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Cosmological Perturbations from Non-Equilibrium Field Dynamics

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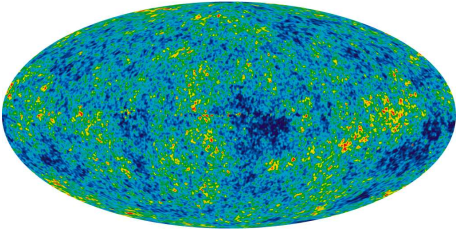
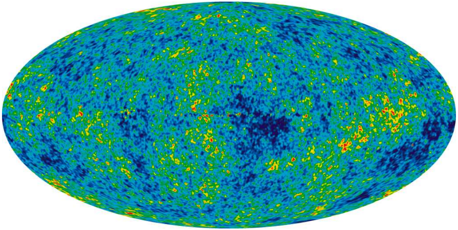
3 April 2012

Cosmic Microwave Background



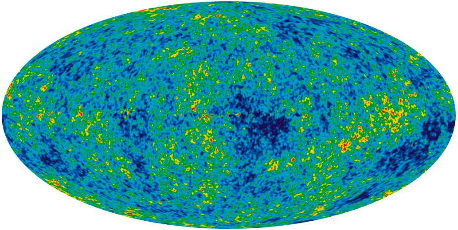
- Penzias&Wilson 1964
- Thermal radiation from recombination $t \approx 300000\text{yrs}$
- Redshifted from $T \approx 4000\text{K}$ to $T \approx 2.7\text{K}$

Cosmic Microwave Background



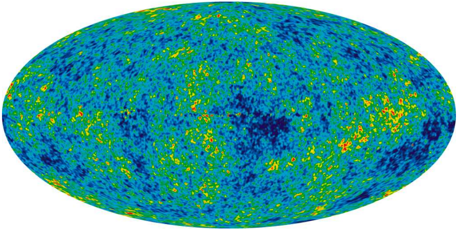
- Temperature anisotropies $\delta T \sim 10^{-5}\mathrm{K}$
- COBE 1992, WMAP 2003–2010, Planck 2012
- The best source of information about the early universe

Primordial Density Perturbations



- Higher temperature = Higher density
- Origin of large-scale structure:
  Galaxies, galaxy clusters

Primordial Density Perturbations



- Highly Gaussian $-10 < f_{\mathrm{NL}} < 74$
- Nearly scale-invariant spectrum $n_{\mathrm{s}} = 0.963 \pm 0.012$
- Just as inflation predicts!

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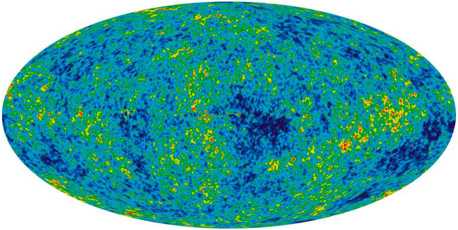
Horizon Problem



- Particle horizon at recombination $\ll$ Observable universe today
- Points separated my more than 2^o not in causal contact

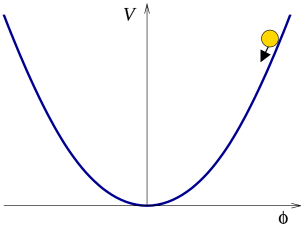
Inflation

- Scalar inflaton field ϕ, with $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$
- Dominates energy density: $\rho \approx V(\phi)$
- Slow roll conditions: $\epsilon = \frac{1}{2}M_{\rm Pl}^2(V'/V)^2 \ll 1$, $|\eta| = M_{\rm Pl}^2|V''/V| \ll 1$

$\Rightarrow$ Exponential expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_{\rm Pl}^2}$$

- Solves horizon, flatness problems



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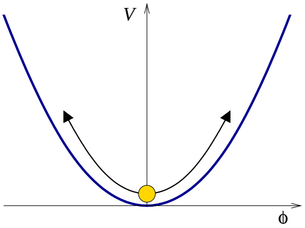

