Energy Momentum Tensor Correlators in Hot Yang-Mills theory

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INT, Seattle, 12-04-12
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Outline

- Introduction & Motivation
- Setup in SU(Nc) Yang-Mills theory
- Results
  - Correlators in UV limit
  - Correlators in coordinate space
  - Spectral densities in bulk channel
- Summary and Outlook

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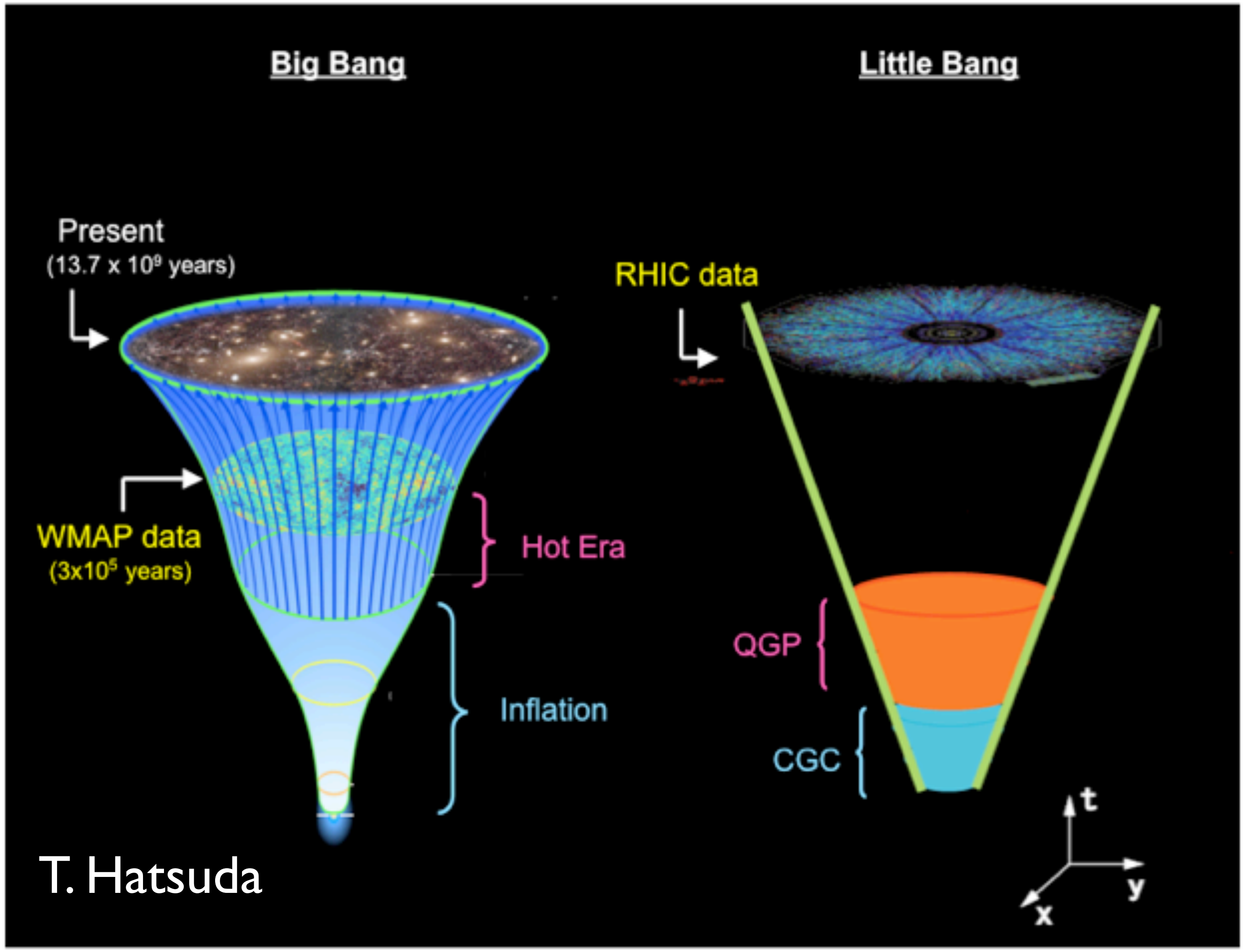
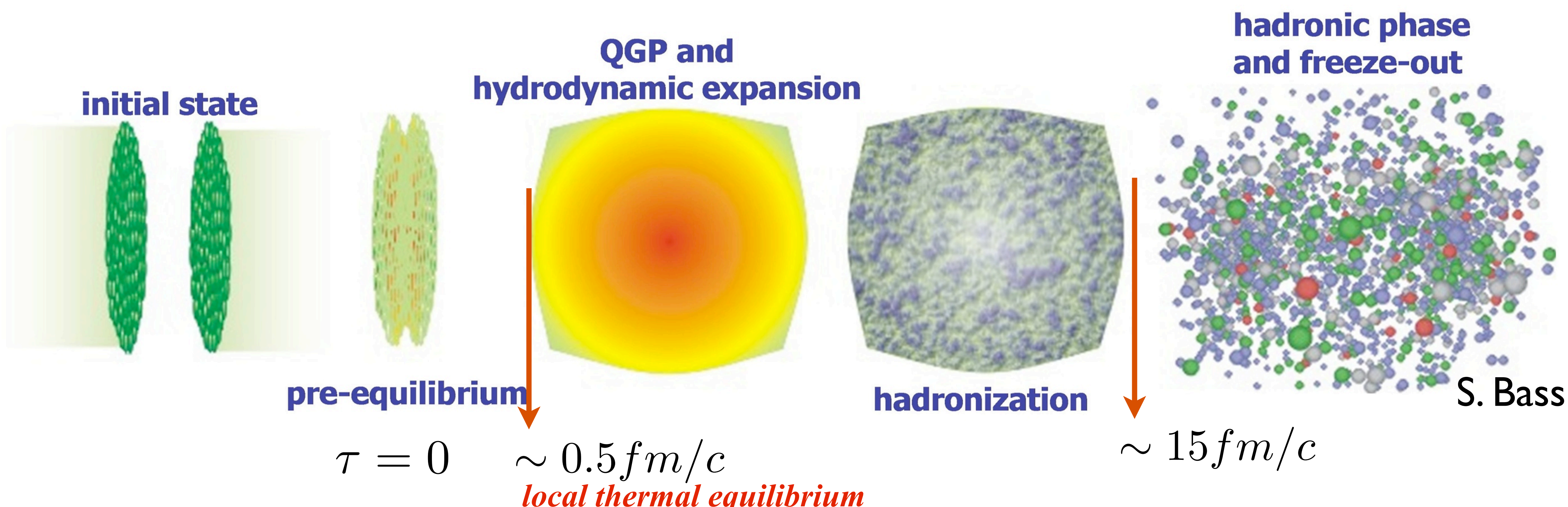
Cosmology and HIC



HIC @ RHIC



CGC/
Glauber

hydro with
small viscosity

Recombination/
Fragmentaion

hadron gas with
large viscosity

Puzzles from RHIC

- Viscous hydro is compatible with experimental data only when

$$\eta/s \lesssim 0.2$$

- Elliptic flow in PbPb @ 2.76TeV at LHC [ALICE: *arXiv:1011.3914 [nucl-ex]]* is identical to AuAu at RHIC

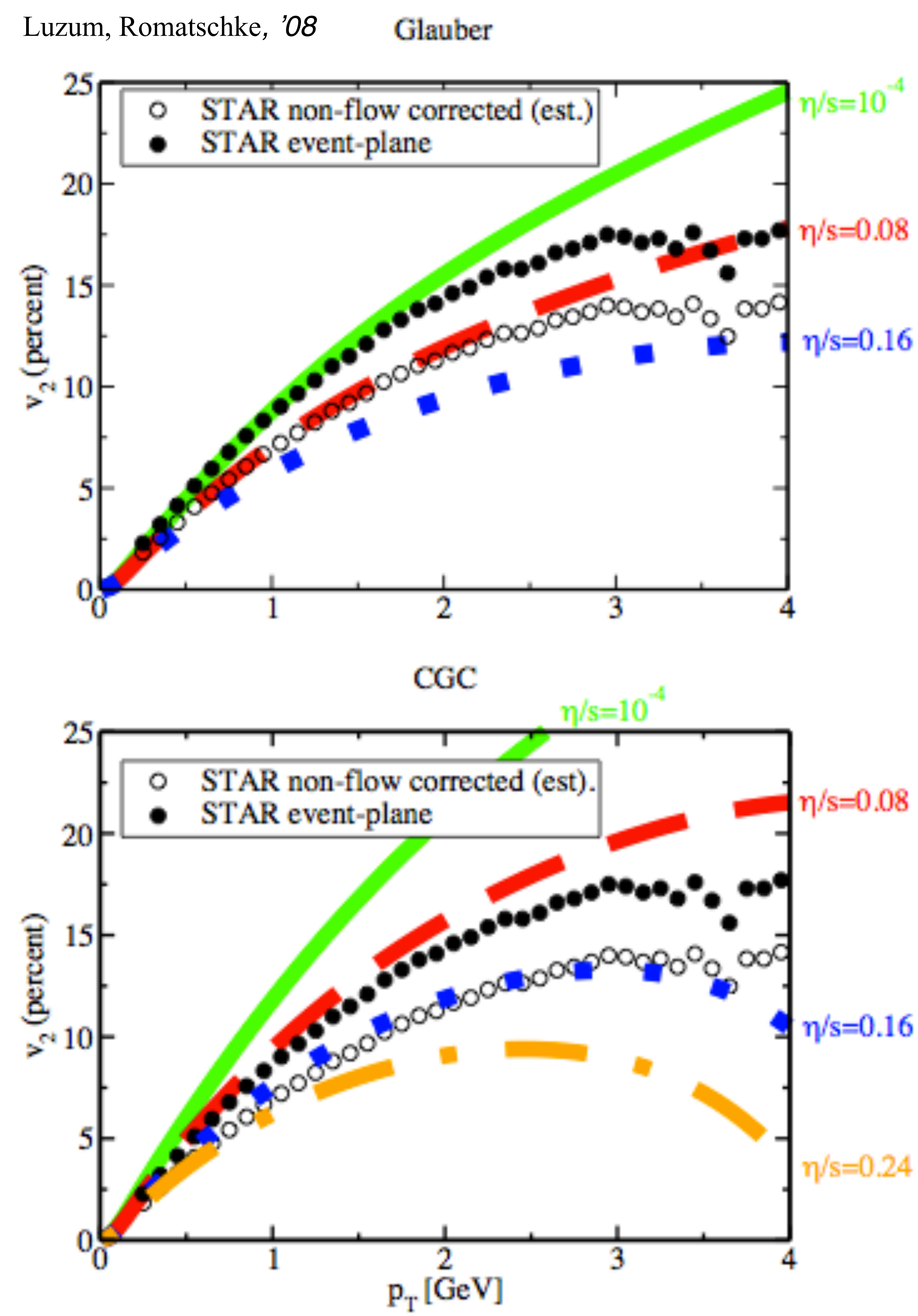
- String theory methods - AdS/CFT with gravity duals:

$$\eta/s = \frac{1}{4\pi}$$

- What are η, ζ,... in QCD? Is the plasma 'strongly coupled'? Is N = 4 SYM really a good model for QGP?

- Ultimate answer only from non-perturbative calculations in QCD!

