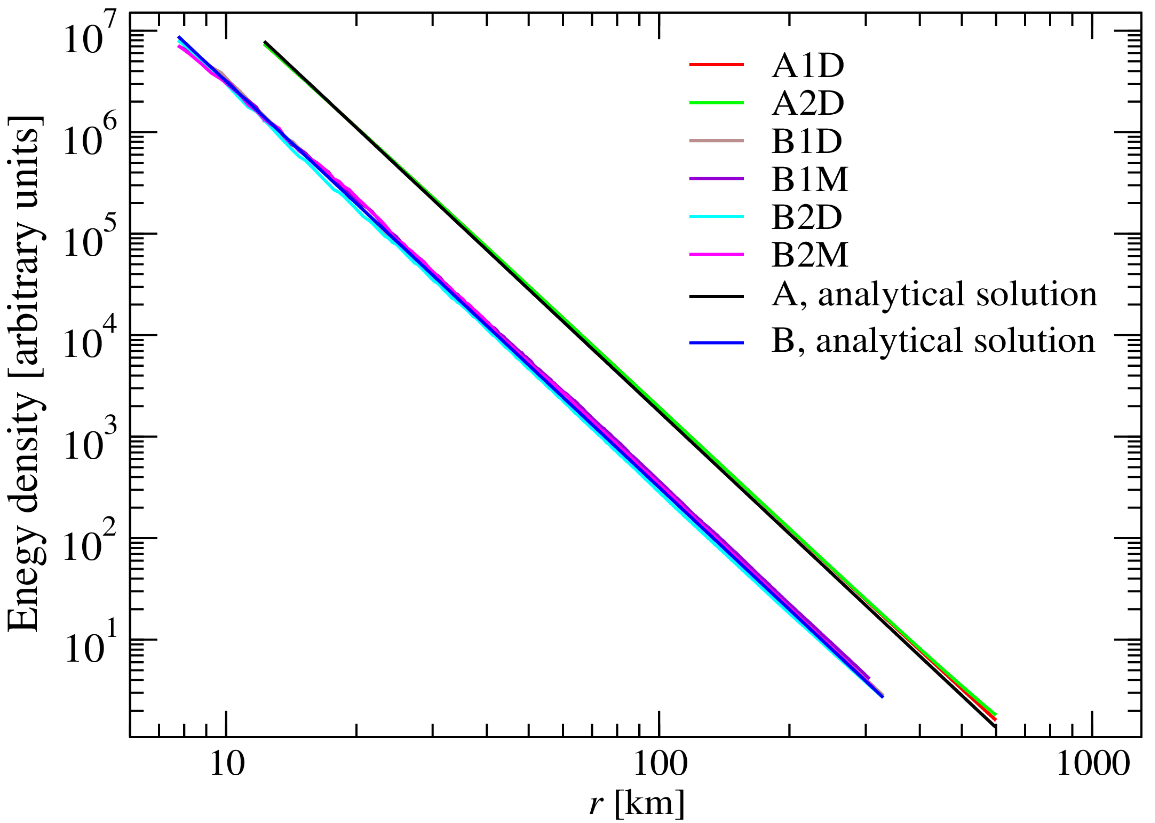
Tests

Homogeneous sphere



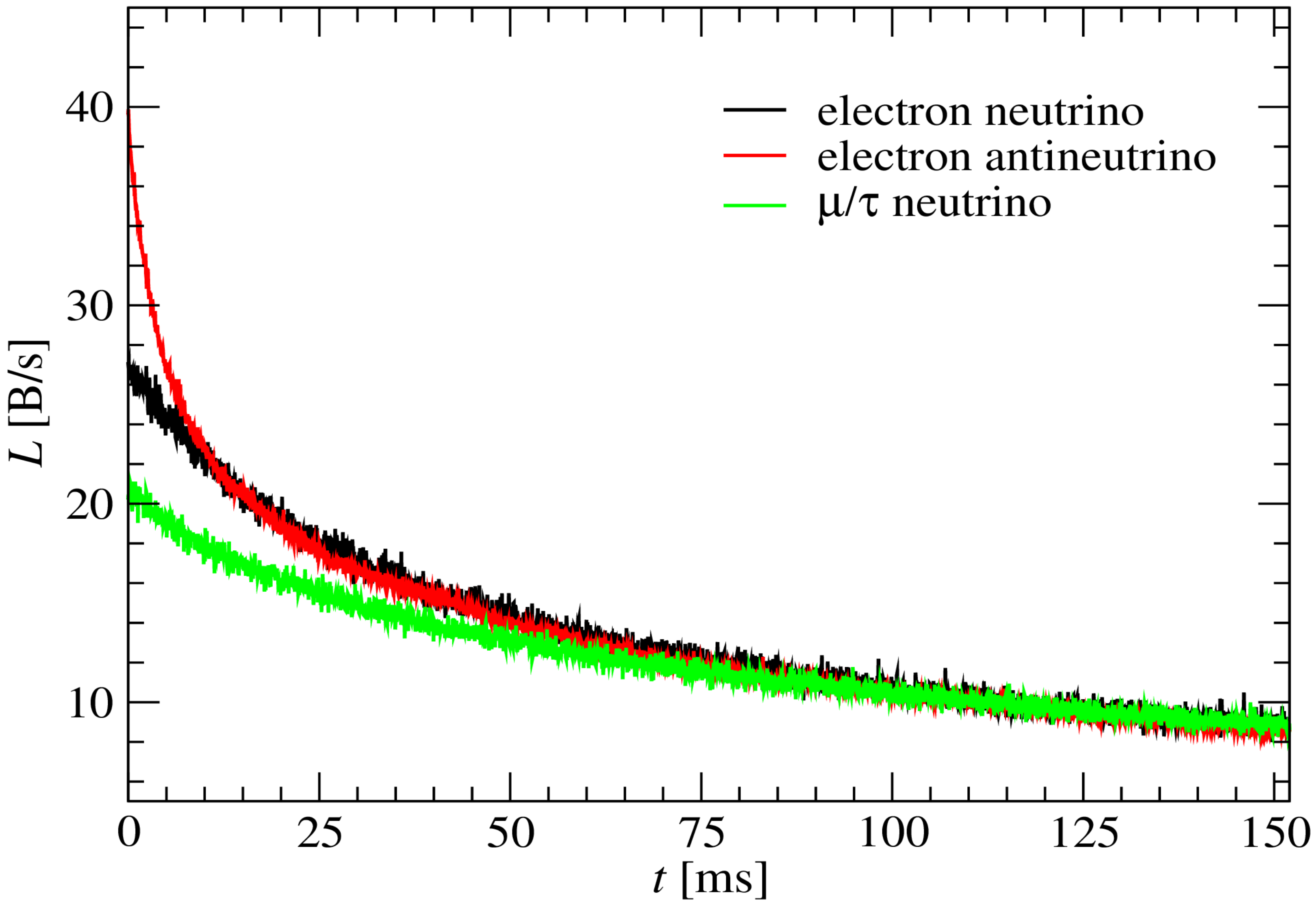
Homologously expanding shell



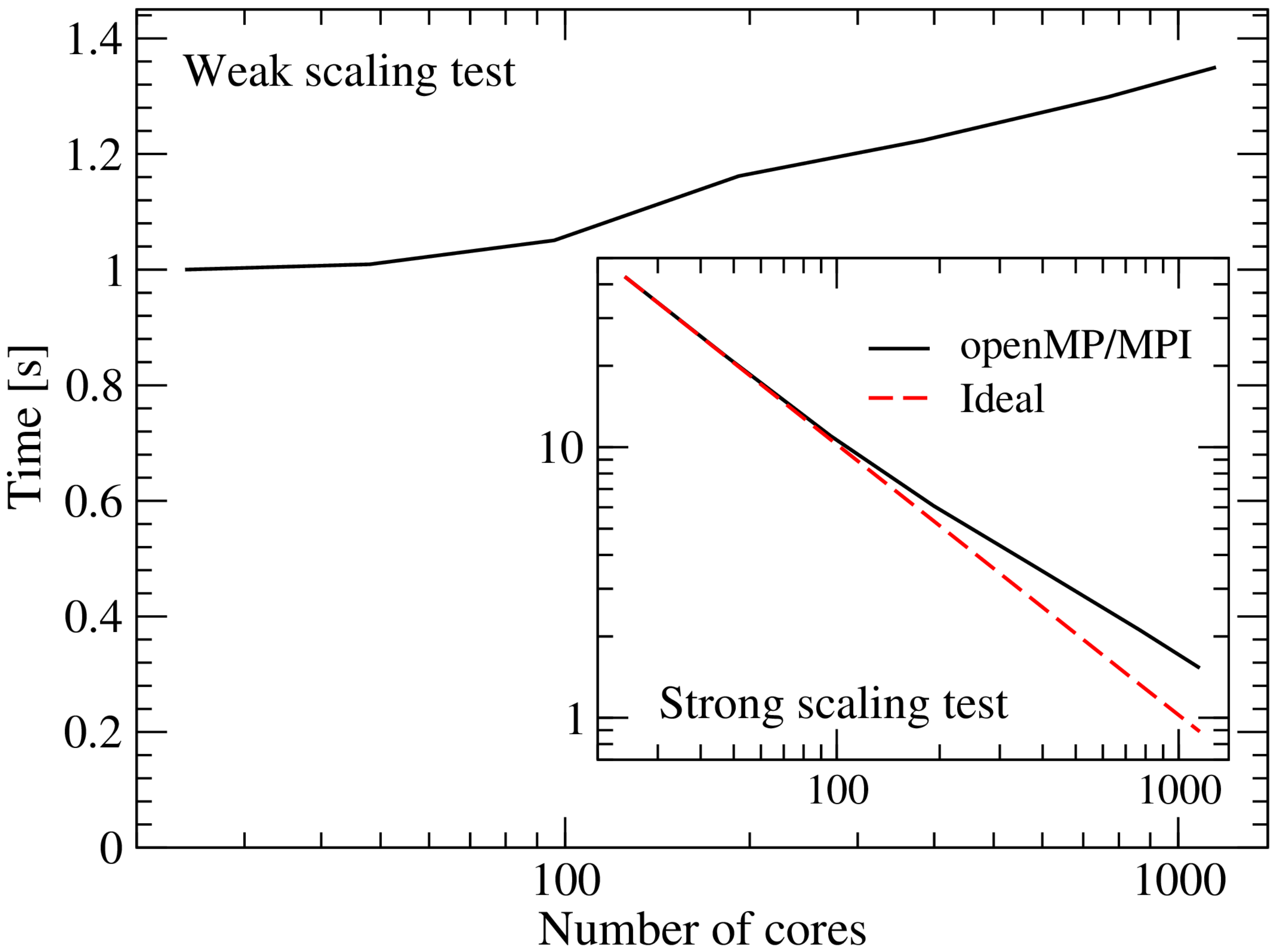


Proto-neutron star cooling

using Ott et al. (2008) PNS model



Parallel scaling
mesh replication method



Monte Carlo Summary

- Implicit MC for neutrino transport