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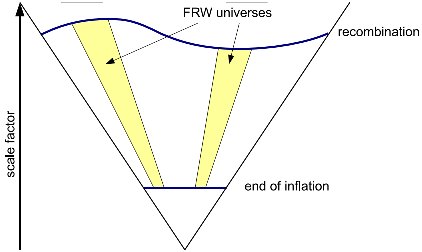
Separate Universes



- Each horizon volume $\sim$ separate FRW universe (Salopek&Bond 1990)
- Described by a separate Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_{Pl}^2}$$

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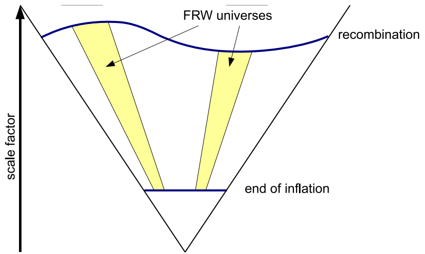
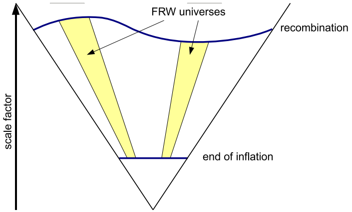
Separate Universes



- Each horizon volume $\sim$ separate FRW universe (Salopek&Bond 1990)
- Curvature perturbation $\zeta = \delta T/T = \delta \ln a|_{\rho=\rho_*}$
- Valid at distances $d \gg 1/H$

Conservation of ζ



- Curvature perturbation $\zeta = \delta T/T = \delta \ln a|_{\rho=\rho_*}$
- If the equation of state $P = P(\rho)$ is the same everywhere,

$$\frac{d\ln a}{d\rho} = -\frac{1}{3(\rho + P(\rho))} \Rightarrow \frac{d\zeta}{d\rho_*} = 0$$

Single-Field Inflation

- Inflation ends when slow roll conditions fail:
  $\phi = \phi_{\rm end}$, $\rho = V(\phi_{\rm end})$
- Later evolution the same in each separate universe:
  Same equation of state $\Rightarrow \zeta$ conserved
- Curvature perturbation:
  Field perturbation at the end of inflation

$$\zeta = \delta \ln a|_{\phi=\phi_{\rm end}} = -\frac{\partial \ln a}{\partial \phi}\delta\phi = -\frac{H}{\dot{\phi}}\delta\phi$$

Field Perturbations

- Inhomogeneous modes (comoving wave number k):

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V''\right)\delta\phi_k \approx 0$$

- Slow roll $\Rightarrow$ light field $V'' \ll H^2$
- Underdamped inside horizon $k/a \gg H$: Stays in vacuum

$$P_\phi(k) = \frac{1}{2k} \quad \text{for} \quad k/a \gg H$$

where $\langle \delta\phi_{k_1}\delta\phi_{k_2}\rangle \equiv P_\phi(k_1)(2\pi)^3\delta(k_1+k_2)$

- Overdamped outside horizon $k/a \ll H$: Freeze out

$$P_\phi(k) = \frac{H^2}{2k^3} \quad \text{for} \quad k/a \ll H$$

Field Perturbations

- Inhomogeneous modes (comoving wave number k):

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V''\right)\delta\phi_k \approx 0$$

- Overdamped outside horizon $k/a \ll H$: Freeze out

$$P_\phi(k) = \frac{H^2}{2k^3} \quad \text{for} \quad k/a \ll H$$

- Power spectrum (contribution from log scale)

$$\mathcal{P}_\phi(k) = \frac{k^3}{2\pi^2} P_\phi(k) = \frac{H^2}{4\pi^2} \;\sim \text{constant}$$

Curvature Perturbations

- Curvature perturbation: Field perturbation at the end of inflation

$$\zeta = \delta \ln a|_{\phi=\phi_{\rm end}} = -\frac{\partial \ln a}{\partial \phi}\delta\phi = -\frac{H}{\dot{\phi}}\delta\phi$$

