The non-linear Glasma

Soeren Schlichting
J.Berges

“Gauge field dynamics in and out of equilibrium”
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RUPRECHT-KARLS-
UNIVERSITÄT
HEIDELBERG

Federal Ministry
of Education
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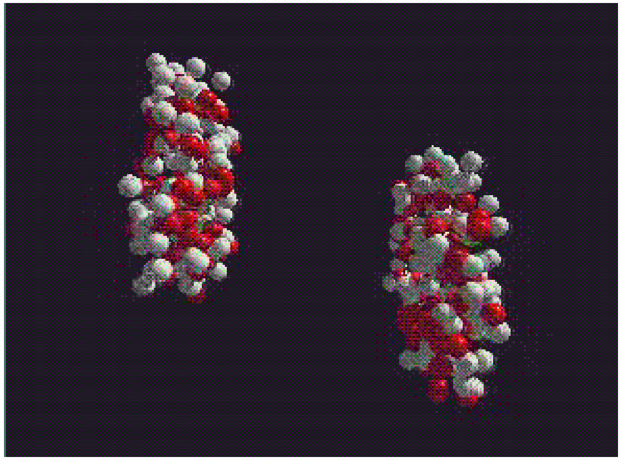
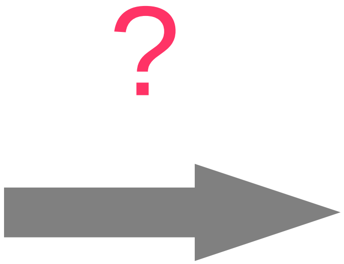
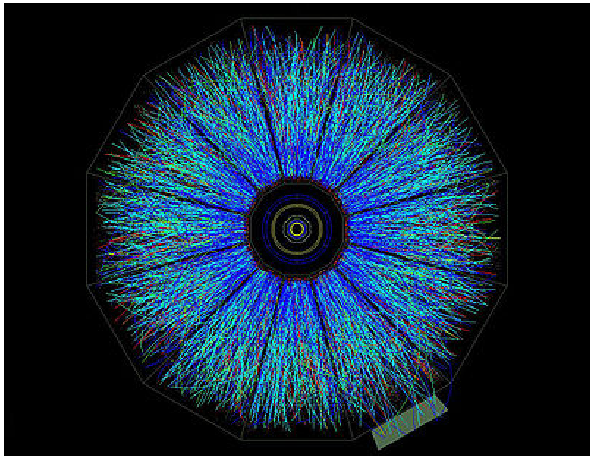
Motivation

- understand thermalization and related questions from first principles
- use ab-initio approach to relativistic heavy-ion collisions
- weak coupling and high energies





