Heavy quarks in the quark–gluon plasma

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INT, 14 March 2012

Outline

Background

Charm

- Temperature dependence
- Reconstructed correlators
- Nonzero momentum
- Towards the physical limit
- Charm diffusion

Beauty

- Correlators
- Spectral functions
- Relativistic beauty

Nonzero density

Summary and outlook

Background

- J/ψ suppression — a probe of the quark–gluon plasma?
- Sequential suppression $\longrightarrow$ quarkonia as QGP thermometers?
- c, b quarks created in primordial collisions, hard probes?
- To what extent do c, b quarks thermalise?
- Quenched lattice results suggest that S-waves survive well into the plasma phase
- Sequential charmonium suppression + recombination explains experimental results?
- Uncertainty about which potential to use in potential models, how to treat continuum
- How reliable are quenched lattice simulations?

Quenched vs dynamical

Are quenched lattice results reliable?

- $T_c^{N_f=0} \approx 1.5\,T_c^{N_f=2+1}$, $T_c^{N_f=2} \approx T_c^{N_f=2+1}$
- No $D - \bar{D}$ threshold in quenched QCD
- Light quarks can catalyse $Q\bar{Q}$ dissociation so it occurs at lower temperature
- Lower T_c, lower T_d — conspire to give the same T_d/T_c?
- Potential models indicate little change in T_d/T_c

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- Potential models indicate little change in T_d/T_c
- Only dynamical lattice calculations can give the answer

Dynamical anisotropic lattices

- A large number of points in time direction required
- For $T = 2T_c$, $\mathcal{O}(10)$ points $\Longrightarrow a_t \sim 0.025$ fm
- Far too expensive with isotropic lattices $a_s = a_t$!
- Fixed-scale approach
  $\longrightarrow$ need only 1 $T = 0$ calculation for renormalisation
- Independent handle on temperature

- Introduces 2 additional parameters
- Non-trivial tuning problem [PRD **74** 014505 (2006)]
  [See also Edwards, Joó, Lin, PRD **78** 054501 (2008)]

Spectral functions

- $\rho_\Gamma(\omega, \overrightarrow{p})$ related to euclidean correlator $G_\Gamma(\tau, \overrightarrow{p})$ according to

$$G_\Gamma(\tau, \overrightarrow{p}) = \int \rho_\Gamma(\omega, \overrightarrow{p}) \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} d\omega$$

- an ill-posed problem
- use Maximum Entropy Method to determine most likely $\rho(\omega)$
- requires a large number of time slices to have any chance of a reliable determination
- must introduce model function $m_0(\omega)$
- we have used continuum free spectral function + others

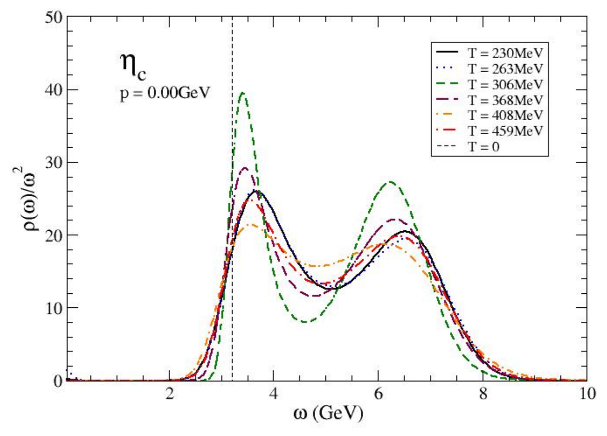
Simulation parameters

[PRD **76** 194513 (2007), arXiv:1005.1209]

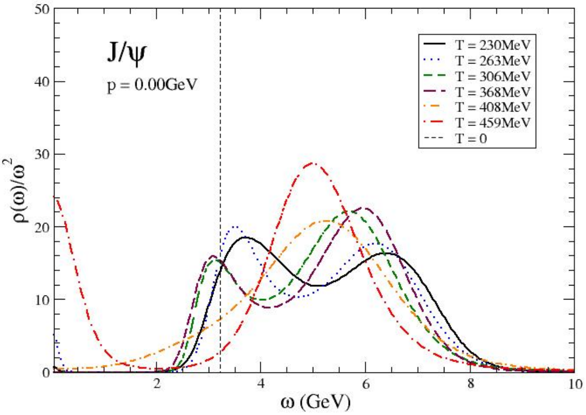
ξ	a_s (fm)	a_t^{-1} (GeV)	m_π/m_ρ	N_s	L_s (fm)
6.0	0.162	7.35	0.54	12	1.94