- CMB temperature power spectrum

$$\mathcal{P}_T \approx \mathcal{P}_\zeta = \frac{H^2}{\dot{\phi}^2}\mathcal{P}_\phi = \frac{H^4}{4\pi^2\dot{\phi}^2}$$

- Independent of any physics after inflation in single-field inflation

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Non-Gaussianity

- Local non-Gaussianity (Komatsu&Spergel 2000)

$$\zeta(x) = \zeta_0(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_0(x)^2 - \langle\zeta_0(x)^2\rangle\right)$$

- Two-point function

$$\langle\zeta(k_1)\zeta(k_2)\rangle = P(k_1)(2\pi)^3\delta(k_1+k_2)$$

- Three-point function

$$\begin{aligned}\langle\zeta(k_1)\zeta(k_2)\zeta(k_2)\rangle &= -\frac{5}{6} f_{\rm NL} \left[P(k_1)P(k_2) + \text{cyclic}\right] \\ &\quad \times(2\pi)^3\delta(k_1+k_2+k_3)\end{aligned}$$

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- Use this as a general definition of $f_{\rm NL}$
  - Generally $f_{\rm NL} = f_{\rm NL}(k_1, k_2, k_3)$

Non-Gaussianity

- Local non-Gaussianity (Komatsu&Spergel 2000)

$$\zeta(x) = \zeta_0(x) - \frac{3}{5} f_{\rm NL} \left(\zeta_0(x)^2 - \langle \zeta_0(x)^2 \rangle \right)$$

- Observations show a hint for $f_{\rm NL} \sim 50$:
  - $-10 < f_{\rm NL} < 74$ (WMAP 2010)
- Planck should detect $f_{\rm NL} \sim 1$
- Single field inflation: $f_{\rm NL} \sim \epsilon \ll 1$ (Maldacena 2003)

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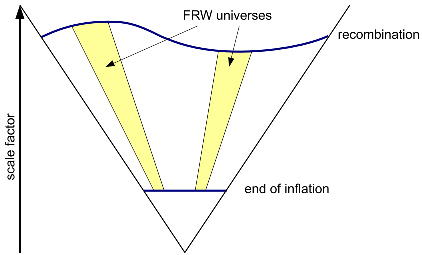
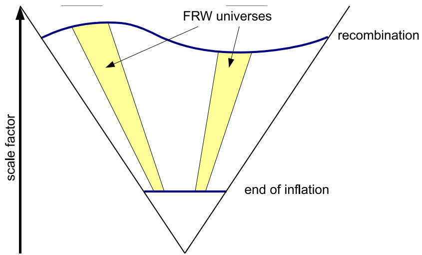
Another Light Scalar χ



- $m_\chi < H \Rightarrow$ Scale-invariant perturbations just like ϕ

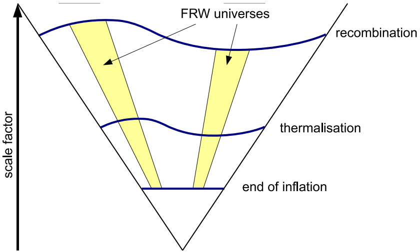
$$\mathcal{P}_\chi(k) \approx \frac{H^2}{4\pi^2}$$

Another Light Scalar χ



- $m_\chi < H \Rightarrow$ Scale-invariant perturbations just like ϕ
- Each separate ”universe” has different initial value χ_0
- Affects expansion $\Rightarrow$ Scale factor depends on χ_0

Another Light Scalar χ



- Thermalisation erases memory of $\chi_0 \Rightarrow P = P(\rho)$
- Curvature perturbation determined at thermalisation:
  $\zeta_{\rm rec} = \zeta_{\rm therm} = \delta \ln a|_{\rho=\rho_{\rm therm}}$

Calculating the Curvature Perturbation

- Solve Friedmann+field eqs for each separate universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho(\phi,\chi)}{3M_{Pl}^2}$$