Luzum, Romatschke, '08



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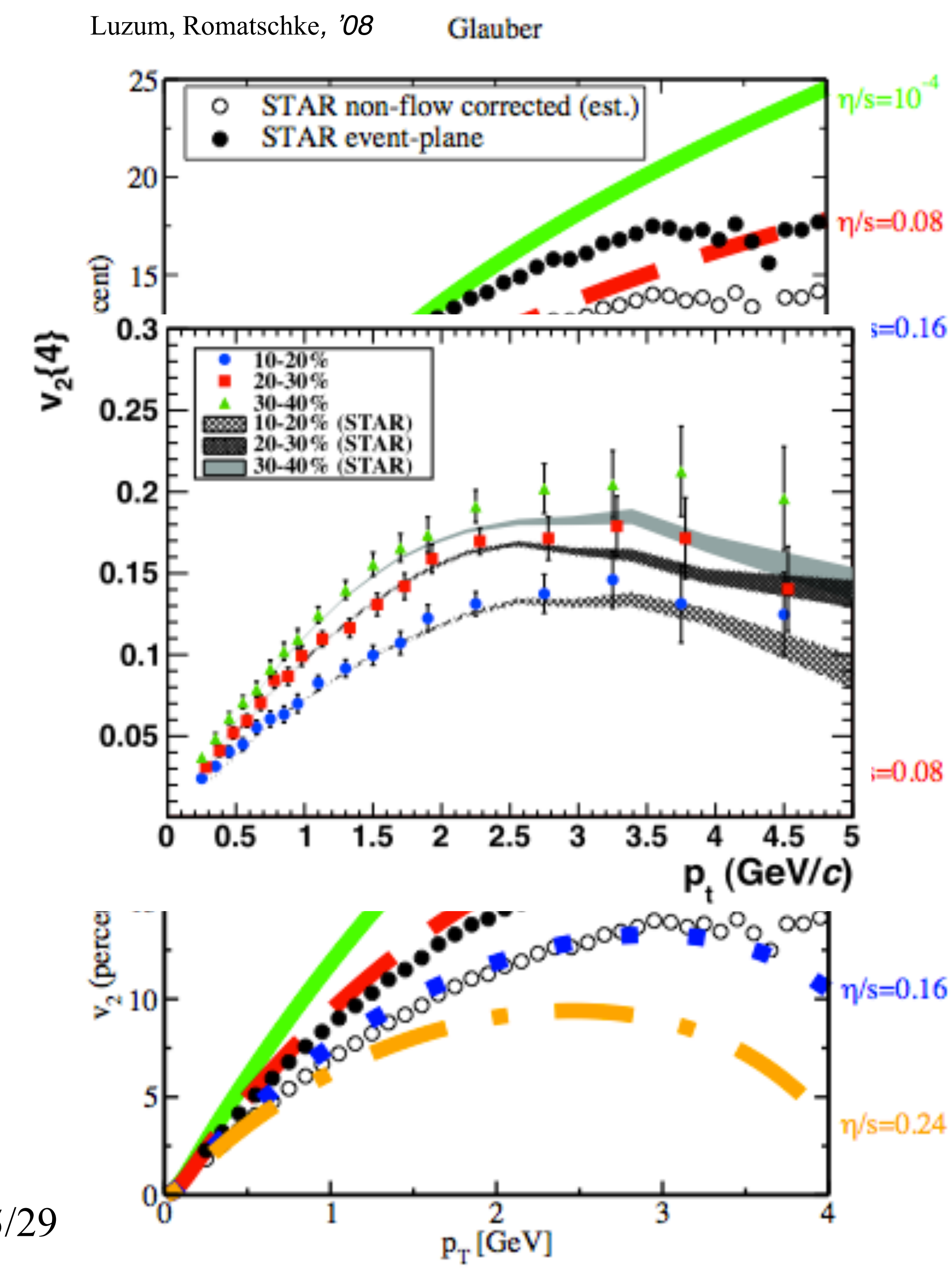

Linearized Viscous Hydrodynamics

- Macroscopic Form of Energy Momentum Tensor:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (e+P)u^{\mu}u^{\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^{\mu}u^{\nu} + \Delta^{\nu}u^{\mu}) + (\frac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_{\rho}u^{\rho}$$

- $\eta, \zeta =$ shear and bulk viscosity
- u^{μ} velocity of energy transport
- $\Delta^{\mu} = \partial_{\mu} - u_{\mu}u^{\beta}\partial_{\beta}, \quad H^{\mu\nu} = u^{\mu}u^{\nu} - g^{\mu\nu}$

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Motivation I Transport Coefficients

- Matching of linearized hydrodynamic and linear response description in QFT---Kubo formulae: Viscosities and other transport coeffs. are obtainable from retarded Minkowski correlators of energy momentum tensor $T^{\mu\nu}$

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho_{12,12}(\omega, \mathbf{k}=\mathbf{0})}{\omega}$$

$$\zeta = \frac{\pi}{9} \sum_{i,j=1}^{3} \lim_{\omega\to 0} \frac{\rho_{ii,jj}(\omega, \mathbf{k}=\mathbf{0})}{\omega}$$

$$\rho_{\mu\nu\rho\sigma} = \mathrm{Im} G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0})$$

$$G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0}) \equiv i \int_0^\infty dt e^{i\omega t} \int d^3x \, \langle [T_{\mu\nu}(t, \mathbf{x}), T_{\rho\sigma}(0, \mathbf{0})] \rangle$$

$$G_R(\omega) = \tilde{G}_E(\omega_\ell \to -i[\omega + i\epsilon])$$

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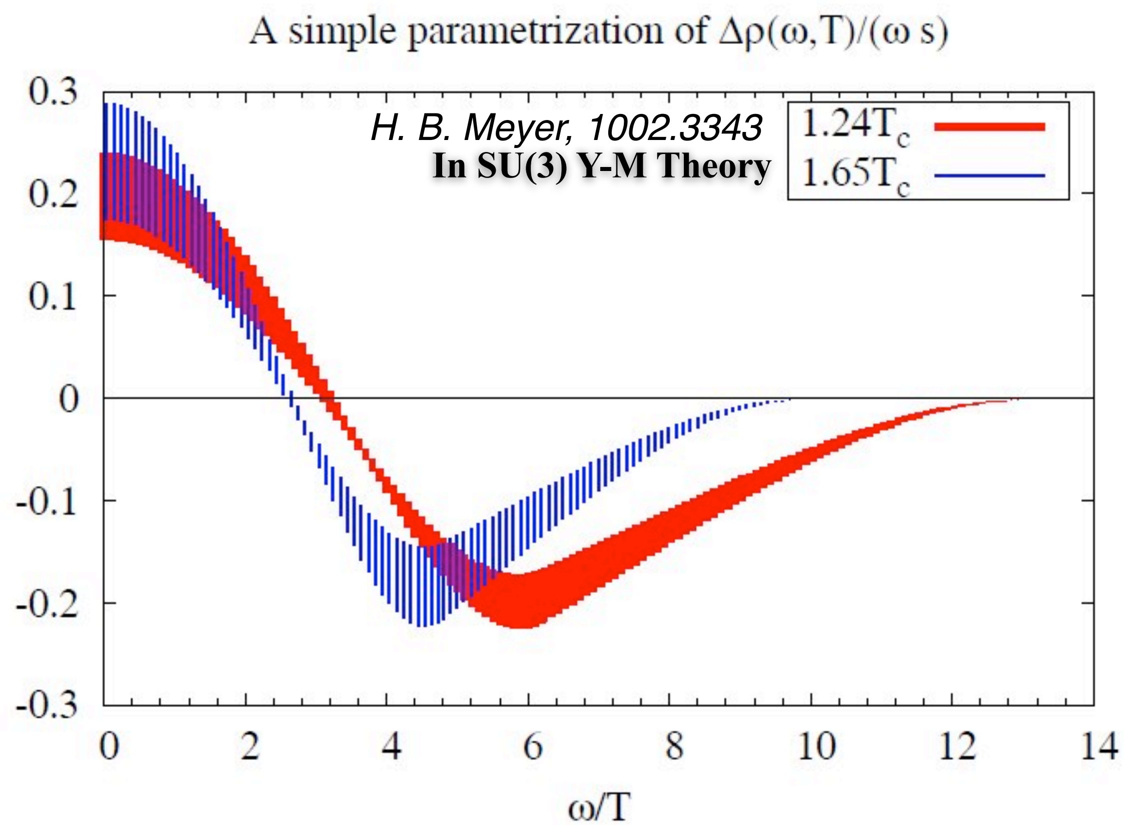
Motivation I
Transport Coefficients

- However, Lattice determines spectral density ρ from Euclidean correlators: Need to invert

$$G(\hat{\tau}) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{1}{2}-\hat{\tau}\right)\beta\omega}{\sinh\frac{\beta\omega}{2}}$$

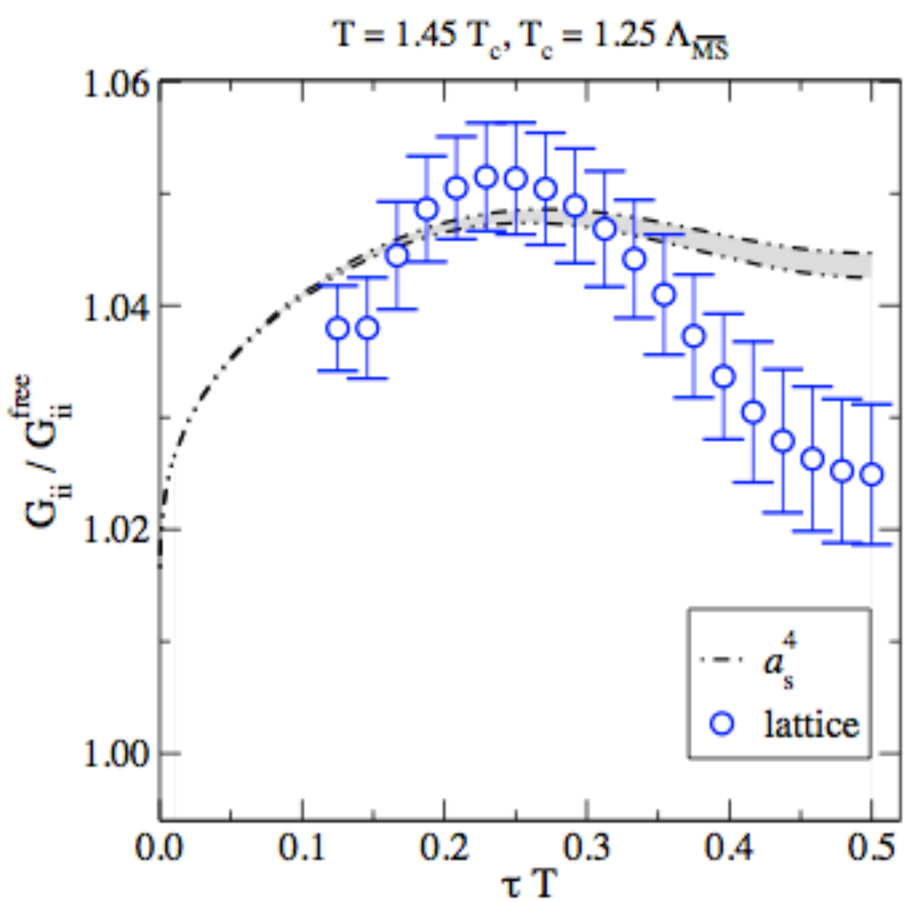


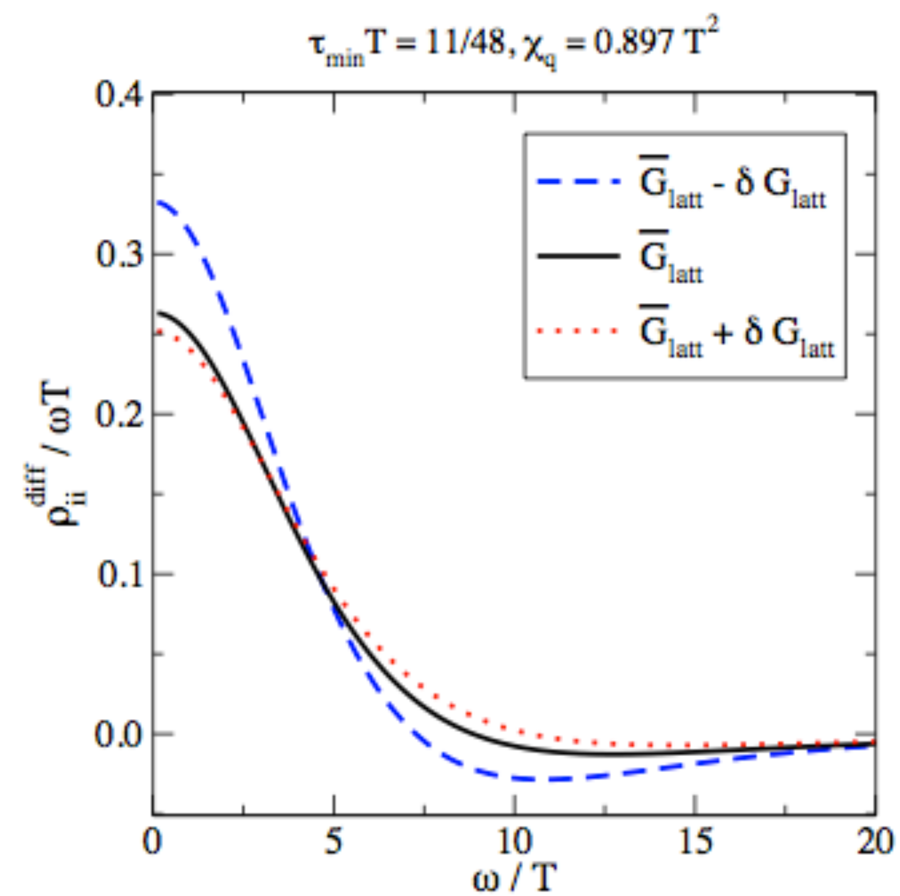
- For extracting IR limit of ρ, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input!

