Cold Electroweak Baryogenesis and Real-Time Fermions

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INT Workshop on
Gauge Field Dynamics In and Out of Equilibrium
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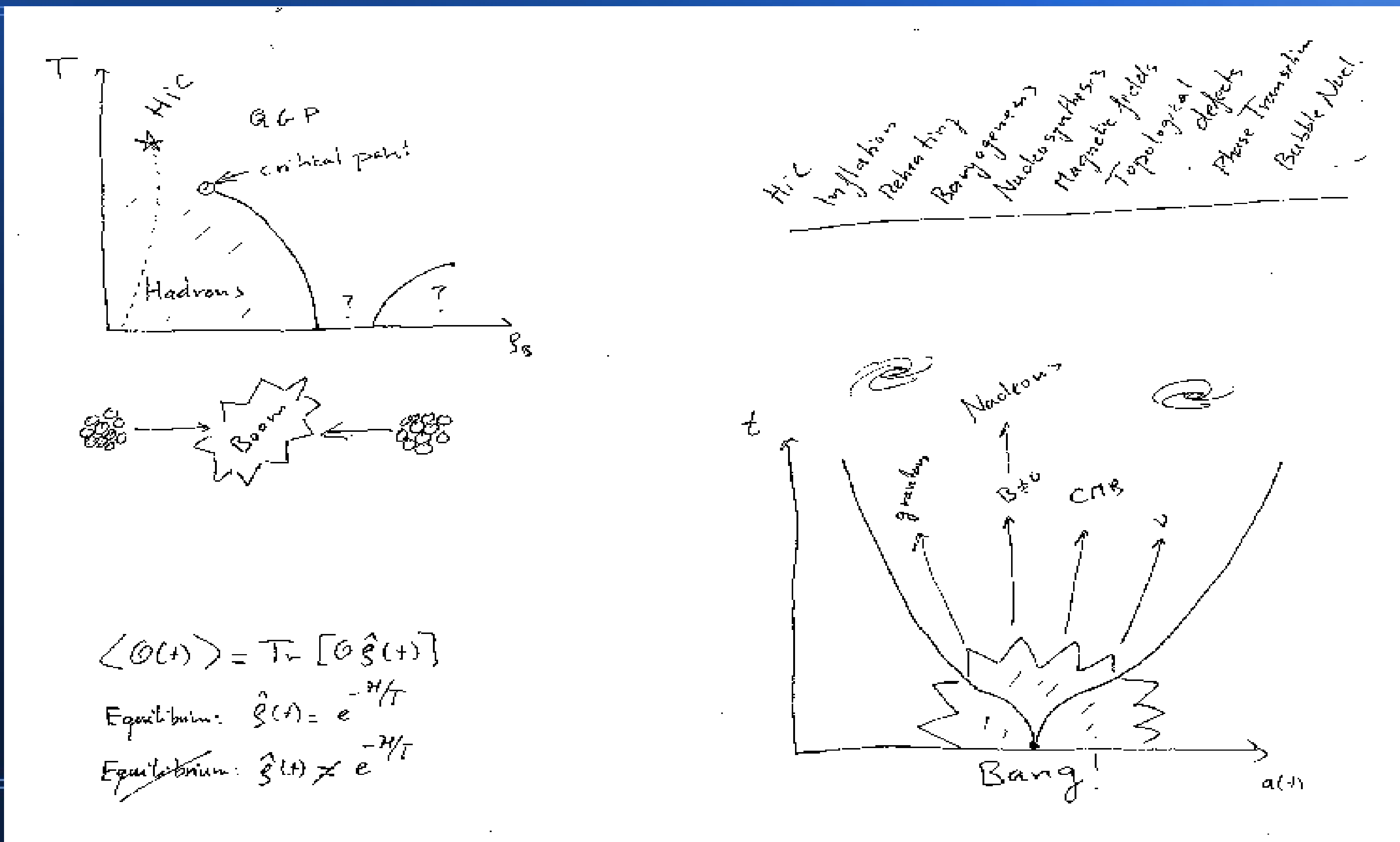
Map



T
HIC
QGP
critical point
Hadrons
?
?
ρ_B
Boom
⟨O(t)⟩ = Tr [O ρ̂(t)]
Equilibrium: ρ̂(t) = e^{-H/T}
Equilibrium: ρ̂(t) ≠ e^{-H/T}
HIC
Inflation
Reheating
Baryogenesis
Nucleosynthesis
Magnetic fields
Topological defects
Phase Transition
Bubble Nucl.
Nucleons
t
gravitons
B≠0
CMB
ν
Bang!
a(t)

Electroweak Baryogenesis

Mechanism for explaining the matter/antimatter asymmetry in the Universe,

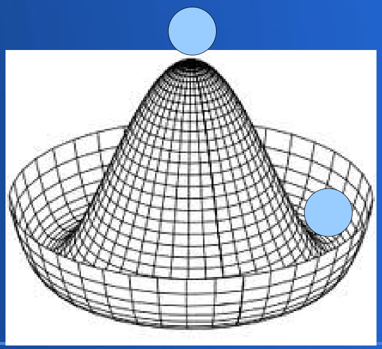
$$\frac{n_B}{n_\gamma} \simeq 6 \times 10^{-10}$$

Require

- Baryon ons number
- Breakin KM matrix
- Out-of-equilibrium dynamics → Electroweak symmetry breaking

Higgs mass = 125 GeV
→ Game over !

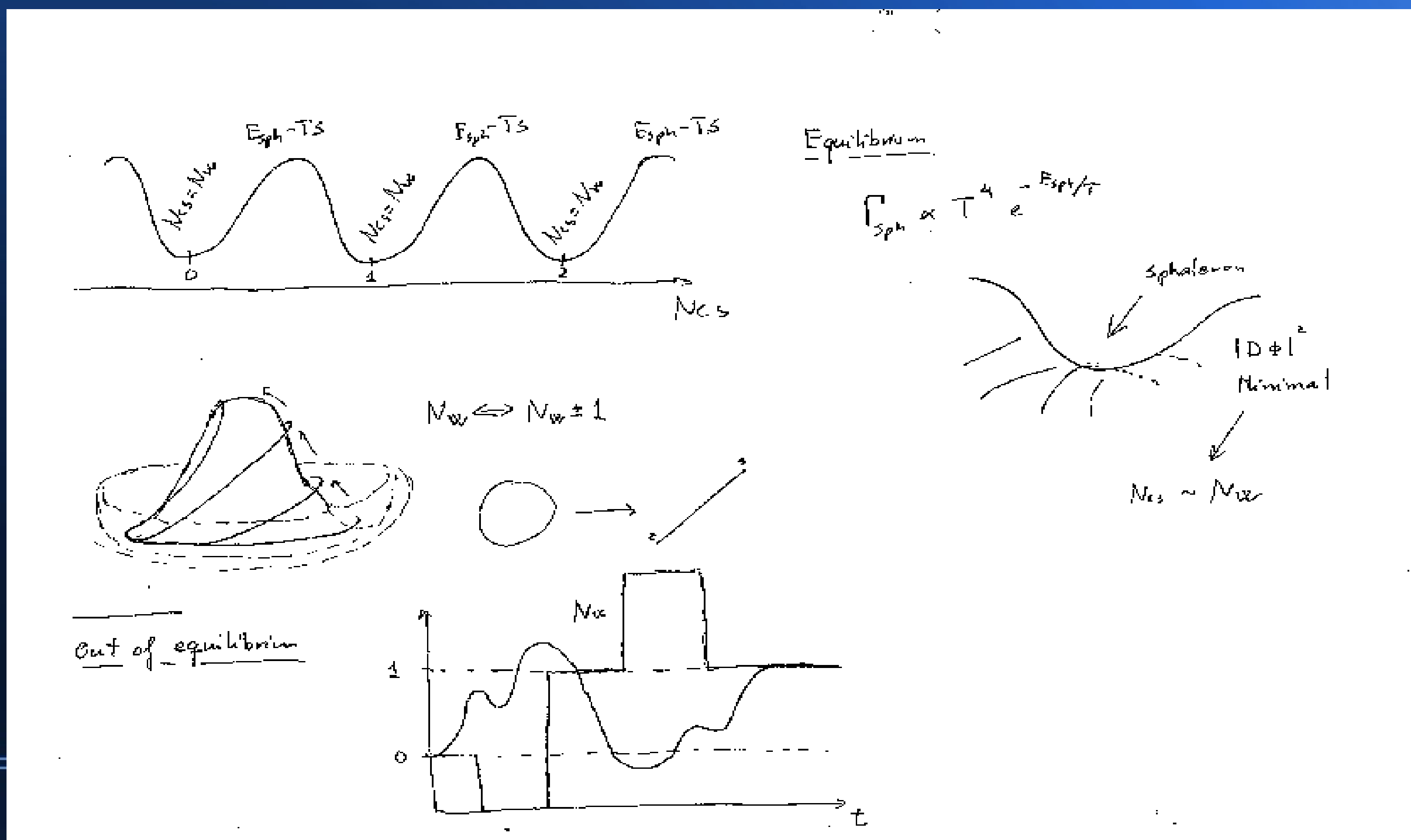
1. order phase transition
2. order phase transition
...
Cross-over transition
Spinodal transition



Kuzmin, Rubakov, Shaposhnikov: 1985
Kajantie, Laine, Rummukainen, Shaposhnikov: 1996

Electroweak baryon number violation

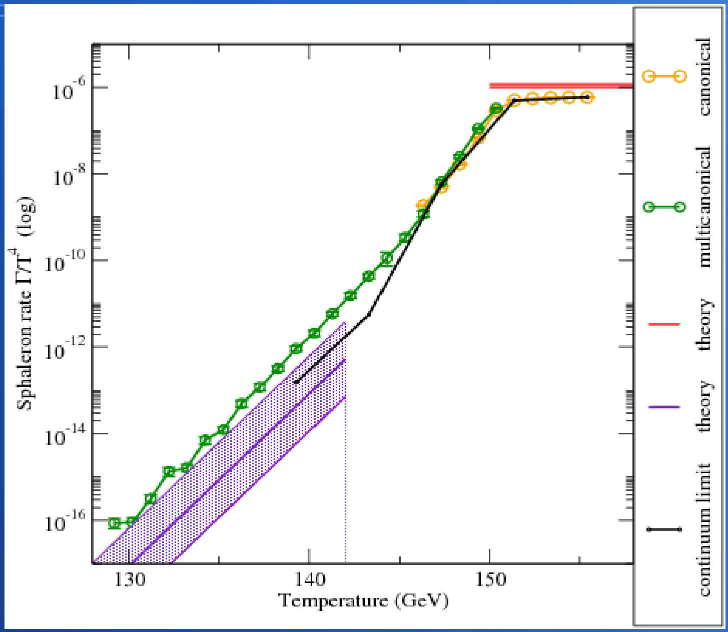
$$B(t) - B(0) = 3[N_{cs}(t) - N_{cs}(0)] = L(t) - L(0)$$



Baryogenesis from Leptogenesis

- Heavy (Majorana) neutrinos decay out of equilibrium.
- Violation of Lepton number conservation → B-L nonzero.
- Universe thermalizes, cools.
- Equilibrium sphaleron processes convert only L to some B and some L.
- No need for out-of-equilibrium electroweak transition.

