Heavy quarkonium in a moving thermal bath

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 - Heavy quarkonium at weak coupling ($\Upsilon(1S), J/\psi$)
 - Weakly coupled QGP
 - Fruitful strategy in the case at rest (Miguel Angel Escobedo, JS; Phys. Rev. A 78, 032520 (2008), Phys. Rev. A 82, 042506 (2010))



(H. Pereira Da Costa, for the PHENIX collaboration, arXiv:1007.3688)

∇ "Gauge Field Dynamics In and Out of Equilibrium" INT UW, Seattle, March 15, 2012 ● Heavy guarkonium in a moving thermal bath – p.2/27



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• The J/ψ suppression depends on the rapidity



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- The J/ψ suppression may depend on the transverse momentum
- Is this due, at least in part, to the fact that the in-medium properties of J/ψ depend on the velocity ?

- $m \neq 0$, T = 0 case:
 - m (hard), electron mass
 - $m\alpha/n$ (soft), inverse Bohr radius, $\alpha = e^2/4\pi$; e, electron charge
 - $m\alpha^2/n^2$ (ultrasoft), binding energy

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- $m \neq 0$, $T \neq 0$ case: what is the interplay among the scales above?



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m ≠ 0, *T* ≠ 0 case: contributions of energies above *T* are exponentially suppressed by Boltzmann factors

Non-Relativistic QED (T=0)

$$\mathcal{L}_{NRQED} = -\frac{1}{4} d_1 F_{\mu\nu} F^{\mu\nu} + \frac{d_2}{m^2} F_{\mu\nu} D^2 F^{\mu\nu} + N^{\dagger} i D^0 N + + \psi^{\dagger} (i D^0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_F e \frac{\boldsymbol{\sigma} \mathbf{B}}{2m} + c_D e \frac{\boldsymbol{\nabla} \mathbf{E}}{8m^2} + + i c_S e \frac{\boldsymbol{\sigma} (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m^2}) \psi$$

(Caswell, Lepage, 1986)

Potential NRQED (T=0)

$$\begin{split} L_{pNRQED} &= -\int d^3 \mathbf{x} \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \int d^3 \mathbf{x} S^{\dagger}(t, \mathbf{x}) \left(i D_0 + \frac{\boldsymbol{\nabla}^2}{2m} + \frac{Z\alpha}{|\mathbf{x}|} + \right. \\ &+ \frac{\boldsymbol{\nabla}^4}{8m^3} + \frac{Ze^2}{m^2} \left(-\frac{c_D}{8} + 4d_2 \right) \delta^3(\mathbf{x}) + i c_S \frac{Z\alpha}{4m^2} \boldsymbol{\sigma} \cdot \left(\frac{\mathbf{x}}{|\mathbf{x}|^3} \times \boldsymbol{\nabla} \right) \right) S(t, \mathbf{x}) \\ &+ \int d^3 \mathbf{x} S^{\dagger}(t, \mathbf{x}) e \mathbf{x} \cdot \mathbf{E} S(t, \mathbf{x}) \,. \end{split}$$

(Pineda, Soto, 1997)

Hard Thermal Loops EFT (m=0)

$$\delta \mathcal{L}_{HTL} = \frac{1}{2} m_D^2 F^{\mu\alpha} \int \frac{d\Omega}{4\pi} \frac{k_\alpha k_\beta}{-(k.\partial)^2} F^{\mu\beta} + m_e^2 \bar{\psi} \gamma^\mu \int \frac{d\Omega}{4\pi} \frac{k_\mu}{k.\partial} \psi$$
$$k = (1, \mathbf{\hat{k}}), \qquad m_D^2 = e^2 T^2/3, \qquad m_e^2 = e^2 T^2/8$$

(Braaten, Pisarsky, 1992)

The $v \neq 0$ case

• Bound state at rest, the medium moves at velocity v (Weldom, 82)

$$f(\beta k^0) \rightarrow f(\beta^{\mu} k_{\mu}) = \frac{1}{e^{|\beta^{\mu} k_{\mu}|} \pm 1}, \ \beta^{\mu} = \frac{\gamma}{T}(1, \mathbf{v})$$

$$v = |\mathbf{v}|$$
, $\gamma = 1/\sqrt{1-v^2}$

- O(3) rotational symmetry is reduced to O(2)
- In light cone coordinates $k_+ = k_0 + k_3$, $k_- = k_0 k_3$

$$\beta^{\mu}k_{\mu} = \frac{1}{2}\left(\frac{k_{+}}{T_{+}} + \frac{k_{-}}{T_{-}}\right), \ T_{+} = T\sqrt{\frac{1+v}{1-v}}, \ T_{-} = T\sqrt{\frac{1-v}{1+v}}$$