  - Gives $a(t)$, $\rho(t)$
  - Non-linear, includes gravity, valid at $d \gg H^{-1}$
- Pick $\rho_* < \rho_{\rm therm}$, and calculate $\zeta = \delta \ln a|_{\rho=\rho_*}$

  $\Rightarrow$ Gives curvature perturbation for given ϕ_0, χ_0

Calculating the Curvature Perturbation

- Solve Friedmann+field eqs for each separate universe
- Pick $\rho_* < \rho_{\rm therm}$, and calculate $\zeta = \delta \ln a|_{\rho=\rho_*}$
  - Two fields: $\zeta = \zeta(\phi_0, \chi_0)$
  - $\delta\phi_0 \Rightarrow$ Usual inflationary perturbations
  - $\delta\chi_0 \Rightarrow$ New contribution - Potentially nonlinear!
- Need to calculate $N(\chi_0) = \ln a(\rho_*, \chi_0)$
  - Perturbative approach: Taylor expand

$$\zeta = \zeta_\phi + \frac{\partial N}{\partial \chi_0}\delta\chi_0 + \frac{1}{2}\frac{\partial^2 N}{\partial \chi_0^2}\delta\chi_0^2 + \ldots$$

Calculating the Curvature Perturbation

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Calculating the Curvature Perturbation

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- Need to calculate $N(\chi_0) = \ln a(\rho_*, \chi_0)$
  - Non-equilibrium dynamics, fully non-linear:
    Solve field and Friedmann eqs numerically
  - Solve for many different initial values χ_0

$\Rightarrow$ Whole non-linear function $N(\chi_0)$ (Bassett&Tanaka 2003, Suyama&Yokoyama 2006)

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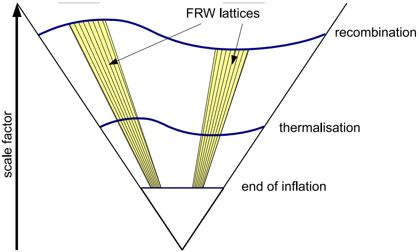
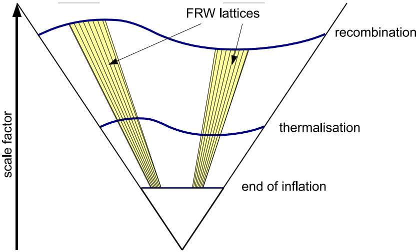
Lattice Calculation



- Describe each "universe" as a lattice (Chambers&AR 2007)
- Inhomogeneous fields, FRW metric
- Lattice size $L \sim 1/H$

Lattice Calculation



- Solve field evolution + Friedmann eq on lattice
- Initial conditions $\chi(x) = \chi_0 + \delta\chi(x)$
- Find curvature perturbation as $\zeta(\chi_0) = \delta \ln a(\chi_0)|_{\rho=\rho_*}$

Initial Conditions

- Sub-horizon modes: Quantum vacuum
  - Gaussian classical fluctuations with (Khlebnikov&Tkachev 1996)

$$\overline{|\chi_k|^2} = \frac{V}{2\omega_k}, \quad \overline{|\dot{\chi}_k|^2} = V\frac{\omega_k}{2}$$

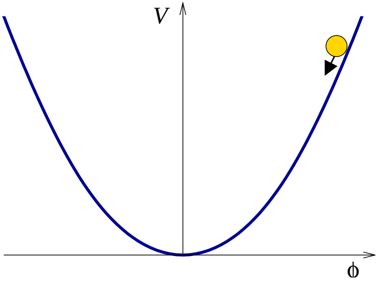
  - Linear dynamics: quantum = classical
  - Non-linear dynamics: quantum $\approx$ classical
- Mean value $\leftrightarrow$ Super-horizon modes
  - Initial field value χ_0
  - Input parameter – we are calculating $\zeta(\chi_0)$, not $\zeta(x)$!

