On calculations of cosmic relic abundances of extremely weakly interacting particles

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INT Seattle, April 10, 2012 Gauge field dynamics in and out of equilibrium

Outline

- Extremely Weakly Interacting Particles (EWIPs)
- Cosmic Relic Abundances (Th.Relic,Th.Prod.)
- Calculation of the Collision Term (Hard + Soft)
- Phenomenological Implications (T_D, DM, BBN)

Extremely Weakly Interacting Particles (EWIPs)

Extremely Weakly Interacting Particles (EWIPs)

[Raffelt, '06] **Bounds on the Peccei-Quinn Scale**

 F_{in} F_{in} F_{out} Bounds from Axion Searches i_{on} bars indicate the search Λ **Cosmological Axion Bounds** ADMX search for galactic dark matter $A_{\text{atmos}-\text{lepol}}$ Astrophysical Axion Bounds

chiral multiplet U(Raffelt, '06] **Bounds on the Peccei-Quinn Scale**

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Peccei-Quinn Scale taken to be real by field redefinitions. The phenomeno-

 $m_a \simeq 0.6 \text{ meV} (10^{10} \text{ GeV}/f_{\text{PQ}})$ Axion Mass $\overline{\mathsf{on} \; \mathsf{Mass}}$

Axion Interactions would form at the set of α (s)quarks, axion-gluon and axion-photon interactions are **obtained as described by the effective Lagrangian Structure Lagrangian Structure Lagrangian Structure Lagrangia** Axion Interactions and axion-photon interactions and axionwill face the case in which axions were in the case i GCstars&Whitedwarfcooling(electrons)

• with gluons $\bullet\text{ with }\mathfrak{g}$ and in the mode dark matter for fa use of the 1011 states for fact the 1011 states for fact the 1011 states for fact the 1011 beled). For (ma^e, TR) combinations within the gray bands, the and in the nominal set model independent uon
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1006 Excess rate of \overline{C} i
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ion Interactions
\n
$$
\mathcal{L}_{agg} = \frac{g_s^2}{32\pi^2 f_a} a G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}
$$
\n
$$
\mathcal{L}_{a\gamma\gamma} = \frac{e^2 C_{a\gamma\gamma}}{32\pi^2 f_a} a F_{\mu\nu} \widetilde{F}^{\mu\nu}
$$
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$$
(or \ \mathcal{L}_{c\gamma\gamma} = \frac{g_{a\gamma\gamma}}{2} a F_{\mu\nu} \widetilde{F}^{\mu\nu})
$$

• with photons **CHUCHL** model dependent de .
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\n
$$
\mathcal{L}_{a\gamma\gamma} = \frac{e^2 C_{a\gamma\gamma}}{32\pi^2 f_a} a F_{\mu\nu} \widetilde{F}^{\mu\nu}
$$
\n
$$
(\text{or } \mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \widetilde{F}^{\mu\nu})
$$

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prics, M librium with the primordial plasma before decoupling at $\frac{dX[O]}{S}$ as a function of the temperature $\frac{G}{S}$. ne axion lifetime $\tau_a = \Gamma^{-1}_{a \to \gamma \gamma} = \frac{Q_{\rm T} T_{a}}{2}$ α axion meanner $a = a \rightarrow \gamma \gamma$ = by ^G!^a $\mathcal{L}_{\mathcal{L}}$ and $\mathcal{L}_{\mathcal{L$ **Figure 2018** and Field strength tensor and Figures of the LOSP, we turn now to the phenomenologically $\frac{1}{2}$ $\frac{1}{2}$ in the second searches
we the system lifetimes E^{-1} 64π is the axion metric $a - I_{a \to \gamma\gamma} - \frac{1}{g_{a \gamma\gamma}^2 m_a^3}$ but crucial for axion searches and governs the axion lifetime

Excessradiation HotDM

^Q [−] ² 4 + z

5

 $\frac{1}{2}$ $\frac{1}{2}$

Axion Searches

Updated figure provided by M. Kuster for [FDS, 0811.3347] see also [Battesti et al., 0705.0015]

Axion Dark Matter FIG. 4. The axion density parameter from thermal processes

xion Condensate: CDM \sum is striggled is still relativistic to \sum in \sum Axion Condensate: CDM $\overline{1+\frac{1}{2}}$ Axion Condonsato: CDM $\frac{1}{\sigma^2}$ T MOIT CONGUISALU. CDTT