N_τ	T (MeV)	T/T_c	# configs
80	92	0.42	250
32	230	1.05	1000
28	263	1.20	1000
24	306	1.40	500
20	368	1.68	1000
18	408	1.86	1000
16	459	2.09	1000

S-wave T dependence (η_c)

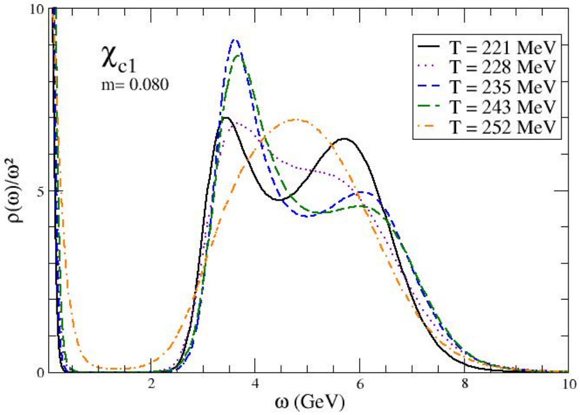


S-wave T dependence (J/ψ)



J/ψ (S-wave) melts at $T \sim 370 - 400$ MeV or $1.7 - 1.9T_c$?

P-waves



P-waves melt at $T < 250$ MeV or $1.2 T_c$?

Reconstructed correlators

Reconstructed correlator is defined as

$$G_r(\tau; T, T_r) = \int_0^\infty \rho(\omega; T_r) K(\tau, \omega, T) d\omega$$

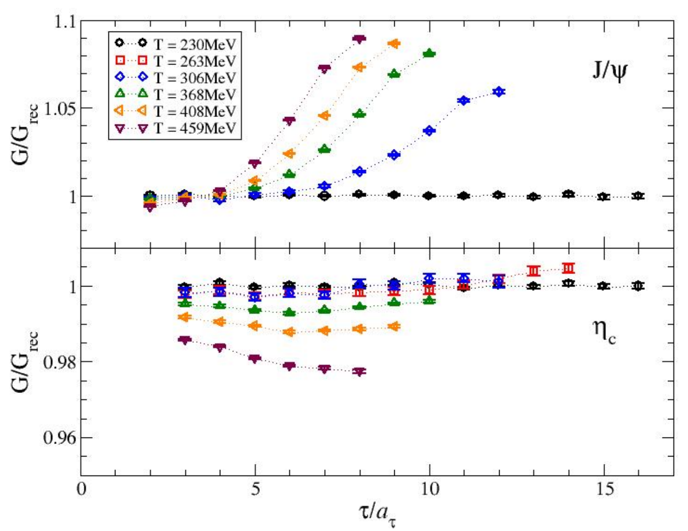
where K is the kernel

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

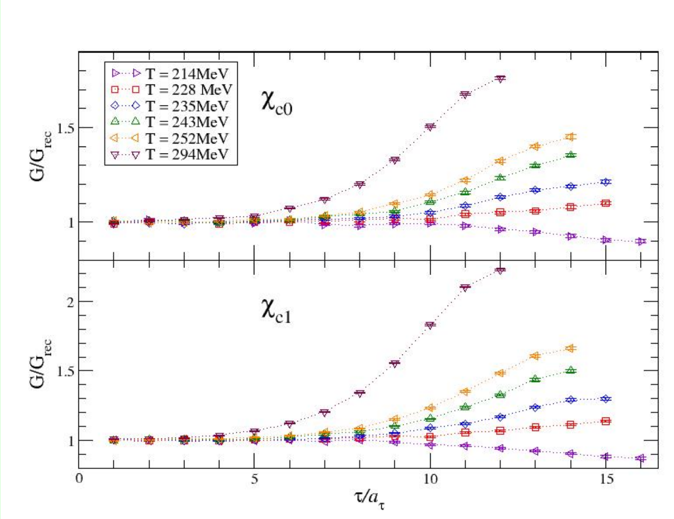
If $\rho(\omega; T) = \rho(\omega; T_r)$ then $G_r(\tau; T, T_r) = G(\tau; T)$

We use $N_\tau = 32$ as our reference temperature
since the spectral function is most reliably determined there