- Multi-group discrete-diffusion
- Velocity-dependence
- Applicable to both neutrinos and photons
- Parallel scaling (in 1D)

Monte Carlo

E. Abdikamalov, A. Burrows, C. D. Ott, F. Löffler, E. O'Connor, J. Dolence, E. Schnetter, 2012, ApJ

Filtered Spherical Harmonics (FP_N)

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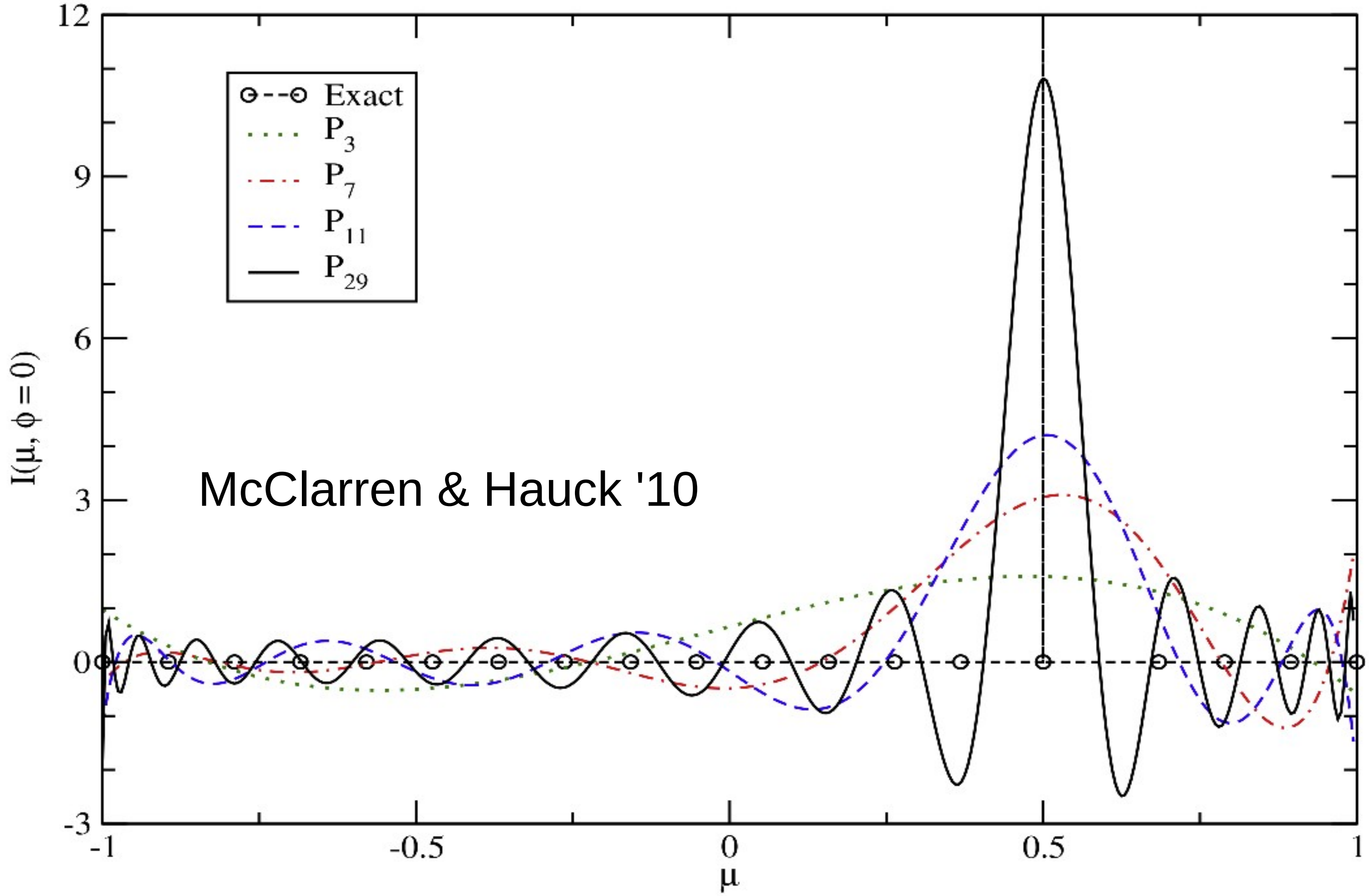
P_N scheme

$$I(x,\nu,\Omega,t) \simeq \sum_{l=0}^{N} \sum_{m=-l}^{l} E^{ml}(x,\nu,t) Y_{ml}(\Omega)$$

$$E^{lm}(x,\nu,t) = \int_{4\pi} I(x,\nu,\Omega,t) Y^{lm}(\Omega) d\Omega$$

- Hyperbolic system ($v \leq c$)
- Rotationally invariant (no ray-effects as in S_N)
- Less memory ($P_{N-1} \sim S_N$)

Oscillations in P_N



(a) Beam: $I(\mu, \varphi) = \delta(\mu - \frac{1}{2})$.

