Turbulence and Bose-Einstein condensation Far from Equilibrium

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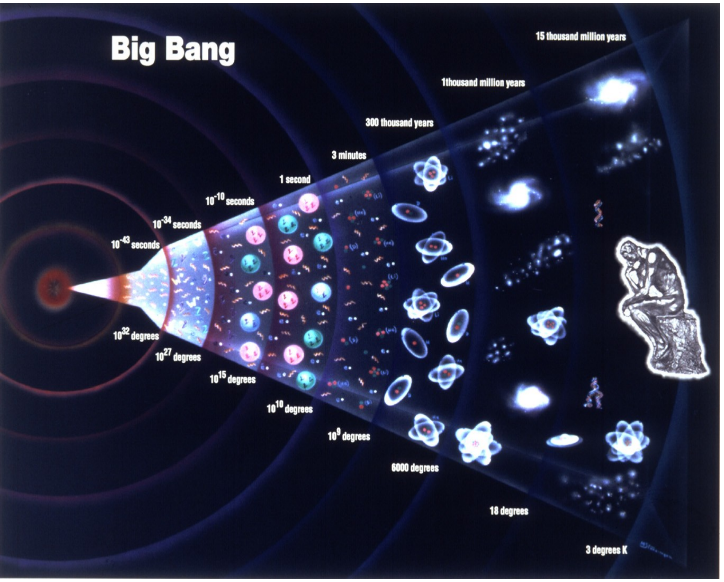
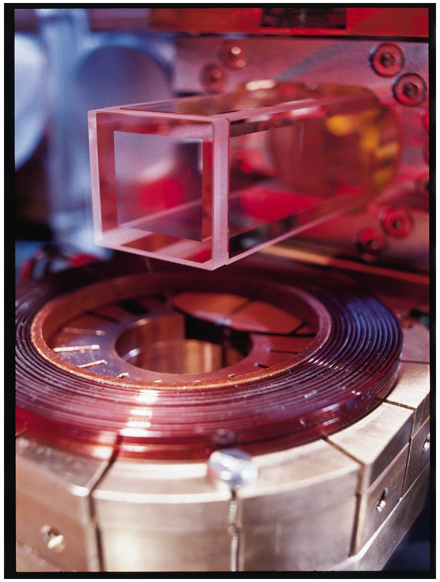
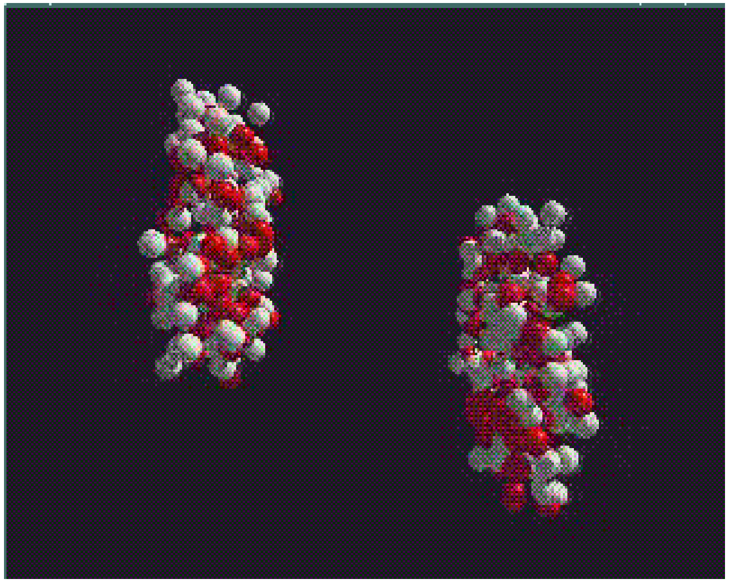
Gauge Field Dynamics In and Out of Equilibrium







