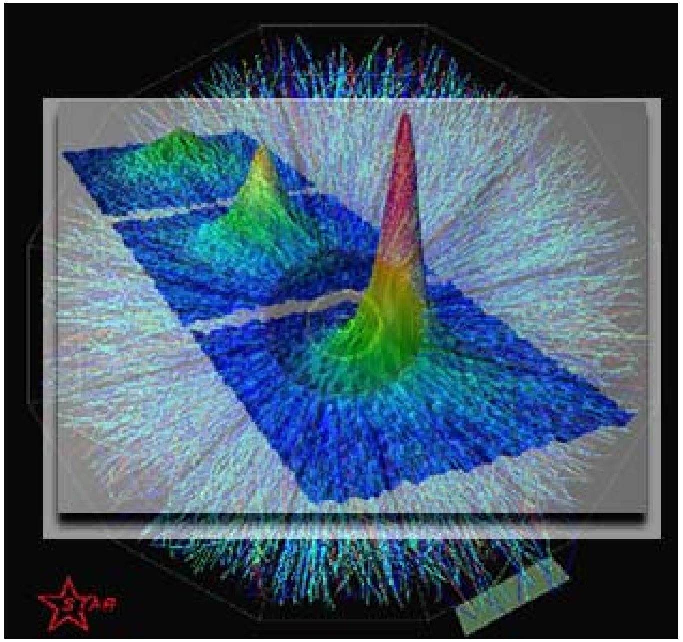
INT Program "Gauge Field Dynamics In and Out of Equilibrium" MAR.20th, 2012

The "First-Few-Fermi" after the "Little-Bang"



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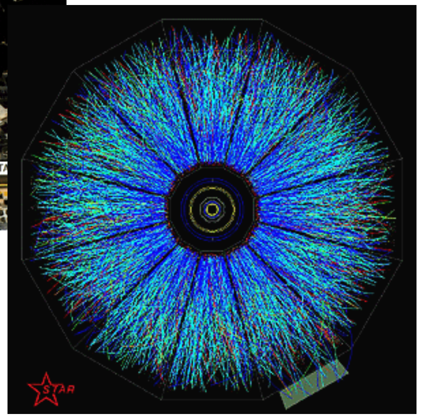
Outline

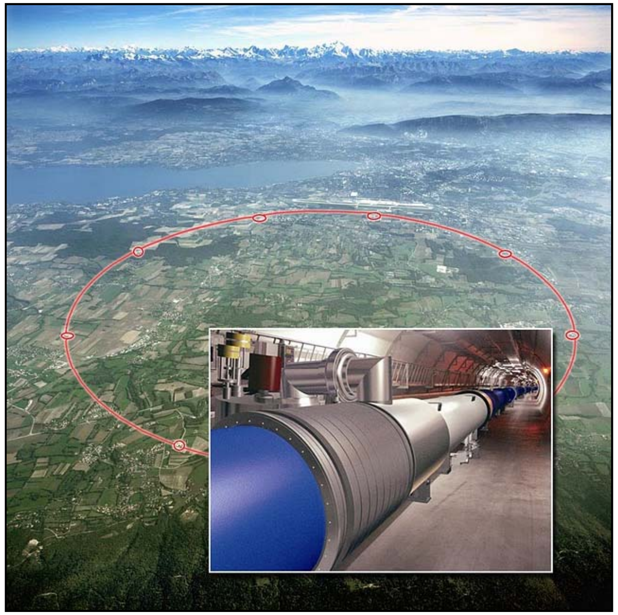
- The Pre-Equilibrium Matter in Heavy Ion Collisions
- Two Important Features: Overpopulation & Uni-Scale
- A Kinetic Approach: Dynamical BEC & Separation of Scales
- Discussions

References:
Blaizot, Gelis, JL, McLerran, Venugopalan, Nucl. Phys. A873, 68 (2012);
Blaizot, JL, McLerran, to appear;
Chiu, Hemmick, Khachatryan, Leonidov, JL, McLerran, arXiv:1202.3679 [nucl-th].

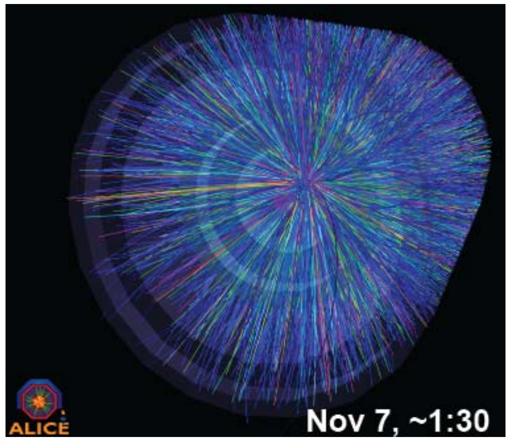
"Little Bang" in the Laboratories



RHIC Event (from STAR)



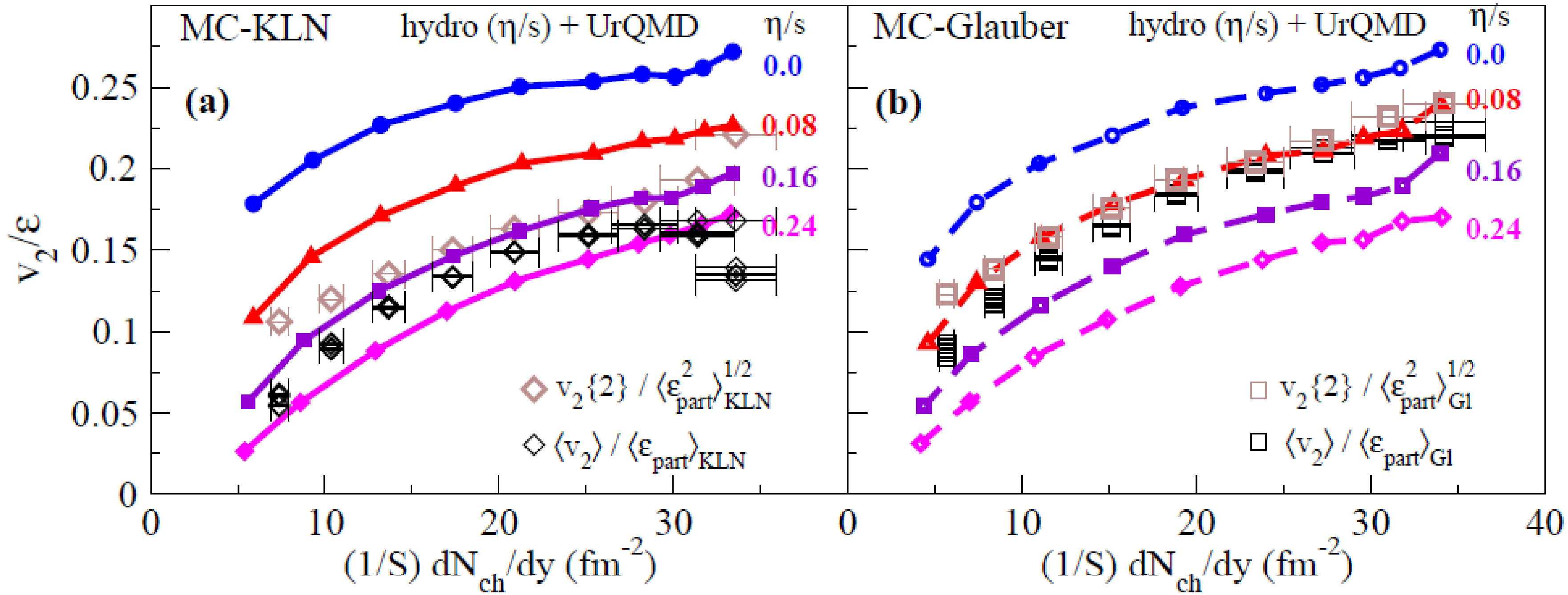
LHC Event (from ALICE)



Currently ongoing
heavy ion collisions programs:
RHIC (BNL), since 2000;
LHC (CERN), since 2010.

Beautiful "little bang" delivered !
T~10^12 K : The hottest matter today!
Furthermore: strongly interacting !

Hot QCD Matter: Nearly Perfect Fluid



Song, Bass, Heinz, Hirano, Shen, 2010

Created matter's explosion appears nearly ideal → strongly coupled

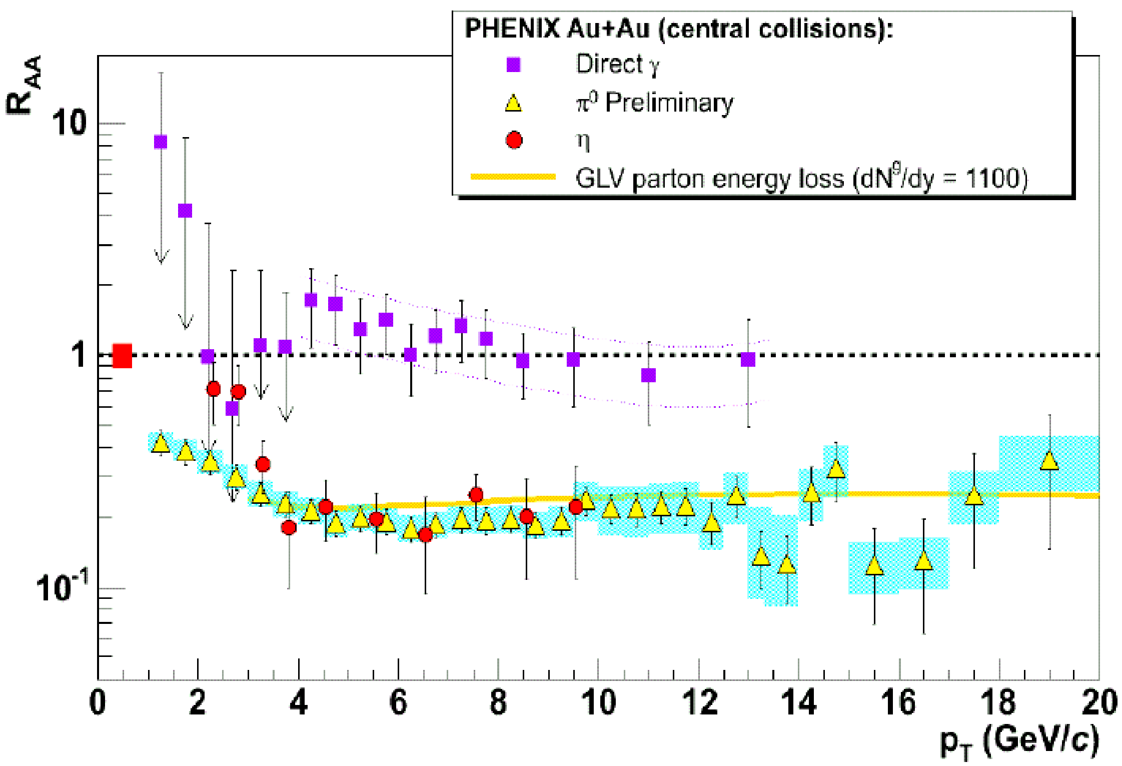
"Coupling-ometer" via transport properties e.g. shear viscosity

