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From Complex to Stochastic Potential: Heavy Quarkonia in the QGP

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In collaboration with
A.R., T.Hatsuda & S.Sasaki: **arXiv:1108.1579**
Y.Akamatsu, A.R.: **arXiv:1110.1203**

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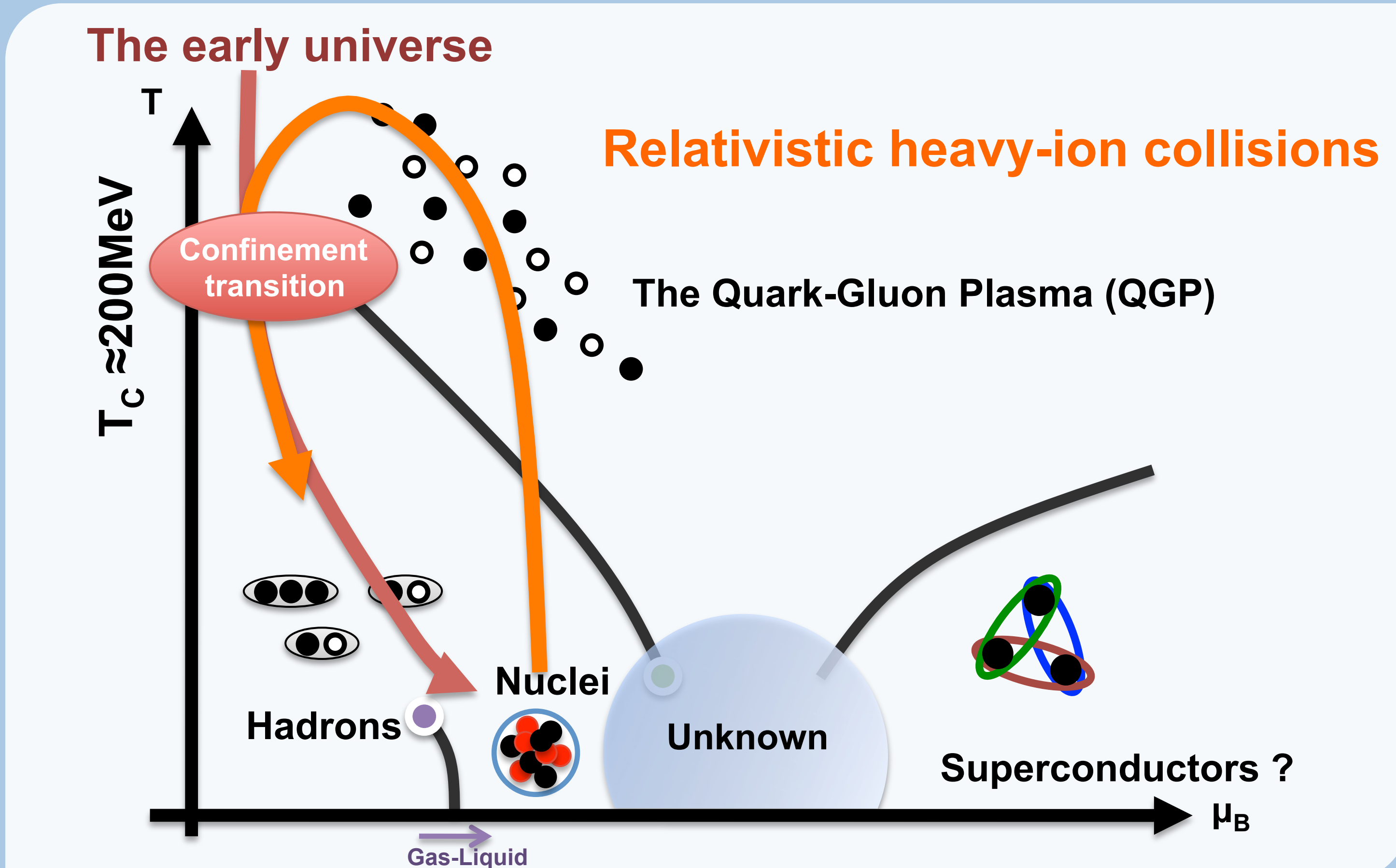
Heavy Quarkonia: Physics Motivation

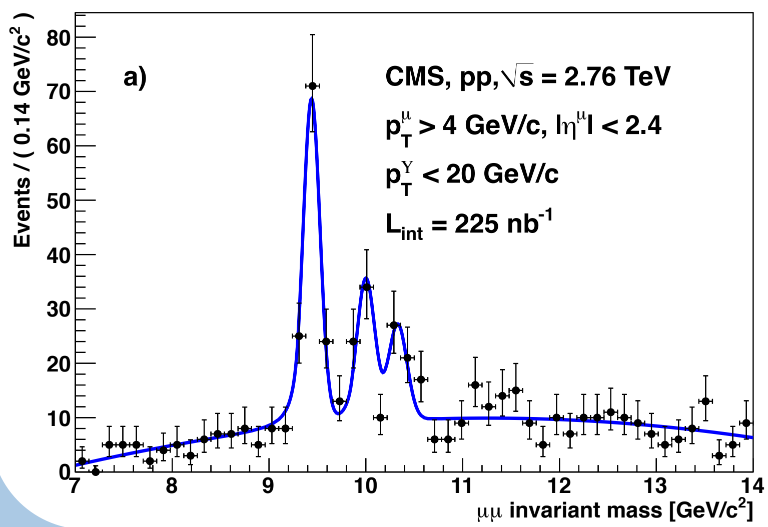


- Explore the physics of the phase transition at $T_C \approx 200$MeV
- Hadronic thermometer: Heavy Quarkonia (J/Ψ,Υ) (Matsui, Satz 1986)

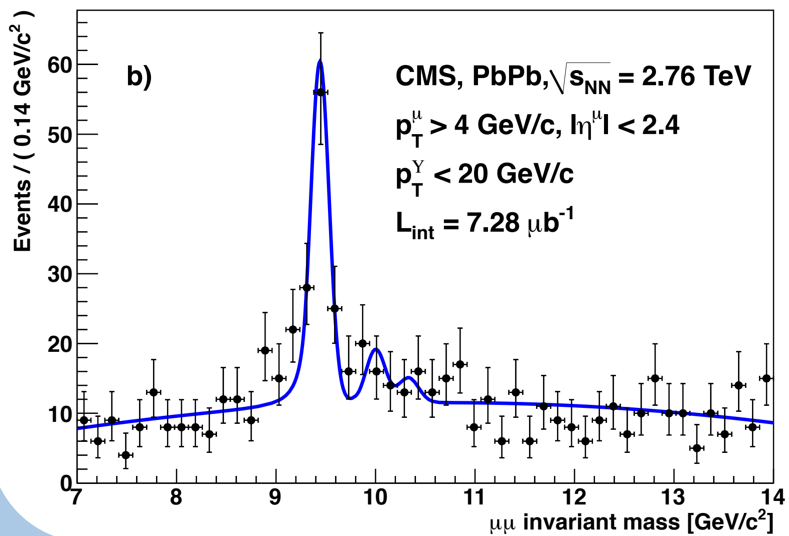
Heavy Quarkonia: Physics Motivation II

- Experiments do measure heavy quarkonium suppression at RHIC and LHC

Bottomonium in p+p collisions



Bottomonium in Pb+Pb collisions



Phys.Rev.Lett.107:052302,2011

- Large quark mass allows a separation of scales
  - ➡ Goal: Derive the potential from first principles QCD
- Need to develop fully dynamical models of QQ suppression
  - ➡ Goal: Treat effects at finite T consistently, e.g. spatial decoherence

Theoretical progress

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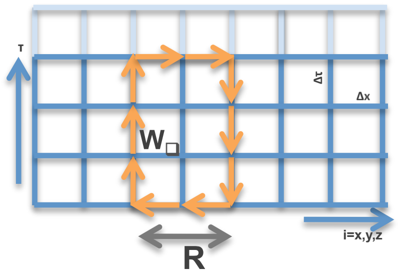
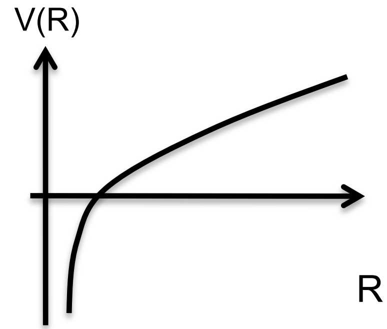
- Goal is to derive a Hamiltonian with: $$H = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V^{(0)}(R) + V^{(1)}(R)\frac{1}{m} + \dots$$
- At **T=0** systematic framework available: NRQCD, pNRQCD **Brambilla et al. 2005**

Lattice QCD (LQCD): Monte Carlo

Derivation of $V^0(R)$: **Wilson Loop**

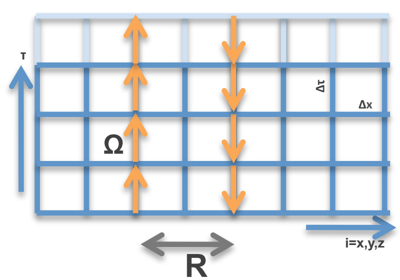


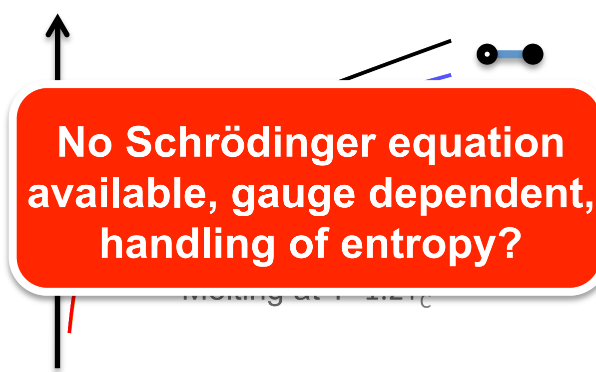


- Potential Models at T>0 **Nadkarni, 1986**

LQCD: Monte Carlo

Ad-hoc choice: Free Energies or Internal Energies





No Schrödinger equation available, gauge dependent, handling of entropy?

