Anisotropic Hydrodynamics

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Gauge Field Dynamics In and Out of Equilibrium
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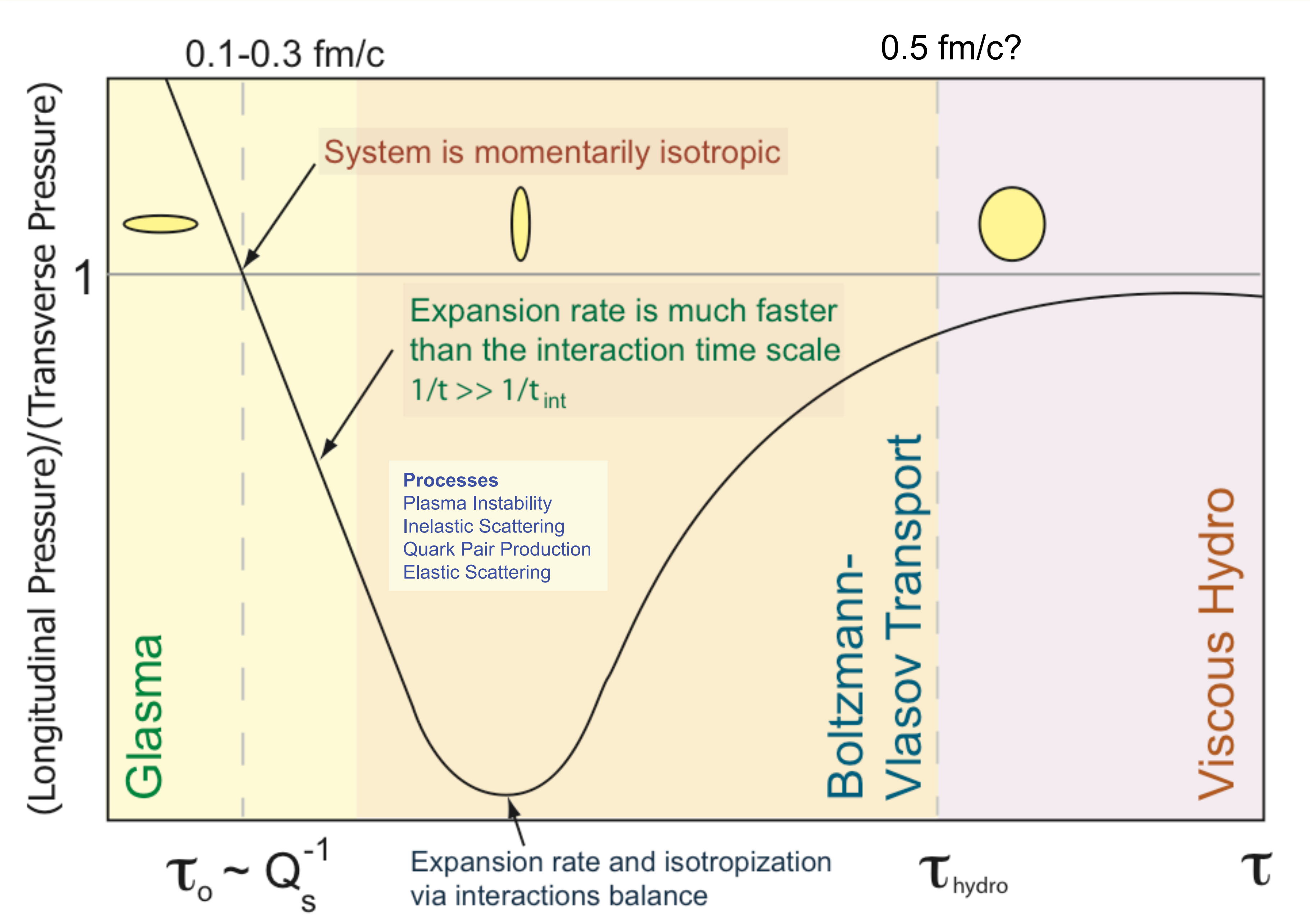
Motivation

- Cannot apply viscous hydrodynamics too early after initial nuclear impact
- Large corrections to ideal EM tensor due to rapid longitudinal expansion
- These corrections grow as η/S increases
- Also breaks down near transverse and longitudinal edges where the system is dilute ~ free streaming
- How can we improve things to make hydro-like theories more quantitatively reliable for HIC?
- In the process we may learn something about the approach to isotropy in the quark gluon plasma and improve phenomenology.

QGP momentum anisotropy



Anisotropic Plasma

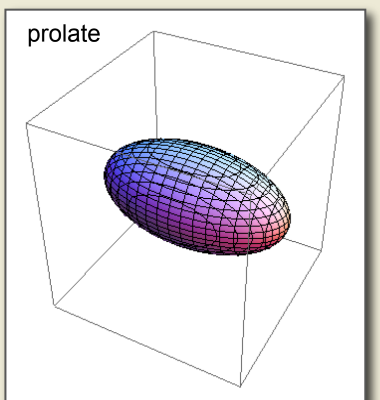
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{RS}(\mathbf{p}, \xi(\tau), p_{\rm hard}(\tau)) \\ = f_{\rm iso}([\mathbf{p}^2 + \xi(\tau) p_z^2]/p_{\rm hard}^2(\tau))$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

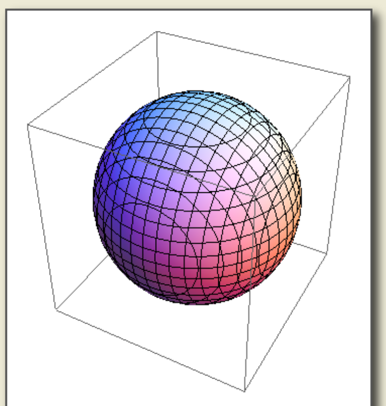
Small Anisotropy Limit (Thermal $f_{\rm iso}$)

$$f \approx f_{\rm iso}(p)\left[1 - \xi \frac{p_z^2}{2\, p_{\rm hard}\, p}\left(1 \pm f_{\rm iso}(p)\right)\right]$$

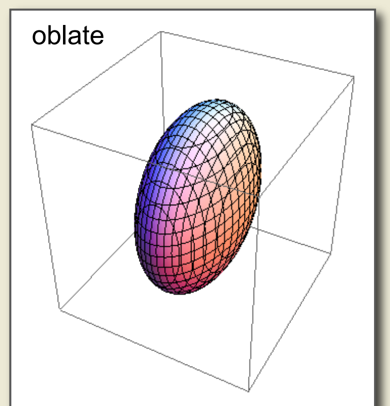
Anisotropy parameter, ξ, is related to pressure anisotropy of the system.



$-1 < \xi < 0$



$\xi = 0$

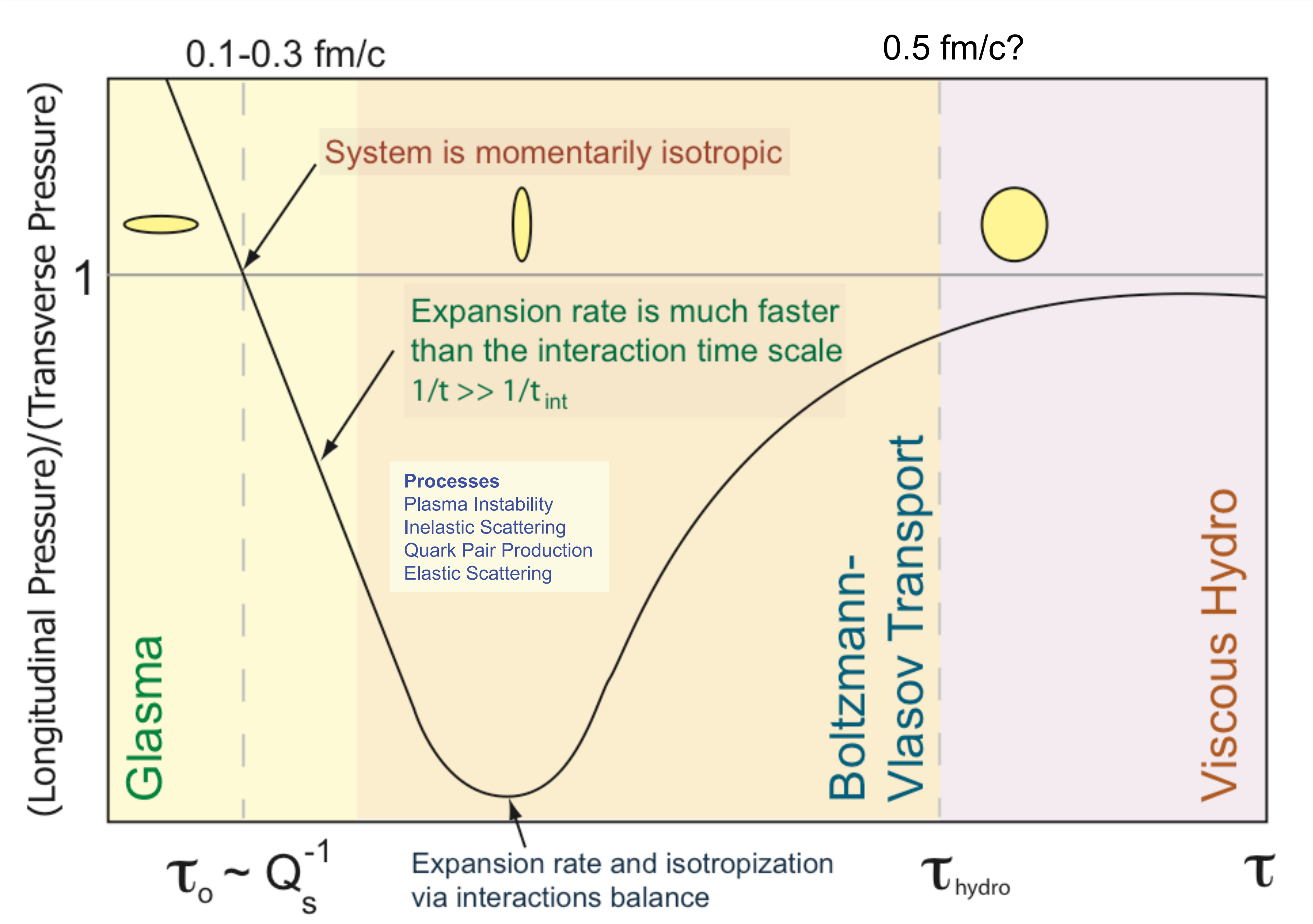


$\xi > 0$

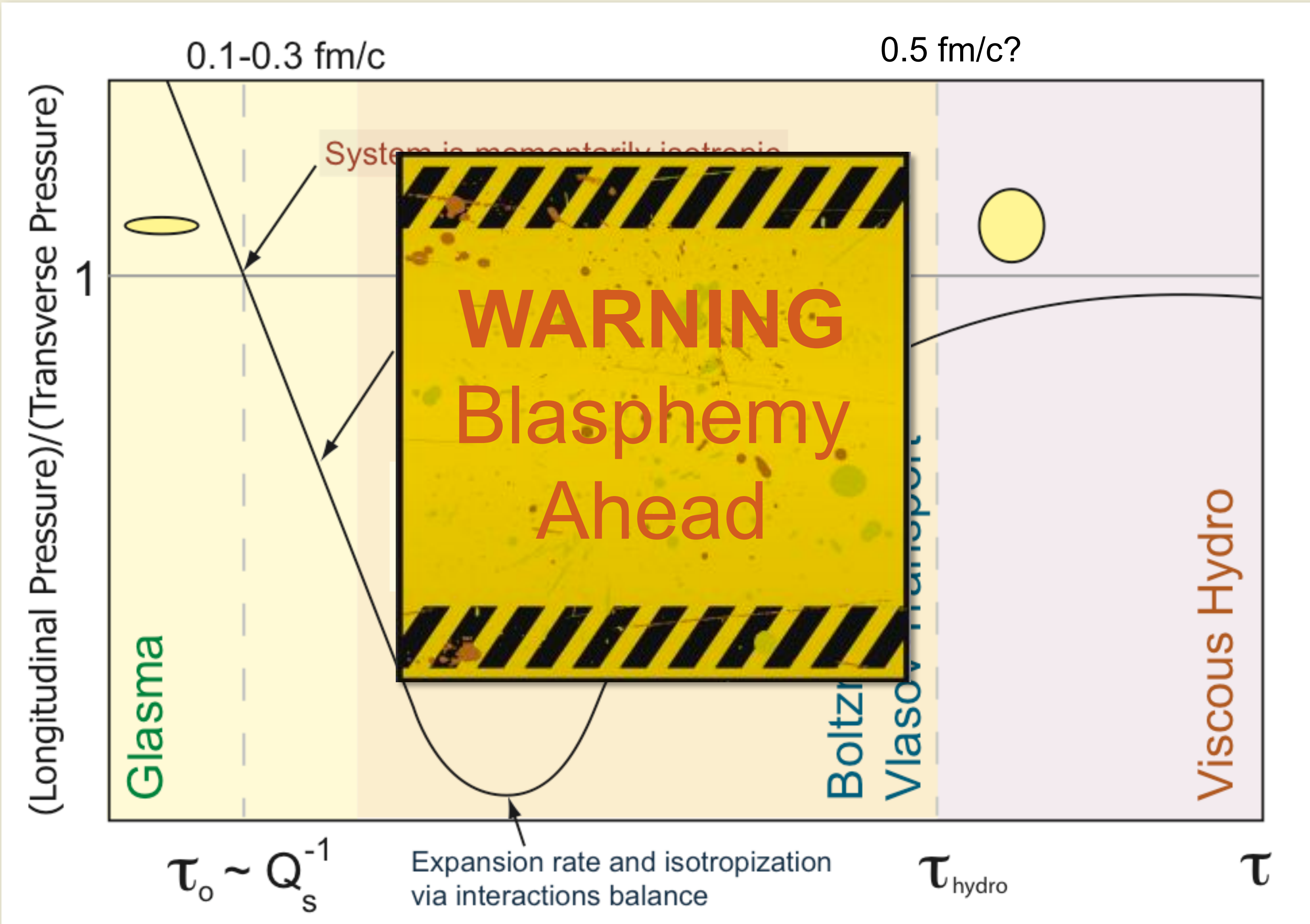
Navier-Stokes Limit

$$\xi \to \frac{10}{T\tau}\frac{\eta}{S}$$

QGP momentum anisotropy



QGP momentum anisotropy

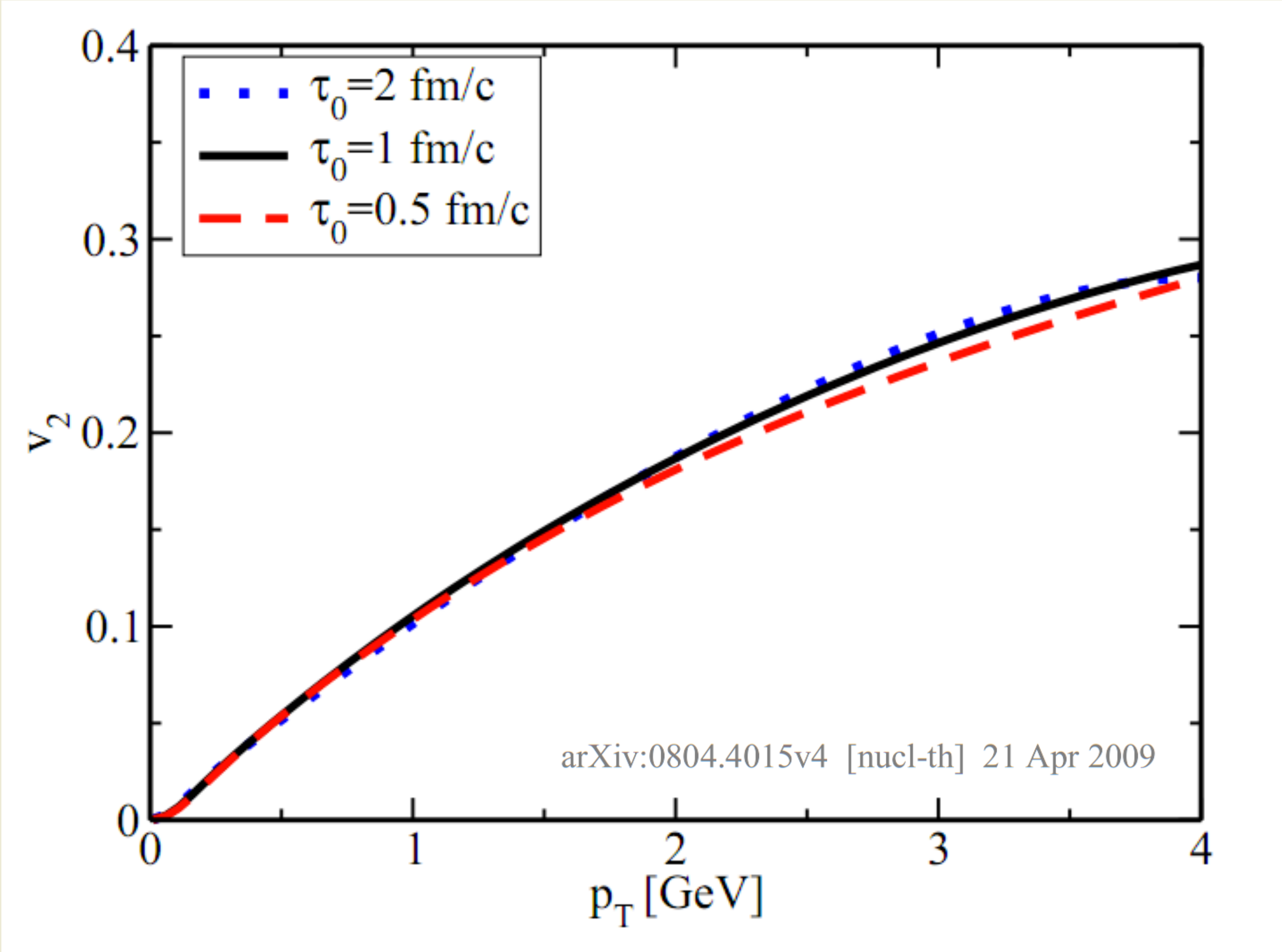


Come ye of little faith …

- It has been taken as gospel that agreement with experimental data for elliptic flow requires early thermalization/ isotropization at times on the order of 0.5 fm/c.
- Is that true within viscous hydro?
- Let’s ask some experts…

Conformal Relativistic Viscous Hydrodynamics: Applications to RHIC results at $\sqrt{s_{NN}} = 200$ GeV

Matthew Luzum[1] and Paul Romatschke[2]

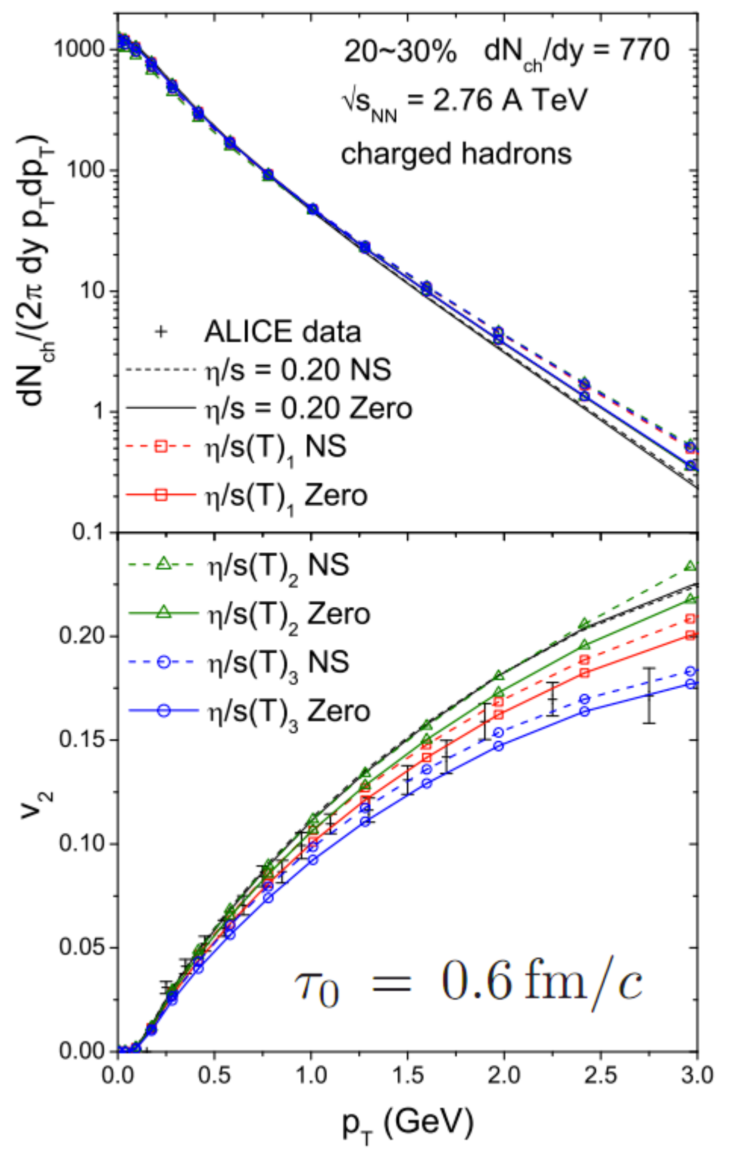


Simulation parameters were $T_i = 0.29$ GeV, $T_f = 0.14$ GeV for $\tau_0 = 2$ fm/c, $T_i = 0.36$ GeV, $T_f = 0.15$ GeV for $\tau_0 = 1$ fm/c, and $T_i = 0.43$ GeV, $T_f = 0.16$ GeV for $\tau_0 = 0.5$ fm/c.