Infinity ——————————————— **0**

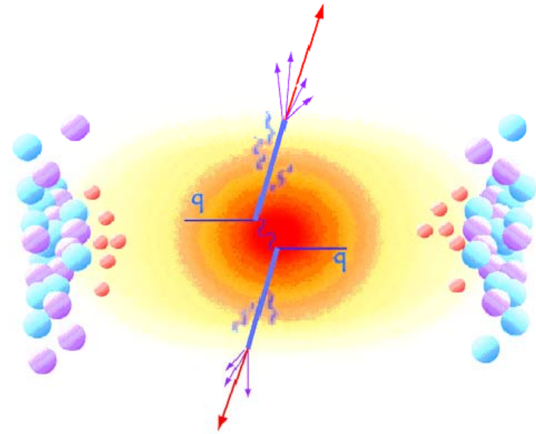
(eta/s=1/4pi) **(infinitely viscous)**

(quantum limit?)

Hot QCD Matter: Strong Jet Quenching



Gyulassy, Wang; ……



$$R_{A\text{-}A}(p_t) \equiv \frac{d^2N^{A\text{-}A}/dp_t d\eta}{T_{A\text{-}A}d^2\sigma^{N\text{-}N}/dp_t d\eta}$$

Jet-medium interaction is very strong→ color-opaque matter!

(high Pt yield (Raa), di-hadron correlation,……)

Strongly Interacting Matter

Strongly Interacting Matter in Heavy Ion Collisions:

Strong flow; opaque to jet;

fast "apparent" thermalization

WHY? Logically I see two possibilities:

- ***coupling constant is large, T ~ \Lambda_QCD***

→ **likely complete re-organization of D.o.F. , emergence**

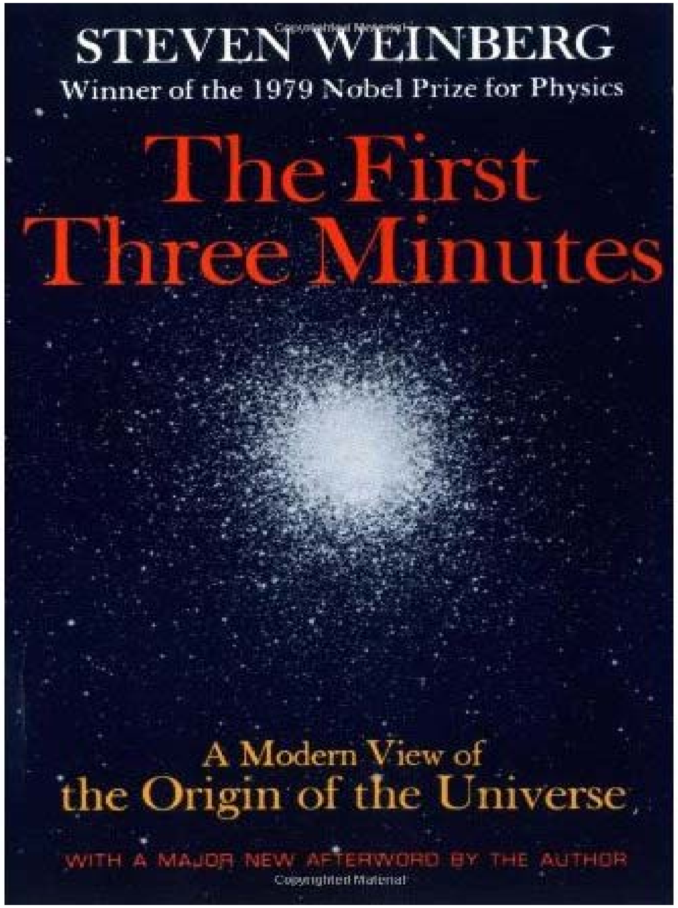
(issues: very early time? higher and higher beam energy…)

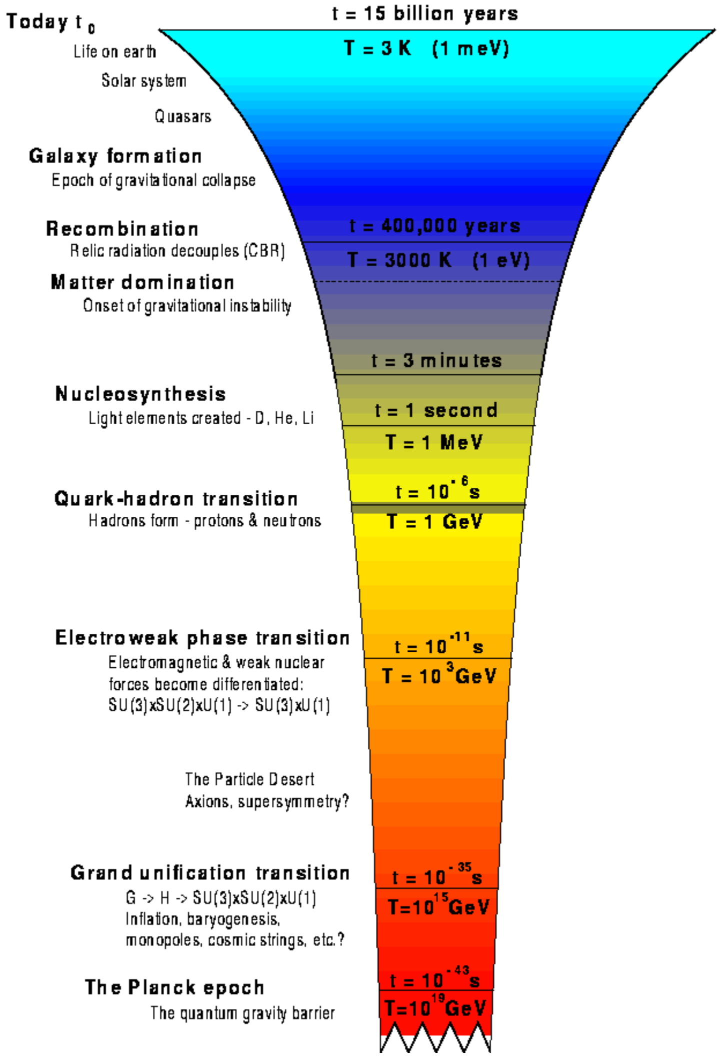
- ***coupling can be moderate/small, but high (phase-space)***