Perturbative derivations of $V^0(R)$

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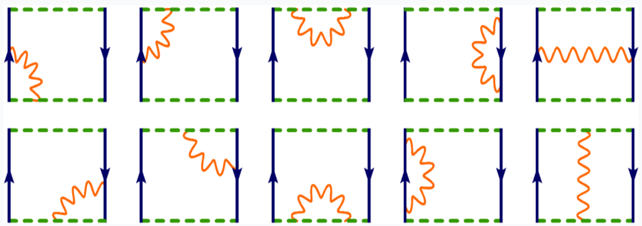
- Direct Calculation of the Wilson loop in Hard Thermal Loop PT

Laine, Philipsen, Romatschke, Tassler JHEP03 (2007) 054; Beraudo et. al. NPA 806:312,2008

$W_\square(t,R) =$



$$V(R) = -\frac{g C_F}{4\pi}\left[m_D + \frac{e^{-m_D R}}{R}\right] - \frac{i g^2 T C_F}{4\pi}\phi(m_D R)$$

$$\phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Debye screening: a cloud of quarks and gluons mitigates the interaction effects

Landau damping: collisions with the deconfined environment

- Effective field theory treatment using perturbation theory

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

  - Treats explicitly all different scales in the system
  - Additional contributions to real and imaginary part: e.g. Singlet to Octet brake-up

→ Question: How to obtain the potential non-perturbatively

A different viewpoint on Heavy Quarkonia

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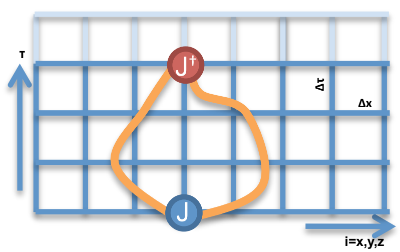
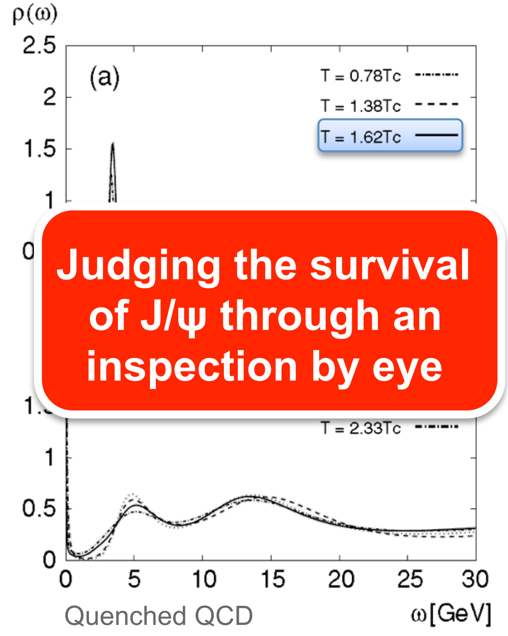
- Determine the spectra of heavy quarkonia directly from Lattice QCD

LQCD: Monte Carlo



Cannot measure spectral function directly

Infer from a measurable quantity instead:
Maximum Entropy Method



Asakawa, Hatsuda, 2004

see also: Umeda, Nomura, Matsufuru 2005; Datta, Karsch, Petreczky 2004; Jakovac, Petreczky, Petrov, Velytsky 2007; Aarts et.al. 2007, 2011; H.~-T.~Ding et. al. 2010; H.~Ohno et al. 2011

➡ Task at hand: To combine the clarity of the potential picture with non-perturbative capabilities of lattice QCD

Overall strategy: Separation of scales

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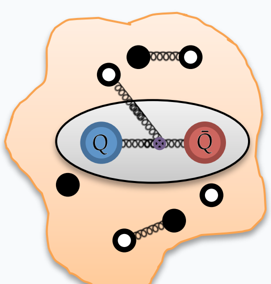
- Use only the following separation of scales

$$\frac{\Lambda_{QCD}}{m_Q c^2} \ll 1, \quad \frac{T}{m_Q c^2} \ll 1, \quad \frac{\mathbf{p}}{m_Q c} \ll 1$$

- Select appropriate degrees of freedom

Relativistic thermal field theory



Derivation of the heavy quark potential →

Dirac fields
$\bar{Q}(x), Q(x)$
$\bar{q}(x), q(x), A^\mu(x)$

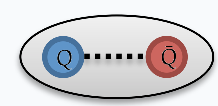
Pauli fields
$\bar{\chi}(x), \chi(x)$
$\bar{q}(x), q(x), A^\mu(x)$

Lattice QCD: Monte Carlo

Position & Momentum
$\mathbf{z}(t), \mathbf{p}(t)$
$\rho_\Box(\omega, R) \Rightarrow V^0(R)$

Maximum Entropy Method

Quantum mechanics



- Obtain a dynamical Schrödinger equation with non-perturbative potential $V^0(R)$

A QQbar wavefunction

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- Relativistic field theory: Meson Currents

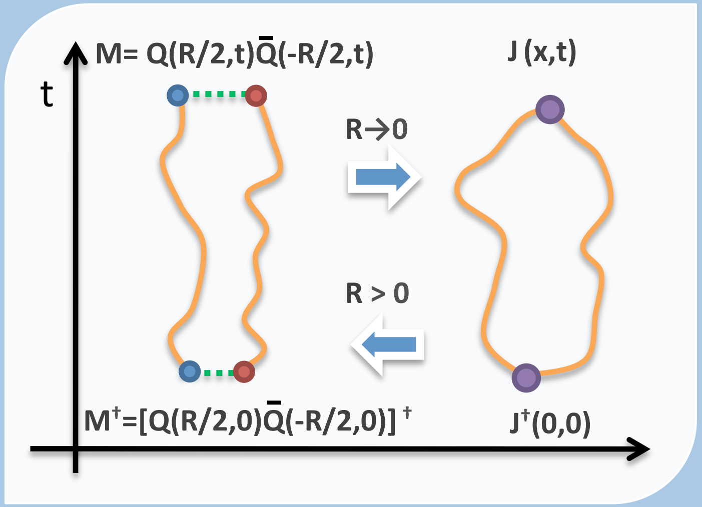
$$J(x) = \bar{Q}(x)\Gamma Q(x)$$

  - Test charges: introduce external separation

$$M(\mathbf{R},t) = \bar{Q}(\mathbf{x},t)\ \Gamma\ W(\mathbf{x},\mathbf{y},t)\ Q(\mathbf{y},t)$$

  - Time evolution: Gauge invariant description

$$D^{>}(\mathbf{R},t) = \langle M(\mathbf{R},t)M^{\dagger}(\mathbf{R},0)\rangle$$



- At T=0 rigorously defined as Nambu-Bethe-Salpeter wavefunction

$$\Psi_{NBS}(\mathbf{R},t) = \langle 0|M(\mathbf{R},t)|Q\bar{Q}\rangle$$

Review: Aoki, Hatsuda, Ishii Prog.Theor.Phys. 123 (2010) 89

- At T>0, attempt a generalization via the Mesic correlators

Barchielli et. al. 1988
Iida, Ikeda PoS(Lat 2011)195

$$\Psi_{Q\bar{Q}}(\mathbf{R},t) \overset{\text{match}}{\equiv} D^{>}(\mathbf{R},t) = \left\langle \mathcal{T}\left[\int \mathcal{D}[\bar{Q},Q]\ \Gamma\bar{\Gamma}\ WW^{\dagger}\ Q(y')\bar{Q}(y)Q(x)\bar{Q}(x')\ e^{iS_{QQ}[Q,\bar{Q},A]}\right]\right\rangle$$

Three steps towards the potential

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- I. Integrate out rest energy: Foldy-Tani-Wouthuysen expansion in $1/mc^2$

$$S^{FTW}_{QQ}[A] = \bar{Q}(x)\left[\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix} D_0 - mc + \frac{g}{2mc^2}\begin{pmatrix}\sigma_i & 0\\ 0 & \sigma_i\end{pmatrix} B^i + \frac{1}{2mc} D_i^2\right] Q(x)$$

  - Upper and lower components decouple: Pauli Spinors Q=(χ,ξ) both contribute

- II. Grassmann Integration: Replace pairs of $\chi\chi^\dagger$, $\xi\xi^\dagger$ with QM Green's functions S
  - No fermion determinant since heavy quarks do not appear in virtual loops

$$D^{>}_{QM} = \left\langle \mathcal{T}\left[W(\mathbf{x},\mathbf{y})\, G\, S(\mathbf{y},\mathbf{y}')\, \bar{G}\, W^\dagger(\mathbf{x}',\mathbf{y}')\, S^\dagger(\mathbf{x},\mathbf{x}')\right.\right\rangle$$

Temperature dependence | 2x2 Matrix Spin Structure | Quantum mechanical Greens function

- III. Write greens functions as QM path integrals (quark propagation amplitude):

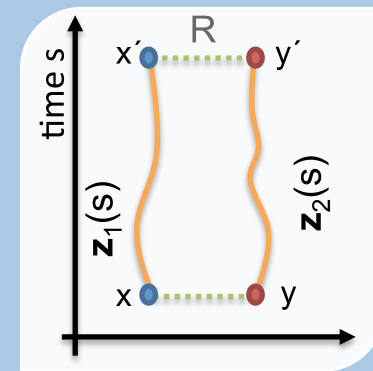
$$S(x,x') = \int_x^{x'} \mathcal{D}[\mathbf{z},\mathbf{p}]\,\mathcal{T}\exp\left[i\int_t^{t'} dt\left(\mathbf{p}(t)\dot{\mathbf{z}}(t) - \frac{1}{2m}\left(\mathbf{p}(t)-\frac{g}{c}\mathbf{A}(\mathbf{z}(t),t)\right)^2 - gA^0(\mathbf{z}(t),t) + \frac{g}{mc}\sigma_i B^i(\mathbf{z}(t),t)\right)\right]$$