S-waves


T = 230MeV
T = 263MeV
T = 306MeV
T = 368MeV
T = 408MeV
T = 459MeV
J/ψ
η_c
G/G_rec
1.1
1.05
1
0.98
0.96
0
5
10
15
τ/a_τ

P-waves

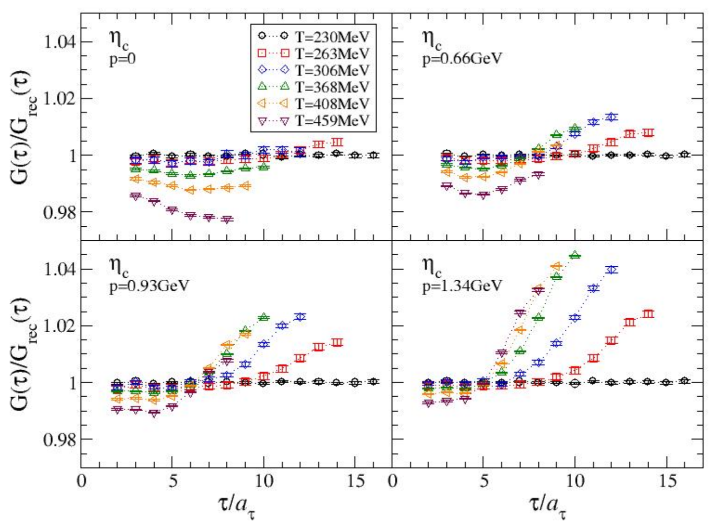

▷ T = 214MeV
□ T = 228 MeV
◇ T = 235MeV
△ T = 243MeV
◁ T = 252MeV
▽ T = 294MeV
χ_{c0}
χ_{c1}
G/G_{rec}
1
1.5
2
0
5
10
15
τ/a_τ

Nonzero momentum

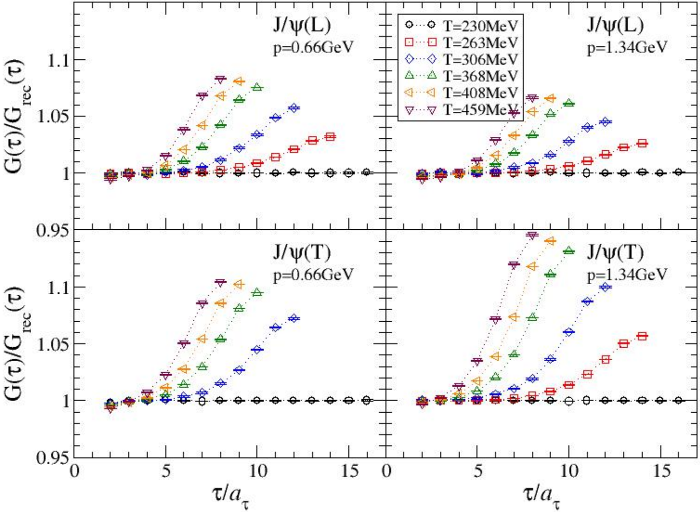
[With MB Oktay, arXiv:1005.1209]

- Charmonium is produced at nonzero momentum
- Transverse momentum (and rapidity) distributions important to distinguish between models
- Momentum dependent binding?
- Gives an additional window to transport properties
- Related to screening masses

Reconstructed correlators



Reconstructed correlators



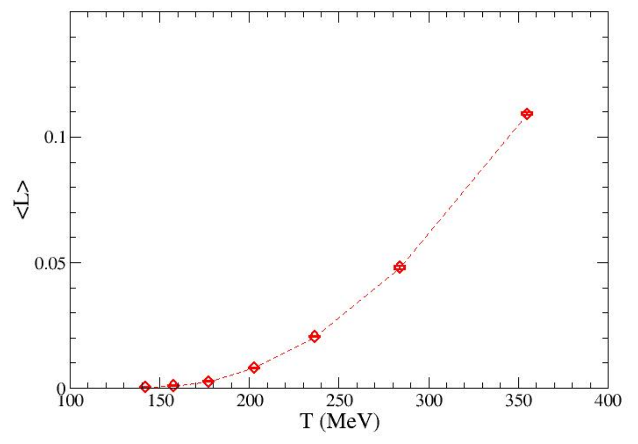
Towards the physical limit

Anisotropic clover-improved Wilson fermions, 2+1 flavours
[HadSpec Collab, PRD **79** 034502 (2009)]