Radial and elliptic flow in Pb+Pb collisions at the Large Hadron Collider from viscous hydrodynamics

Chun Shen,[1,*] Ulrich Heinz,[1,†] Pasi Huovinen,[2,‡] and Huichao Song[3,§]

arXiv:1105.3226v2 [nucl-th] 9 Sep 2011



η/s model	$\pi_0^{\mu\nu}$	s_0 (fm^{-3})	T_0 (MeV)
$\eta/s = 0.2$	0	191.6	427.9
	NS	172.4	413.9
$(\eta/s)_1(T)$	0	179.6	419.2
	NS	119.3	368.7
$(\eta/s)_2(T)$	0	179.6	419.2
	NS	115.6	365.1
$(\eta/s)_3(T)$	0	175.2	416.0
	NS	116.6	366.1

NS = Navier Stokes

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1 \qquad \xi_{\rm NS} = \frac{10}{T\tau}\frac{\eta}{\mathcal{S}}$$

Using values from the paper I obtain

$$\xi_{0,\rm NS} \simeq 1.3$$

Which corresponds to

$$\mathcal{P}_L/\mathcal{P}_T \simeq 0.51$$

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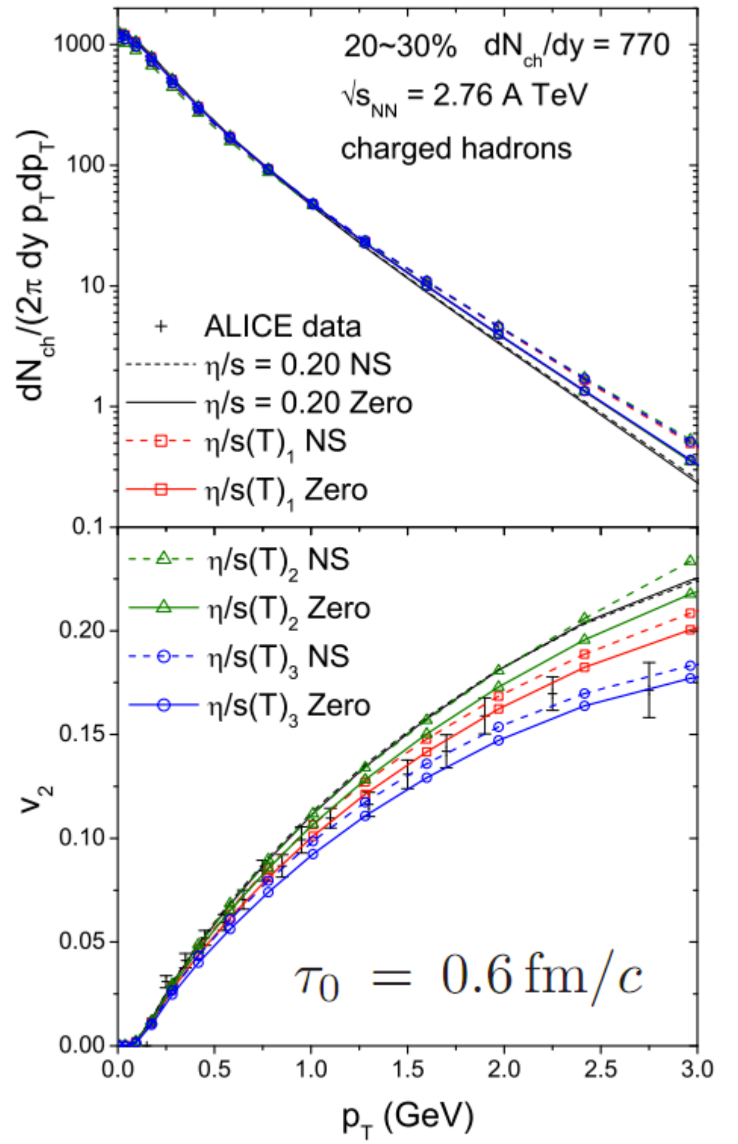

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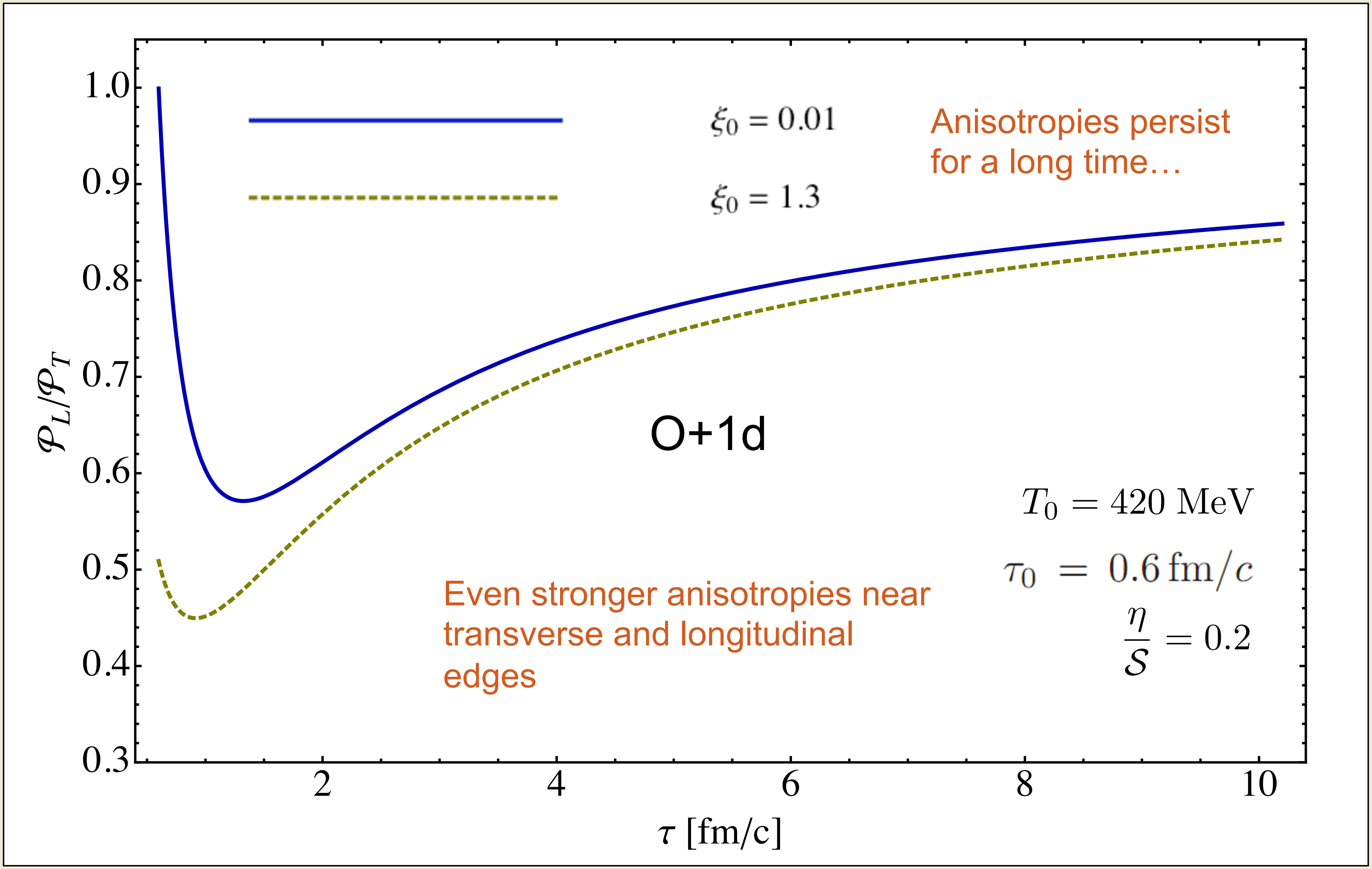
NS = Navier Stokes

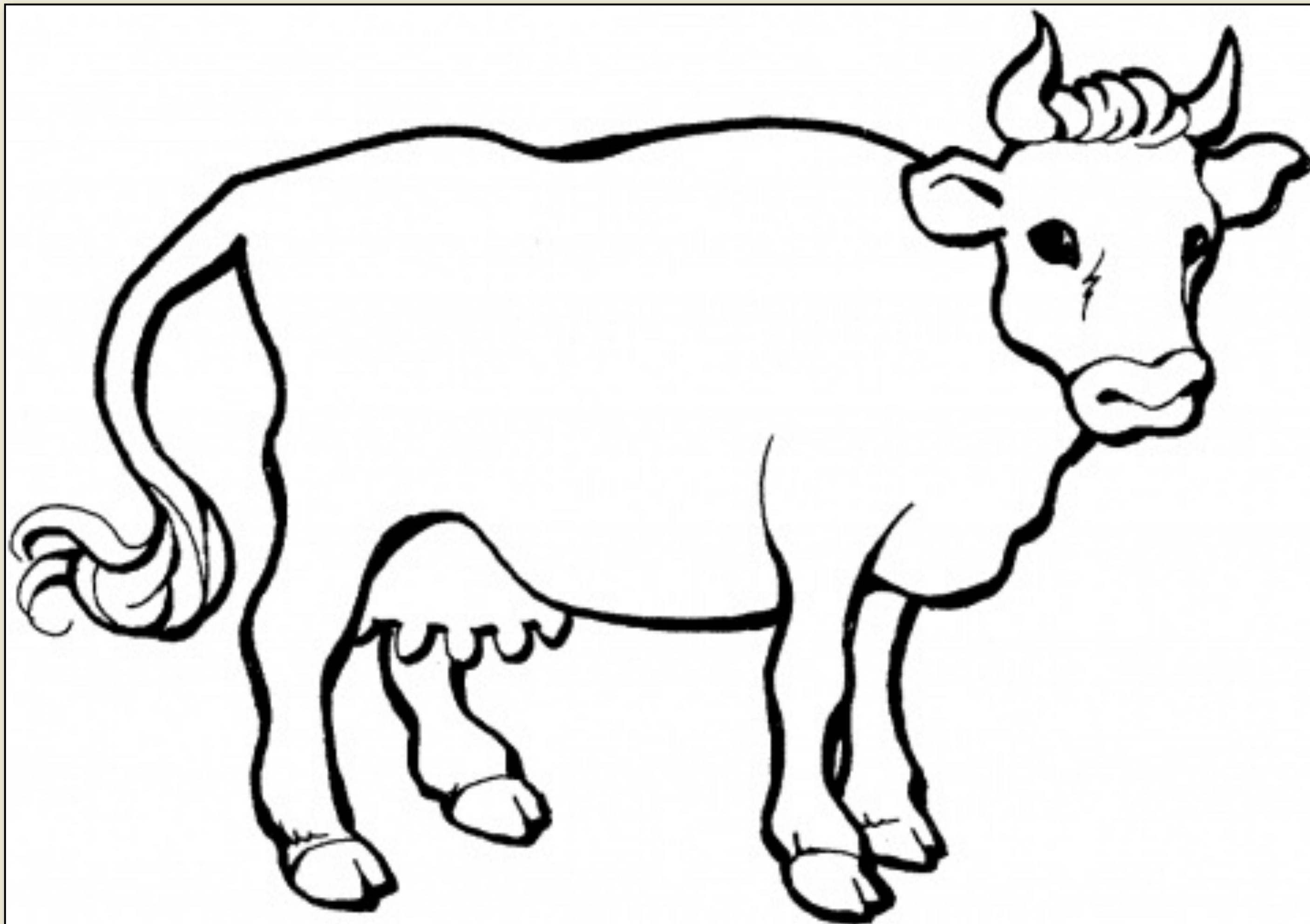
Viscous Hydro situation is even worse than this would make it seem. For Navier-Stokes at the initial time shown, the longitudinal pressure is negative