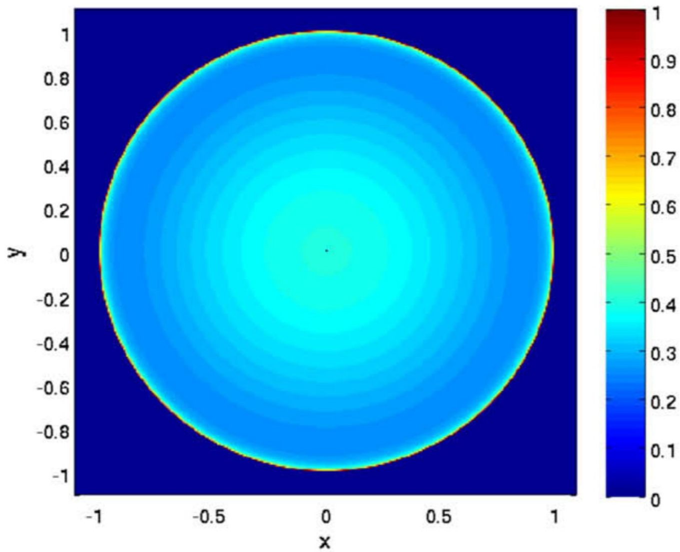
Filtered P_N

McClarren & Hauck 2010

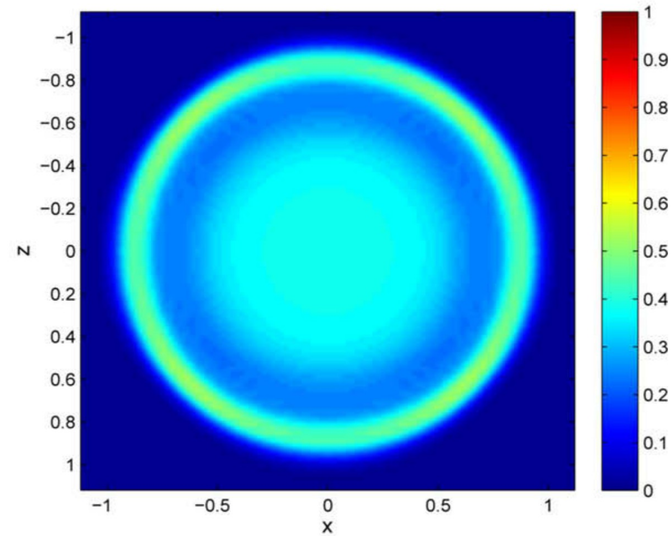
- Use filters to remove oscillations
- Preserves rotational invariance
- Converges to the transport solution
- Efficient and accurate