- For $v \not\sim 1$ (moderate velocities), $T_+ \sim T \sim T_-$
- For $v \sim 1$ (relativistic velocities), $T_+ \gg T \gg T_-$
 - Collinear region, $k_+ \sim T_+$, $k_- \sim T_-$
 - Soft (ultrasoft) region, $k_+ \sim k_- \sim T_-$

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 - Thermal effects are encoded in the ultrasoft photons



• For $T = \beta^{-1} \ll m \alpha^2 / n^2$:

$$\delta E_n = -\frac{4\pi^3 \alpha}{45\beta^4} A_{ij}(v) \langle n | x^i \frac{\bar{P}_n}{(H_0 - E_n)} x^j | n \rangle \left(1 + \mathcal{O}\left((\frac{n^2}{\beta m \alpha})^2 \right) \right)$$

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• For
$$T=eta^{-1}\gg mlpha^2/n^2$$
, $l=0$:

$$\delta E_n = \frac{\alpha \pi T^2}{3m_e} - \frac{4Z\alpha^2}{3} \frac{|\phi_n(\mathbf{0})|^2}{m_e^2} \left(-\frac{1}{2v} \log\left(\frac{1+v}{1-v}\right) + 1 - \gamma \right) + \frac{2\alpha}{3\pi m_e^2} \sum_r |\langle n|\mathbf{p}|r\rangle|^2 (E_n - E_r) \log\left(\frac{2\pi T}{|E_n - E_r|}\right) \times (1 + \mathcal{O}\left((\frac{\beta m\alpha}{n^2})^2\right)$$

$$\delta\Gamma_n = \frac{2Z^2\alpha^3 T\sqrt{1-v^2}}{3n^2v} \log\left(\frac{1+v}{1-v}\right) \times \left(1 + \mathcal{O}\left(\frac{\beta m\alpha}{n^2}\right)\right)$$

• For $T = \beta^{-1} \gg m\alpha^2/n^2$, $l \neq 0$:

$$\delta E_{nlm} = \frac{\alpha \pi T^2}{3m_e} + \frac{2\alpha}{3\pi m_e^2} \sum_r |\langle n|\mathbf{p}|r\rangle|^2 (E_n - E_r) \log\left(\frac{-E_1}{|E_n - E_r|}\right) \\ - \frac{Z^3 \alpha^2 \langle 2l00|l0\rangle \langle 2l0m|lm\rangle}{2\pi m_e^2 a_0^3 n^3 l(l+\frac{1}{2})(l+1)} \rho(v)$$

$$\delta\Gamma_{nlm} = \frac{Z^2 \alpha^3 T \sqrt{1 - v^2}}{3n^2 v} \left(2 \log \left(\frac{1 + v}{1 - v} \right) + \left(\left(1 - \frac{3}{v^2} \right) \log \left(\frac{1 + v}{1 - v} \right) + \frac{6}{v} \right) \langle 2l00|l0\rangle \langle 2l0m|lm\rangle \right)$$

$$\rho(v) = \frac{1}{2v} \left(1 - \frac{1}{v^2} \right) \log \left(\frac{1+v}{1-v} \right) - \frac{2}{3} + \frac{1}{v^2}$$

 \bullet The decay width decreases as v increases !

• The $T \ll m$ case: NRQED can be used as a starting point • The potentials depend on T:

$$\begin{split} \delta \mathcal{L}_{pNRQED} &= \int d^3 \mathbf{x} \left(\frac{\alpha \pi T^2}{3m_e} \psi^{\dagger} \psi - \frac{\pi \alpha T^2}{6m_e^3} \nabla \psi^{\dagger} \nabla \psi \right) \\ &+ \int d^3 \mathbf{x_1} \, d^3 \mathbf{x_2} N^{\dagger}(t, \mathbf{x_2}) N(t, \mathbf{x_2}) \left[-\frac{4Z\alpha}{3m_e^2} \left(\log \left(\frac{\mu}{2\pi T} \right) - \log 2 \right) \right. \\ &+ \gamma + \frac{3}{8v} \left(1 + \frac{1}{3v^2} \right) \log \left(\frac{1+v}{1-v} \right) - \frac{1}{4v^2} \right) \delta^3(\mathbf{x_1} - \mathbf{x_2}) \\ &+ \frac{\alpha \rho(v) v^i v^j \partial_{ij}^2 V_c(r)}{4\pi m_e^2 v^2} \right] \psi^{\dagger}(t, \mathbf{x_1}) \psi(t, \mathbf{x_1}) \end{split}$$