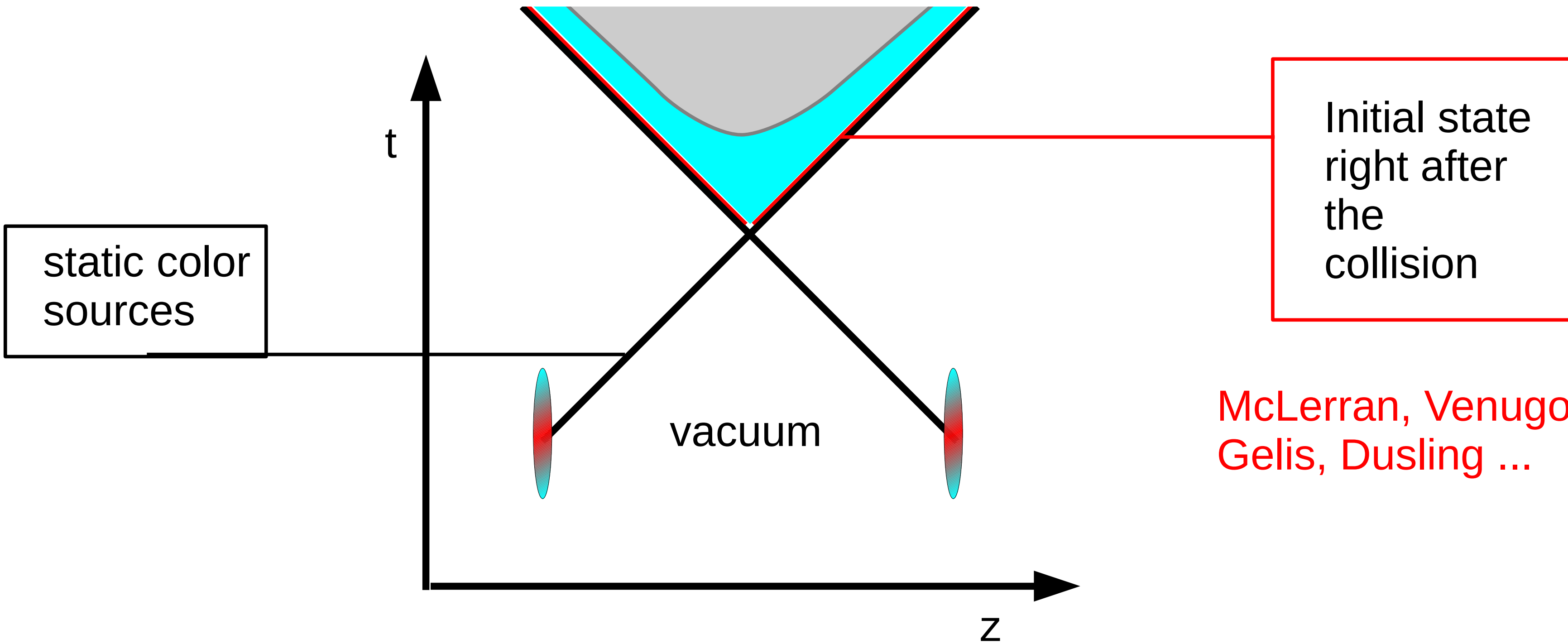


Outline

- Introduction to CGC framework and the Glasma
- Plasma instabilities and non-linear dynamics
- Summary & Outlook

CGC and the Glasma

- eikonal approximation – problem becomes particle production in the presence of strong sources



McLerran, Venugopalan, Gelis, Dusling ...

$$\boxed{J^{\mu}_{a}(t,x_{\perp},z)=\delta^{\mu +}\varrho^{a}_{(1)}(x_{\perp})\delta(x^{-})+\delta^{\mu -}\varrho^{a}_{(2)}(x_{\perp})\delta(x^{+})}$$

CGC and the Glasma

Initial value problem in QFT

$J^{\mu}_{a}(x)$ ~ 1/g => $A^{a}_{\mu}(x)$ ~ 1/g

$$F^{ab}_{\mu\nu}(x,y) = \frac{1}{2}\langle\left\{\hat{A}^{a}_{\mu}(x), \hat{A}^{b}_{\nu}(y)\right\}\rangle - \mathcal{A}^{a}_{\mu}(x)\mathcal{A}^{b}_{\nu}(y)$$

$$\rho^{ab}_{\mu\nu}(x,y) = i\langle\left[\hat{A}^{a}_{\mu}(x), \hat{A}^{b}_{\nu}(y)\right]\rangle$$ ~ 1

~ 1 (initially)
unstable in forward lightcone

Quantum evolution equations:

$$\frac{\delta S[J,A]}{\delta A^{a}_{\mu}(x)} = -J^{\mu}_{a}(x)$$ + loop corrections

$$iG_0^{-1}[x;A]\ \rho(x,y) = 0$$
$$iG_0^{-1}[x;A]\ F(x,y) = 0$$ + loop corrections

Closed set of coupled integro-differential equations

Glasma and Fluctuations

Weak coupling, small fluctuations

$$\frac{\delta S[J,A]}{\delta A_\mu^a(x)} = -J_a^\mu(x) \quad \cancel{\text{+ loop corrections}}$$

=> recovers classical field solution
(McLerran,Venugopalan,Fukushima,Gelis,Lappi,...)

