Chiral and deconfinement transition in QCD

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Deconfinement and chiral symmetry restoration are expected to happen at some temperature

Deconfinement transition is analogous to the transition from ionized gas to plasma with no well defined transition temperature

Chiral transition is analogous to the feromagnet (spin system) transition in external magnetic fields

Earlier lattice results ~~and the controversy~~:
RBC-Bielefeld: ~~$T_c=192(7)(4)$~~ MeV (deconfinement and chiral)
Budapest-Wuppertal: $T_c=151(3)(3)$ MeV (chiral) √ and ~~$T_c=175(4)(3)$ MeV~~ (deconfinement)

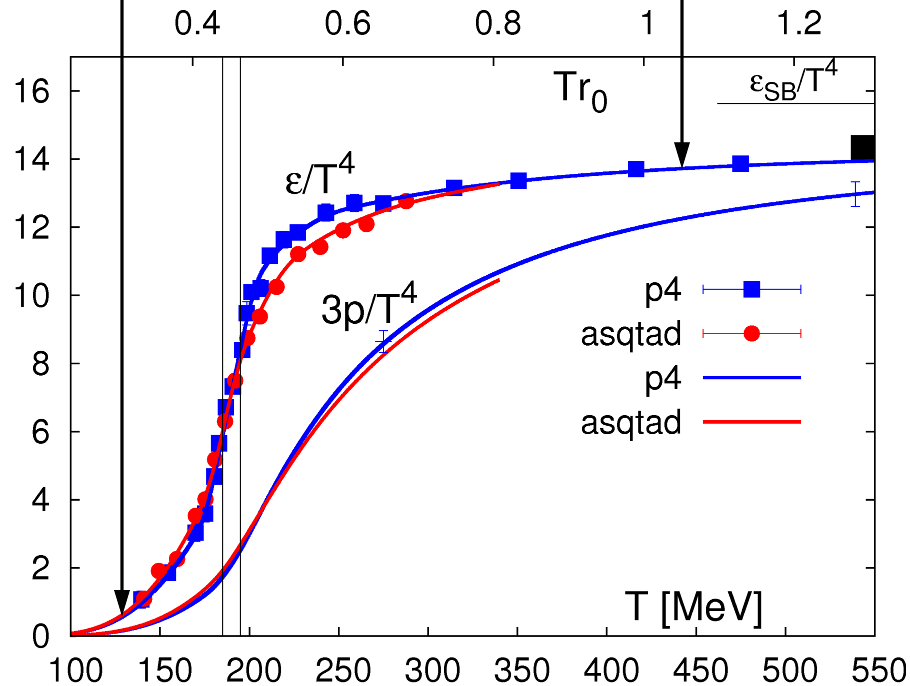
HotQCD : Phys. Rev. D85 (2012) 054503; arXiv:1203.0784, also P.P. arXiv:1203.5320

INT, April 6, 2012

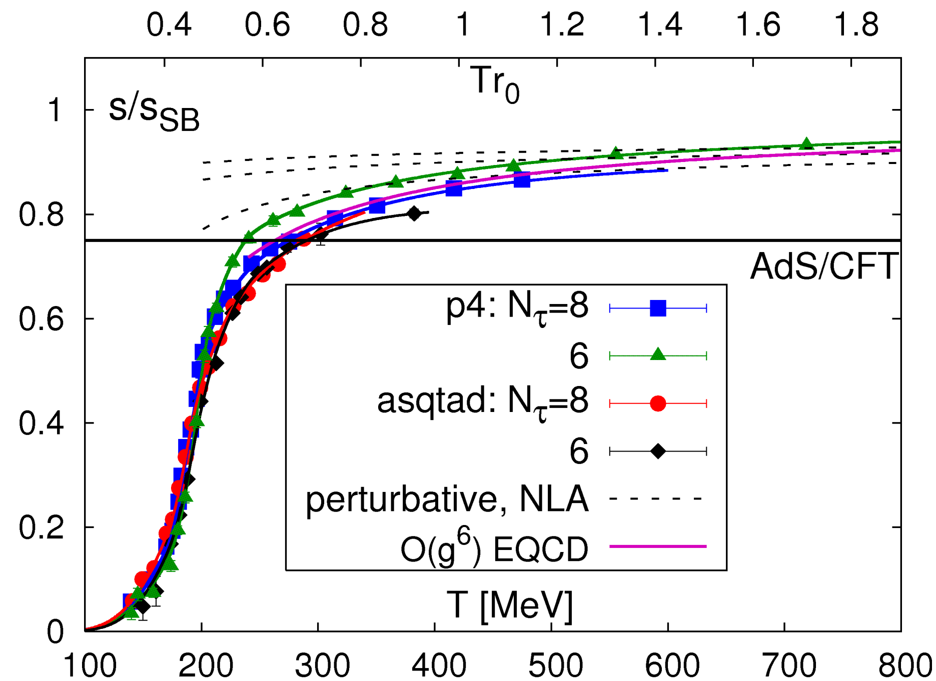
Deconfinement : entropy, pressure and energy density

meson gas = 3 light d.o.f.

free gas of quarks and gluons = 18 quark+18 anti-quarks +16 gluons =52 mass-less d.o.f