 $\text{IS } 2 \quad 0.15 \frac{\partial^2 f}{\partial x^2}$ /10¹² C₀V V ⁷/6 $\begin{equation} \begin{array}{ll} \frac{1}{2} & \Omega_a^{\rm MIS}h^2 \sim 0.15 \, \theta_i^2 (f_{\rm PQ}/10^{12}\,{\rm GeV})^{7/6} \end{array} \end{equation}$ $\begin{bmatrix} \frac{1}{2} \end{bmatrix}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]

Axion Dark Matter FIG. 4. The axion density parameter from thermal processes

xion Condensate: CDM \sum is striggled is still relativistic to \sum in \sum Axion Condensate: CDM $\overline{1+\frac{1}{2}}$ Axion Condonsato: CDM \bigodot with condensate. CDT

 $\text{IS } 2 \quad 0.15 \frac{\partial^2 f}{\partial x^2}$ /10¹² C₀V V ⁷/6 $\Omega_a^{\rm MIS} h^2 \sim 0.15 \theta_i^2 (f_{\rm PQ}/10^{12}\,{\rm GeV})^{7/6}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]

\mathbf{A} for many formation \mathbf{A} Axions can provide CDM a h2 for higher TR with Axions can provide CDM ter condensate with the axions from thermal processes,

Frank D. Steffen (Max-Planck-Institute of Physics, Munich) On calculations of cosmic relic abundances of EWIPs 9 $\frac{1}{2}$ $\frac{1}{2}$ from hard grad from hard gluons with $\frac{1}{2}$ $\frac{$ $\frac{1}{2}$. The respective soft and $\frac{1}{2}$. The respective soft and $\frac{1}{2}$ summing soft and $\frac{1}{2}$ soft and $\frac{1}{2}$ summing soft and $\frac{1}{2}$ summing soft and $\frac{1}{2}$ summing soft and $\frac{1}{2}$ summing soft

On calculations of cosmic relic abundances of EWIPs

Cosmic Relic Abundances

 \lceil details \rightarrow blackboard]

• TR $>$ T_D: $1+2 \rightleftarrows 3+X$ reheating temp. θ decoupling temp. of X

> $T > T_D$: X in thermal eq. with the primordial plasma T ~ T_D: X decouples as a **hot thermal relic**

• T_R > T_D : $1+2$ \rightarrow 3+X

• Hard Thermal Loop (HTL) Resummation &

[details \rightarrow blackboard] kcut such that game that game that game the weak coupling that game the weak coupling the weak coupling that g Γ dotoile \rightarrow −1 | details of (5) follows Ref. [10]. The leading order contribution to

 $\overline{\mathsf{F}}$

[Graf, FDS,1008.4528]

Thermal Axion Production in the Hot QGP 2

 $=$ E g6 $=$ E \pm

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 $\sum_{n=1}^{\infty}$ cancer

 \sim

 \cdot rank D. Steffen $\,$ (Max-Planck-Institute of Physics, Munich) $\,$ $\,$

Axion Dark Matter FIG. 4. The axion density parameter from thermal processes

xion Condensate: CDM \sum is striggled is still relativistic to \sum in \sum Axion Condensate: CDM $\overline{1+\frac{1}{2}}$ Axion Condonsato: CDM \bigodot with condensate. CDT

 $\text{IS } 2 \quad 0.15 \frac{\partial^2 f}{\partial x^2}$ /10¹² C₀V V ⁷/6 $\Omega_a^{\rm MIS} h^2 \sim 0.15 \theta_i^2 (f_{\rm PQ}/10^{12} \, {\rm GeV})^{7/6}$ $\begin{bmatrix} \frac{1}{2} \end{bmatrix}$ [..., Sikivie, '08; Kim, Carosi, '08, ...]

Axion Dark Matter cesses Started to differ by more than 5% from Y equality of the Started to differ by more than 5% from Y equal $\overline{}$ FIG. 4. The axion density parameter from thermal processes **The axion conduction co**

Axion Dark Matter cesses Started to differ by more than 5% from Y equality of the Started to differ by more than 5% from Y equal $\overline{}$ **The axion conduction co**

Extremely Weakly Interacting Particles (EWIPs)

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Axino Interactions would form at the set of α Axino–Interactions

● with gluons and gluinos **by a model indeperated** her temperatures. The reduced independent TR and a such that and axions were neglected model independent ι
α 8nnnos
8nnnos

$$
\mathcal{L}_{\widetilde{a}\widetilde{g}g}=i\,\frac{g_{\rm s}^2}{64\pi^2f_a}\,\bar{\widetilde{a}}\,\gamma_5\,[\gamma^\mu,\gamma^\nu]\;\widetilde{g}^a\,G^a_{\mu\nu}
$$