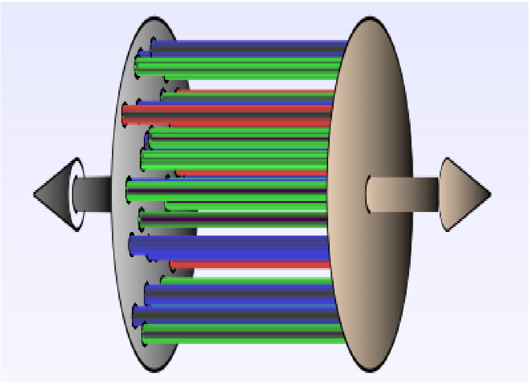
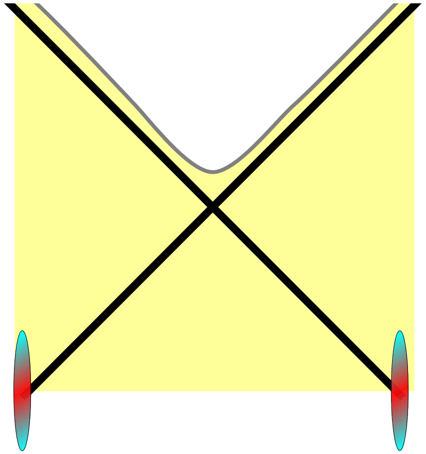


Fig. by F. Gelis

$$iG_0^{-1}[x;A]\ \rho(x,y) = 0$$
$$iG_0^{-1}[x;A]\ F(x,y) = 0 \quad \cancel{\text{+ loop corrections}}$$

equivalent to linearized classical evolution equations
=> spectrum of fluctuations right after the collision

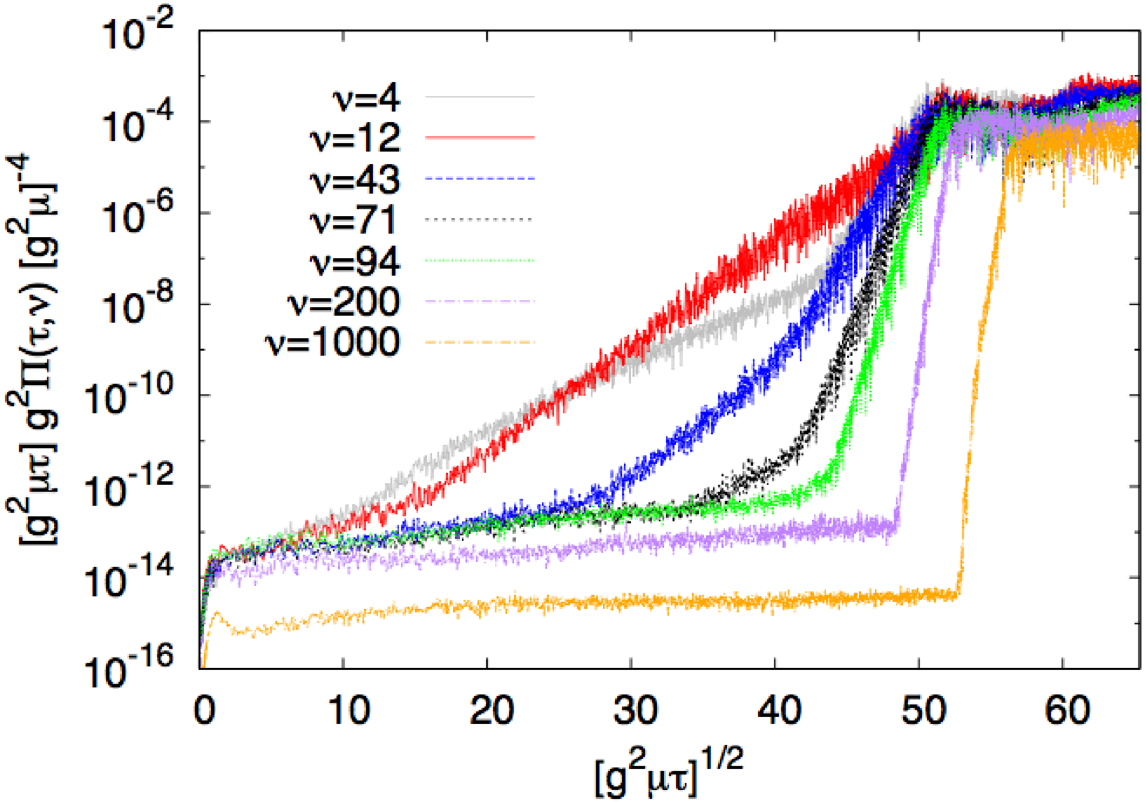


Dusling, Gelis, Venugopalan (2011)