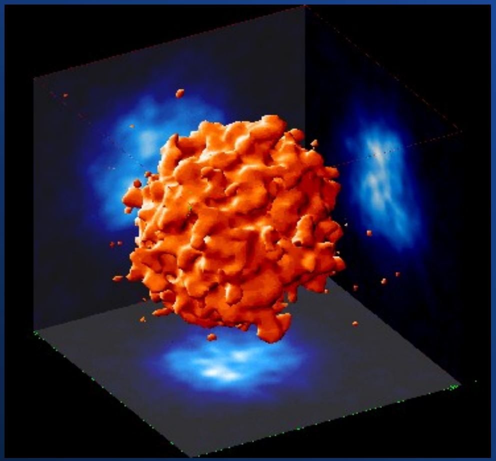
Fukugita, Yanagida: 1986
Luty: 1992



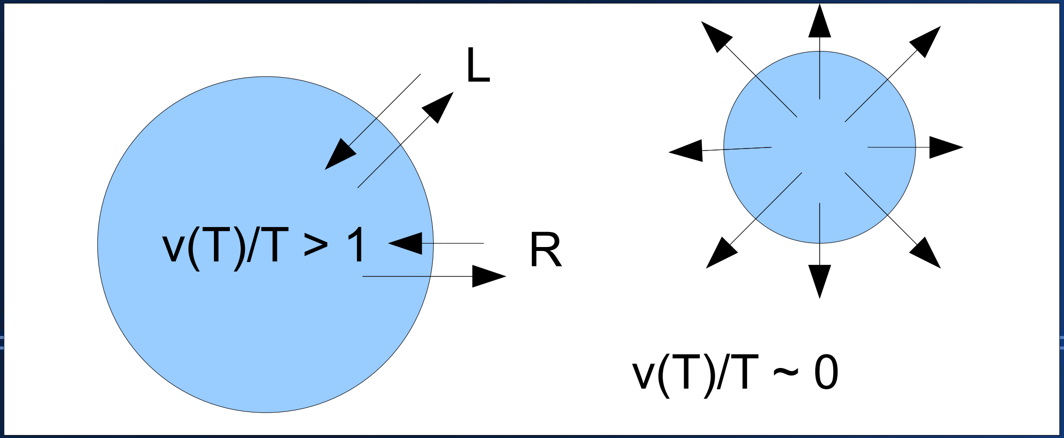
Ambjorn, Askgaard, Porter, Shaposhnikov: 1991
Philipsen: 1993, 1995
Ambjorn, Krasnitz: 1993, 1995
Moore: 1996-2000
& Rummukainen: 1999
& Bödeker: 1999
Shanahan, Davis: 1998
Smit, Tang: 1996
D'Onofrio, Rummukainen, AT: 2012
Arnold, Yaffe, Son, Kajantie, Laine, Burnier...

“Hot” Electroweak Baryogenesis

Kuzmin, Rubakov, Shaposhnikov: 1985.
Cohen, Kaplan, Nelson: 1991.



K. Rummukainen: 2001



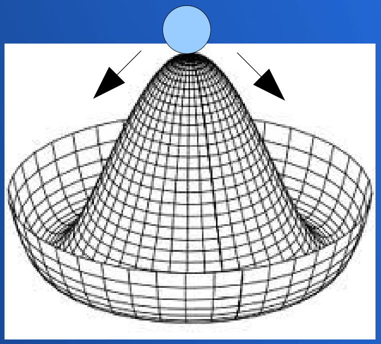
- Enlarge scalar sector to give strong finite T phase transition.
- Bubble nucleation.
- Advancing bubble wall interacts with plasma, breaking CP, net CP asymmetry inside and outside bubble.
- Sphalerons active outside; suppressed inside. Convert to B asymmetry.
- Bubbles eventually cover space.

"Cold" Electroweak Baryogenesis

- Enlarged scalar sector allows for super-cooling of Universe...
- ...and then rapid quench.
- → low-T spinodal transition.
- → quench speed determines out-of-equilibrium-ness.
- Thermalization to T<Mw.
- = "Tachyonic preheating".

$$\ddot{\phi}_k + (k^2 - m^2 + g^2\sigma^2(t))\phi_k = 0$$

$$\phi_k \to e^{\sqrt{m^2-k^2}t}, \quad n_k \to e^{2\sqrt{m^2-k^2}t}$$



$$V(0) - V(v) = \frac{\pi^2}{30} g^* T_{\rm reh}^4, \quad T_{\rm reh} \simeq 40\,{\rm GeV}$$

Two Mechanisms

Krauss and Trodden: 1999

(and Turok and Zadrozny: 1990-1)

- Symmetry breaking → Kibble mechanism.
- Net density of localized Higgs field textures.
- Average winding zero, average Ncs zero.
- Asymmetric unwinding under CP-violation.
- → Net asymmetry in Nw and Ncs.

Also: Copeland, Lyth, Rajantie, Trodden: 2001

Garcia-Bellido, Grigoriev, Kusenko, Shaposhnikov: 1999

- Spinodal transition → unstable IR modes in Higgs field.
- Energy driven into gauge field.
- Growth of gauge field under CP-bias → Net Chern-Simons number.
- <Ncs^2> → non-equilibrium “diffusion” rate.
- → Net asymmetry in Nw and Ncs.

Also: Garcia-Bellido, Gonzalez-Arroyo, Garcia Perez 2002-2003-2004

What actually happens

Krauss and Trodden: 1999

(and Turok and Zadrozny: 1990-1)

- Symmetry breaking → Kibble mechanism.
- Net density of localized Higgs field textures.
- Average winding zero, average Ncs zero.
- Asymmetric unwinding under CP-violation, nonzero Ncs and oscillating Higgs field.
- → Net asymmetry in Nw and Ncs.

Garcia-Bellido, Grigoriev, Kusenko, Shaposhnikov: 1999

- Spinodal transition → unstable IR modes in Higgs field.
- Energy driven into gauge field.
- Growth of gauge field under CP-bias → Net Chern-Simons number.
- <Ncs^2> → non-equilibrium "diffusion" rate.
- → Net asymmetry in Nw and Ncs.

AT, Smit; Skullerud, Smit, AT; AT, Smit, Hindmarsh; 2003-2006

How do we know?

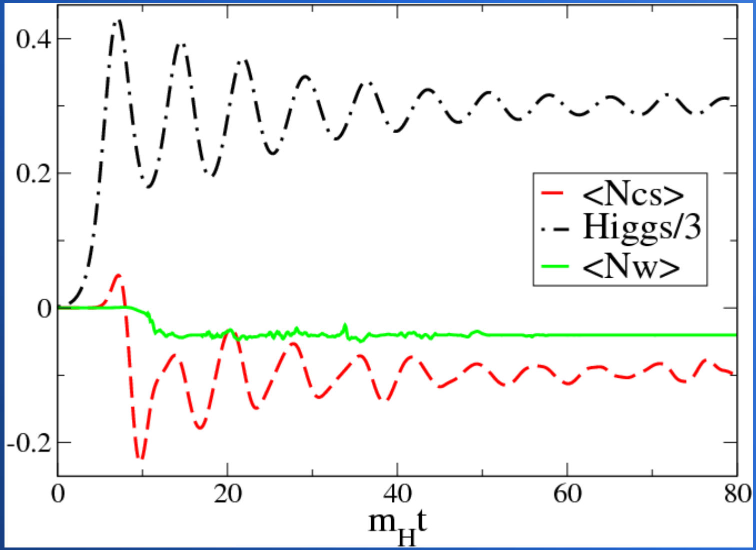
Lattice simulations:

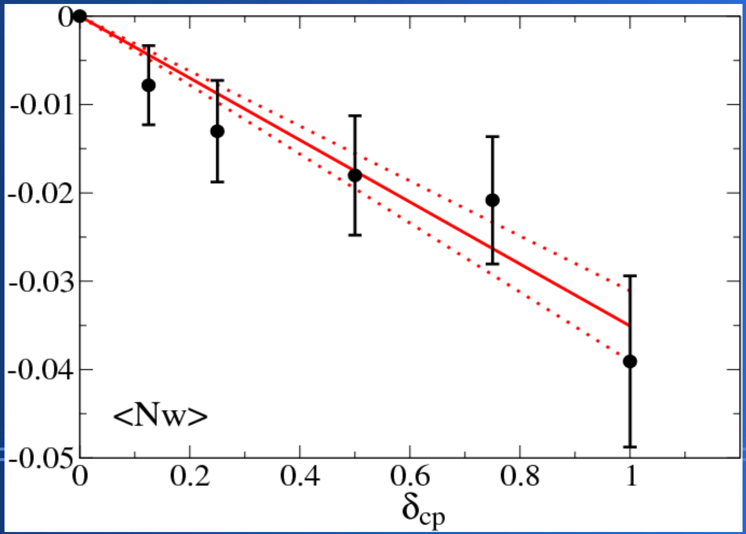
- Classical dynamics of...
- ...SU(2) + Higgs + CP-violation.
- Cold initial conditions.
- Fast quench (flip the mass).
- Average over ensemble.

