On calculations of cosmic relic abundances of extremely weakly interacting particles

# Frank D. Steffen





Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

# Gauge field dynamics in and out of equilibrium INT Seattle, April 10, 2012

# Outline

- Extremely Weakly Interacting Particles (EWIPs)
- Cosmic Relic Abundances (Th.Relic, Th.Prod.)
- Calculation of the Collision Term (Hard + Soft)
- Phenomenological Implications (T<sub>D</sub>, DM, BBN)

#### **Extremely Weakly Interacting Particles (EWIPs)**



#### **Extremely Weakly Interacting Particles (EWIPs)**



#### [Raffelt, '06] Bounds on the Peccei-Quinn Scale



Bounds from Axion Searches Cosmological Axion Bounds

Astrophysical Axion Bounds

#### [Raffelt, '06] Bounds on the Peccei-Quinn Scale



Bounds from Axion Searches Cosmological Axion Bounds Astrophysical Axion Bounds

Peccei-Quinn Scale $f_a \gtrsim 6 \times 10^8 \,\mathrm{GeV}$ 

**Axion Mass**  $m_a \simeq 0.6 \, \mathrm{meV} \, (10^{10} \, \mathrm{GeV} / f_{\mathrm{PQ}})$ 

# **Axion Interactions**

 with gluons model independent

$$\mathcal{L}_{agg} = \frac{g_{\rm s}^2}{32\pi^2 f_a} \, a \, G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$$

• with photons model dependent

$$\mathcal{L}_{a\gamma\gamma} = \frac{e^2 C_{a\gamma\gamma}}{32\pi^2 f_a} a F_{\mu\nu} \widetilde{F}^{\mu\nu}$$
  
(or  $\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \widetilde{F}^{\mu\nu}$ )

but crucial for axion searches and governs the axion lifetime  $\tau_a = \Gamma_{a \to \gamma\gamma}^{-1} = \frac{64\pi}{q_{a\gamma\gamma}^2 m_a^3}$ 

5

# **Axion Searches**



Updated figure provided by M. Kuster for [FDS, 0811.3347] see also [Battesti et al., 0705.0015]

Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs



Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs

7



Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs



Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs





#### Axion Condensate: CDM

 $\Omega_a^{\rm MIS} h^2 \sim 0.15 \, \theta_i^2 (f_{\rm PQ}/10^{12}\,{\rm GeV})^{7/6}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]



Axion Condensate: CDM

 $\Omega_a^{\rm MIS} h^2 \sim 0.15 \, \theta_i^2 (f_{\rm PQ}/10^{12}\,{\rm GeV})^{7/6}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]

Axions can provide CDM



Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs

## **Cosmic Relic Abundances**

 $[details \rightarrow blackboard]$ 

•  $T_R > T_D$ :  $I+2 \rightleftharpoons 3+X$ 

 $T > T_D$ : X in thermal eq. with the primordial plasma T ~ T<sub>D</sub>: X decouples as a **hot thermal relic** 

•  $T_R > T_D$ :  $I+2 \rightarrow 3+X$ 



collision term



• Hard Thermal Loop (HTL) Resummation

 $[details \rightarrow blackboard]$ 

[Graf, FDS, 1008.4528]

# **Thermal Axion Production in the Hot QGP**



 $10^{12}$ 



Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

#### Axion Condensate: CDM

 $\Omega_a^{\rm MIS} h^2 \sim 0.15 \, \theta_i^2 (f_{\rm PQ}/10^{12}\,{
m GeV})^{7/6}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]





#### **Extremely Weakly Interacting Particles (EWIPs)**



#### **Extremely Weakly Interacting Particles (EWIPs)**



## **Axino Interactions**

with gluons and gluinos model independent

$$\mathcal{L}_{\widetilde{a}\widetilde{g}g} = i \, \frac{g_{\rm s}^2}{64\pi^2 f_a} \, \overline{\widetilde{a}} \, \gamma_5 \left[\gamma^\mu, \gamma^\nu\right] \, \widetilde{g}^a \, G^a_{\mu\nu}$$

