The role of plasma instabilities in thermalization

Aleksi Kurkela
with Guy Moore

Arxiv:
- 1107.5050 — thermalization in generic setup, complete treatment of plasma instabilities
- 1108.4684 — specialized to heavy ion collisions

869 order of magnitude estimates in total

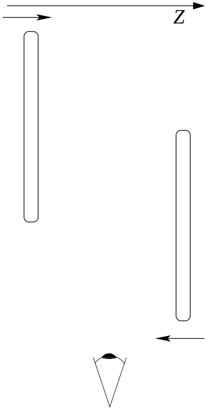
Motivation

In:

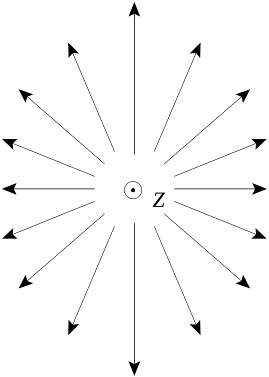
Pb+Pb @ few TeV per nucleon

Out:

Anisotropic yield of $\sim 10^4$ hadrons ("v_2")



$\Rightarrow$

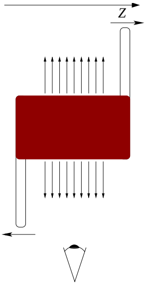


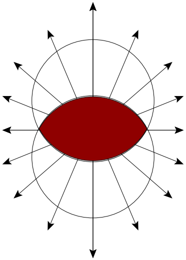
Motivation

Model:

Hydro flow of nearly thermal fluid:

- Collective flow turns spatial anisotropy to momentum anisotropy





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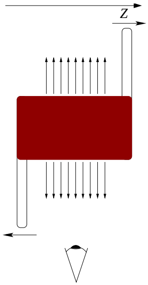

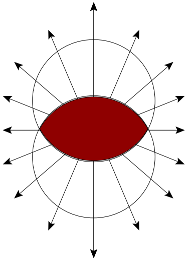

- Assumes: Local thermal equilibrium $T_{\mu\nu}^{r.f.} \approx \mathrm{diag}(\mathrm{e}, \mathrm{p}, \mathrm{p}, \mathrm{p})$
- Inputs:
  - Equation of state, viscosities...
  - Initial geometry
  - Thermalization time $\tau_0 \sim 0.4 \ldots 1.2\mathrm{fm/c}$

Motivation

Objective:

- Why is $\tau_0 \lesssim 0.4 \ldots 1.2\text{fm/c}$???
- What happens before? Maybe observable in experiment

Method:

- Extremely big: $N_{nucl} \to \infty$
- Extremely high energy: $\sqrt{s} \to \infty$
  - weak coupling: $\alpha_s \ll 1$
  - $\Rightarrow$ Separation of scales: Kinetic theory, Hard loops, Vlasov equations, ...
  - Might be still non-perturbative ($\alpha f \gtrless 1$)
- Purely parametric: counting powers of α_s

Warning: all scales logarthmic

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Objective of this talk: What are the relevant physical process that lead to thermalization in a HIC?

Warning: all scales logarthmic

Initial condition: $t \sim Q_s^{-1}$

At weak coupling, well understood: Color Glass Condensate

- Characteristic, *saturation* scale: Q_s
- High occupancy: $f(p < Q_s) \sim 1/\alpha$ (need something to cancel α in $\sigma_{\rm large\ angle}$!)

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Distribution of gluons: $f(p) \sim \alpha^{-c}\theta(Q_s - p)\theta(\underbrace{\alpha^d}_{\delta} Q_s - p_z)$

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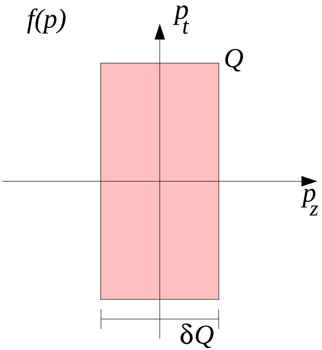
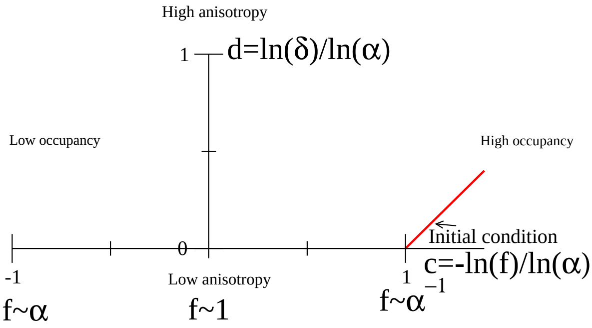
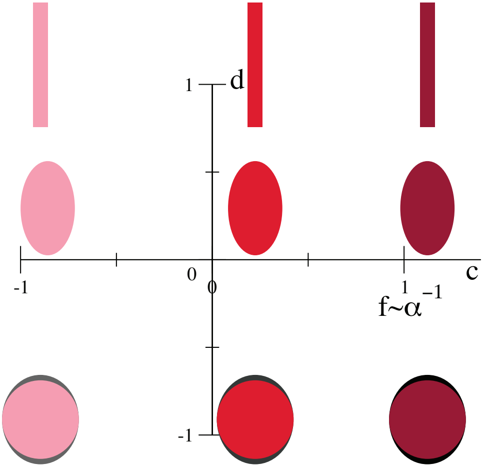