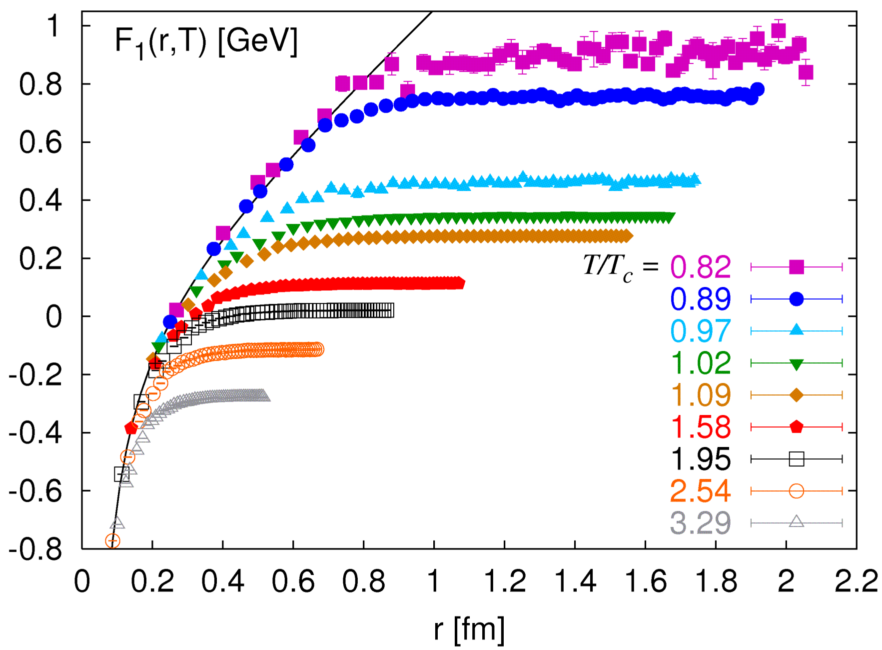
Bazavov et al (HotQCD), PRD 80 (09) 14504

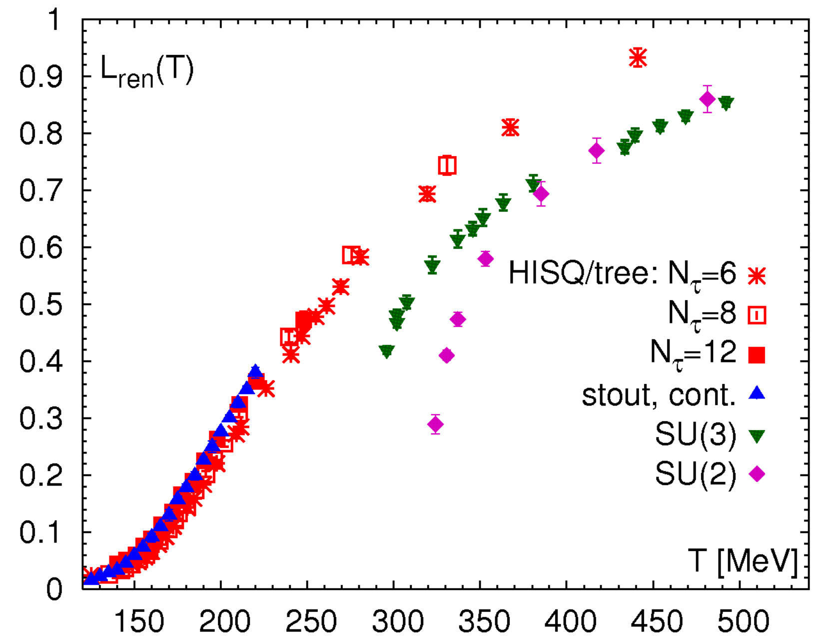


Petreczky, NPA 830 (10) 11c

- rapid change in the number of degrees of freedom at *T=170-200*MeV: deconfinement
- deviation from ideal gas limit is about 10% at high *T* consistent with the perturbative result
- no large discretization errors in the pressure and energy density at high *T*
- no continuum limit yet !

Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T), m_D \sim T$$

$$r_{bound} > 1/m_D$$

melting of bound states of heavy quarks => quarkonium suppression at RHIC

Polyakov loop

$$L_{ren} = \exp(-F_Q(T)/T)$$

free energy of a static quark

infinite in the pure glue theory or large in the "hadronic" phase ~600MeV

decreases in in the deconfined phase

$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

Chiral symmetry of QCD in the vacuum and for T>0

Nobel Prize 2008

• Chiral symmetry : For light quarks $m_{u,d} \ll \Lambda$ and QCD Lagrangian

$SU_A(2)$ symmetry $\psi \to e^{i\phi T^a \gamma_5}\psi$ $\qquad \psi_{L,R} \to e^{i\phi_{L,R}T^a}\psi_{L,R}$

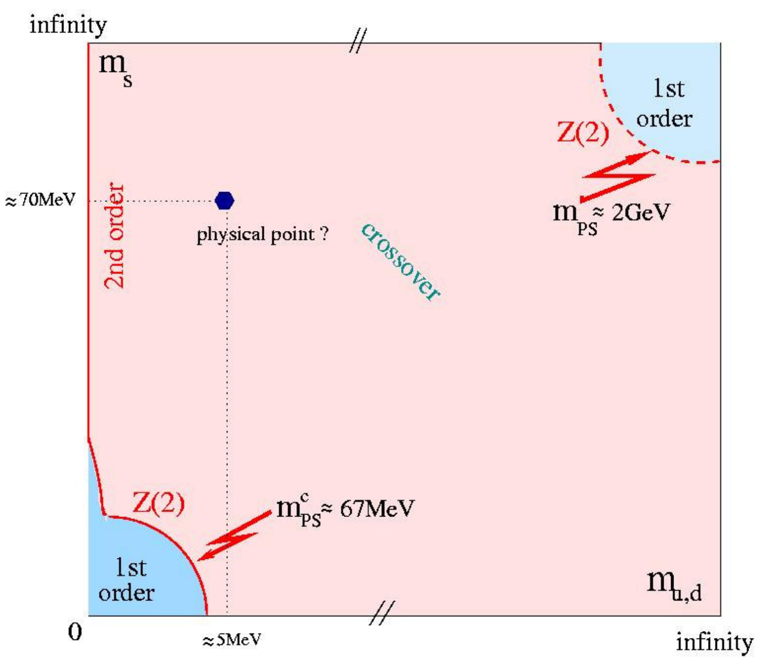
The vacuum (ground state) is not $\langle\bar\psi\psi\rangle = \langle\bar\psi_L\psi_R\rangle + \langle\bar\psi_R\psi_L\rangle \neq 0$

spontaneous symmetry breaking or Nambu-Goldstone realization of the symmetry

⇩

hadrons with opposite parity have very different masses

Chiral symmetry is expected to get restored at high T: $\langle\bar\psi\psi\rangle = 0$



• For vanishing u,d-quark masses the chiral transition is either 1st order or 2nd order phase transition.
• For physical quark masses there could be a 1st order phase transition or crossover