• with photons/Z-bosons and binos in model dependent **arished**
are $\vec{a} \gamma$
 $\frac{YY}{f_a}$
 $\frac{YY}{f_a}$ ns and binos model dependent \sim $t_i\left[\gamma_\mu,\gamma_\nu\right] B\,\left(\cos\theta_W F_{\mu\nu}-\sin\theta_W Z_{\mu\nu}\right)$ $\overline{\tilde{a}}\sim_{\overline{a}}\overline{a}$ α Gaussian Company (6) $16\pi f_a$ ⁰ $\frac{1}{2}$ **•** with photons/Z-bosons $\mathcal{L}_{\approx \widetilde{P}}$ in $\overline{Z} = i \frac{\alpha_Y C_{\text{a}YY}}{\widetilde{a}} \overline{\widetilde{q}} \gamma_5 [\gamma_{\mu}, \gamma_{\nu}] \widetilde{B} (\cos \theta_W F_{\mu\nu} {\cal L}_{\widetilde{a}\widetilde{B}\gamma/Z}=i\,\frac{\widetilde{\omega_1\circ a_{1}}\,{\mathrm i}}{16\pi f_a}\,\tilde{a}\,\gamma_5\,\left[\gamma_\mu,\gamma_\nu\right]\,B\,\left(\cos\theta_W F_{\mu\nu}-\right)$ της είναι τους από το προϊόντα στη συνεργασία τους από το προϊόντα στη συνεργασία τους από το προϊόντα στη συν
Επιχειρηματικές συνεργασίας στη συνεργασία τους από το προϊόντα στη συνεργασία τους από το προϊόντα στη συνεργ \bullet with photons^{7} bosons and h $\alpha_{\rm Y} C_{\rm aYY}$ $16\pi f_a$ $\tilde{a}\,\gamma_5\, \left[\gamma_\mu,\gamma_\nu\right]\, B\, \left(\cos\theta_W F_{\mu\nu} - \sin\theta_W Z_{\mu\nu}\right)$

with axion far exing contractions but crucial ion axino searches but crucial for axino searches Few man diagrams of the dominant contributions to the dominant contributions to the dominant contributions to the dominant contributions of the dominant contributions to the dominant contributions of the dominant contribut

4 + z

[Brandenburg, FDS, '04]

Thermal Production of

Axino Dark Matter

in the Early Universe

Axino Number Density for $f_a > T_D > T_R \gtrsim 10^4$ GeV

• Boltzmann equation: time evolution of axino density $n_{\tilde{a}}$ in the thermal bath

$$
\frac{dn_{\tilde{a}}}{dt} + 3Hn_{\tilde{a}} = C_{\tilde{a}} = \int d^3p \frac{d\Gamma_{\tilde{a}}}{d^3p} \quad \leftarrow \quad \text{generation of } \tilde{a} - \text{annihilation of } \tilde{a}
$$

 $(C_{a+b\rightarrow c+\tilde{a}} \in C_{\tilde{a}})$ • collision term for $a(p_1) + b(p_2) \rightarrow c(p_3) + \tilde{a}(p)$:

$$
C_{a+b\to c+\tilde{a}} = \int \frac{d^3p}{(2\pi)^3 2E} \int \left[\prod_{i=1}^3 \frac{d^3p_i}{(2\pi)^3 2E_i} \right] (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p)
$$

$$
\times \left[|M_{a+b\to c+\tilde{a}}|^2 f_a f_b (1 \pm f_c) (1 - f_{\tilde{a}}) - |M_{c+\tilde{a}\to a+b}|^2 f_c f_{\tilde{a}} (1 \pm f_a) (1 \pm f_b) \right]
$$

• phase space densities: $f_i \longrightarrow$ number densities: $n_i = \int \frac{d^3p_i}{(2\pi)^3 2E_i} g_i f_i(E_i)$

a, b, and c:
$$
f_i = f_i^{\text{eq}} = f_{B/F} = \frac{1}{\exp(E_i/T) \mp 1}
$$
, axis: $f_{\tilde{a}} \approx 0$

[Kim, '79; Shifman, Vainshtein, Zakharov, '80] Axino Interactions <- Hadronic (KSVZ) Axion Models

• axino-gluino-gluon interaction:

$$
\mathcal{L}_{\tilde{a}\tilde{g}g} = i \, \frac{\alpha_{\rm s}}{16\pi (f_a/N)} \, \bar{\tilde{a}} \, \gamma_5 \, \left[\gamma^\mu, \gamma^\nu \right] \, \tilde{g}^a \, G^a_{\mu\nu}
$$