Preview - Pressure Anisotropy as a Function of Time

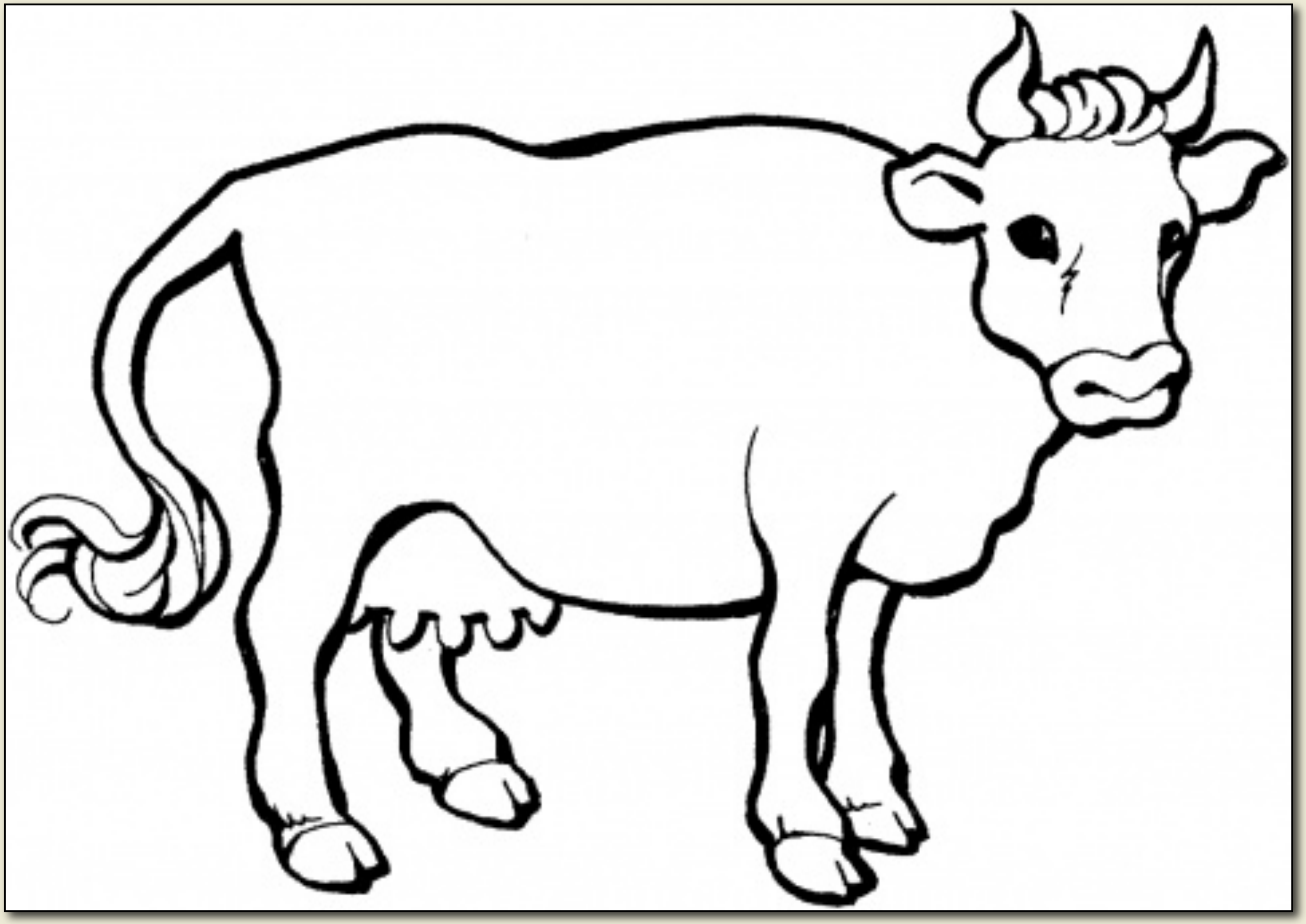


--- Physics 101 ---

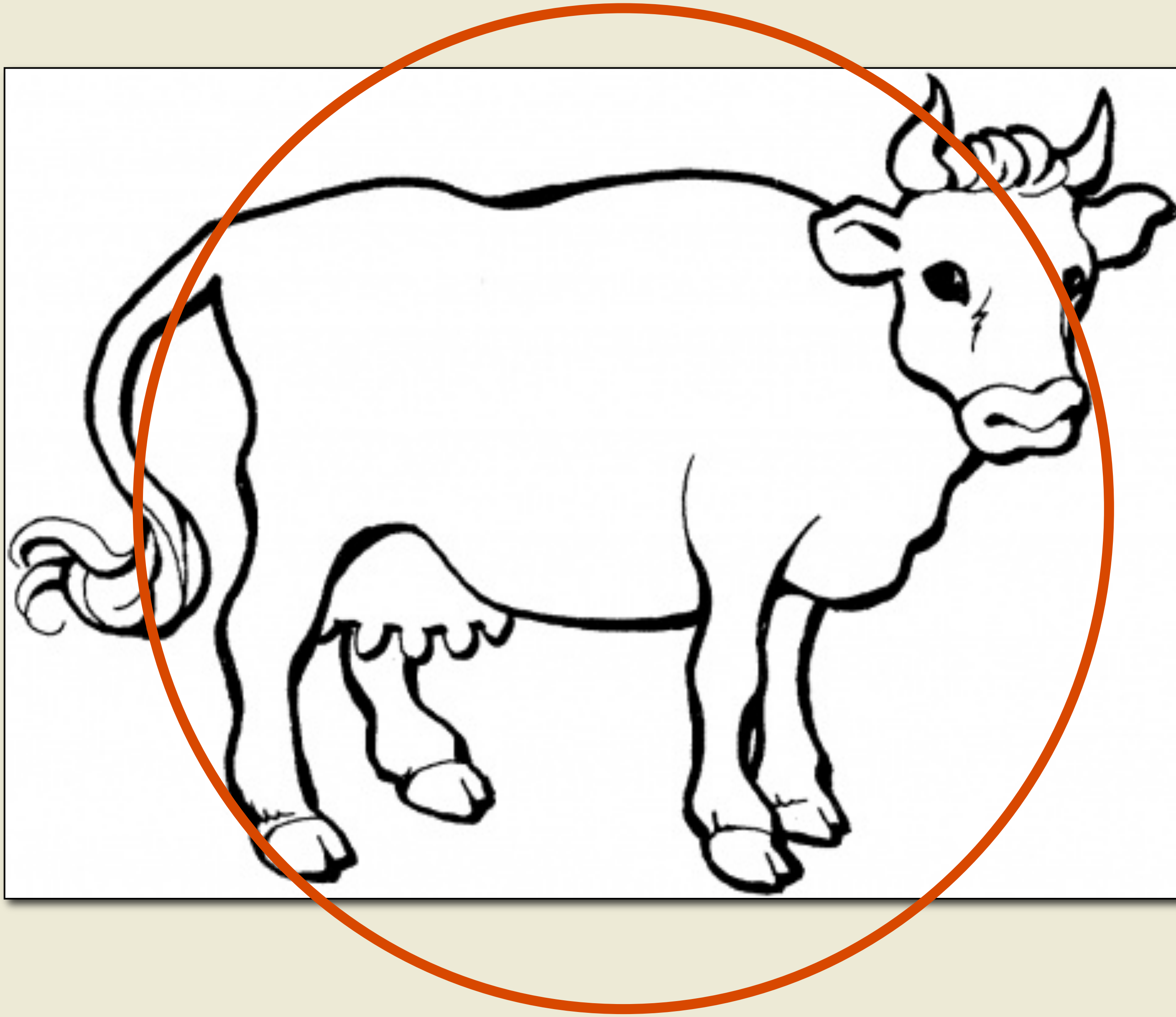
Consider a cow…



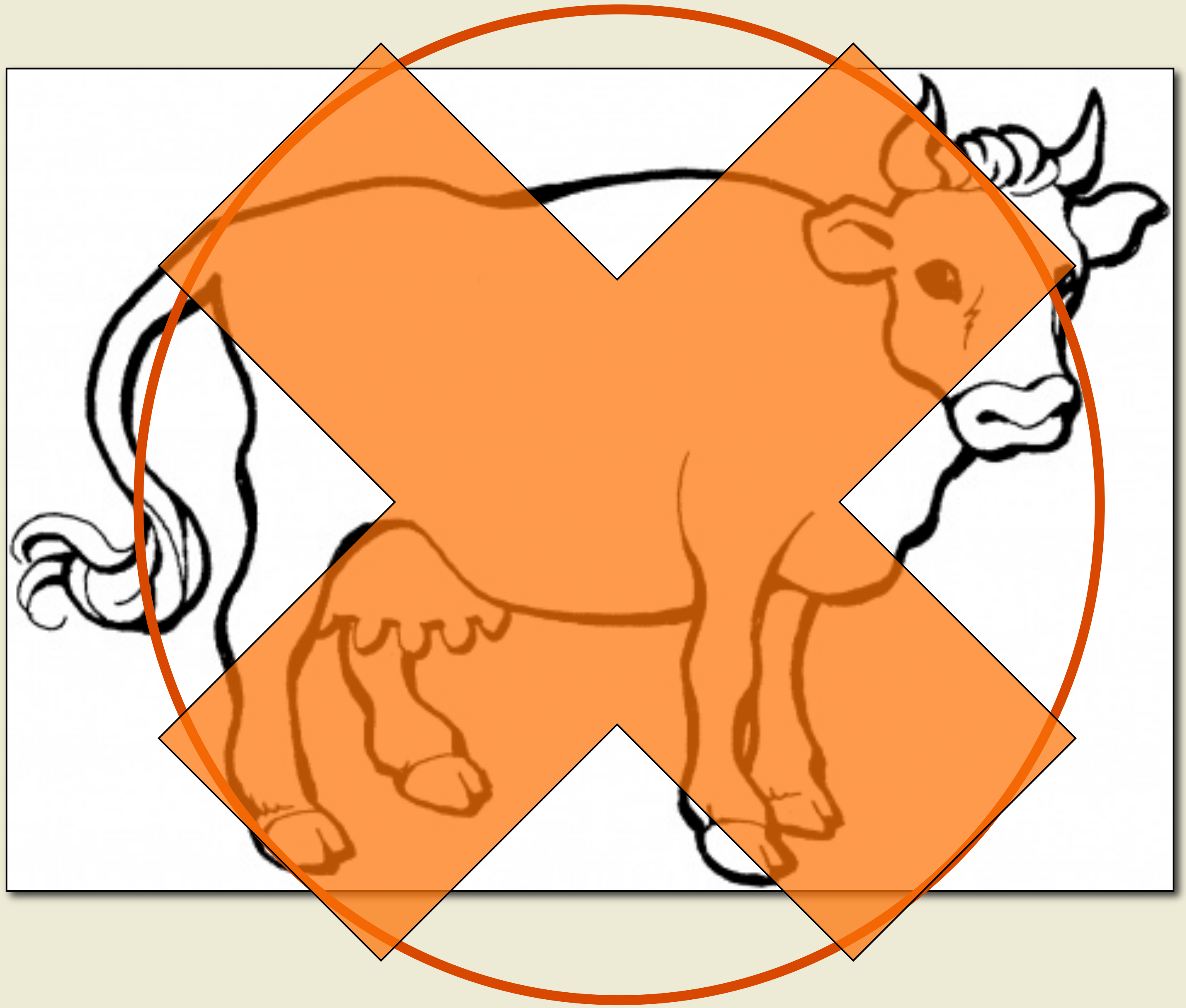
Cows are spheres



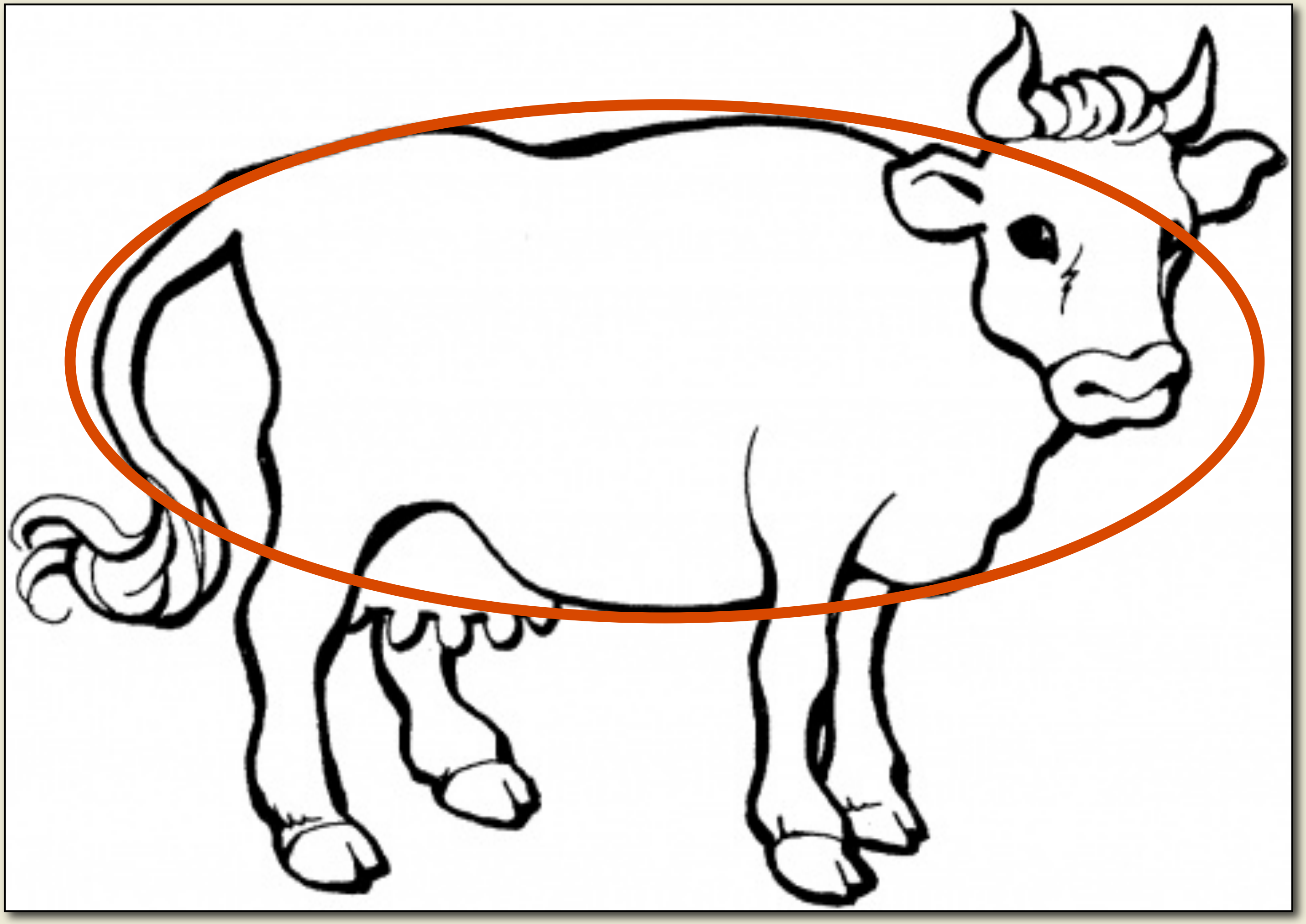
Cows are spheres?



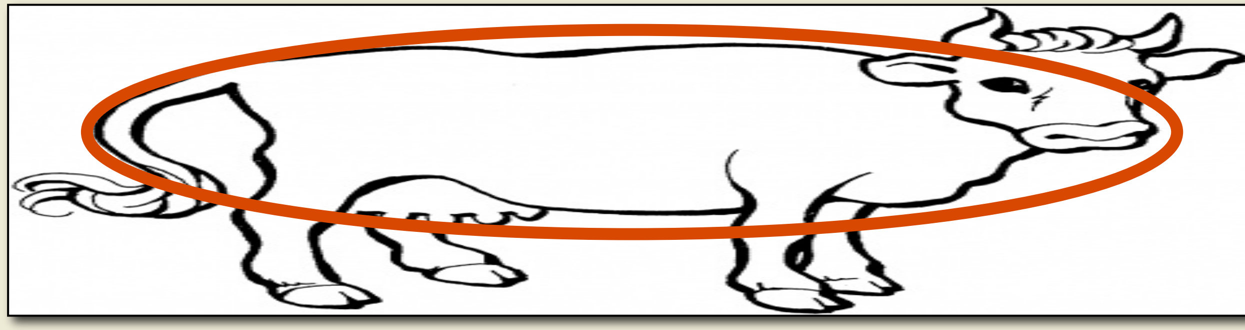
Cows are not spheres



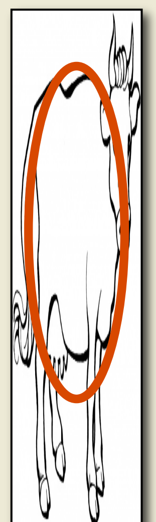
Cows are more like ellipsoids!



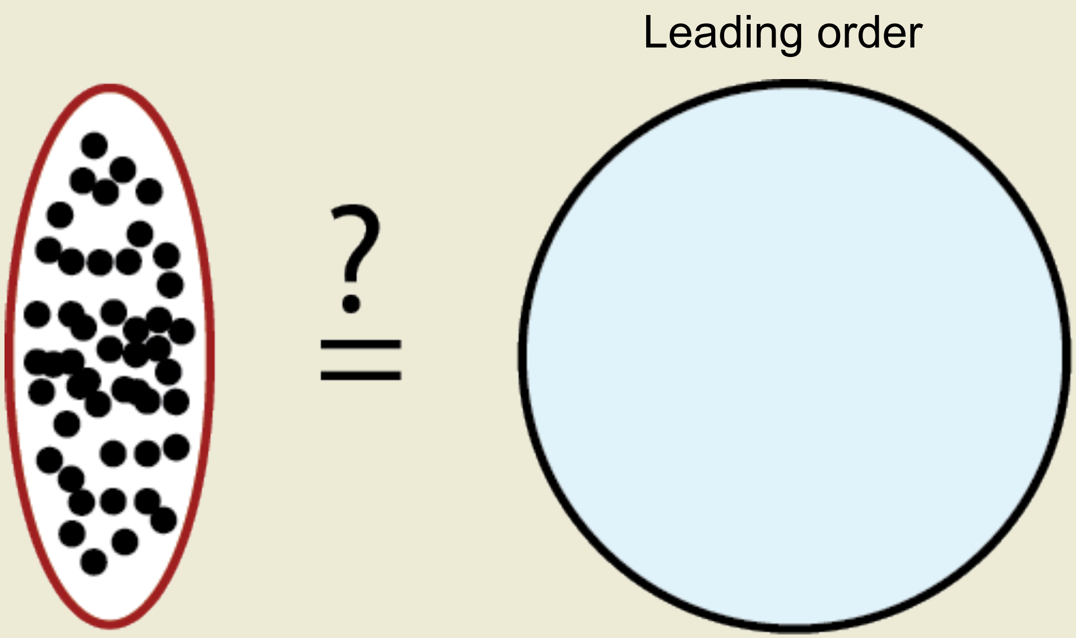
Especially very short cows…



or very tall cows…



Why is this Seussian parable relevant?



Viscous hydro says that we should approximate our particle momentum-space distribution to first order by a sphere. However, if the system is highly anisotropic in momentum space, this will result in large corrections…

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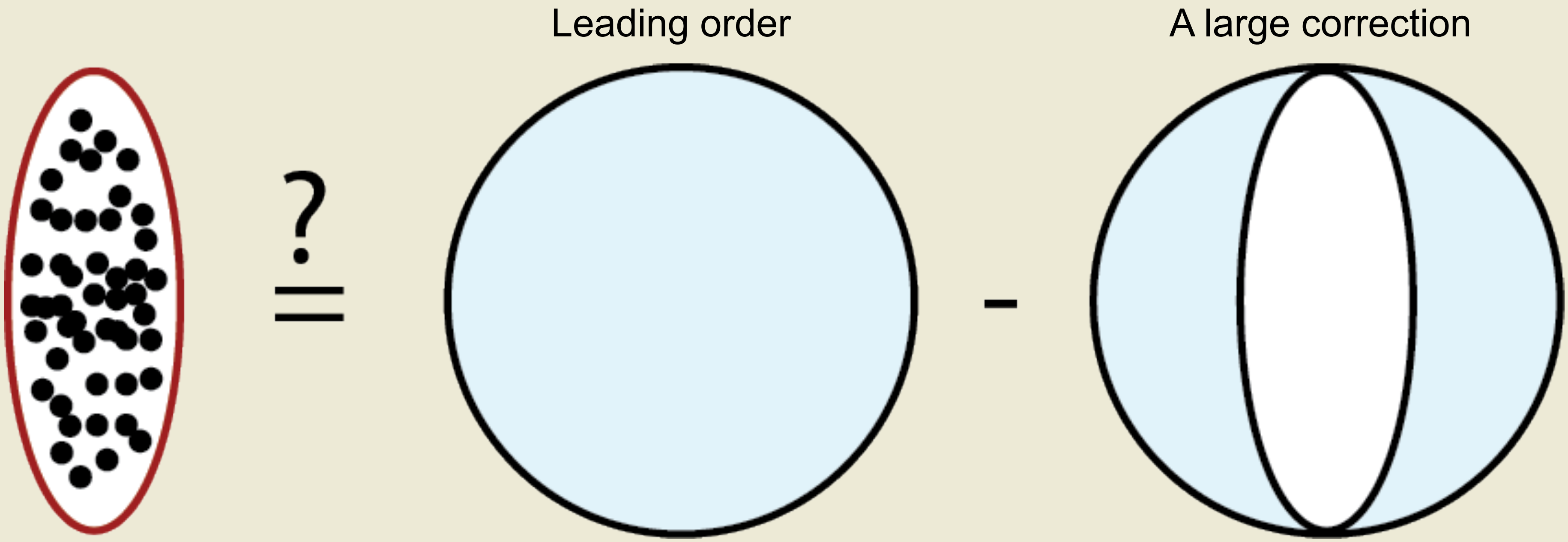


Viscous hydro says that we should approximate our particle momentum-space distribution to first order by a sphere. However, if the system is highly anisotropic in momentum space, this will result in large corrections…

--- Hydro From Transport ---

Near Equilibrium QGP Evolution

- If the system is close to equilibrium and has pressures in the local rest frame which are approximately isotropic ($P_T \cong P_L$) then we might try to use relativistic viscous hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \Pi^{\mu\nu}$$

- The ideal stress tensor is thermal and isotropic
- Large amplitudes of the shear tensor compared to the ideal stress tensor indicate a problem with the hydrodynamic expansion itself

Relativistic Hydro from Transport

• Describe evolution of the system using the Boltzmann equation

$$p^\alpha \partial_\alpha f = -C[f]$$

C[f] = Collisional Kernel

• Can extract hydro equations from the Boltzmann equation by taking "moments" of the equation using an integral operator

$$\hat{I}^{\mu\nu\cdots\sigma} \equiv \int \frac{d^3p}{2E}\, p^\mu p^\nu \cdots p^\sigma$$

eg.

$$\hat{I} \equiv \int \frac{d^3p}{2E}$$

0^{th} moment operator

$$\hat{I}^\mu \equiv \int \frac{d^3p}{2E}\, p^\mu$$

1^{st} moment operator

0^{th} Moment

N^α : Particle Number and Current

$$\partial_\alpha N^\alpha = -\int \frac{d^3p}{E} C[f]$$

if number conserving collisional kernel →

$$\partial_\alpha N^\alpha = 0$$

Number conservation

If particle number changing processes in kernel, eg 2 → 3, RHS is nonzero

1^{st} Moment

$T^{\alpha\beta}$: Energy-Momentum Tensor

$$\partial_\alpha T^{\alpha\beta} = -\int \frac{d^3p}{E} p^\beta C[f]$$

if energy conserving collisional kernel →

$$\partial_\alpha T^{\alpha\beta} = 0$$

Energy-momentum conservation!

2^{nd} Order Viscous Hydro

- The first two moments are enough to generate equations of motion for ideal hydrodynamics.

- In number conserving theories the second moment gives the first non-trivial (dissipative) equation of motion and can be used to derive 2^{nd}-order viscous hydro using transport theory.

- If the system is homogeneous in the transverse directions, the energy-momentum tensor in the local rest frame has the following form

$$T^{\alpha\beta} = \begin{pmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P}_T & 0 & 0 \\ 0 & 0 & \mathcal{P}_T & 0 \\ 0 & 0 & 0 & \mathcal{P}_L \end{pmatrix}$$

$\mathcal{E}$: Energy Density

$\mathcal{P}_T$: Transverse Pressure

$\mathcal{P}_L$: Longitudinal Pressure

Boost Invariant 1d Hydro

- Consider a boost invariant system that is homogeneous in the transverse directions.
- Expand the energy momentum tensor to first order around an isotropic state → 1d second order viscous hydro.
- The 1d second order viscous hydro equations can be written in terms of the isotropic energy density/pressure and the rapidity-rapidity (ς-ς) component of the shear tensor $\Pi = \Pi^{\varsigma}{}_{\varsigma}$

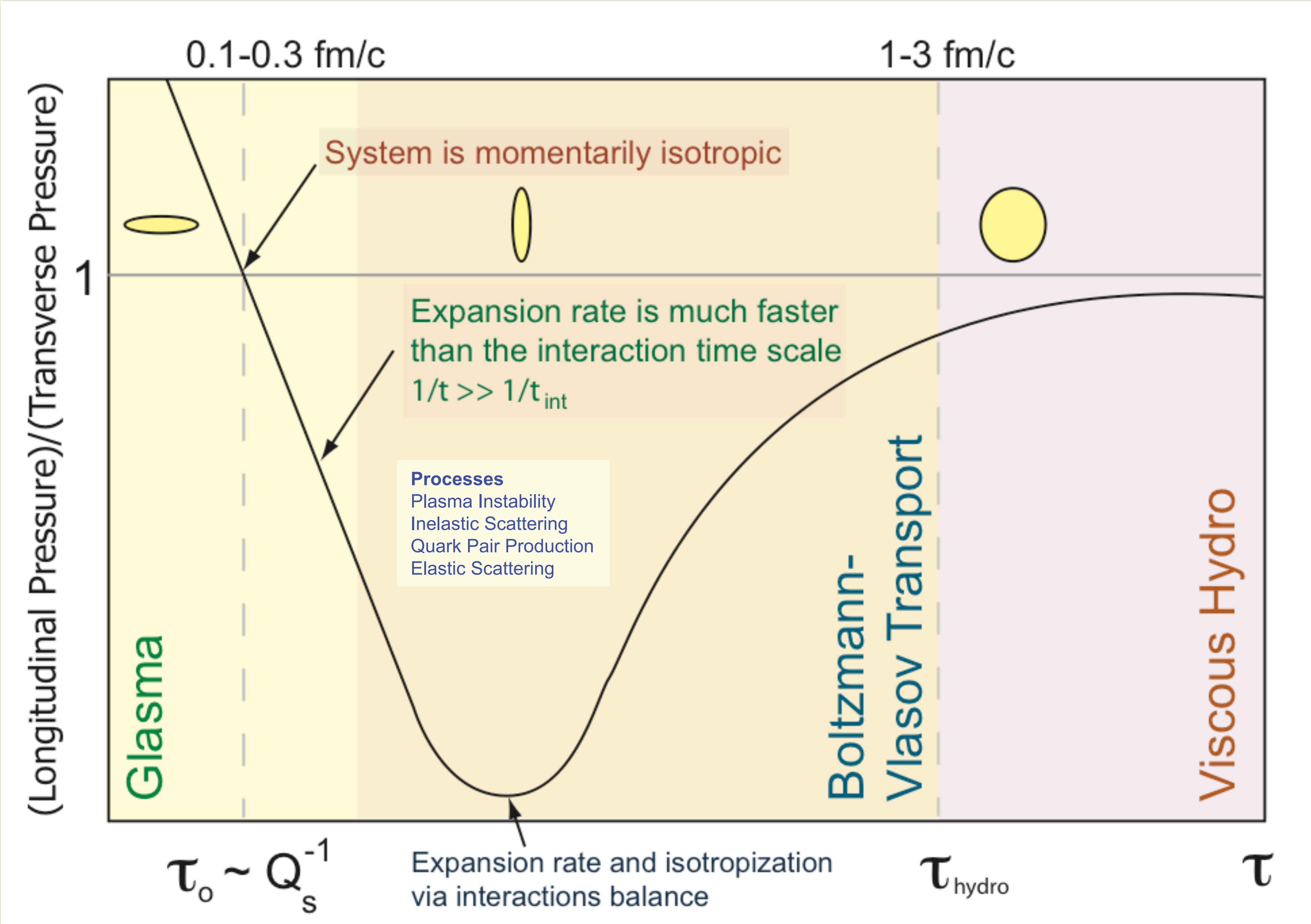
$$\partial_\tau \mathcal{E} = -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau}$$

$$\partial_\tau \Pi = -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau}$$