$$\frac{3\,\delta_{\rm cp}}{16\pi^2 m_W^2}\phi^\dagger\phi\,{\rm Tr}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$n_B = 3\,n_{\rm cs} = 3\,n_{\rm W}$$

$$\frac{n_B}{n_\gamma} = -(0.32 \pm 0.04)\times 10^{-4}\times\delta_{\rm cp}$$





AT, Smit: **JHEP 0608:012,2006**

Sources of CP-violation

Standard Model:

- CKM matrix

Shaposhnikov: 1987

$$\delta_{\rm cp} \propto J \frac{\Delta}{T^{12}}, \qquad T > m_q$$

$$\delta_{\rm cp} \propto J \frac{\Delta}{v^{12}}, \qquad T \simeq 0$$

$$\begin{aligned} J &= 3 \times 10^{-5}, \\ \Delta &= \Pi_{(d,s,b),(u,c,t)}(m_i^2 - m_j^2) \end{aligned}$$

$$\delta_{\rm cp} \simeq 10^{-20}$$

Standard Model + Higgs:

- CKM matrix
- 2-Higgs potential

$$\begin{aligned} V(\phi_1, \phi_2) = {} & \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 \\ & + \mu_{12}^2 \phi_1^\dagger \phi_2 + \mu_{12}^{2,*} \phi_2^\dagger \phi_1 \\ & + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ & + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\ & + \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2 \end{aligned}$$

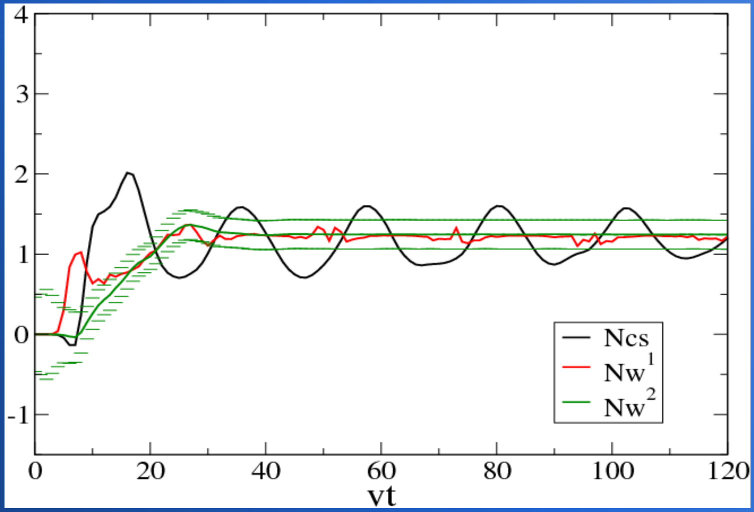
CEWBaG in 2HDM

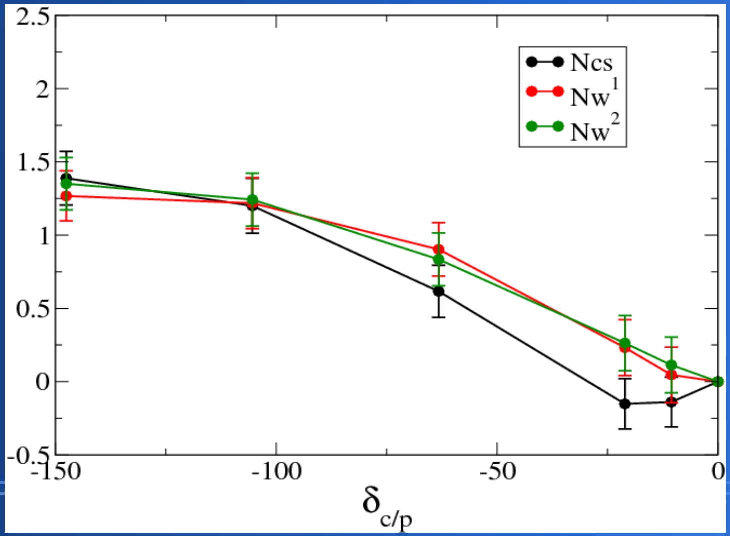
- Ncs is P and CP odd.
- Potential is C and CP odd.
- Fermion-gauge interaction is C and P odd.
- Integrate out fermion → C/P odd bosonic terms.

$$\frac{\delta_{C/P}}{16\pi^2 m_W^2} i(\phi_2^\dagger \phi_1 - \phi_1^\dagger \phi_2) \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\frac{n_B}{n_\gamma} = -(0.28 \pm 0.12) \times 10^{-5} \times \delta_{C/P}$$

AT, Bin Wu: 1203.5012 (Last Friday)





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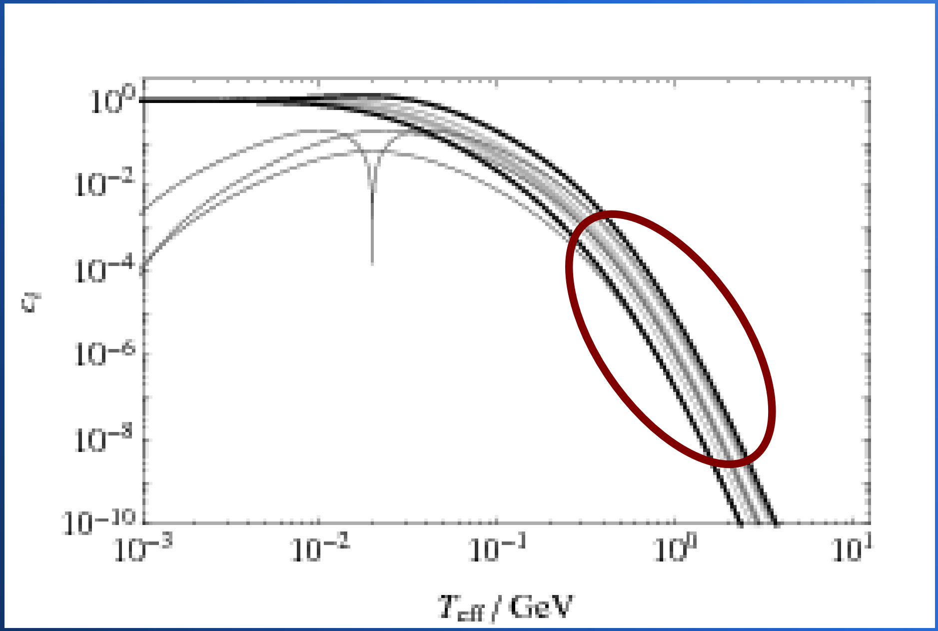
Standard Model + Higgs:

- CKM matrix
- 2-Higgs potential

$$\begin{aligned} V(\phi_1, \phi_2) = \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 \\ + \mu_{12}^2 \phi_1^\dagger \phi_2 + \mu_{12}^{2,*} \phi_2^\dagger \phi_1 \\ + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\ + \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_5^* (\phi_2^\dagger \phi_1)^2 \end{aligned}$$

Bosonized Standard Model

- Integrate out fermions at finite T from the SM → TrLog.
- Expand in covariant derivatives.
- → resums Higgs field insertions.
- C(P)-violation at order 6, 4 W and 2 Z/$\partial\phi$.
- For bosonic simulations, we also need C/P violating sector.



$$\Gamma_{\rm CP} = -\frac{i}{2} N_c J G_F \kappa_{\rm cp} \int_0^{1/T} dx_0 \int d^3\mathbf{x} \left(\frac{v}{\phi}\right)^2 \mathcal{O}[W, Z, \partial\phi, c_i(vT/\phi)]$$

Brauner, Taanila, AT, Vuorinen: **Phys.Rev.Lett.108:041601,2012**

See also: Smit: 2004,
Salcedo: 2011
Garcia-Recio, Salcedo: 2009
Hernandez, Konstandin, Schmidt: 2008

And now for something completely different...

...but related.

Bosonic field

$$\hat{\phi}(\mathbf{x},t) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{2\,\omega_\mathbf{k}}} \left(\hat{a}_\mathbf{k} f_\mathbf{k}(\mathbf{x},t) + \hat{a}^\dagger_\mathbf{k} f^*_\mathbf{k}(\mathbf{x},t) \right)$$

$$\langle \hat{\phi}(\mathbf{x},t)^2 \rangle = \int \frac{d^d \mathbf{k}}{(2\pi)^d} \frac{\langle a_\mathbf{k} a^\dagger_\mathbf{k} + a^\dagger_\mathbf{k} a_\mathbf{k} \rangle}{2\,\omega_\mathbf{k}} |f_\mathbf{k}(\mathbf{x},t)|^2, \qquad \langle a_\mathbf{k} a^\dagger_{\mathbf{k}'} \rangle \propto \delta(\mathbf{k},\mathbf{k}')$$

- Numerical effort O(nx^2d)!
- Gaussian truncations of Schwinger-Dyson hierarchy (Hartree, “Large N”= 1/N LO, …)
- Large occupation numbers → Effectively classical dynamics of ensemble of random numbers

$$\hat{a}_\mathbf{k} \to \xi_k, \qquad \langle \xi_\mathbf{k} \xi^\dagger_\mathbf{k} + \xi^\dagger_\mathbf{k} \xi_\mathbf{k} \rangle = \langle \hat{a}_\mathbf{k} \hat{a}^\dagger_\mathbf{k} + \hat{a}^\dagger_\mathbf{k} \hat{a}_\mathbf{k} \rangle$$

- Generalizes to full, interacting, non-perturbative classical approximation to quantum dynamics. Numerical effort O(nx^d * Nqb).
- Works well except for equilibrium → truncated quantum dynamics, SD/KB/2PI.