density that coherently amplifies interaction

→ **More is different !** → **crucial for *early time evolution*!**

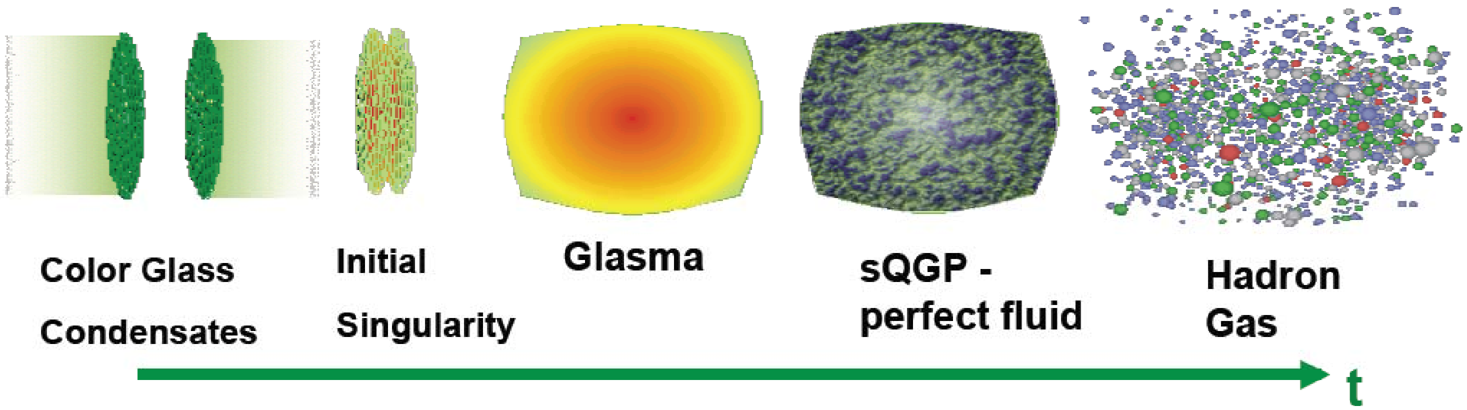
The First Three Minutes





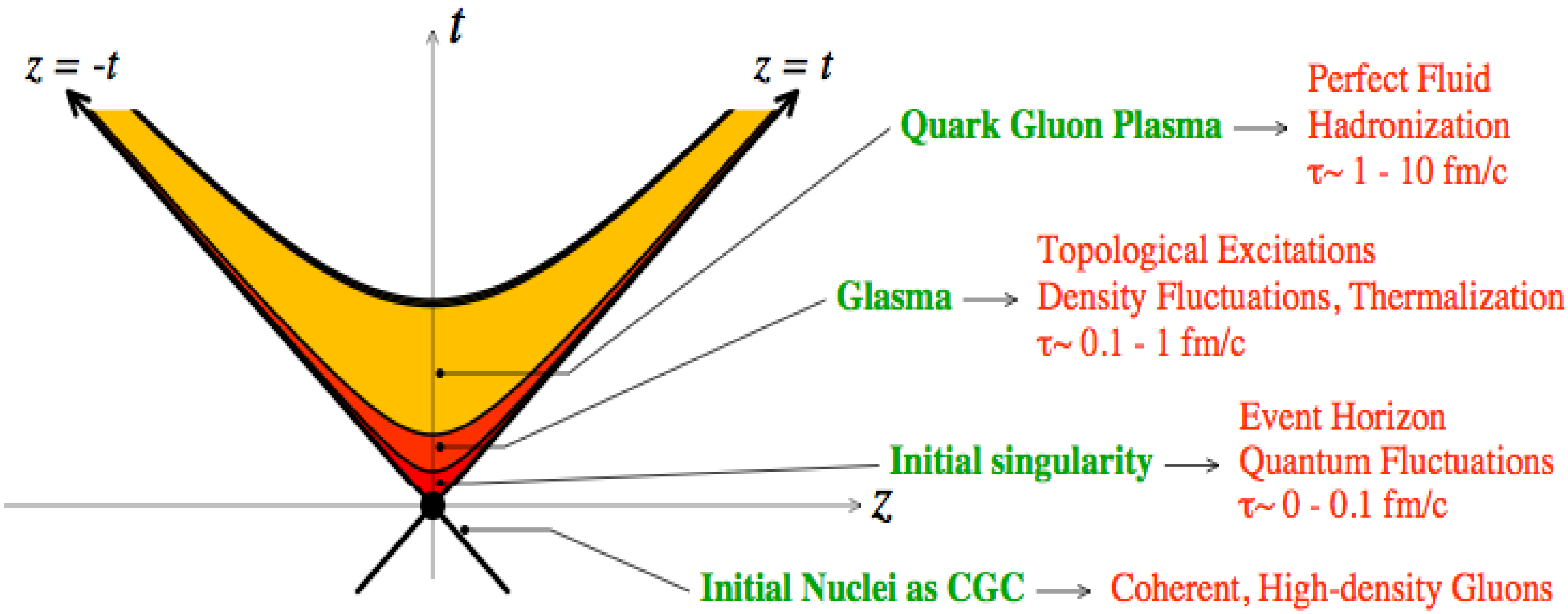
After the Big Bang ---

Just After the "Little Bang"???

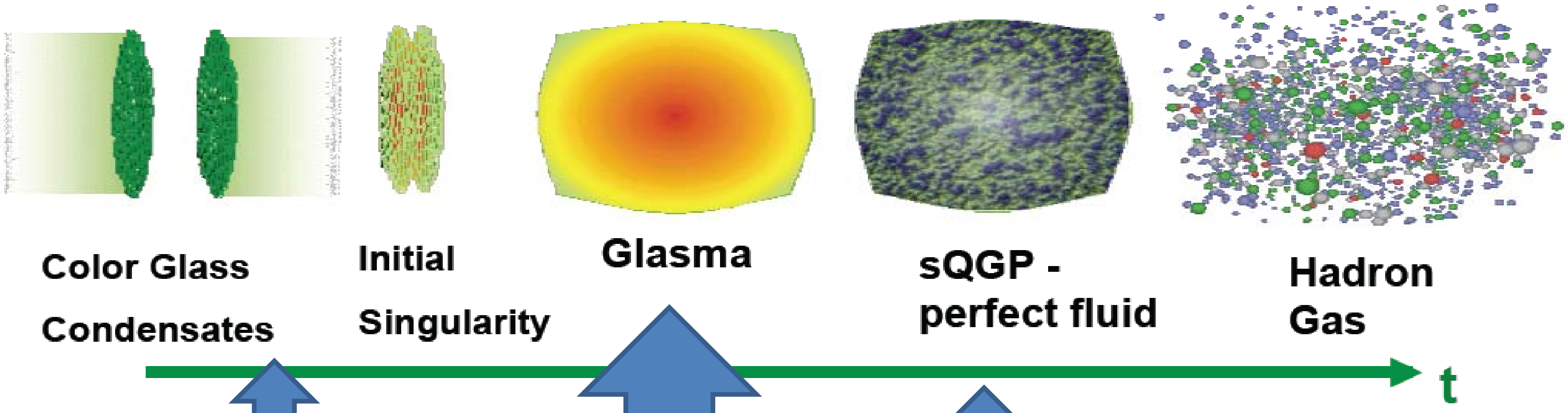


In analog to the "Big Bang" :

yet with the pre-equilibrium evolution the least understood open problem



The Pre-Equilibrium Matter



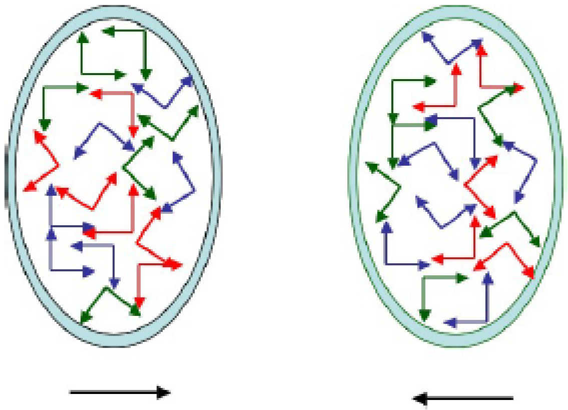
Strong constraint from
the initial condition:
saturation

Pre-equilibrium matter
(We focus on this!)

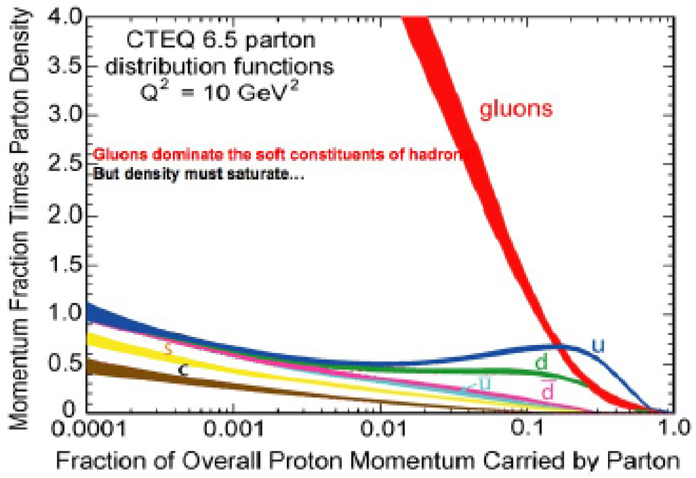
Strong constraint from
Hydro modeling and empirical data:
fast “thermalization” ~ fm/c
(to the extent of justifying hydro as effective description)

The Pre-Pre-Equilibrium

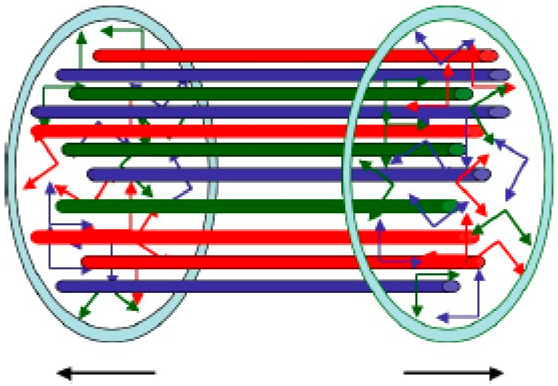
Before the collision: saturation



$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$



Right after the collision



Strong longitudinal expansion → anisotropy;

Instabilities play an important role:

Isotropization of energy-momentum tensor;

rapid growth toward large occupation of soft modes;

on the time scale ~ 1/Qs .

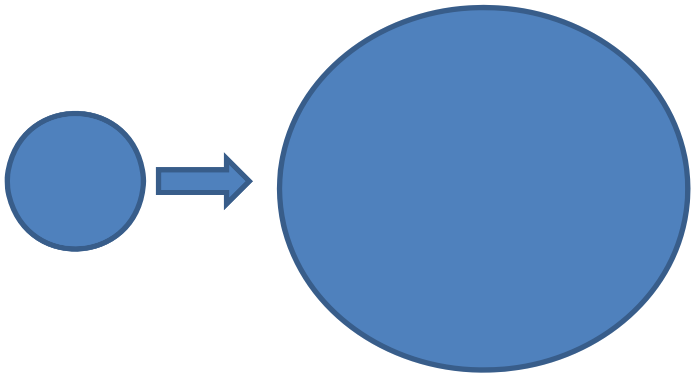
A High Density Gluon System

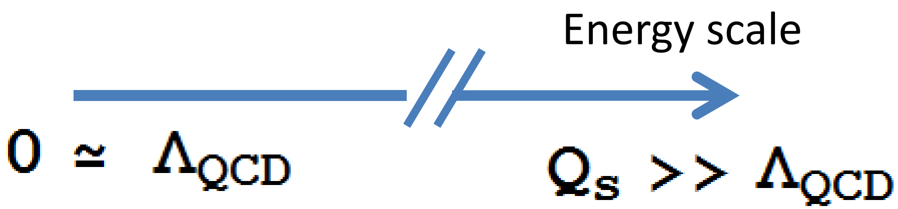
We consider a high density gluon system, starting at a time $\tau_0 \sim \frac{1}{Q_s}$