Non-equilibrium initial state

Turbulent flow

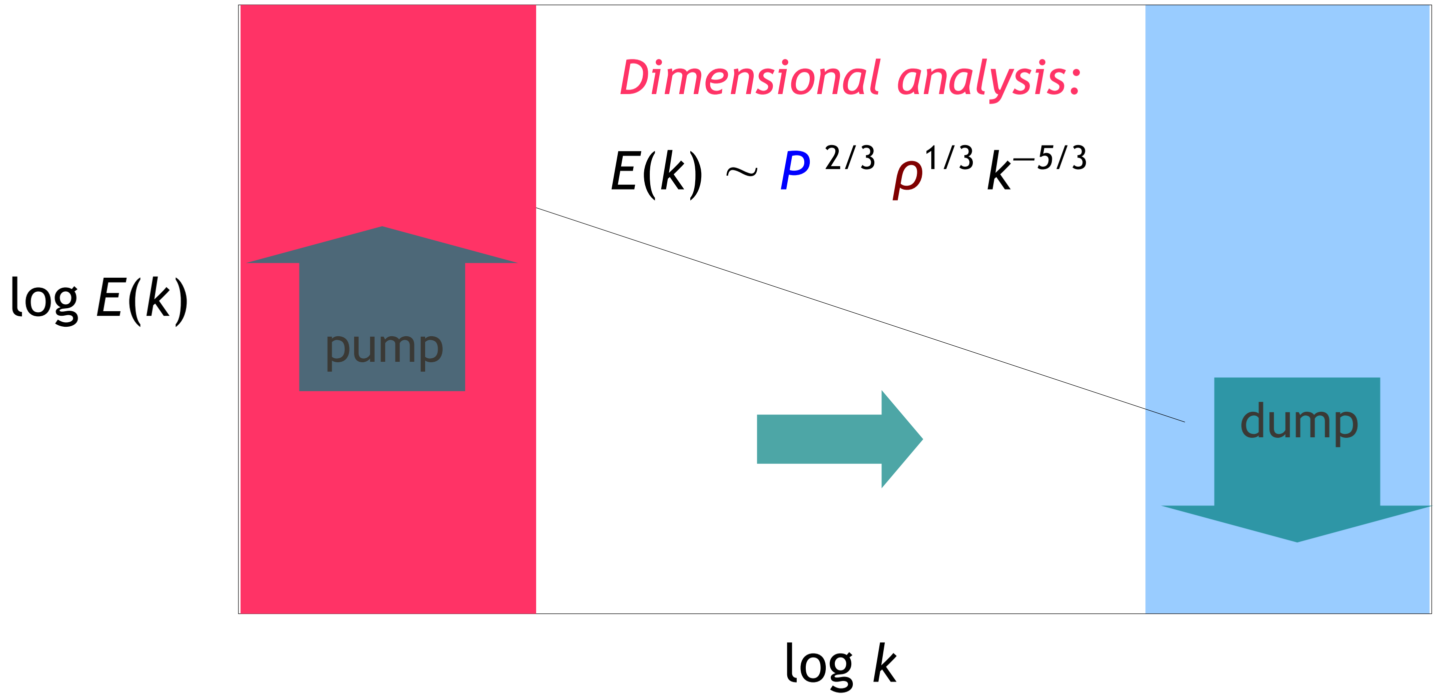
Thermal equilibrium

Kolmogorov turbulence

"local" interactions in momentum space

Constant flux in momentum space

Scale invariant transport

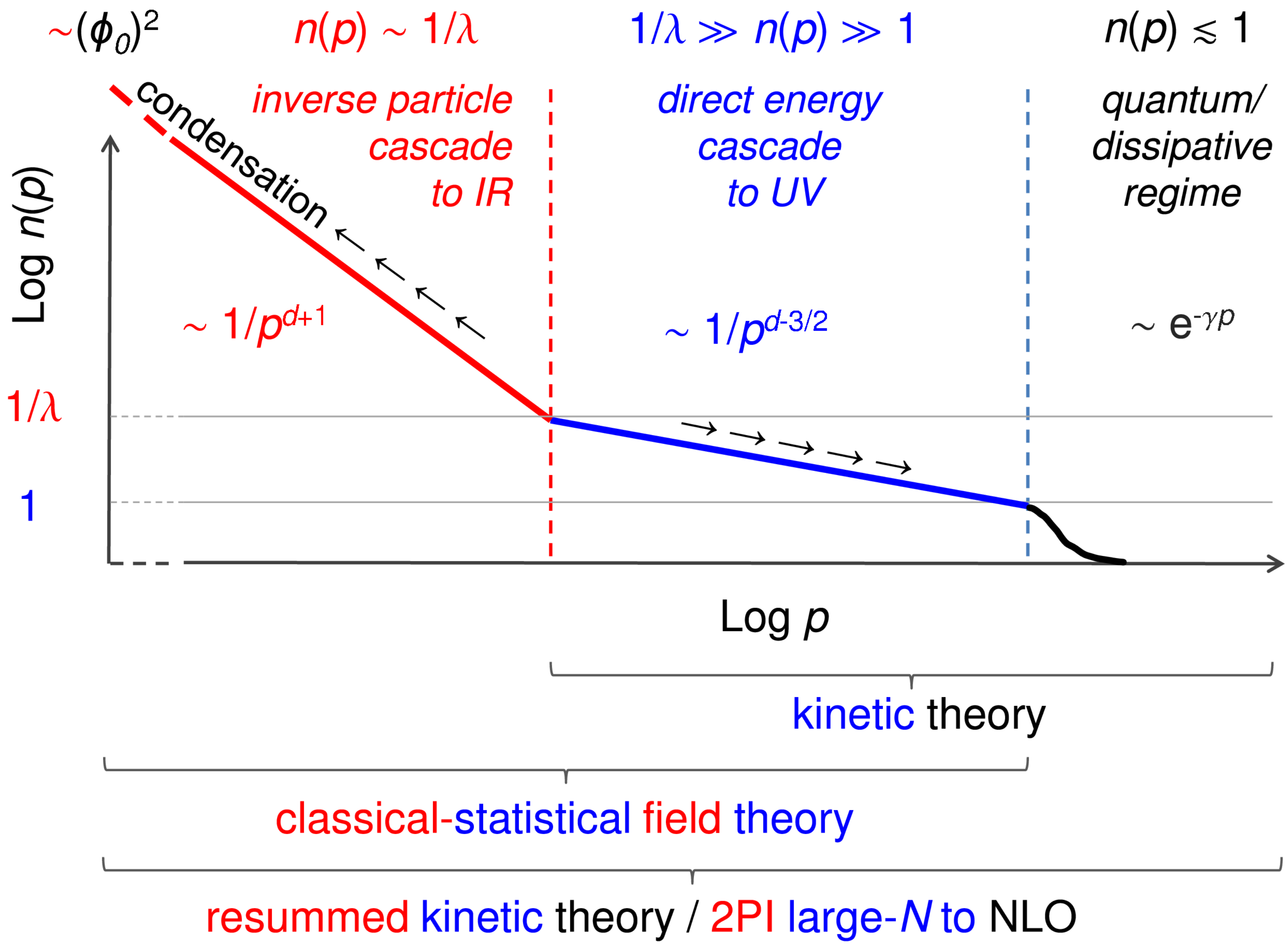


Turbulence in an incompressible fluid

3D:

Radial energy density	E	[kg s^{-2}]
Radial energy flux	P	[kg m^{-1} s^{-3}]
Density	ρ	[kg m^{-3}]