Fermionic field

- Fermion fields are always quantum. No classical limit, no large occupation numbers.
- Fermion fields are always bilinear in the action.

$$\psi = \frac{1}{\sqrt{2}}[\psi_1 - i\psi_2]$$

$$\hat{\psi}_i(\mathbf{x},t) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{\sqrt{2\,\omega_{\mathbf{k}}}} \left(\hat{b}_{\mathbf{k}} U_{\mathbf{k}} f_{\mathbf{k}}(\mathbf{x},t) + \hat{b}^\dagger_{\mathbf{k}} V_{\mathbf{k}} f^*_{\mathbf{k}}(\mathbf{x},t)\right)$$

$$\begin{aligned} D_{\alpha\beta}(x,y) \quad &= \quad \frac{1}{2}\left(\langle \hat{\psi}_\alpha(x)\hat{\psi}_\beta(y) - \hat{\psi}_\beta(y)\hat{\psi}_\alpha(x)\rangle\right) = \\ & \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{2\omega_{\mathbf{k}}}\left[U_{k\alpha}V_{k\beta}e^{ik(x-y)} - V_{k\alpha}U_{k\beta}e^{-ik(x-y)}\right], \\ & D^*_{\alpha\beta} = -D_{\alpha\beta} \end{aligned}$$

Aarts and Smit: 1998

- Numerical effort O(nx^2d)!

Male and Female

- Complex numbers do not anticommute. Use two ensembles of fields.

$$\psi^M(\mathbf{x},t) = \frac{1}{\sqrt{2}}\int \frac{d^dk}{(2\pi)^d}\frac{1}{\sqrt{2\,\omega_{\mathbf{k}}}}\left(\eta_{\mathbf{k}}U_{\mathbf{k}}f_{\mathbf{k}}(\mathbf{x},t)+\eta^*_{\mathbf{k}}V_{\mathbf{k}}f^*_{\mathbf{k}}(\mathbf{x},t)\right),$$

$$\psi^F(\mathbf{x},t) = \frac{i}{\sqrt{2}}\int \frac{d^dk}{(2\pi)^d}\frac{1}{\sqrt{2\,\omega_{\mathbf{k}}}}\left(\eta_{\mathbf{k}}U_{\mathbf{k}}f_{\mathbf{k}}(\mathbf{x},t)-\eta^*_{\mathbf{k}}V_{\mathbf{k}}f^*_{\mathbf{k}}(\mathbf{x},t)\right),$$

$$D_{\alpha\beta}(x,y) \rightarrow i\langle\psi_{M\alpha}(x)\psi_{F\beta}(y)\rangle \qquad \hat{b}_{\mathbf{k}}\rightarrow\eta_k, \qquad \langle\eta_{\mathbf{k}}\eta^\dagger_{\mathbf{k}}\rangle=\langle\hat{b}_{\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}}\rangle$$

$$= \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{2\omega_{\mathbf{k}}}\left[U_{k\alpha}V_{k\beta}e^{ik(x-y)}-V_{k\alpha}U_{k\beta}e^{-ik(x-y)}\right],$$

- Male-Female correlators reproduce all fermion bilinears. Effort O(2nx^d * Nqf).
- Classical, nonlinear bosonic fields. Fermion evolution linear in bosonic background.
- Bilinear quantum fermion sources to boson eoms → M-F correlators.

Hindmarsh, Borsanyi: 2009

1+1D model: U(1)-Higgs+fermions

$$
\begin{aligned}
S &= -\int d^2x\,[(D_\mu\phi)^\dagger(D^\mu\phi)+\lambda(\phi^\dagger\phi-v^2/2)^2] \\
&\quad -\int d^2x\,\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} \\
&\quad -\int d^2x\,[\bar\psi\gamma^\mu(\partial_\mu+iA_\mu\gamma_5)\psi+G\bar\psi(\phi^*P_L+\phi P_R)\psi]
\end{aligned}
$$

- Anomaly equation:

$$N_f=\int dx\, j^0=-\frac{1}{2\pi}\int dx\, A_1(x)=N_{\rm cs}$$

- Higgs winding:

$$N_{\rm W}=\frac{1}{2\pi}\int dx\,\partial_1\theta(x),\qquad \phi(x)=|\phi(x)|e^{i\theta(x)}$$

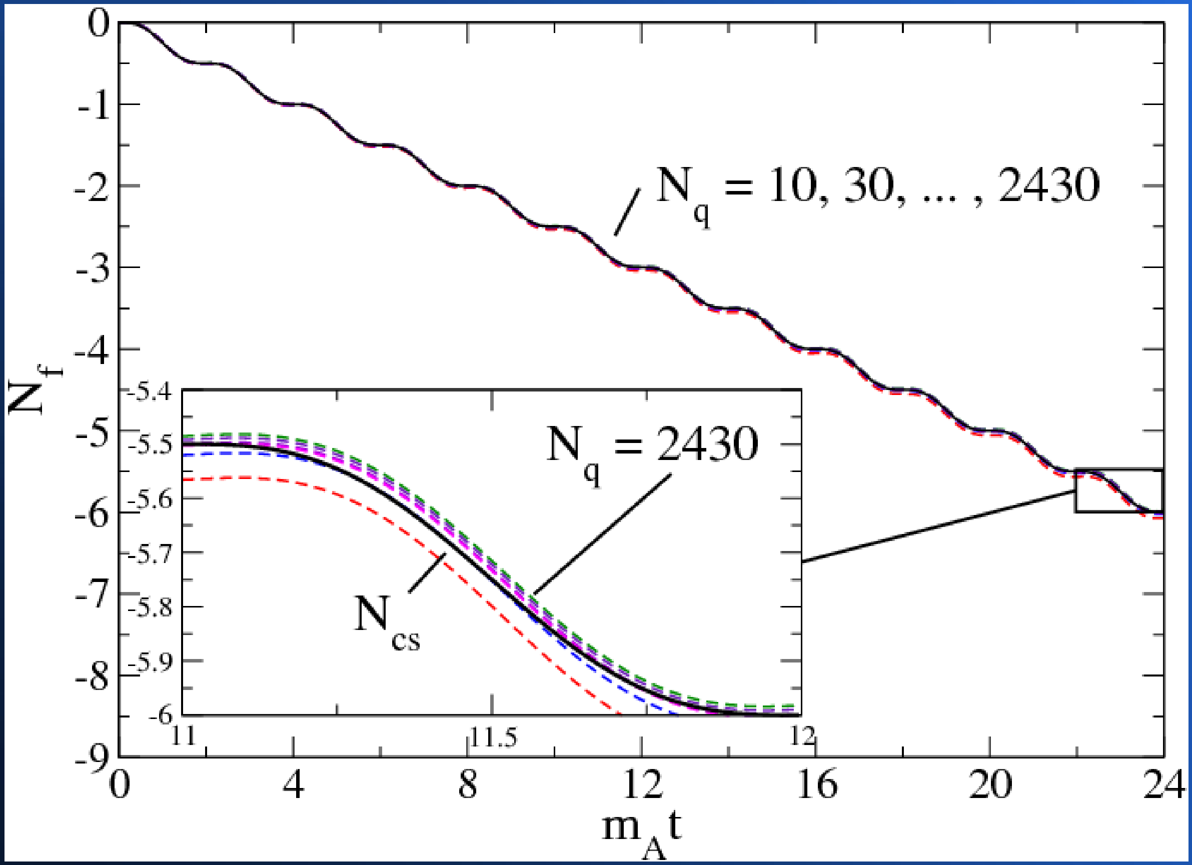
Technical stuff

- Lattice implementation:
  - 1D spatial lattice.
  - Non-compact U(1) gauge field.
  - Wilson fermions in space.
  - Timelike fermion doublers not initialized → stay unexcited.

$$\gamma^\mu D_\mu \psi \to \gamma^\mu D_\mu \psi + \frac{r_\mu}{2} D_\mu D^\mu \psi$$

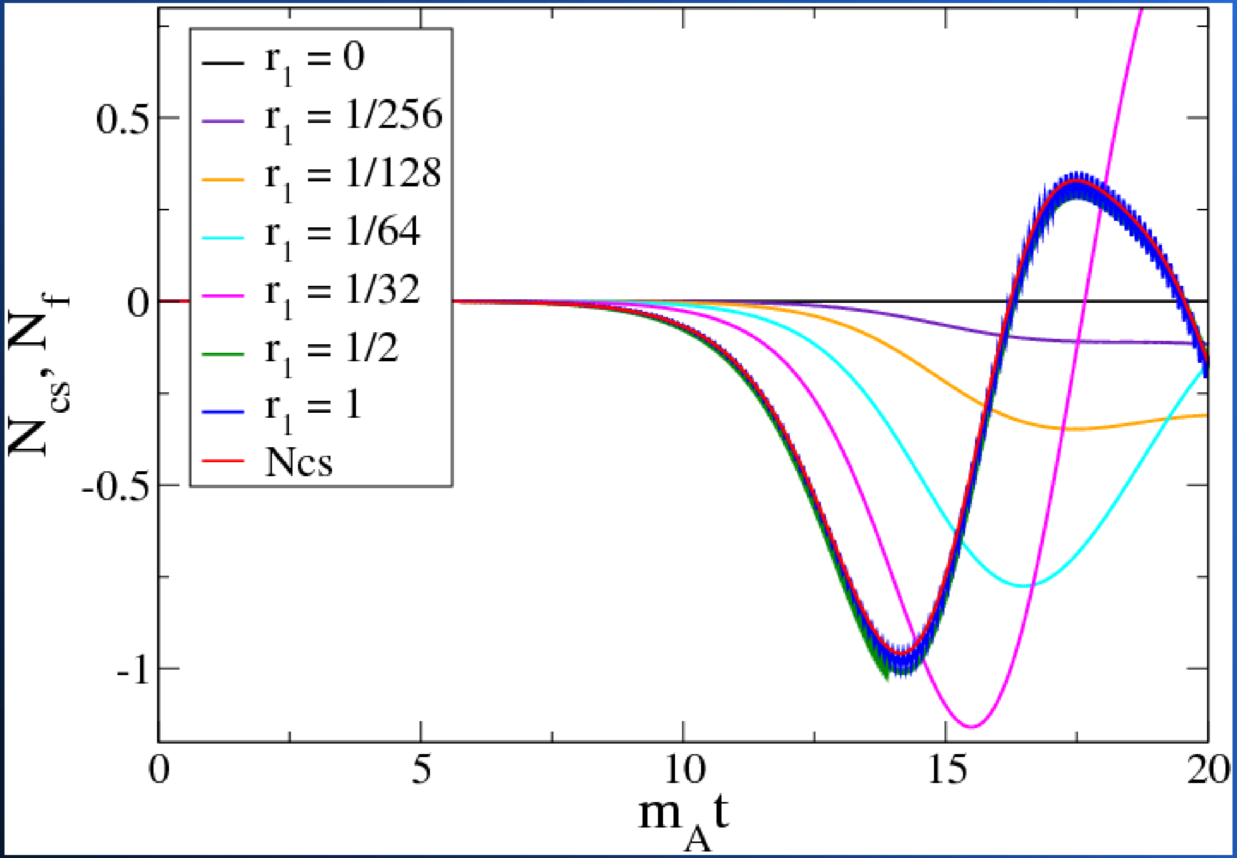
- Charge conjugation on upper fermion component:
  - Axial → vector.
  - Dirac mass → Majorana mass.
  - Fermion current ↔ Axial current.