Filtered P_N: 2D line problem

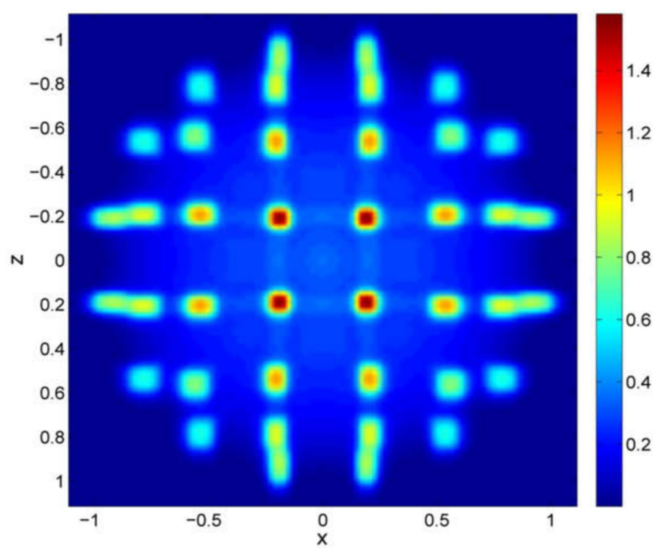
McClarren & Hauck 2010



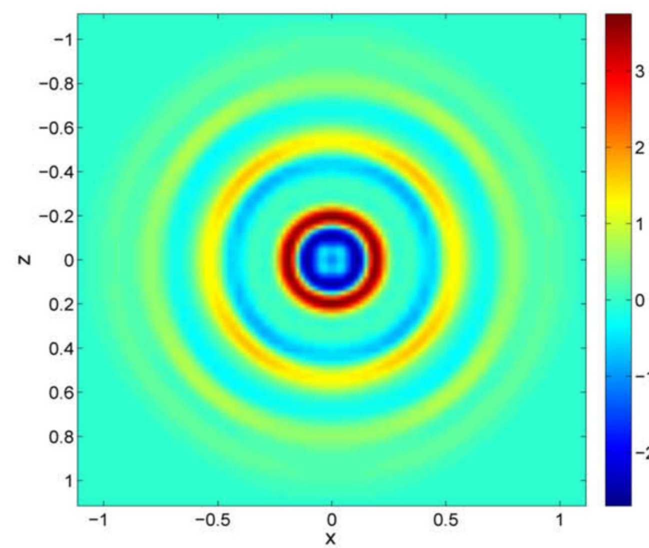
(a) Transport



(b) FP_7



(c) S_8



(d) P_7

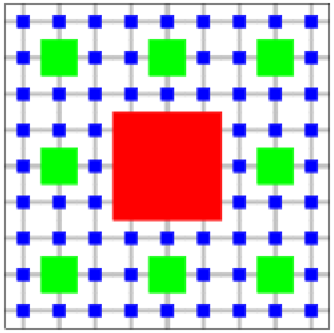
Filtered P_N

Open issues

- Other filters?
- What about 3D?
- Filtering as a continuum operation

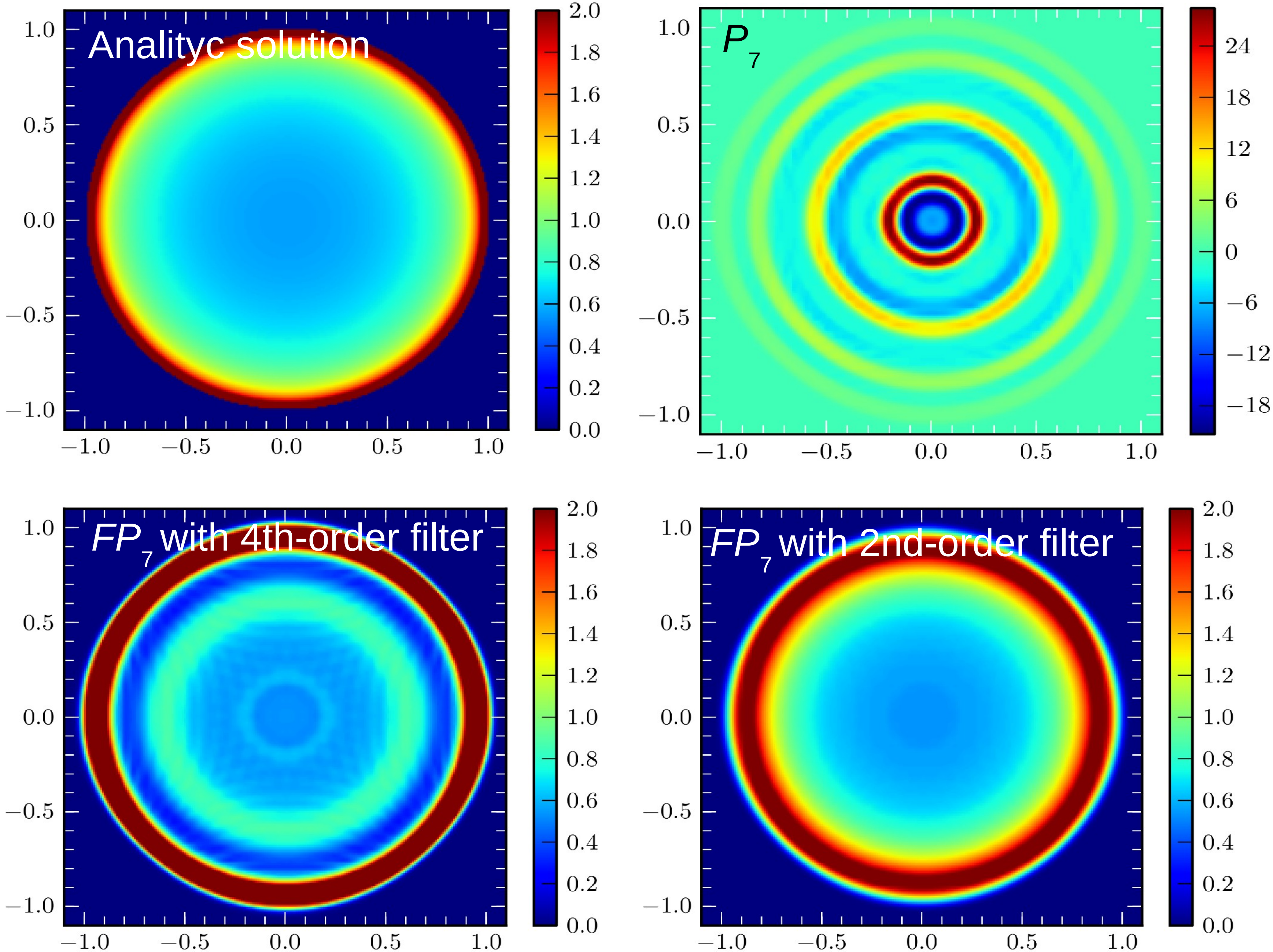
Charon code

- 3D (using Cactus and Carpet)
- Space discretization: AP DG scheme
  (Mcclarren & Lowrie '08)
- Semi-implicit time integration
  (McClarren+ '06)
- 2nd- and 4th-order filters
  (continuum limit)