Barchielli et. al. 1988

Defining the Heavy Quark Potential

- Combine the path integrals for each single quark/antiquark

$$D^{>}_{QM} = \exp[-2imc^2t]\int \mathcal{D}[\mathbf{z}_1,\mathbf{p}_1]\int \mathcal{D}[\mathbf{z}_2,\mathbf{p}_2]\times$$
$$\exp\left[i\int_t^{t'} ds \sum_i \left(\mathbf{p}_i(s)\dot{\mathbf{z}}_i(s) - \frac{\mathbf{p}_i^2(s)}{2m}\right)\right]\left\langle \frac{1}{N}\mathrm{Tr}\left[\mathcal{P}_\mathcal{C}\exp\left[\frac{ig}{c}\oint_\mathcal{C} dx^\mu A_\mu(x)\right]\right]\right\rangle$$



- Use the transfer matrix to read off the Hamiltonian

$$\langle \mathrm{Tr}[\exp[\oint A]\,]\rangle \equiv \exp\left[i\int_t^{t'} ds\, U(\mathbf{z}_1(s),\mathbf{z}_2(s),\mathbf{p}_1(s),\mathbf{p}_2(s),s)\right]$$

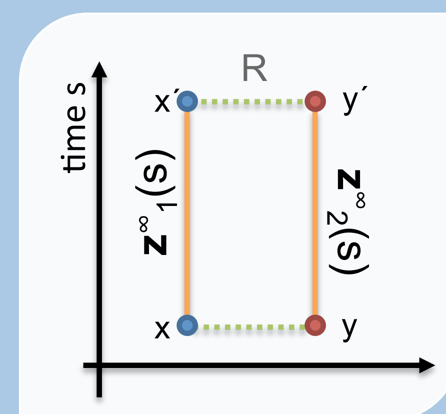
$p \longrightarrow 0$

- Systematic expansion of the potential in p/mc

$$i\log\left[\left\langle W(z(t),t)\right\rangle\right] = \int_t^{t'} ds\left(V^0(z,s)\big|_{p=0} + V_n^{1,i}(z,s)\big|_{p=0}\frac{p_n^i(s)}{mc} + \dots\right)$$

- In the static limit: rectangular Wilson loop contour $W_\square(\mathbf{R},t)$
- Take the time derivative to obtain the potential $V^0(R)$ at late t

$$\lim_{t\to\infty}\frac{i\partial_t W_\square(\mathbf{R},t)}{W_\square(\mathbf{R},t)} = V^0(\mathbf{R})$$



The Potential and Spectral Functions

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- Make time dependence of the Wilson loop explicit (here in real-time)

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)$$

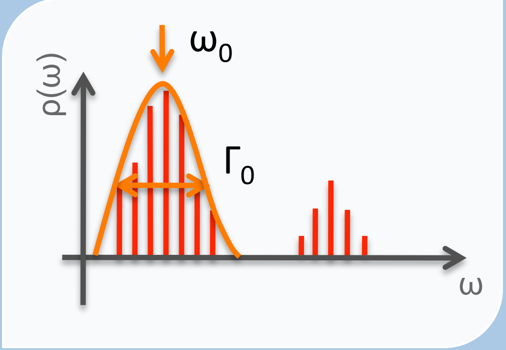
  - In the infinite mass limit: $\rho(R,\omega)=\rho_\square(R,\omega)>0$

$$V^0(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)}$$

  - At each R, lowest lying peak determines the potential
  - Two analytically solvable cases: Breit-Wigner and Gaussian

$$\rho_{BW}(R,\omega) \propto \frac{\Gamma(R)}{\Gamma^2(R)+(\omega_0(R)-\omega)^2} \quad V^0_{BW}(R) = \omega_0(R) - i\Gamma(R)$$

$$\rho_G(R,\omega) \propto \mathrm{Exp}\Big[-\frac{(\omega_0(R)-\omega)^2}{2\Gamma^2(R)}\Big] \quad V^0_G(R) = \omega_0(R) - i\Gamma^2(R)t$$



Extracting the Potential from Lattice QCD I

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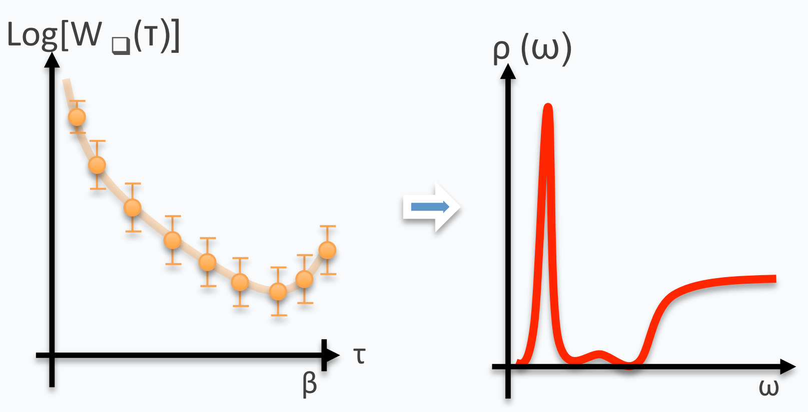
- We can measure neither $\rho_\square(R,\omega)$ nor $W_\square(R,t)$ directly in Lattice QCD

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \iff W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

O(10) + noise **O(1000)**

  - Simple χ^2 fitting is ill defined



- Maximum Entropy Method
  - Regularize χ^2 fitting by incorporating prior information
  - Check what parts of the spectrum depend on the regularization!

Extracting the Potential from Lattice QCD II

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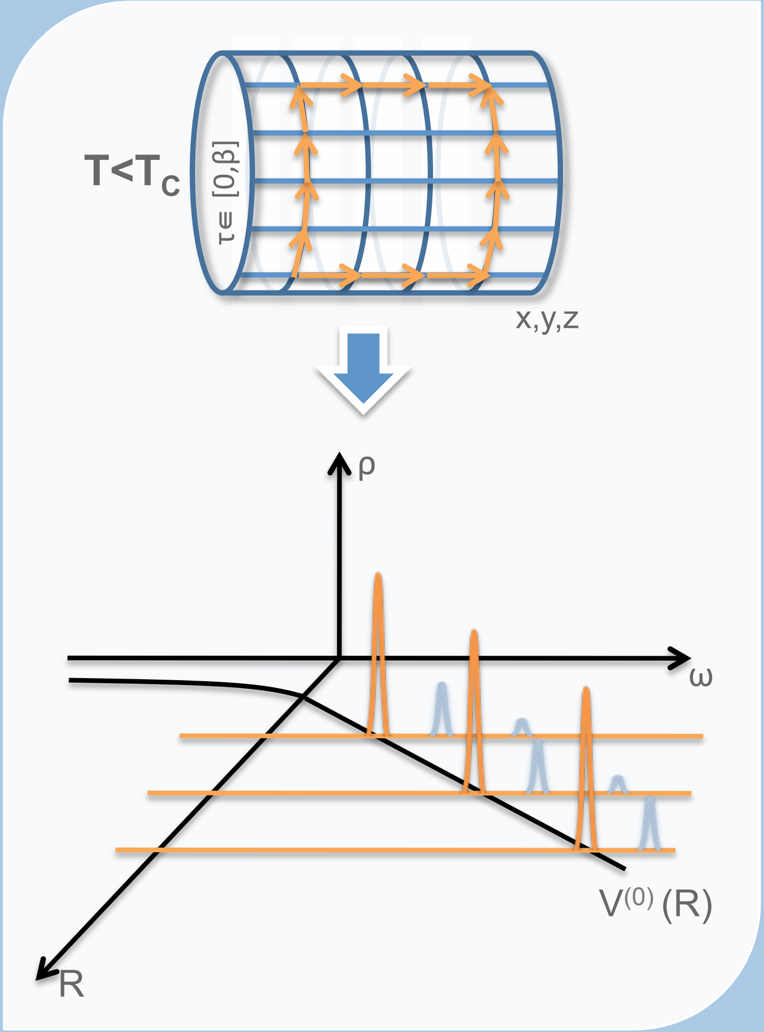