By-hand sphaleron transitions



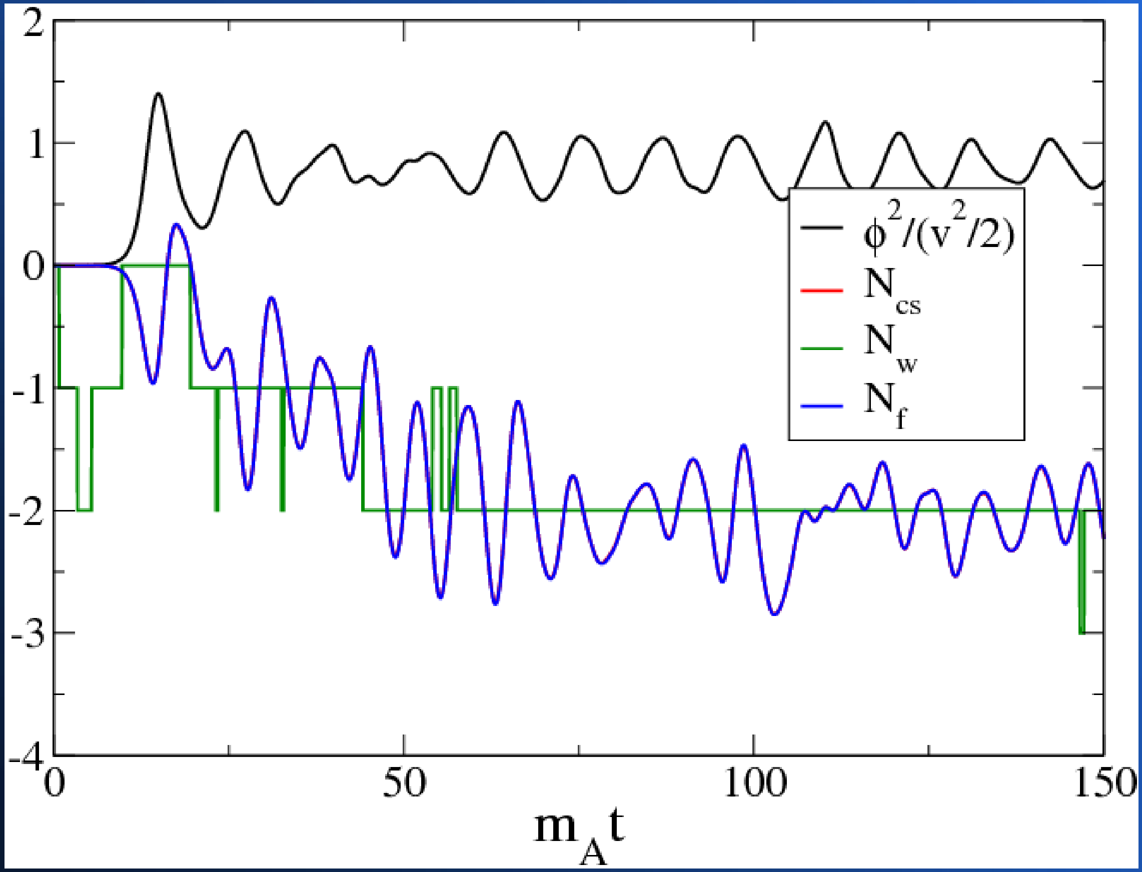
Saffin, AT, JHEP 1107 (2011) 066

Space-like doublers



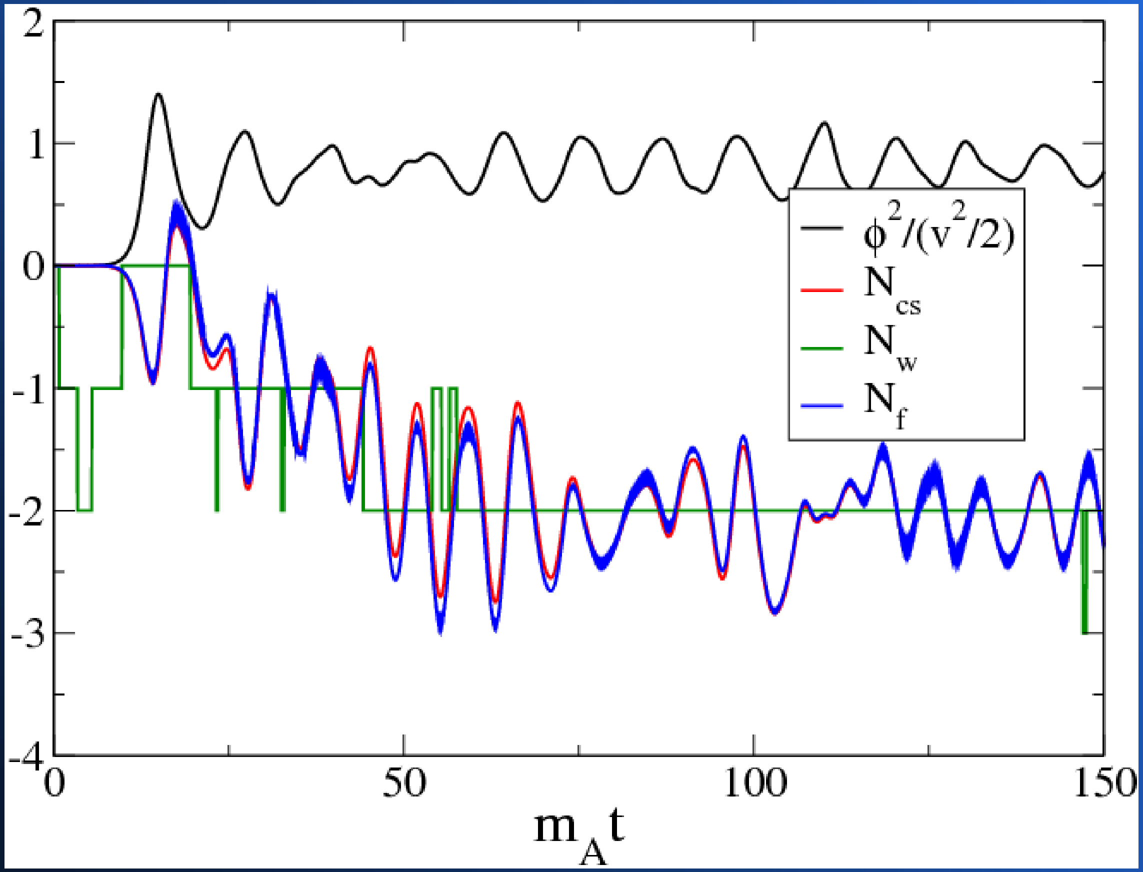
Saffin, AT, JHEP 1107 (2011) 066

Spinodal transition



Saffin, AT, JHEP 1107 (2011) 066

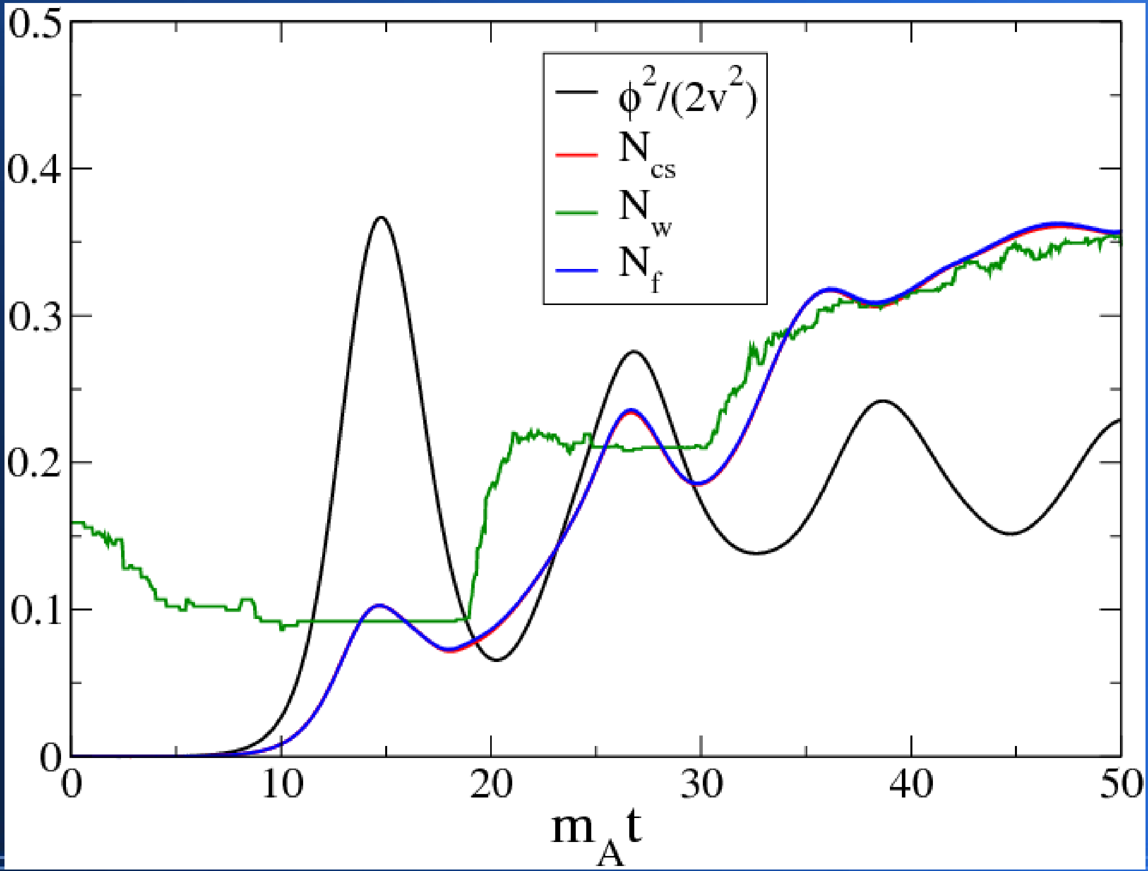
Yukawa couplings



Saffin, AT, JHEP 1107 (2011) 066

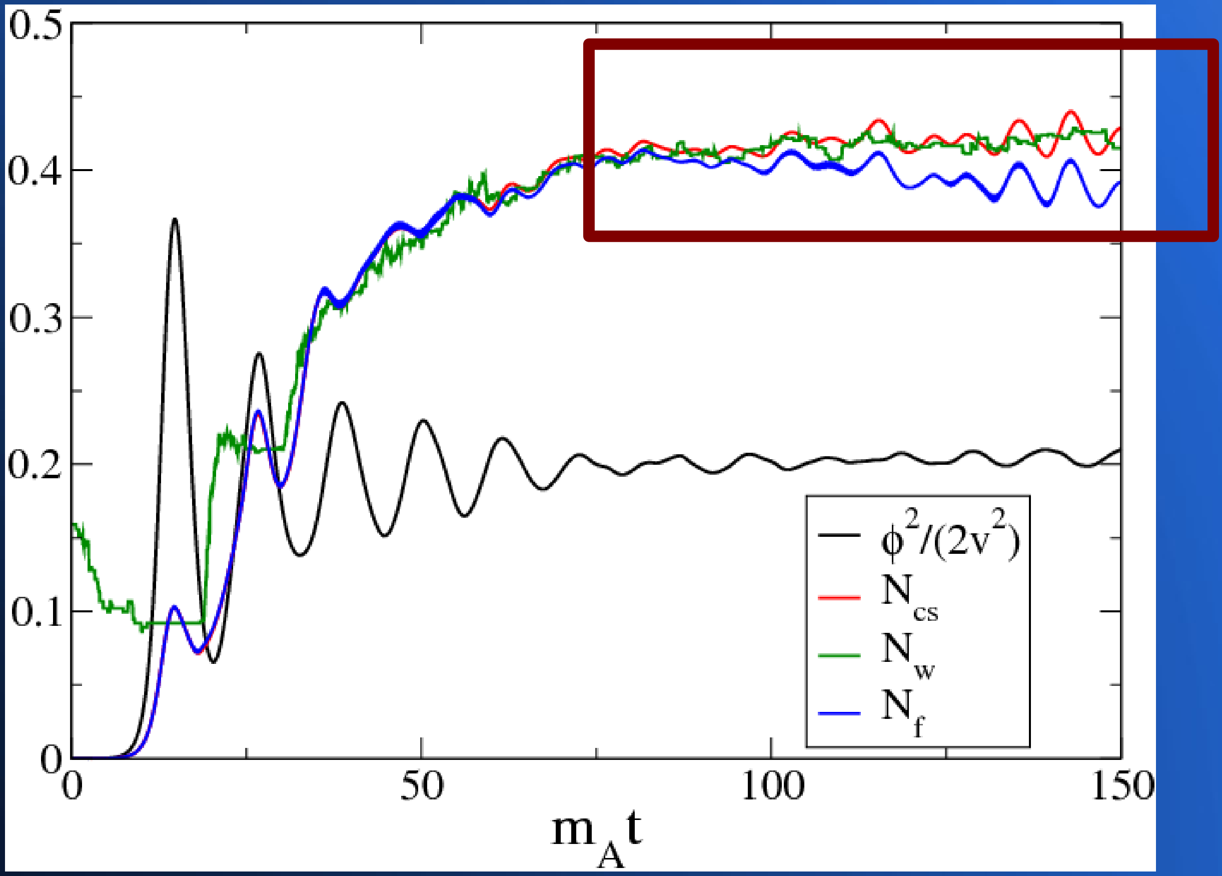
With CP-violation: Average

$$S \to S - \int d^2x \, \frac{\kappa}{4\pi} \phi^\dagger \phi \epsilon_{\mu\nu} F^{\mu\nu}$$



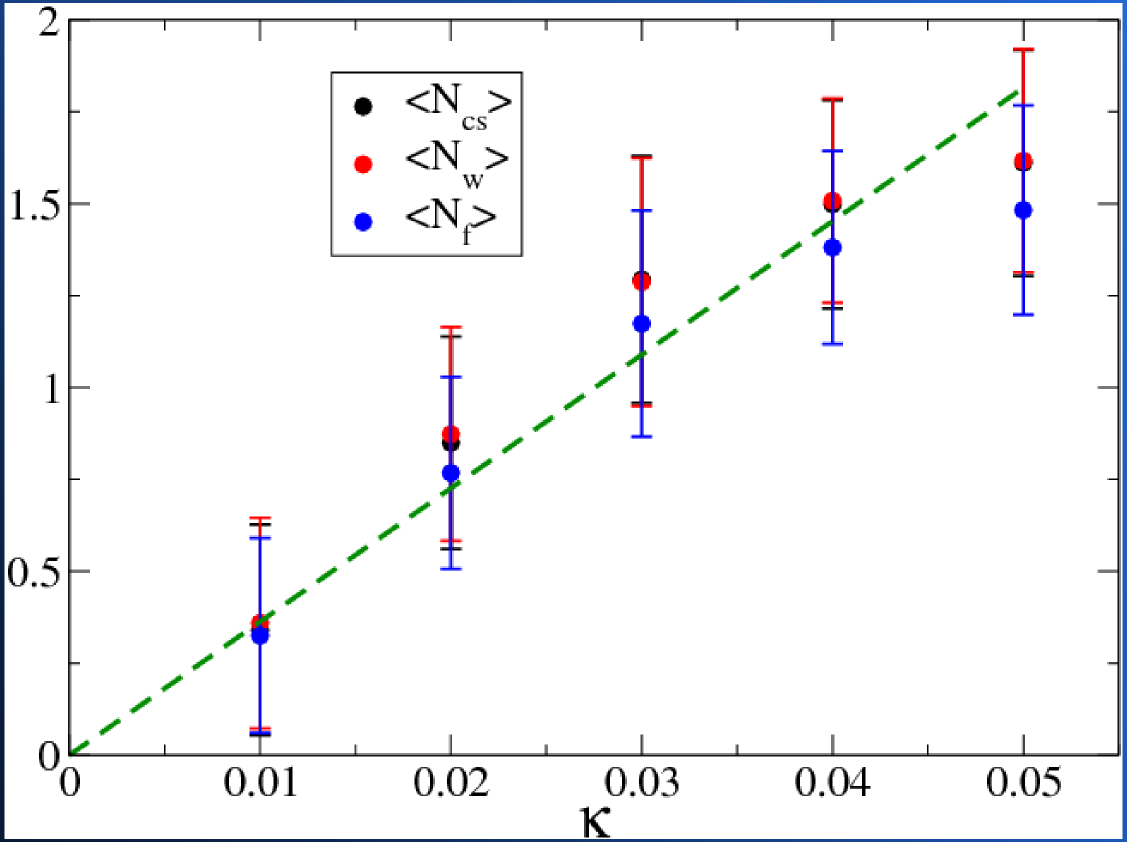
Saffin, AT, JHEP 1107 (2011) 066

With CP-violation: Average



Saffin, AT, JHEP 1107 (2011) 066

Baryon asymmetry



Saffin, AT, JHEP 1107 (2011) 066

3+1D: SU(2)+Higgs+Fermions

$$
\begin{aligned}
S &= S_H + S_W + S_F + S_Y \\