Plasma instabilities

Consider boost non-invariant fluctuations
=> Grow exponentially in the forward light-cone

Initially small fluctuations become ***large***



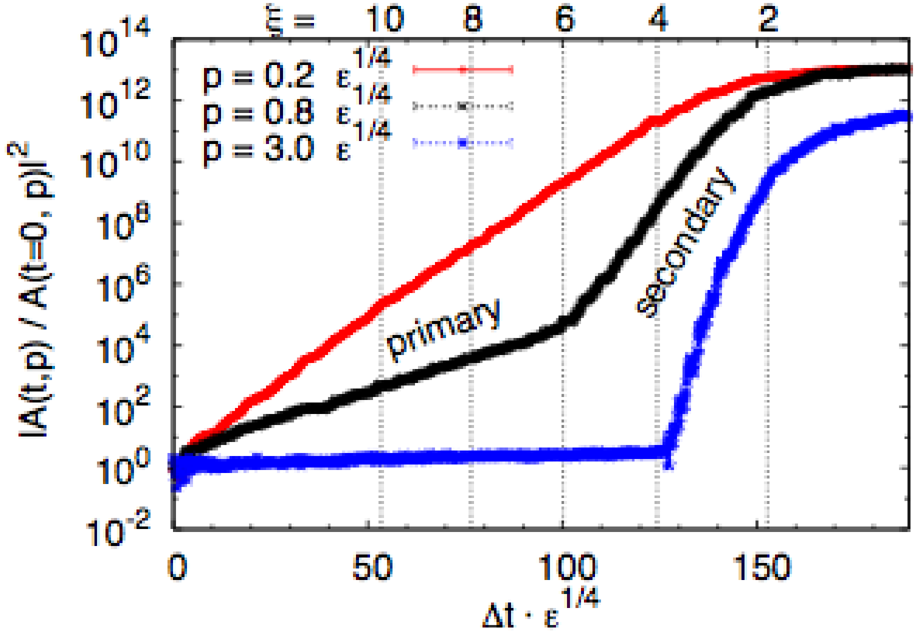
Classical statistical lattice simulation

- CGC initial conditions
(MV model)

- simplified fluctuations

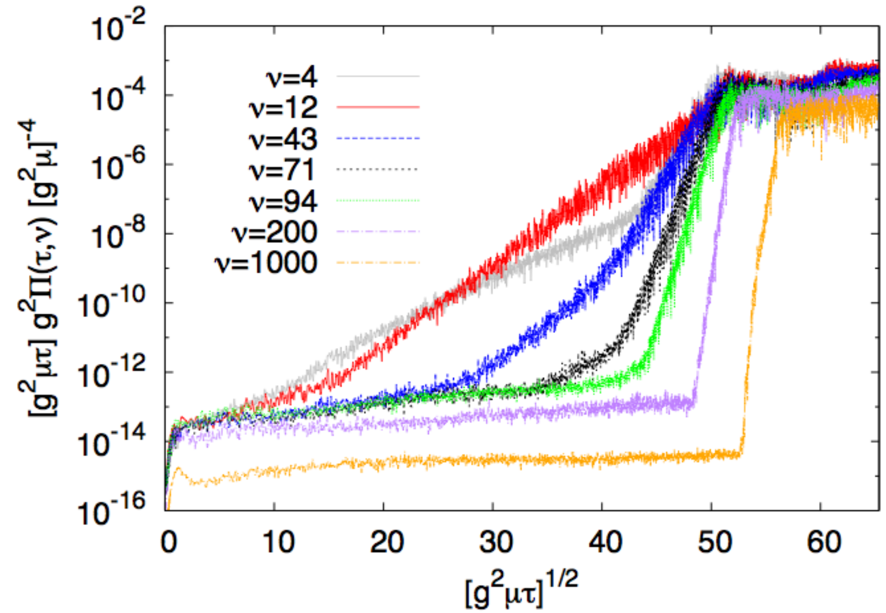
Romastschke,Venugopalan (2006); Fukushima,Gelis (2011); SS,Berges (in preparation)