$$f \sim \frac{1}{\alpha_s} \quad , \quad p < Q_s$$

$$f \sim 0 \quad , \quad p > Q_s$$

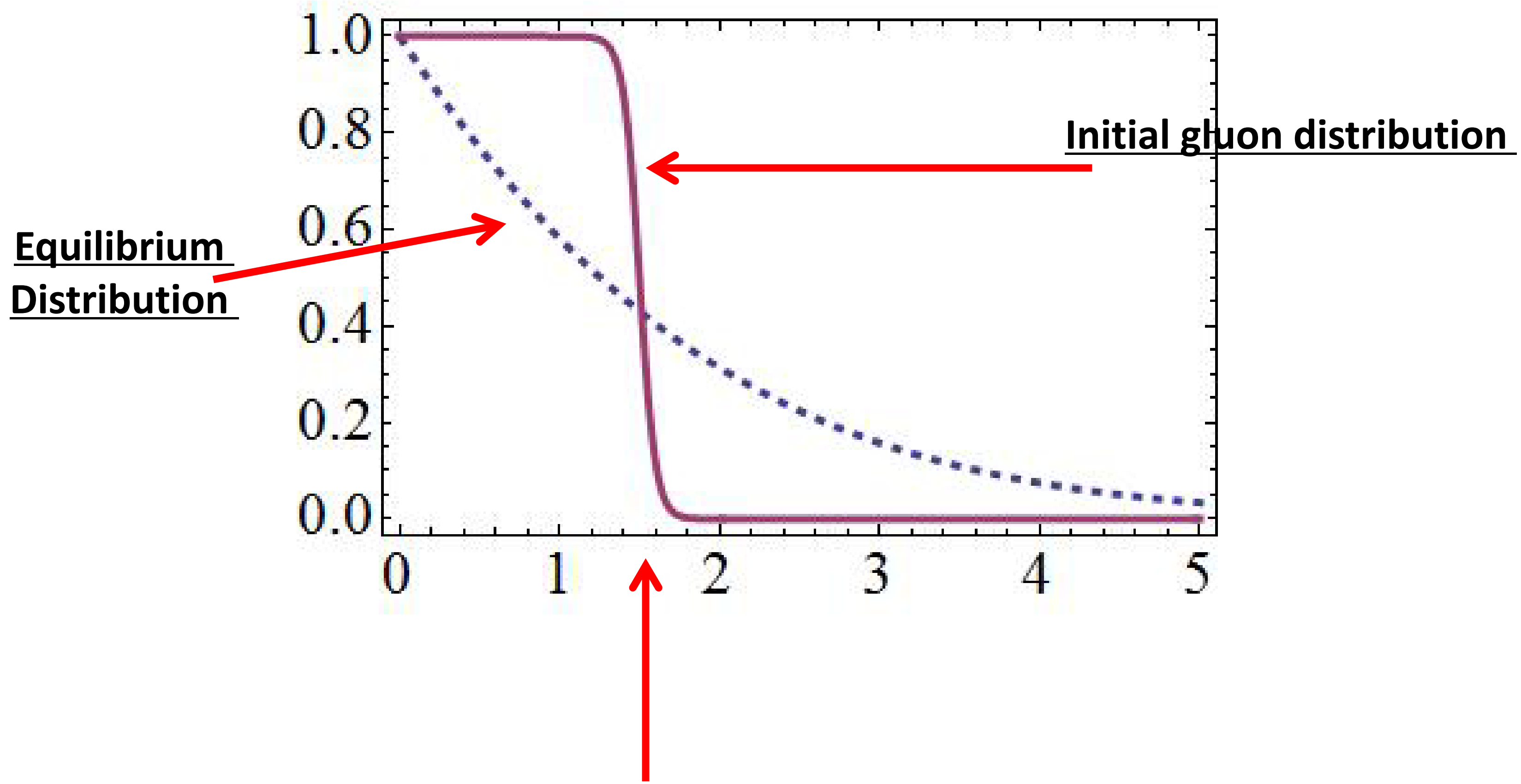
Some idealization concerning real heavy ion collisions:
very large transversely; very high energy , i.e. large Qs and small coupling





Far from Equilibrium…

The initial gluon system is far from equilibrium ! (Plotted here: p* f(p))



Saturation Scale
Q ~ 1 GeV or larger
Weakly coupled

OVERPOPULATION

We consider a high density gluon system, starting at a time $\tau_0 \sim \frac{1}{Q_s}$

$$f \sim \frac{1}{\alpha_s} \ , \ p < Q_s \qquad f \sim 0 \ , \ p > Q_s$$

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s}$$

Overpopulation parameter: $n_0\, \epsilon_0^{-3/4}$

For our initial gluon system: $n_0\, \epsilon_0^{-3/4} \sim 1/\alpha_s^{1/4}$

In contrast, for equilibrated QGP:

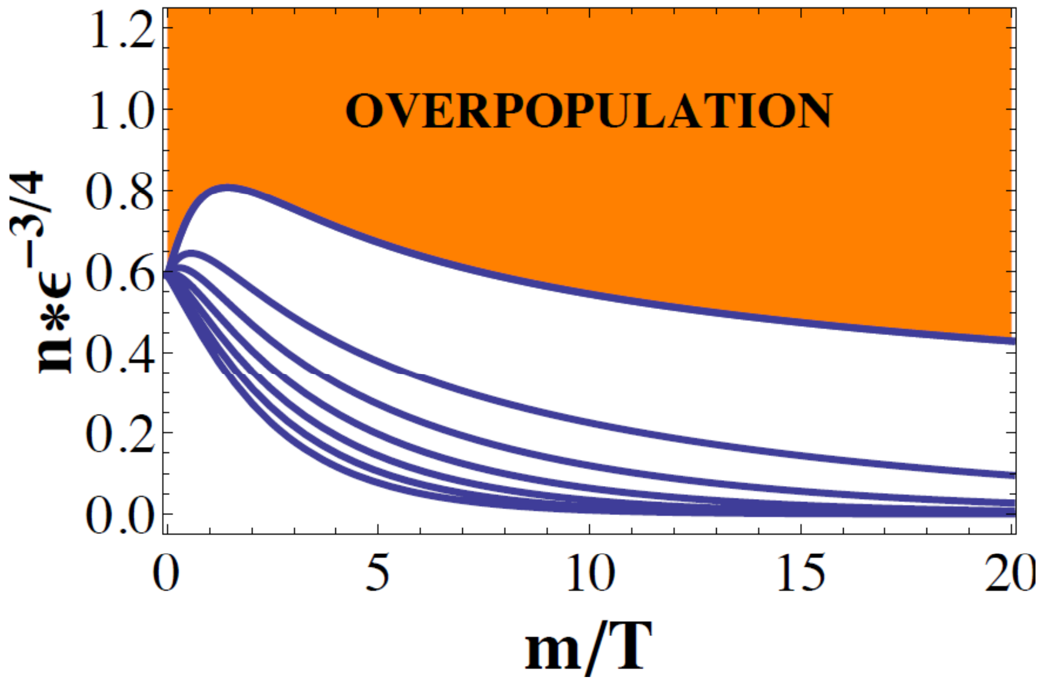
$$\epsilon_{eq} \sim T^4 \qquad n_{eq} \sim T^3 \qquad n_{eq}\, \epsilon_{eq}^{-3/4} \sim 1$$

OVERPOPULATION

Does building up a chemical potential help? NO!
The maximum gluons that can be accommodated by a Bose-Einstein distribution with \mu:

$$f_{eq}(\boldsymbol{k}) \equiv \frac{1}{e^{\beta(\omega_k - \mu)} - 1}$$

$$\mu \leq \omega_{p=0} = m \neq 0$$



For the given amount of initial energy, our gluon system has too many gluons

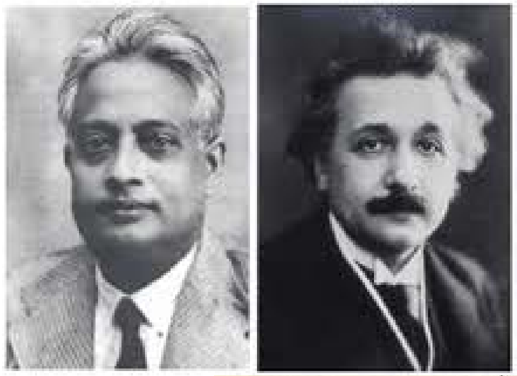
→Actually a "cold" dense system of gluons initially

→ can NOT equilibrate to a BE distribution by default (assuming elastic dominance)