Evidence for 2nd order transition in the chiral limit => universal properties of QCD transition:

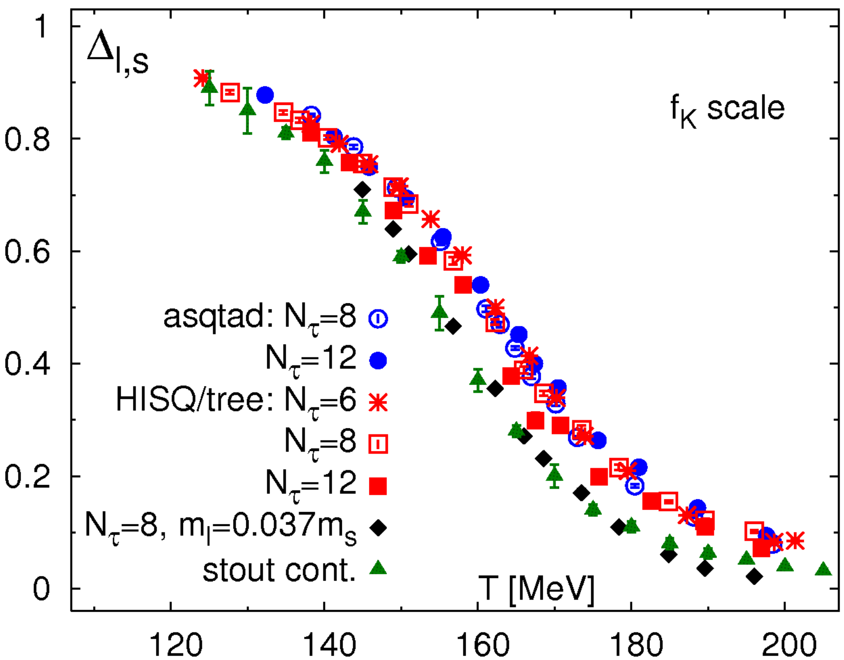
$SU_A(2) \sim O(4)$
relation to spin models

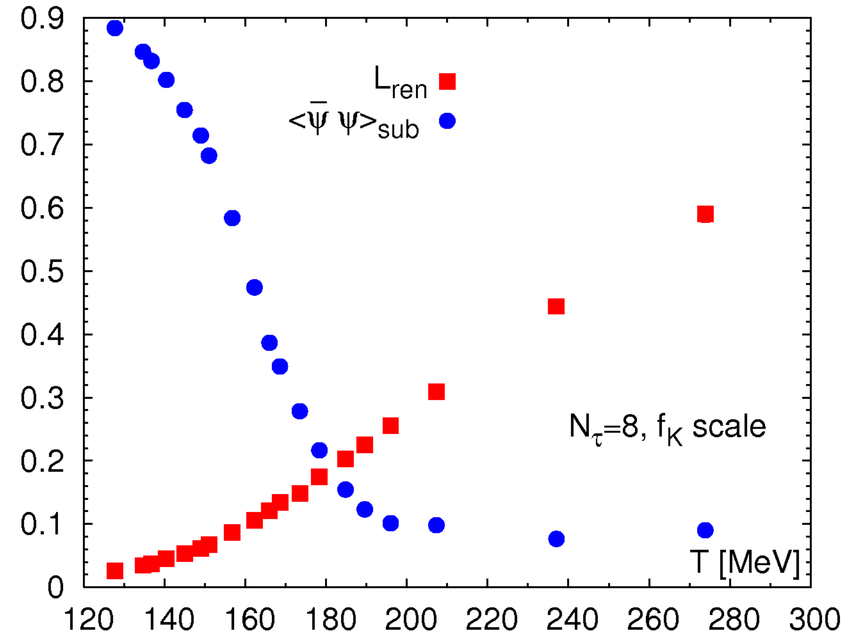
The temperature dependence of chiral condensate

Chiral condensate needs multiplicative and additive renormalization for non-zero quark mass

$$\langle\bar{\psi}\psi\rangle_l \Rightarrow \langle\bar{\psi}\psi\rangle_{sub} = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,T=0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T=0}}$$

P.P. arXiv:1203.5320





- Cut-off effects are significantly reduced when f_K is used to set the scale
- After quark mass interpolation based on *O(N)* scaling the HISQ/tree results agree with the stout continuum result !
- The deconfinement in terms of color screening sets in at temperatures higher than the chiral transition temperature

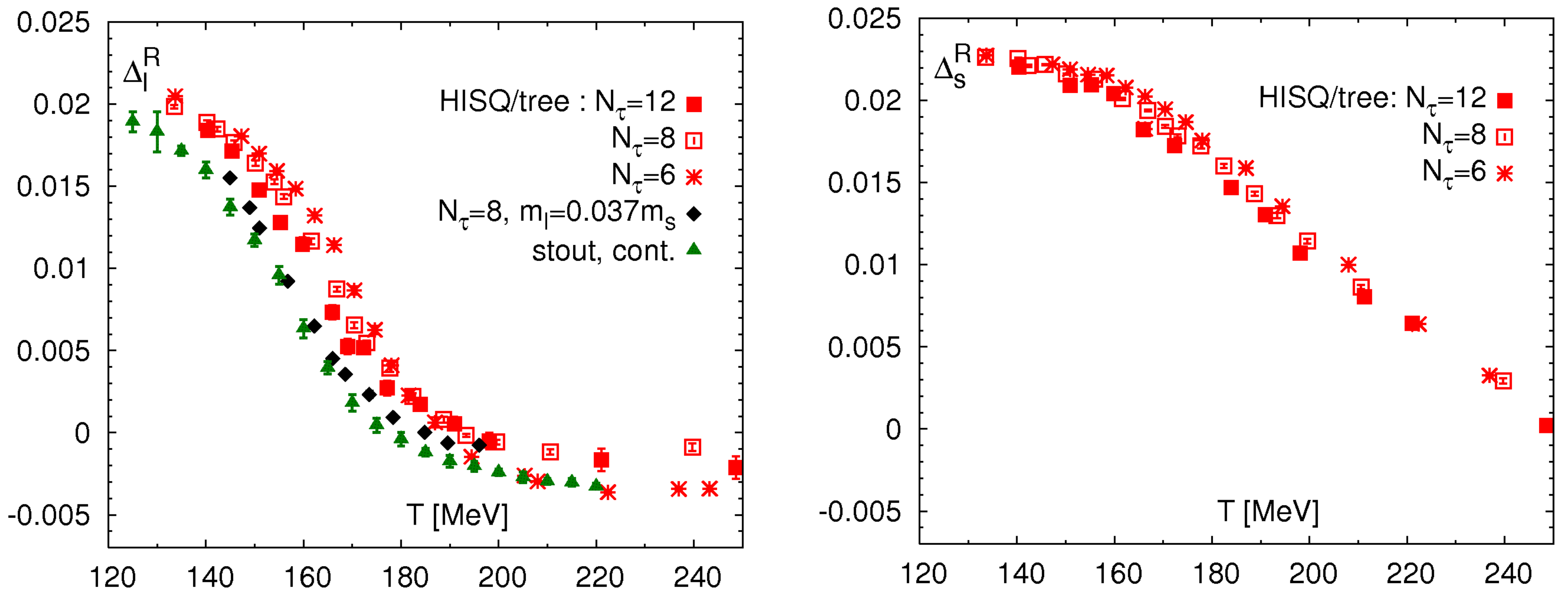
The temperature dependence of chiral condensate

Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar\psi\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar\psi\psi\rangle_{q,T} - \langle\bar\psi\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar\psi\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503



• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results !

• strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l) + f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

governed by universal *O(4)* scaling $$M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ \ z = t/h^{1/\beta\delta}$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta-1} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \qquad \chi_{t,t} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$T_{m,l} \quad = \quad T_{t,l} \quad = \quad T_{t,t} \quad = \quad \boldsymbol{T_c^0}$$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1} f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z=z_p$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)} + \ldots$$

Caveat : staggered fermions O(2)

$m_l \to 0$, $a > 0$,

proper limit $a \to 0$, before $m_l \to 0$

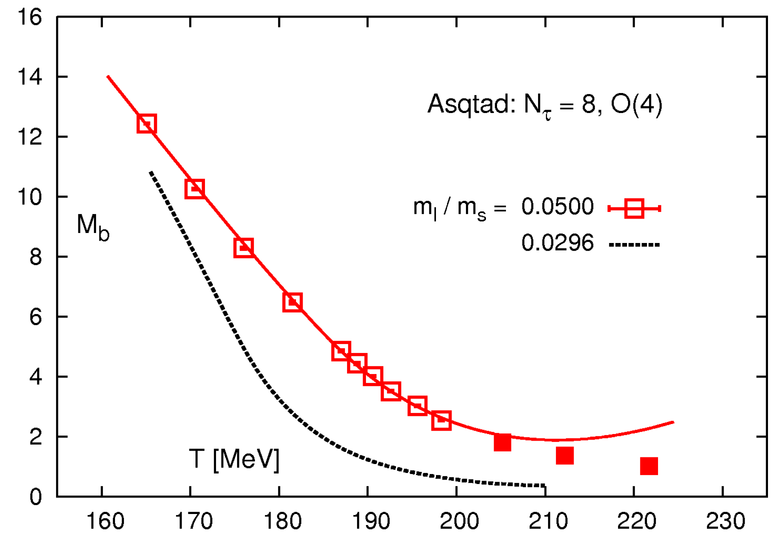
O(N) scaling and the transition temperature

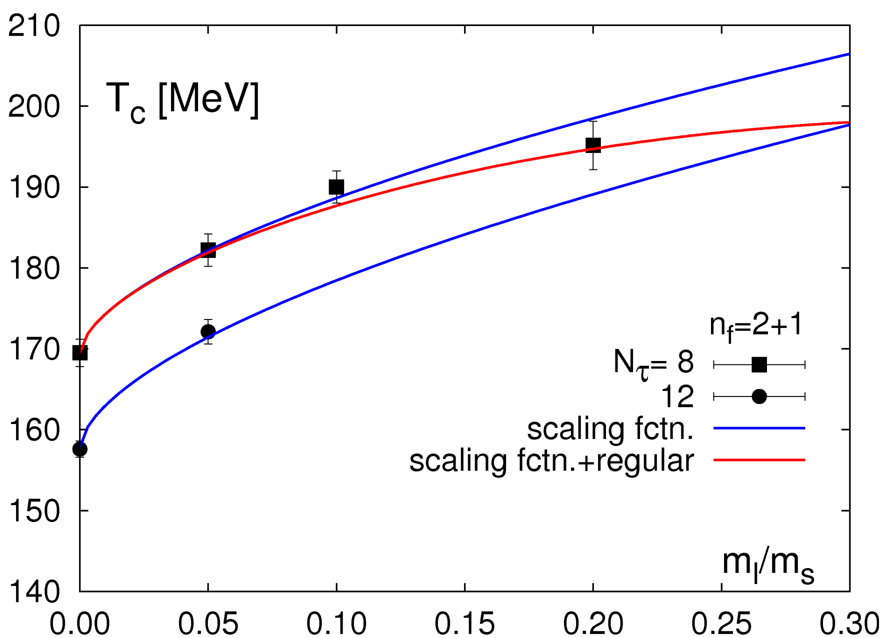
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