Non-linear amplification



SU(2) – fixed box

Berges, Scheffler, Sexty (2008)



SU(2) – CGC expanding

Berges, SS (2012)

Non-linear amplification

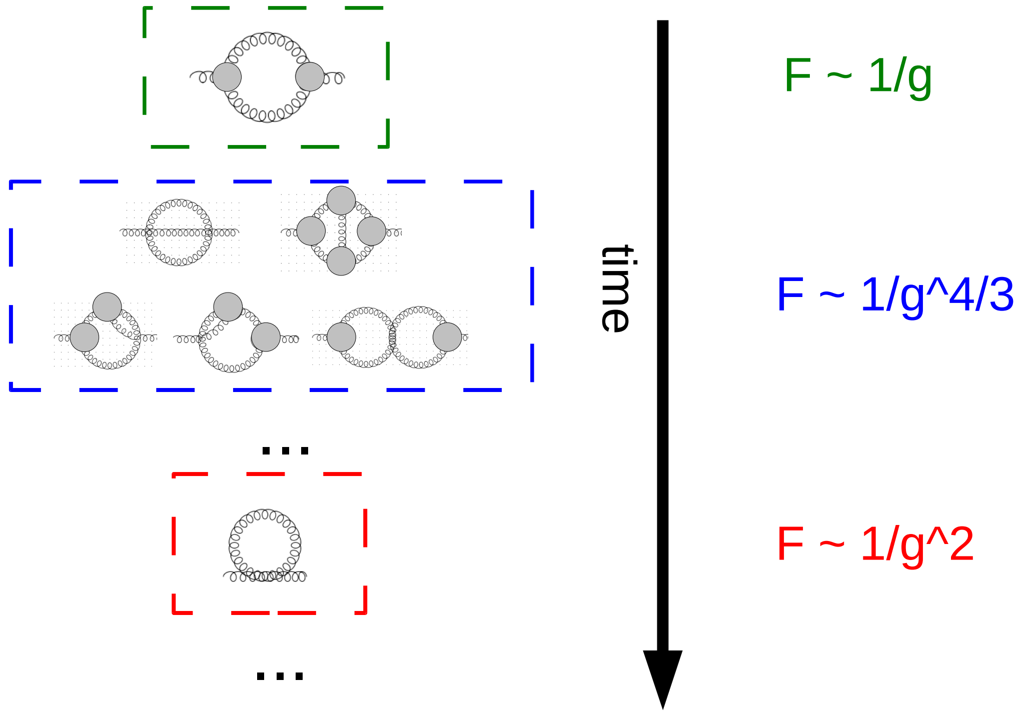
Weak coupling, fluctuations grow with time

$$\frac{\delta S[J,A]}{\delta A_\mu^a(x)} = -J_a^\mu(x)$$ + loop corrections

$$iG_0^{-1}[x;A]\ \rho(x,y) = 0$$
$$iG_0^{-1}[x;A]\ F(x,y) = 0$$
+ loop corrections

Q: What are the relevant loop corrections?

Power counting



power counting
g^2 F^2, g^4F^3,...g^2F,...

Take into account enhancement due to large fluctuations

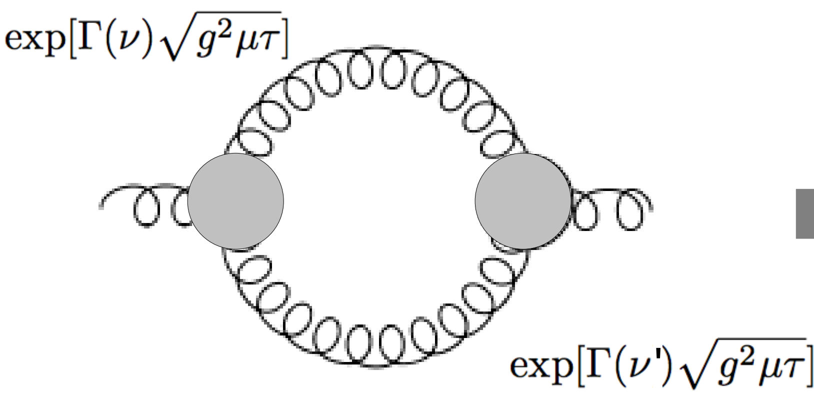
Can distinguish different dynamical regimes where higher order corrections are suppressed by at least a fractional power of the coupling constant