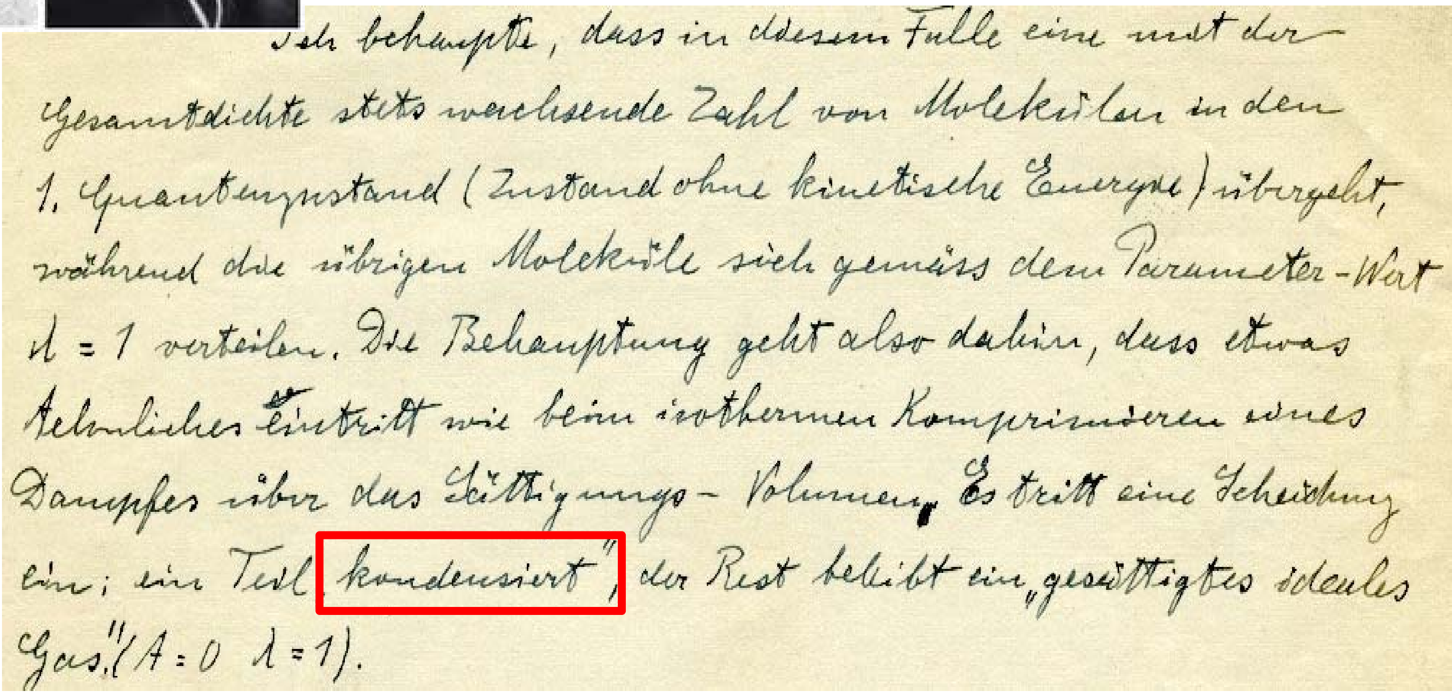
OVERPOPULATION

Overpopulation → Bose-Einstein condensation for equilibrium state

All the extra gluons get absorbed into zero momentum mode.



$$f_{\mathrm{eq}}(\boldsymbol{k}) = n_{\mathrm{c}}\delta(\boldsymbol{k}) + \frac{1}{e^{\beta(\omega_{\boldsymbol{k}} - m_0)} - 1}$$

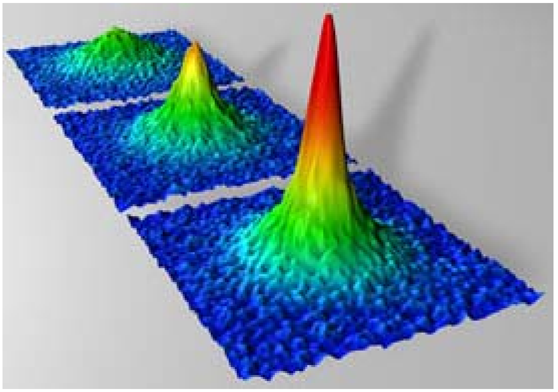


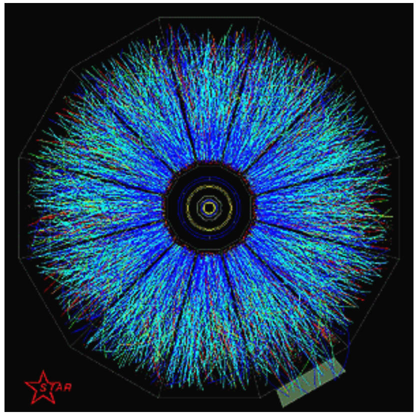
Ich behaupte, dass in diesem Falle eine mit der
Gesamtdichte stets wachsende Zahl von Molekülen in den
1. Quantenzustand (Zustand ohne kinetische Energie) übergeht,
während die übrigen Moleküle sich gemäss dem Parameter-Wert
λ = 1 verteilen. Die Behauptung geht also dahin, dass etwas
Ähnliches eintritt wie beim isothermen Komprimieren eines
Dampfes über das Sättigungs-Volumen. Es tritt eine Scheidung
ein; ein Teil „kondensiert", der Rest bleibt ein „gesättigtes ideales
Gas" (A = 0 λ = 1).

Bose-Einstein Condensation from Overpopulation

Initial overpopulation + Energy & Number conservation → BEC !!

$$n_0 \, \epsilon_0^{-3/4}$$





Atomic BEC: for given number of atoms, reduce energy (cooling) toward overpopulation

Dynamical formation of BEC at the early stage after heavy ion collisions:
born to have too many gluons for the available energy , really fascinating !
(Caveat: assuming elastic process dominance on certain time scale; discuss later…)

$$T \sim Q_s / \alpha_s^{1/4} \qquad n_c = n - n_g \qquad n_c \sim \frac{Q_s^3}{\alpha_s} \left(1 - \alpha_s^{1/4}\right)$$

Initially Uni-Scale

There is ONLY ONE SCALE initially, i.e. the Qs

$$f \sim \frac{1}{\alpha_s}\ , \ p < Q_s \qquad f \sim 0\ ,\ p > Q_s$$

This distribution is highly un-desired thermodynamically, i.e. by examining the entropy:

$$s \sim \int_p [\,(1+f) * Ln\,(1+f) - f * Ln\,(f)\,]$$

$$[\,] \sim O\,(1 + Ln\,(f))\ ,\ f >> 1$$

$$[\,] \sim O\,(1)\ ,\ f \sim 1$$

$$[\,] \sim O\,(f * Ln\,(1/f))\ ,\ f << 1$$

Thermalization → maximization of entropy →

More beneficial to distribute the gluons

to wider region in p-space with f ~ 1

Separation of Scales

We introduce two scales --- momentum cutoff scale & the saturated scale

$$\Lambda : \; f << 1 \text{ for } p > \Lambda \qquad \Lambda_s : \; f \sim \frac{1}{\alpha_s}$$

There is ONLY ONE SCALE initially, $\Lambda \sim \Lambda_s \sim Q_s$

$$f \sim \frac{1}{\alpha_s}\;, \quad p < Q_s \qquad f \sim 0\;, \quad p > Q_s$$

Toward thermalization, the two scales must be separated!

By how much?

Again, for a very weakly coupled equilibrated QGP

$$\Lambda \sim T \qquad \Lambda_s \sim \alpha_s * T$$

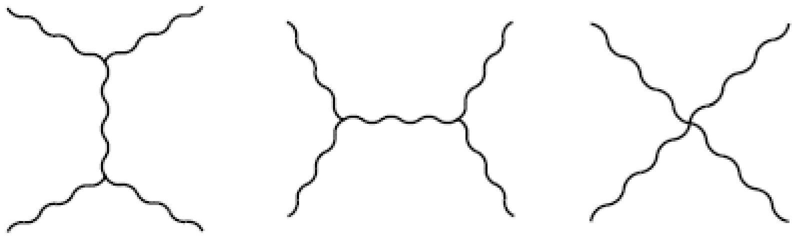
Therefore, thermalization requires specific separation of the two scales:

$$\frac{\Lambda_s}{\Lambda} \sim \alpha_s$$

Amplified Scattering

The initial gluon system is ***highly occupied → change the power-counting***

e.g. for the collision integral in kinetic evolution



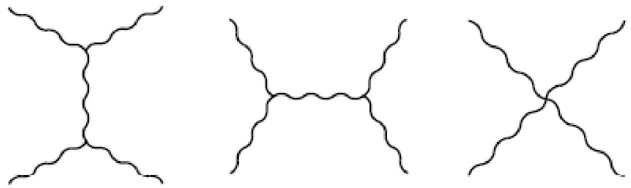
$$f * f * \alpha_s^2 \sim O(1)$$

$$f(p) \sim \frac{1}{\alpha_s}$$

Coherent amplification of scattering →
Fast thermalization from overpopulation?!

Kinetic Evolution

With 2→2 gluon scattering
& small-angle approximation
(here for isotropic case;
anisotropic case also derived and similar)



$$\mathcal{D}_t f(p) = \xi \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[f'(p) + \left(\frac{\alpha_S}{\Lambda_s} \right) f(p)[1+f(p)] \right] \right\}$$

(BE as fixed point)

$$\Lambda \left(\frac{\Lambda_s}{\alpha_S} \right)^2 \equiv \int_0^\infty dp\, p^2 f(p) \left[1 + f(p) \right]$$

$$\Lambda \frac{\Lambda_s}{\alpha_S} \equiv - \int_0^\infty dp\, p^2 \frac{d}{dp} \left[f(p) \right] = 2 \int_0^\infty dp\, p^2 \frac{f(p)}{p}$$

For highly occupied initial condition:
coupling constant disappears **in the scales!**

$$f \sim \frac{1}{\alpha_s} \qquad \Lambda_s^2 * \Lambda \sim O(1)$$

In contrast, for thermalized BE distribution:

$$\Lambda \sim T \qquad \Lambda_s \sim \alpha_s * T \qquad \Lambda_s^2 * \Lambda \sim O(\alpha_s^2)$$

Recap of the Scenario

Bose–Einstein condensation and thermalization of the quark–gluon plasma

Jean-Paul Blaizot [a], François Gelis [a], Jinfeng Liao [b,*], Larry McLerran [b,c], Raju Venugopalan [b]

A highly overpopulated gluon system evolving toward equilibrium:

Strong overpopulation → ***dynamical BEC formation***

Initial uni-scale → ***separation of the two scales by coupling \alpha_s***

High occupancy

→ ***coherent amplification of scattering, ~O(1) effect despite coupling***

NEXT: analytic analysis of scaling solutions ; ultimately nuermically solving the equation