• with photons/Z-bosons and binos model dependent  $\mathcal{L}_{\tilde{a}\tilde{B}\gamma/Z} = i \frac{\alpha_{\rm Y} C_{\rm aYY}}{16\pi f_a} \,\overline{\tilde{a}} \,\gamma_5 \, [\gamma_\mu, \gamma_\nu] \,\widetilde{B} \, (\cos \theta_W F_{\mu\nu} - \sin \theta_W Z_{\mu\nu})$ 

but crucial for axino searches

[Brandenburg, FDS, '04]

# **Thermal Production of**

# **Axino Dark Matter**

# in the Early Universe

Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

On calculations of cosmic relic abundances of EWIPs

#### Axino Number Density for $f_a > T_D > T_R \gtrsim 10^4 \text{ GeV}$

• Boltzmann equation: time evolution of axino density  $n_{\tilde{a}}$  in the thermal bath

$$\frac{dn_{\tilde{a}}}{dt} + 3Hn_{\tilde{a}} = C_{\tilde{a}} = \int d^3p \, \frac{d\Gamma_{\tilde{a}}}{d^3p} \quad \longleftarrow \quad \text{generation of } \tilde{a} - \text{annihilation of } \tilde{a}$$

• collision term for  $a(p_1) + b(p_2) \to c(p_3) + \tilde{a}(p)$ :  $(C_{a+b\to c+\tilde{a}} \in C_{\tilde{a}})$ 

$$C_{a+b\to c+\tilde{a}} = \int \frac{d^3p}{(2\pi)^3 2E} \int \left[ \prod_{i=1}^3 \frac{d^3p_i}{(2\pi)^3 2E_i} \right] (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p) \\ \times \left[ |M_{a+b\to c+\tilde{a}}|^2 f_a f_b (1\pm f_c)(1-f_{\tilde{a}}) - |M_{c+\tilde{a}\to a+b}|^2 f_c f_{\tilde{a}} (1\pm f_a)(1\pm f_b) \right]$$

• phase space densities:  $f_i \longrightarrow$  number densities:  $n_i = \int \frac{d^3 p_i}{(2\pi)^3 2E_i} g_i f_i(E_i)$ 

$$a, b, \text{ and } c: f_i = f_i^{eq} = f_{B/F} = \frac{1}{\exp(E_i/T) \mp 1}$$
, axino:  $f_{\tilde{a}} \approx 0$ 

[Kim, '79; Shifman, Vainshtein, Zakharov, '80] Axino Interactions  $\leftarrow$  Hadronic (KSVZ) Axion Models

• axino–gluino–gluon interaction:

$$\mathcal{L}_{\tilde{a}\tilde{g}g} = i \, \frac{\alpha_{\rm s}}{16\pi (f_a/N)} \, \bar{\tilde{a}} \, \gamma_5 \, \left[\gamma^{\mu}, \gamma^{\nu}\right] \, \tilde{g}^a \, G^a_{\mu\nu}$$

18

#### Thermal Axino Production in SUSY QCD

• A:  $q^a + q^b \to \tilde{q}^c + \tilde{a}$ • B:  $g^a + \tilde{g}^b \to g^c + \tilde{a}$  (crossing of A) • C:  $\tilde{q}_i + g^a \to \tilde{q}_j + \tilde{a}$ • D:  $g^a + q_i \rightarrow \tilde{q}_j + \tilde{a}$  (crossing of C) • E:  $\bar{q}_i + q_j \to g^a + \tilde{a}$  (crossing of C) • F:  $\tilde{g}^a + \tilde{g}^b \to \tilde{g}^c + \tilde{a}$  $\widetilde{g}^{a}$  $\widetilde{\mathbf{g}}^{\mathrm{b}}$ • G:  $q_i + \tilde{g}^a \to q_j + \tilde{a}$ g<sup>a</sup>  $\widetilde{\mathbf{g}}^{a}$ • H:  $\tilde{q}_i + \tilde{g}^a \to \tilde{q}_j + \tilde{a}$  $\widetilde{\mathbf{q}}_{i}$ 