Motivation I
Successful example

- For the vector-current correlator, 5-loop vacuum limit and accurate lattice data available ⇒ Model-independent analytic continuation of Euclidean correlator à la [Burnier, Laine, Mether; *EPJC 71*] possible

- Result: Estimate for flavor current spectral density and flavour diffusion coefficient [Burnier, Laine; *EPJC 72*]





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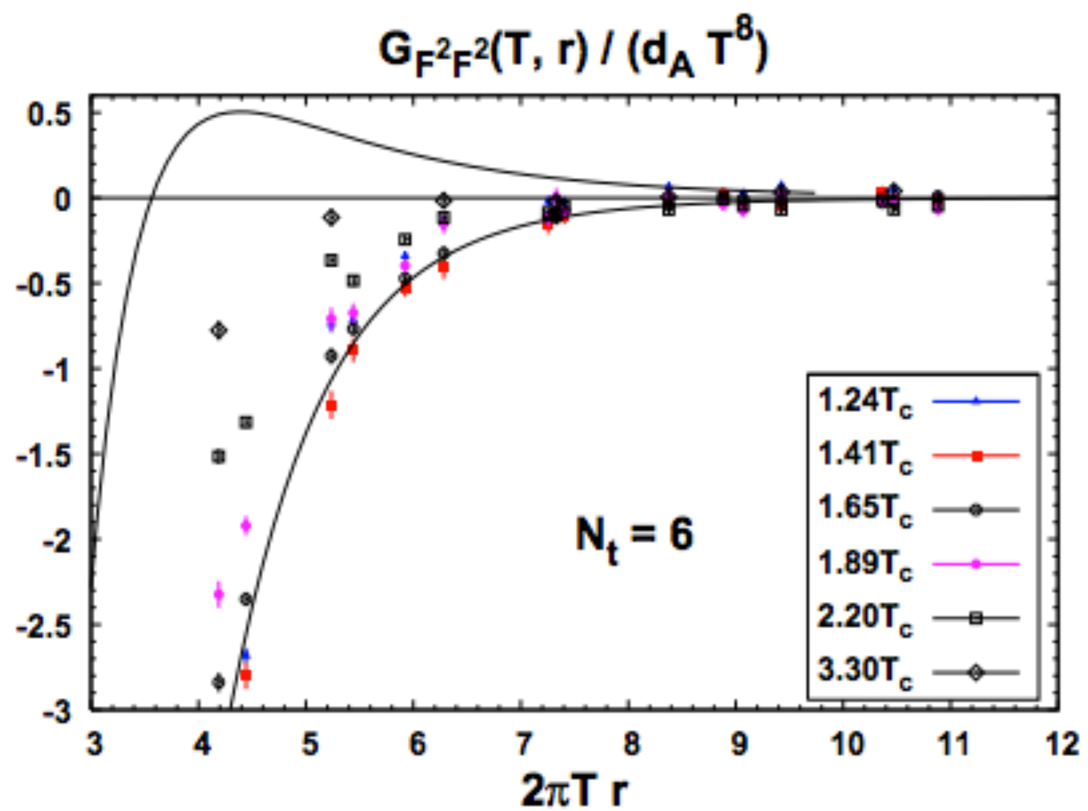
Motivation II
Correlators

- Spatial correlators measure screening in medium ⇒ Comparison between lattice QCD, pQCD and AdS/CFT results offers insights into structure and properties of the QGP
- Iqbal & Meyer (0909.0582): Lattice data for correlators of $\mathrm{Tr}F^{\mu\nu}F_{\mu\nu}$ in semi-quantitative agreement with strongly coupled N = 4 SYM, while leading order pQCD result completely off. How about NLO?



Setup

- Work within SU(Nc) Y-M theory $S_E = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^{3-2\epsilon}\mathbf{x} \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$
- Operators: $\theta \equiv c_\theta \, g_{\mathrm{B}}^2 F^a_{\mu\nu} F^a_{\mu\nu} \, , \quad \chi \equiv c_\chi \, \epsilon_{\mu\nu\rho\sigma} g_{\mathrm{B}}^2 F^a_{\mu\nu} F^a_{\rho\sigma} \, ,$
- Define:
  Euclidean correlators
  - $G_\theta(x) \equiv \langle \theta(x) \theta(0) \rangle_c \, ,$
  - $G_\chi(x) \equiv \langle \chi(x) \chi(0) \rangle \, ,$
  - $G_\eta(x) \equiv 2 c_\eta^2 X_{\mu\nu,\alpha\beta}(x) \, \langle T_{\mu\nu}(x) \, T_{\alpha\beta}(0) \rangle_c \, ,$

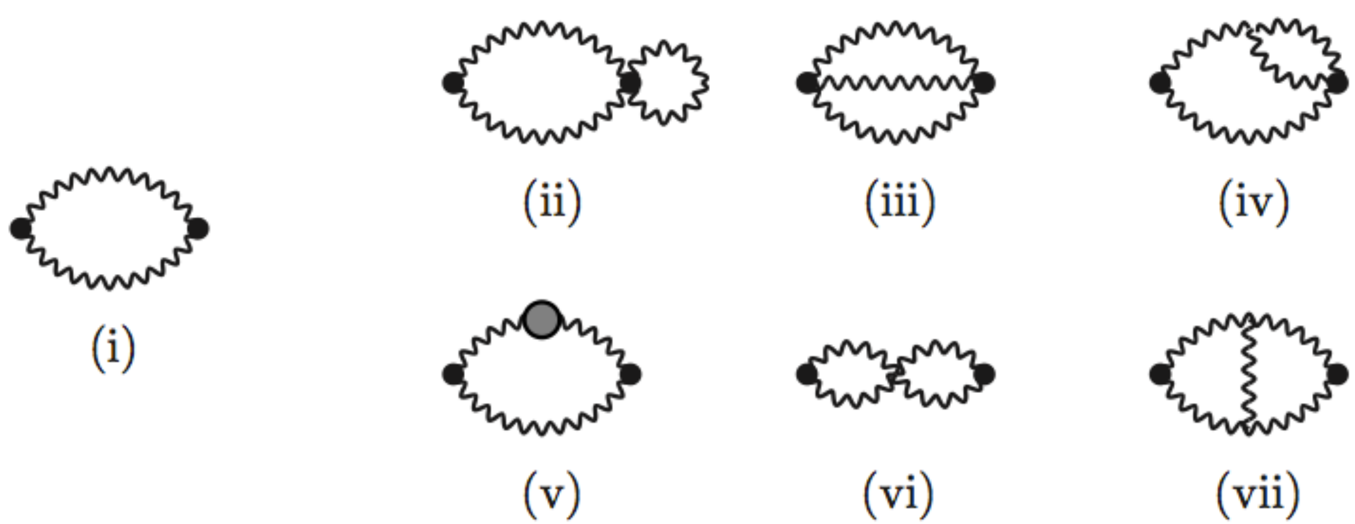
  where $X_{\mu\nu,\alpha\beta} \equiv P^T_{\mu\nu} P^T_{\alpha\beta} - \frac{D-2}{2} (P^T_{\mu\alpha} P^T_{\nu\beta} + P^T_{\mu\beta} P^T_{\nu\alpha}) \, ,$

  $$T_{\mu\nu} = \frac{1}{4} \delta_{\mu\nu} F^a_{\alpha\beta} F^a_{\alpha\beta} - F^a_{\mu\alpha} F^a_{\nu\alpha} \, ,$$

  $$G_\eta(x) = -16 c_\eta^2 \, \langle T_{12}(x) \, T_{12}(0) \rangle_c \, .$$

Correlators to NLO

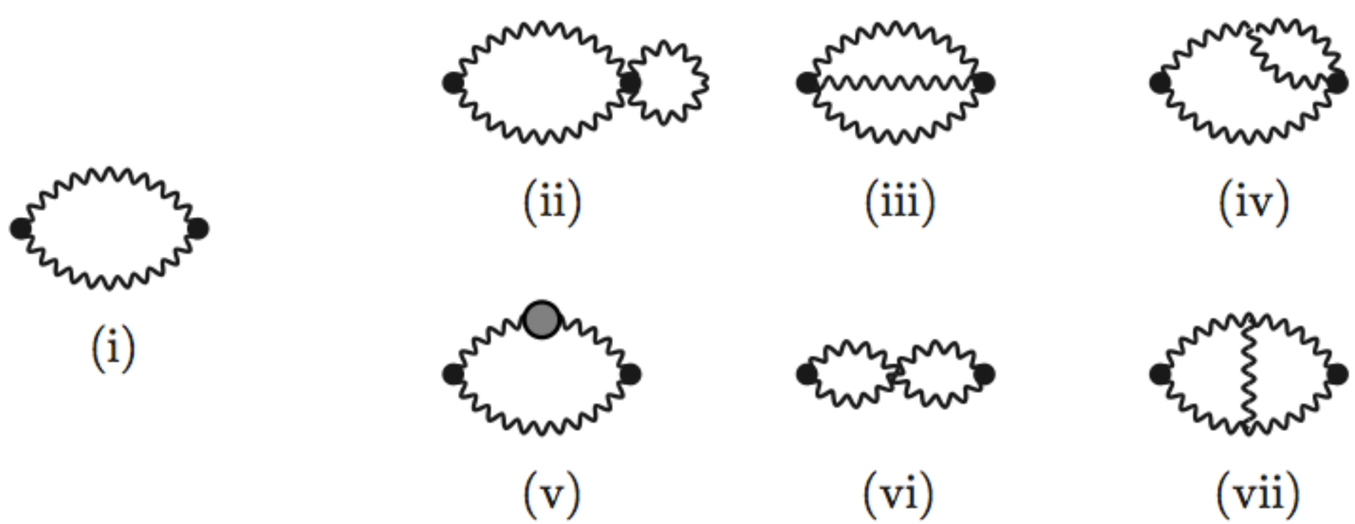
The LO and NLO Feynman graphs contributing to the correlators



- Write down diagrammatic expansions for Euclidean correlators in momentum space $\tilde{G}_\alpha(P) \equiv \int_x e^{-iP\cdot x}\tilde{G}_\alpha(x)$
- Carry out Matsubara Sums by 'cutting' thermal lines and evaluate remaining 3d integrals
- Extract the spectral densities with $\rho_\alpha(\omega) = \mathrm{Im}\tilde{G}_\alpha(p_0 = -i\omega + 0^+, \mathbf{p} = \mathbf{0})$

Correlators to NLO

The LO and NLO Feynman graphs contributing to the correlators



When can perturbation theory be expected to converge?

$$\bar{\Lambda}_{x,T} \simeq \sqrt{(\bar{\Lambda}_x)^2 + (\bar{\Lambda}_T)^2} \sim \sqrt{\frac{1}{x^2} + (2\pi T)^2}$$

At least, if either $x \ll 1/\Lambda_{QCD}$ ($\omega \gg \Lambda_{QCD}$) or $T \gg \Lambda_{QCD}$!

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Wilson coefficients for OPE

- In UV, define $\Delta\tilde{G}_\alpha(P) \equiv \tilde{G}_\alpha(P) - \tilde{G}_\alpha^{T=0}(P)$

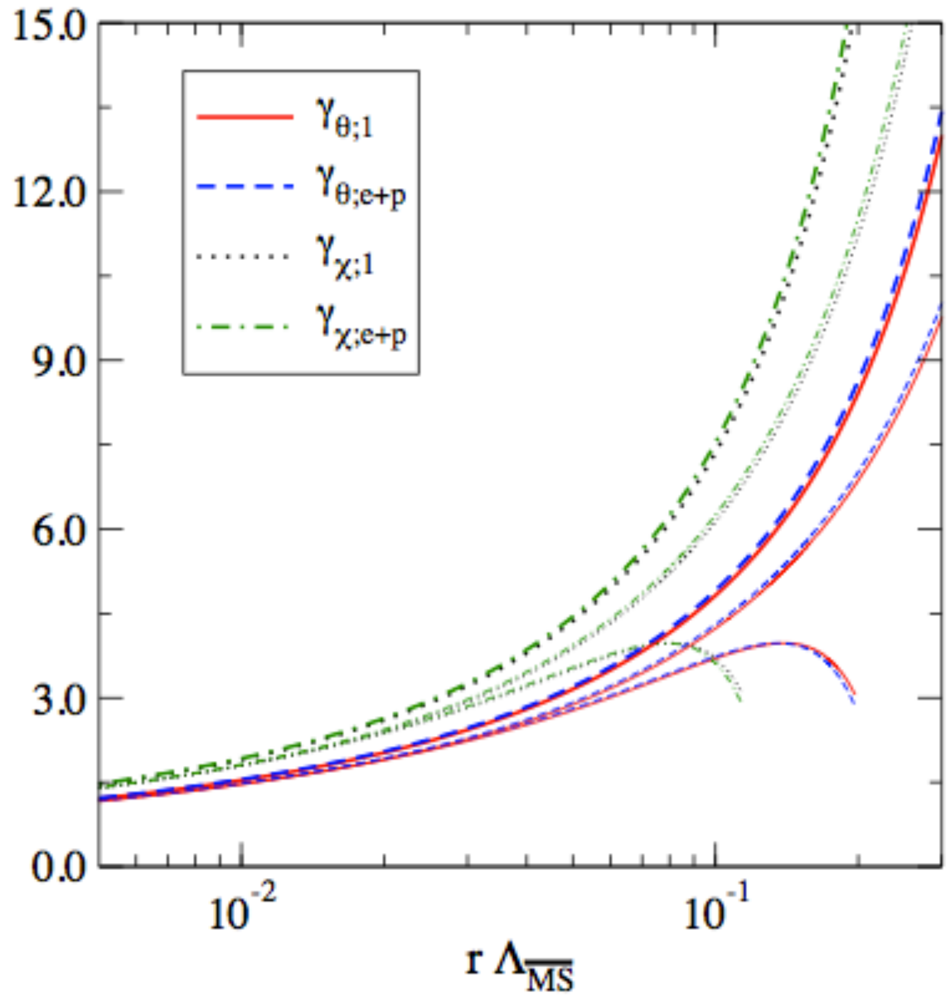
$$\frac{\Delta\tilde{G}_\theta(P)}{4c_\theta^2 g^4} = \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{203}{18}\right)\right](e+p)(T)$$
$$-\frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\theta P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