Out of equilibrium systems: descriptors



High anisotropy:

$$f(p) \sim \alpha^{-c}\theta(Q_s - p)\theta(\alpha^d Q_s - p_z)$$

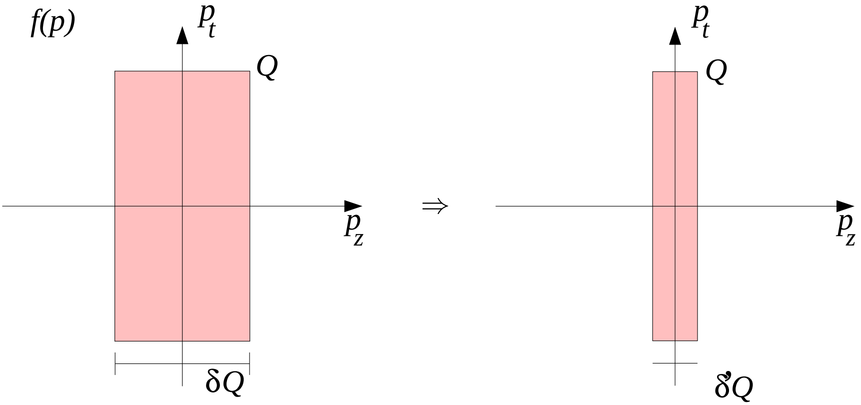
Small anisotropy:

$$f(p) \sim \alpha^{-c}(1 + \alpha^{-d} F(\hat{p}))$$

Longitudinal expansion

Spatial expansion translates into redshift in $p_z \sim \delta Q_s$

- Changes only p_z, Q_s stays constant
- Changes $\delta = \alpha^d$
- If $p_z \ll p_t$, $\varepsilon(t) \sim \alpha^{-1} Q_s^4/(Q_s t)$



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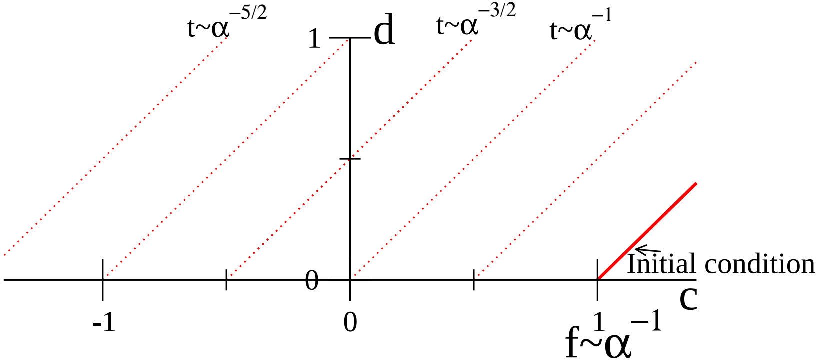

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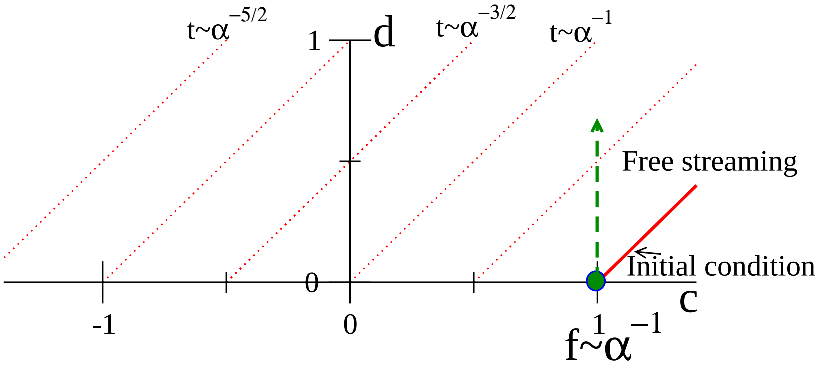
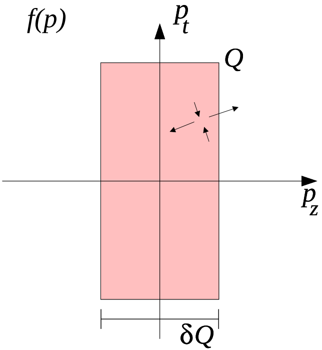
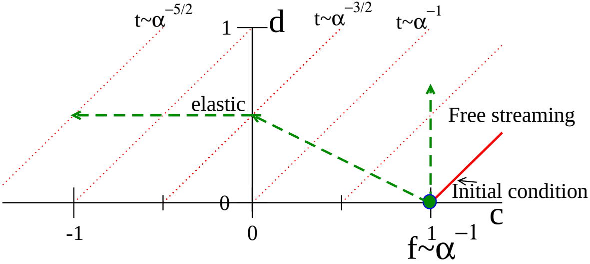