Turbulence and condensation in scalar field theories

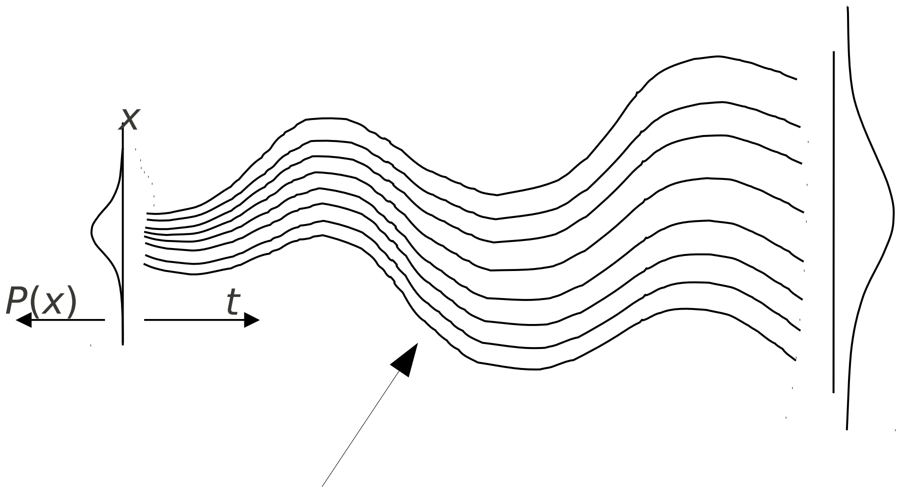


Classical statistical field theory

Classical fluctuations dominate over quantum for $n > 1$

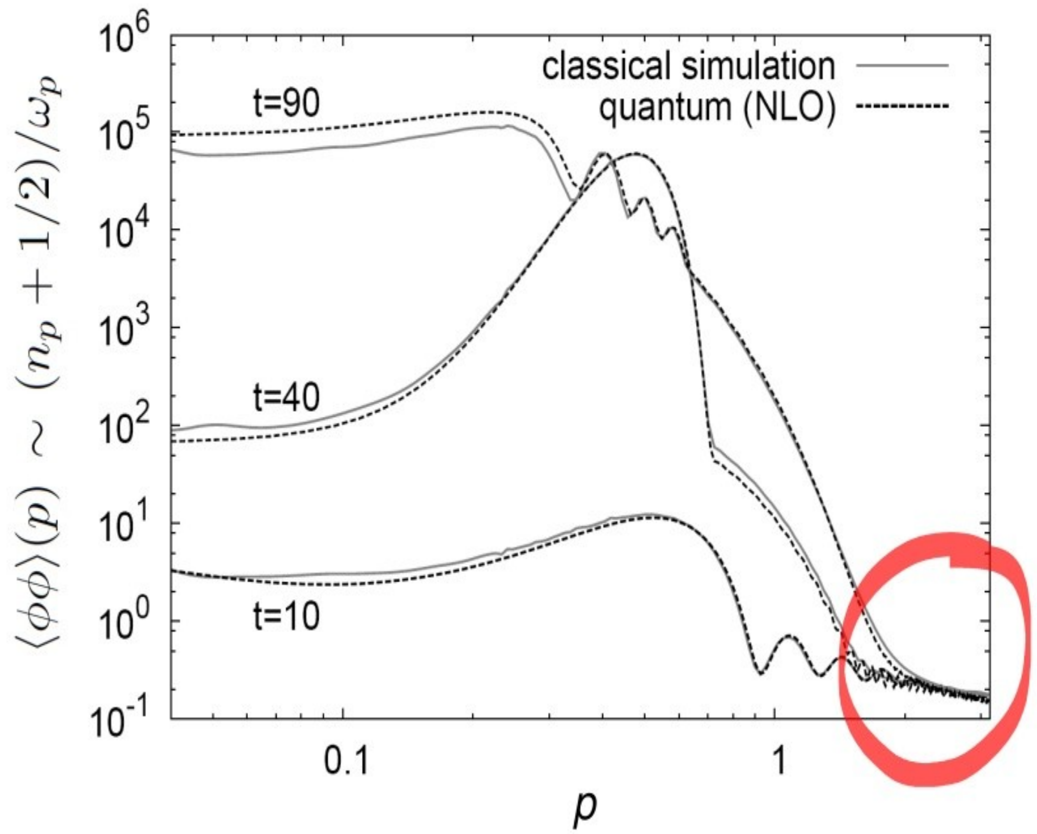
$$F = \langle \{ \phi, \phi \} \rangle \gg \langle [\phi, \phi] \rangle = \rho$$

$$\langle O(\phi, \pi) \rangle = \int D\phi(0) D\pi(0) O(\phi(t), \pi(t))$$



Using classical EOM

$$\ddot{\phi} - \nabla^2 \phi + \frac{1}{2} m^2 \phi + \frac{\lambda}{6} \phi^3 = 0$$



[Berges, Schmidt, Rothkopf 2008]

What is a condensate?

In equilibrium: $N = V \int \frac{d^3 k}{2\pi^3} \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}$ Maximum at $\mu = \epsilon_0$

Condensation: $N > N_{max}$

Macroscopic occupation of the zero mode

Condensate fraction $\frac{N_0}{N}$

Particle distribution: $n_k = \delta^3(k) n_0 + n'_k$

In terms of 2point function $F(x, y) = \{\phi(x), \phi(y)\}$

$$n_k = F(k)\omega_k \Rightarrow \quad F(k=0) \sim V$$

condensate $= \left(\frac{\int d^3 x \, \phi(x)}{V} \right)^2 = \frac{F(k=0)}{V}$ Independent of the volume

Condensation in bose gas

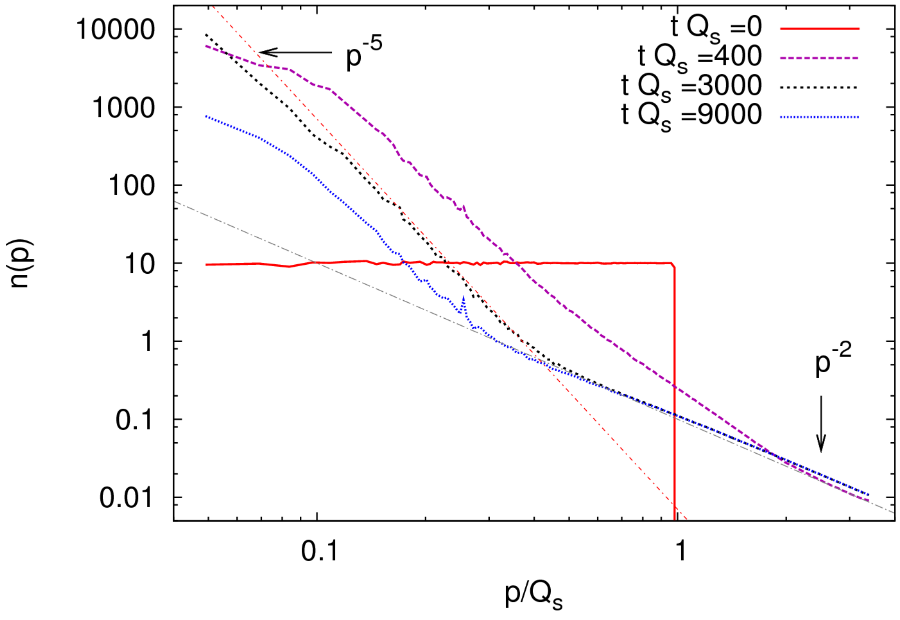
[Berges, Sexty (2012)]

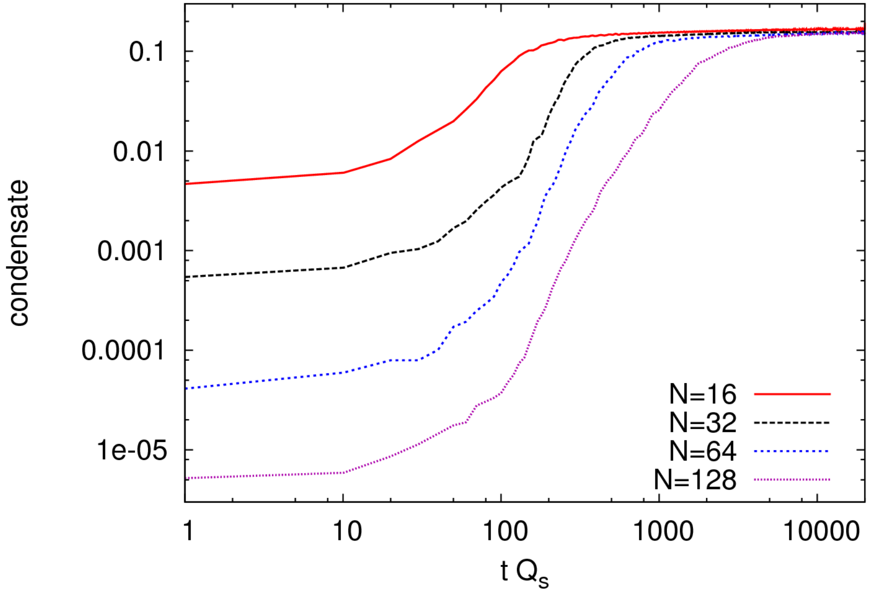
Non relativistic scalars described by complex field $\Psi(x,t)$