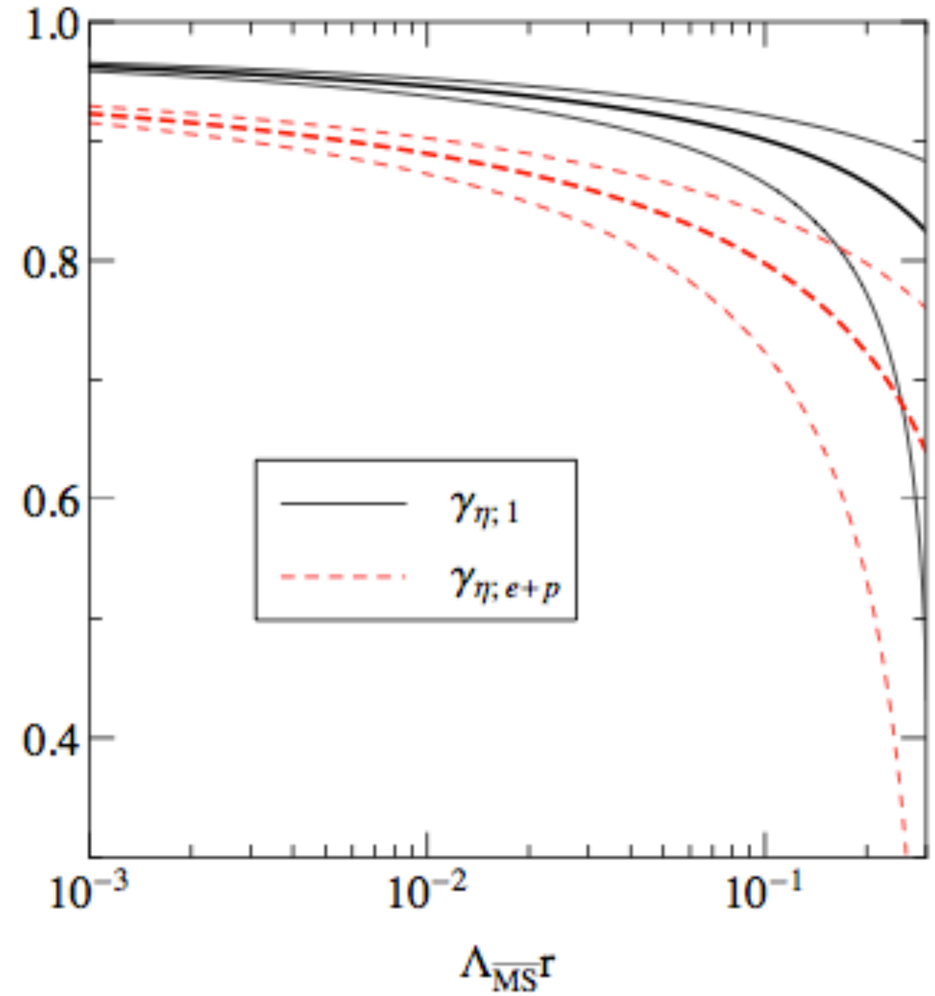
$$\frac{\Delta\tilde{G}_\chi(P)}{-16c_\chi^2 g^4} = \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{347}{18}\right)\right](e+p)(T)$$
$$+\frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\chi P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

$$\frac{\Delta\tilde{G}_\eta(P)}{4c_\eta^2} = -\left\{1+\frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)-\frac{1}{3}\frac{g^2N_c}{(4\pi)^2}\left[22+\frac{41}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\right]\right\}(e+p)(T)$$
$$+\frac{4}{3g^2b_0}\left[1-g^2b_0\ln\zeta_\eta\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

Note the absence of logs of $\bar{\mu}$ in the shear result.

Spatial Correlators in short distance





$$\frac{G_\chi(r)}{-16c_\chi^2} = \frac{12d_A}{\pi^4 r^8}\gamma_{\chi;\,1}(r) - \frac{2(e+p)}{\pi^2 r^4}\gamma_{\chi;\,e+p}(r) + \frac{2(e-3p)}{\pi^2 r^4}\gamma_{\chi;\,e-3p}(r) + \mathcal{O}\left(\frac{T^6}{r^2}\right)$$

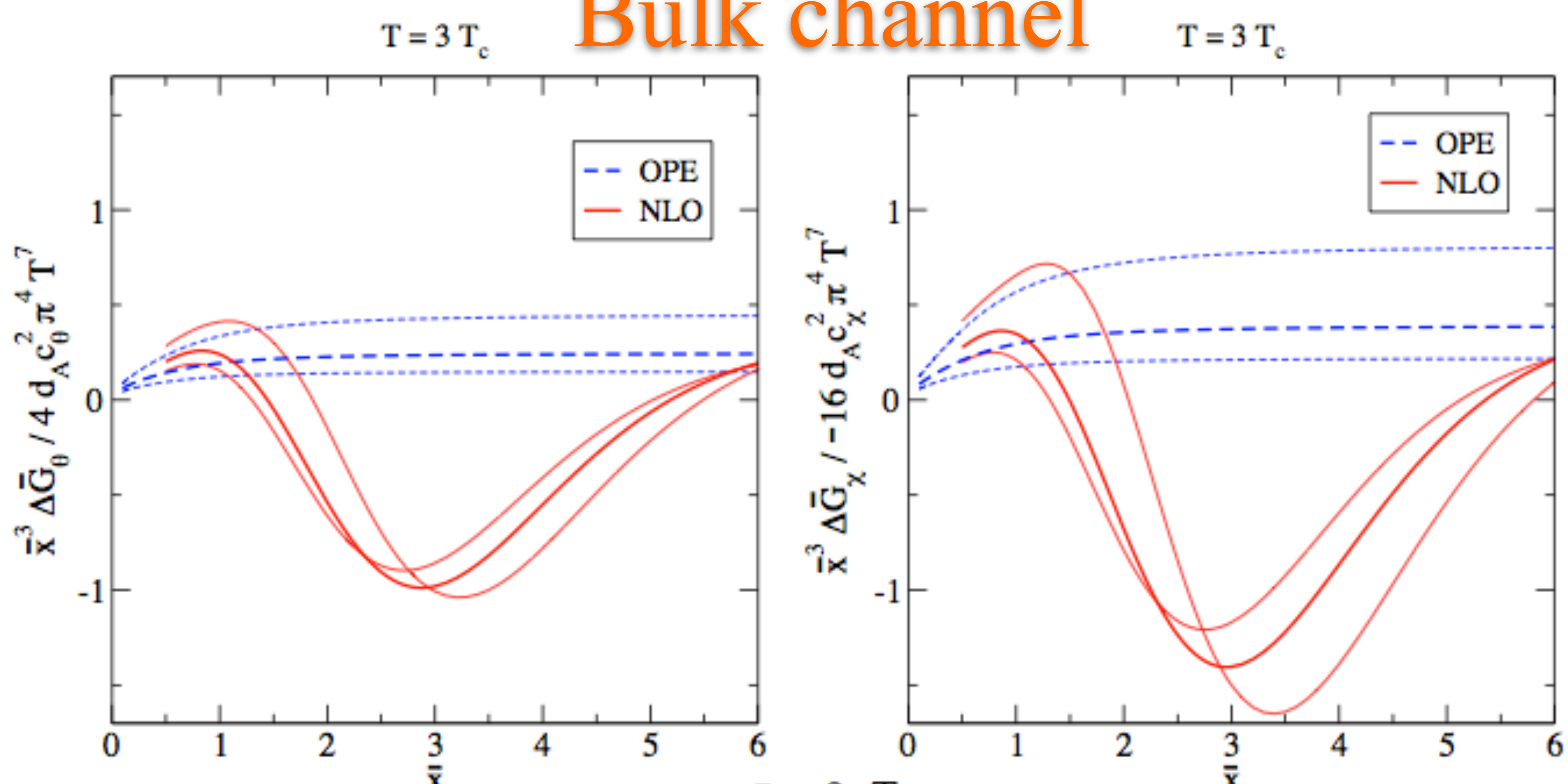
$$\frac{G_\theta(r)}{4c_\theta^2} = \frac{12d_A}{\pi^4 r^8}\gamma_{\theta;\,1}(r) - \frac{2(e+p)}{\pi^2 r^4}\gamma_{\theta;\,e+p}(r) - \frac{2(e-3p)}{\pi^2 r^4}\gamma_{\theta;\,e-3p}(r) + \mathcal{O}\left(\frac{T^6}{r^2}\right)$$

$$\frac{G_\eta(r)}{4c_\eta^2} = -\frac{24d_A}{5\pi^4 r^8}\gamma_{\eta;1}(r) + \frac{2(e+p)(T)}{\pi^2 r^4}\gamma_{\eta;e+p}(r) + \frac{(e-3p)(T)}{\pi^2 r^4}\gamma_{\eta;e-3p}(r) + \mathcal{O}\left(\frac{T^6}{r^2}\right)$$

Time averaged spatial correlators

$$\bar{G}_\alpha(x) \equiv \int_0^\beta d\tau\ G_\alpha(X) = \int_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}}\ \tilde{G}_\alpha(p_n = 0, \mathbf{p})$$

Bulk channel

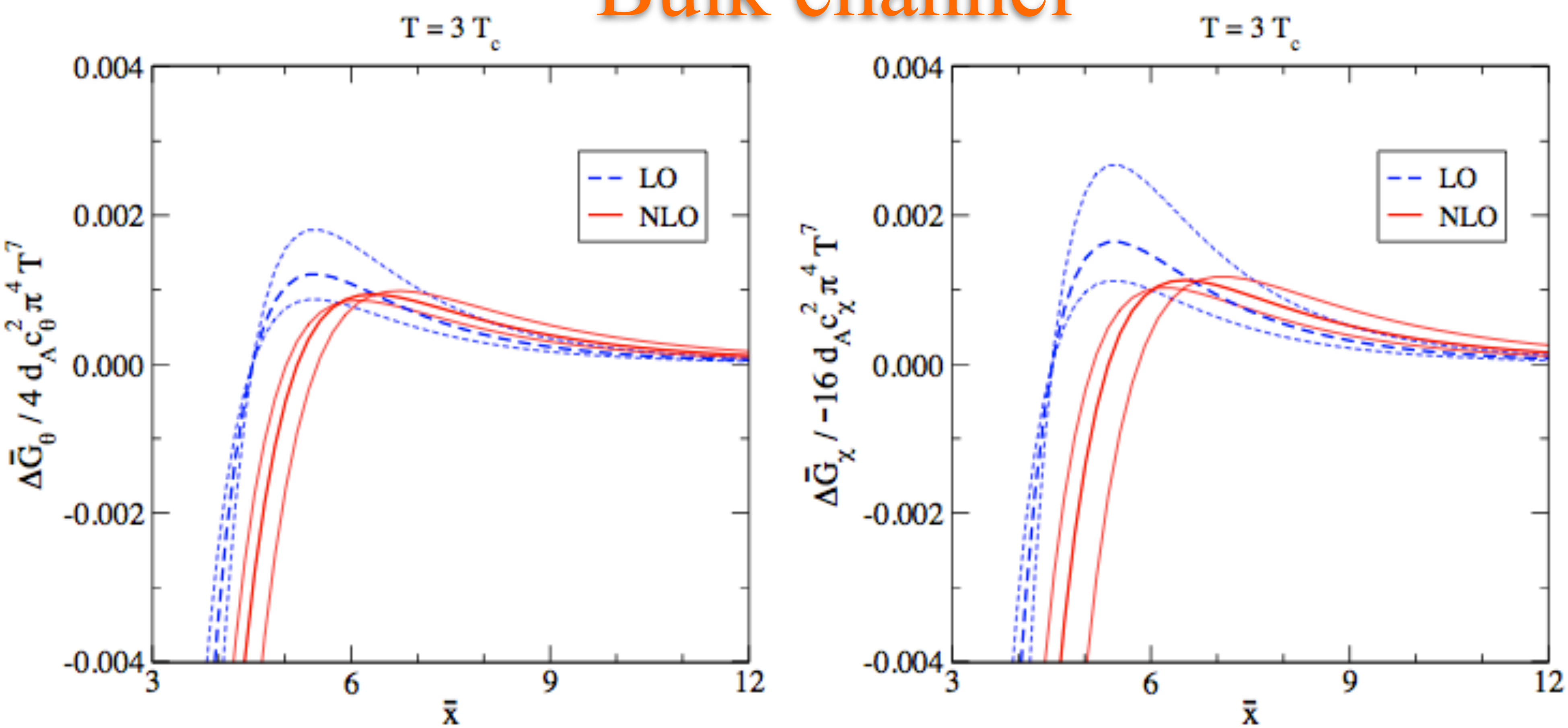


$\bar{x} \equiv 2\pi T x$

- OPE results are applicable only in the range $\bar{x} \lesssim 1$
- There is a visible difference between the two channels in the regime $1 \lesssim \bar{x} \lesssim 5$.