 μ, factorization scale arising from IR divergences, which cancels against the one of ultrasoft contributions in physical observables



 $v \not\sim 1$: the $T \ll m$ case

• In the ultrasoft contributions,

$$1/(e^{|\beta^{\mu}k_{\mu}|}-1) \to 1/|\beta^{\mu}k_{\mu}|-1/2 + \cdots$$

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$$\delta E_{nlm} = \frac{\alpha \pi T^2}{3m_e} - \frac{\pi \alpha^3 T^2}{6m_e n^2} + \frac{4Z\alpha^2}{3m_e^2} \left(\gamma + \frac{3}{8v} \left(1 + \frac{1}{3v^2}\right) - \frac{1}{4v^2}\right) |\phi_n(\mathbf{0})|^2 - \frac{\alpha \rho(v) v^i v^j}{4\pi m_e^2 v^2} \langle n |\partial_{ij}^2 V_c(r)|n \rangle + \frac{2\alpha}{3\pi m_e^2} \sum_r |\langle n|\mathbf{p}|r \rangle|^2 (E_n - E_r) \left(\log\left(\frac{2\pi T}{|E_n - E_r|}\right) + \frac{5}{6}\right)$$

$$\delta\Gamma_{nlm} = \frac{Z^2 \alpha^3 T \sqrt{1 - v^2}}{3n^2 v} \left(2 \log\left(\frac{1 + v}{1 - v}\right) + \left(\left(1 - \frac{3}{v^2}\right) \log\left(\frac{1 + v}{1 - v}\right) + \frac{6}{v}\right) \langle 2l00|l0\rangle \langle 2l0m|lm\rangle \right)$$

Relativistic velocity ($v \sim 1$)

- The $T_+ \sim m\alpha/n \gg T_- \gg m\alpha^2/n^2$ case: NRQED can be used as a starting point
 - The collinear photons $(k_+ \sim T_+, k_- \sim T_-)$ have virtualities $\sim T^2 \ll (m\alpha/n)^2$ and hence must be kept in the effective theory: pNRQED \rightarrow pNRQED + SCET

$$\delta \mathcal{L}_{SCET} = c_1 \frac{\psi^{\dagger} \psi}{m_e} \frac{\bar{n}^{\mu} F_{\mu i}}{(\bar{n}\partial)} \frac{\bar{n}^{\nu} F_{\nu i}}{(\bar{n}\partial)} + c_2 \frac{\psi^{\dagger} \psi}{m_e} \frac{\bar{n}^{\mu} n^{\nu} F_{\mu \nu}}{(\bar{n}\partial)} \frac{\bar{n}^{\alpha} n^{\beta} F_{\alpha\beta}}{(\bar{n}\partial)} + \cdots$$

 $F_{\mu\nu}$, collinear photons; ψ , NR electron field n = (1, 0, 0, 1), $\bar{n} = (1, 0, 0, -1)$

The pNRQED Lagrangian for ultrasoft photons remains the same

 $v \sim 1$: the $T_+ \sim m\alpha/n \gg T_- \gg m\alpha^2/n^2$ case

Calculation in pNRQED+ SCET

•
$$\delta E_n^{\text{col}} = \frac{\pi \alpha T^2}{3m_e}$$
, $\delta \Gamma_n^{\text{col}} = 0$

• There are three relevant regions in the us contribution:

•
$$k_+, k_- \sim T_-$$

- $k_+, k_- \sim m\alpha^2/n^2$
- $k_{+} \sim m\alpha^{2}/n^{2}$ and $k_{-} \sim m\alpha^{2}/n^{2}(T_{-}/T_{+})$

$$\delta E_{n00}^{\rm us} = -\frac{4Z\alpha^2}{3} \left(1 - \gamma + \frac{1}{2} \log\left(\frac{1-v}{1+v}\right) \right) \frac{|\phi_n(0)|^2}{m_e^2} \\ -\frac{2\alpha}{3\pi m_e^2} \sum_r |\langle n|\mathbf{p}|r\rangle|^2 (E_n - E_r) \log\left(\frac{|E_n - E_r|}{2\pi T}\right) \\ \delta \Gamma_{n00}^{\rm us} = \frac{4Z^2\alpha^3 T}{3n^2} \sqrt{\frac{1-v}{1+v}} \log\left(\frac{1+v}{1-v}\right)$$