Gross-Pitaevski equation: $$i\partial_t \Psi(x,t) = \left(-\frac{\partial_i^2}{2m} + g|\Psi(x,t)|^2 \right) \Psi(x,t)$$

conserved particle number $$n_{tot} = \int d^3x |\Psi(x,t)|^2$$

occupation in zero mode: condensate = $$\left| \frac{\int d^3x \Psi(x,t)}{V} \right|^2$$





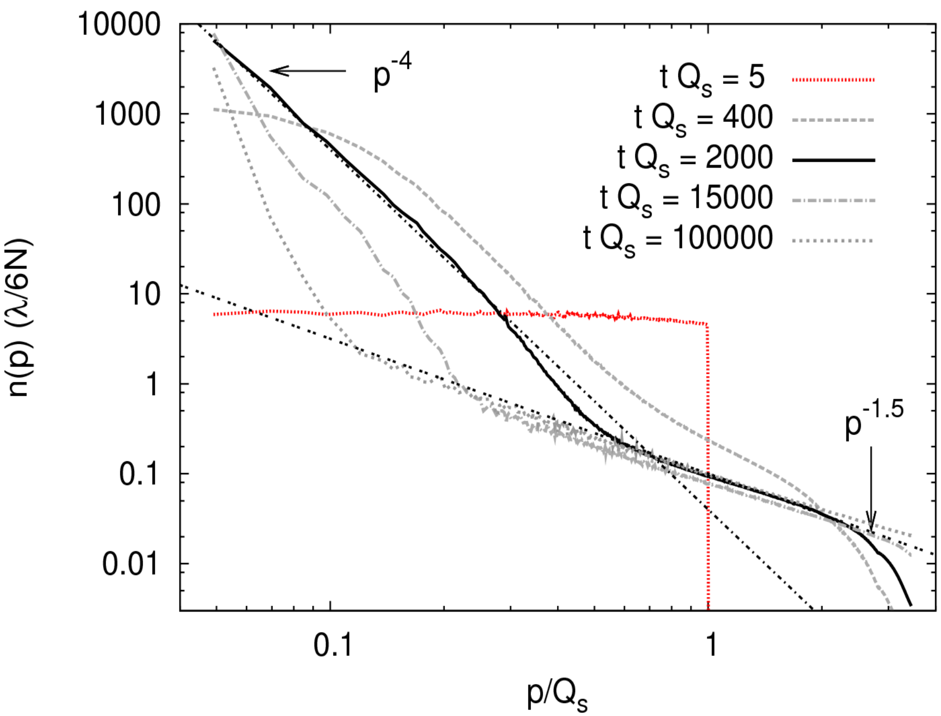
Non-equilibrium Bose condensation

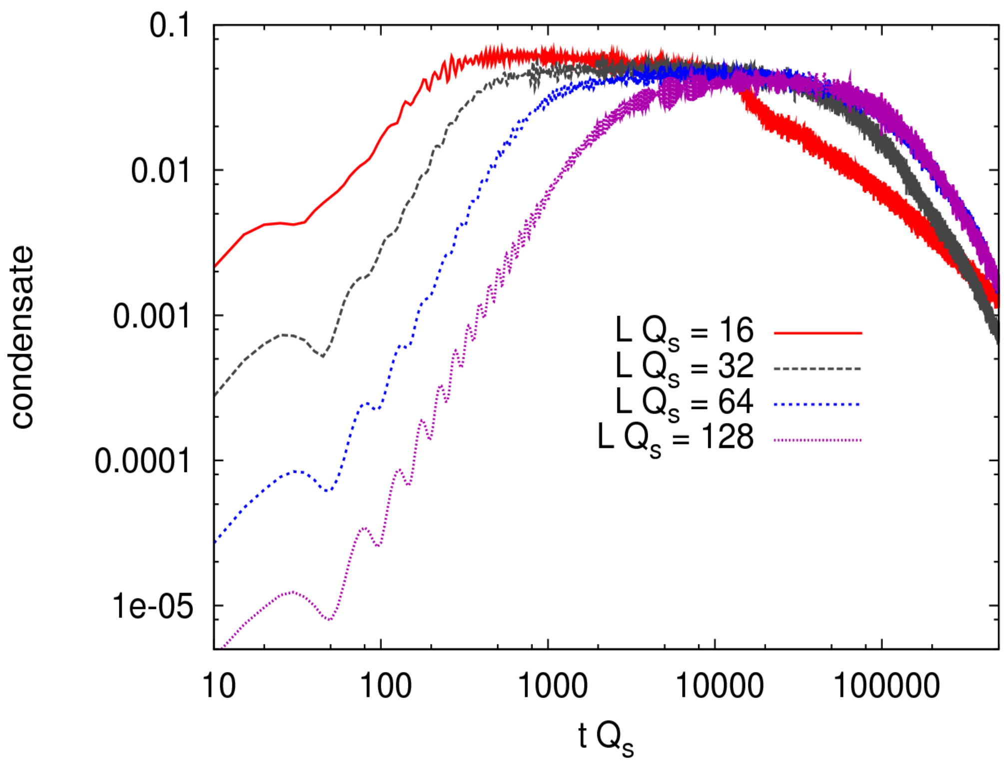
[Berges, Sexty (2012)]

O(4) massless relativistic scalars

Initial conditions: overpopulation

condensate $= \left\langle \left| \frac{\int d^3x\, \phi_a(x)}{V} \right|^2 \right\rangle_{ens}$