• I:  $q_i + \bar{q_j} \to \tilde{g}^a + \tilde{a}$  (crossing of G)

	process $i$	$ \mathcal{M}_i ^2 / rac{g^6}{128 \pi^4 (f_a/N)^2}$
А	$g^a + g^b  o \tilde{g}^c + \tilde{a}$	$4(s+2t+2\frac{t^2}{s}) f^{abc} ^2$
В	$g^a + \tilde{g}^b  ightarrow g^c + \tilde{a}$	$-4(t+2s+2rac{s^2}{t}) f^{abc} ^2$
С	$\tilde{q}_i + g^a \to q_j + \tilde{a}$	$2s T^a_{ji} ^2$
D	$g^a + q_i  o \tilde{q}_j + \tilde{a}$	$-2t T^a_{ji} ^2$
Е	$\bar{\tilde{q}}_i + q_j \to g^a + \tilde{a}$	$-2t T^a_{ji} ^2$
F	$\tilde{g}^a + \tilde{g}^b  o \tilde{g}^c + \tilde{a}$	$-8 \frac{(s^2 + st + t^2)^2}{st(s+t)}  f^{abc} ^2$
G	$q_i + \tilde{g}^a  o q_j + \tilde{a}$	$-4(s+\frac{s^2}{t}) T^a_{ji} ^2$
Η	$ ilde{q}_i +  ilde{g}^a  o  ilde{q}_j +  ilde{a}$	$-2(\frac{t}{2}+2s+2\frac{s^2}{t}) T^a_{ji} ^2$
Ι	$q_i + \bar{q}_j \to \tilde{g}^a + \tilde{a}$	$-4(t+\frac{t^2}{s}) T^a_{ji} ^2$
J	$\tilde{q}_i + \bar{\tilde{q}}_j  o \tilde{g}^a + \tilde{a}$	$2(\frac{s}{2} + 2t + 2\frac{t^2}{s}) T^a_{ji} ^2$

B, F, G, & H: Logarithmic IR Singularity



• Thermal Production Rate: \* complete to LO in g , \* finite , \* indep. of  $\Lambda$ 

$$E \left. \frac{d\Gamma_{\tilde{G}}}{d^3 p} \right|_{\text{LO in } g} = E \left. \frac{d\Gamma_{\tilde{G}}}{d^3 p} \right|_{\text{soft}} + E \left. \frac{d\Gamma_{\tilde{G}}}{d^3 p} \right|_{\text{hard}} = A_{\text{soft}} + A_{\text{hard}} + B \ln \left[ \frac{1}{g} \right]$$

#### The Collision Term to Leading Order in the Coupling g

• Collision Term: 
$$C_{\tilde{a}}(T) = \int d^3p \left( \frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{soft}} + \frac{d\Gamma_{\tilde{a}}}{d^3p} \bigg|_{\text{hard}} \right)$$

$$C_{\tilde{a}}(T) = \frac{(N_c^2 - 1)}{(f_a/N)^2} \frac{3\zeta(3)g^6 T^6}{4096\pi^7} \left[ \ln\left(\frac{1.380 T^2}{m_g^2}\right) (N_c + n_f) + 0.4336 n_f \right]$$

• Thermal Gluon Mass in the "QGSGP":

$$m_g^2 = \frac{g^2 T^2}{6} (N_c + n_f)$$
 with  $N_c = 3$  and  $n_f = 6$ 

• Running of the Strong Coupling in the MSSM:

$$g(T) = \left(g^{-2}(M_Z) + \frac{3}{8\pi^2} \ln\left[\frac{T}{M_Z}\right]\right)^{-1/2} \longrightarrow \quad 0.85 \text{ for } T \approx 10^{10} \text{ GeV}$$

#### Solving the Boltzmann Equation $\rightarrow$ Axino Abundance

• Boltzmann equation

$$\frac{dn_{\tilde{a}}}{dt} + 3Hn_{\tilde{a}} = C_{\tilde{a}}$$

• conservation of entropy

$$sR^3 = \text{const.}$$

• yield  $\rightarrow$  scale out expansion

$$Y_{\tilde{a}} = \frac{n_{\tilde{a}}}{s}$$

• Boltzmann equation

$$\frac{d}{dt}Y_{\tilde{a}} = \frac{C_{\tilde{a}}}{s}$$

• radiation dominated epoch

$$dt = -\frac{dT}{H(T)T}, \ H(T) = \sqrt{\frac{g_*(T)\pi^2}{90}} \frac{T^2}{M_{\rm Pl}}$$
  