Scattering: Elastic

Elastic scattering makes the distribution fluffier





- Along the attractive solution scattering and expansion compete
- When typical occupancies $f \ll 1$: loss of Bose enhancement
- At late times: Fixed anisotropy, dilute away

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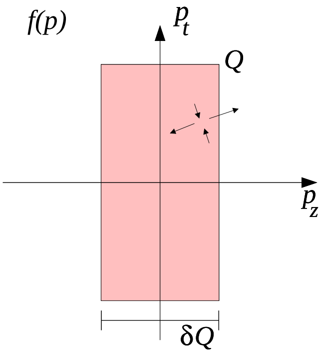

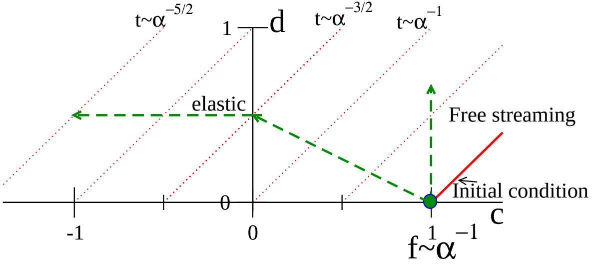

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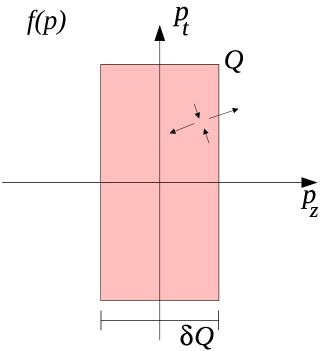

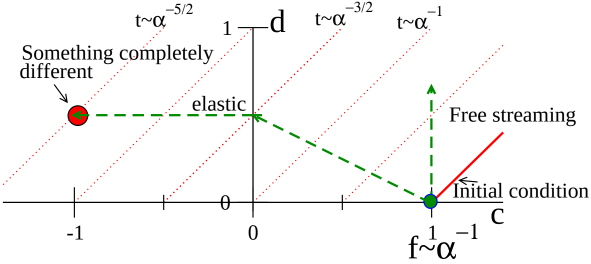

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Elastic scattering not enough for thermalization!

Scattering: Inelastic

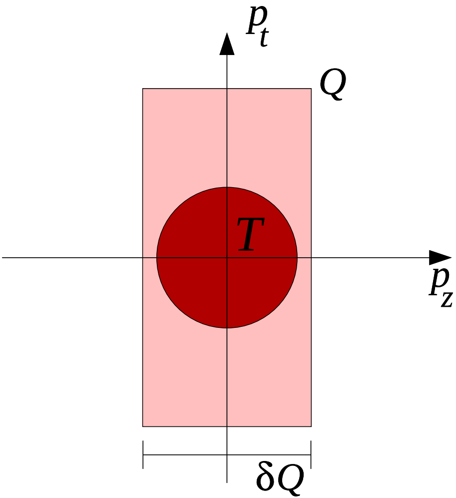
Inelastic scattering plays two significant roles (Baier, Mueller, Schiff & Son 2000)

1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles

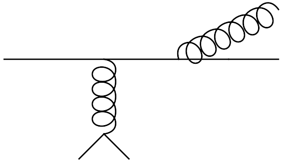
Scattering: Inelastic

Baier, Mueller, Schiff & Son 2000

Soft splitting: Creation of soft thermal bath



- Soft modes quick to emit
  $n_s \sim \alpha n_{\rm col}$

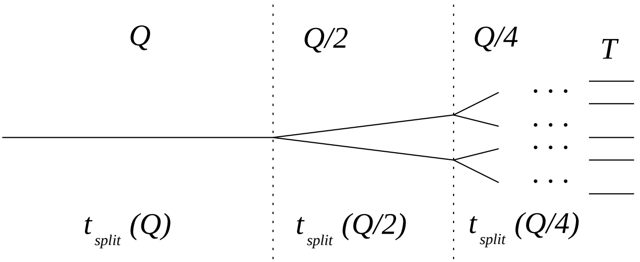


- Low p: easy to bend
  $\Rightarrow$thermalize quickly
- *Can dominate dynamics!*
  (i.e. scattering, screening, ...)

Scattering: Inelastic

Baier, Mueller, Schiff & Son 2000

Hard splitting: Q_s modes break before they bend!