Turbulent cascade

Conserved charge → Stationary power law solution with k-independent flow

$$\partial_k n_k = 0$$

2->2 dominates: particle number effectively conserved

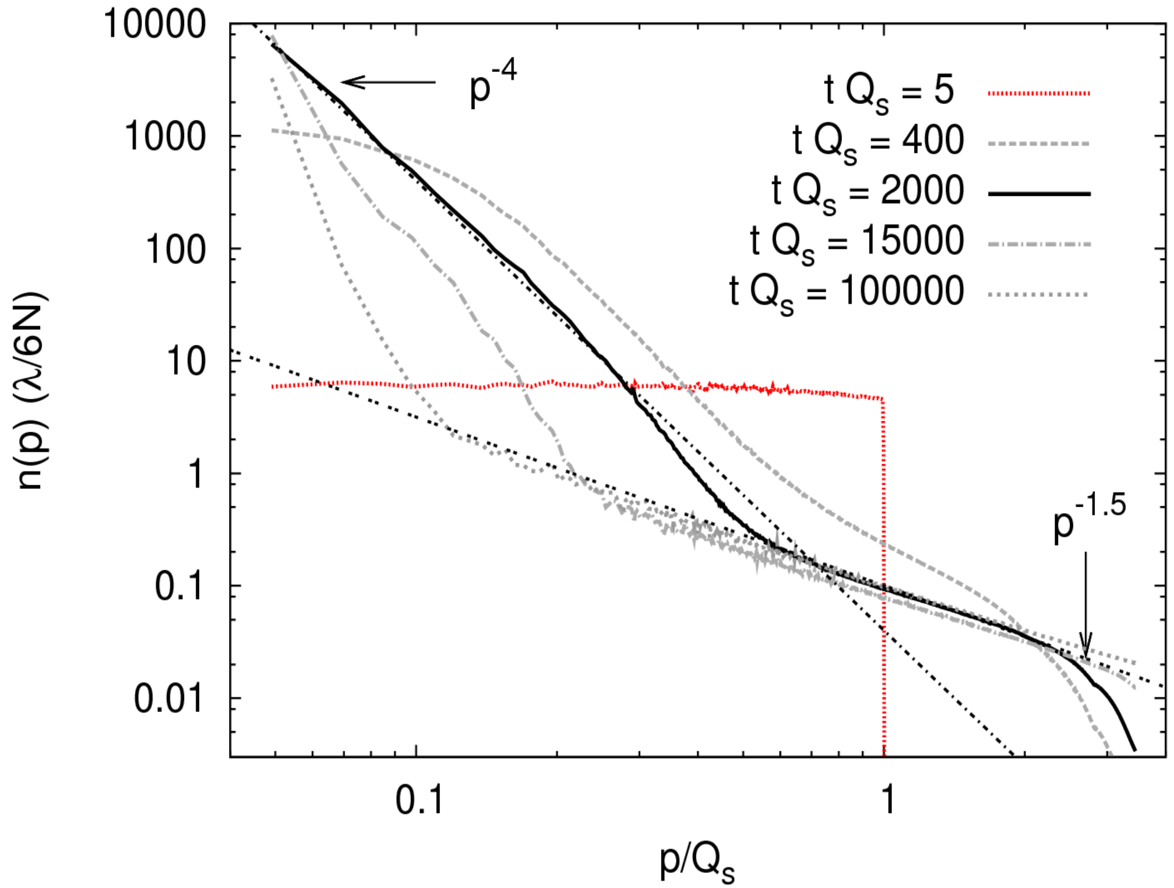
Dual cascade: particles to IR
energy to UV

Particle flow   Energy flow

$$\kappa_{IR} = d+1 \text{ or } \kappa_{IR} = d+2$$

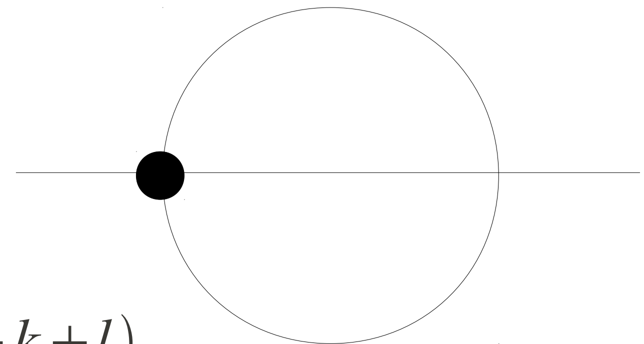
$$\kappa_{UV} = d-2 \text{ or } \kappa_{UV} = d-3/2$$

$$n_k \sim k^{-\kappa}$$



IR resummation – Strong turbulence

1/N resummation: effective vertex



$$\Sigma(p)=\int_{kql}\lambda_{eff}(p+q)G(q)G(k)G(l)\delta^{(4)}(p+q+k+l)$$

$$\lambda_{eff}(p)=\frac{\lambda}{(1+\Pi^R(p))(1+\Pi^A(p))}$$

With one loop bubble:

$$\Pi(p)=\int_q G(p)G(p-q)$$

In the IR: $\Pi(p)\gg 1$

The vertex scales:

$\lambda_{eff}(s\,p)=s^{2r}\lambda_{eff}(p)$ with $r=3+\kappa-d$

In the UV: $\lambda_{eff}=\lambda$

$$\lambda(sp)=\lambda(p)$$

Strong turbulence in the IR: $\kappa=4$ or 5 (in d=3)

From 2PI to kinetic equations

Using Wigner coordinates

$$F_p(X) = \int d^4 s \exp(-ip_\mu s^\mu) F(X + s/2, X - s/2)$$

Gradient expansion, spatially homogeneous ensemble:

$$\partial_t \rho_p(X) = 0$$

$$2 p_0 \partial_t F_p(X) = \Sigma_p^\rho(X) F_p(X) - \Sigma_p^F(X) \rho_p(X)$$

Define:

$$F_p(X) = (n_p(X) + 1/2) \rho_p(X)$$

$$n_{eff}(t, \mathbf{p}) = \int_0^\infty \frac{dp_0}{2\pi} 2 p_0 \rho_p(X) n_p(X)$$