The "Static Box" Problem

A schematic scaling distribution characterized by the two evolving scales:

$$f(p) \sim \frac{1}{\alpha_s} \text{ for } p < \Lambda_s, \qquad f(p) \sim \frac{1}{\alpha_s}\frac{\Lambda_s}{\omega_p} \text{ for } \Lambda_s < p < \Lambda, \qquad f(p) \sim 0 \text{ for } \Lambda < p$$

$$n_g \sim \frac{1}{\alpha_s}\Lambda^2\Lambda_s \qquad \epsilon_g \sim \frac{1}{\alpha_s}\Lambda_s\Lambda^3 \qquad n = n_c + n_g$$

Time evolution of the distribution, i.e. of the two scales → need two conditions

At $\tau_0 \sim 1/Q_s$: $\Lambda \sim \Lambda_s \sim Q_s$

Essential points here:

- Energy is conserved, but not gluon number → absorption into condensate
- Collision time scale (from transport equation) ~ \tau for scaling solutions

$$t_{\text{scat}} = \frac{\Lambda}{\Lambda_s^2}$$

Thermalization in The "Static Box"

Two conditions fixing the time evolution:

$$\Lambda_s \Lambda^3 \sim \text{constant} \qquad t_{\text{scat}} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

The scaling solution:

$$\Lambda_s \sim Q_s \left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

Upon thermalization: separation of scales; entropy production; eliminating over-pop.

$$\Lambda_s \sim \alpha_s \Lambda \quad \Rightarrow \quad \boxed{t_{\text{th}} \sim \frac{1}{Q_s}\left(\frac{1}{\alpha_s}\right)^{7/4}}$$

$$T \sim \Lambda \sim Q_s / \alpha_s^{1/4} \qquad s \sim \Lambda^3 \sim Q_s^3 / \alpha_s^{3/4} >> s_0 \sim Q_s^3$$

$$n * \epsilon^{-3/4} \sim \left(\frac{\Lambda_s}{\alpha_s * \Lambda}\right)^{1/4} \to O(1)$$

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$$n * \epsilon^{-3/4} \sim \left(\frac{\Lambda_s}{\alpha_s * \Lambda}\right)^{1/4} \rightarrow O(1)$$

Condensate in The "Static Box"

Condensate dominates the number density:

$$n_c \sim n_0[1 - (t_0/t)^{1/7}] \qquad n_g \sim n_0\left(\frac{t_o}{t}\right)^{1/7}$$

While gluons dominate the energy density

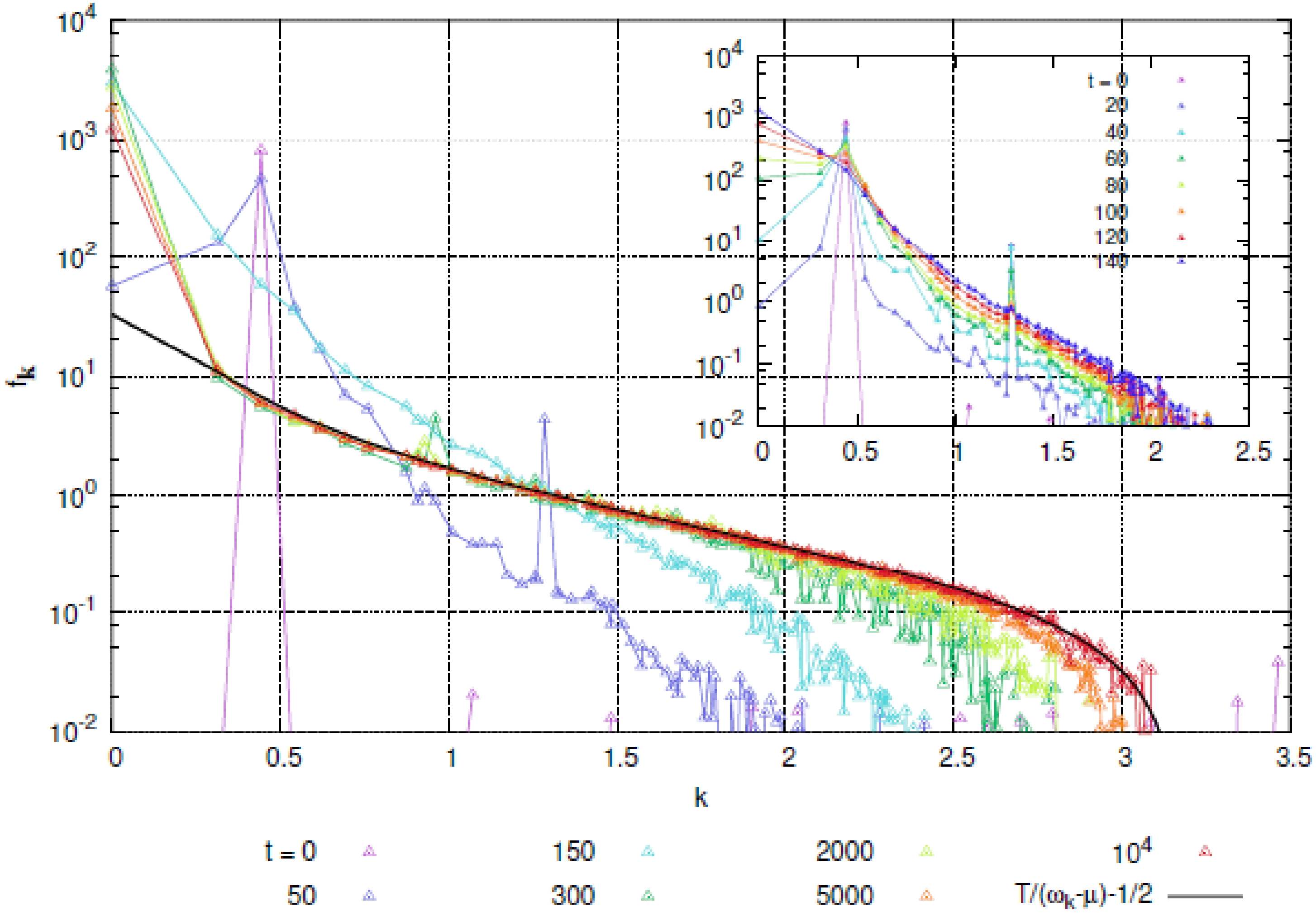
$$\epsilon_c \sim n_c m \sim n_c \sqrt{\Lambda\Lambda_s}$$

$$m^2 \sim \alpha_s \int dp\, p^2 \frac{df(p)}{d\omega_p} \sim \Lambda\Lambda_s$$

$$\frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{1/7}$$

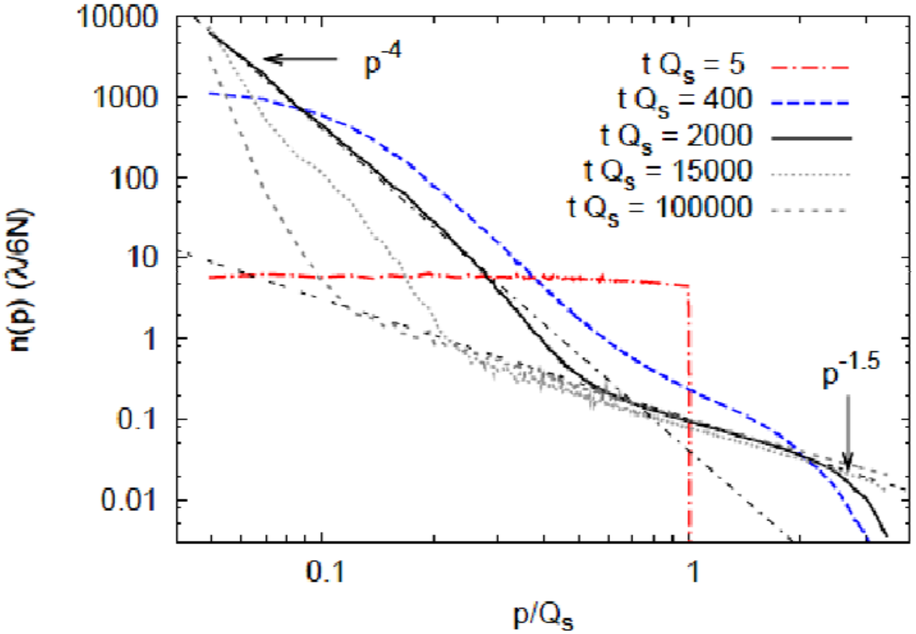
A robust, though transient ultimately, Bose-Einstein condensate can be dynamically formed and maintained during the thermalization!

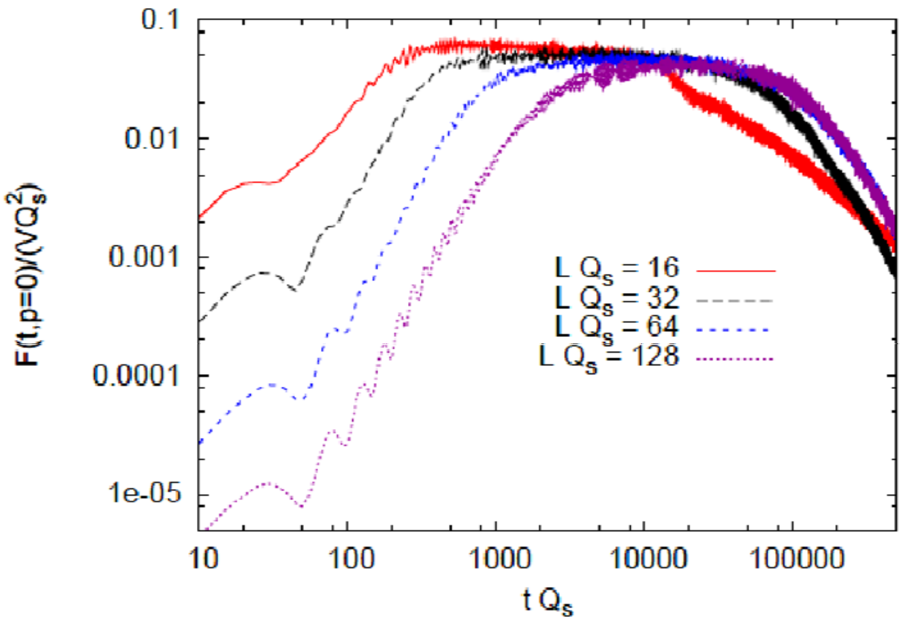
Evidence of BEC from Scalar Field Theory Computation