Most important diagrams contained in classical statistical lattice gauge theory

Non-linear amplification

dominated by ***soft unstable*** modes

secondary instabilities



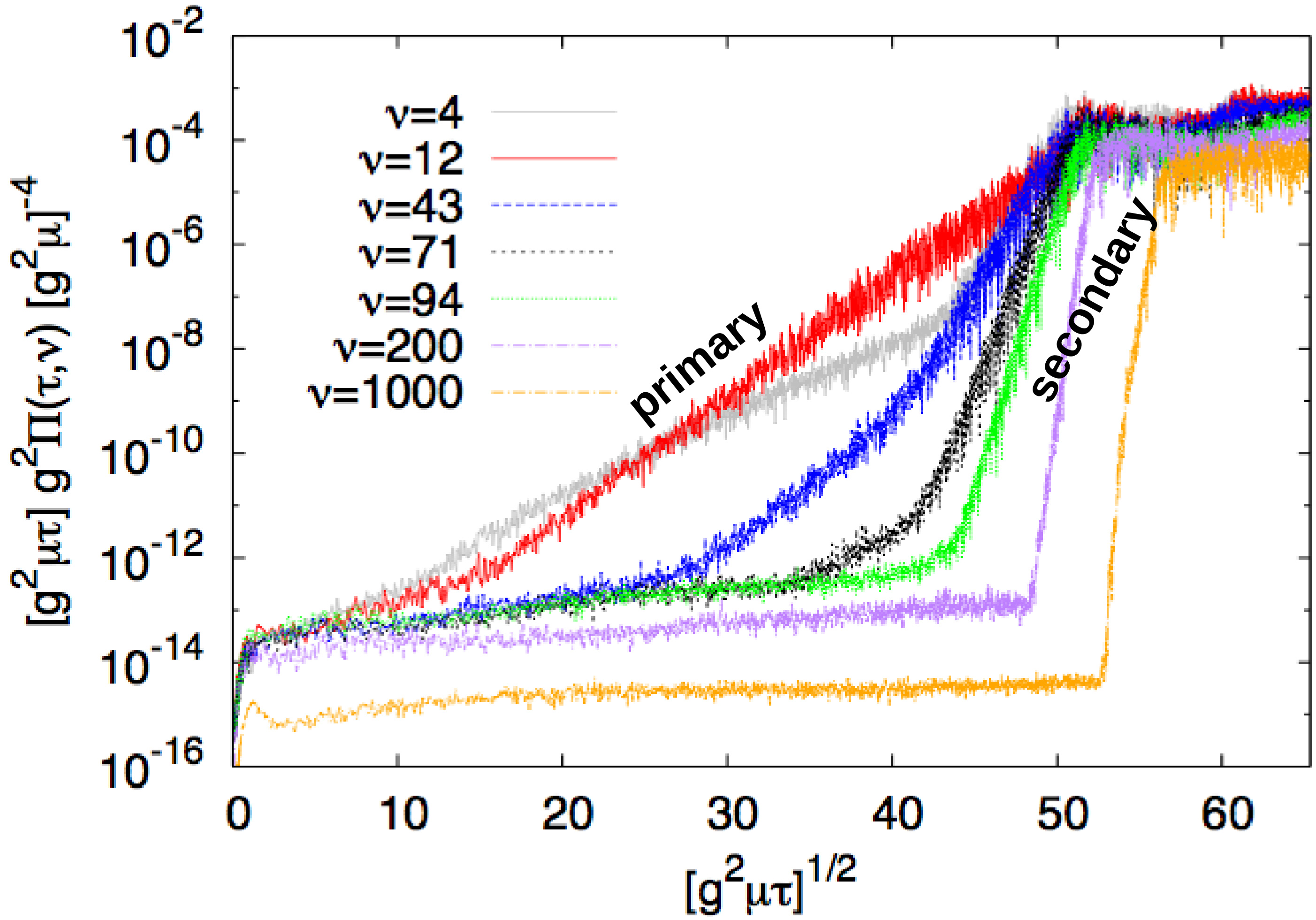
$$\exp[\,2\,\bar{\Gamma}\sqrt{g^2\mu\tau}\,]$$

relevant time scale **parametrically**

$$\sqrt{g^2\mu\tau_{\mathrm{Sec}}} \overset{g\ll 1}{\sim} \frac{1}{2\Gamma_0}\ln g^{-2}$$