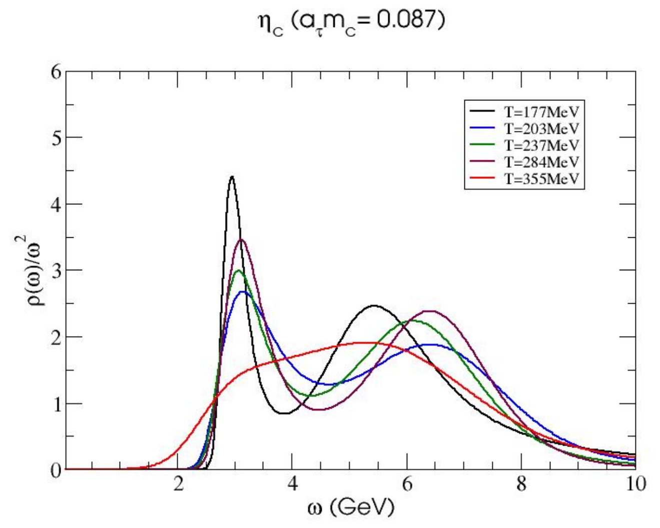
ξ	a_s (fm)	a_t^{-1} (GeV)	m_π/m_ρ	N_s	L_s (fm)
3.5	0.122	5.68	0.45	24	2.93

N_τ	T (MeV)	T/T_c	# configs	used (c)	used (b)
125	35	0.25	100	—	100
40	142	0.8	380	—	103
36	158	0.9	193	—	67
32	177	1.0	1000	38	680
28	203	1.1	835	100	703
24	237	1.3	1000	57	735
20	284	1.6	1000	539	1000
16	355	2.0	395	102	290

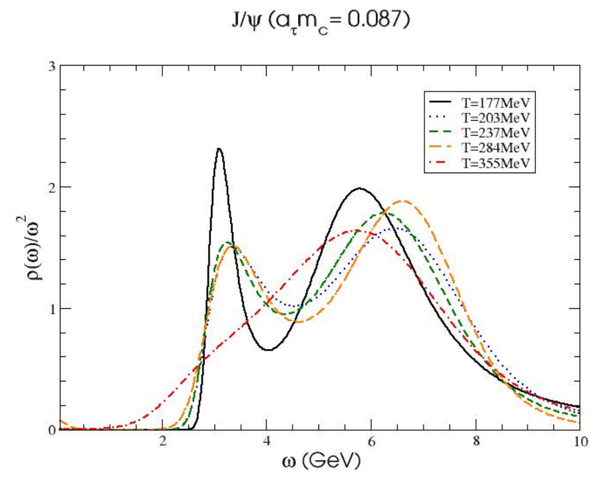
Polyakov loop (Unrenormalised)



Pseudoscalar spectral function



Vector spectral function



Charm diffusion

How fast do charm quarks thermalise?

The heavy quark diffusion constant D is given by

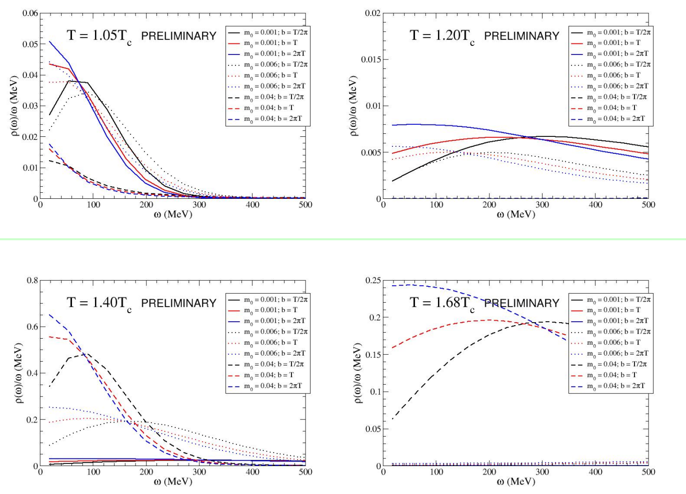
$$D = \frac{1}{\chi^{00}} \lim_{\omega \to 0} \frac{\rho_V(\omega)}{\omega} \, ,$$