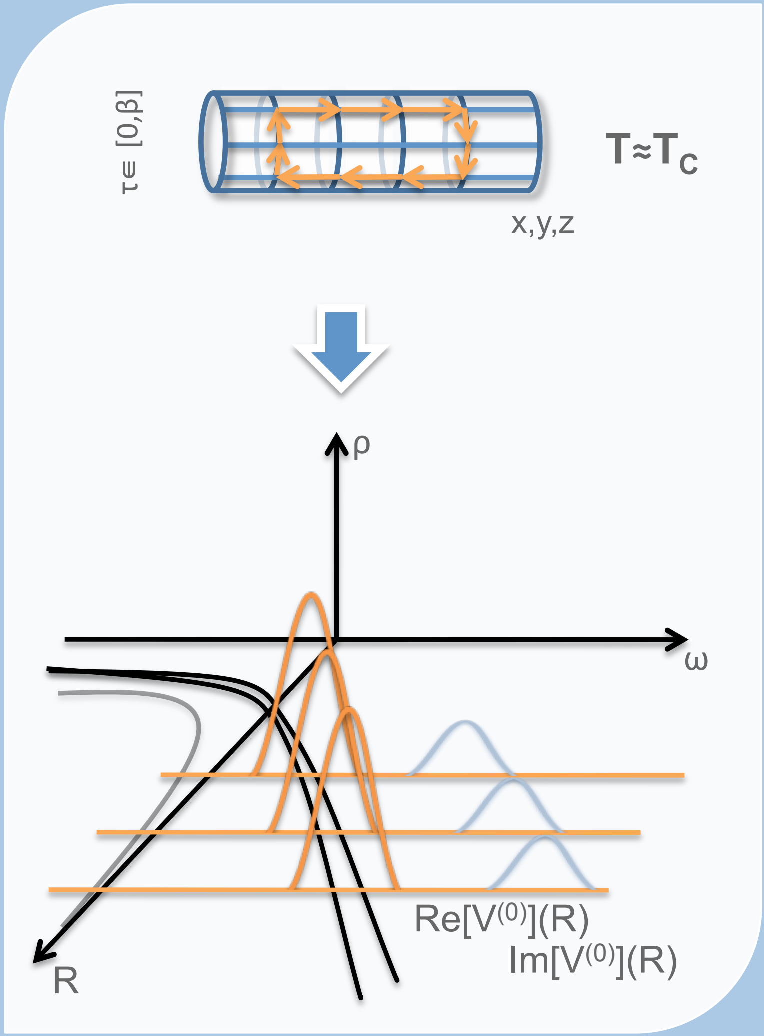

First Numerical results

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Quenched QCD Simulations

- Anisotropic Wilson Plaquette Action
- T=0.78TC, 1.17TC, 2.33TC
- NX=20 β=6.1 ξ_b=3.2108 NT=36, 24, 12
- Box Size: 2fm Lattice Spacing: 0.1fm
- HB:OR 1:4 with 200 sweeps/readout

Spectral Peak Fit

- Breit-Wigner and Gaussian shape
- Error bars from prior dependence

Maximum Entropy Method

- N_ω=1500, Prior: m_0/ω, varied over 4 orders
- Extended search space: decoupled from N_τ
- Arbitrary precision artihmetic: 384bit
- Test I: Vary the prior amplitude
- TestII: Use different N_τ

The Potential at T=0.78T_C

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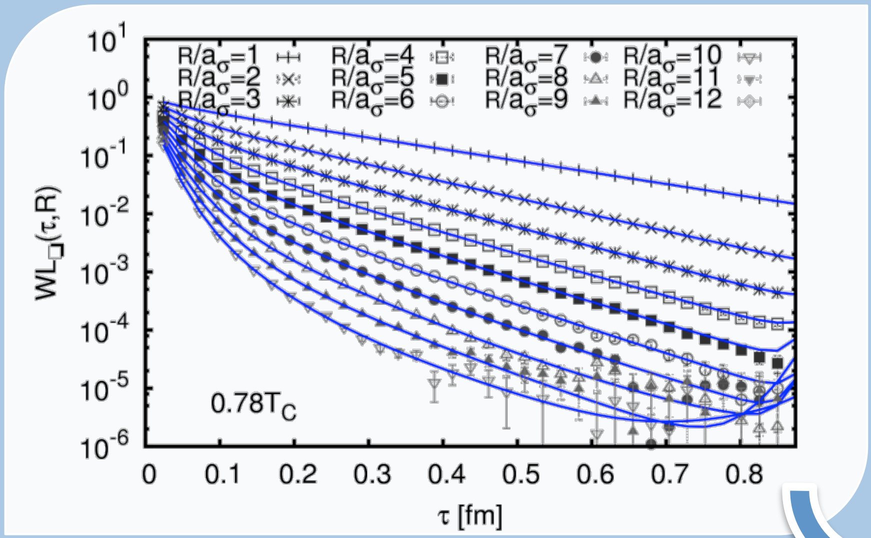