Can be calculated explicitly when primary instabilities are described analytically (e.g. Berges, Serreau; Berges, Boguslavski ,SS (scalar field theory))

Non-linear amplification

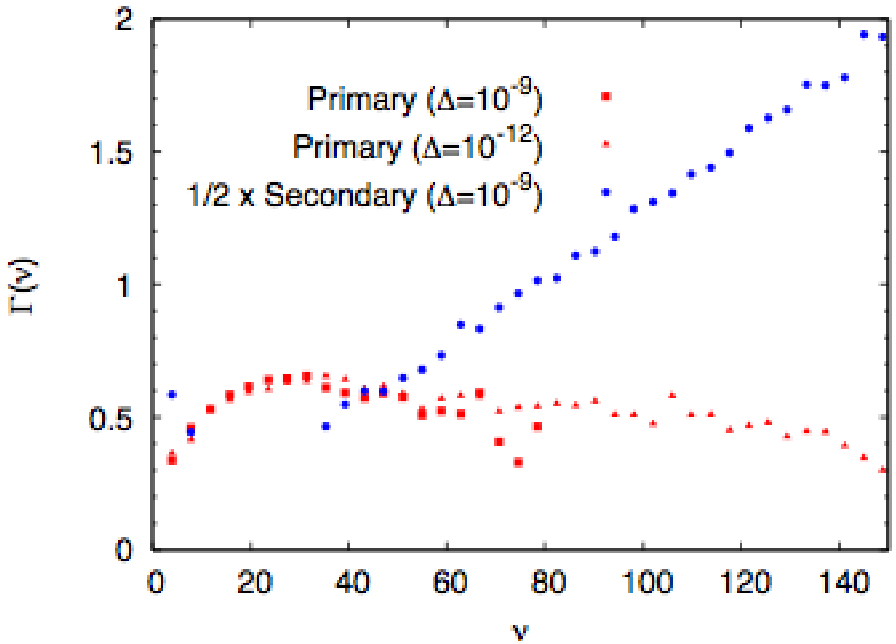


MV model, spectrum of fluctuations simplified

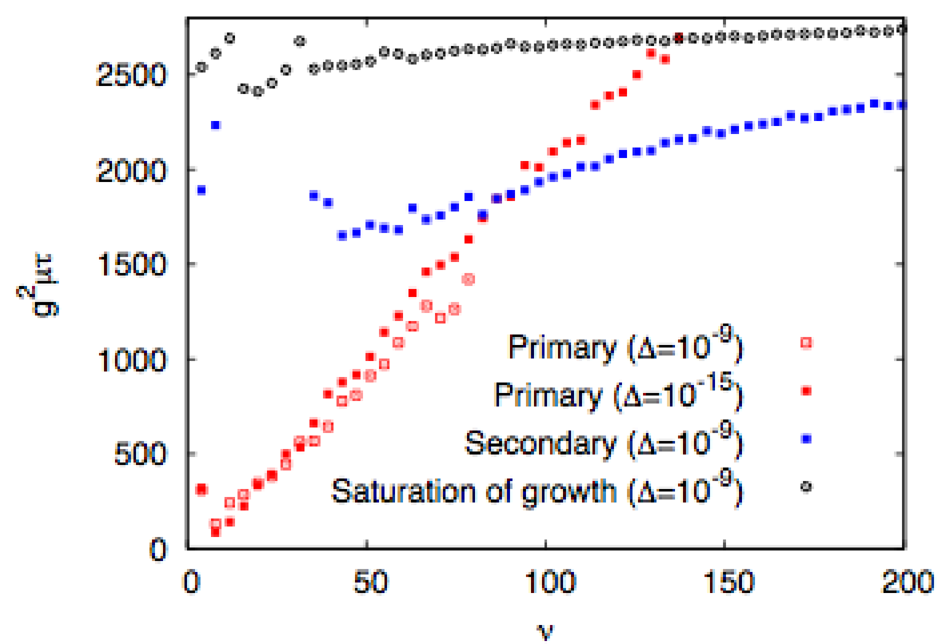
SS,Berges (in preparation)