Time averaged spatial correlators

Bulk channel



- Qualitatively, NLO results considerably closer to lattice than LO ones in bulk channel
- **However:** We computed time averaged correlator, not equal time
- AdS computation of same correlator in large-Nc YM underway

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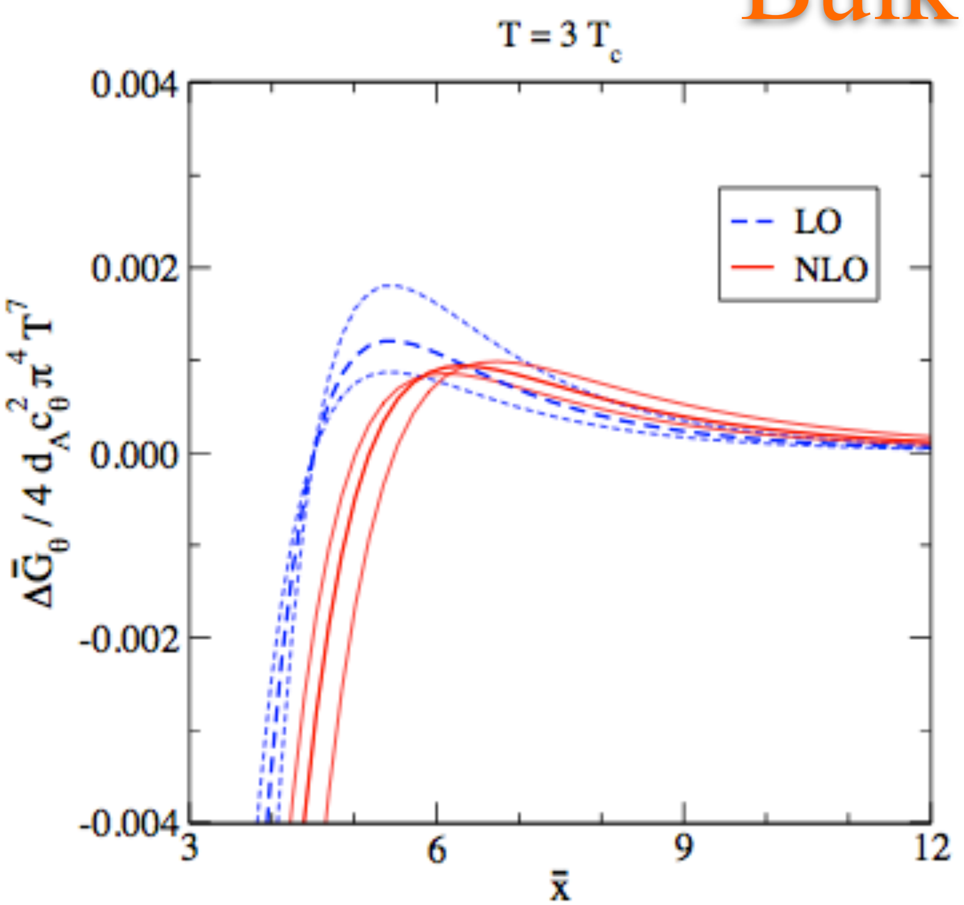
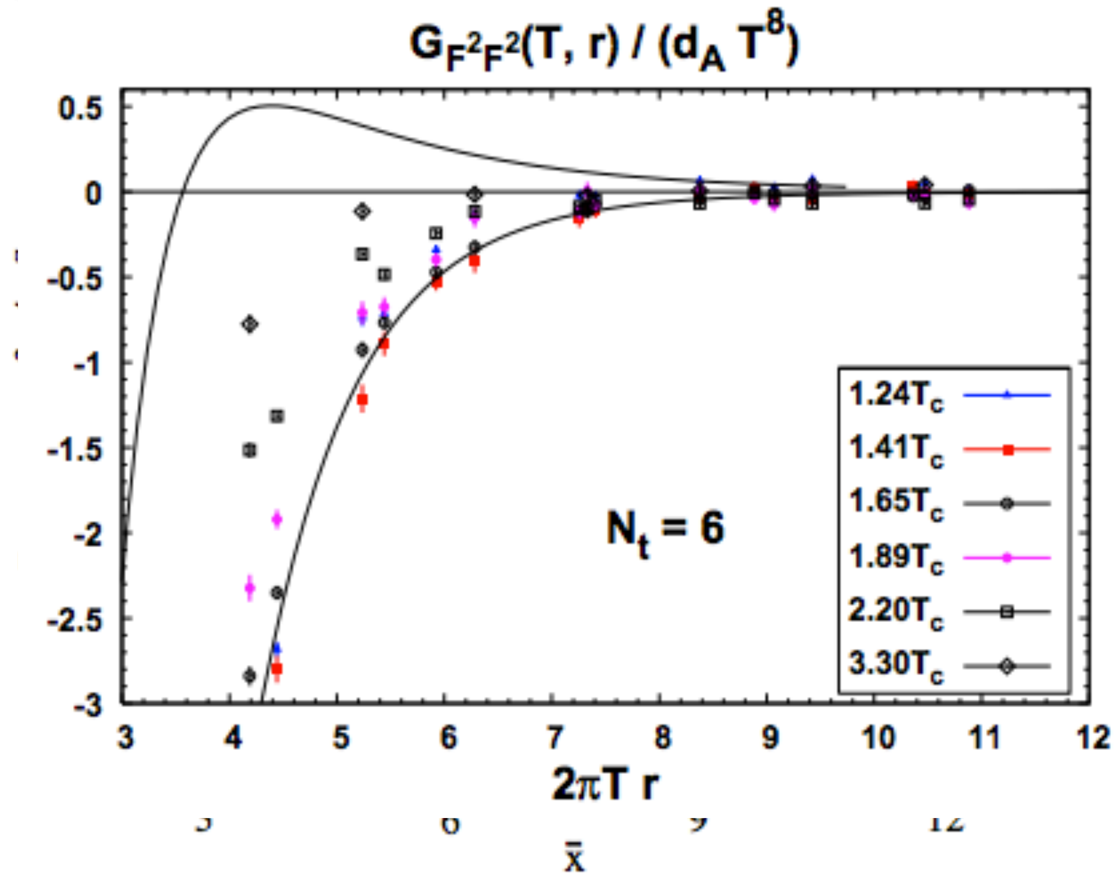
Time averaged spatial correlators

Bulk channel





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One way to cal. SPF to NLO

Spectral Functions

$$\rho(\omega) = \mathrm{Im}\Big[\tilde{G}(P)\Big]_{P\to(-i[\omega+i0^+],\mathbf{0})} .$$

- After Matsubara Sums, the imaginary part can be extracted with

$$\frac{1}{\omega \pm i0^+} = \mathbb{P}\left(\frac{1}{\omega}\right) \mp i\pi\delta(\omega) .$$

- Example:

$$\mathcal{I}_{\rm j}(P) \equiv \sum\!\!\!\!\!\!\!\int_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2} .$$

Denoting $E_q \equiv q\ ,\quad E_r \equiv r\ ,\quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2}\ ,$

Spectral Functions

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$$\frac{\tilde{G}_\theta(P)}{4d_A c_\theta^2} = g_\mathrm{B}^4(D-2)\Big[-\mathcal{J}_\mathrm{a} + \frac{1}{2}\mathcal{J}_\mathrm{b}\Big]$$
$$+ \quad g_\mathrm{B}^6 N_\mathrm{c}\Big\{2(D-2)\Big[-(D-2)\mathcal{I}_\mathrm{a} + (D-4)\mathcal{I}_\mathrm{b}\Big] + (D-2)^2\Big[\mathcal{I}_\mathrm{c} - \mathcal{I}_\mathrm{d}\Big]$$
$$+ \frac{34-13D}{3}\mathcal{I}_\mathrm{f} - \frac{(D-4)^2}{2}\mathcal{I}_\mathrm{g} + (D-2)\Big[-\mathcal{I}_\mathrm{e} + 3\mathcal{I}_\mathrm{h} + 2\mathcal{I}_{\mathrm{i}'} - \mathcal{I}_\mathrm{j}\Big]\Big\}$$

$$\mathcal{I}_\mathrm{j}(P) \equiv \SumInt_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2} .$$

Denoting $E_q \equiv q\,, \quad E_r \equiv r\,, \quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2}\,,$

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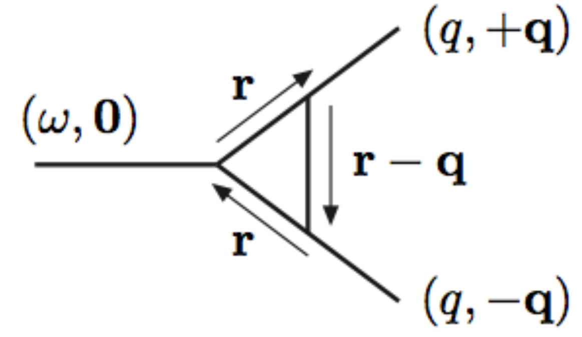
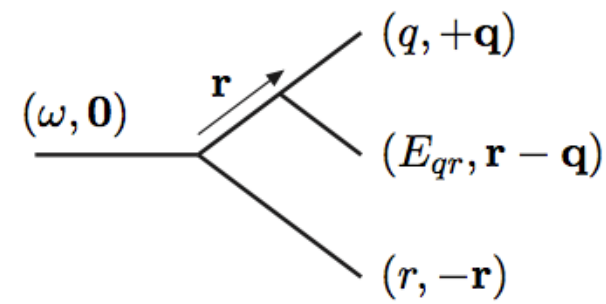
$\rho_{\mathcal{I}_j}(\omega)$

$$
\begin{aligned}
\rho_{\mathcal{I}_j}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6\pi}{4qrE_{qr}} \Bigg\{ \\
& \frac{1}{8q^2}\Big[\delta(\omega-2q)-\delta(\omega+2q)\Big] \times \\
& \times\Bigg[\bigg(\frac{1}{(q+r-E_{qr})(q+r)}-\frac{1}{(q-r+E_{qr})(q-r)}\bigg)(1+2n_q)(n_{qr}-n_r) \\
& \quad +\bigg(\frac{1}{(q+r+E_{qr})(q+r)}-\frac{1}{(q-r-E_{qr})(q-r)}\bigg)(1+2n_q)(1+n_{qr}+n_r)\Bigg] \\
+ & \frac{1}{8r^2}\Big[\delta(\omega-2r)-\delta(\omega+2r)\Big] \times \\
& \times\Bigg[\bigg(\frac{1}{(q+r-E_{qr})(q+r)}-\frac{1}{(q-r-E_{qr})(q-r)}\bigg)(1+2n_r)(n_{qr}-n_q) \\
& \quad +\bigg(\frac{1}{(q+r+E_{qr})(q+r)}-\frac{1}{(q-r+E_{qr})(q-r)}\bigg)(1+2n_r)(1+n_{qr}+n_q)\Bigg] \\
+ & \Big[\delta(\omega-q-r-E_{qr})-\delta(\omega+q+r+E_{qr})\Big]\frac{(1+n_{qr})(1+n_q+n_r)+n_qn_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
+ & \Big[\delta(\omega-q-r+E_{qr})-\delta(\omega+q+r-E_{qr})\Big]\frac{n_{qr}(1+n_q+n_r)-n_qn_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
+ & \Big[\delta(\omega-q+r-E_{qr})-\delta(\omega+q-r+E_{qr})\Big]\frac{n_r(1+n_q+n_{qr})-n_qn_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \\
+ & \Big[\delta(\omega+q-r-E_{qr})-\delta(\omega-q+r+E_{qr})\Big]\frac{n_q(1+n_r+n_{qr})-n_rn_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}\Bigg\}
\end{aligned}
$$