ρ_V is the spectral function of the conserved-current operator $V_i(\overrightarrow{x}, t)$

$$\chi^{00} = \frac{1}{T} \int \langle V_0(\overrightarrow{x}, t) V_0(\overrightarrow{0}, t) d^3x$$

Preliminary results using default model $m(\omega) = m_0 \omega (b + \omega)$

Results


T = 1.05T_c PRELIMINARY
T = 1.20T_c PRELIMINARY
T = 1.40T_c PRELIMINARY
T = 1.68T_c PRELIMINARY
m_0 = 0.001; b = T/2π
m_0 = 0.001; b = T
m_0 = 0.001; b = 2πT
m_0 = 0.006; b = T/2π
m_0 = 0.006; b = T
m_0 = 0.006; b = 2πT
m_0 = 0.04; b = T/2π
m_0 = 0.04; b = T
m_0 = 0.04; b = 2πT
ρ(ω)/ω (MeV)
ω (MeV)
0
100
200
300
400
500
0.01
0.02
0.03
0.04
0.05
0.06
0.005
0.015
0.2
0.4
0.6
0.8
0.05
0.1
0.15
0.2
0.25

Beauty (and the beast?)

- Many *b* quarks will be produced at LHC
  → Recent results from CMS, ATLAS (+ STAR)
- Cold nuclear matter effects, recombination less important
  → cleaner probes?
- $T_d^{\Upsilon} \sim 3-5\,T_c$ — hard to do on the lattice
- χ_b melts at $T_d^{\chi_b} \lesssim 1.2\,T_c$?
- Use NRQCD and relativistic action,
  compare two approaches

NRQCD

Scale separation $M_Q \gg T, M_Q v$

Integrate out hard scales $\longrightarrow$ Effective theory

Expand in orders of heavy quark velocity $\mathbf{v}$; we use $\mathcal{O}(\mathbf{v}^4)$ action

Advantages

- No temperature-dependent kernel, $G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$
- No zero-modes
- Longer euclidean time range
- Appropriate for probes not in thermal equilibrium

Disadvantages

- Not renormalisable, requires $Ma_s \gtrsim 1$
- Does not incorporate transport properties

Correlators

[PRL **106** 061602 (2011)]

Bound state

$$G(\tau) \sim e^{-\Delta E \tau}$$

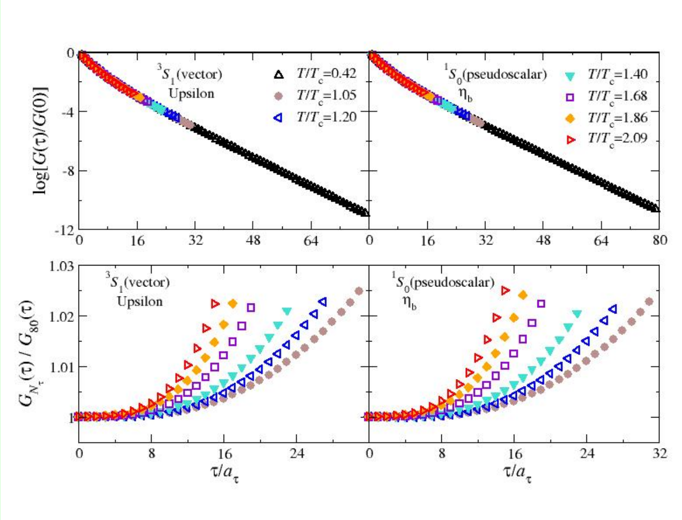
Effective mass $a_\tau m_{\text{eff}}(\tau) = \log(G(\tau - a_\tau)/G(\tau))$

Noninteracting quarks

S-waves:	$G_S(\tau) \sim \tau^{-3/2}$
P-waves:	$G_P(\tau) \sim \tau^{-5/2}$

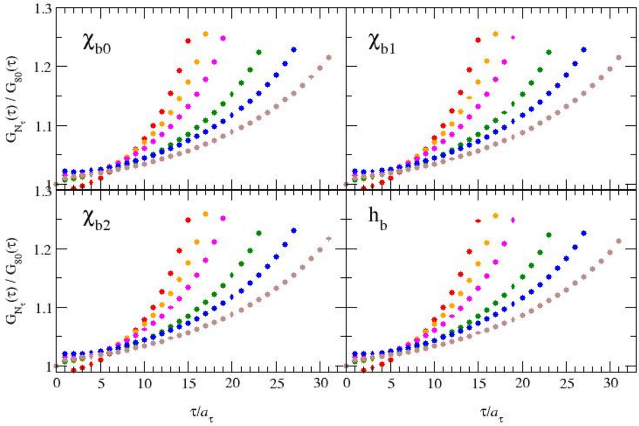
Effective power $\alpha_{\text{eff}}(\tau) = -\tau G'(\tau)/G(\tau)$