S_H &= -\int d^4x \; \left[D_\mu \phi^\dagger D^\mu \phi + \lambda(\phi^\dagger \phi - v^2/2)^2\right], \\
S_W &= -\int d^4x \; \frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu}, \\
S_F &= -\int d^4x \; \left[\bar{q}_L \gamma^\mu D_\mu q_L + \bar{u}_R \gamma^\mu D_\mu u_R + \bar{d}_R \gamma^\mu D_\mu d_R \right. \\
&\qquad \left. + \bar{l}_L \gamma^\mu D_\mu l_L + \bar{\nu}_R \gamma^\mu D_\mu \nu_R + \bar{e}_R \gamma^\mu D_\mu e_R \right], \\
S_Y &= -\int d^4x \; \left[G^u \bar{q}_L \phi u_R + G^d \bar{q}_L \phi d_R + G^e \bar{l}_L \phi e_R + G^\nu \bar{l}_L \phi \nu_R \right. \\
&\qquad + \hat{G}^u \bar{q}_L \tilde{\phi} u_R + \hat{G}^d \bar{q}_L \tilde{\phi} d_R + \hat{G}^e \bar{l}_L \tilde{\phi} e_R + \hat{G}^\nu \bar{l}_L \tilde{\phi} \nu_R \\
&\qquad \left. + h.c. \right]
\end{aligned}
$$