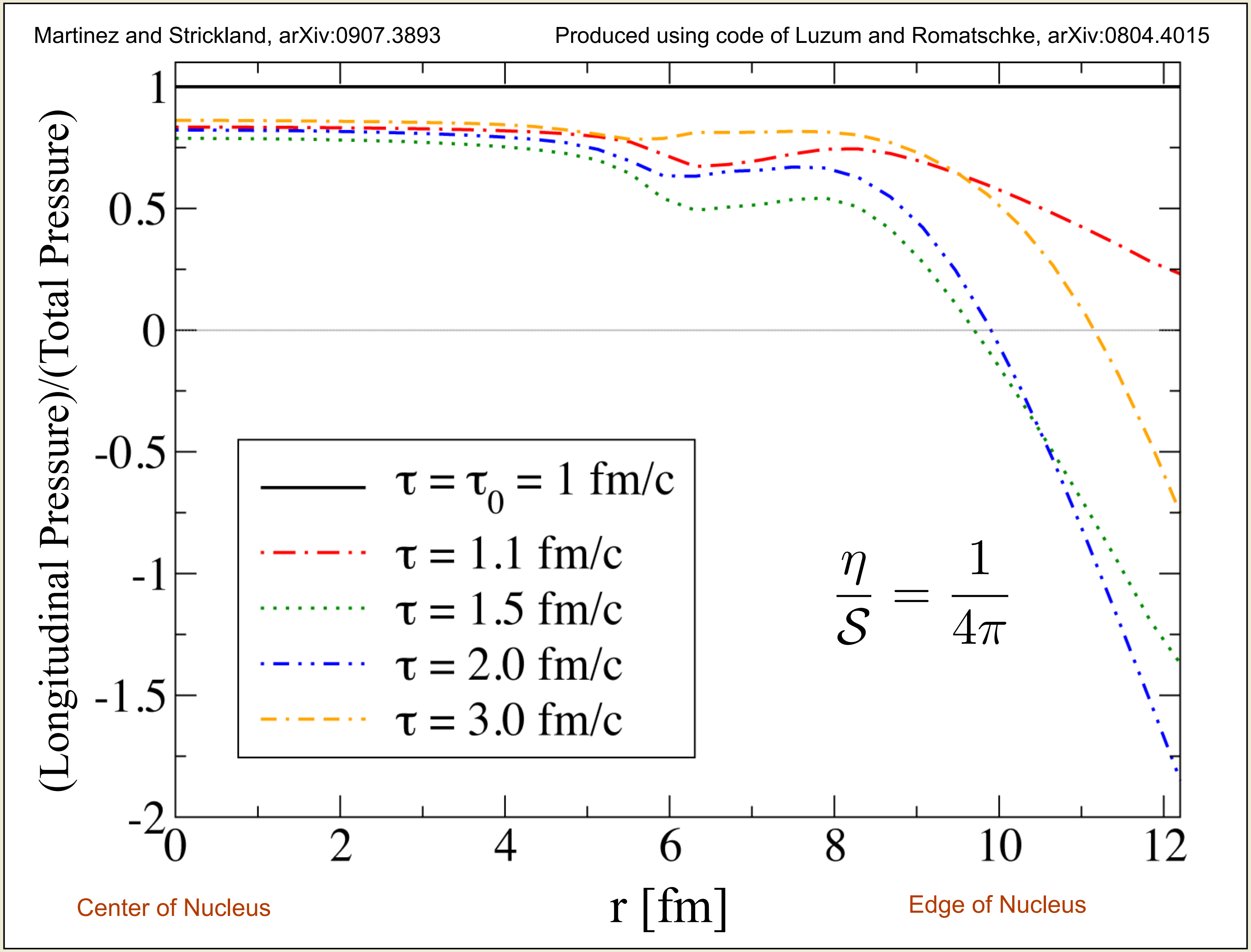
$\eta =$ Shear viscosity

$\tau_\pi =$ Shear relaxation time

QGP momentum anisotropy



Hydro Results - Strong Coupling



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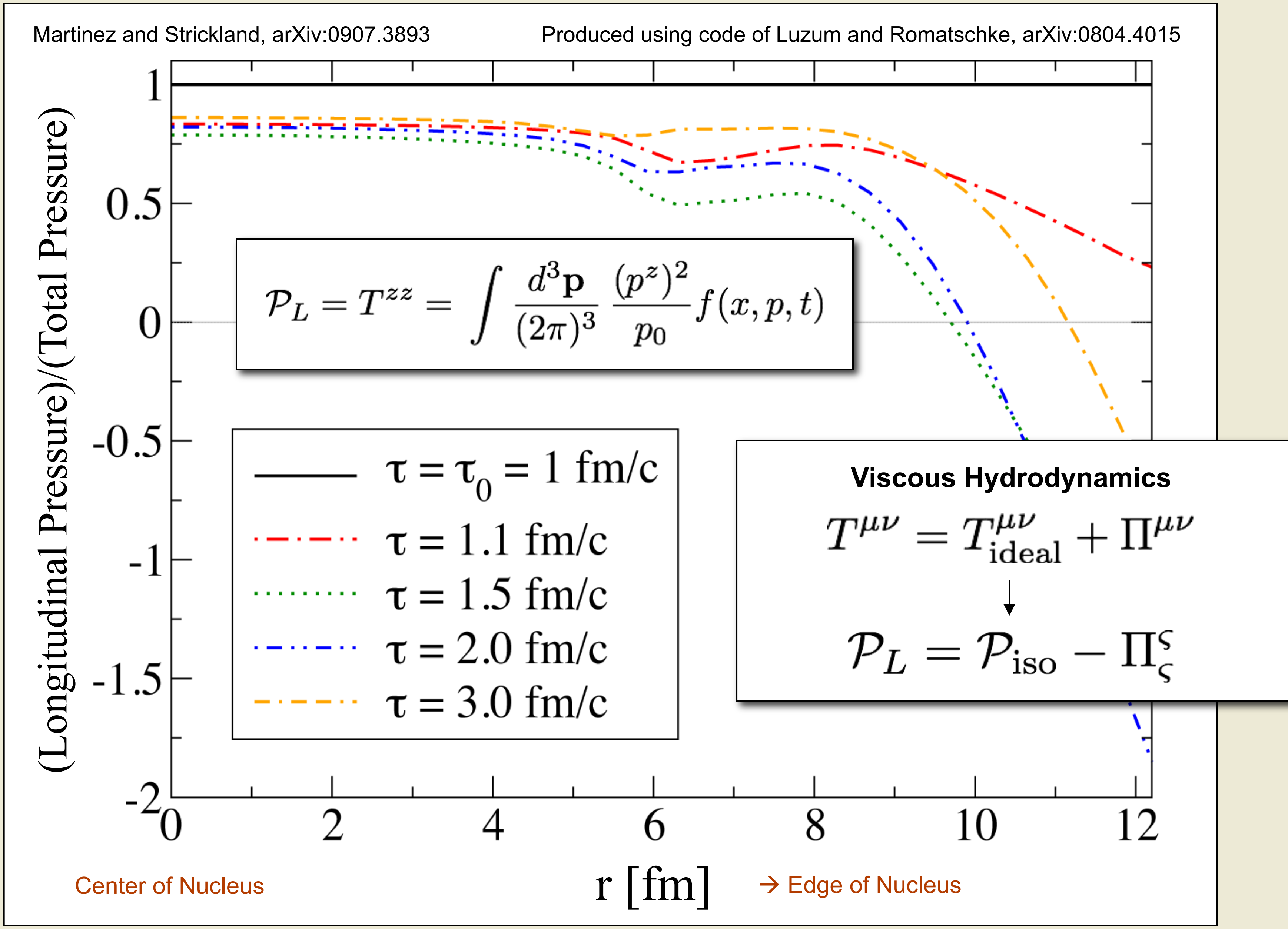
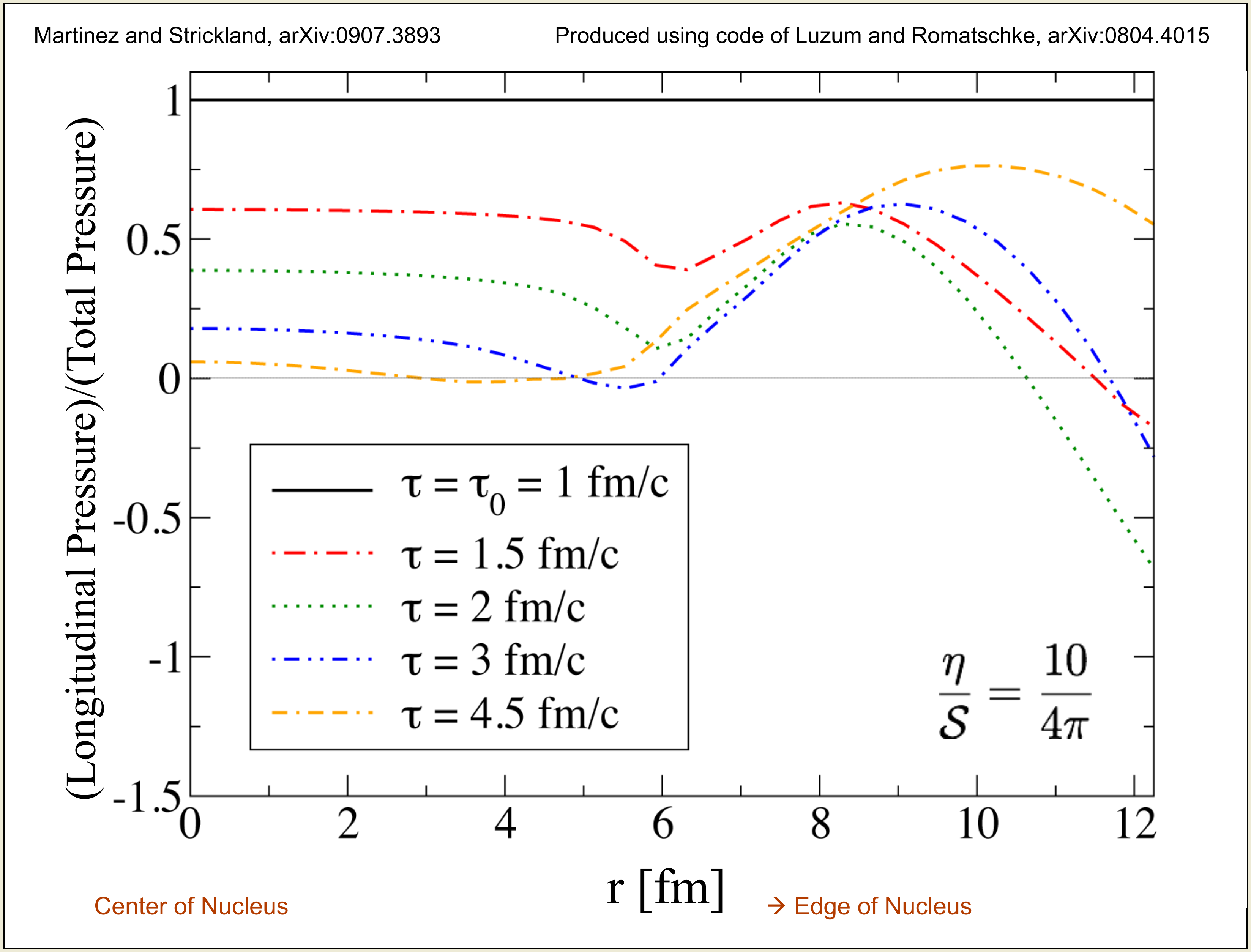


Hydro Results - Weak Coupling



Start over from scratch

Viscous Hydrodynamics Expansion

$$f(\mathbf{x},\mathbf{p},\tau) = \underline{f_{\rm eq}(|\mathbf{p}|, T(\tau))} + \delta f_1 + \delta f_2 + \cdots$$

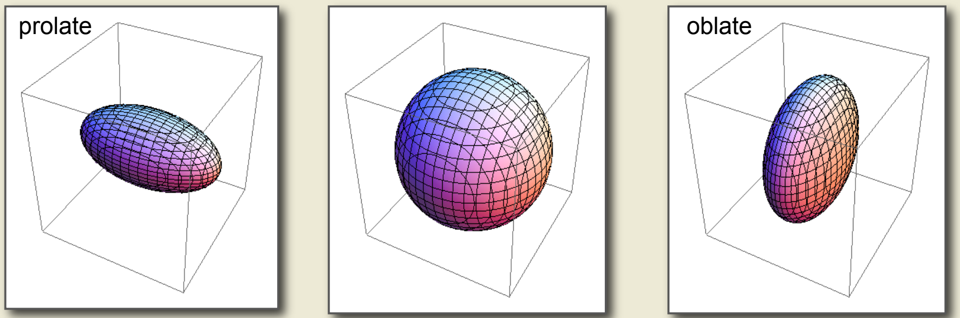
Isotropic in momentum space

Anisotropic Hydrodynamics (AHYDRO) Expansion

$$f(\mathbf{x},\mathbf{p},\tau) = f_{\rm aniso}(\mathbf{p}, p_{\rm hard}(\tau), \xi(\tau)) + \delta f_1' + \delta f_2' + \cdots$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

$$\begin{aligned} f(\tau,\mathbf{x},\mathbf{p}) &= f_{RS}(\mathbf{p}, \xi(\tau), p_{\rm hard}(\tau)) \\ &= f_{\rm iso}\left([\mathbf{p^2} + \xi(\tau) p_z^2]/p_{\rm hard}^2(\tau)\right) \end{aligned}$$



$-1 < \xi < 0$ $\xi = 0$ $\xi > 0$

Collisional Kernel

$$p^\alpha \partial_\alpha f = -C[f]$$

- Relaxation time approximation

$$\mathcal{C}[f(t,z,\mathbf{p})] = p_\mu u^\mu \, \Gamma \; [f(t,z,\mathbf{p}) - f_{\rm eq}(t,z,|\mathbf{p}|,T(\tau))]$$

- Where Γ is the relaxation rate
- Γ will be fixed by matching to 2nd order viscous hydro in the weak anisotropy limit
- $T(\tau)$ is the self-consistent isotropic temperature which can be fixed by requiring energy conservation at all proper times [Baym '84]

Using relaxation-time approximation scattering kernel gives

0th Moment of Boltzmann EQ

$$\partial_\alpha N^\alpha \neq 0$$

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

1st Moment of Boltzmann EQ

$$\partial_\alpha T^{\alpha\beta} = 0$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

where

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

Linearized Equations

If we expand and keep only the lowest non-vanishing order in the anisotropy parameter we find

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

and the coupled nonlinear differential equations reduce to

$$\partial_\tau \mathcal{E} = -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau}$$

$$\partial_\tau \Pi = -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau}$$

$$\Gamma = \frac{2}{\tau_\pi}$$

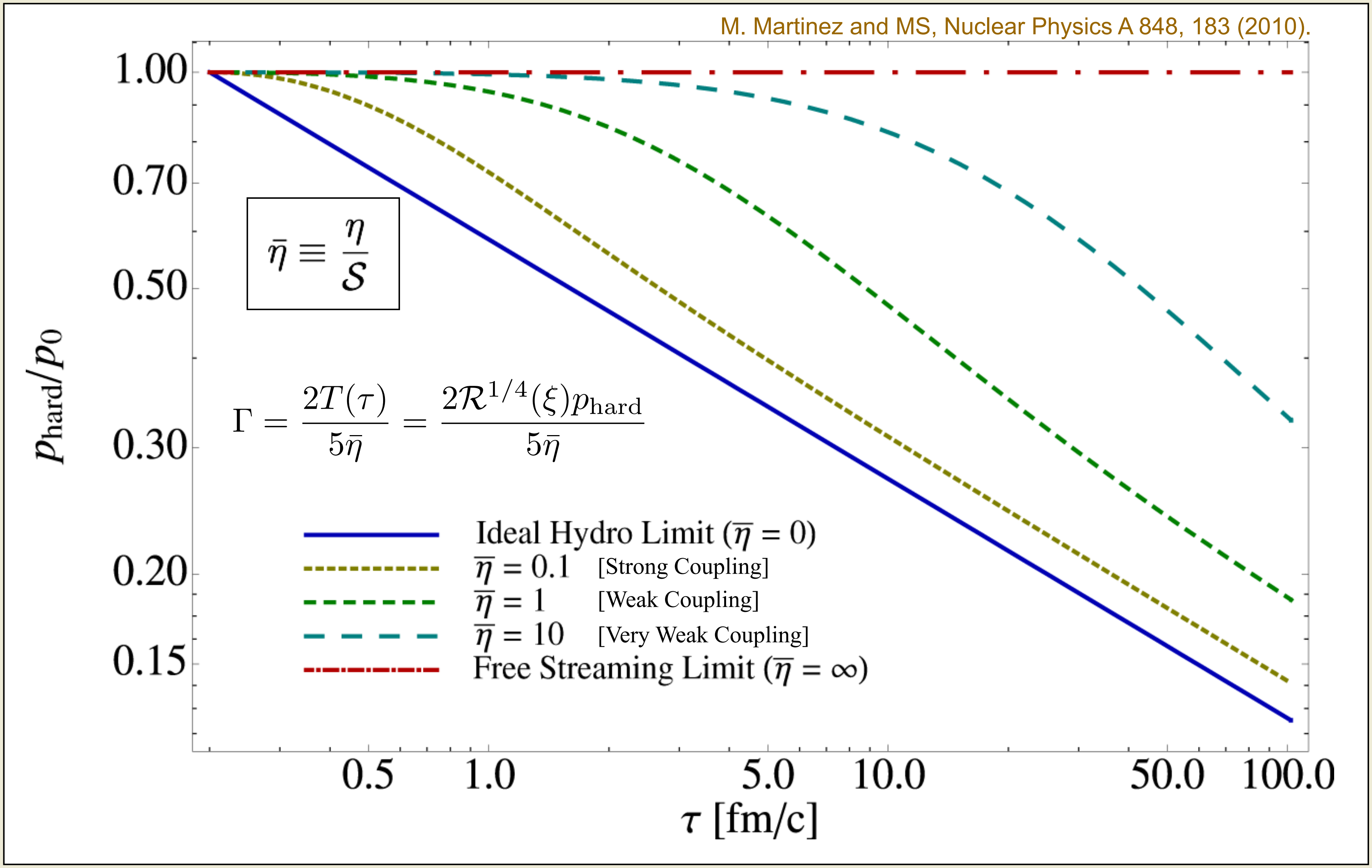
$$\tau_\pi = \frac{5}{4}\frac{\eta}{\mathcal{P}}$$

Reproduces 2nd order viscous hydro in small anisotropy limit!