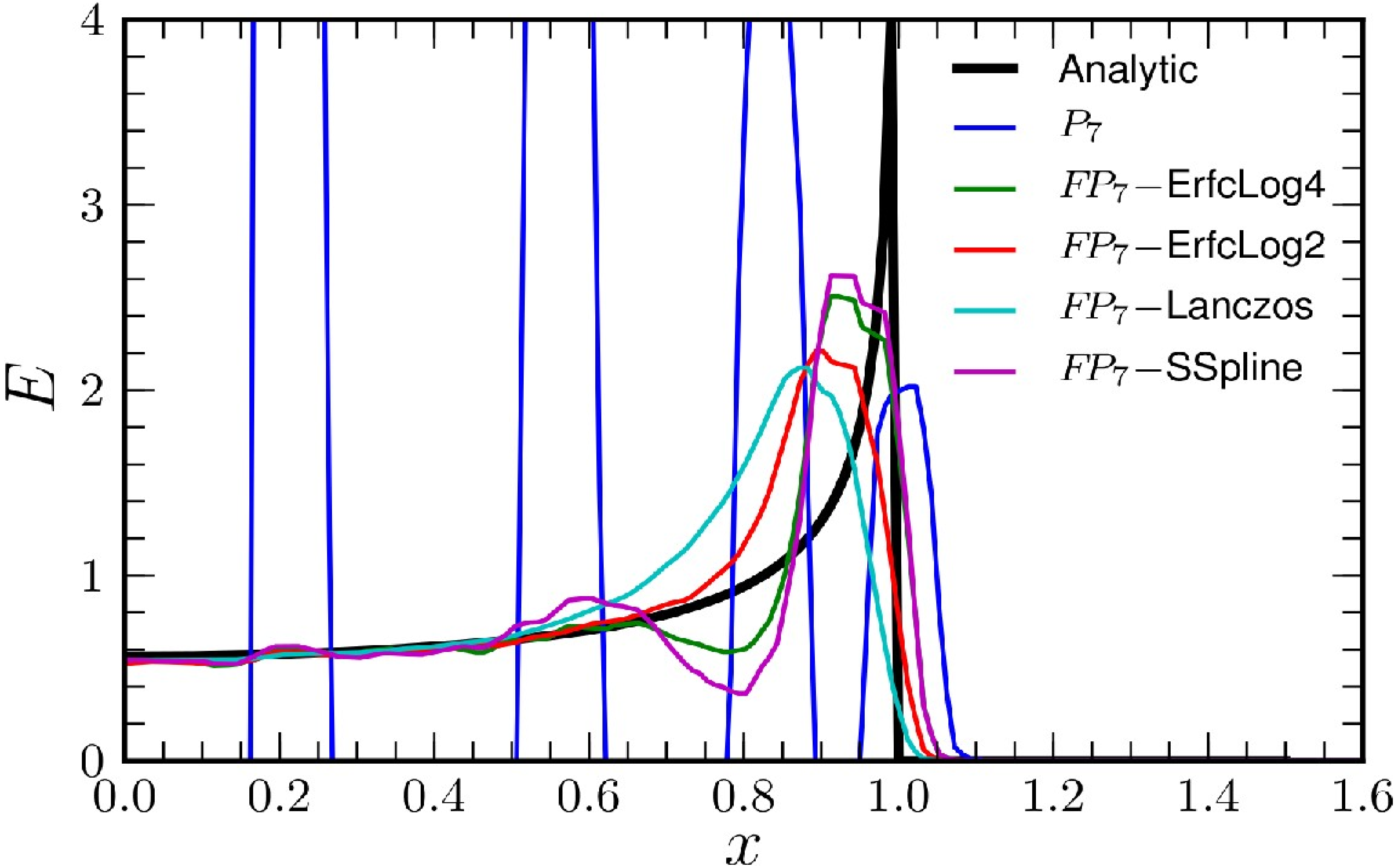


- Special relativity
- No velocity dependence

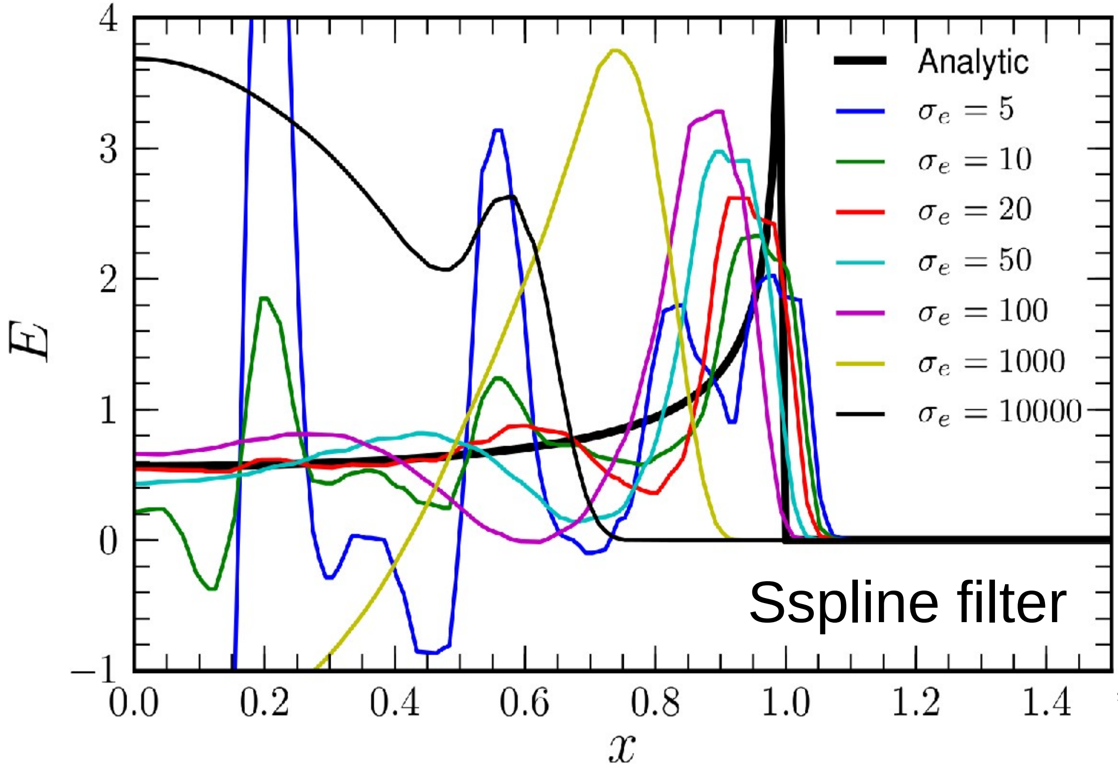
Line problem



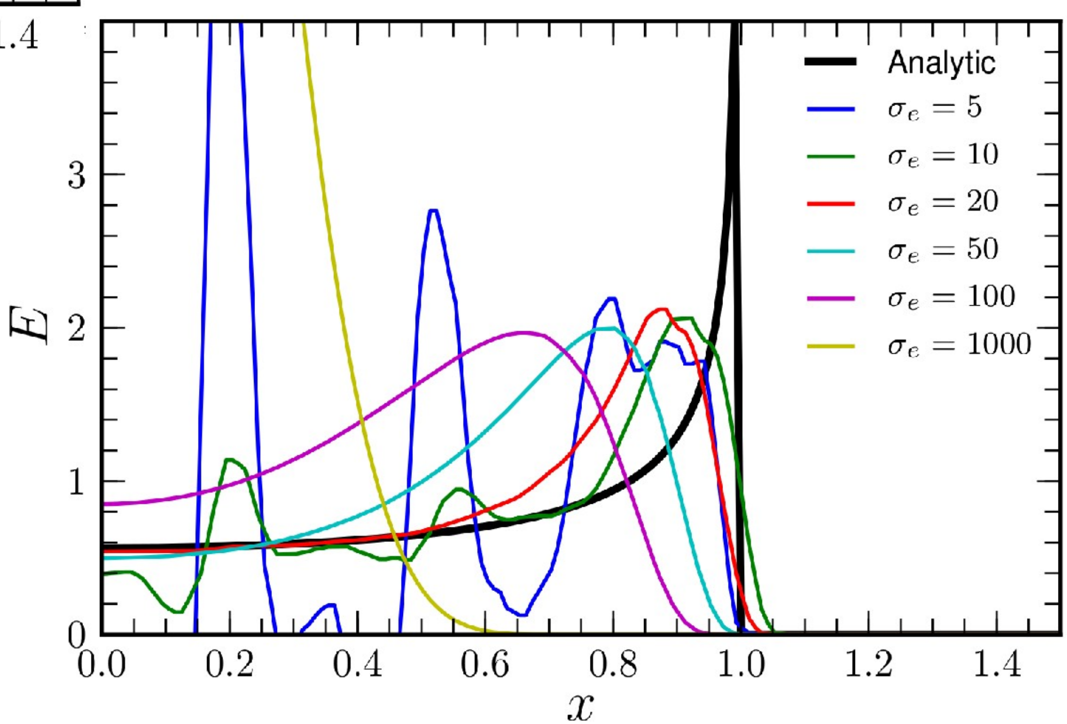
Line problem



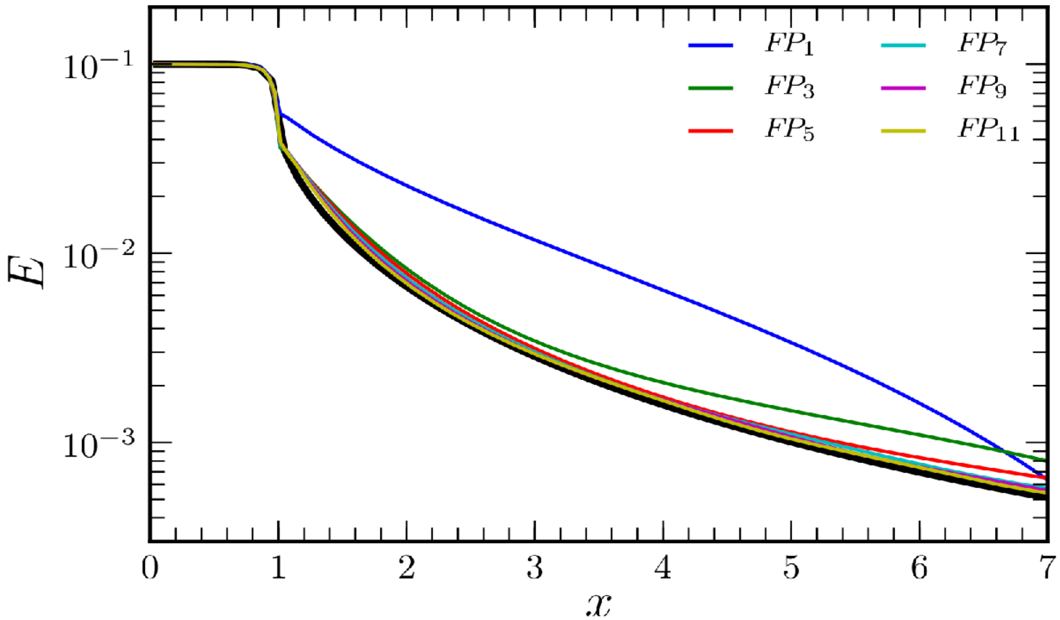
Line problem

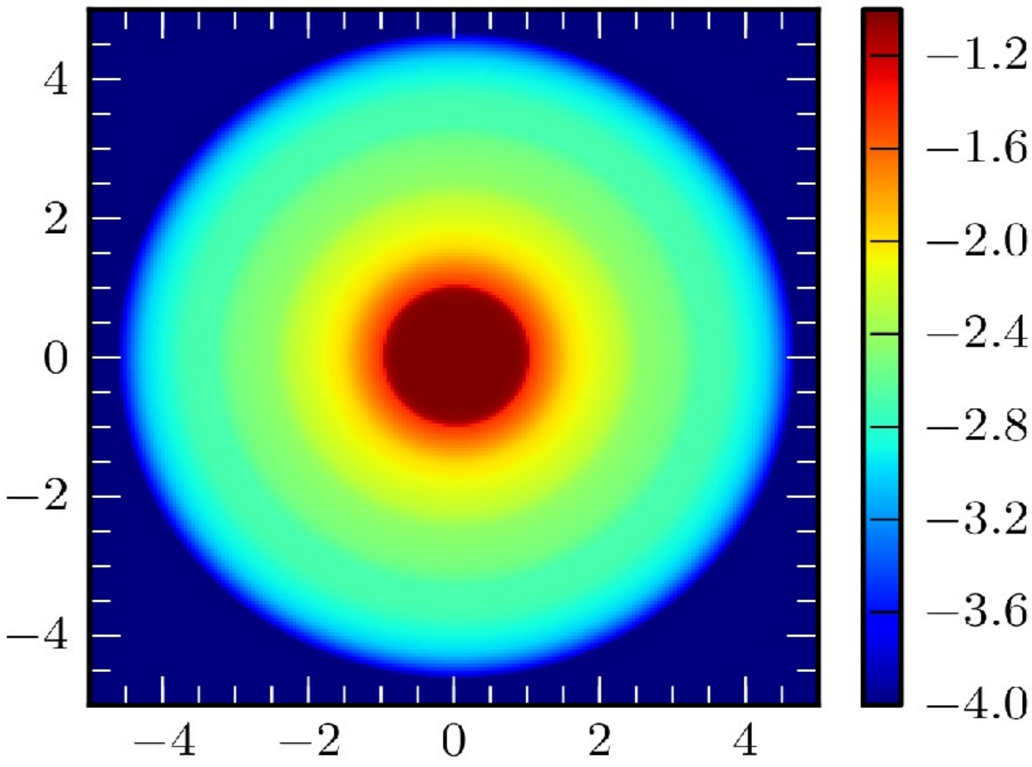


Lanzcos filter



Homogeneous sphere test





Conclusion

- 3D filtered spherical harmonics
- Continuum filter formulation
- 2nd-order filters are better
- Overall, FP_N is a promising approach to 3D radiation transport

Supplemental Material

Velocity-dependent discrete-diffusion: operator splitting

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{\partial H_0}{\partial x} = \kappa_0 (B - J_0)$$

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{\varepsilon_0}{3c}\frac{\partial J_0}{\partial \varepsilon_0}\frac{D \ln \rho}{Dt} = 0$$

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{v}{c}\frac{\partial J_0}{\partial r} + \frac{J_0}{c}\frac{\partial v}{\partial r} = 0$$

Implicit Monte Carlo for Photons

[Fleck & Cummings '71]

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n}\cdot\nabla I = \kappa(B - I)$$

$$\downarrow \qquad \downarrow$$

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n}\cdot\nabla I = \kappa_{ea,n}(B_n - I)$$
$$+\chi_n \int\int \kappa_{es,n} I d\Omega d\varepsilon - \kappa_{es,n} I$$

Much larger timesteps!