Thermal Axino Production in SUSY QCD

g c

a

• A: $q^a + q^b \rightarrow \tilde{q}^c + \tilde{a}$ a g a g a g a g a \widetilde{a} a c g + $\begin{array}{ccc} \widetilde{\mathbf{g}}^{\mathrm{a}} & \mathbb{S} & + & \widetilde{\mathbf{g}}^{\mathrm{b}} \mathbb{S} \end{array}$ + $\qquad \qquad +$ + $\mathrm{\widetilde{g}}^\mathrm{b}$ b c g^{b} regges and \widetilde{g}^{c} g^{b} regges to $\mathrm{\mathring{g}}^{\mathrm{c}}$ g b g $\widetilde{\mathrm{g}}$ • B: $g^a + \tilde{g}^b \rightarrow g^c + \tilde{a}$ (crossing of A) • C: $\tilde{q}_i + g^a \rightarrow \tilde{q}_j + \tilde{a}$ $\widetilde{\mathbf{q}}_\text{i}$ a a $\widetilde{\mathrm{g}}$ a g q_j • D: $g^a + q_i \rightarrow \tilde{q}_j + \tilde{a}$ (crossing of C) • E: $\overline{q}_i + q_j \rightarrow g^a + \tilde{a}$ (crossing of C) • F: $\tilde{g}^a + \tilde{g}^b \rightarrow \tilde{g}^c + \tilde{a}$ g a g a a a a $\widetilde{\mathrm{g}}$ a c g g a b + + g b c $\widetilde{\textbf{g}}^{\text{b}}$ assoc $\widetilde{\textbf{g}}^{\text{c}}$ b c $\widetilde{\mathrm{g}}$ $\widetilde{\mathrm{g}}$ $\widetilde{\mathrm{g}}$ $\widetilde{\mathrm{g}}$ • G: $q_i + \tilde{g}^a \rightarrow q_j + \tilde{a}$ q_i a a g g a q_j • H: $\tilde{q}_i + \tilde{g}^a \rightarrow \tilde{q}_j + \tilde{a}$ $\widetilde{\mathbf{q}}_i$ a ag a $\widetilde{\mathrm{g}}$ $\widetilde{\text{q}}_\text{j}$

- I: $q_i + \bar{q}_j \rightarrow \tilde{g}^a + \tilde{a}$ (crossing of G)
- J: $\tilde{q}_i + \overline{q}_j \rightarrow \tilde{g}^a + \tilde{a}$ (crossing of H)

B, F, G, & H: Logarithmic IR Singularity

$$
E\frac{d\Gamma_{\tilde{G}}}{d^3p}\bigg|_{\text{LO in }g} = E\frac{d\Gamma_{\tilde{G}}}{d^3p}\bigg|_{\text{soft}} + E\frac{d\Gamma_{\tilde{G}}}{d^3p}\bigg|_{\text{hard}} = A_{\text{soft}} + A_{\text{hard}} + B\ln\left[\frac{1}{g}\right]
$$

BBN

3.7 × 10[−]¹¹

=

 $\mathcal{L} = 2.44 \pm 0.000$. The decay analysis: mGeV $\mathcal{L} = 2.44 \pm 0.000$ and $\mathcal{L} = 2.44 \pm 0.000$. The decay analysis: $\mathcal{L} = 2.44 \pm 0.000$

! ^m^G^e

=

! ^m^G^e

MPl ⁼ ².⁴⁴ [×] ¹⁰¹⁸ GeV ... ⁺ ^τ^e decay analysis: ^m^Ge, ^MPl (?), ...

3.7 × 10[−]¹¹

! ^m^G^e

=

<u>T e seu a component de la pa</u>

3.7 × 10[−]¹¹

eq.

 $\overline{}$ NLSP ! LSP + SM

 $\overline{}$, the decay analysis $\overline{}$. The decay analysis $\overline{}$

The Collision Term to Leading Order in the Coupling g

• Collision Term:
$$
C_{\tilde{a}}(T) = \int d^3p \left(\frac{d\Gamma_{\tilde{a}}}{d^3p} \Big|_{\text{soft}} + \frac{d\Gamma_{\tilde{a}}}{d^3p} \Big|_{\text{hard}} \right)
$$

$$
C_{\tilde{a}}(T) = \frac{(N_c^2 - 1)}{(f_a/N)^2} \frac{3\zeta(3)g^6 T^6}{4096\pi^7} \left[\ln\left(\frac{1.380\,T^2}{m_g^2}\right) (N_c + n_f) + 0.4336\,n_f \right]
$$