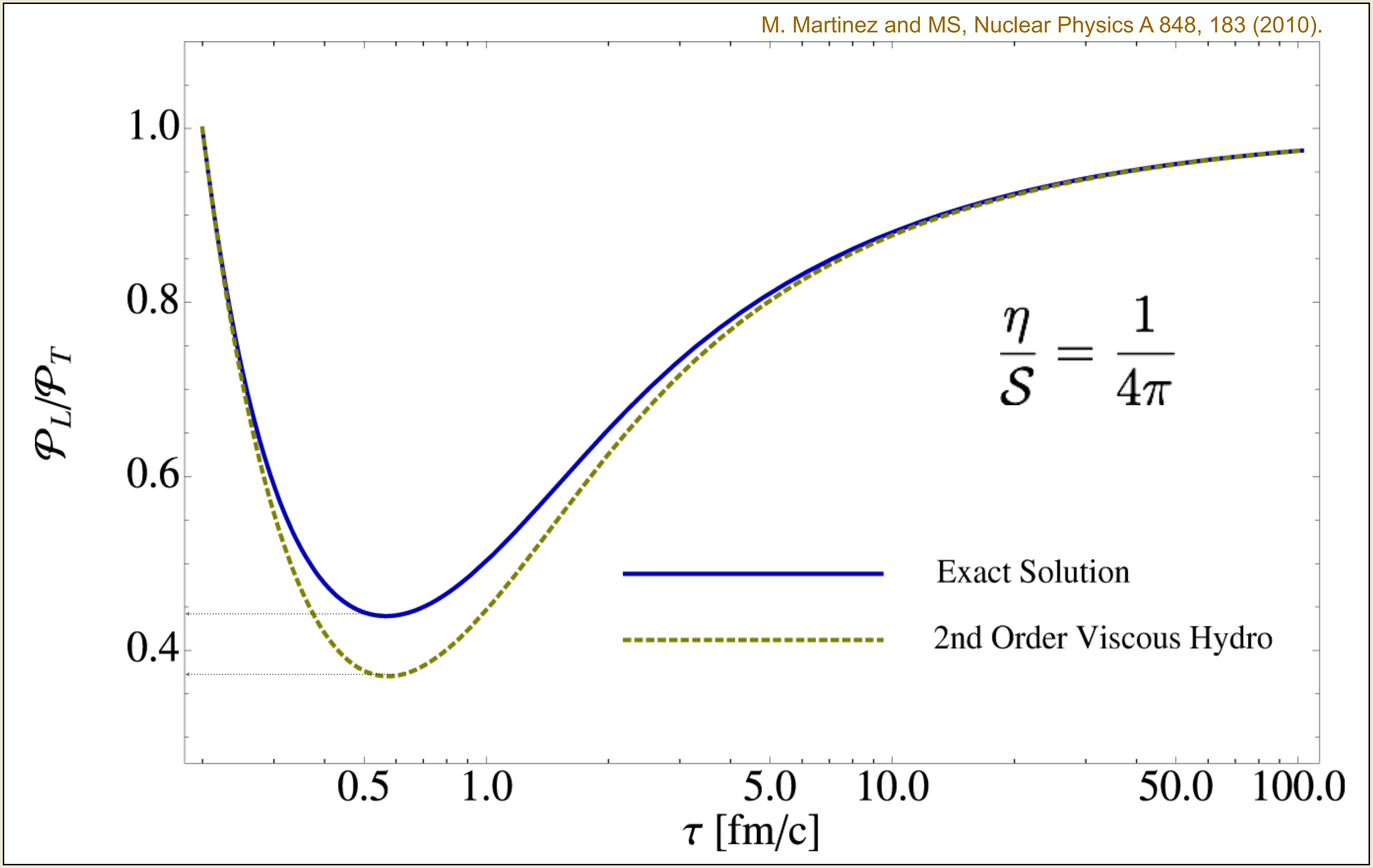
Hard Momentum vs Time

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

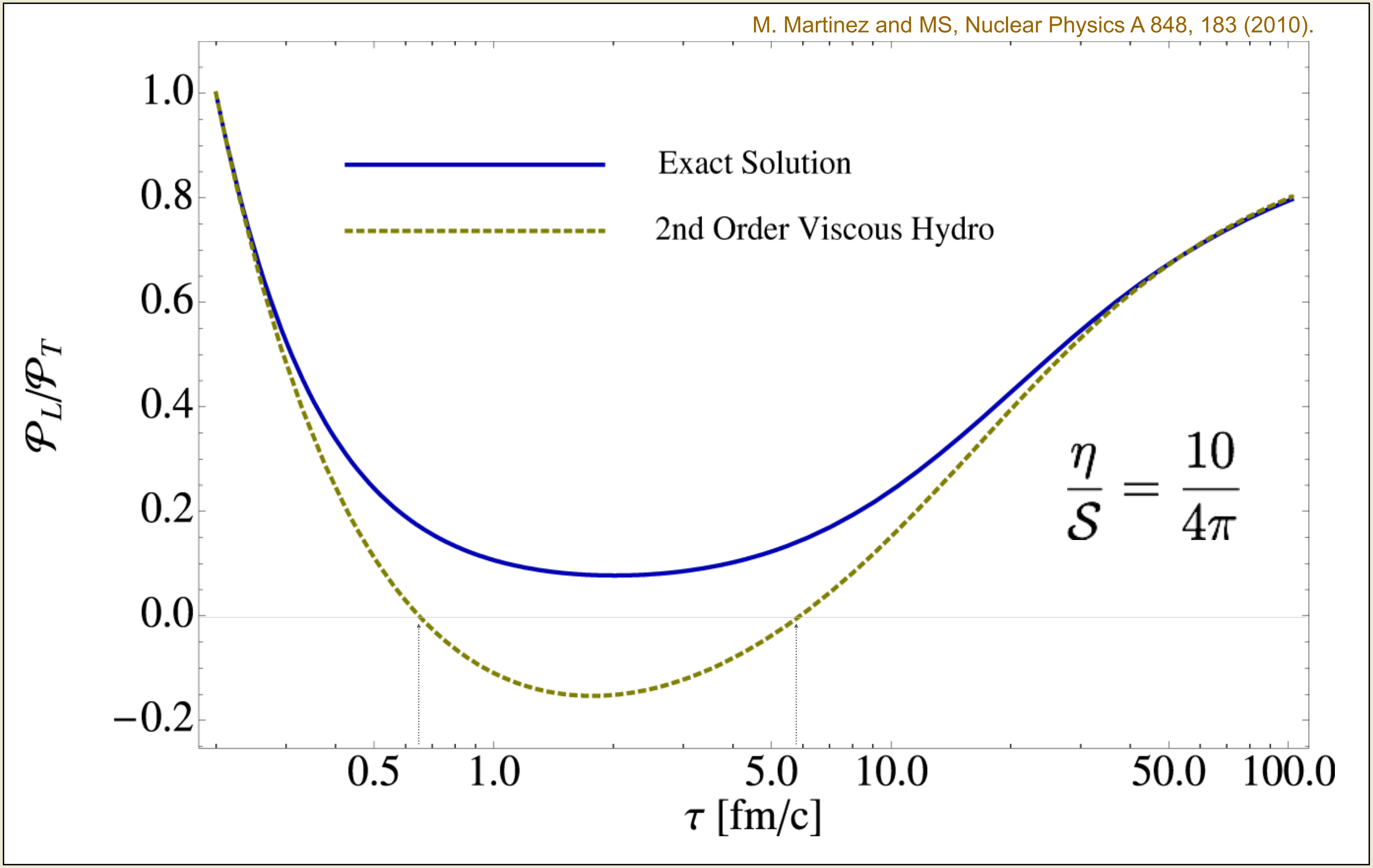
$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$



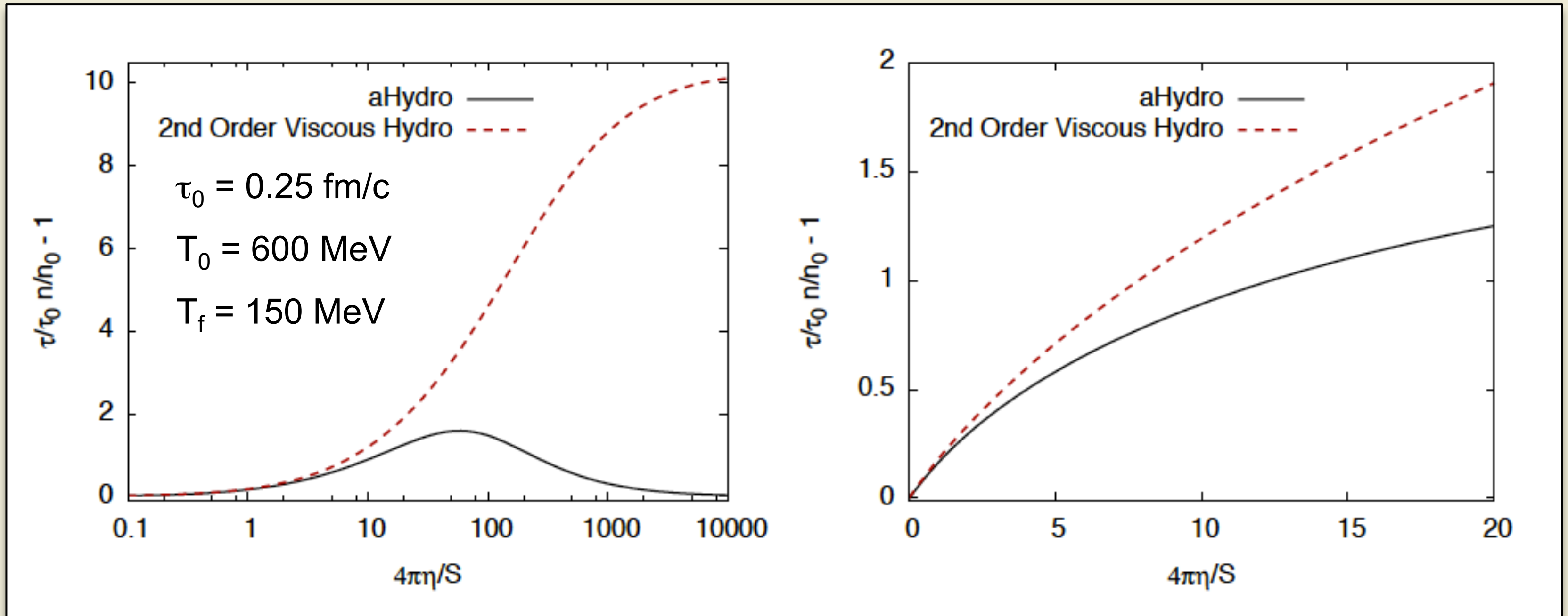
Hydro vs AD : Strong Coupling

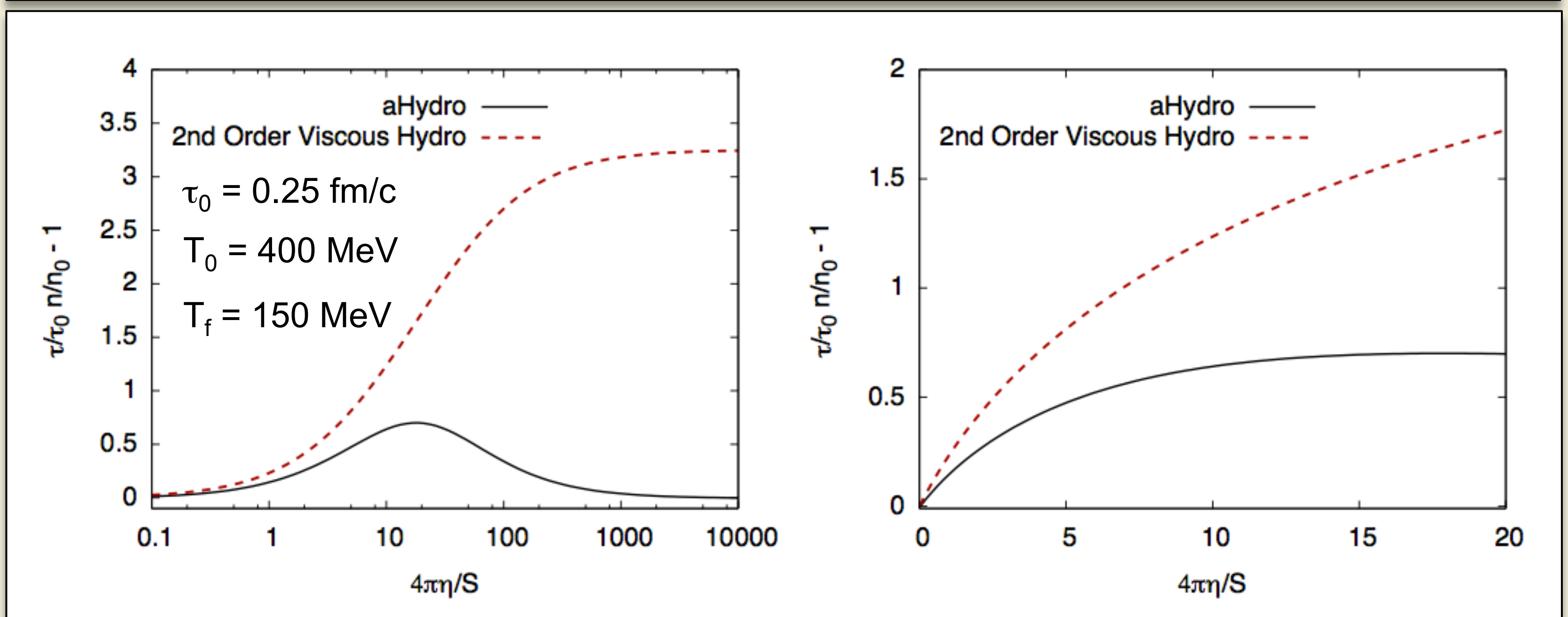


Hydro vs AD : Weak Coupling



Entropy Production





N=4 SUSY using AdS/CFT

- In 0+1 case there are now numerical solutions of Einstein's equations to compare with.
  [Heller, Janik, and Witaszczyk, arXiv:1103.3452]
- They study a wide variety of initial conditions and find a kind of universal lower bound for the thermalization time

RHIC 200 GeV/nucleon:
T_0 = 350 MeV, τ_0 > 0.35 fm/c

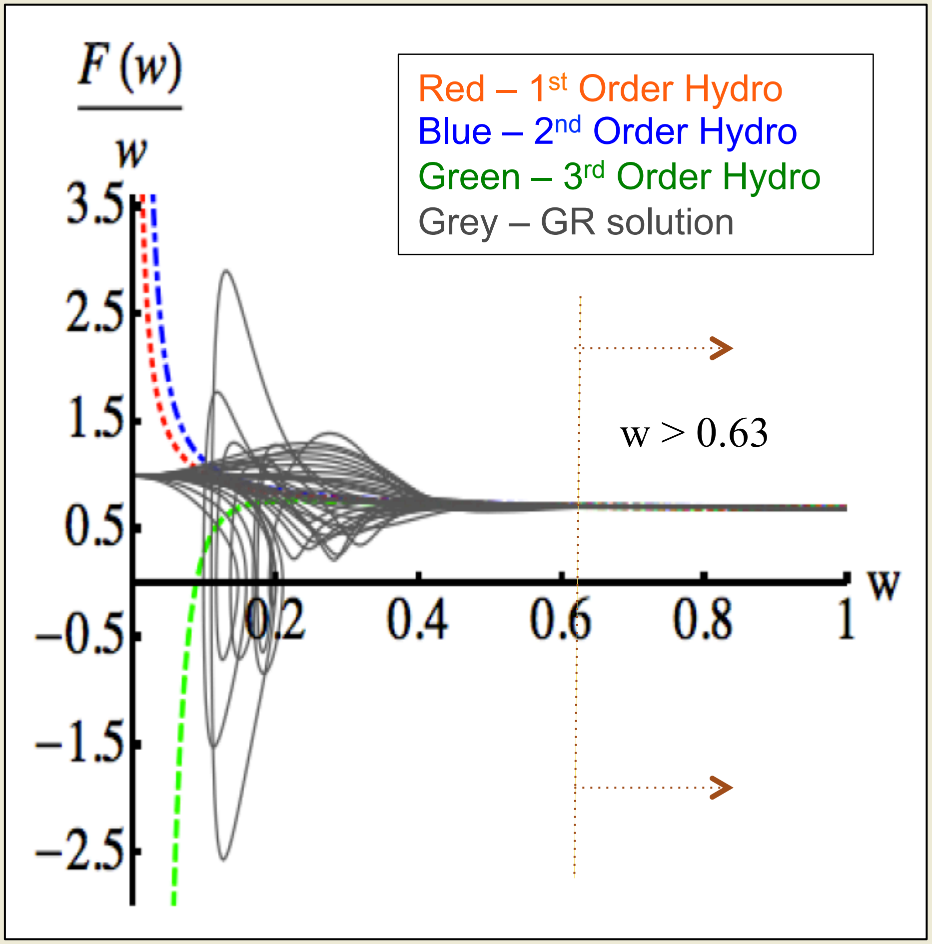
LHC 2.76 TeV/nucleon:
T_0 = 600 MeV, τ_0 > 0.2 fm/c

$$\langle T_{\tau\tau}\rangle \equiv \varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8}\pi^2 \cdot T_{eff}^4 \cdot$$

$$w = T_{eff} \cdot \tau$$

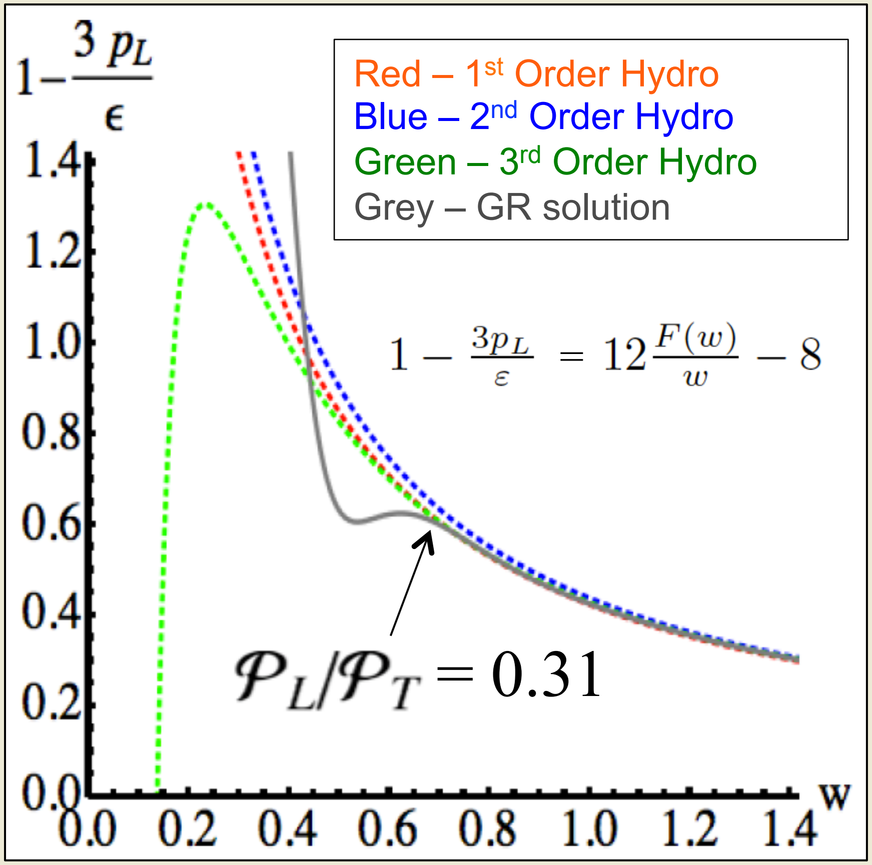
$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w},$$

F_{hydro} known up to 3rd order hydro analytically

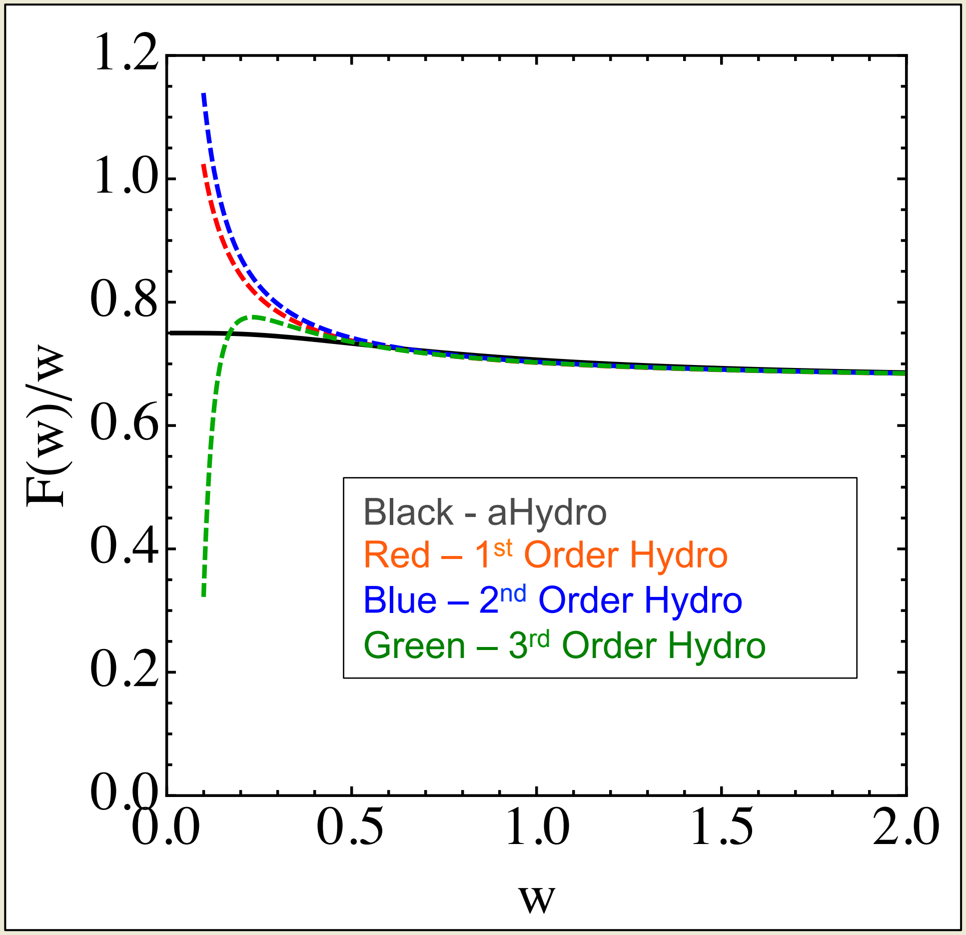


N=4 SUSY using AdS/CFT

However, at that time the system is not isotropic and remains anisotropic for the entirety of the evolution



How well does the `aHydro` framework compare?