On-shell limit, only 2->2 contributes

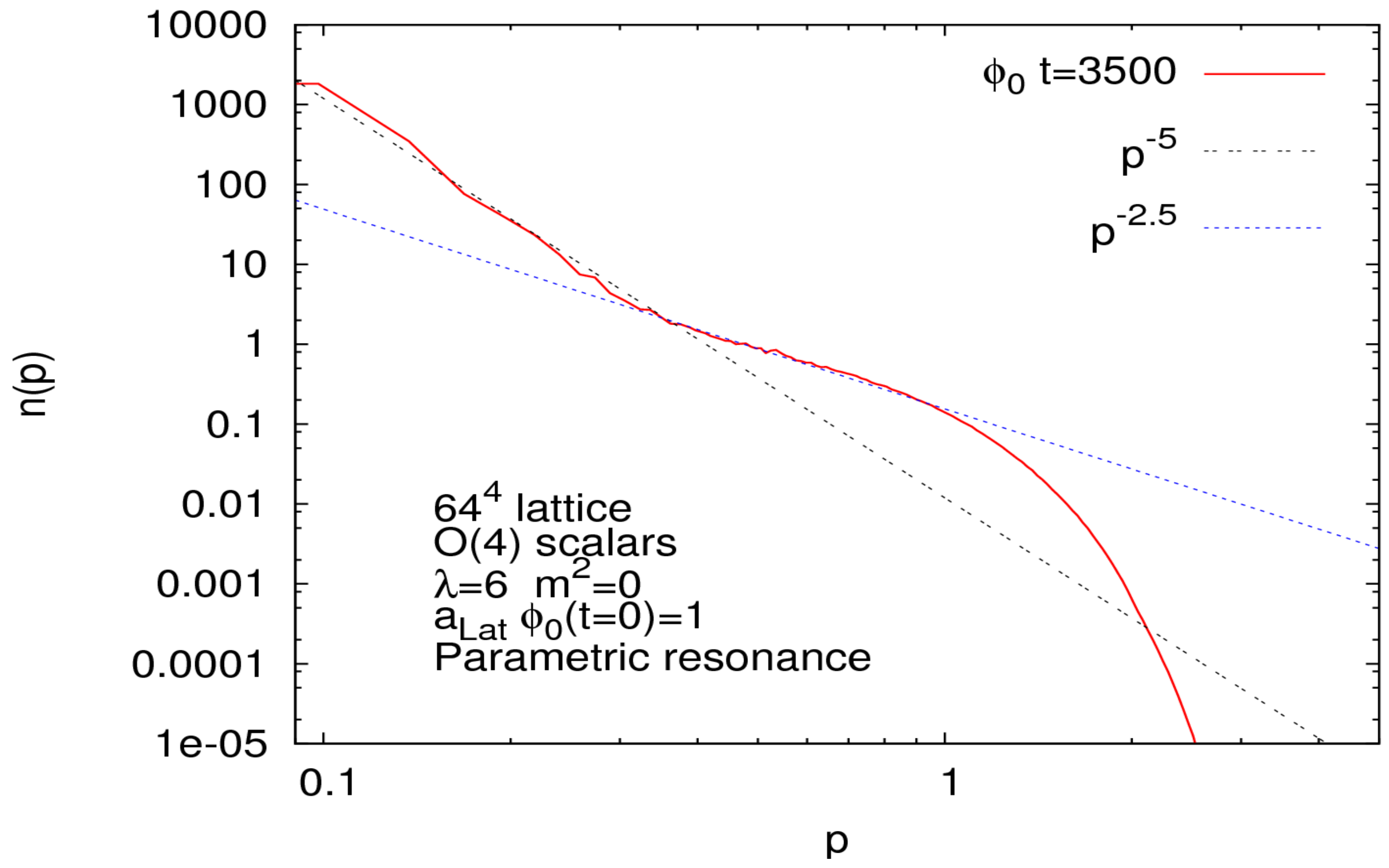
$$\partial_t n_{eff}(t, \mathbf{p}) = \int d\Omega_{2\to 2} \left[(1+n_p)(1+n_l) n_q n_r - n_p n_l (1+n_q)(1+n_r) \right] \lambda_{eff}(p+l)$$

Effective kinetic description also valid at $\lambda n \gtrsim 1$

[Berges, Sexty (2011)]

Turbulence in d=4

$$\kappa_{UV} = d - \frac{3}{2} \qquad \kappa_{IR} = d + 1$$



Bose-Einstein Condensation and Thermalization of the Quark Gluon Plasma

[Blaziot et al, 2011]

Initial CGC: $\epsilon_0 \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 \sim \frac{Q_s^3}{\alpha_s} \quad \rightarrow \quad n_0 e_0^{-3/4} \sim \alpha_s^{-1/4}$

Thermal eq: $\epsilon_{eq} \sim T^4 \qquad n_{eq} \sim T^3 \quad \rightarrow \quad n_0 e_0^{-3/4} \sim 1$

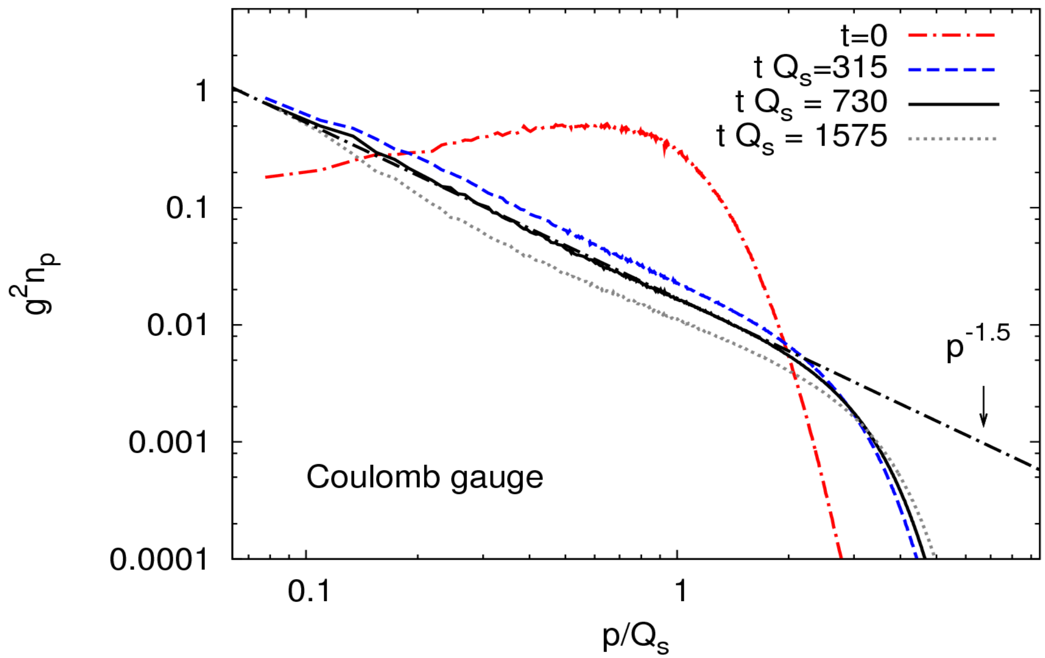
Elastic processes dominate

Particles pile up in the IR

Overpopulation leads to emergence of condensate

Gauge theory turbulence

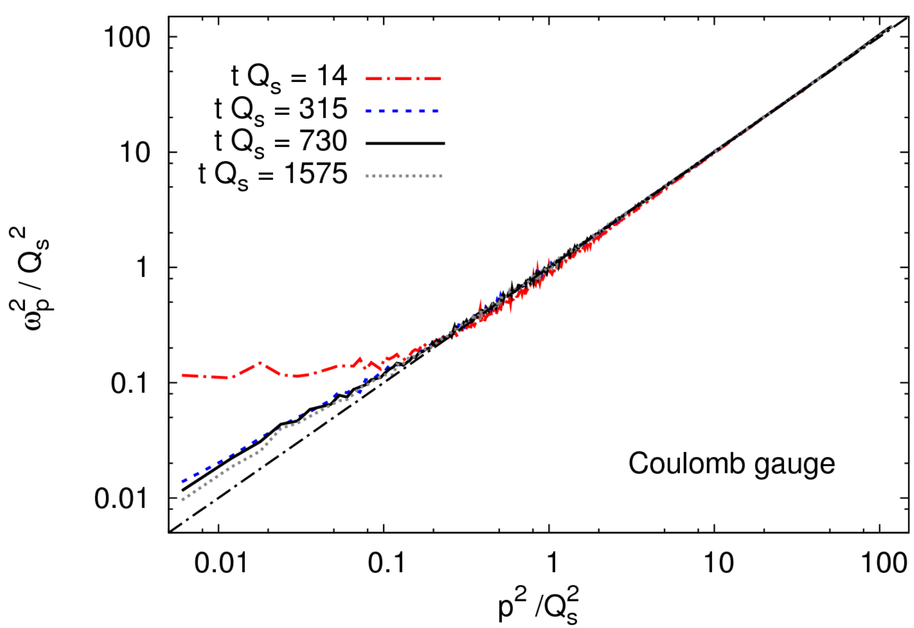
Pure SU(2) gauge theory   overpopulated initial condition