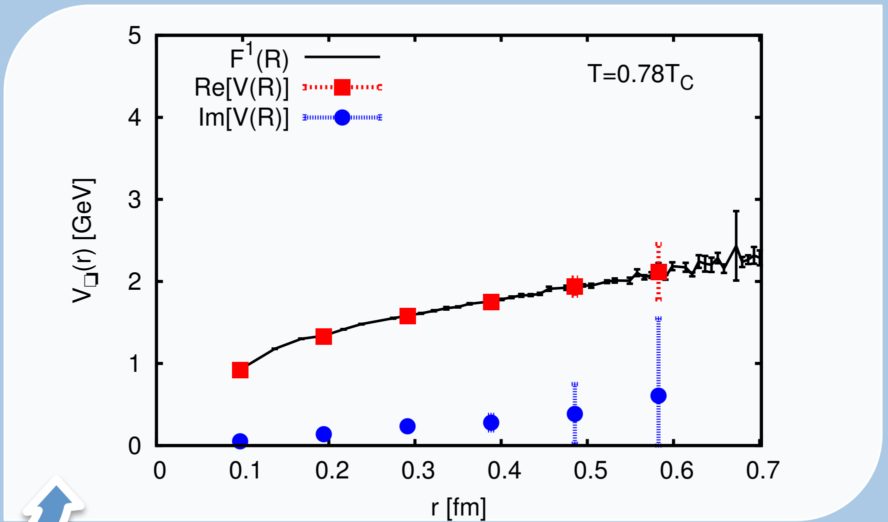

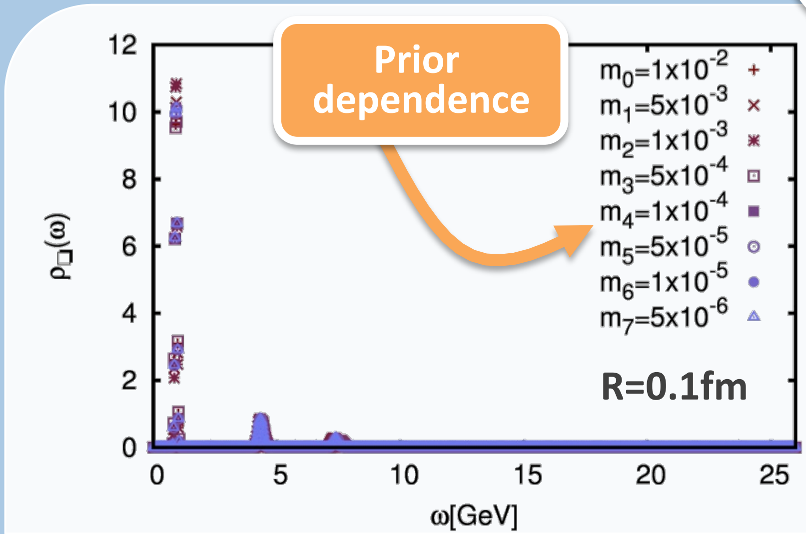

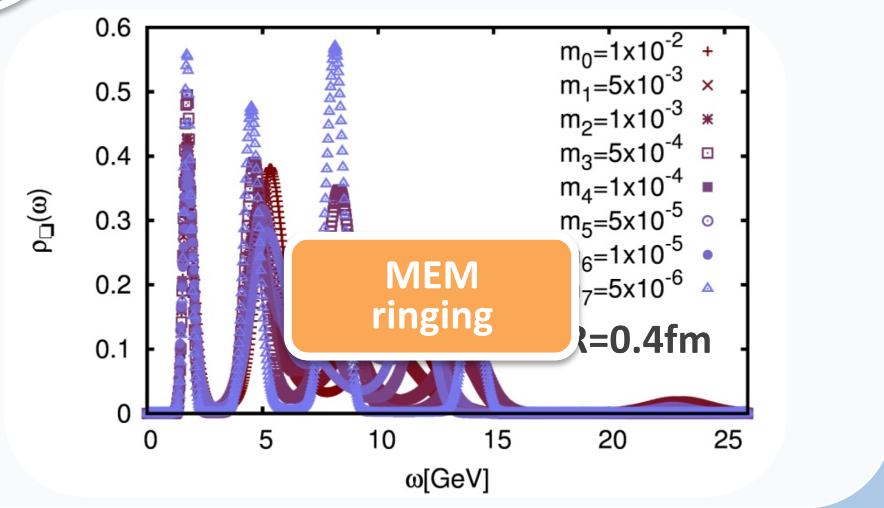

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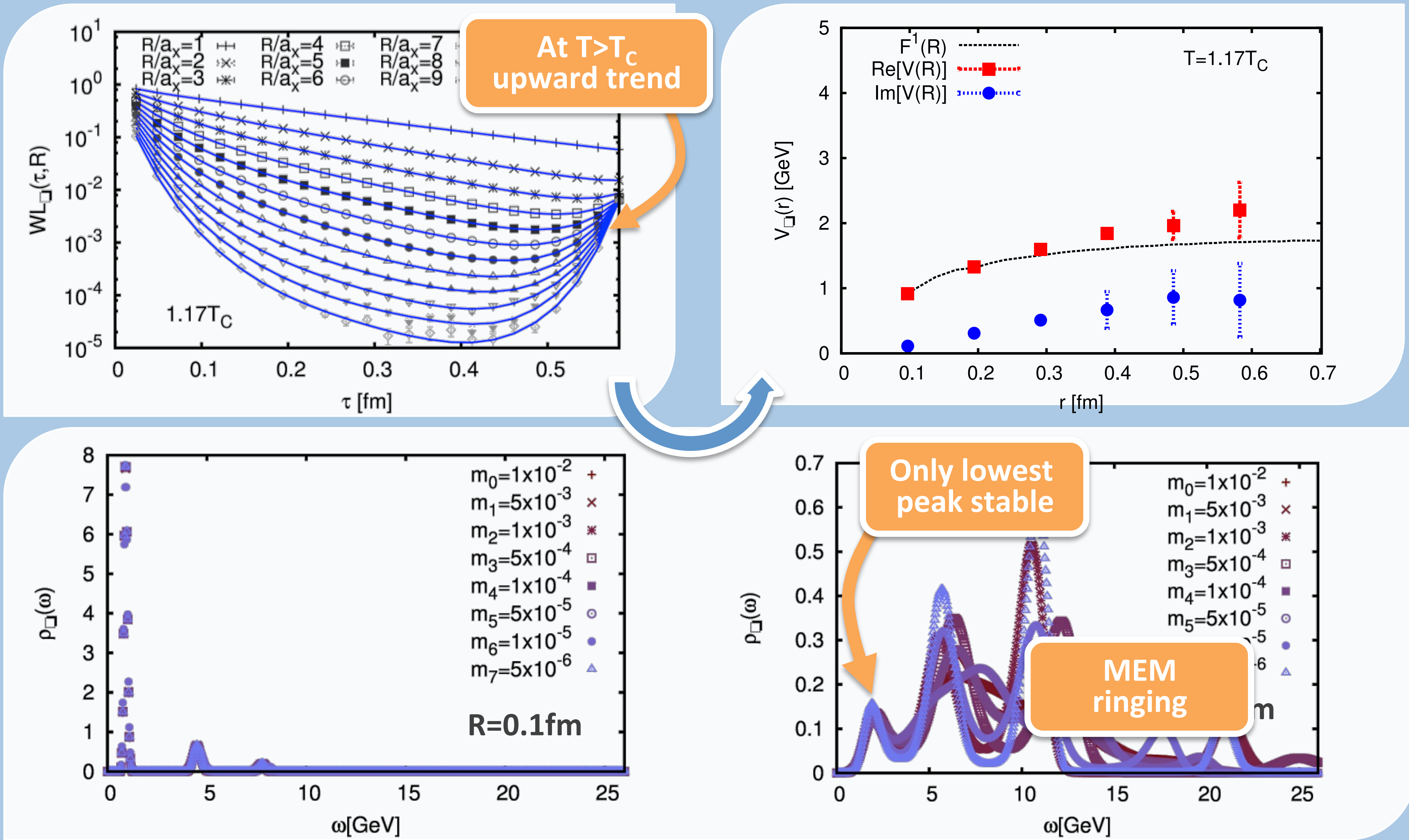
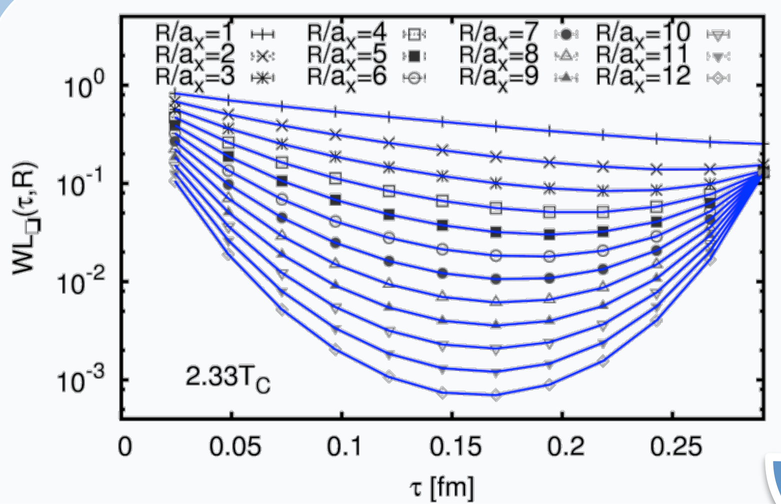
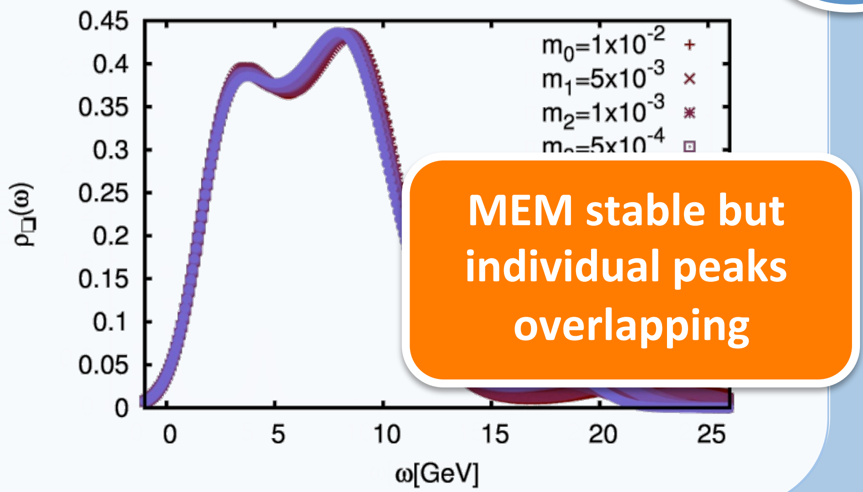
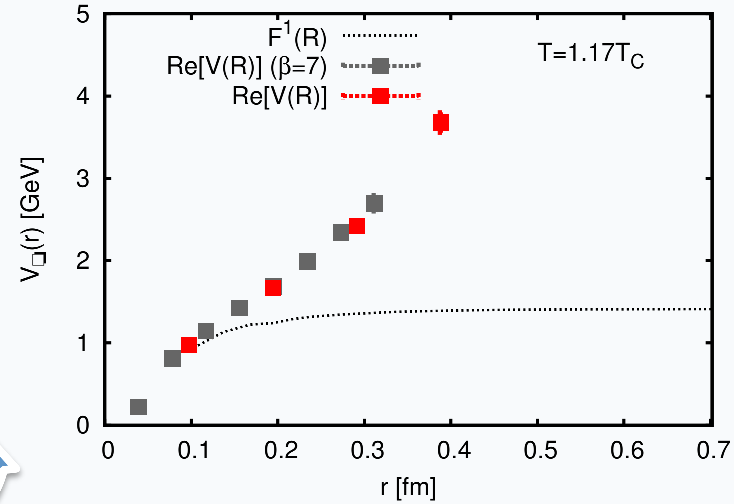
The Potential at T=1.17T_C

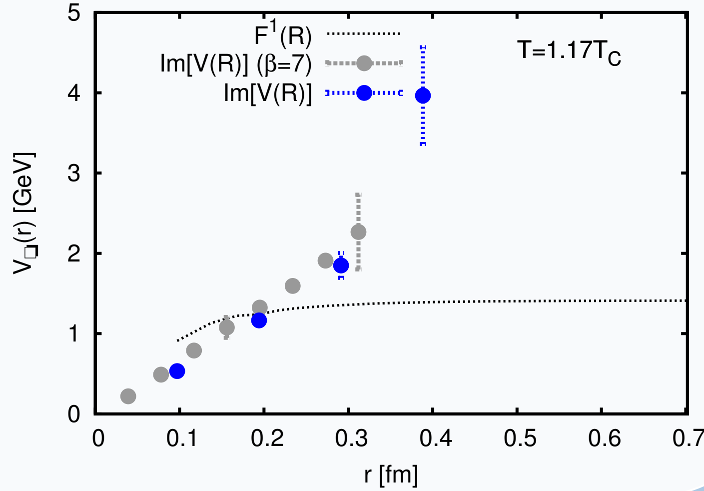


The Potential at $T=2.33T_C$









Complex Potential Conclusion

- Static heavy quark potential derived from QCD
  - Values can be extracted from Lattice QCD: Wilson loop spectral functions
  - Spectral width / Imaginary part present above T_C
  - Current numerical evaluation seems to favor strong imaginary part at $T>T_C$

→ Test extraction from spectral functions on HTL Wilson loop data/spectra

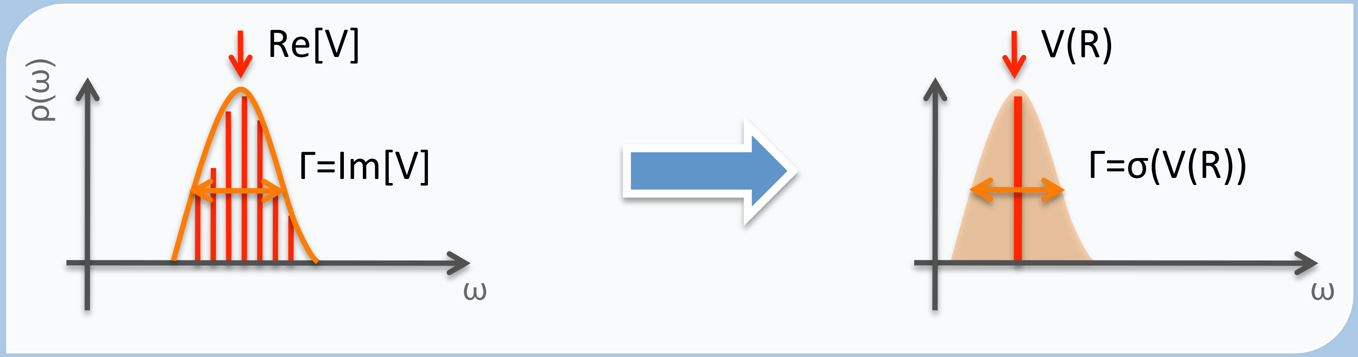
work in progress with Y. Burnier

Include dynamical fermions for more realistic screening effects

work in progress with O. Kaczmarek

A stochastic potential in the QGP

- A new proposal: width in the spectral function ⟺ uncertainty in Re[V(R)]