• entropy density MSSM

$$s(T) = \frac{2\pi^2}{45} g_{*S}(T) T^3, \quad g_{*S} = g_* = \frac{915}{4}$$

• Axino Yield from Thermal Production  

$$Y_{\tilde{a}} \approx \frac{C_{\tilde{a}}(T_R)}{s(T_R)H(T_R)} = 2.0 \times 10^{-7} g^6 \ln\left(\frac{1.108}{g}\right) \left(\frac{10^{11} \text{ GeV}}{f_a/N}\right)^2 \left(\frac{T_R}{10^4 \text{ GeV}}\right)$$
• Axino Density from Thermal Production  

$$\Omega_{\tilde{a}}h^2 = m_{\tilde{a}}Y_{\tilde{a}}s(T_0)h^2/\rho_c = 5.5 g^6 \ln\left(\frac{1.108}{g}\right) \left(\frac{m_{\tilde{a}}}{0.1 \text{ GeV}}\right) \left(\frac{10^{11} \text{ GeV}}{f_a/N}\right)^2 \left(\frac{T_R}{10^4 \text{ GeV}}\right)$$

# **Axino LSP Case**



# **Axino LSP Case**



#### **Extremely Weakly Interacting Particles (EWIPs)**



#### **Extremely Weakly Interacting Particles (EWIPs)**







$$\Omega_{\tilde{G}}^{\text{TP}}h^{2} = m_{\tilde{G}}Y_{\tilde{G}}^{\text{TP}}(T_{0}) s(T_{0}) h^{2}/\rho_{c}$$

$$= \sum_{i=1}^{3} \omega_{i} g_{i}^{2}(T_{\text{R}}) \left(1 + \frac{M_{i}^{2}(T_{\text{R}})}{3m_{\tilde{G}}^{2}}\right) \ln\left(\frac{k_{i}}{g_{i}(T_{\text{R}})}\right)$$

$$\times \left(\frac{m_{\tilde{G}}}{100 \text{ GeV}}\right) \left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}}\right)$$
The dark matter density
$$\Omega_{\text{dm}}^{3\sigma} h^{2} = 0.105_{-0.030}^{+0.021}$$
probes
the reheating temperature

Frank D. Steffen (Max-Planck-Institute of Physics, Munich)



$$\Omega_{\tilde{G}}^{\text{TP}}h^{2} = m_{\tilde{G}}Y_{\tilde{G}}^{\text{TP}}(T_{0}) s(T_{0}) h^{2}/\rho_{c}$$

$$= \sum_{i=1}^{3} \omega_{i}g_{i}^{2}(T_{R}) \left(1 + \frac{M_{i}^{2}(T_{R})}{3m_{\tilde{G}}^{2}}\right) \ln\left(\frac{k_{i}}{g_{i}(T_{R})}\right)$$

$$\times \left(\frac{m_{\tilde{G}}}{100 \text{ GeV}}\right) \left(\frac{T_{R}}{10^{10} \text{ GeV}}\right)$$
The dark matter density
$$\Omega_{dm}^{3\sigma} h^{2} = 0.105_{-0.030}^{+0.021}$$
probes
the reheating temperature

Frank D. Steffen (Max-Planck-Institute of Physics, Munich)

## **Thermal Leptogenesis**



## **Thermal Leptogenesis**



# The gravitino can become a problem ...



#### **Upper Limits on the Reheating Temperature**



#### Thermal Leptogenesis requires T >10<sup>9</sup> GeV

#### **Upper Limits on the Reheating Temperature**



#### Thermal Leptogenesis requires T >10° GeV

# Conclusion

Refined calculations of the thermal production of extremely weakly interacting particles (EWIPs) are worth pursuing

32