Factorized int./
Virtual correction



Phase space int./
Real correction



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Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_\mathrm{j}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$

$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \quad \omega > 0$$

$$\times \Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$

$$+\left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)$$

- $$\rho^{(\mathrm{fz,p})}_{\mathcal{I}_\mathrm{j}}(\omega) \approx \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}}) \int_\lambda^{\frac{\omega}{2}} \frac{\mathrm{d}q}{q} \ln\left|\frac{q+\sqrt{q^2-\lambda^2}}{q-\sqrt{q^2-\lambda^2}}\right| .$$
- $$\rho^{(\mathrm{fz,e})}_{\mathcal{I}_\mathrm{j}}(\omega) = \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}})\Bigg\{ \int_0^\infty \mathrm{d}q\, n_q\, \mathbb{P}\left[\frac{1}{q+\frac{\omega}{2}}\ln\left|\frac{\lambda^2}{2q\omega-\lambda^2}\right| + \frac{1}{q-\frac{\omega}{2}}\ln\left|\frac{\lambda^2}{2q\omega+\lambda^2}\right|\right]$$
$$+ \int_\lambda^\infty \mathrm{d}q\, n_q\left[\frac{1}{q}\ln\left|\frac{q+\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q+\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right| + \frac{1}{q}\ln\left|\frac{q-\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q-\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right|\right]\Bigg\} .$$

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Real Correction

$$\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6\pi}{4qrE_{qr}}\Big\{$$

$0 < \lambda < \omega$

$$+ \Big[\delta(\omega-q-r-E_{qr}) - \delta(\omega+q+r+E_{qr})\Big]\frac{(1+n_{qr})(1+n_q+n_r)+n_qn_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega-q-r+E_{qr}) - \delta(\omega+q+r-E_{qr})\Big]\frac{n_{qr}(1+n_q+n_r)-n_qn_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega-q+r-E_{qr}) - \delta(\omega+q-r+E_{qr})\Big]\frac{n_r(1+n_q+n_{qr})-n_qn_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}$$

$$+ \Big[\delta(\omega+q-r-E_{qr}) - \delta(\omega-q+r+E_{qr})\Big]\frac{n_q(1+n_r+n_{qr})-n_rn_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}\Big\} .$$

$$\begin{aligned}
(1+n_{qr})(1+n_q+n_r)+n_qn_r &= n_qn_rn_{qr}\left(e^{q+r+E_{qr}}-1\right) ,\\
n_{qr}(1+n_q+n_r)-n_qn_r &= n_qn_rn_{qr}\left(e^{q+r}-e^{E_{qr}}\right) ,\\
n_r(1+n_q+n_{qr})-n_qn_{qr} &= n_qn_rn_{qr}\left(e^{q+E_{qr}}-e^{r}\right) ,\\
n_q(1+n_r+n_{qr})-n_rn_{qr} &= n_qn_rn_{qr}\left(e^{r+E_{qr}}-e^{q}\right) ,
\end{aligned}$$

$$\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \frac{2\omega^4}{(4\pi)^3}\int_0^\infty \mathrm{d}q\int_0^\infty \mathrm{d}r\int_{E^-_{qr}}^{E^+_{qr}}\mathrm{d}E_{qr}\, n_qn_rn_{qr}\Big\{$$

$$\text{(i)}\quad \frac{\delta(\omega-q-r-E_{qr})}{(2r-\omega)(2q-\omega)}\left(1-e^{q+r+E_{qr}}\right)$$

$$\text{(ii)}\quad + \frac{\delta(\omega-q-r+E_{qr})}{(2r-\omega)(2q-\omega)}\left(e^{E_{qr}}-e^{q+r}\right)$$

$$\text{(iii)}\quad + \frac{\delta(\omega+q-r-E_{qr})}{(2r-\omega)(2q+\omega)}\left(e^{r+E_{qr}}-e^{q}\right)$$

$$\text{(iv)}\quad + \frac{\delta(\omega-q+r-E_{qr})}{(2r+\omega)(2q-\omega)}\left(e^{q+E_{qr}}-e^{r}\right)\Big\} .$$

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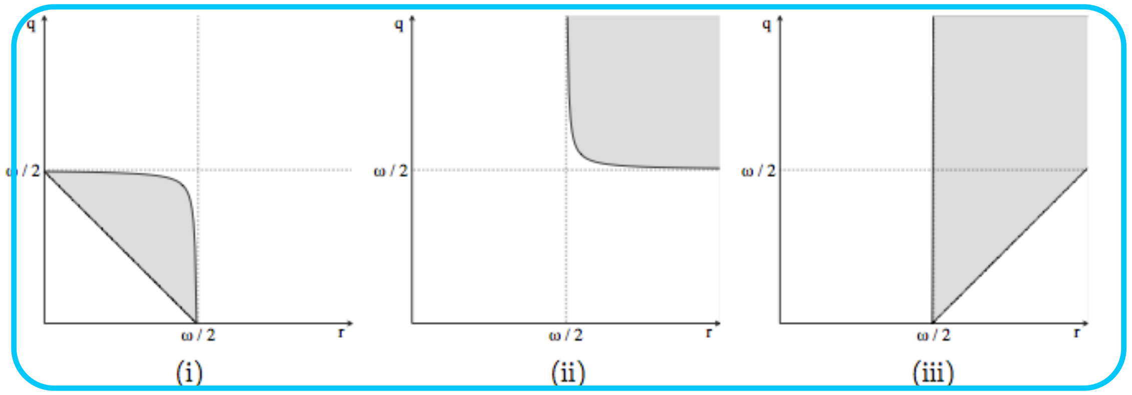
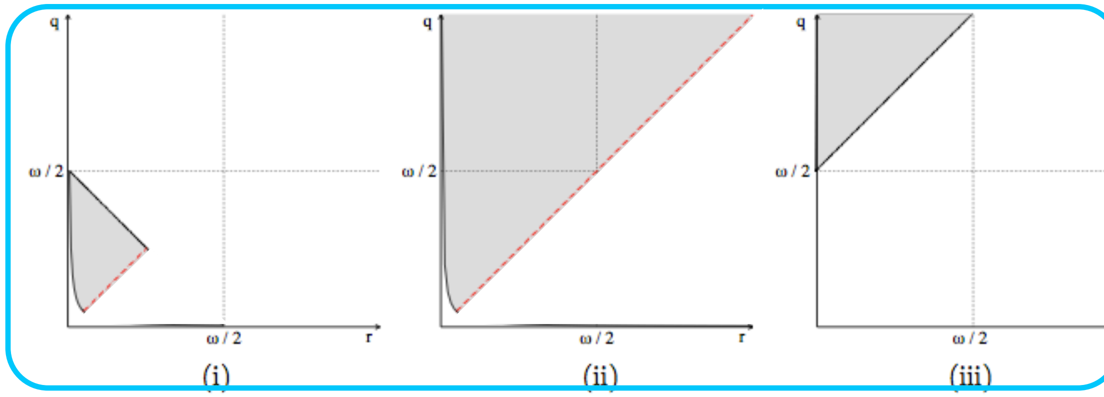
Real Correction

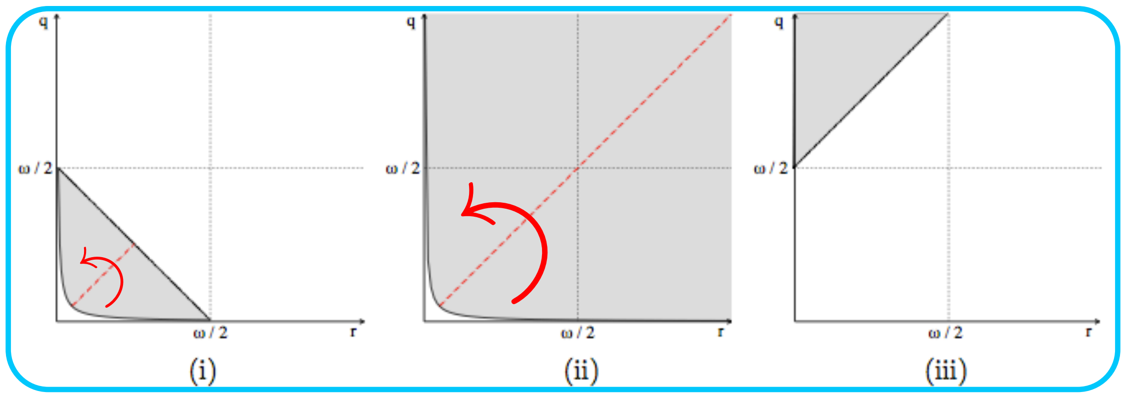
$\lambda = \omega/10$





$$(\text{i}): \quad q \to \frac{\omega}{2} - q\,, \quad r \to \frac{\omega}{2} - r\,,$$
$$(\text{ii}): \quad q \to \frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,,$$
$$(\text{iii}): \quad q \to -\frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,,$$

$$(\text{i}) \quad n_{\frac{\omega}{2}-q}\, n_{\frac{\omega}{2}-r}\, n_{q+r}(1-e^{\omega}) = -(1+2n_{\frac{\omega}{2}})\left[1 + n_{q+r} + n_{\frac{\omega}{2}-q} + (1+n_{\frac{\omega}{2}-r})\frac{n_{q+r}n_{\frac{\omega}{2}-q}}{n_r^2}\right].$$
$$(\text{ii}) \quad n_{\frac{\omega}{2}+q}\, n_{\frac{\omega}{2}+r}\, n_{q+r}\, e^{q+r}(1-e^{\omega}) = (1+2n_{\frac{\omega}{2}})\left[-n_{q+r} + n_{q+\frac{\omega}{2}} - (1+n_{q+\frac{\omega}{2}})\frac{n_{q+r}n_{r+\frac{\omega}{2}}}{n_r^2}\right].$$
$$(\text{iii}) \quad n_{q-\frac{\omega}{2}}\, n_{\frac{\omega}{2}+r}\, n_{q-r}\, e^{q-\frac{\omega}{2}}(e^{\omega}-1) = (1+2n_{\frac{\omega}{2}})\left[n_{q-\frac{\omega}{2}} - n_q - n_{q-\frac{\omega}{2}}\frac{(1+n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}}\right].$$



$\rho_{\mathcal{I}_j}(\omega)$

- Collect every part together and simplify them with $\lambda \ll \omega$,
- All the divergent terms cancel each other, we can set $\lambda \to 0$ in the end.

$$
\begin{aligned}
&\frac{(4\pi)^3 \rho_{\mathcal{I}_j}(\omega)}{\omega^4(1+2n_{\frac{\omega}{2}})} = \\
&\quad \int_0^{\frac{\omega}{4}} \mathrm{d}q\, n_q \left[\left(\frac{1}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\left(1-\frac{2q}{\omega}\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right)\right] \\
&+ \int_{\frac{\omega}{4}}^{\frac{\omega}{2}} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\left(1-\frac{2q}{\omega}\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right) - \frac{1}{q-\frac{\omega}{2}}\ln\left(\frac{2q}{\omega}\right)\right] \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{2}{q}\right)\ln\left(\frac{2q}{\omega}-1\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right) + \left(\frac{1}{q} - \frac{1}{q-\frac{\omega}{2}}\right)\ln\left(\frac{2q}{\omega}\right)\right] \\
&+ \int_0^{\frac{\omega}{2}} \mathrm{d}q \int_0^{\frac{\omega}{4}-|q-\frac{\omega}{4}|} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{\frac{\omega}{2}-q}\, n_{q+r}(1+n_{\frac{\omega}{2}-r})}{n_r^2} \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q \int_0^{q-\frac{\omega}{2}} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{q-\frac{\omega}{2}}(1+n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}} \\
&+ \int_0^{\infty} \mathrm{d}q \int_0^{q} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{(1+n_{q+\frac{\omega}{2}})n_{q+r} n_{r+\frac{\omega}{2}}}{n_r^2} + \mathcal{O}(\lambda \ln \lambda)\,.
\end{aligned}
$$