- At each time step the **purely real** potential V(R) is distorted by thermal fluctuations
- Construct unitary stochastic time evolution (neglects back reaction on medium)

$$\Psi_{Q\bar{Q}}(\mathbf{R},t) = \mathcal{T}\exp\left[-\frac{i}{\hbar}\int_0^t dt \left\{-\frac{\nabla^2}{2m_Q} + V(\mathbf{R}) + \Theta(\mathbf{R},t)\right\}\right]\Psi_{Q\bar{Q}}(\mathbf{R},0)$$

$$\langle\Theta(\mathbf{R},t)\rangle = 0, \quad \langle\Theta(\mathbf{R},t)\Theta(\mathbf{R}',t')\rangle = \hbar\Gamma(\mathbf{R},\mathbf{R}')\delta_{tt'}/\Delta t$$

Heavy Quarkonia as Open Quantum System

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- Underlying theoretical framework: Open Quantum Systems

  see e.g. H.-P. Breuer, F. Petruccione, Theory of Open Quantum Systems

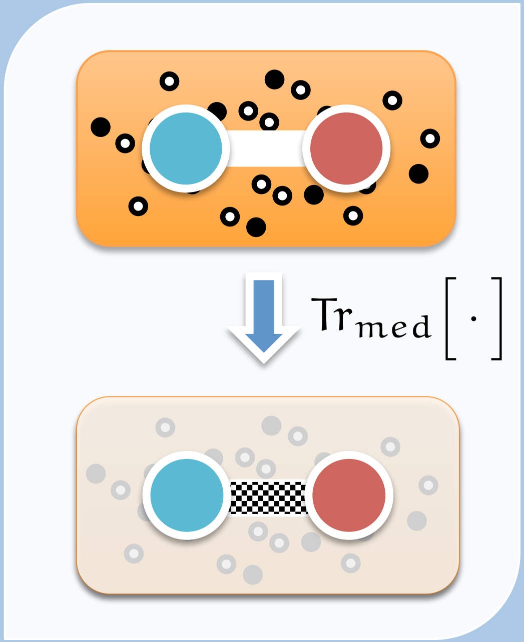
  - Change of notation: ρ(t) density matrix of states

$$H = H_{sys} \otimes I_{med} + I_{sys} \otimes H_{med} + H_{int}$$

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H, \rho(t)]$$ unitary evolution: $H=H^\dagger$

  - Interested in the dynamics of the QQbar system only

$$\rho_{Q\bar{Q}}(t, \mathbf{R}, \mathbf{R}') = \mathrm{Tr}_{med}\left[\rho(t, \mathbf{R}, \mathbf{R}')\right] = \langle \Psi_{Q\bar{Q}}(\mathbf{R}, t)\Psi^*_{Q\bar{Q}}(\mathbf{R}', t)\rangle_\Theta$$



  - Interaction with the medium induces a stochastic element into the dynamics (similar concepts are quantum state diffusion, quantum jumps, etc..)

    see also: N.Borghini, C.Gombeaud: arXiv:1109.4271

  - **Decoherence**: Interaction between medium and QQbar select a basis of states in which ρ_{QQ} becomes diagonal over time

Time evolution of Heavy Quarkonium

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- Evolution on the level of the wavefunction:

$$i\frac{d}{dt}\Psi_{Q\bar{Q}}(\mathbf{R},t) = \Big(-\frac{\nabla^2}{2\mu} + V(\mathbf{R}) + \Theta(\mathbf{R},t) - i\frac{\Delta t}{2}\Theta^2(\mathbf{R},t)\Big)\Psi_{Q\bar{Q}}(\mathbf{R},t)$$

  - Average wavefunction only depends on diagonal correlations

$$i\frac{d}{dt}\langle\psi_{Q\bar{Q}}(\mathbf{R},t)\rangle_\Theta = \Big(-\frac{\nabla^2}{2\mu} + V(\mathbf{R}) - \frac{i}{2}\Gamma(\mathbf{R},\mathbf{R})\Big)\langle\psi_{Q\bar{Q}}(\mathbf{R},t)\rangle_\Theta$$

- Evolution on the level of the density matrix:
  - Averaged quantity whose time evolution depends on off-diagonal Γ(R,R')

→ How to extract infromation about Γ(R,R') from the lattice

- Survival probability of Heavy Quarkonia (Vacuum: H^{vac} QGP: H)
  - Admixture c^{ν}_{nm} of initial bound eigenstates Φ_n of H^{vac} at the current time

$$c^{\nu}_{nm}(t) = \int d\mathbf{R}d\mathbf{R}'\ \Phi^*_n(\mathbf{R})\ \langle\Psi_{Q\bar{Q}}(\mathbf{R},t)\Psi^*_{Q\bar{Q}}(\mathbf{R}',t)\rangle_\Theta\ \Phi_n(\mathbf{R}') \Rightarrow P^{\nu}(t) = \sum_{n\ \text{bound}} c^{\nu}_{nn}(t)$$

First numerical 1-d calculations

- QQbar in vacuum: Cornell potential with string breaking $\sigma=(0.4\text{GeV})^2$ $r_{sb}=1.5\text{fm}$

Determine vacuum ground state

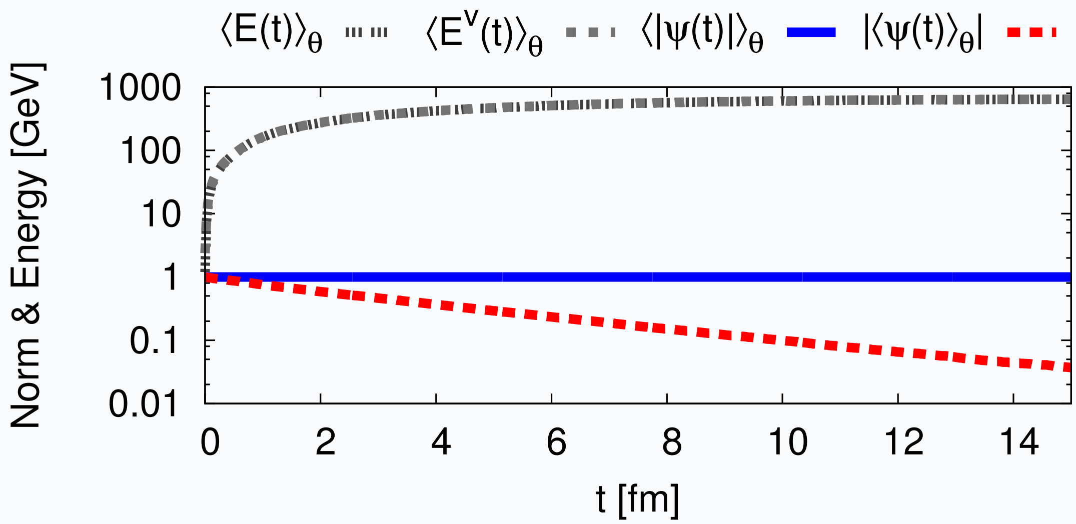
- **PETSC+SLEPC eigensystems**

- Dynamics governed by different model potentials ($T=2.33T_C$):
  - All thermal effects in the noise, real part similar to vacuum
  - Debye screening and small noise ($m_D\approx1\text{GeV}$) **(Laine et. al. 2007)**
  - Pure Debye screening without noise ($m_D=5\text{GeV}$) **(Matsui, Satz, 1986)**

Stochastic Dynamics

- **Crank-Nicholson algorithm in 1 dimension**
- **N=512, dx=0.01fm, dt=dx/100 fm**
- **Diagonal noise: too small correlation length**

Generic features of the simulation



- Unitarity is preserved in each member of the stochastic ensemble $<|\Psi_{QQ}|>=1$
- Imaginary part emerges after averaging $|<\Psi_{QQ}>|<1$
- Diagonal noise: Correlation length $l_{corr}= dx << 2\pi/T$