- In vacuum: on-shell particles, no splitting
- In medium: Particles receive small kicks frequently
  - For stochastic uncorrelated kicks: Brownian motion in p-space

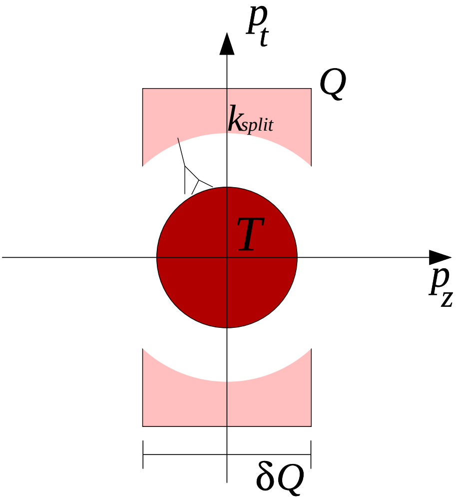
$$\Delta p_\perp^2 \sim \hat{q}\, t, \qquad t_{\rm split}(k) \sim \alpha \underbrace{\sqrt{\hat{q}/k}}_{t_{\rm form}} \qquad \text{(LPM)}$$

  - Momentum diffusion coefficient $\hat{q}$ describes how the medium wiggles a hard parton

Bottom-Up

Baier, Mueller, Schiff & Son 2000

Thermal bath eats the hard particles away:



- Scales below k_{split} have cascaded down to T-bath

  $$t_{\mathrm{split}}(k_{\mathrm{split}}) \sim t \Rightarrow k_{\mathrm{split}} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat up the thermal bath

  $$T^4 \sim k_{\mathrm{split}} \int d^3p f(p)$$

- Thermalization when Q_s gets eaten

  $$k_{\mathrm{split}} \sim Q_s$$

Needs $\hat{q}$ as an input

Old bottom up:
Baier, Mueller, Schiff & Son 2000

$\hat{q}$ dominated by elastic scattering with thermal bath??

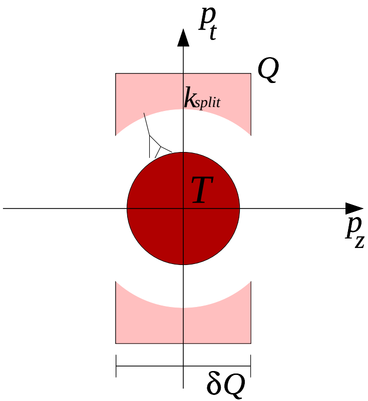
BMSS assumed: $\hat{q} \sim \hat{q}_{\text{elastic}} \sim \alpha^2 T^3$
Solve self-consistently:

$$\begin{cases} k_{\text{split}} & \sim & \alpha^2 \hat{q} t^2 \\ T^4 & \sim & k_{\text{split}} (Q_s^3/(Q_s t)) \\ \hat{q} & \sim & \alpha^2 T^3 \end{cases}$$

for:

$$\Rightarrow \begin{cases} T & \sim & \alpha^3 Q_s (Q_s t) \\ k_{\text{split}} & \sim & \alpha^{13} (Q_s t)^5 Q \\ \tau_0 & \sim & \alpha^{-13/5} Q_s^{-1} \end{cases}$$

But: Is this all there is?

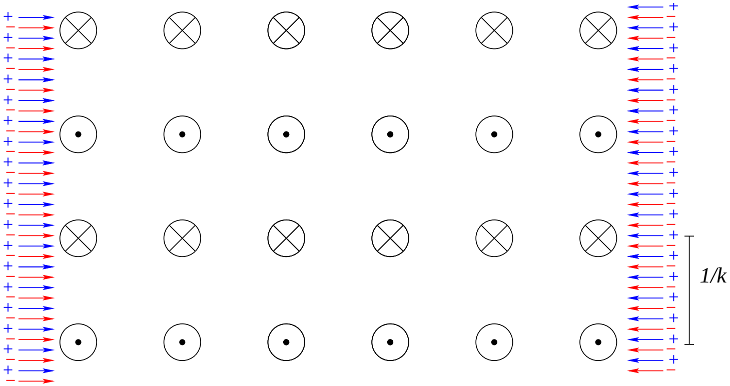


$\hat{q}$ dominated (always!) by

plasma instabilities

Plasma instabilities: Idea

Exponential growth of (chromo)-magnetic fields in anisotropic plasmas



How do particles deflect?

Plasma instabilities: Idea

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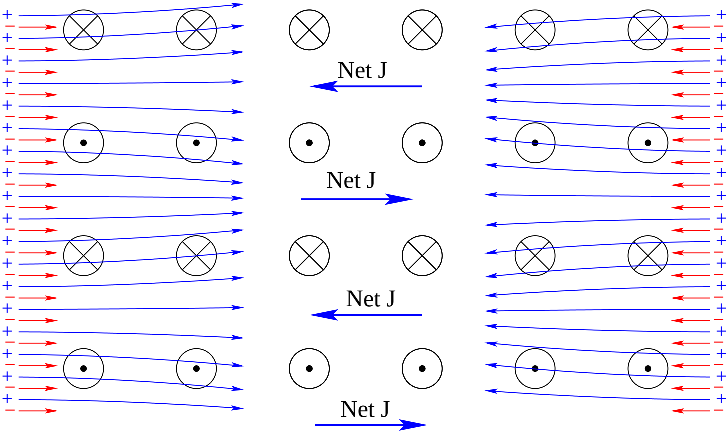
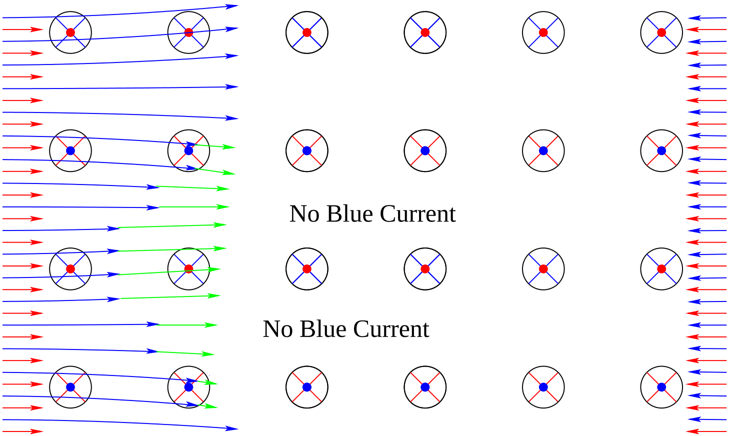