Growth rates & time scales

Growth rates

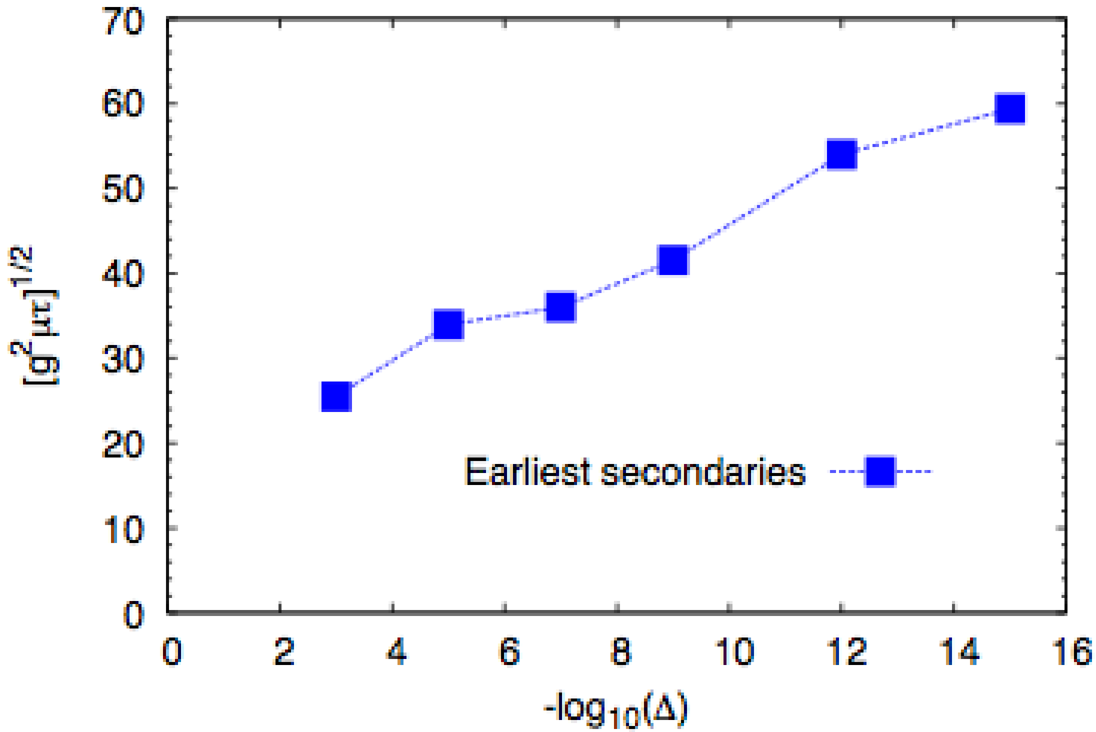


Set-in times



SS,Berges (in preparation)

Coupling dependence



Confirms logarithmic behavior

$$\sqrt{g^2\mu\tau_{\mathrm{Sec}}} \overset{g\ll 1}{\simeq} \frac{1}{2\Gamma_0}\ln g^{-2}$$

Subleading corrections from:

- delayed set-in of primary instability
- Spectrum of initial fluctuations

Δ^2 : size of intial fluctuations ~ g^2

SS,Berges (in preparation)

Summary

Different dynamical regimes of a system undergoing instabilities:

- linear instability regime
- non-linear amplification regime
- saturation of growth

Non-linear effects occur before saturation and are dominant for high-momentum modes

Relevant time scale depends only logarithmically on g^2

$$\sqrt{g^2\mu\tau_{\mathrm{Sec}}} \overset{g\ll 1}{\sim} \frac{1}{2\Gamma_0}\ln g^{-2}$$