Heavy quarkonium in the QGP I

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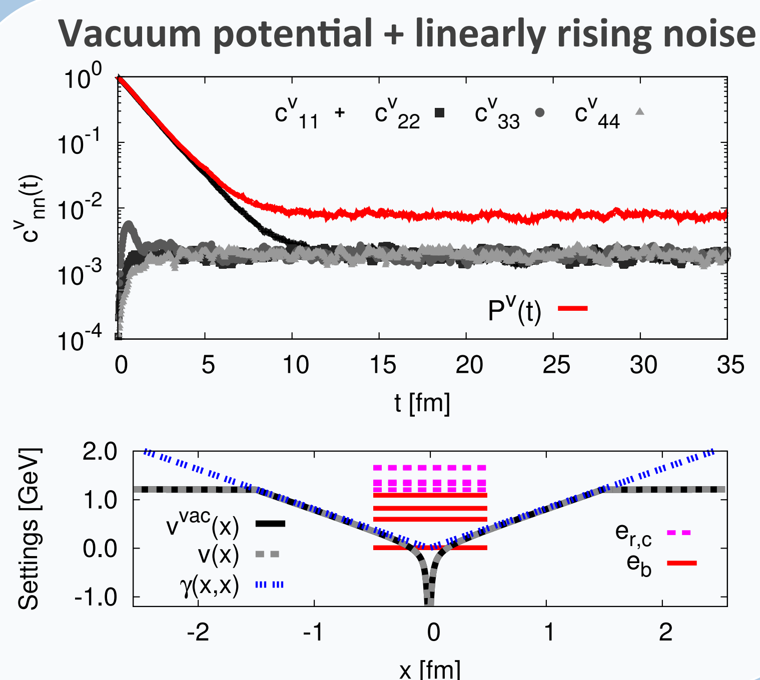
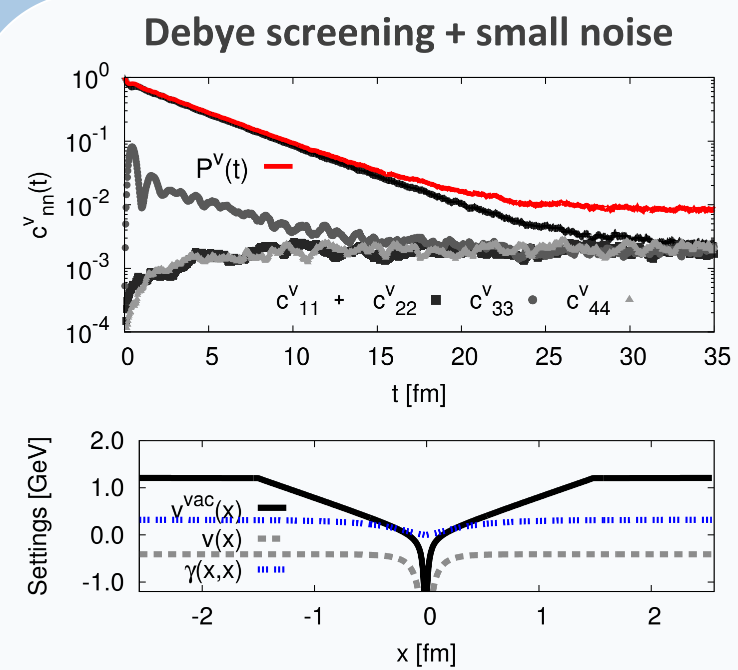
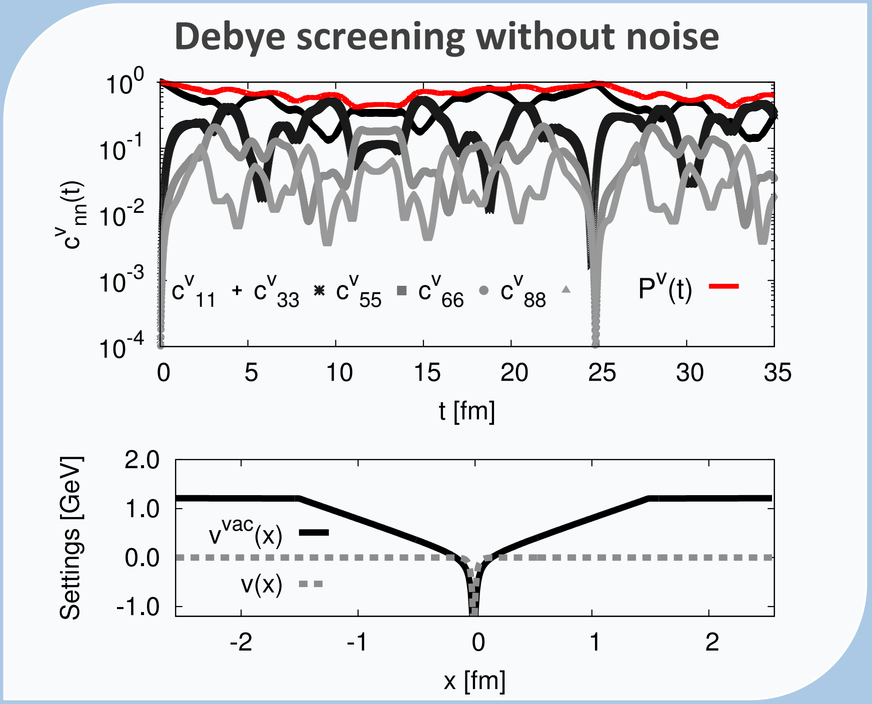




- Exponential suppression in $P^V(t)$: v(x) and Γ(x,x) determine speed
- Populating of higher states: Noise vs. mixing through h(x)
- Very different parameter sets give similar asymptotic $P^V(t)$ -> artificial

Heavy quarkonium in the QGP II



- Mixing through h(x) with m_D=5GeV
- Observed suppression not exponential, not even monotonous
- Only parity even eigenstates are excited

Conclusion and Outlook

- Static heavy quark potential derived from QCD
  - Values can be extracted from Lattice QCD: Wilson loop spectral functions
  - Spectral width / Imaginary part present above T_C
  - Current numerical evaluation seems to favor strong imaginary part at $T>T_C$

→ Test extraction from spectral functions on HTL Wilson loop data/spectra

work in progress with Y. Burnier

Include dynamical fermions for more realistic screening effects

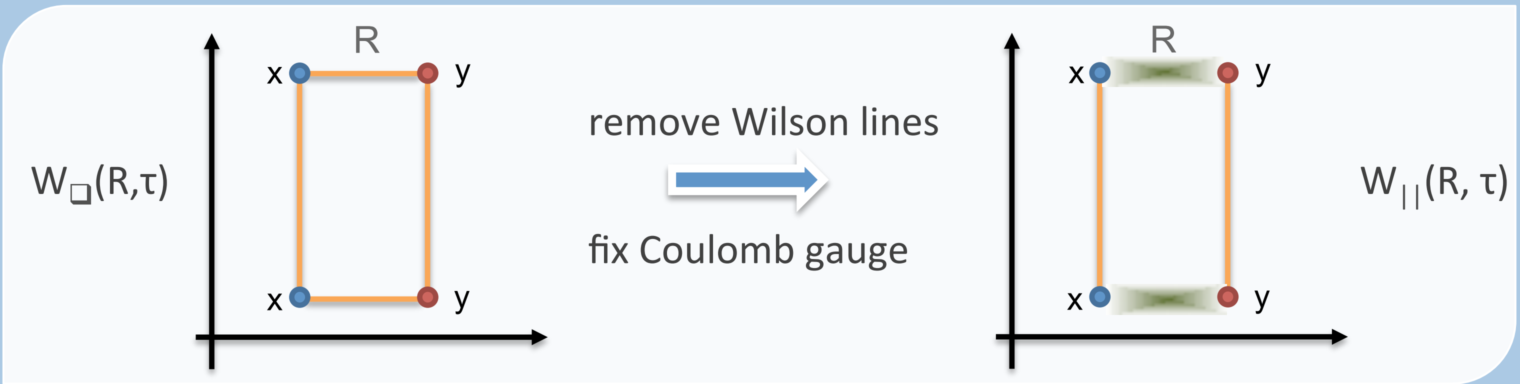
work in progress with O. Kaczmarek

- Stochastic evolution of Heavy Quarkonia in the QGP
  - Instead of imaginary part: uncertainty in the real part of the potential
  - Microscopic evolution fully unitary, Im[V] obtained after ensemble average

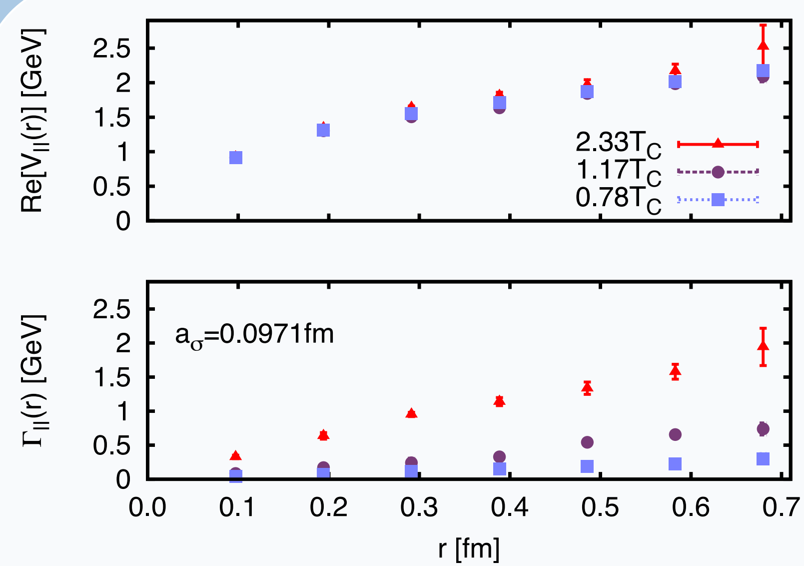
→ 3d-simulation, LQCD determination of Γ(R,R'), incorporate drift term …

Additional Slides

An alternative observable



- Derivation of V(R) remains the same
- Lattice data much less noisy
- Real part and width are smaller
  - Possible tradeoff to ensure same physics outcome?



Extracting the Potential from Lattice QCD I

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- We can measure neither $\rho_\square(R,\omega)$ nor $W_\square(R,t)$ directly in Lattice QCD

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

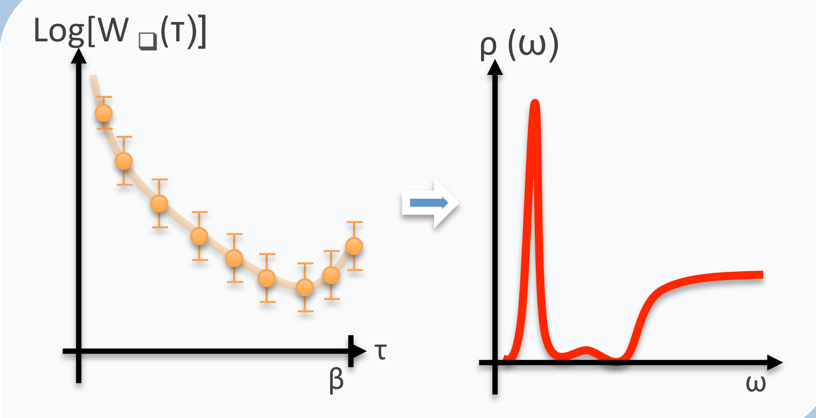
O(10) + noise **O(1000)**

  - Simple χ^2 fitting is ill defined

- Bayes Theorem (Maximum Entropy Method)

$$P[\rho|Dh] = \frac{P[D|\rho h]\, P[\rho|h]}{P[D|h]}$$

  - Regularize χ^2 fitting through entropy



$$\propto \mathrm{Exp}\Big[-\frac{1}{2}\sum_{ij}\big(D(\tau_i) - D_\rho(\tau_i)\big)C^{-1}_{ij}\big(D(\tau_j) - D_\rho(\tau_j)\big)\Big]$$