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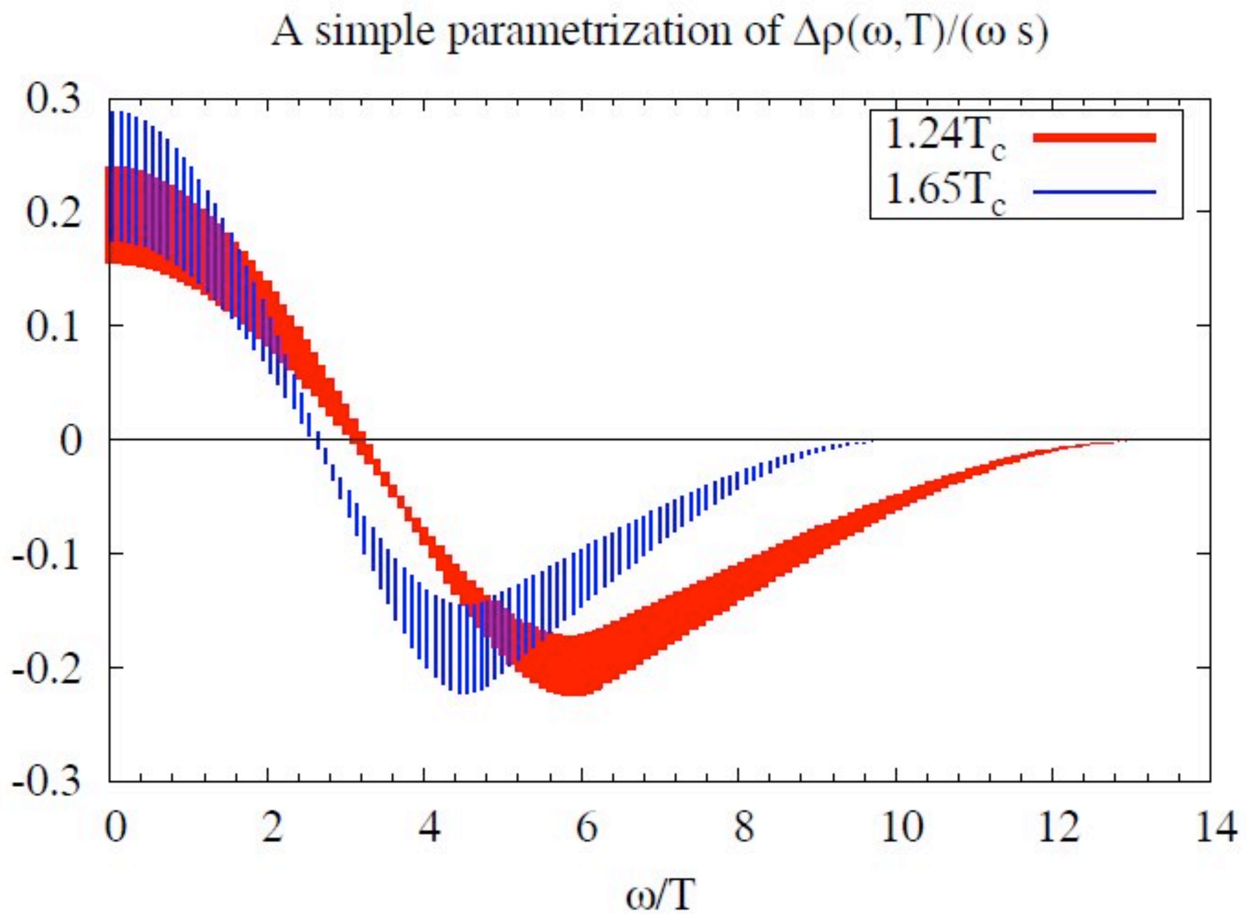
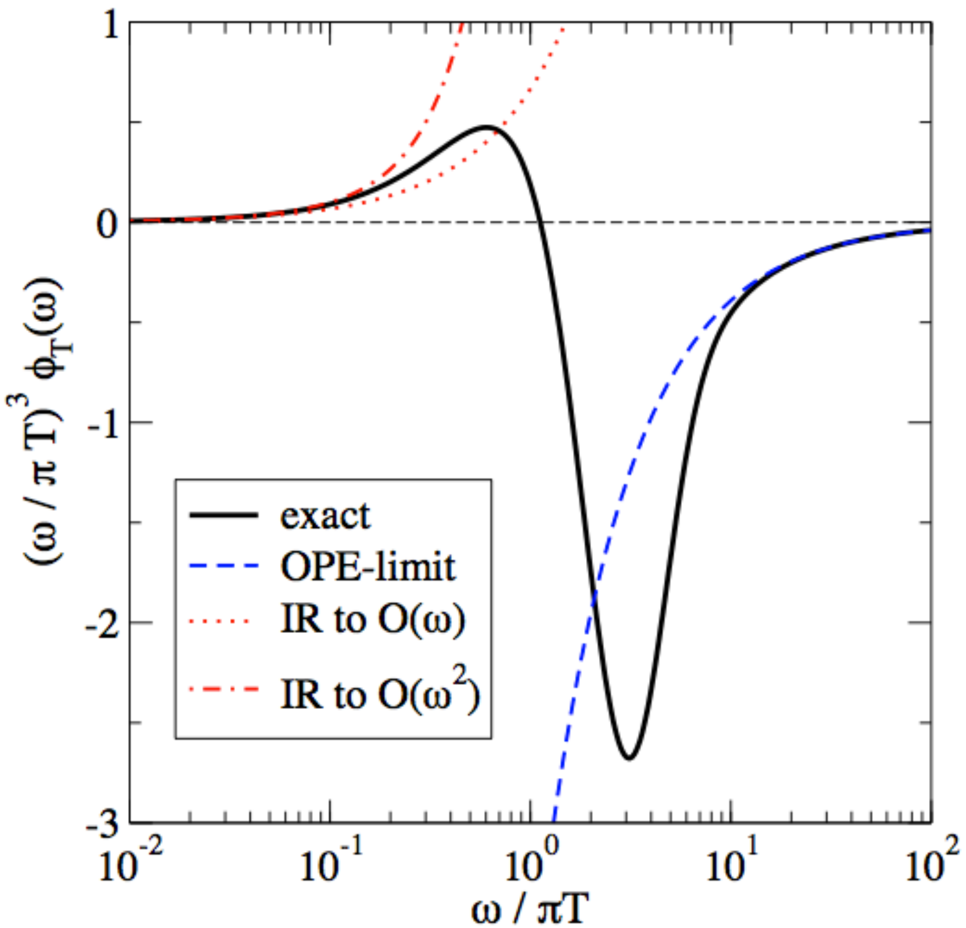
Spectral functions

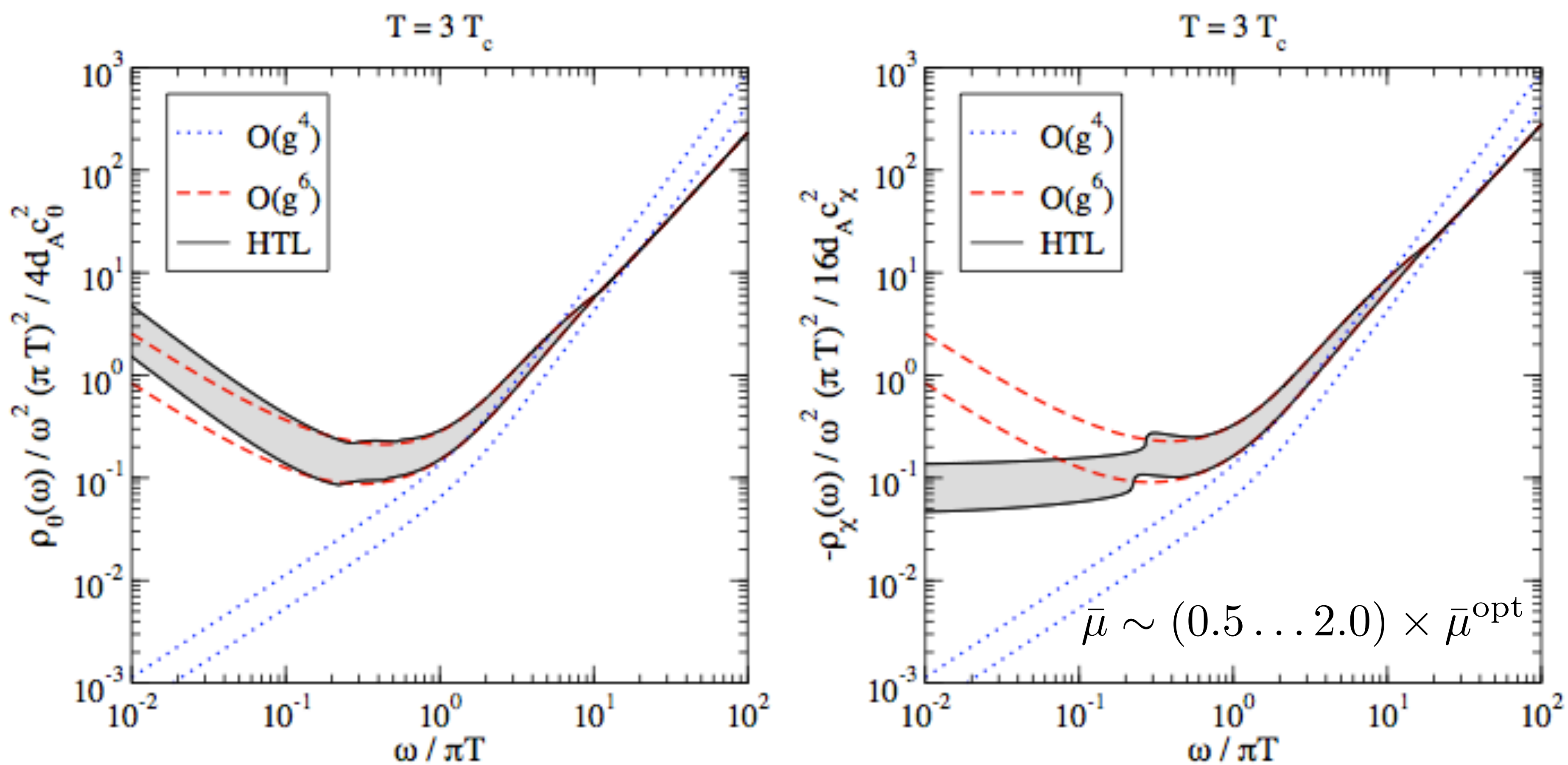
H. B. Meyer, 1002.3343





- Lattice result is compatible with NLO calculation in perturbative part!
- The full results are hoped to aid lattice determination of transport coefficients by providing non-trivial, dominant perturbative part of spectral density in $\omega \gtrsim T$.

Spectral Functions



$$\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}(1+2n_{\frac{\omega}{2}})\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16 d_A c_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}(1+2n_{\frac{\omega}{2}})\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\rho^{\rm QCD}_{\rm resummed} = \rho^{\rm QCD}_{\rm resummed} - \rho^{\rm HTL}_{\rm resummed} + \rho^{\rm HTL}_{\rm resummed} \approx \rho^{\rm QCD}_{\rm naive} - \rho^{\rm HTL}_{\rm naive} + \rho^{\rm HTL}_{\rm resummed}\,.$$

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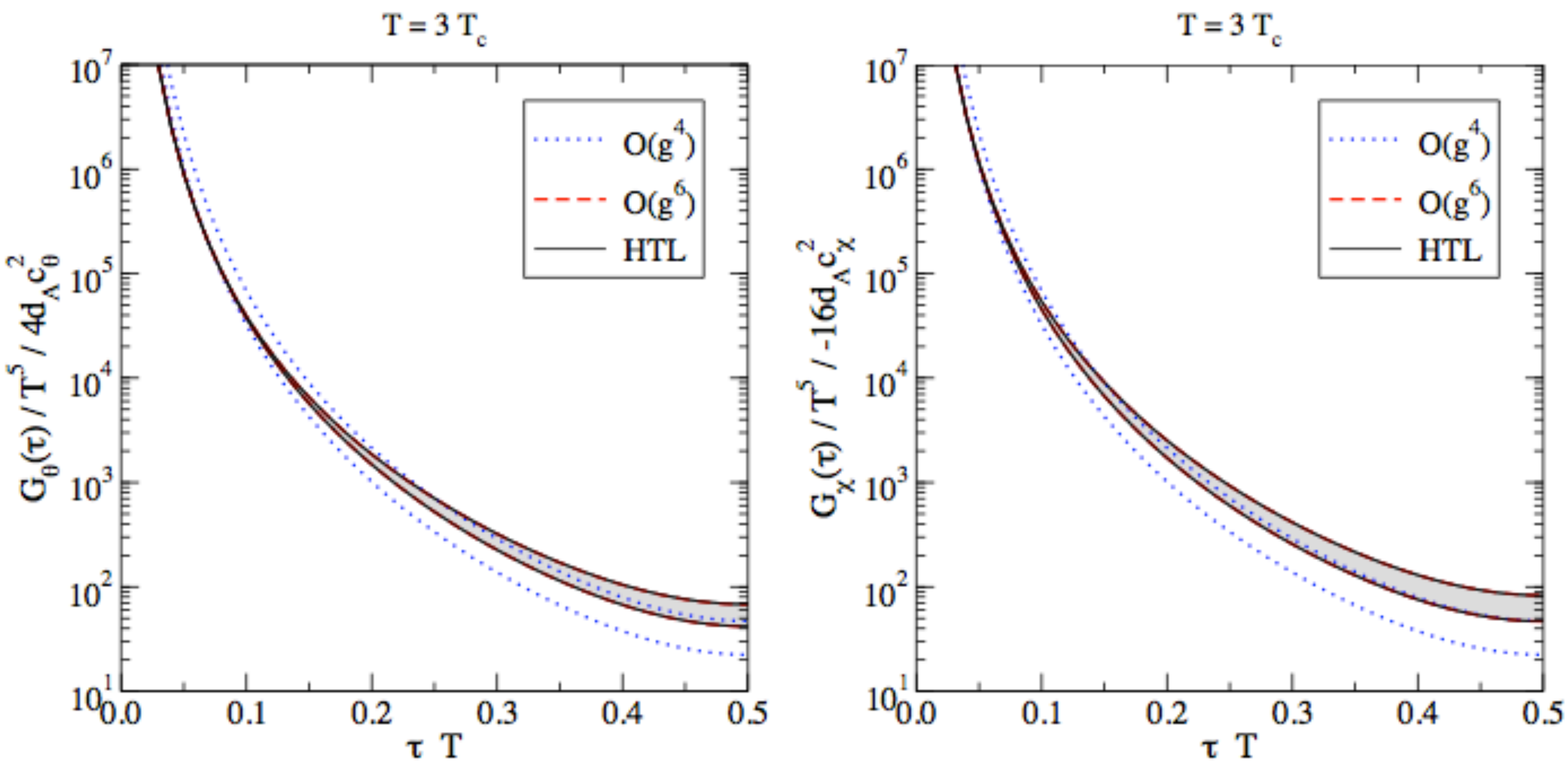
Imaginary-time Correlators