Massless Preheating

- Chaotic inflation + massless scalar χ (Prokopec&Roos 1997)

$$V(\phi,\chi)=\frac{1}{4}\lambda\phi^4+\frac{1}{2}g^2\phi^2\chi^2, \quad \lambda\sim 7\times 10^{-14}$$

- Inflation: $\phi > 2\sqrt{3}M_{\rm Pl}$



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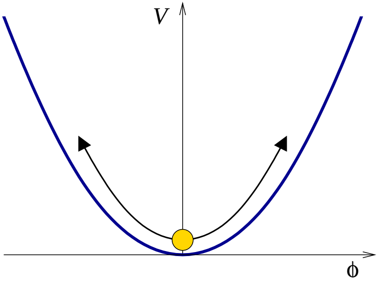
- After inflation: Radiation domination $a \propto t^{1/2}$



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$$V(\phi,\chi)=\frac{1}{4}\lambda\phi^4+\frac{1}{2}g^2\phi^2\chi^2, \quad \lambda\sim 7\times 10^{-14}$$

- After inflation: Radiation domination $a\propto t^{1/2}$
- Inflaton zero mode $\ddot{\phi}+3H\dot{\phi}+\lambda\phi^3=0$
  - Rescale the field $\phi=a^{-1}\tilde{\phi}$
  - Rescale time $d\tau=a^{-1}\lambda^{1/2}\tilde{\phi}_{\rm ini}dt$

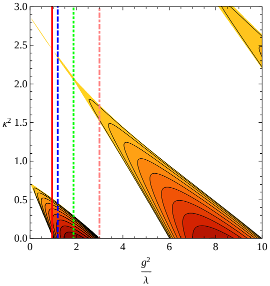
$\Rightarrow$ $\tilde{\phi}''+\lambda\tilde{\phi}^3=0\Rightarrow\tilde{\phi}(\tau)=\tilde{\phi}_{\rm ini}{\rm cn}(\tau;1/\sqrt{2})$ (Jacobi cosine)

- Inhomogeneous χ modes $\chi_k=a^{-1}\tilde{\chi}_k$

$$\tilde{\chi}_k''+\left[\kappa^2+\frac{g^2}{\lambda}{\rm cn}^2(\tau;1/\sqrt{2})\right]\tilde{\chi}_k=0, \quad \kappa^2=\frac{k^2}{\lambda\tilde{\phi}^2_{\rm ini}}$$

Parametric Resonance

- Floquet theorem:
  - $\tilde{\chi}_k(\tau) = e^{\mu\tau} f(\tau)$ with periodic $f(\tau)$
  - Imaginary μ: Oscillates
  - Real μ: Exponential growth = resonance
- Resonance transfers energy to χ
- Resonant modes for every g^2/λ
- When χ grows enough, evolution becomes non-linear
  $\Rightarrow$ Resonance ends – Thermalisation
- Amount of growth needed depends on initial χ
  $\Rightarrow$ Curvature perturbation (Chambers&AR,2007)
- Effect strongest when $\kappa = 0$ dominates the resonance



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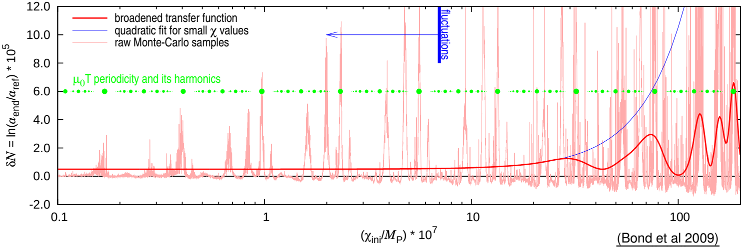

(Bond et al 2009)

- Chaotic behaviour: Highly non-Gaussian (Chambers&AR 2007)
- Not well approximated by a quadratic $f_{\rm NL}$
- Amplitude $\sim 10^{-4}$ – Observable!?