From: Epelbaum & Gelis

Evidence of BEC from Scalar Field Theory Computation

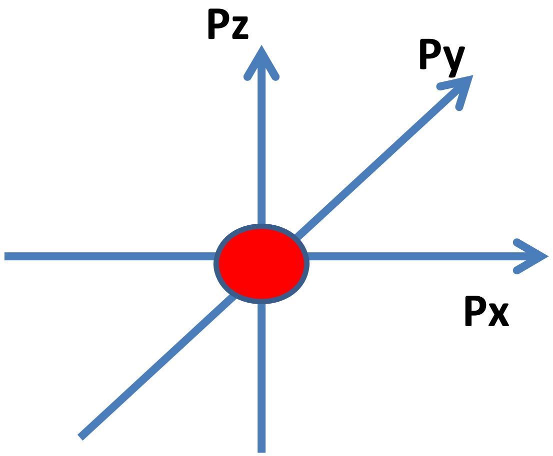




From: Berges & Sexty

Numerical Transport Study

$$\mathcal{D}_t f(p) = \xi \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[f'(p) + \left(\frac{\alpha_S}{\Lambda_s} \right) f(p)[1 + f(p)] \right] \right\}$$



We numerically solve this equation,
but cutting out a "small hole" near the origin:
avoiding divergence due to condensate ;
measuring the particle flux toward condensate
in the momentum space !

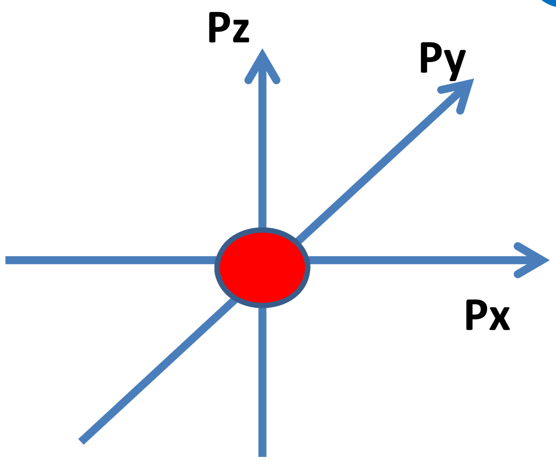
$$f(p) = \lambda \quad , \quad p \le Q_s$$

$$f(p) = \lambda\, e^{-20(p-Q_s)^2/Q_s^2} \quad , \quad p > Q_s$$

$$n\,\epsilon^{-3/4} \approx \lambda^{1/4}$$

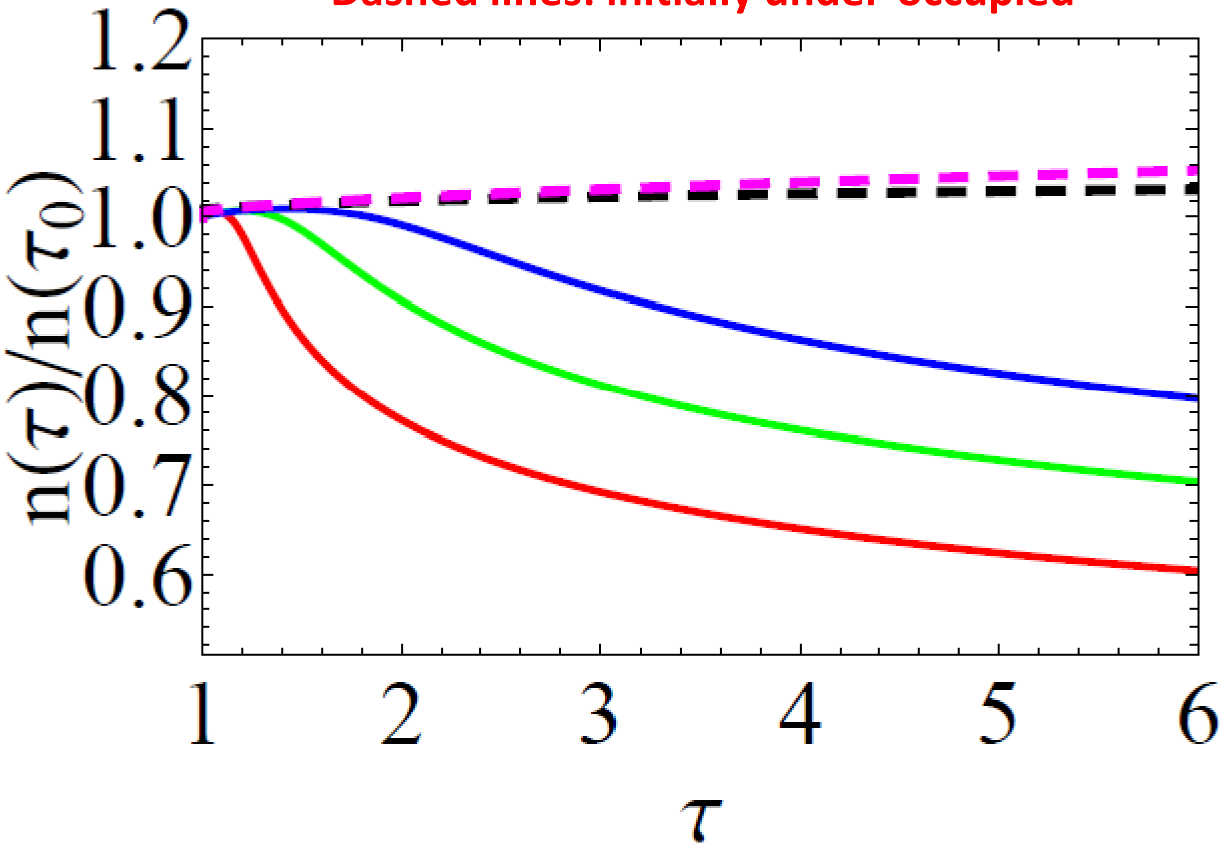
$$\text{Glasma : } \lambda = \frac{1}{\alpha_s}$$