• Agreement with the $v \rightarrow 1$ limit of the $v \not\sim 1$ case

Relativistic velocity ($v \sim 1$)

- The $T_+ \sim m \gg m \alpha / n \gg T_- \gg m \alpha^2 / n^2$ case: QED must be used as a starting point
 - The collinear photons $(k_+ \sim T_+, k_- \sim T_-)$ have virtualities $\sim T^2 \ll m^2$ and hence must be kept in the effective theory: NRQED \rightarrow NRQED + SCET
 - The collinear photons have virtualities $\gg T_{-}^2$ and can be integrated out in the matching to pNRQED. They produce a global energy shift.
 - The pNRQED Lagrangian for ultrasoft photons remains the same
 - Agreement with the $v \rightarrow 1$ limit of the $v \not\sim 1$ case

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- One obtains a v-dependent HTL Lagrangian in the photon sector and v-dependent temperature corrections to the NRQCD matching coefficients
- One next matches to pNRQED, obtaining a v and T dependent potential

$$V(r,T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T \phi(m_D r) + \mathcal{O}\left(\frac{\alpha T^2}{m_\mu}\right)$$

$$\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2+1)^2} \left[\frac{\sin(zx)}{zx} - 1 \right]$$

- It has an imaginary part ! (Laine, Philipsen, Romatschke, Tassler, 06; Escobedo, JS, 08; Brambilla, Ghiglieri, Vairo, Petreczky, 08)
- The disociation temperature becomes

$$T_d \sim m_\mu \alpha^{2/3} / \ln^{1/3} \alpha < m_\mu \alpha^{1/2}$$

where $m_{\mu}\alpha^{1/2}$ is the scale of the disociation temperature for the screening mechanism (Matsui, Satz, 86)

- $\Re V(r,T)$ calculated before (Chu, Matsui, 89)
- V(r,T) is given by the Fourier transform of the longitudinal photon propagator $\Delta_{11}(k)$ at $k^0 = 0$ in the *v*-dependent HTL Lagrangian

$$\Delta_{11}(k) = \frac{1}{2} [\Delta_R(k) + \Delta_A(k) + \Delta_S(k)]$$

 $\Delta_R^*(k) = \Delta_A(k)$, $\Delta_S(k)$ contains the imaginary part

• $\Delta_S(k)$ must be calculated through the following formula, which differs from the one of the v = 0 case (Carrington, Hou, Thoma, 97)

$$\Delta_S(k,u) = \frac{\Pi_S(k,u)}{2i\Im\Pi_R(k,u)} (\Delta_R(k,u) - \Delta_A(k,u))$$

 $u = \gamma(1, \mathbf{v})$. Recall that in the real time formalism $\Pi_R = \Pi_{11} + \Pi_{12}$, $\Pi_S = \Pi_{11} + \Pi_{22}$, $\Delta_R(k) = 1/(\mathbf{k}^2 - \Pi_R)$

• $\Pi_R(k, u)$ is a (complex) function of v and θ , $\mathbf{kv} = |\mathbf{k}| v \cos \theta$, that reduces to $-m_D^2$ when v = 0

We obtain

$$\Pi_S(k,u) = \frac{i2\pi m_D^2 T (1-v^2)^{3/2} (1+\frac{v^2}{2}\cos^2\theta)}{|\mathbf{k}|(1-v^2\sin^2\theta)^{5/2}}$$



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 - Let m_d be the momentum scale for which the real and imaginary part of the potential are equal. If $m_d \gg m_D$, as in the v = 0 case, we find (for $\theta \approx \pi/2$) $m_d \sim \alpha^{1/3}T\sqrt{1-v^2}$. As long as $m_D \ll m_d$, the Landau damping remains the dominant mechanism

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 - Since m_D remains non-vanishing when $v \to 1$, there will be some critical v for which $m_d = m_D$ and both mechanisms Landau damping and screening compete
 - For v sufficiently close to 1 the imaginary part is neglegible and the dominant mechanism for dissociation is screening

v = 0





v = 0.5



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v = 0.9





v = 0.99



Conclusions

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 The decay width of a heavy quarkonium moving in a QGP decreases with its velocity

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- The decay width of a heavy quarkonium moving in a QGP decreases with its velocity
- The dominant mechanism for dissociation changes from Landau damping to screening as the velocity increases