$$n_p \sim p^{-1.5}$$

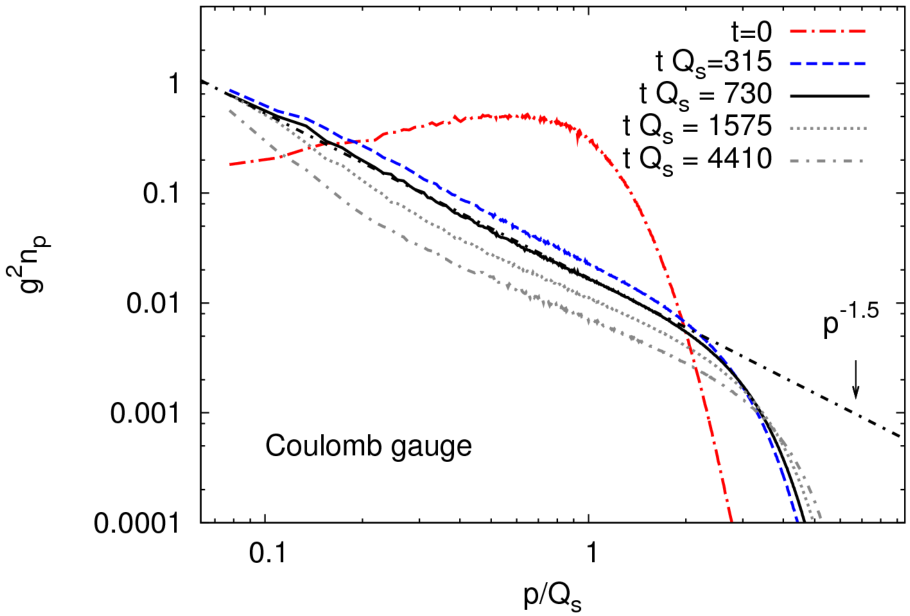
same as scalar UV exponent

Dispersion

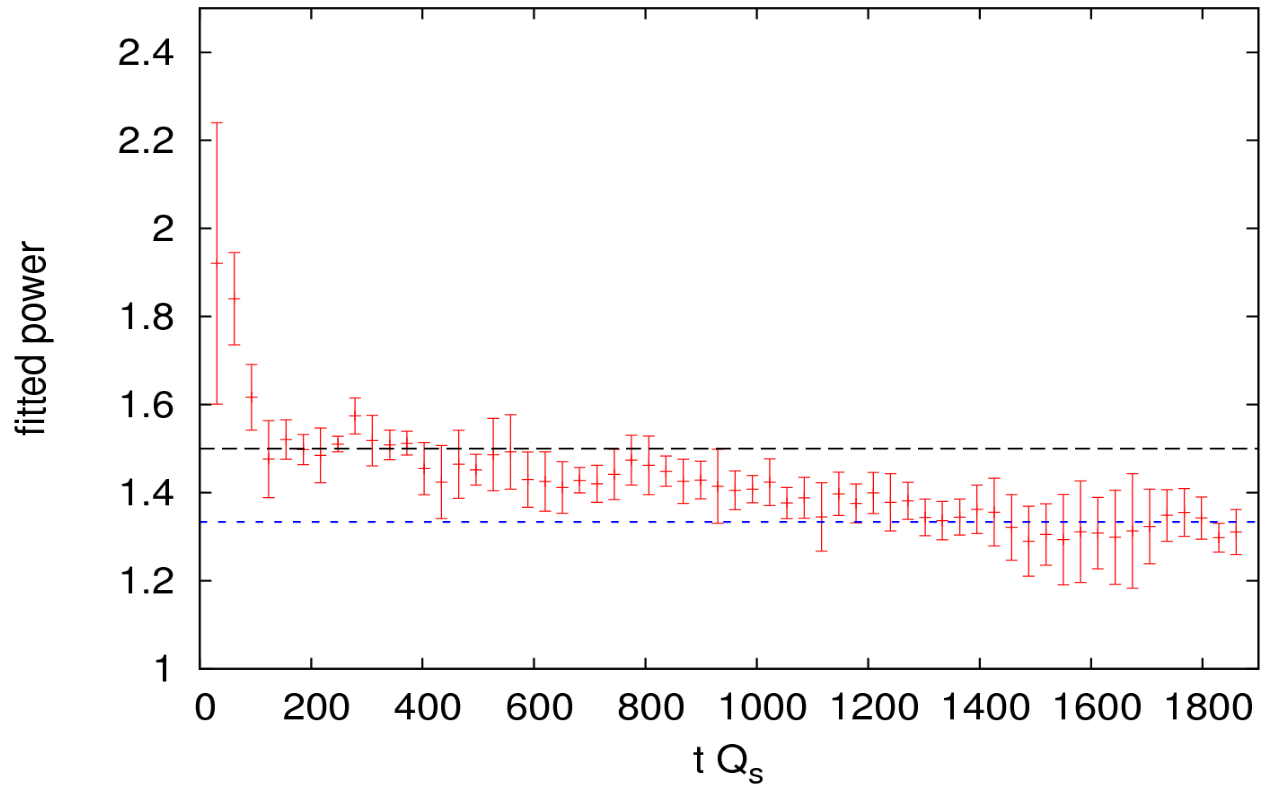


[Berges, Schlichting, Sexty, arXiv:1203.4646]

Time dependence of gauge theory exponent



Fit $n(p) \sim p^{-\kappa}$ [0.4: 1.0]



Wave Turbulence

In terms of corrleation functions

$$\phi = \phi_a \quad \text{or} \quad A_\mu^a$$

$$F(x,y) = \{\phi(x), \phi(y)\}$$

$$\rho(x,y) = [\phi(x), \phi(y)]$$

Stationarity condition:

(Collision integral vanishes)

$$\Pi_\rho(p) F(p) - \Pi_F(p) \rho(p) = 0$$

With self energy: $\Pi(p)$

Scaling ansatz

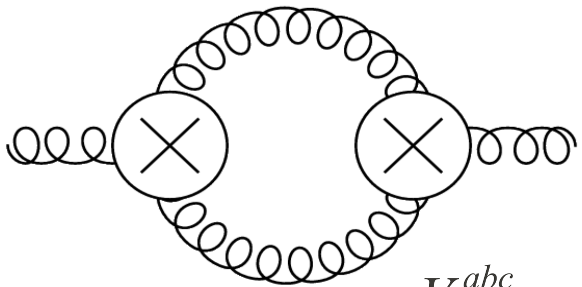
$$F(s^z \omega, s p) = |s|^{-2-\kappa} F(\omega, p)$$

$$\rho(s^z \omega, s p) = |s|^{2-\eta} \rho(\omega, p)$$

Classicality condition

$$F(p) \gg \rho(p)$$

Lowest order contribution
to self energy:



$V^{abc}_{\mu\nu\gamma} \sim g$

$$V^{abc}_{\mu\nu\gamma} = V^{abc}_{0,\mu\nu\gamma} + V^{abc}_{A,\mu\nu\gamma}$$

$$V^{abc}_{0,\mu\nu\gamma}(p,q,k) = g f^{abc}\left(g_{\mu\nu}(p-q)_\gamma + g_{\nu\gamma}(q-k)_\mu + g_{\gamma\mu}(k-p)_\nu\right)$$

$$V^{abc}_{A,\mu\nu\gamma}(x,y,z) = \left(C_{ac,bd}\, g_{\mu\nu} A^d_\gamma(x) + C_{ab,dc}\, g_{\nu\gamma} A^d_\mu(x) + C_{ab,cd}\, g_{\mu\nu} A^d_\nu(x)\right)$$

with $A^d_\delta(x) \sim 1/g$ background field

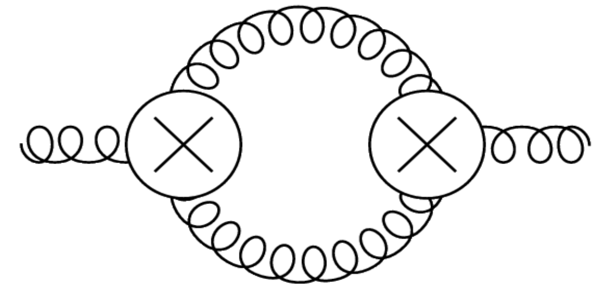
$$C_{ab,cd} = (f^{abe} f^{cde} + f^{ade} f^{cbe})$$
$$g^2 \delta^{d+1}(x-y)\,\delta^{d+1}(x-z)$$

V_0 Kinematically forbidden on shell

Stationarity condition:

$$\Pi_F(p)\rho(p) - \Pi_\rho(p)F(p) = 0$$

Classical part of stationarity condinition:



$$\Pi_F \rho - \Pi_\rho F = \int_{pqk} (2\pi)^4 \delta^{(4)}(p+q+k) V^2$$
$$\left(\rho(p) F(q) F(k) + F(p) \rho(q) F(k) + F(p) F(q) \rho(k)\right)$$

Scaling ansatz:

$$F(sp) = |s|^{-(2+k)} F(p) \qquad \rho(sp) = |s|^{-2} \operatorname{sgn}(s) \rho(p) \qquad V(sp, sq, sk) = s^{\nu} V(p, q, k)$$

Transformation (swapping and rescaling)

$$q \rightarrow \frac{p_0}{k_0} q, \qquad k \rightarrow \frac{p_0}{k_0} p, \qquad p \rightarrow \frac{p_0}{k_0} k$$

$$\Pi_F \rho - \Pi_\rho F = \int_{pqk} (2\pi)^4 \delta^{(4)}(p+q+k) V^2$$
$$\rho(p) F(q) F(k) \left(1 + \left| \frac{p_0}{k_0} \right|^{\Delta} \operatorname{sgn}\left(\frac{p_0}{k_0} \right) + \left| \frac{p_0}{q_0} \right|^{\Delta} \operatorname{sgn}\left(\frac{p_0}{q_0} \right) \right)$$

Solution:

$$\Delta = -1 \qquad \kappa = \frac{3}{2}$$

Conclusions

Scalar case well understood

Dual cascade

Condensation

Weak and strong wave exponents
from kinetic theory (with resummation)

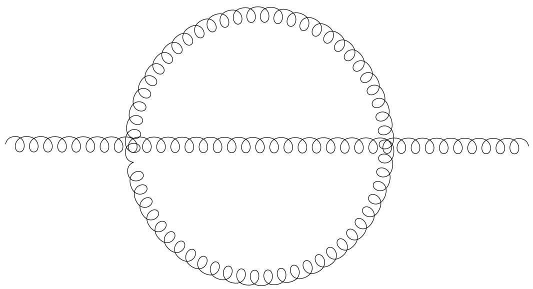
Gauge theory

Numerical inditcation of scaling behaviour with $\kappa = 3/2$

May be explained with background field

g^2 contribution similar to scalars

Scaling analysis with sunset diagram



$$\Sigma(p) = \int_{qkl} G(q) G(k) G(l) \delta^{(4)}(p+q+k+l)$$

Classical part of the stationarity condition:

$$0 = \int_{\boldsymbol{p}\, qkl} V(p,q,k,l)^2 \delta^{(4)}(p+q+k+l) \Big[F(p)F(q)F(k)\rho(l) + F(p)F(q)\rho(k)F(l) + F(p)\rho(q)F(k)F(l) + \rho(p)F(q)F(k)F(l) \Big]$$

Zakharov transformation:

swapping momenta

$$l' = \xi p; \quad p' = \xi l; \quad k' = \xi k; \quad l' = \xi l$$

$$F(p)F(q)F(k)\rho(l) \Rightarrow \rho(p)F(q)F(k)F(l)$$

$$0 = \int_{pqkl} V(p,q,k,l)^2 \delta^{(4)}(p+q+k+l) \rho(p) F(q) F(k) F(l)$$

$$\left[1 + \left|\frac{p_0}{q_0}\right|^{\Delta} \mathrm{sgn}\left(\frac{p_0}{q_0}\right) + \left|\frac{p_0}{k_0}\right|^{\Delta} \mathrm{sgn}\left(\frac{p_0}{k_0}\right) + \left|\frac{p_0}{l_0}\right|^{\Delta} \mathrm{sgn}\left(\frac{p_0}{l_0}\right) \right]$$

Solutions:

$\Delta = -1$

$\Delta = 0$ On shell limit 2->2 dominates

$$\kappa = \frac{5}{3} \text{ and } \kappa = \frac{4}{3}$$