Correlator ratios (S-waves)

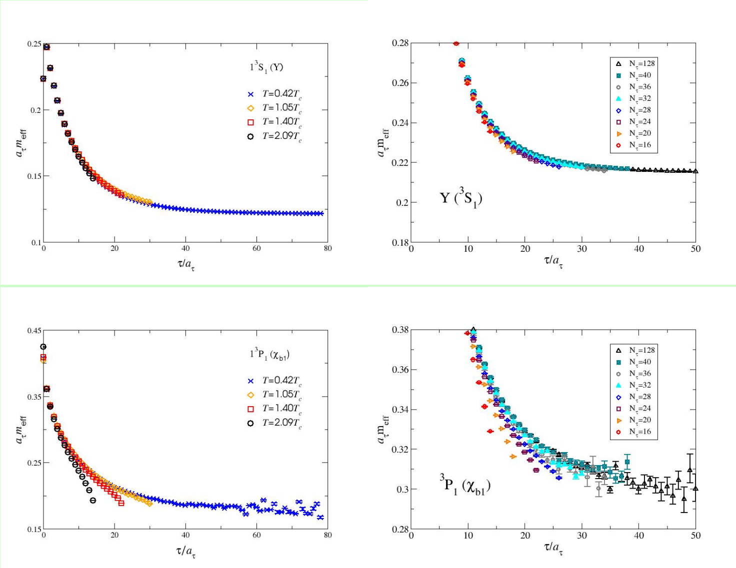


Note: Changes are entirely due to changes in spectral density

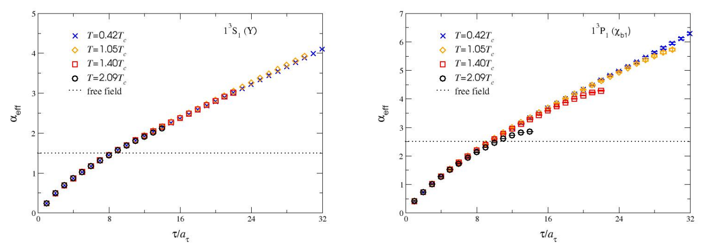
Correlator ratios (P-waves)



Effective mass



Effective power



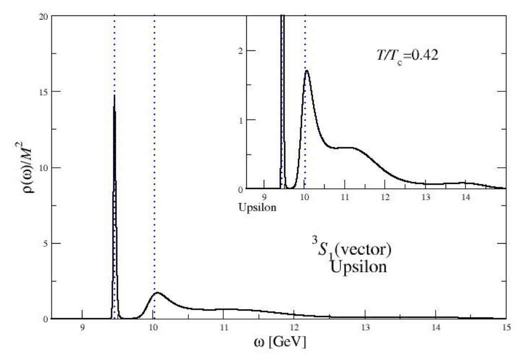
For bound state: $G(\tau) \sim A\exp(-\Delta E\tau) \Longrightarrow \alpha_{\text{eff}}(\tau) \sim \Delta E\tau$

S-waves consistent with bound state, minimal thermal effects

P-waves approach constant α_{eff} with noninteracting value at highest T.

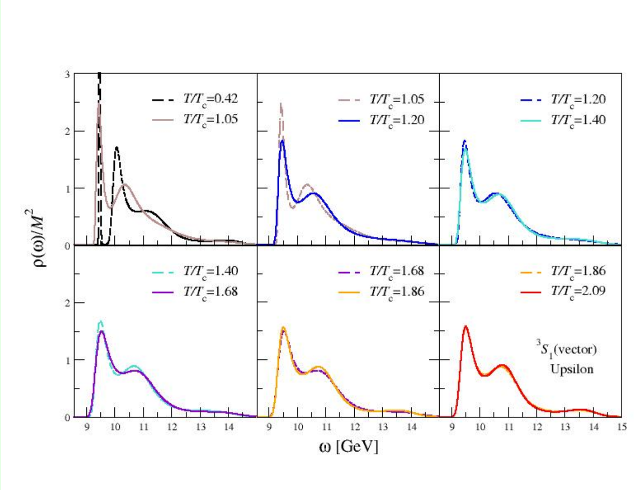
Spectral functions

[JHEP **1111** 103 (2011)]