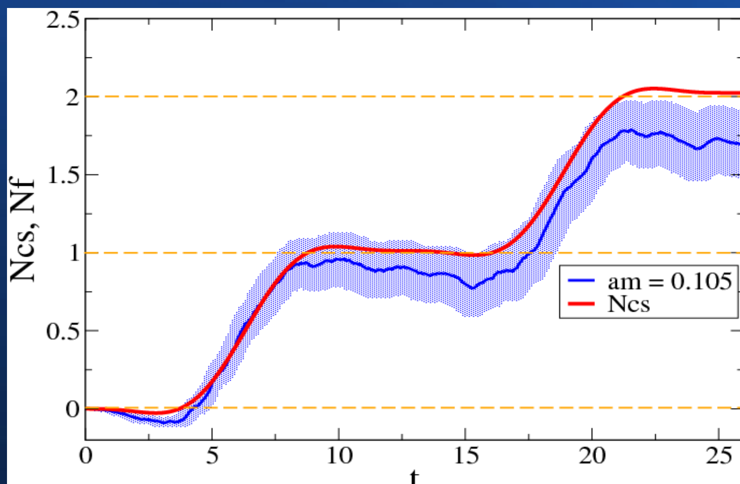
Technical stuff

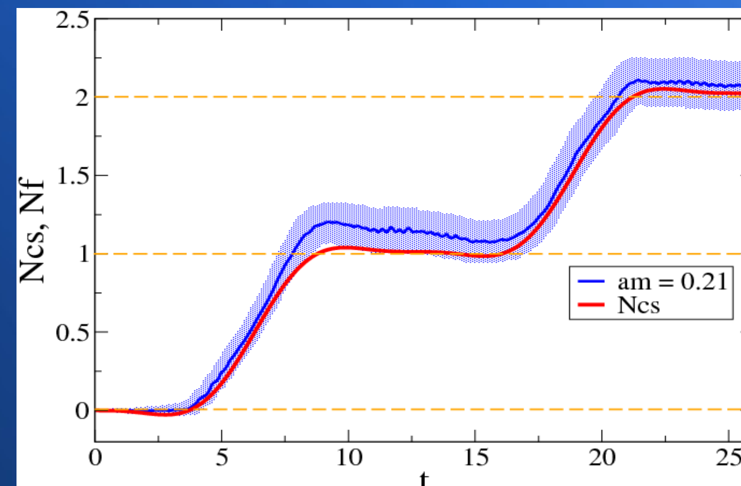
- Lattice implementation:
  - 3D spatial lattice.
  - SU(2) gauge Wilson action.
  - Wilson fermions in space.
  - Timelike fermion doublers not initialized → stay unexcited.

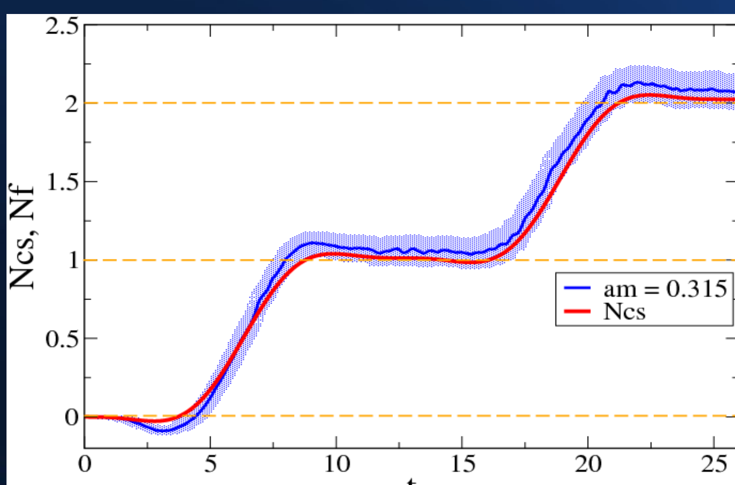
$$\gamma^\mu D_\mu \psi \to \gamma^\mu D_\mu \psi + \frac{r_\mu}{2} D_\mu D^\mu \psi$$

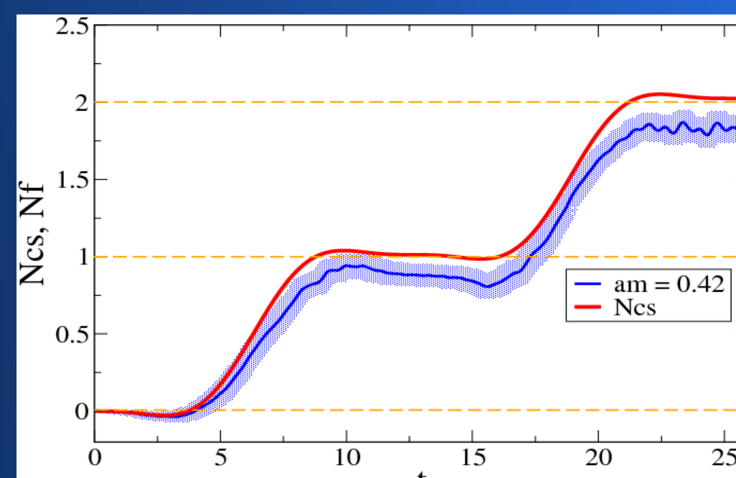
- Charge conjugation on upper fermion component and regrouping:
  - Axial → vector.
  - 2 L-H doublets + 4 R-H singlets → 1 doublet + 2 singlets
  - Fermion current (old fields) ↔ Axial current (new fields).