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Including Transverse Dynamics

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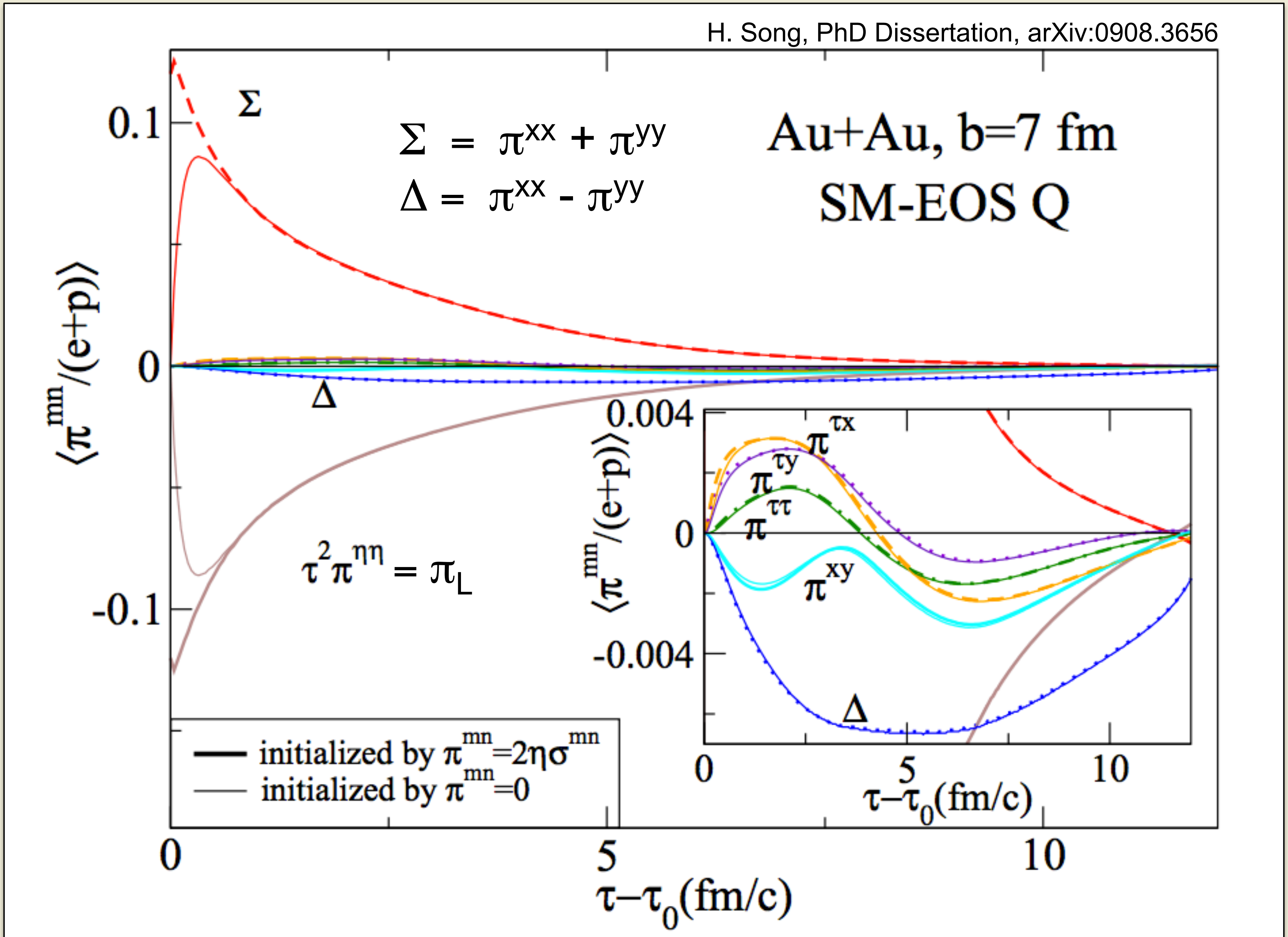


Including Transverse Dynamics

M. Martinez, R. Ryblewski, and MS, forthcoming.

- Now we consider boost invariant dynamics with transverse flow.
- Four equations for four variables u_x, u_y, ξ, and Λ.

$$T^{\mu\nu} = (\mathcal{E}+\mathcal{P}_\perp)u^\mu u^\nu - \mathcal{P}_\perp g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_\perp)z^\mu z^\nu$$

$$D = u^\mu \partial_\mu$$

$$\tilde{\Delta} = \partial_\mu u^\mu$$

$$Dn + n\tilde{\Delta} = J_0$$

$$D\mathcal{E} + (\mathcal{E}+\mathcal{P}_\perp)\tilde{\Delta} + (\mathcal{P}_L - \mathcal{P}_\perp)\frac{u_0}{\tau} = 0$$

$$(\mathcal{E}+\mathcal{P}_\perp)Du_x + \partial_x \mathcal{P}_\perp + u_x D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_x}{\tau} = 0$$

$$(\mathcal{E}+\mathcal{P}_\perp)Du_y + \partial_y \mathcal{P}_\perp + u_y D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_y}{\tau} = 0$$

$$n(\xi,\Lambda) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\rm RS} = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda,\xi) = T^{\tau\tau} = \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_\perp(\Lambda,\xi) = \frac{1}{2}(T^{xx}+T^{yy}) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_L(\Lambda,\xi) = -T^{\varsigma}_{\varsigma} = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

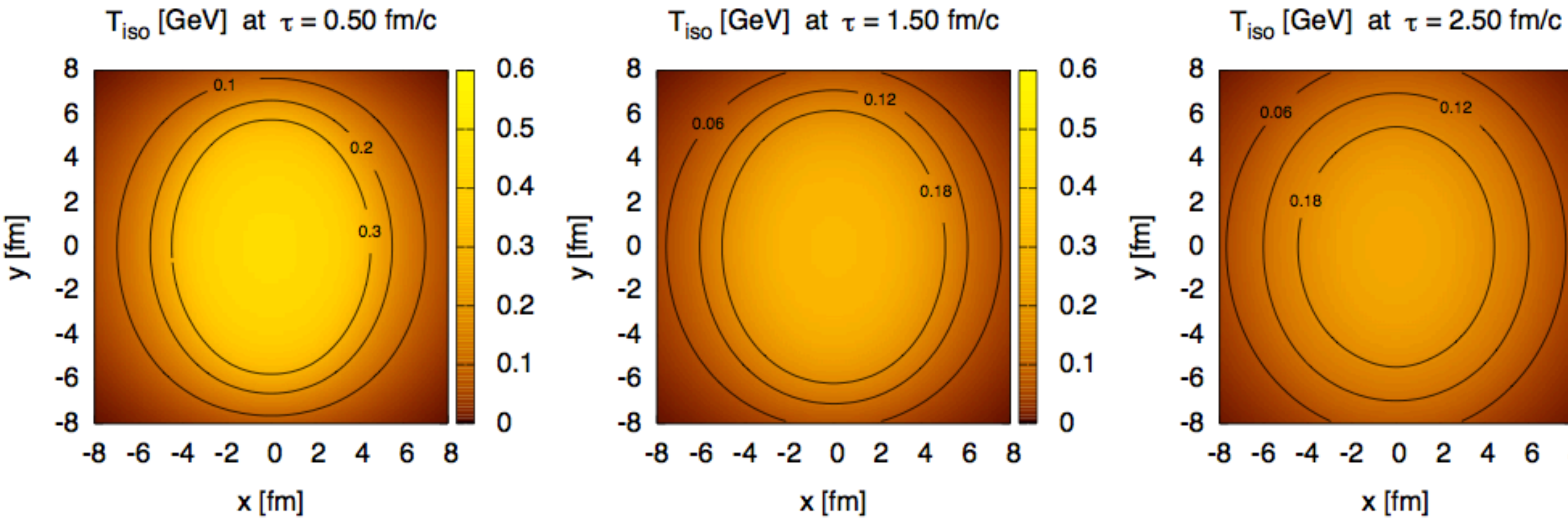
$$\mathcal{R}_\perp(\xi) \equiv \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)\mathcal{R}(\xi)}{\xi+1}\right)$$

$$\mathcal{R}_L(\xi) \equiv \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi)-1}{\xi+1}\right)$$

Transverse Dynamics

Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

$$\frac{\eta}{\mathcal{S}} = \frac{1}{4\pi}$$



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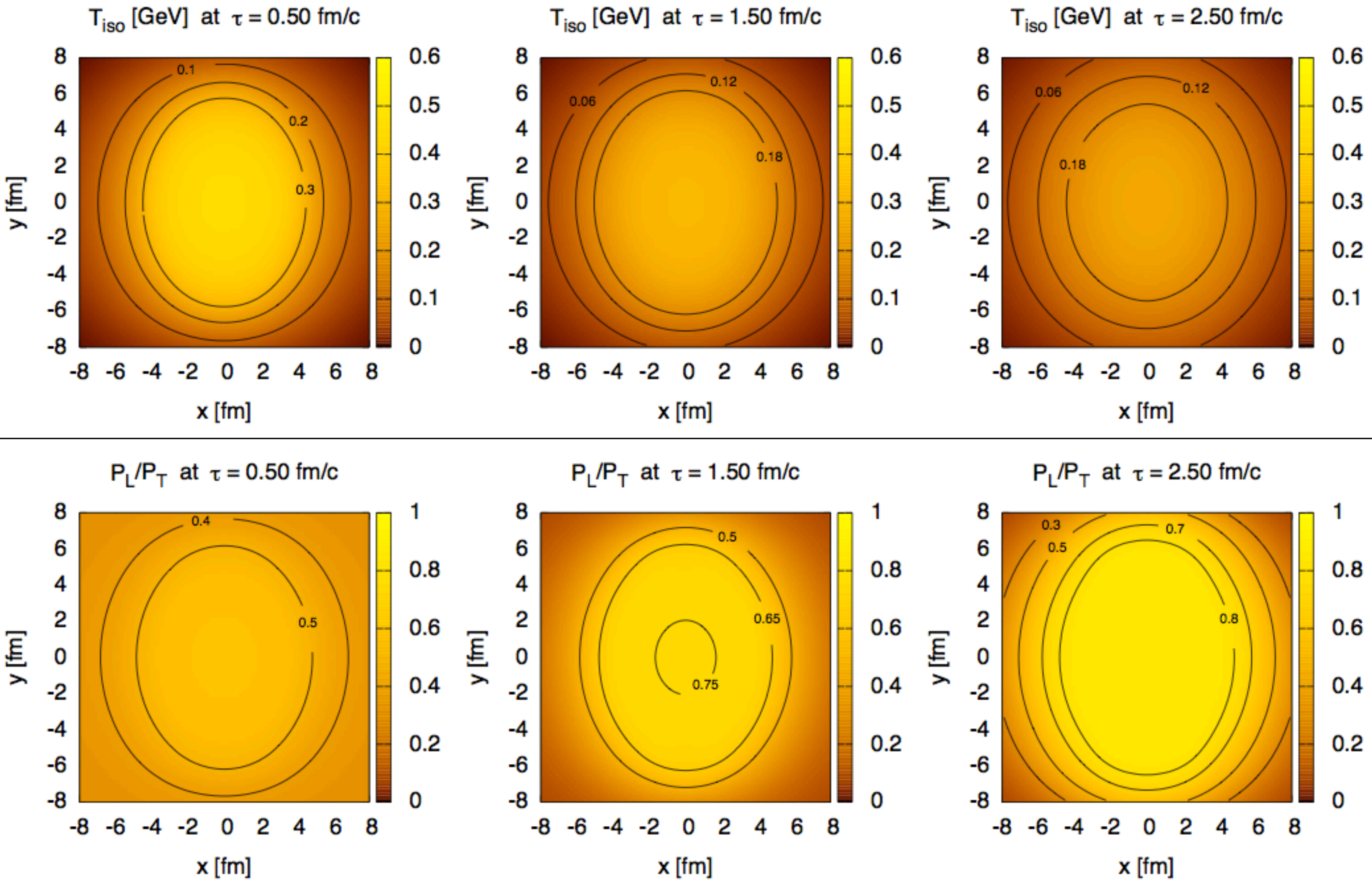
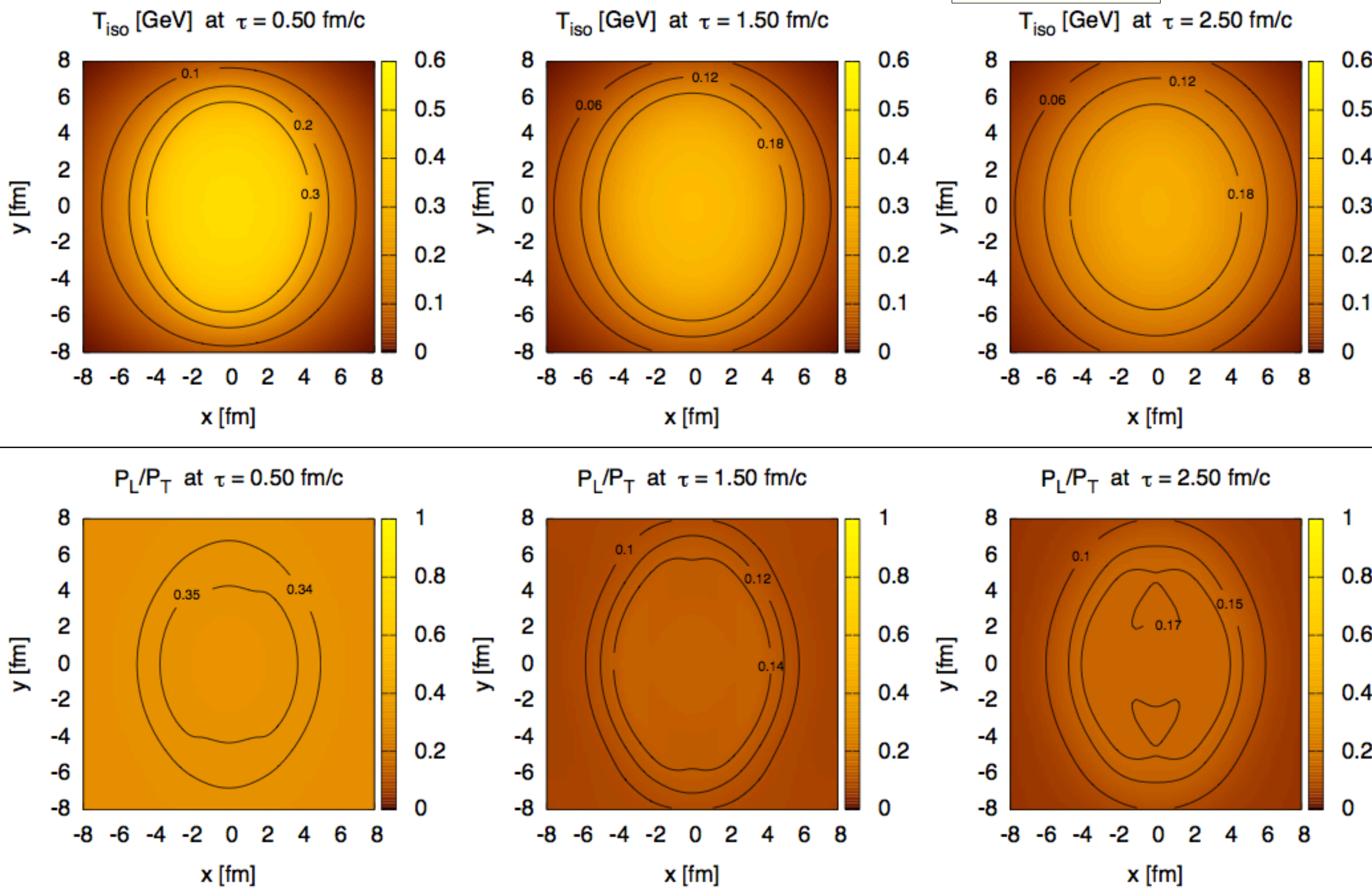
$$\frac{\eta}{\mathcal{S}} = \frac{1}{4\pi}$$



Transverse Dynamics

Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

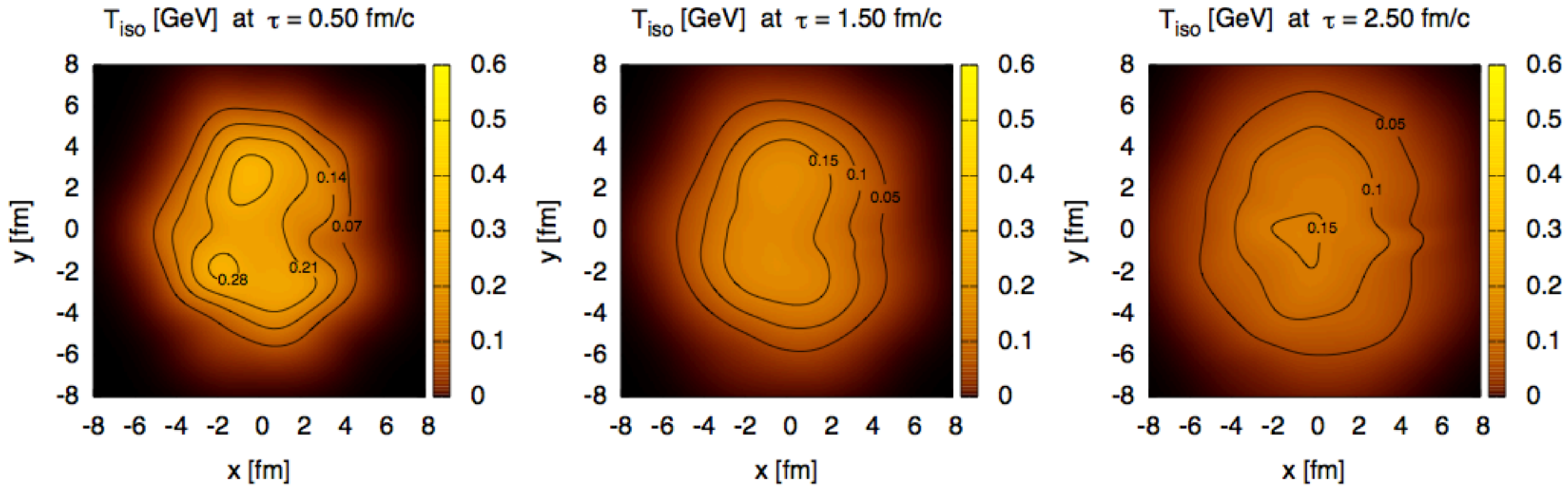
$$\frac{\eta}{\mathcal{S}} = \frac{10}{4\pi}$$