ϒ (1S),
ϒ (2S)
clearly
identified

Spectral functions — T-dependence



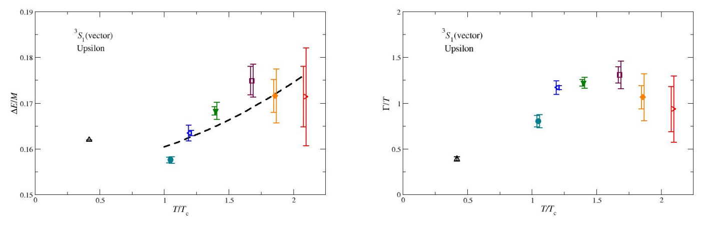
Υ (2S) melts, but ground state remains robust

Mass shift and width

Fit (left side of) peaks to gaussian

$\longrightarrow$ determine peak position (mass) and width

Width is upper bound

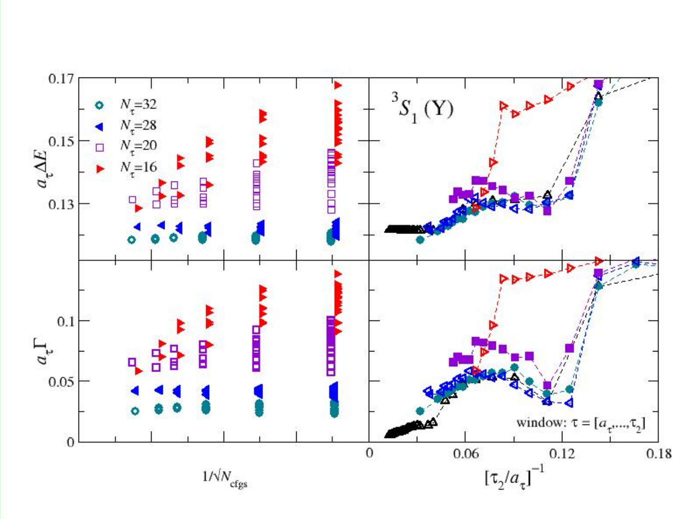


Results are consistent with perturbation theory,

$$\frac{\Gamma}{T} = \frac{1156}{81}\alpha_s^3\,, \qquad \frac{\delta E}{M} = \frac{17\pi}{9}\alpha_s T^2 M^2\,,$$

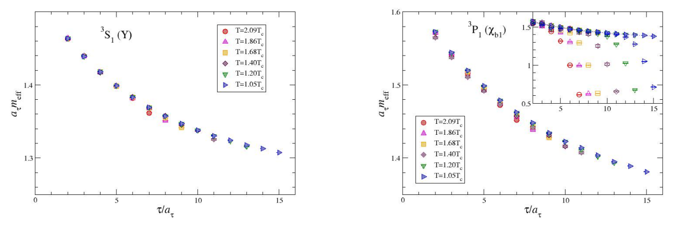
with $\alpha_s \sim 0.4$.

Mass shift and width: uncertainties



Results from relativistic beauty

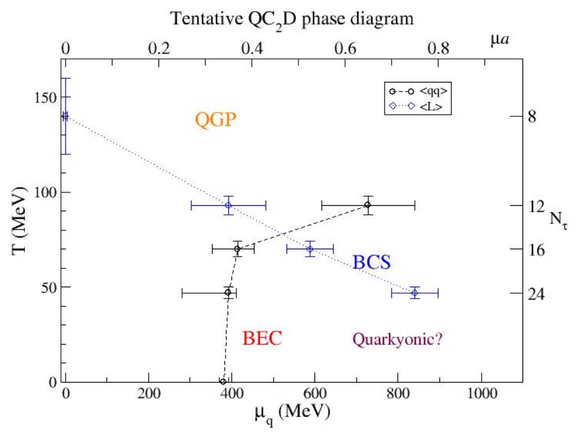
- Used the same action as for charm (and light quarks)
- Used both point and derivative operators for P-waves



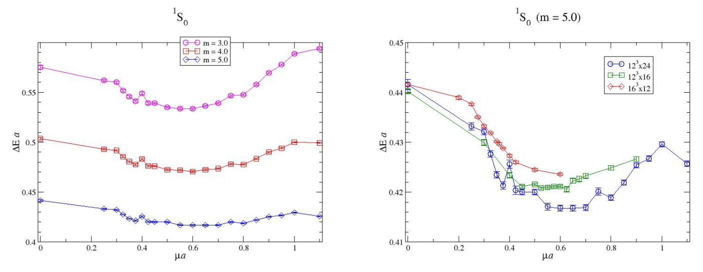
Qualitative agreement with NRQCD results
Derivative operators better behaved — smaller constant mode?