By-hand transitions



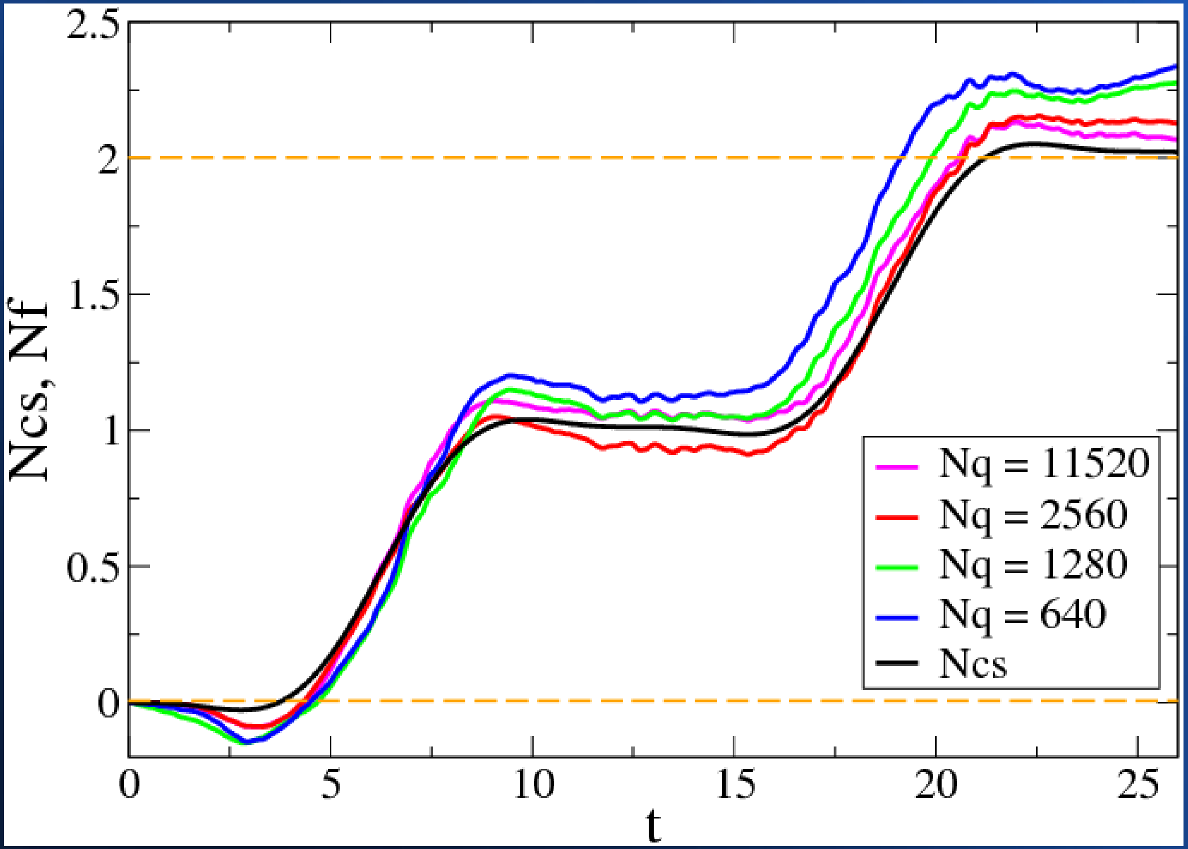






Saffin, AT: **JHEP 1202:102,2012**

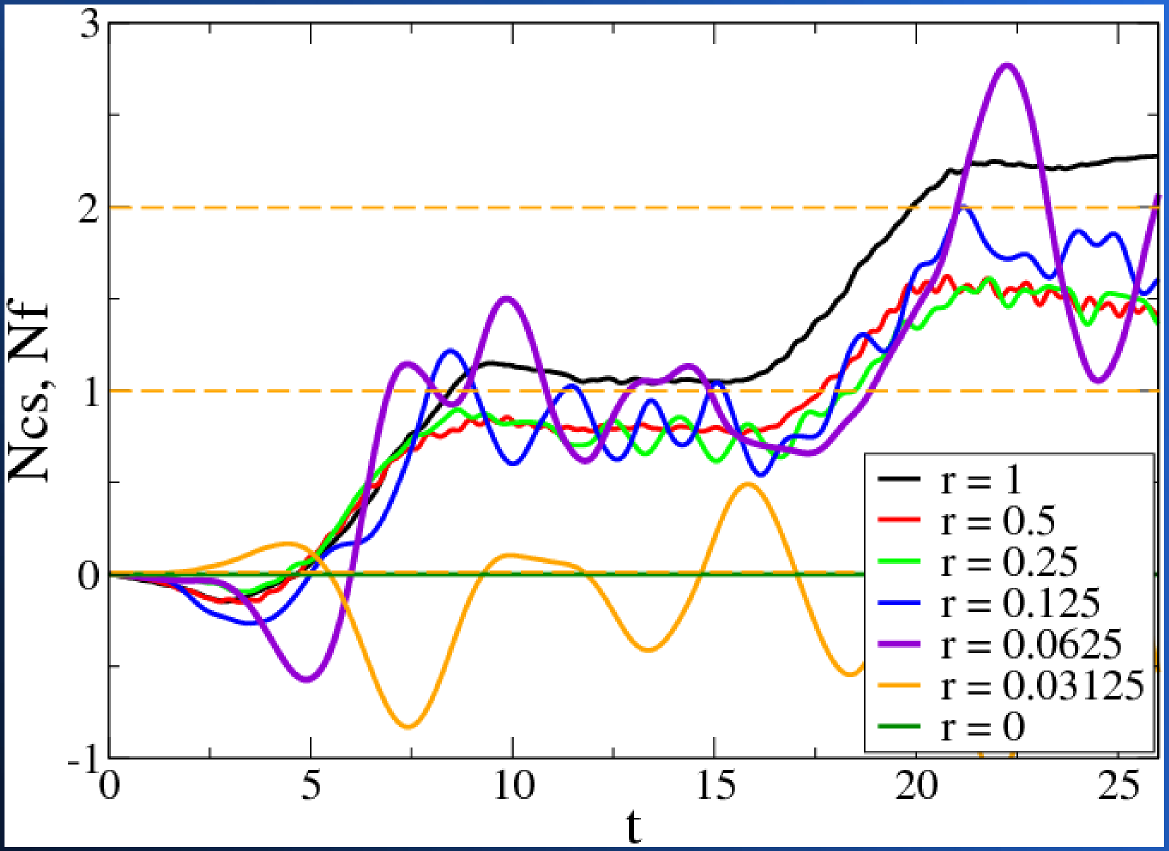
Ensemble size



Saffin, AT: **JHEP 1202:102,2012**

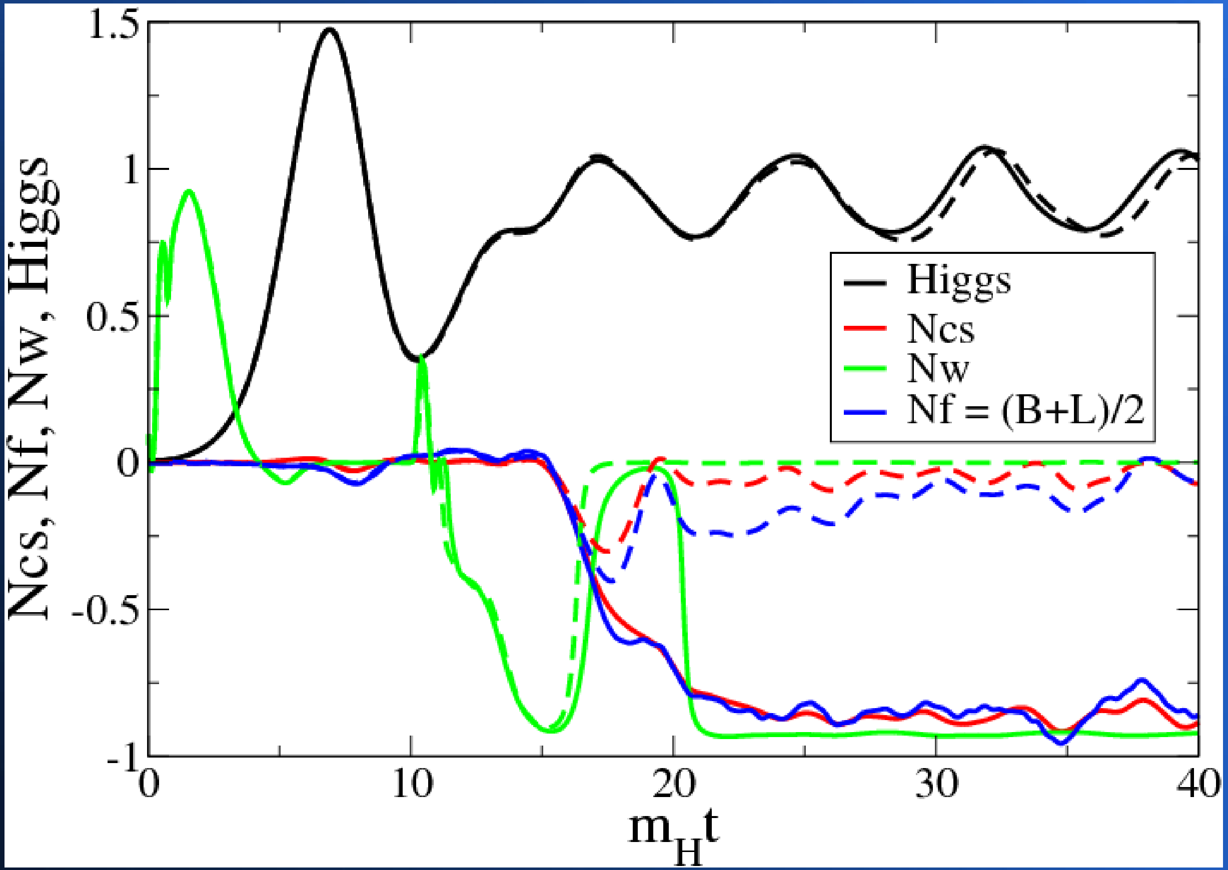
- Nq = 20, nx = 32 fits on 1-2GB memory.
- Nq = 10240 on 512 procs, running 8 hours.
- Computer intensive!
- Would like nx → 64
- Would like t → 100
- Would like Nq → 20000
- Would like 3 colours
- Would like 3 generations
- → factor 400(!)

Spatial doublers



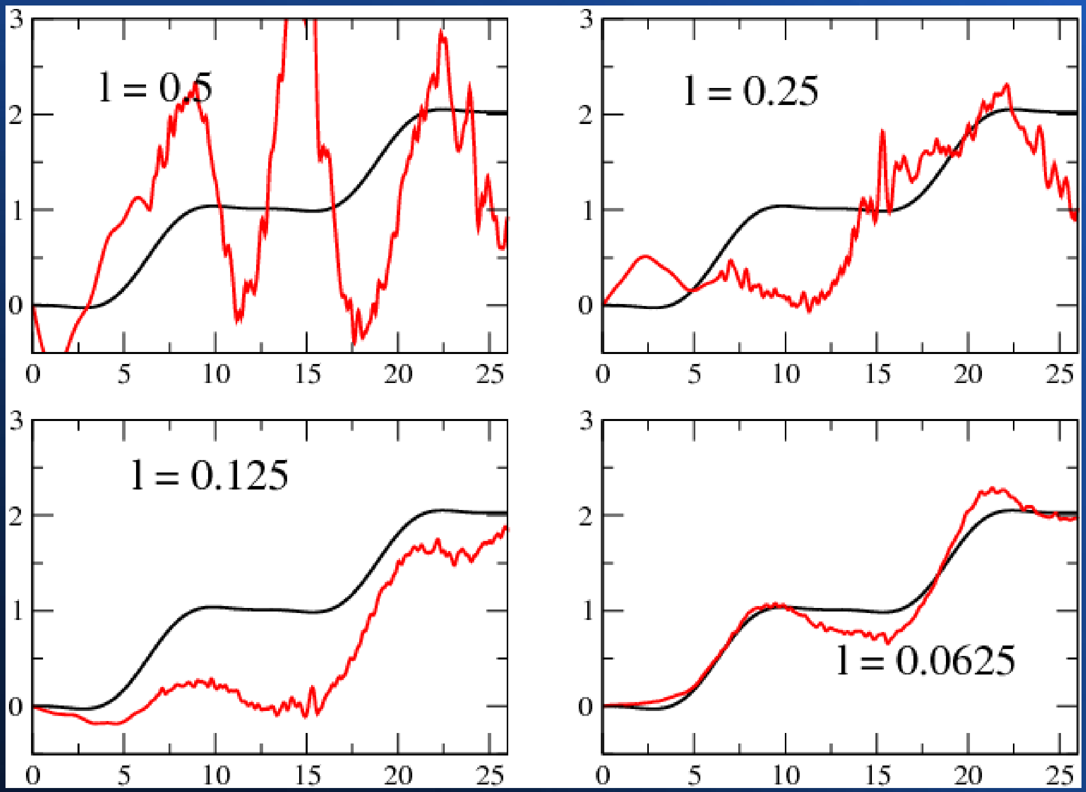
Saffin, AT: **JHEP 1202:102,2012**

Spinodal instability



Saffin, AT: **JHEP 1202:102,2012**

With Yukawa coupling



Saffin, AT: **JHEP 1202:102,2012**

- SM CP-violation is encoded in the CKM quark mixing matrix.
- We need Yukawa couplings of all the 3 generations of quarks to generate a baryon asymmetry!
- Need more statistics/smaller timestep.
- Note: top-mass = 173 GeV corresponds to l = 1.
- Almost there!

Conclusions I

- The SM cannot provide baryogenesis → no out-of-equilibrium.
- 3 main contenders based on SM anomaly: Lepto, “Hot”, “Cold”.
- 3 sources of out-of-equilibrium
  - Out-of-equilibrium decay
  - Bubble nucleation
  - Spinodal instability

Simulations of bosonized CEWBaG

  - CKM (maybe) enough if effective temperature ~1 GeV
  - Dim-6 operator works if coefficient ~10^-5
  - 2HDM with Dim-6 works if coefficient ~10^-4

Conclusions II

- 3 (sofar) proposals to trigger super-cooled, fast spinodal transition
  - Extra scalar field, which is the inflaton   vanTent, Smit, AT: 2004
  - Extra scalar field, which is not the inflaton   Enqvist, Stephens, Taanila, AT: 2010
  - First order phase transition in special potential   Konstandin, Servant: 2011
- Full dynamics with fermions:
  - Anomaly equation holds.
  - Numerically hard.
  - Large Yukawa couplings even harder.
  - In principle possible to do 3 generations with full CKM matrix. Separation of masses? Tune Yukawa couplings to maximize signal? Can it be seen on the lattice?