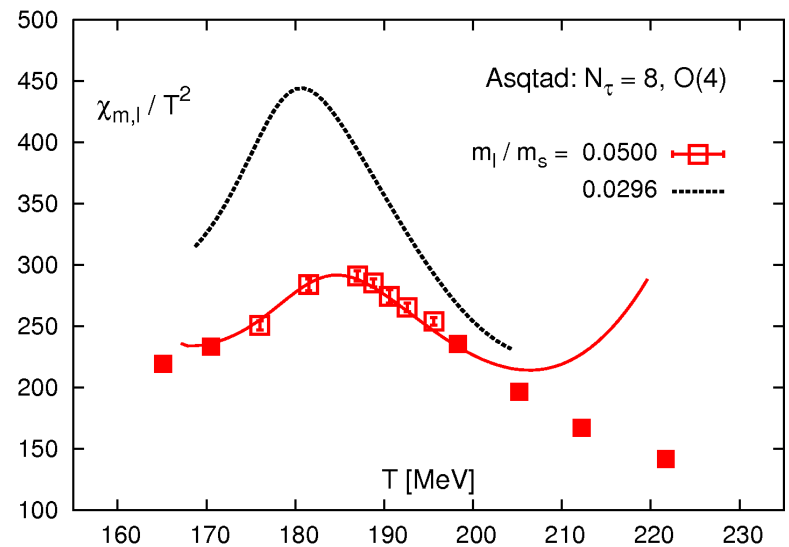
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

$$f_{reg}(T,H) = (a_1(T-T_c^0) + a_2(T-T_c^0)^2 + b_1)H$$

6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1







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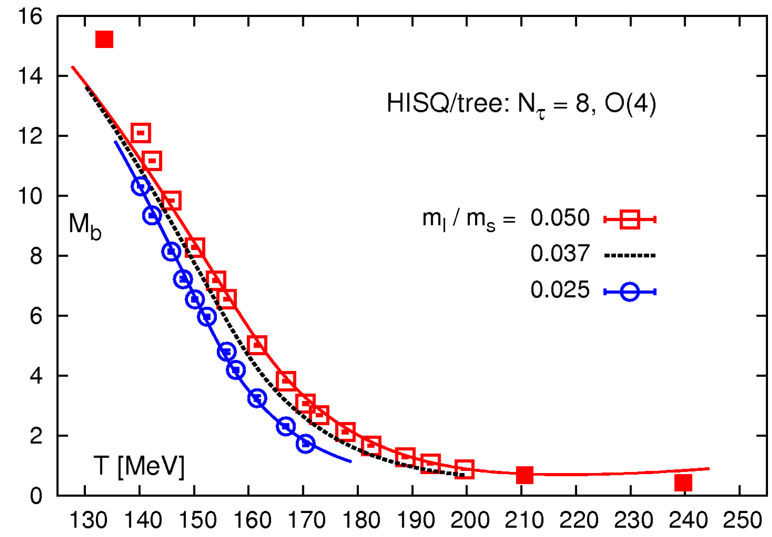

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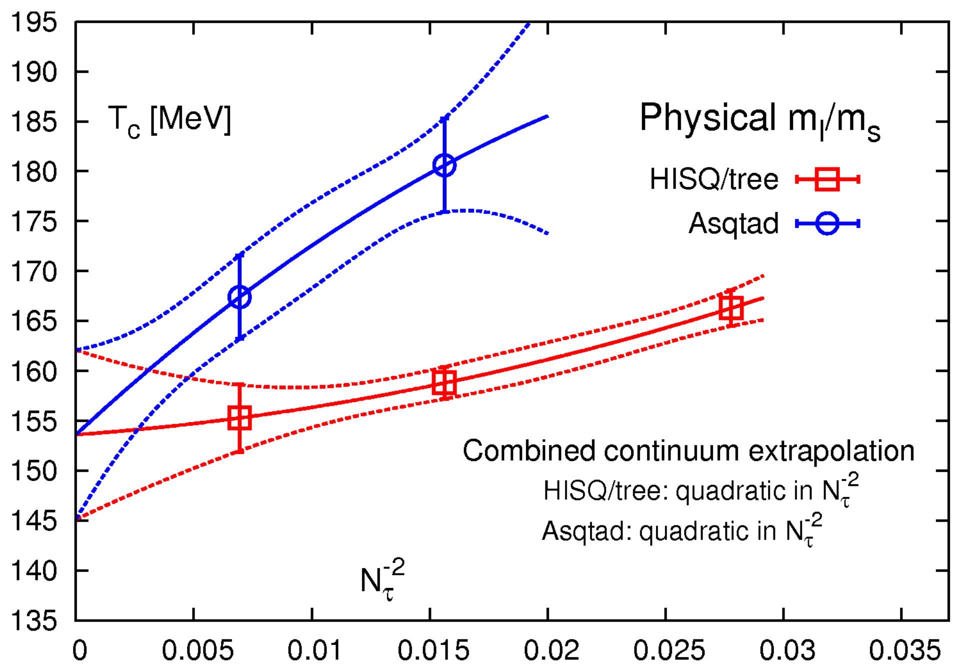
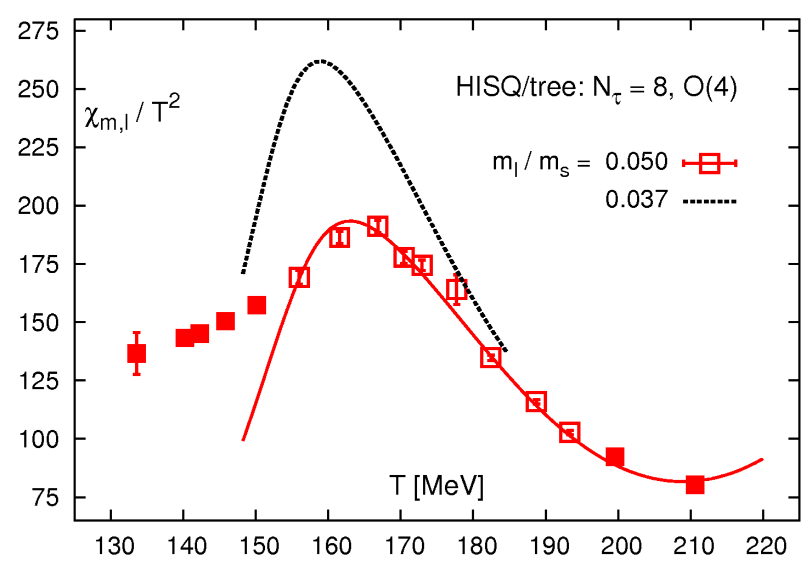
6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1

$$T_c = (154 \pm 8 \pm 1(scale))\,\text{MeV}$$





QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_S,\mu_I)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BSI} \cdot \mu_B^i \cdot \mu_S^j \cdot \mu_I^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \mu_u^i \cdot \mu_d^j \cdot \mu_s^k$$ quark

$$\chi_{ijk}^{abc} = \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\frac{\chi_2^X}{T^2} = \frac{\chi_X}{T^2} = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \frac{\chi_{11}^{XY}}{T^2} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