Overpopulation



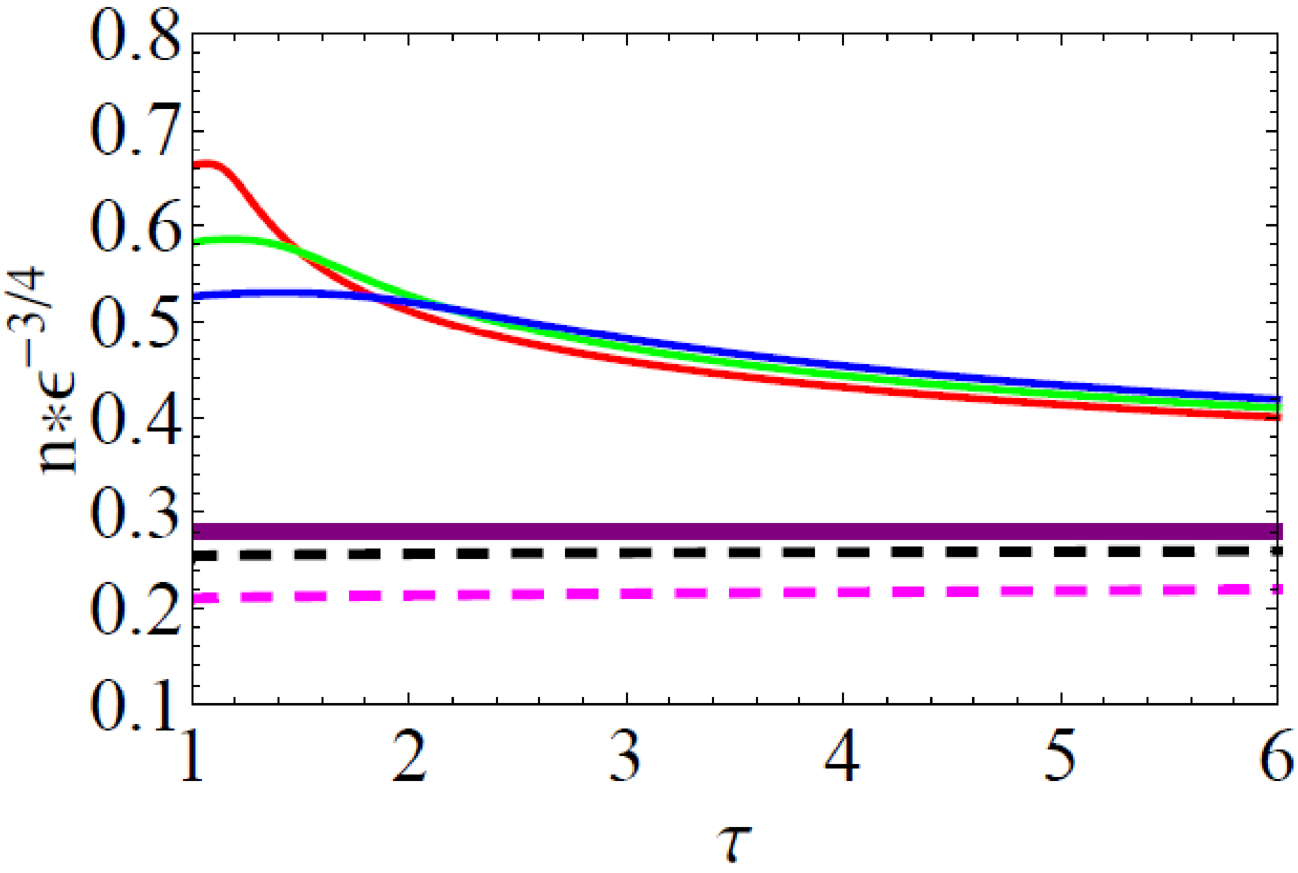
Solid lines: initially over-occupied

Dashed lines: initially under-occupied



Overpopulation → a flux in momentum space toward zero → source term for BEC

Overpopulation



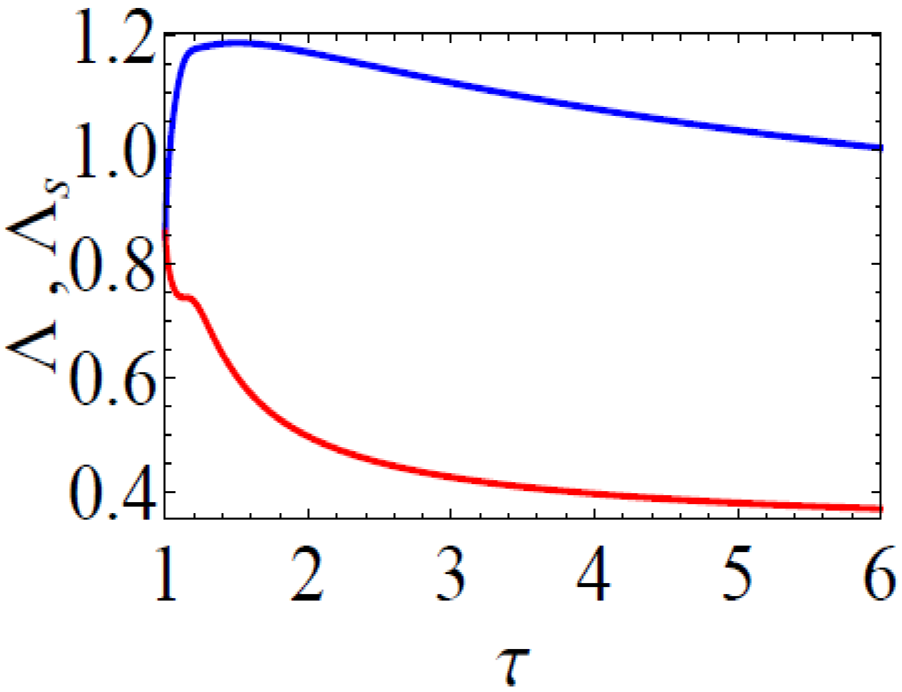
Solid lines: initially over-occupied

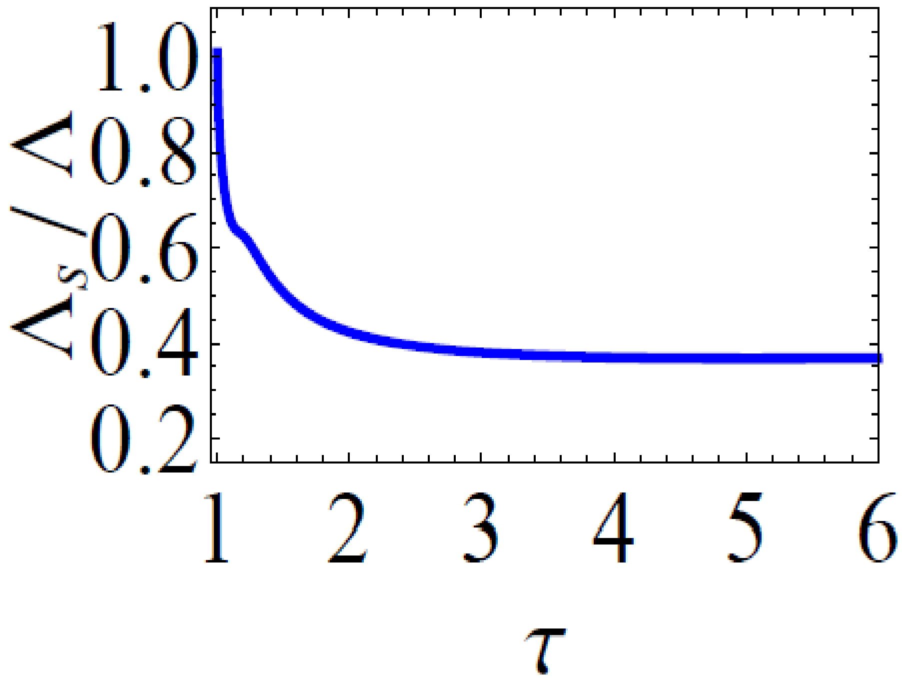
Purple line: value for the equilibrium B.E. distribution

Dashed lines: initially under-occupied

Separation of Scales

$$\text{Glasma}: \lambda = \frac{1}{\alpha_s}$$





Effect of Longitudinal Expansion

Assuming boost-invariance and studying mid-rapidity

$$\partial_t f - \frac{p_z}{t}\partial_{p_z} f = \left.\frac{df}{dt}\right|_{p_z t} = C[f]$$

$$\partial_t \epsilon + \frac{\epsilon + P_L}{t} = 0. \qquad \partial_t n + \frac{n}{t} = 0.$$

Further assuming certain fixed anisotropy

$$P_L = \delta\epsilon$$

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta}$$

δ = 0 : free streaming

δ = 1 / 3 : isotropic

Effect of Longitudinal Expansion

Two conditions fixing the time evolution:

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta} \qquad t_{scat} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

The scaling solution:

$$\Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}.$$

Upon thermalization: separation of scales; entropy production; eliminating over-pop.

$$\Lambda_s \sim \alpha_s \Lambda \quad \Longrightarrow \quad \boxed{\left(\frac{t_{th}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{7/(3-\delta)}}$$

$$T \sim Q_s * \alpha_s^{\frac{1+2\delta}{3-\delta}} \qquad s \sim T^3 \sim Q_s^3 * \alpha_s^{\frac{3*(1+2\delta)}{3-\delta}}$$

Condensate in Expanding Case

NOTE: anisotropic expansion reduces overpopulation

Condensate still can exist and dominate density: for $\delta > 1/5$

$$n_c \sim \frac{Q_s^3}{\alpha_s}\left(\frac{t_0}{t}\right)\left[1-\left(\frac{t_0}{t}\right)^{(-1+5\delta)/7}\right]$$

Energy carried by condensate is subleading:

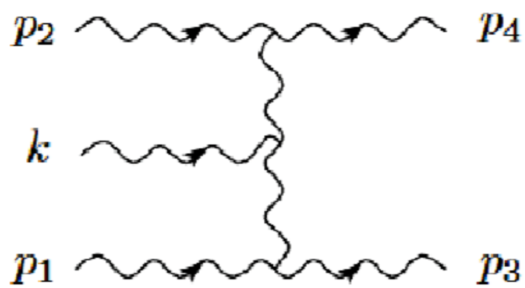
$$\frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{(5-11\delta)/14}$$

The more isotropic the expansion is, the stronger the condensation will be.

Would be great to test in scalar/gauge field simulations aand kinetic computation.

Discussions: Inelastic Processes

Point-1:
The inelastic rate could be very different in far-from-equilibrium system (e.g. Glasma) as compared with near-equilibrium system (e.g. in calculating transport coef.)



$$t_{\rm scat} = \frac{\Lambda}{\Lambda_s^2}$$

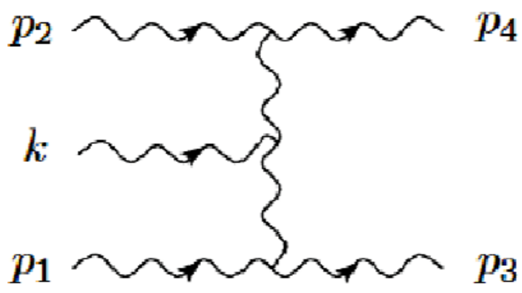
$$\frac{1}{t_{scat}} \sim \alpha_s^{n+m-2} \left(\frac{\Lambda_s}{\alpha_s}\right)^{n+m-2} \left(\frac{1}{m^2}\right)^{n+m-4} \Lambda^{n+m-5}$$

$$m^2 \sim \Lambda_s \Lambda$$

In Glasma case: the rate of inelastic happens to be on the same order as the elastic → transient BEC still, but more subtle now

Discussions: Inelastic Processes

Point-2:
Fast rate of inelastic processes does NOT necessarily imply fast change of particle number!



Examine the factorized limit
Dominant contribution to I.R. enhancement
Singularity or not depending on mass and spectrum shape at small energy

$$C_{1,2\leftrightarrow 3,4\|5} \sim C_{1,2\leftrightarrow 3,4} \times \int_{z'\to 0} dz' \frac{2C_A}{z'} f(z')$$

However the change in particle number density vanishes in this limit!

$$\frac{dn}{dt} = \int_{p_1} C_{1,2\leftrightarrow 3,4\|5} \sim \left[\int_{p_1} C_{1,2\leftrightarrow 3,4}\right] \times \int_{z'\to 0} dz' \frac{2C_A}{z'} f(z')$$

This is ZERO

Summary: a Scenario for Thermalization

- The pre-equilibrium matter starts with high gluon density and one scale.
- Strong overpopulation enforces the system toward BEC.
- Separation of scales must happen toward thermalization.
- Elastic scattering is coherently enhanced despite the small coupling, therefore leading to strongly interacting matter even at early time and driving the thermalization.
- Expansion may proceed with asymmetry → anisotropic hydro?

Outlook

A number of important things to do

- **The role of instabilities:**

  can we incorporate that into the transport approach? How?

  understanding the relations among different approaches

- **The role of inelastic scatterings:**

  need a thorough and quantitative investigate to clarify

- **The role of a condensate:**

  new transport framework including both condensate & particles

- **Full numerical solutions to transport in expanding case**

- **Phenomenological implications:**

  e.g. Flow, jet quenching, EM production in pre-equilibrium matter

Thank you !