Nonzero density (but not QCD. . .)

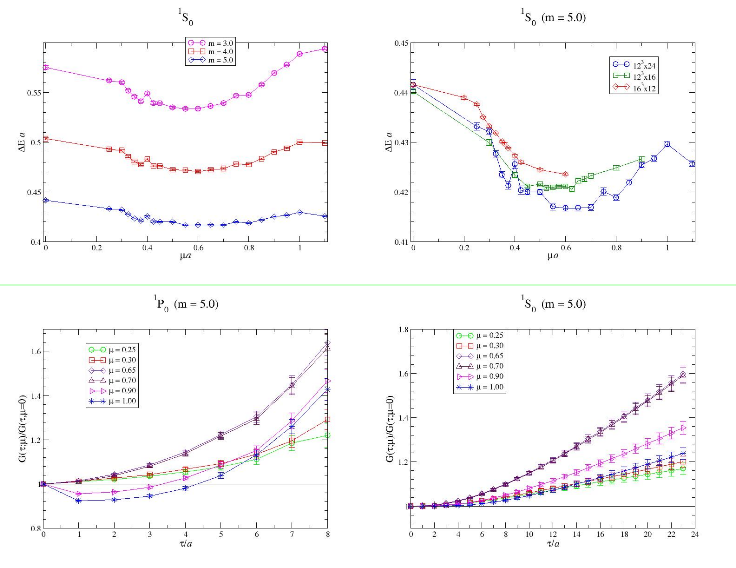
[with S.Hands, S.Kim, arXiv:1202.4353]



NRQC$_2$D correlators and energies



NRQC$_2$D correlators and energies

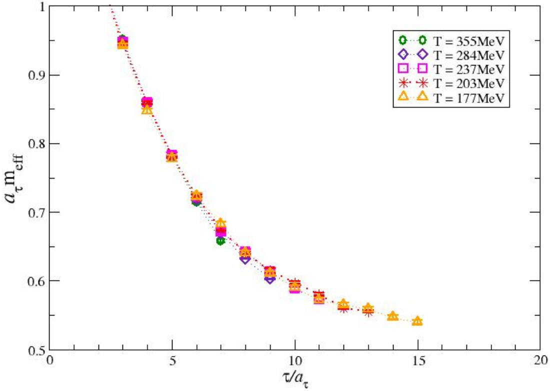


Summary

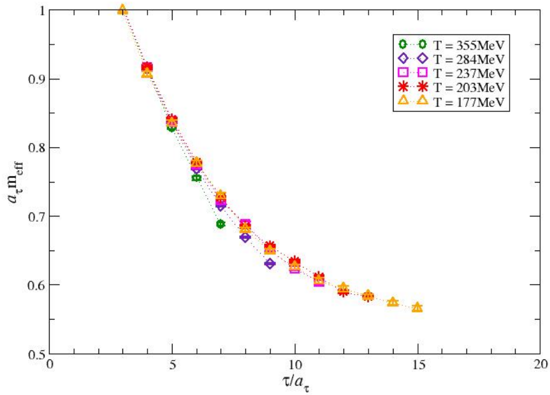
- Charmonium S-waves survive to $T \sim 1.6 - 2T_c$
- P-waves melt at $T < 1.3T_c$
- Significant momentum dependence in reconstructed correlators
- Transverse vector correlators are more sensitive to temperature and momentum
- Beautonium S-wave ground states survive up to $T \gtrsim 2T_c$
- Mass shift and width consistent with perturbation theory
- P-waves approach free power-law behaviour at $T \sim 2T_c$
- Relativistic beauty results compatible with NRQCD
- Simulations on finer lattices with realistic quark content underway
- Charm diffusion calculation in progress
- 2+1 flavours with larger anisotropy planned → higher T

Backup slides

Pseudoscalar effective mass



Vector effective mass



Default model, t-range dependence