Induced current feeds the magnetic field

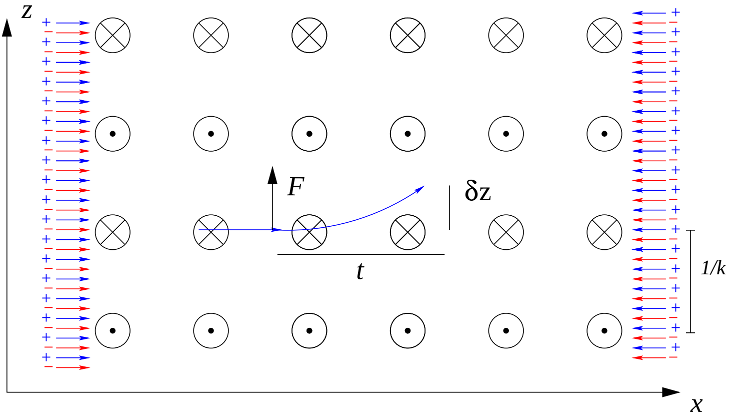
Plasma instabilities: Idea

When chromo-magnetic field becomes strong enough to mix colors, currents no longer "feed" the magnetic field. Growth is cut off.



Plasma instabilities: Slightly more quantitive

- Lorentz force: $F \sim gB$
- Displacement: $\delta z \sim gBt^2/p$



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$$J \sim \underbrace{g}_{charge} \underbrace{\int d^3p f(p)}_{\text{\# of part.s}} \underbrace{[\delta z\, k]}_{\text{disp. fraction}} \sim kBt^2 \underbrace{\alpha \int \frac{d^3p}{p} f(p)}_{m^2} \sim kB m^2 t^2$$

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- Competes in Maxwell's equation with $\nabla \times B \sim k\, B$

$$\boxed{\Rightarrow \text{Exponential growth if } m^2 t^2 > 1}$$

$$\Rightarrow \begin{cases} k_z^{inst} \sim any \\ k_x^{inst} < m \end{cases}$$

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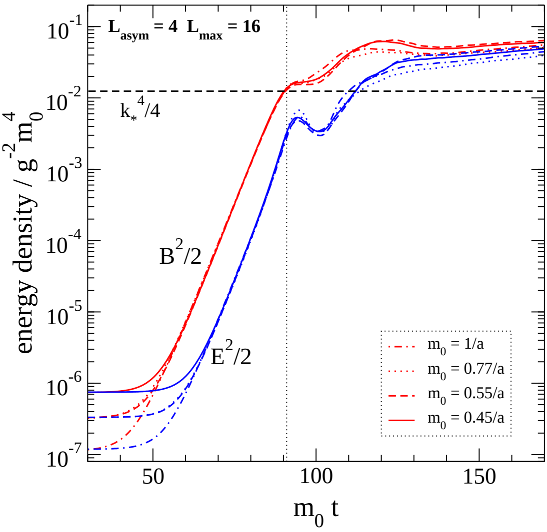
- Saturation when competition in $D_\mu = k_{\mathrm{inst}} + igA_\mu$
  $\Rightarrow A \sim k_{\mathrm{inst}}/g$, or $B \sim k_{\mathrm{inst}} A \sim k_{\mathrm{inst}}^2/g$, or $f(k_{\mathrm{inst}}) \sim 1/\alpha$

Don't take my word for it.

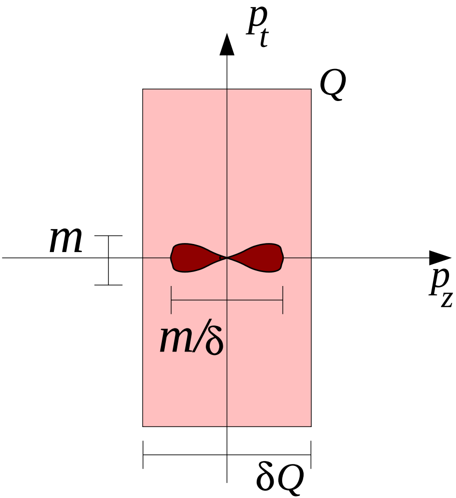


Bödeker and Rummukainen (0705.0180) and many others. . .