 \bullet Thermal Gluon Mass in the "QGSGP":

$$
m_g^2 = \frac{g^2 T^2}{6} (N_c + n_f) \quad \text{with} \quad N_c = 3 \quad \text{and} \quad n_f = 6
$$

• Running of the Strong Coupling in the MSSM:

$$
g(T) = \left(g^{-2}(M_Z) + \frac{3}{8\pi^2} \ln\left[\frac{T}{M_Z}\right]\right)^{-1/2} \longrightarrow 0.85 \text{ for } T \approx 10^{10} \text{ GeV}
$$

Solving the Boltzmann Equation \rightarrow Axino Abundance

Boltzmann equation

$$
\frac{dn_{\tilde{a}}}{dt} + 3Hn_{\tilde{a}} = C_{\tilde{a}}
$$

• conservation of entropy

$$
sR^3 = \text{const.}
$$

• yield \rightarrow scale out expansion

$$
Y_{\tilde{a}} = \frac{n_{\tilde{a}}}{s}
$$

Boltzmann equation

$$
\frac{d}{dt}Y_{\tilde{a}} = \frac{C_{\tilde{a}}}{s}
$$

• radiation dominated epoch

$$
dt = -\frac{dT}{H(T)T}, \ H(T) = \sqrt{\frac{g_*(T)\pi^2}{90}} \frac{T^2}{M_{\text{Pl}}}
$$

• entropy density \t\t\tMSSM

$$
s(T) = \frac{2\pi^2}{45} g_{*S}(T) T^3, \quad g_{*S} = g_* = \frac{915}{4}
$$

\n- Axiino Yield from Thermal Production
$$
Y_{\tilde{a}} \approx \frac{C_{\tilde{a}}(T_R)}{s(T_R)H(T_R)} = 2.0 \times 10^{-7} g^6 \ln\left(\frac{1.108}{g}\right) \left(\frac{10^{11} \text{ GeV}}{f_a/N}\right)^2 \left(\frac{T_R}{10^4 \text{ GeV}}\right)
$$
\n- Axino Density from Thermal Production $\Omega_{\tilde{a}} h^2 = m_{\tilde{a}} Y_{\tilde{a}} s(T_0) h^2 / \rho_c = 5.5 g^6 \ln\left(\frac{1.108}{g}\right) \left(\frac{m_{\tilde{a}}}{0.1 \text{ GeV}}\right) \left(\frac{10^{11} \text{ GeV}}{f_a/N}\right)^2 \left(\frac{T_R}{10^4 \text{ GeV}}\right)$
\n

Axino LSP Case

Axino LSP Case

Extremely Weakly Interacting Particles (EWIPs)

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$$
\Omega_{\tilde{G}}^{\text{TP}}h^2 = m_{\tilde{G}} Y_{\tilde{G}}^{\text{TP}}(T_0) s(T_0) h^2/\rho_c
$$
\n
$$
= \sum_{i=1}^3 \omega_i g_i^2(T_{\text{R}}) \left(1 + \frac{M_i^2(T_{\text{R}})}{3m_{\tilde{G}}^2}\right) \ln\left(\frac{k_i}{g_i(T_{\text{R}})}\right)
$$
\n
$$
\times \left(\frac{m_{\tilde{G}}}{100 \text{ GeV}}\right) \left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}}\right)
$$
\nThe dark matter density

\n
$$
\Omega_{\text{dm}}^{3\sigma} h^2 = 0.105_{-0.030}^{+0.021}
$$
\nprobes

\nthe reheating temperature

Thermal Leptogenesis

dark matter

!DM = 20 %

Thermal Leptogenesis

The gravitino can become a problem ...

Upper Limits on the Reheating Temperature

$p = \frac{1}{2}$ ${\tt requires\ T>10^9\ GeV}$ requires the upper bound on True case is sensitive to the upper bound on True case is sensitive to the observation of the obser **Thermal Leptogenesis requires T >10⁹ GeV** $A = \frac{1}{2}$ Thermal Leptogenesis requires T>

Upper Limits on the Reheating Temperature Upper Limits on

${\tt requires\ T>10^9\ GeV}$ requires the upper bound on True case is sensitive to the upper bound on True case is sensitive to the observation of the obser ϵ quires T $>$ 10⁹ GeV **Thermal Leptogenesis requires T >10° GeV** ral gravitino LSP mass range in gravity-mediated super-Phys. Lett. B588, 7 (2004).

Conclusion

Refined calculations of the thermal production of extremely weakly interacting particles (EWIPs) are worth pursuing