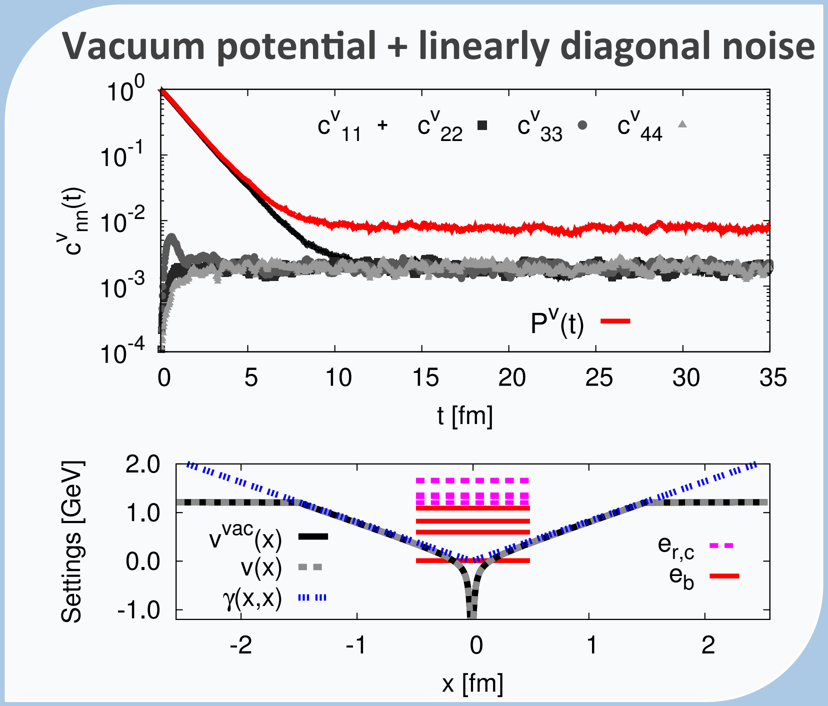
Likelihood: the usual χ^2 fitting term

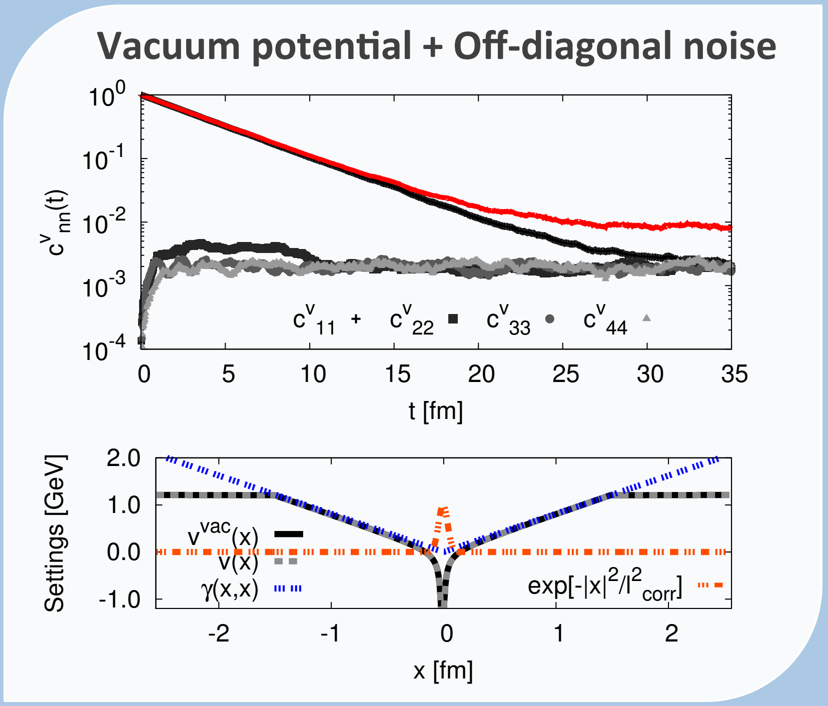
$$\propto \mathrm{Exp}\Big[\alpha \int_{-\infty}^{\infty} \Big\{\rho(\omega) - h(\omega) - \rho(\omega)\mathrm{Log}\Big(\frac{\rho(\omega)}{h(\omega)}\Big)\Big\}d\omega\Big]$$

Prior probability: Shannon-Janes entropy

$$\Rightarrow \quad \frac{\delta}{\delta\rho} P[\rho|Dh] \stackrel{!}{=} 0 \quad \Rightarrow$$ Extended search space: decouple from N_τ

Towards more realistic noise





- On the right: correlation length l_{corr}=4dx $\Gamma(x,x') \propto \mathrm{Exp}[-|x-x'|^2/l^2_{corr}]$
- Noise less localized: ground state suppression slower
- Population of states follows eigenenergy scale (t<10fm)

Comparison: Stochastic V(R) vs. Im[V](R)

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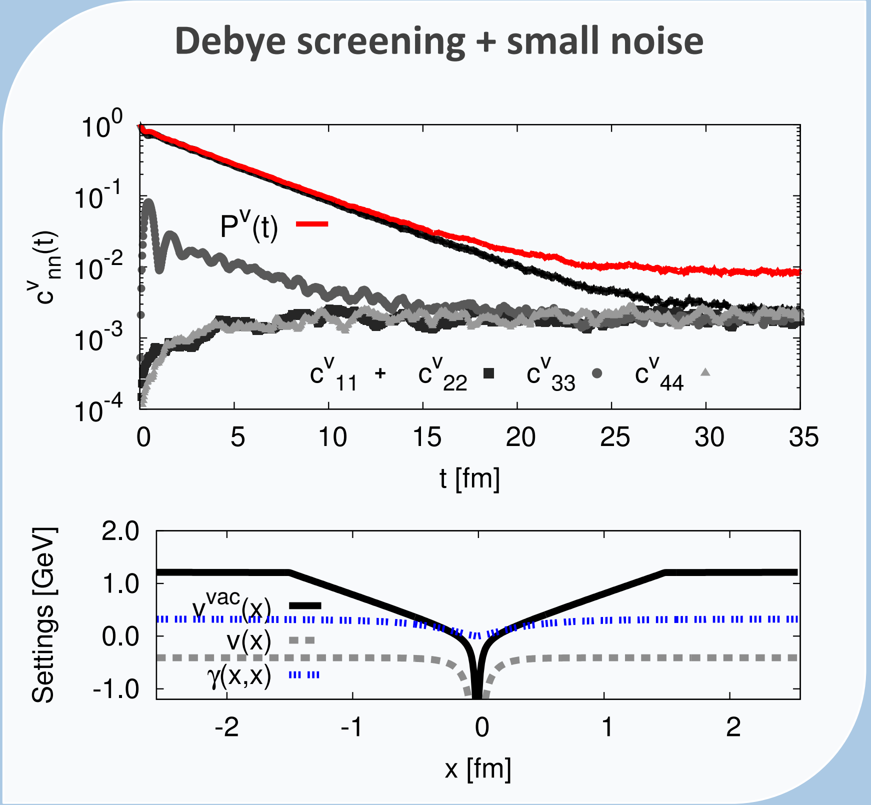

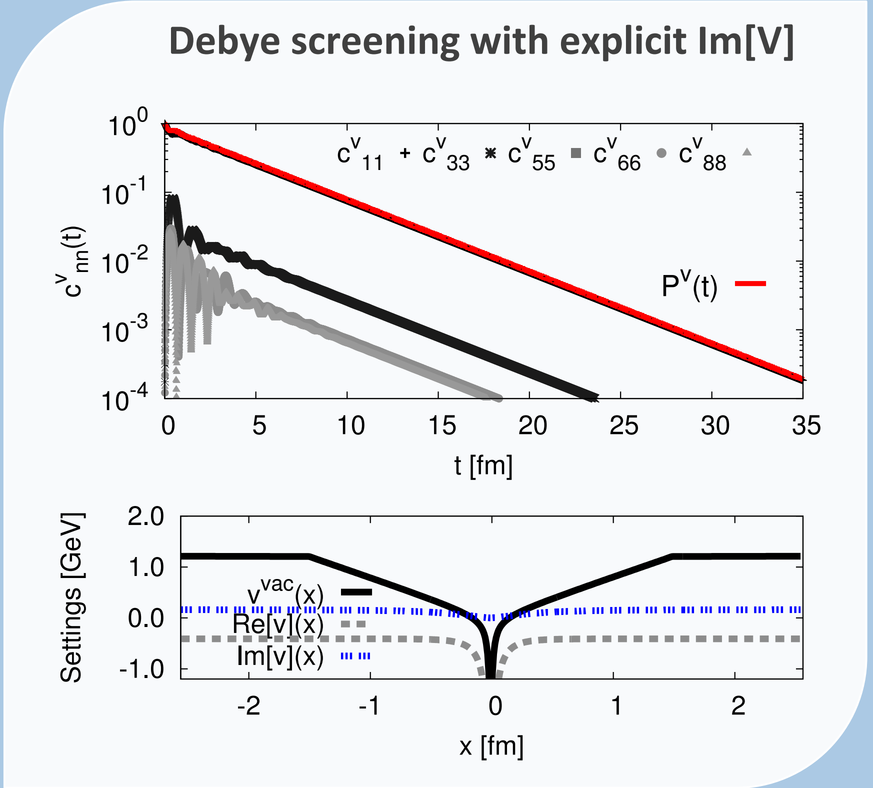

- At early times: similar evolution
- At late times very distinct: finite suppression vs. unabated decrease
- Need to include backreaction physics to understand $P^v(t)$