Plasma instabilites: More complicated distribtutions

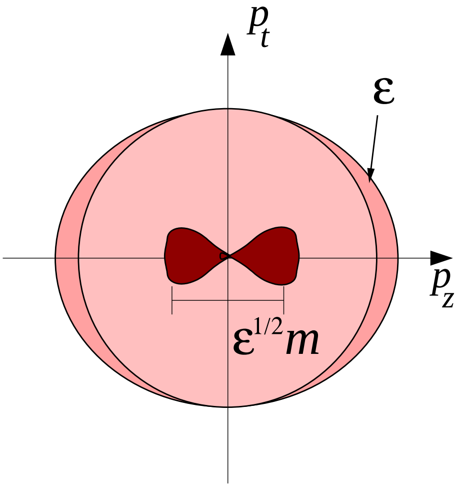
Strong anisotropy:

$$f(\vec{p}) \sim \alpha^{-c}\Theta(Q_s - p)\Theta(\delta Q_s - p_z)$$



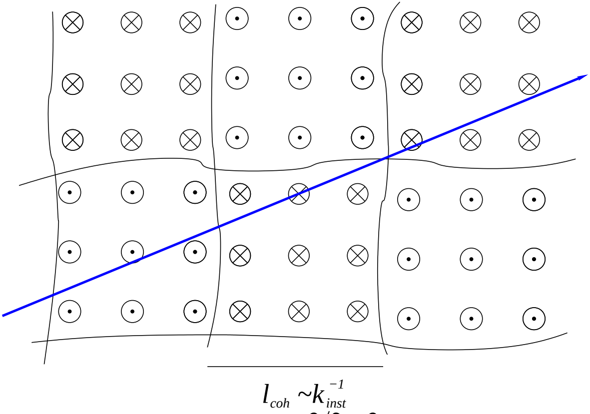
Weak anisotropy:

$$f(\vec{p}) \sim f_0(|\vec{p}|)(1 + \epsilon F(\vec{p}))$$



Plasma instabilites: Momentum transfer

Hard parton traveling through magnetic fields receives coherent kicks from patches of same-sign magnetic fields



- $\Delta p_{\mathrm{kick}} \sim g\, B\, l_{\mathrm{coh}}$
- $\hat{q}\; t \sim \underbrace{N_{\mathrm{kick}}}_{t/l_{\mathrm{coh}}} (\Delta p_{\mathrm{kick}})^2$

  $\hat{q} \sim \alpha\, l_{\mathrm{coh}}\, B^2$

Weak anisotropy: $\hat{q}_\epsilon \sim \epsilon^{3/2} m^3$
Strong anisotropy: $\hat{q}_\delta \sim m^3/\delta^2$

The new bottom-up

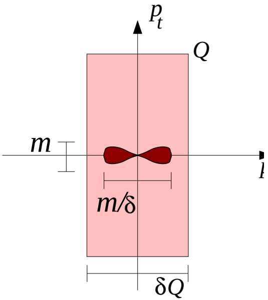
Important physics:

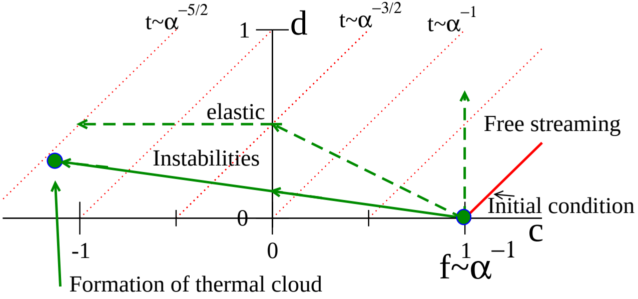
1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles
3. Plasma instabilities, screening

In particular, elastic scattering irrelevant

The new bottom-up: Early stages: $Q_s t < \alpha^{-12/5}$

Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale Q_s





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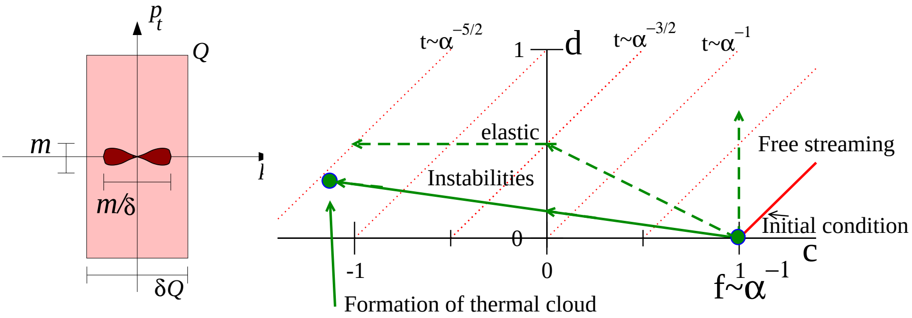

- Even the instabilities are not strong enough to thermalize when $f \gg 1$ $\Rightarrow$ classical theory does not thermalize under longitudinal expansion!