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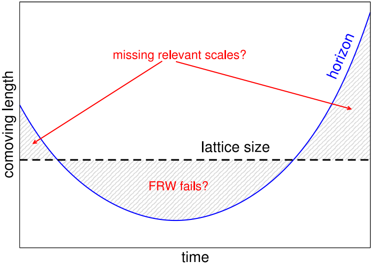
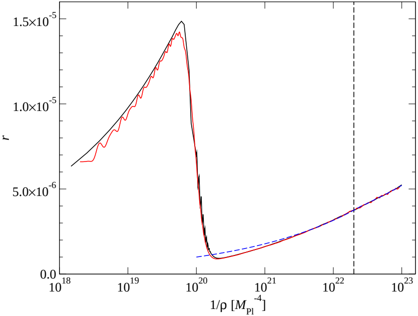
Massless Preheating – Caveat!



- Finite-volume error especially when long wavelengths dominate
- Dynamical metric with Einstein eq. (Bastero-Gil,AR&Szmigiel, in progress)

Curvaton Resonance



(Chambers,Nurmi&AR, 2009)

- Light scalar field, decays through parametric resonance (Enqvist et al 2009)
- Contribution to e.o.s. depends on χ_0 (Chambers,Nurmi&AR 2009)

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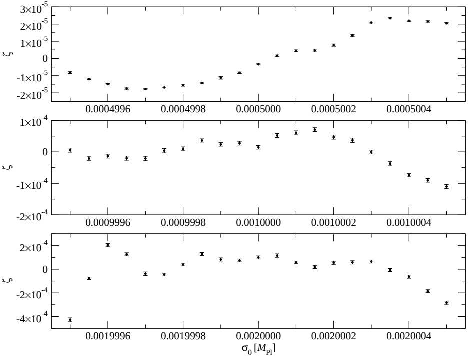
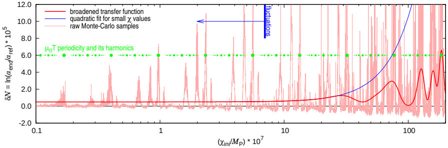
Curvaton Resonance

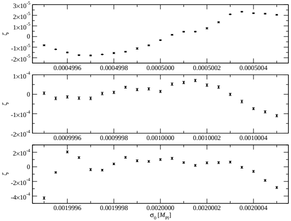


(Chambers,Nurmi&AR, 2009)

Summary

- Non-linear calculation of curvature perturbation due to non-equilibrium physics
  - Separate universes + lattice field theory
  - Works with any (bosonic) field dynamics
- Massless preheating, Curvaton resonance:
  - Possibly observable effects $\Delta\zeta \sim 10^{-5}$
  - Highly non-linear
    $\Rightarrow$ Highly non-Gaussian





Gauge Fields

- So far only scalar fields
- Inflaton/curvaton can couple to gauge fields or fermions
  - E.g. curvaton with electric charge (D'Onofrio,Lerner&AR, in progress)
- Perturbations from other fields?
  - Clearly needs a light field $m \lesssim H$
  - Fermions: Pauli exclusion principle, No classical dynamics
  - Classical gauge fields:
    Conformal invariance $\Rightarrow$ Do not feel expansion
- Quantum gauge fields?
  - Masses of excitations $\gtrsim \Lambda_{\rm QCD}$
  - What happens when $\Lambda_{\rm QCD} < H$?

Gauge Fields in de Sitter

- de Sitter temperature $T_H = H/2\pi$
- Deconfinement when $T_H > T_c \sim \Lambda_{\rm QCD}$ (Marolf et al 2011)
- Hot Yang-Mills: Longest length scale $1/g^2 T_H \gg 1/H$
  - Does this scale exist in dS?
- Otherwise, correlations can exist on even cosmological scales
  - Free Abelian gauge field at temperature T:

$$\langle B_i(0) B_j(x) \rangle = -\left(\delta_{ij} - 3\frac{x_i x_j}{r^2} \right) \frac{T}{4\pi r^3}$$

$\Rightarrow$ Curvature perturbations? Other observable effects?