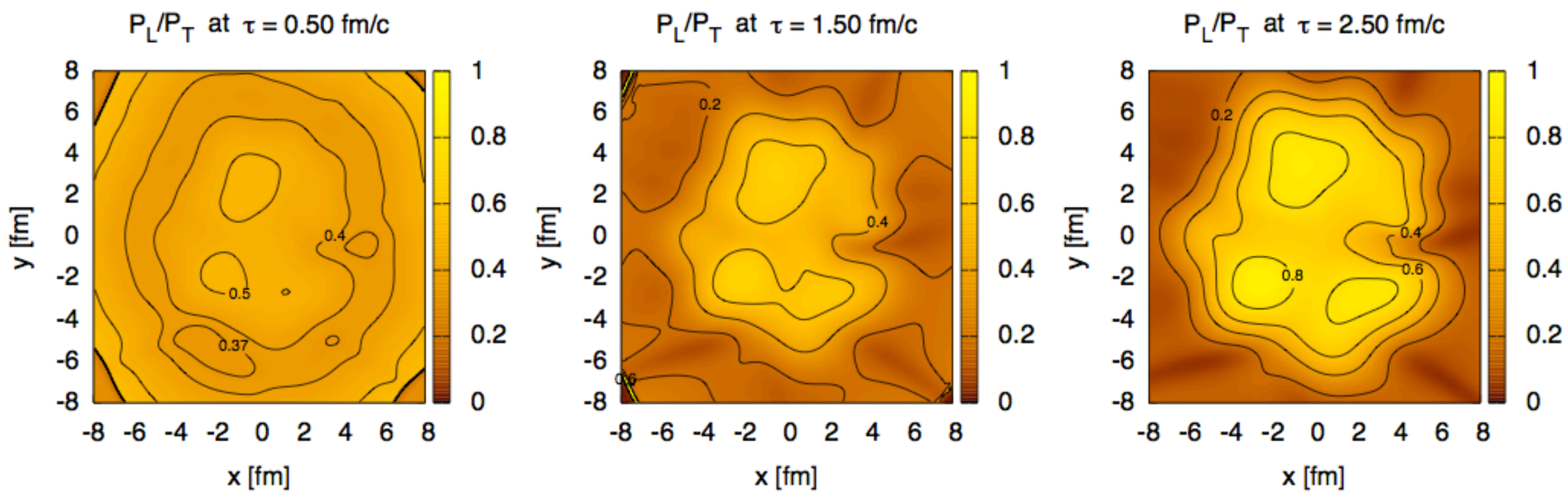


Transverse Dynamics

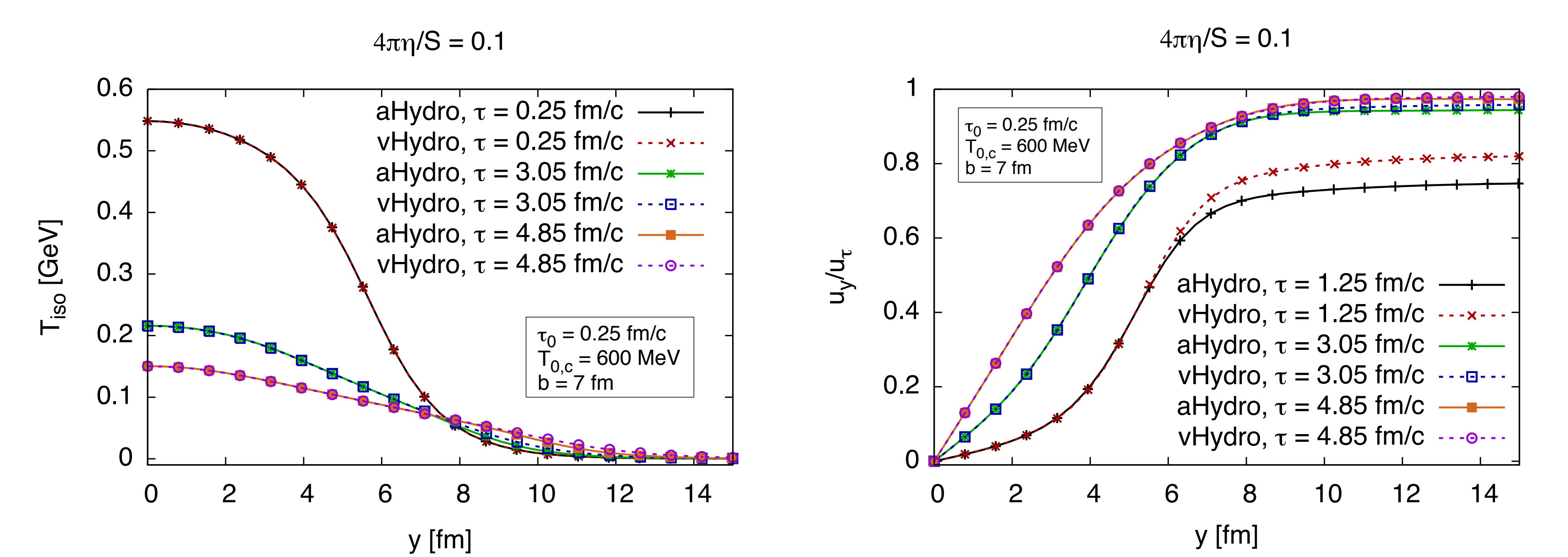
Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

$$\frac{\eta}{\mathcal{S}} = \frac{1}{4\pi}$$

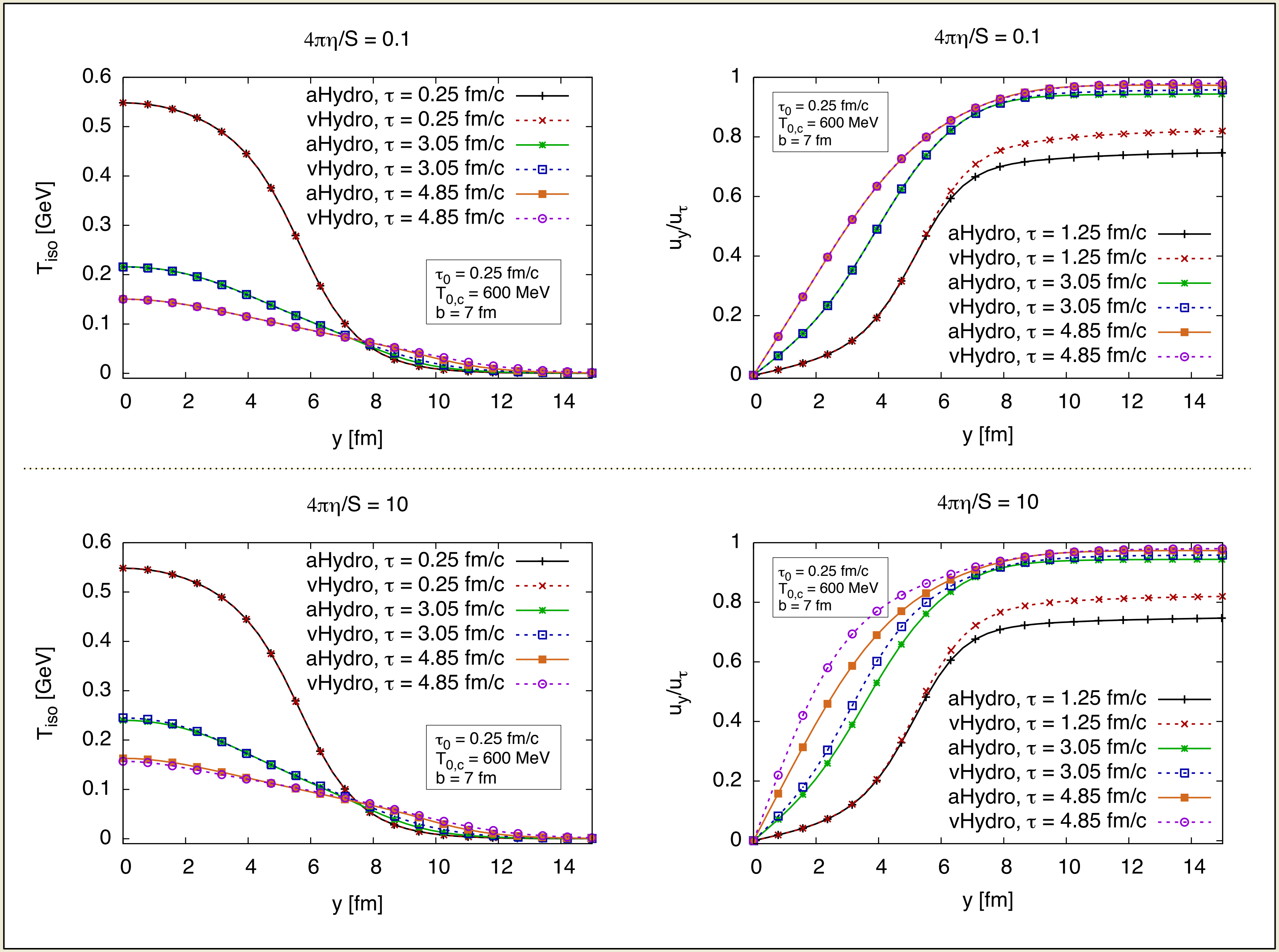




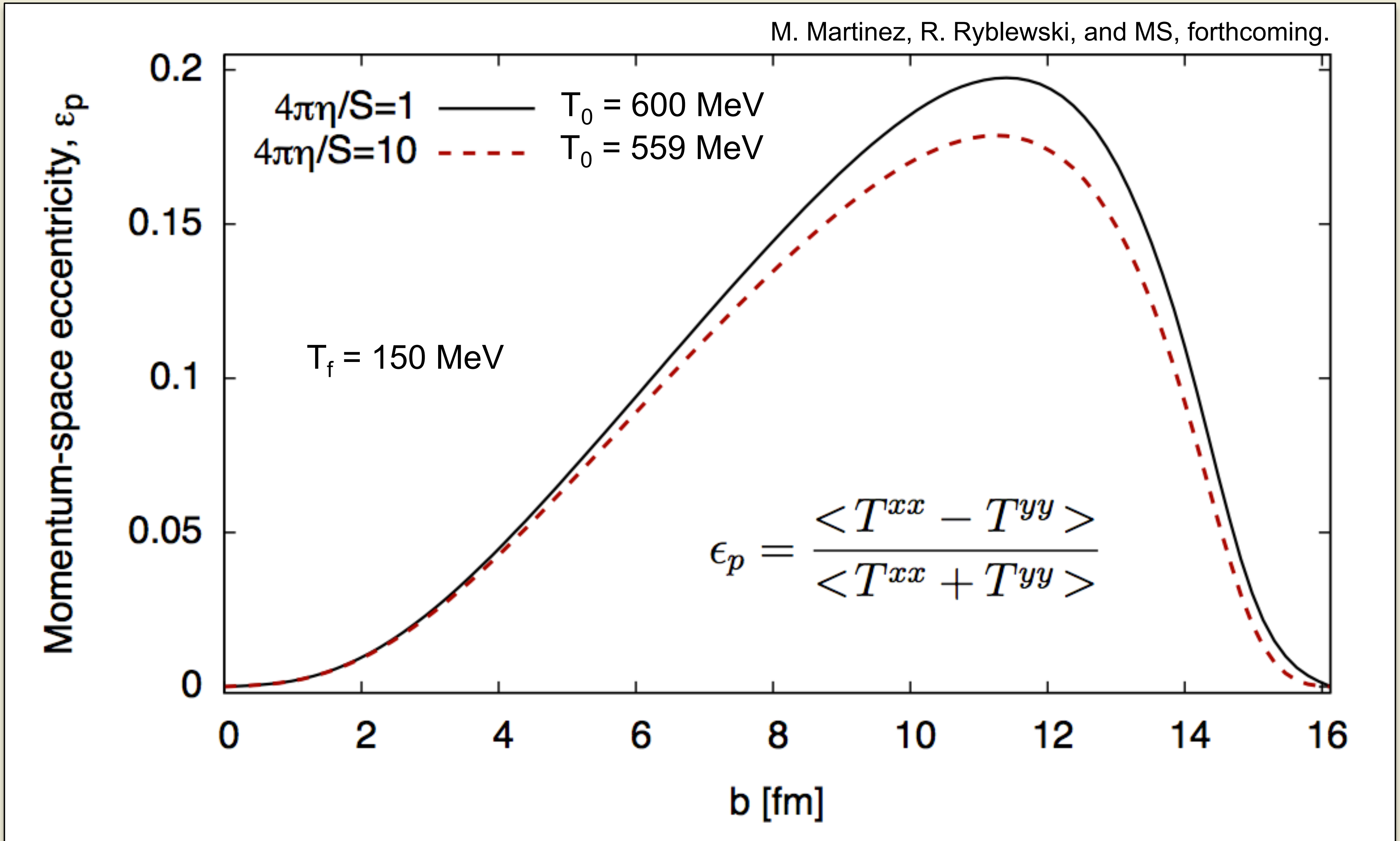
Check against Viscous Hydro



Check against Viscous Hydro



Collective Flow



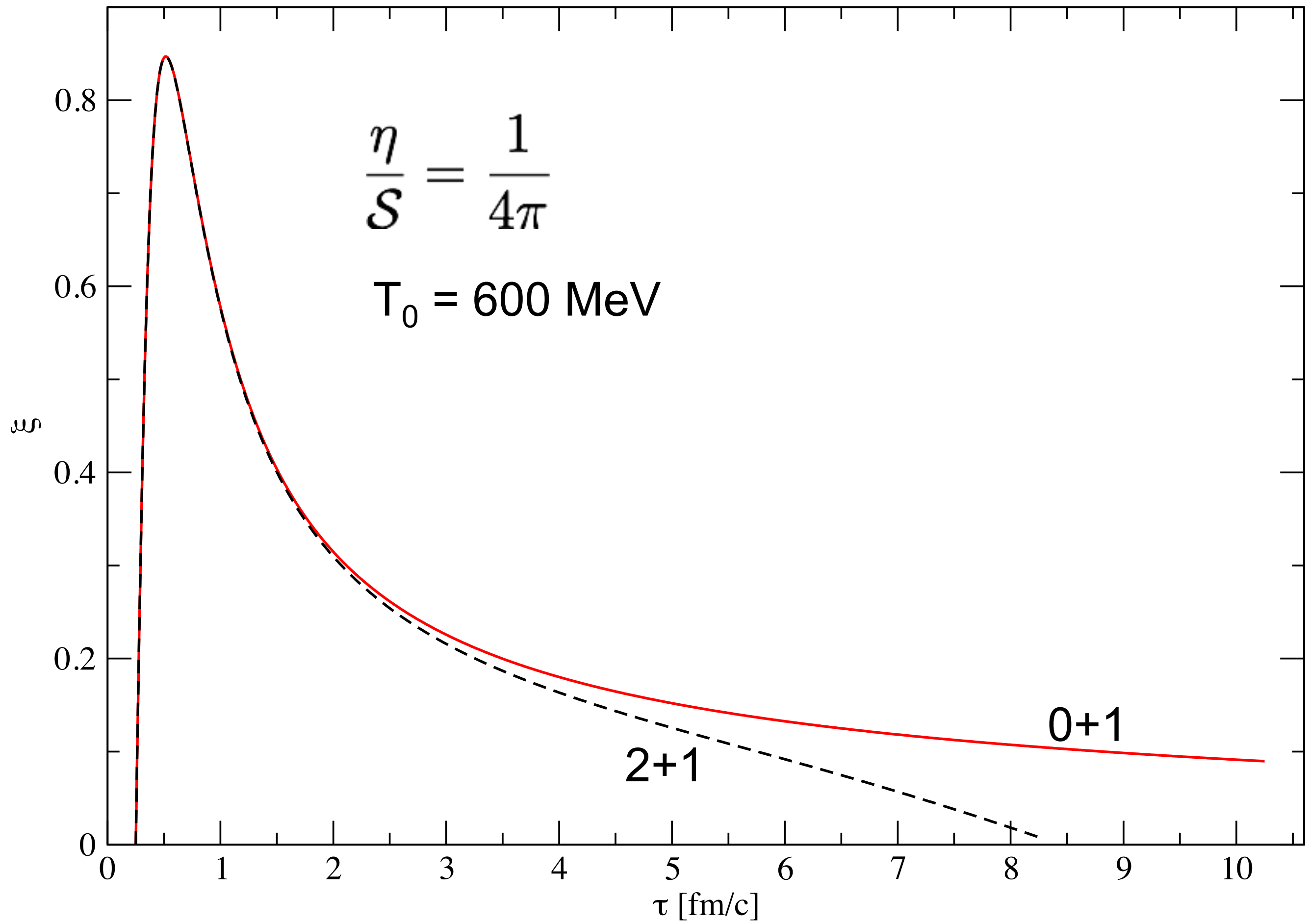
Conclusions

> *Now, to be certain that I have this straight... I'll re-Recapitulate.* Mr. Finklebein the Fish

- Plasma need not be isotropic in momentum space to describe the data
- Large momentum-space anisotropies cause trouble for traditional viscous hydrodynamical approaches
- Particularly worrisome near edges
- A practical way out is to change the expansion point and consider fluctuations around that → "aHydro"
- Results in a more reliable tool to compute the dependence of observables on momentum-space anisotropy; first application, bottomonium suppression [MS arXiv:1106.2571, MS and D. Bazow arXiv:1112.2761]

--- Backup Slides ---

Effect of Transverse Expansion on Isotropization



Numerical Checks 1

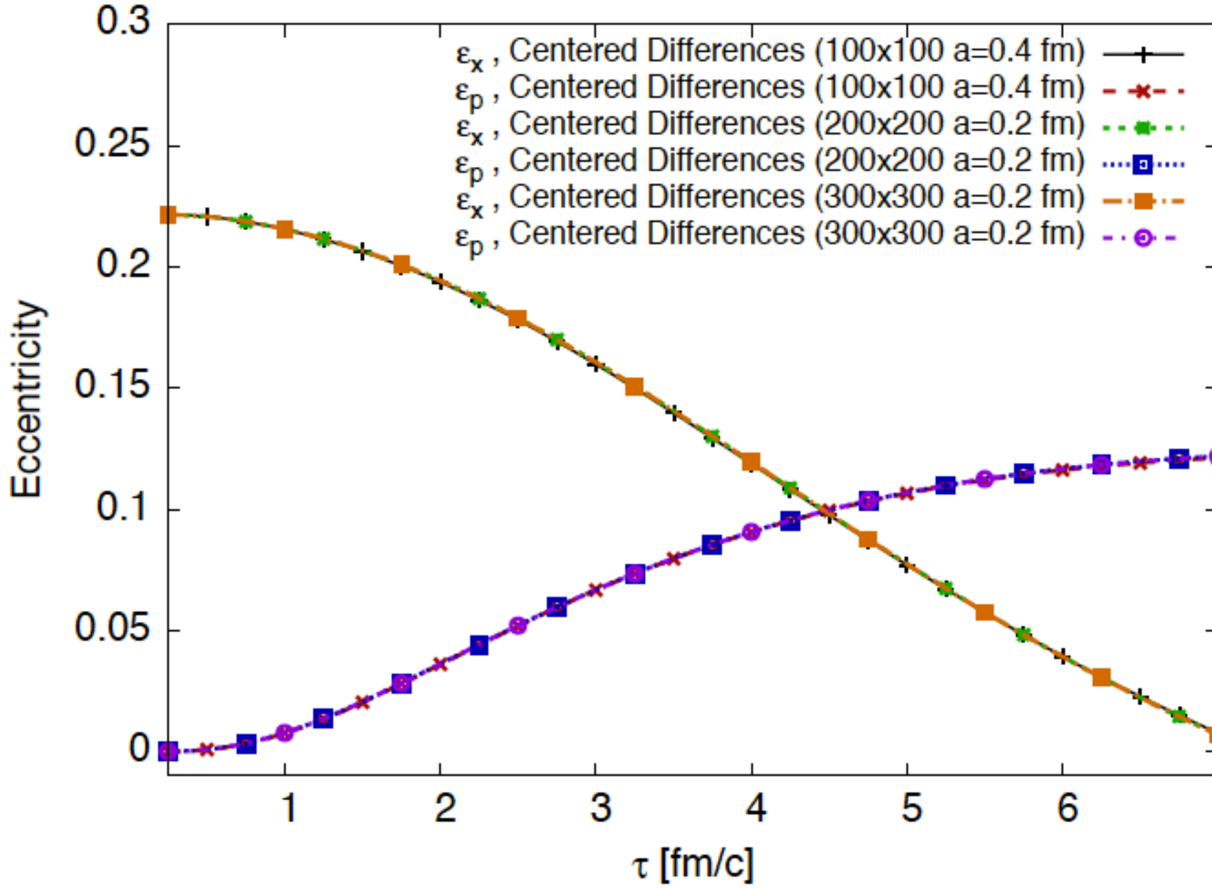


FIG. 1: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. In all three cases we used a RK4 temporal step size of $\epsilon = 0.01$ fm/c.