Computation of Taylor expansion coefficients reduces to calculating the product of inverse fermion matrix with different source vectors

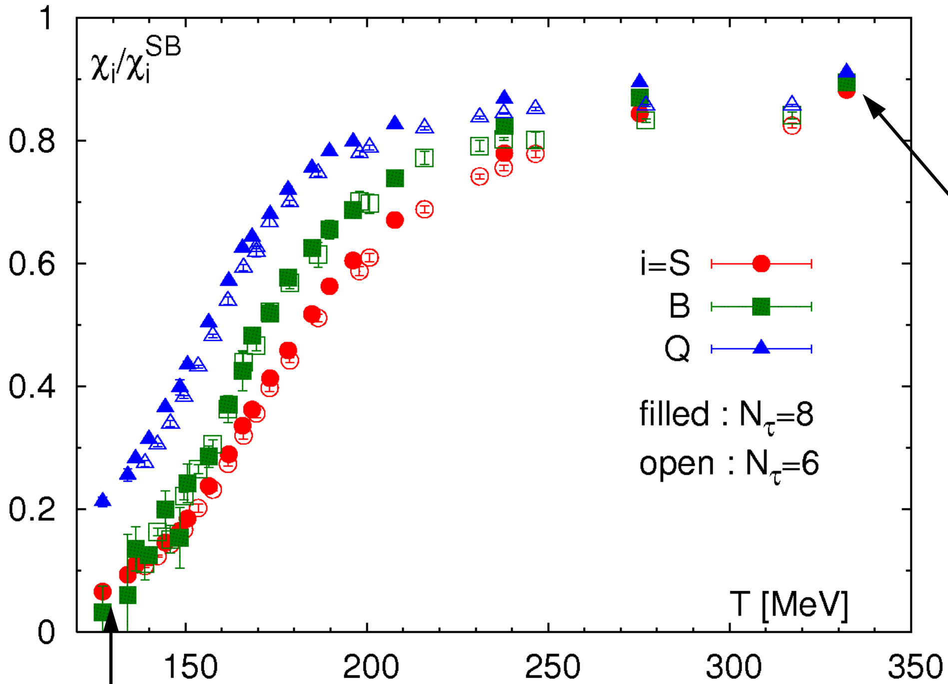
⇨ can be done very effectively on single GPUs

Deconfinement : fluctuations of conserved charges

$$\frac{\chi_B^{SB}}{T^2} = \frac{1}{VT^3}(\langle B^2\rangle - \langle B\rangle^2)$$ baryon number

$$\frac{\chi_Q^{SB}}{T^2} = \frac{1}{VT^3}(\langle Q^2\rangle - \langle Q\rangle^2)$$ electric charge

$$\frac{\chi_S^{SB}}{T^2} = \frac{1}{VT^3}(\langle S^2\rangle - \langle S\rangle^2)$$ strange quark number (strangeness)



Ideal gas of quarks :

$$\chi_B^{SB} = \frac{T^2}{3} \quad \chi_Q^{SB} = \frac{2T^2}{3}$$

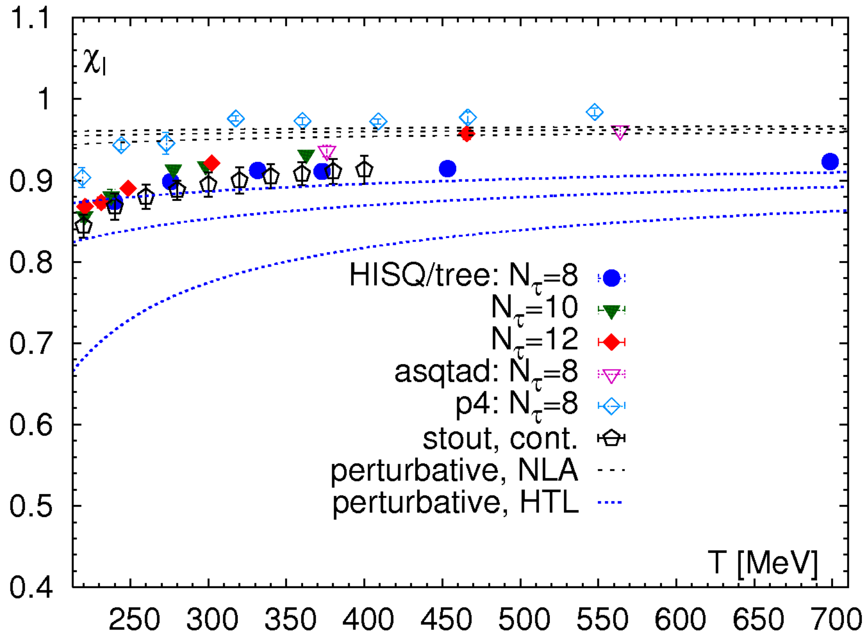
$$\chi_B^{SB} = T^2$$

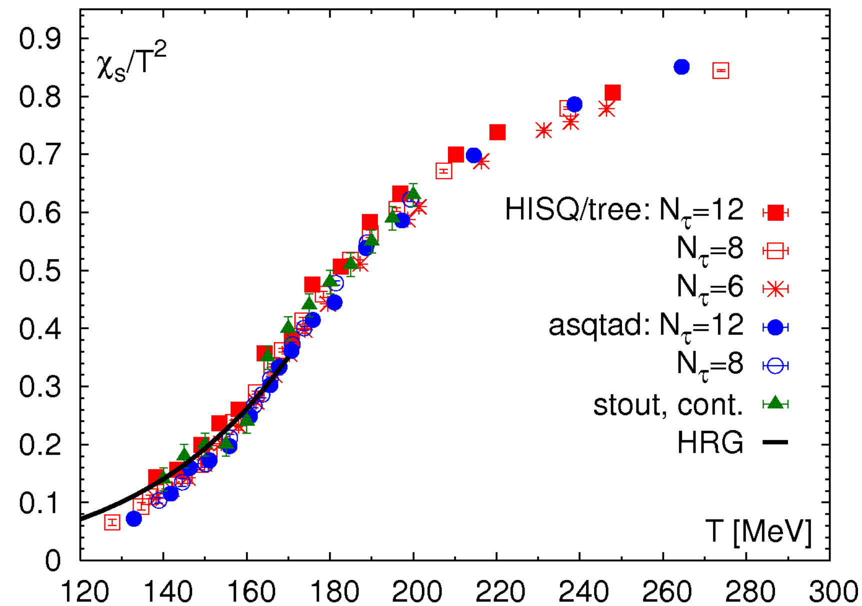
conserved charges carried by light quarks