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, \mathbf{0}) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}} .$$



- Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.

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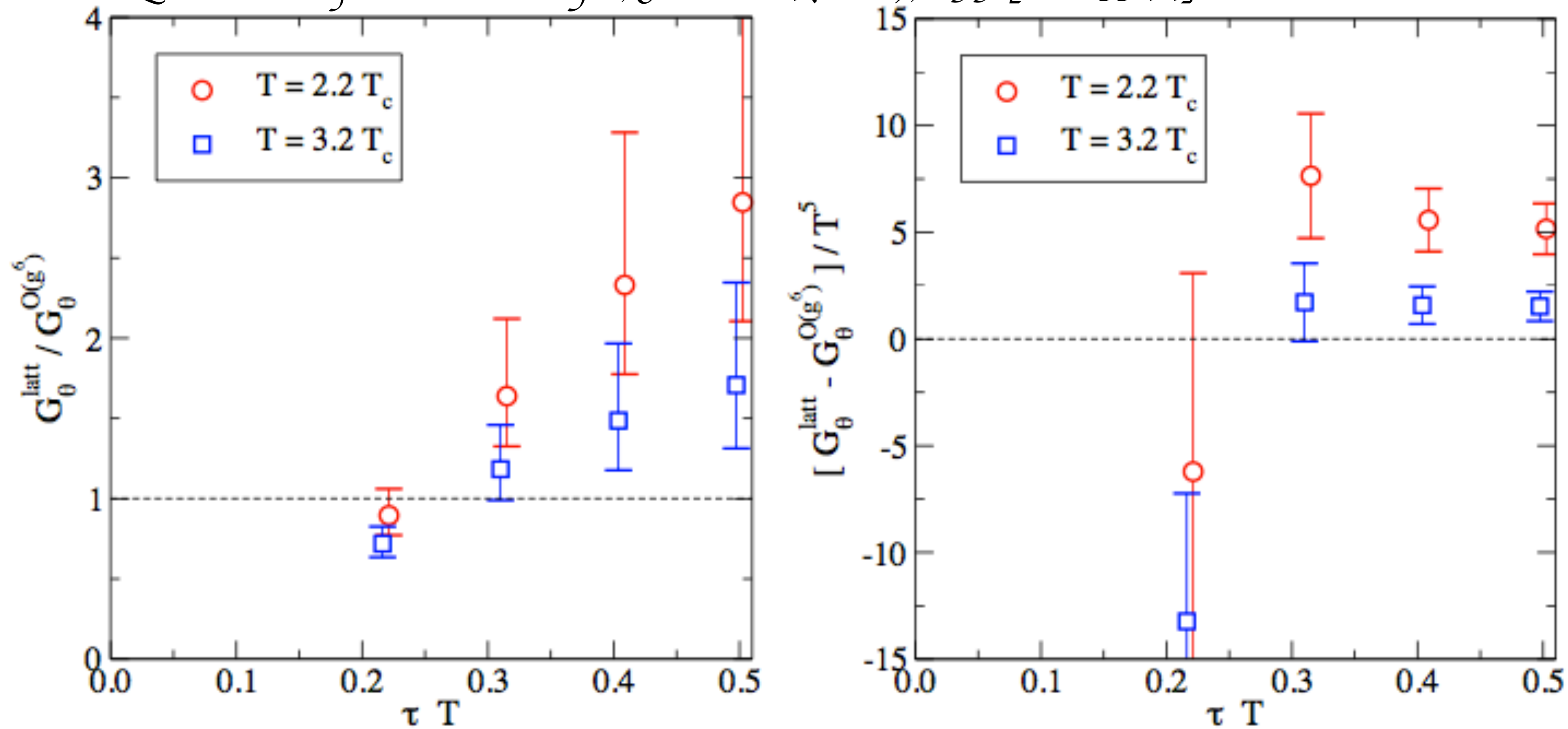
Lattice vs pQCD

Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]



- The ratio shows good agreement at short distance.
- The difference no longer shows the short distance divergence. A model independent analytic continuation could be attempted.

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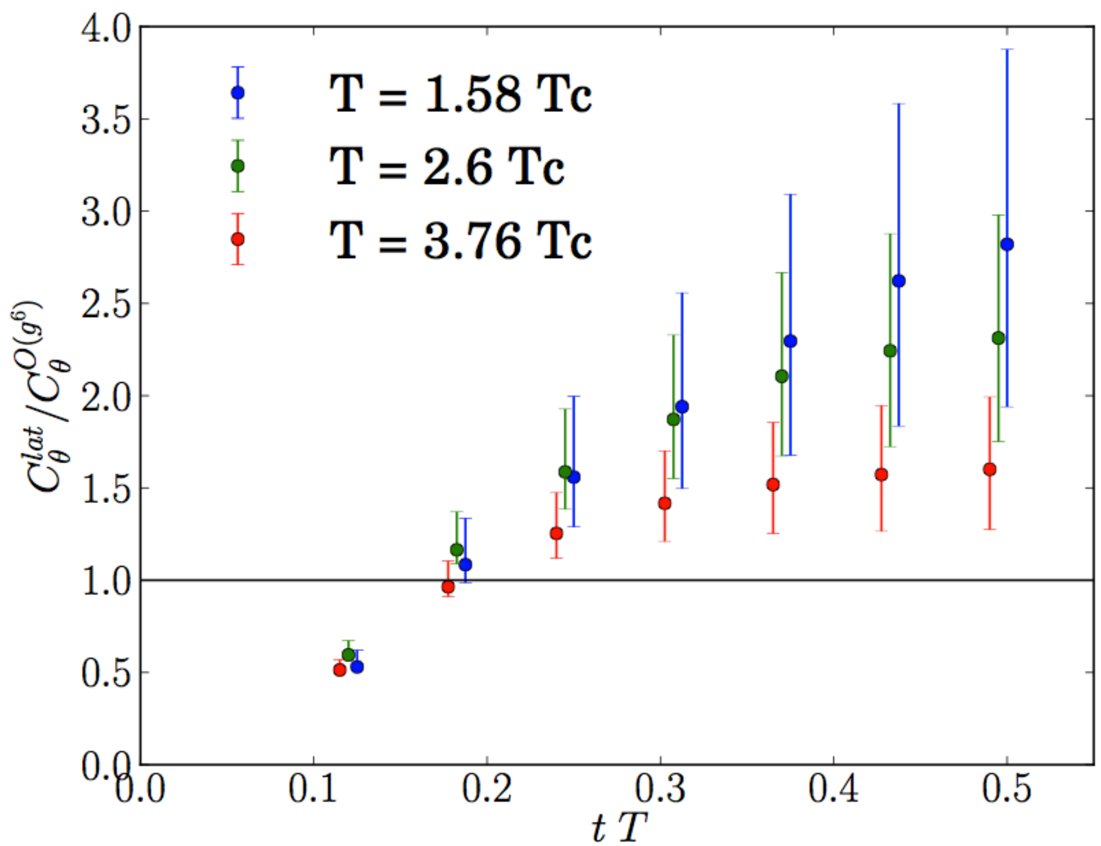
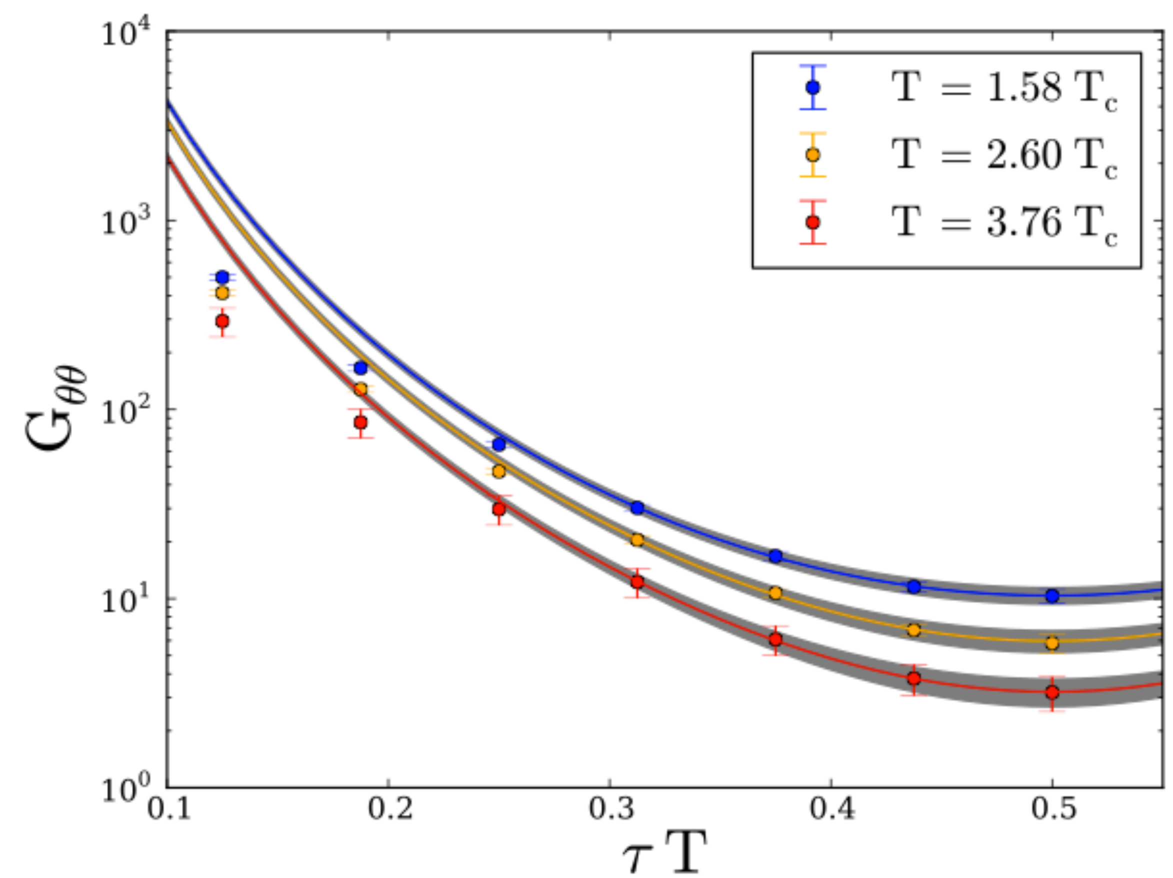
Lattice vs pQCD

Chuan Miao(CPOD2011), H. B. Meyer



Chuan Miao, H.B. Meyer (Preliminary)



- Right panel: Fit correlators with Breit Wigner formula (low frequency) + NLO results (high frequency), the width of the B-W is fixed to 0.5πT.
- NLO perturbative input is rather helpful.

Summary and Outlook

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
  - Spatial correlators a highly useful way test lattice, pQCD and holographic predictions
- NLO results in the bulk channel completed, shear channel underway
  - Results promising, but quantitative comparisons await
- If pure YM results useful, inclusion of fermions straightforward

Thanks for your attention!