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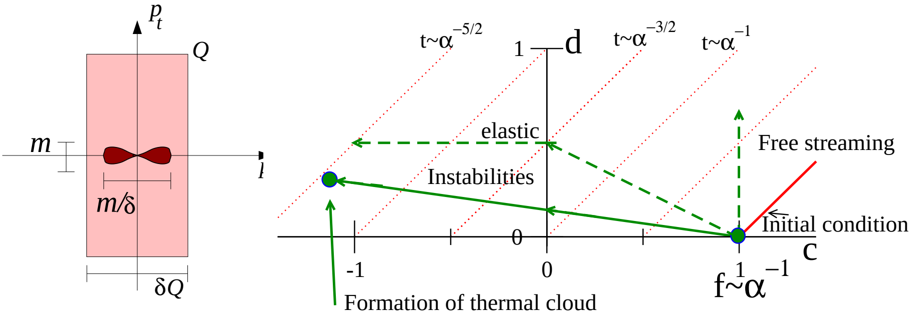
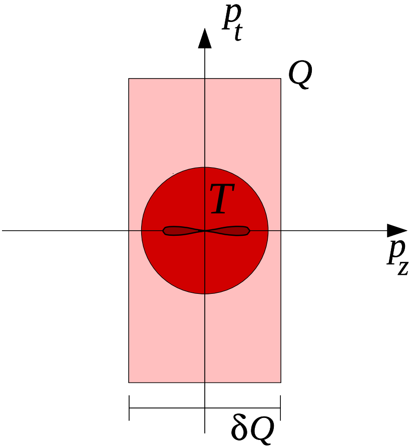


$$\hat{q}_\delta \sim m^3/\delta^2, \quad m^2 \sim \alpha\,\delta\, f\, Q_s^2, \quad \delta \sim p_z/Q_s \sim \sqrt{\hat{q}_\delta t}/Q_s, \quad f \sim 1/(\alpha\delta Q_s t)$$

$$\Rightarrow \delta \sim (Q_s t)^{-1/8}, \quad f(Q_s) \sim (Q_s t)^{-7/8}, \quad \hat{q}_\delta \sim Q_s^3 (Q_s t)^{-5/4}$$

The new bottom-up: Late stages: $\alpha^{-12/5} < Qt < \alpha^{-5/2}$:



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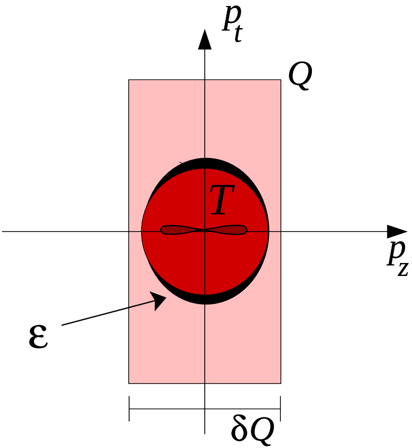

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- The expansion and anisotropic rain make T-bath anisotropic
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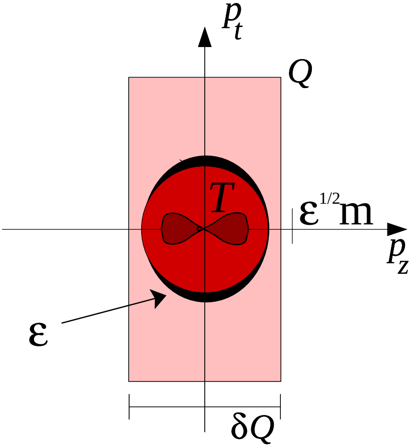

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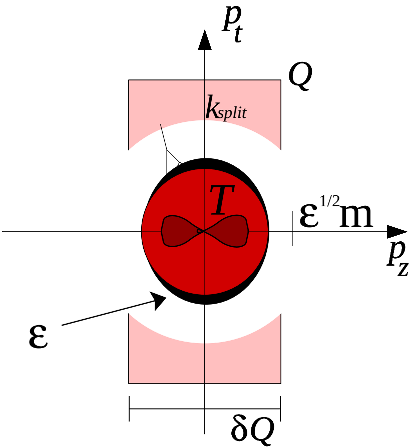

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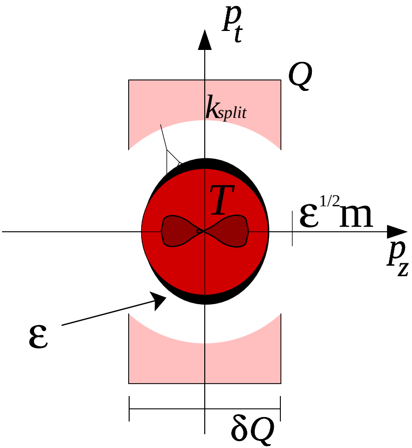