conserved charges are carried by massive hadrons

HotQCD: arXiv:1203.0784

Fluctuations at low and high temperatures





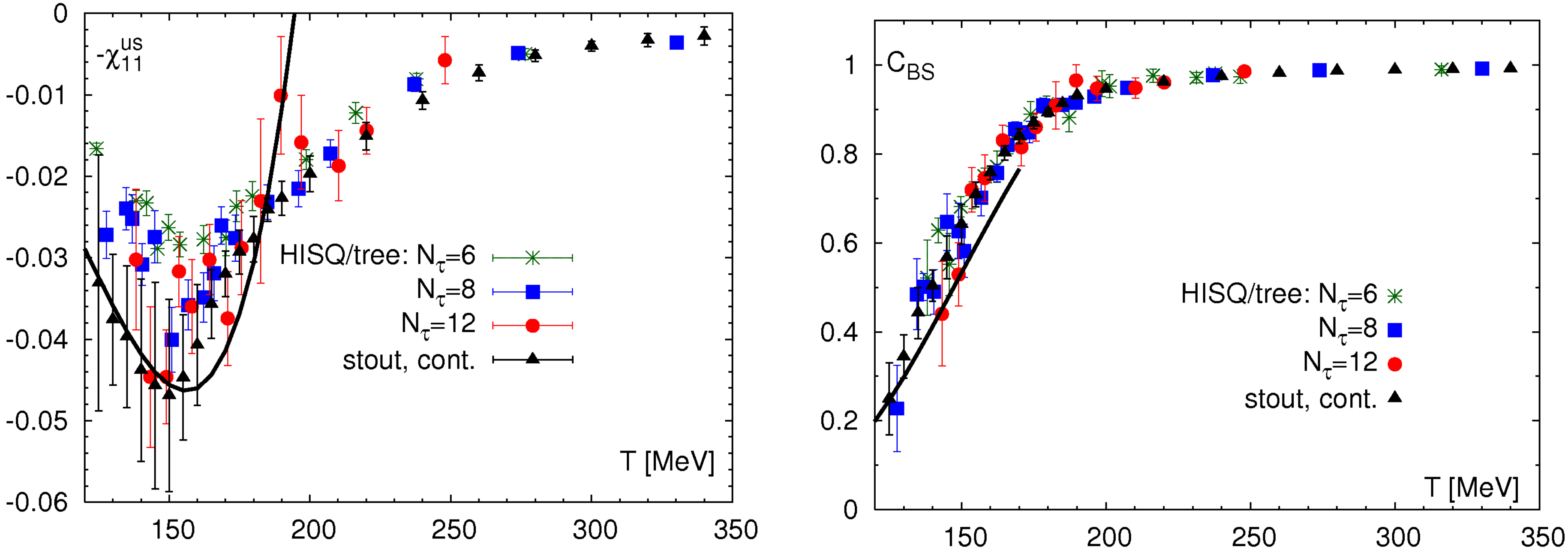
At sufficiently high *T* fluctuations can be described by perturnation theory because of asymptotic freedom

The quark number susceptibilities for *T>300*MeV agree with resummed petrurbative predictions
A. Rebhan, arXiv:hep-ph/0301130
Blaizot et al, PLB 523 (01) 143

hadrons are the relevant d.o.f. at low *T*
⇒ hadron gas + interactions
(approximated by s-channel resonances)
⇒ non-interacting hadron resonance gas
(HRG)

Reasonable agreement between lattice results and HRG the remaining discrepancies are due to the lack of continuum extrapolation