Numerical Checks 2

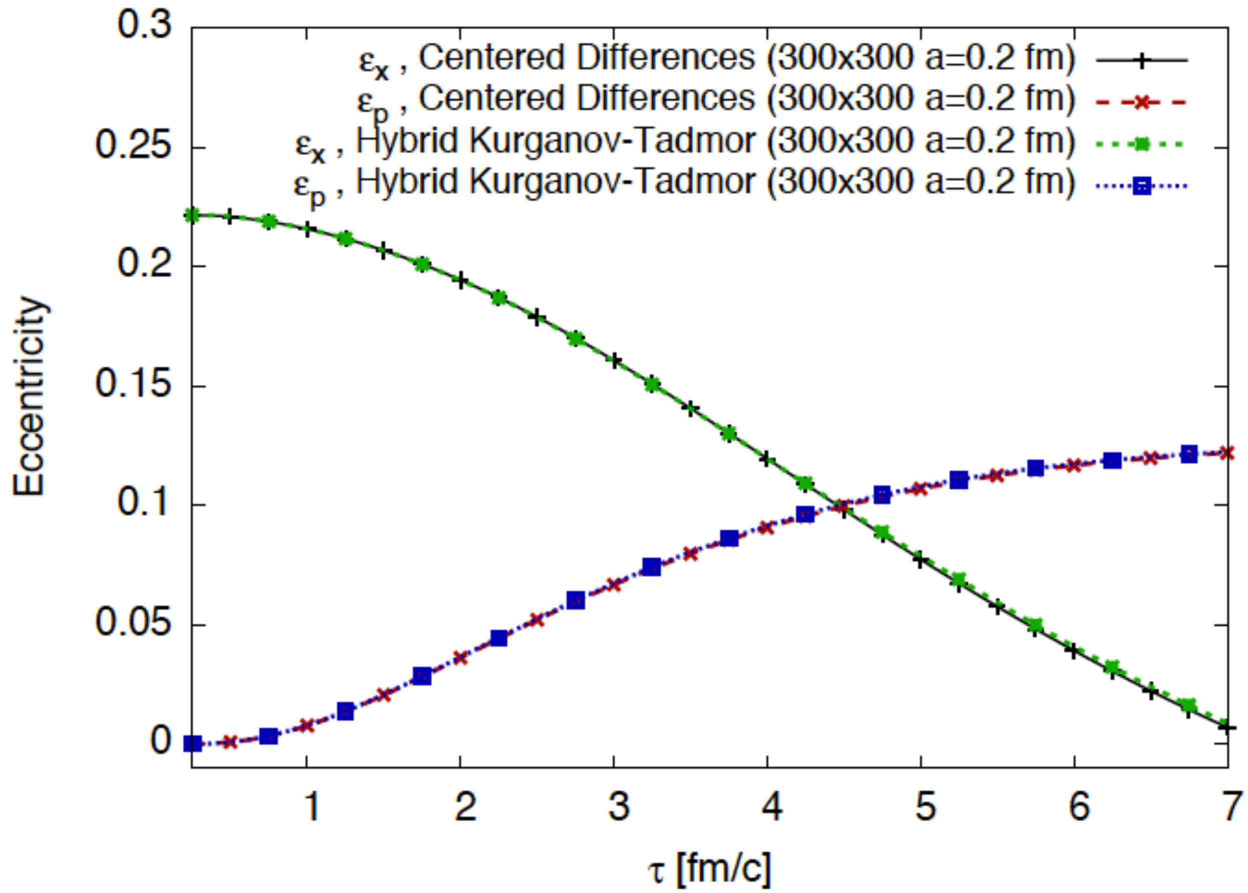


FIG. 2: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we compare the centered differences and Hybrid Kurganov-Tadmor algorithms.

Numerical Checks 2

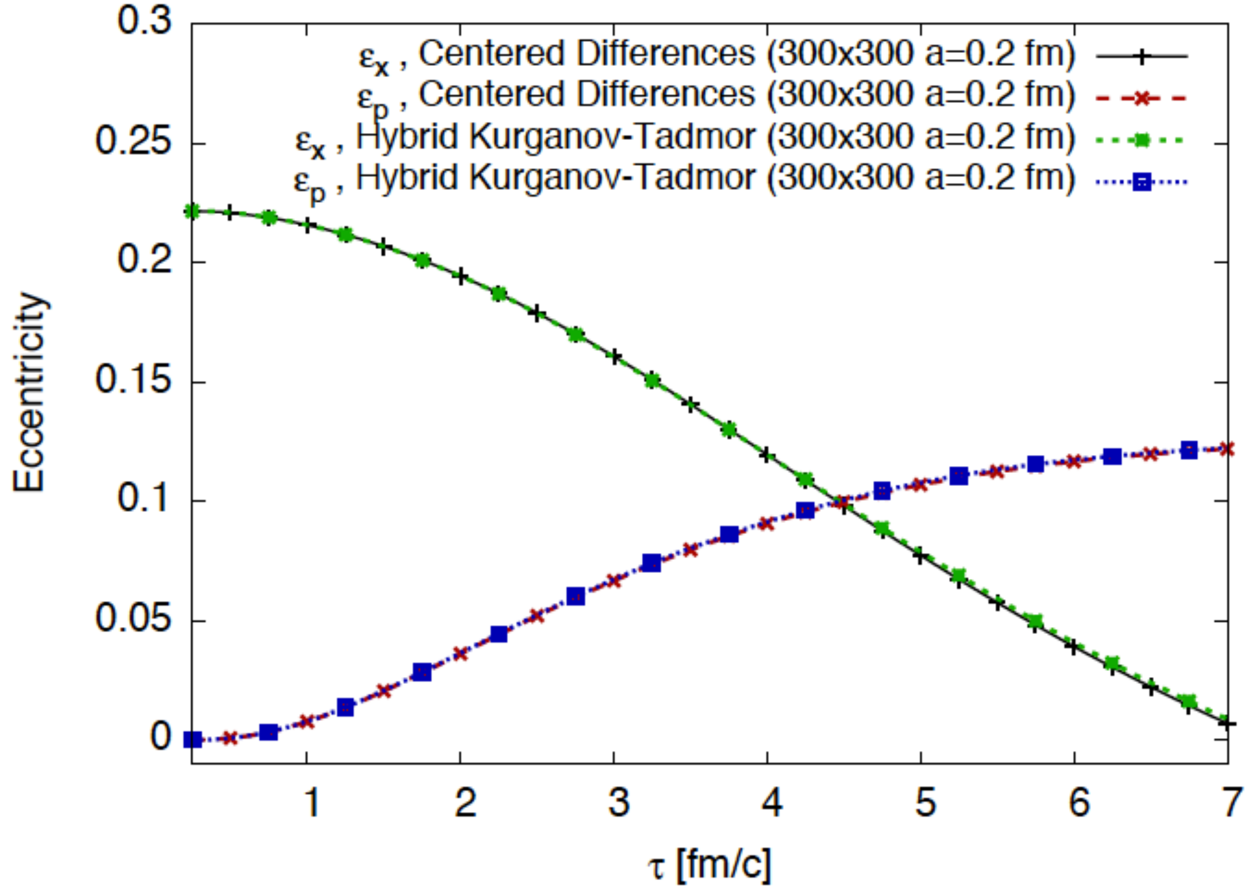


FIG. 2: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we compare the centered differences and Hybrid Kurganov-Tadmor algorithms.

Numerical Checks 3

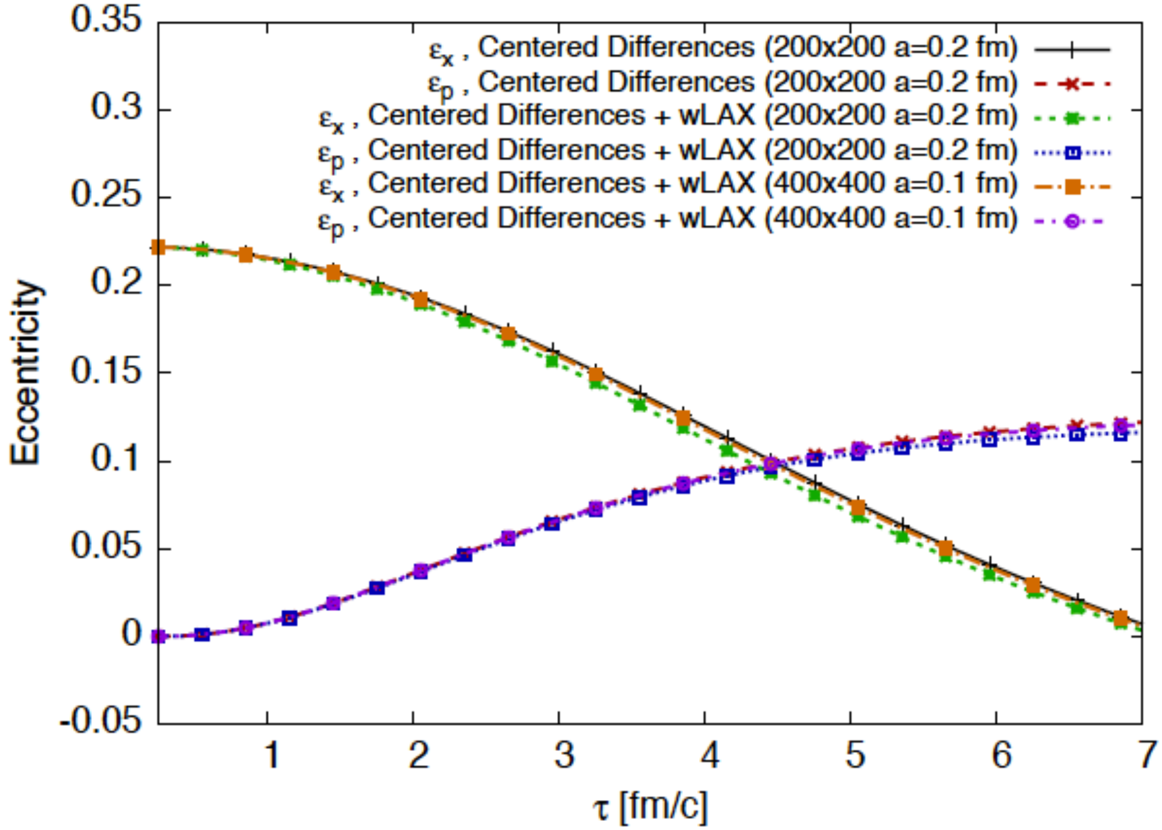
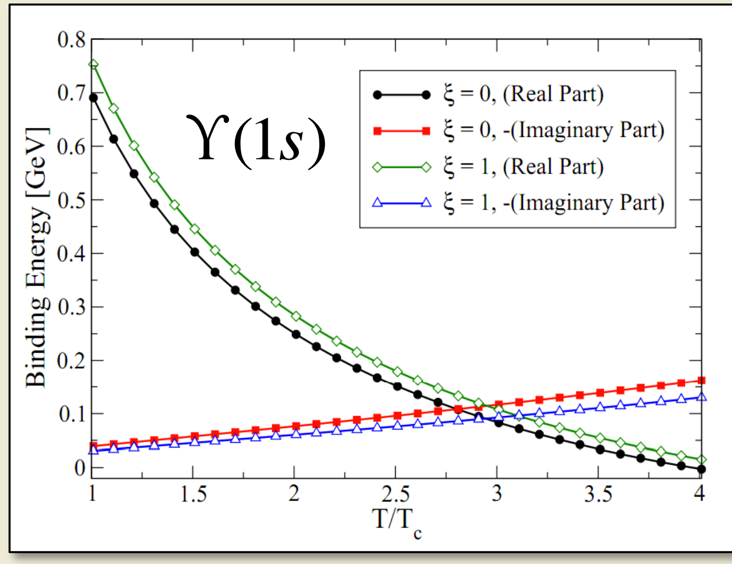


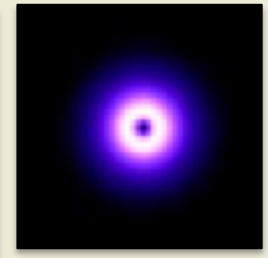
FIG. 10: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we demonstrate the convergence of the wLAX algorithm with $\lambda = 0.05$ to the result obtained without any spatial averaging as one decreases the lattice spacing. In all cases RK4 with a temporal step size $\epsilon = 0.01$ fm/c was used.

Quarkonium Binding

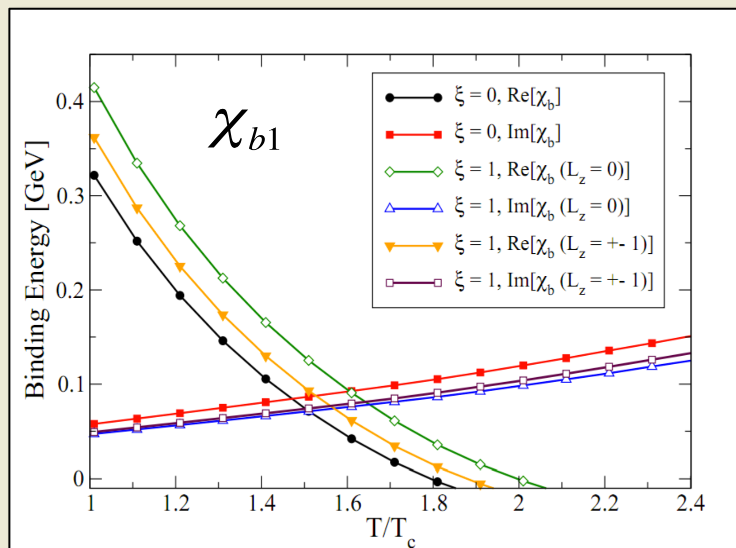
- First theoretical calculation of dependence of states' complex-valued binding energies.
- Imaginary part of the binding energy gives the decay rate (width)
- Width increases with temperature → **in-medium suppression**
- Calculation performed allowing for anisotropy in momentum space (ξ≠0)

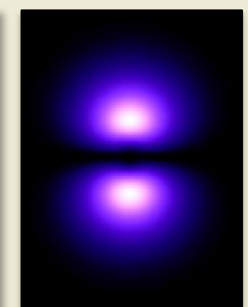
MS et al, Phys. Rev. D 83, 105019 (2011)





1s

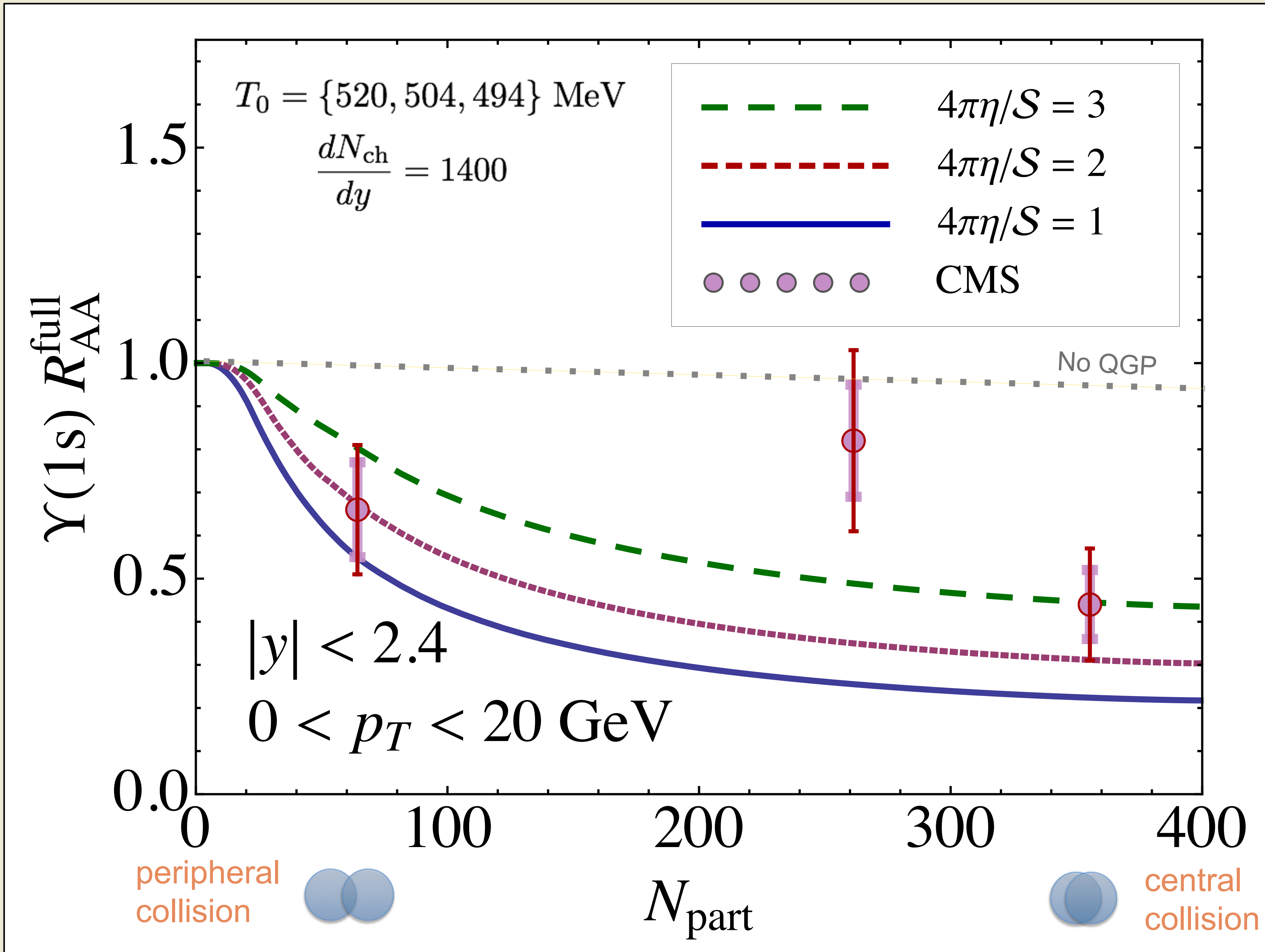




1p

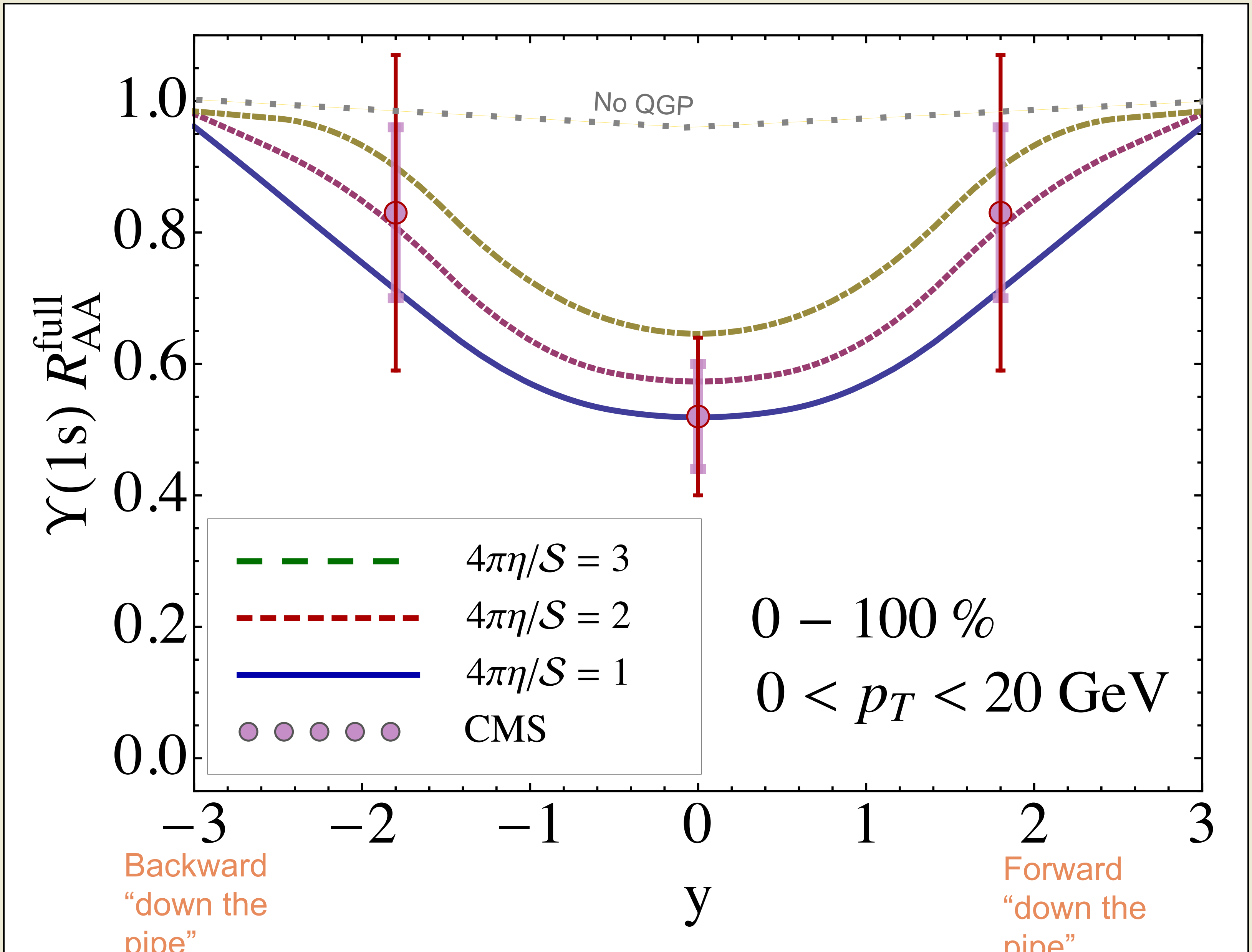
Centrality Dependence - LHC

MS, Phys. Rev. Lett. 107, 132301 (2011).



Rapidity Dependence - LHC

MS, Phys. Rev. Lett. 107, 132301 (2011).



Centrality Dependence - RHIC