Correlations of conserved charges



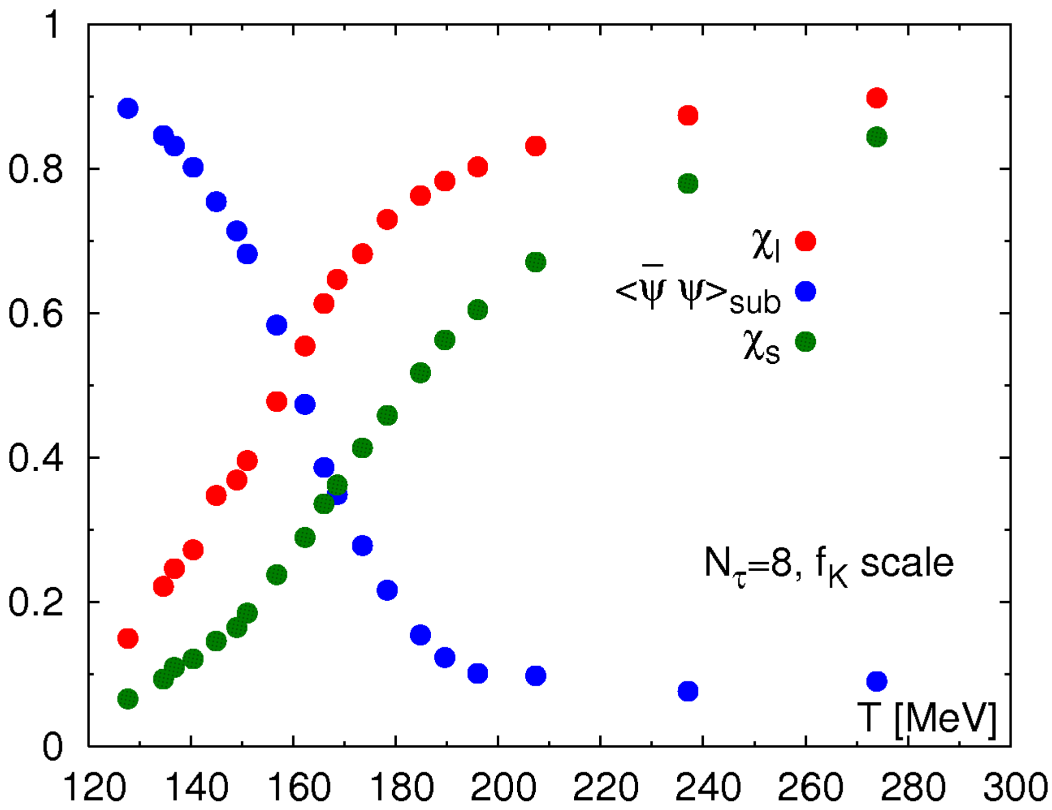
P.P. arXiv:1203.5320

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* => weakly interacting quark gas

- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

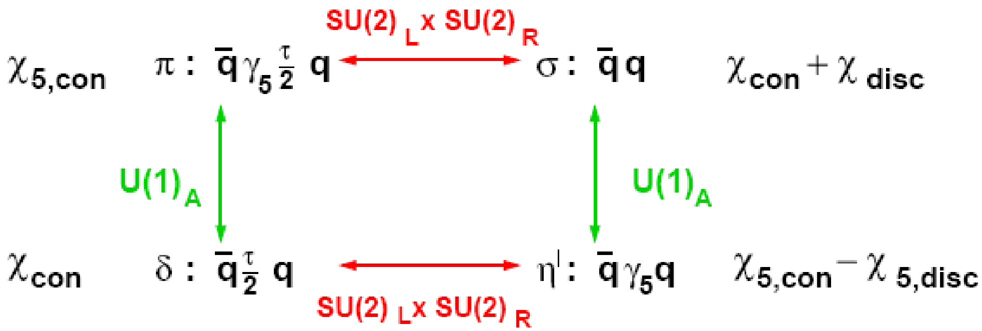
- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ 50 MeV

Deconfinement and chiral transition again



Deconfinement transition in terms of light quark number fluctuations coincides with the chiral transition => no deconfinement transition temperature can be defined

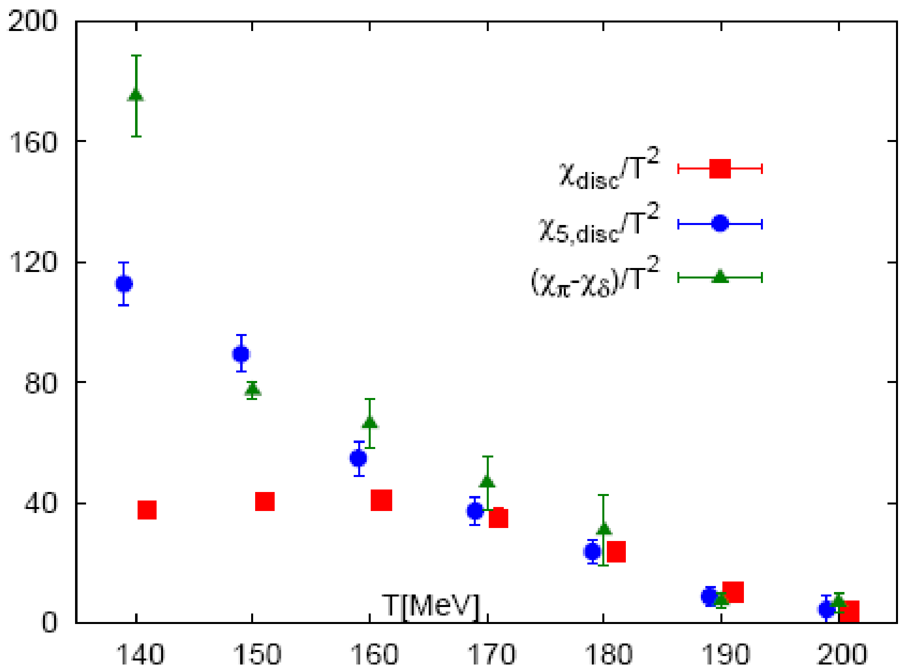
$U_A(1)$ symmetry restoration



$$\chi_{\sigma,\mathrm{disc}} = \langle (\overline{\psi}\psi)^2 \rangle - \langle (\overline{\psi}\psi) \rangle^2 \equiv \chi_{\mathrm{disc}}$$

$$\langle (\overline{\psi}\gamma_5\psi)^2 \rangle \equiv \chi_{5,\mathrm{disc}}$$

HotQCD : Domain Wall Fermions (in progress)



chiral:

$$\chi_\pi = \chi_\delta + \chi_{\mathrm{disc}}$$

$$\chi_\delta = \chi_\pi - \chi_{5,\mathrm{disc}}$$

$$\chi_{\mathrm{disc}} = \chi_{5,\mathrm{disc}}$$

axial:

$$\chi_\pi = \chi_\delta$$

$$\chi_\delta + \chi_{\mathrm{disc}} = \chi_\pi - \chi_{5,\mathrm{disc}}$$

$$\chi_{\mathrm{disc}} = -\chi_{5,\mathrm{disc}}$$

axial symmetry is till broken at *T=200* MeV !

Summary

• Lattice QCD show that at high temperatures strongly interacting matter undergoes a transition to a new state QGP characterized by deconfinement and chiral symmetry restoration

• We see evidence that provide evidence that the relevant degrees of freedom are quarks and gluons; lattice results agree well with perturbative calculations, while at low *T* thermodynamics can be understood in terms of hadron resonance gas
The deconfinement transition can understood as transition from hadron resonance gas to quark gluon gas it is gradual and analagous to ionized gas – plasma transition (implications for sQGP and early thermalization at RHIC ?)

• The chiral aspects of the transition are very similar to the transition in spin system in external magnetic fields: it is governed by universal scaling

• Different calculations with improved staggered actions agree in the continuum limit resulting in a chiral transition temperature *(154 ± 9)* MeV