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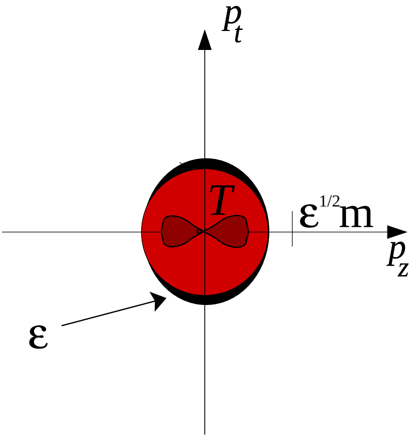

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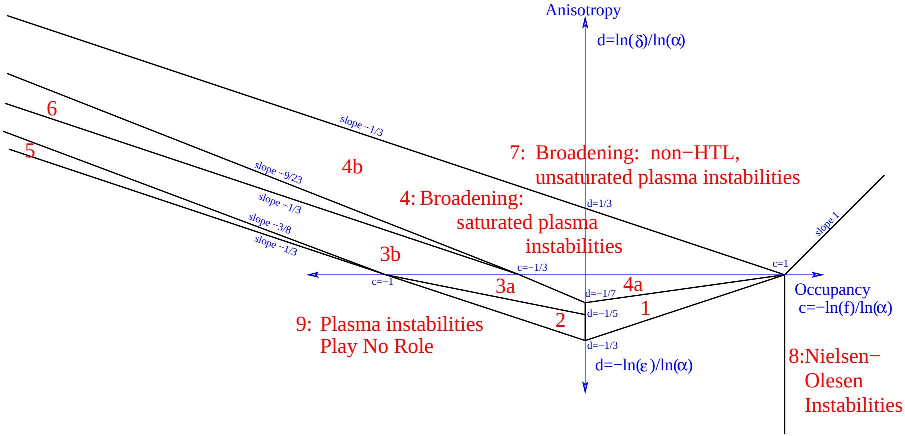
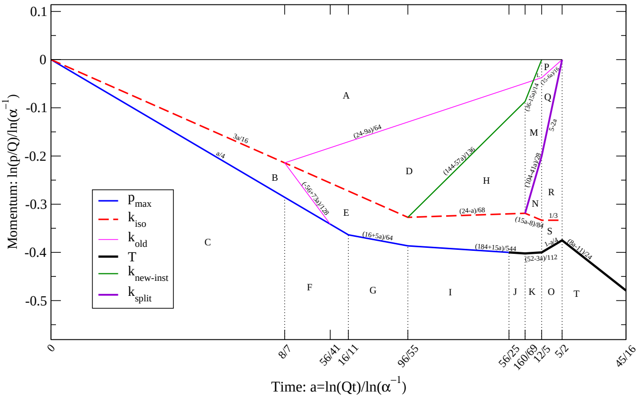
- At $(Q_s t) \sim \alpha^{-5/2}$, $k_{\rm split} \sim Q_s$. Plasma instabilities continue to dominate until $(Q_s t) \sim \alpha^{-45/16}$.

Summary, complete catalog:



We identified the relevant physics in occupancy-anisotropy plane...

Summary, complete (parametric) description of HIC:



Featuring:

- Obscure powers: $\alpha^{\frac{184+15a}{554}}$
- Redshifted relics
- Synchrotron absorption
- Complicated angle dependence
- . . . and many more

...applied to the case of longitudinal expansion.

Conclusions:

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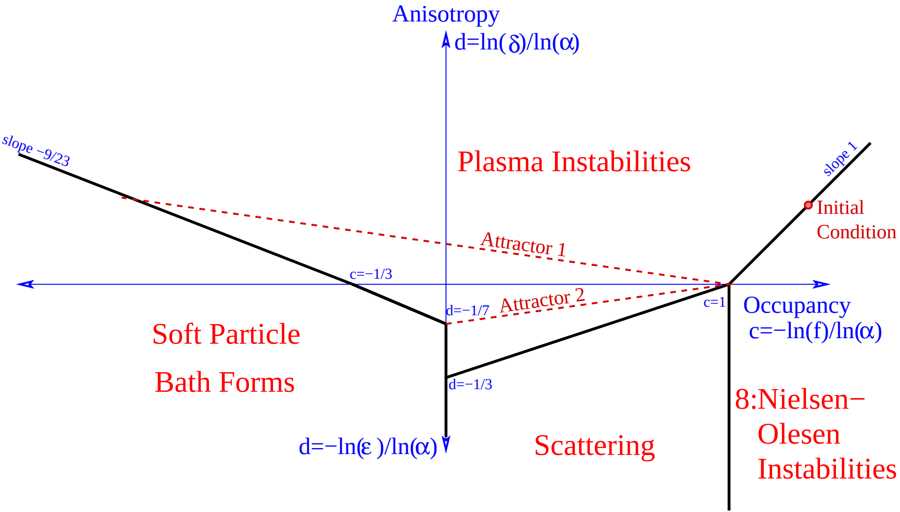
- Apply same methods to
  - Cosmology, reheating
  - Neutron stars, anomalous viscosities

Second attractor?

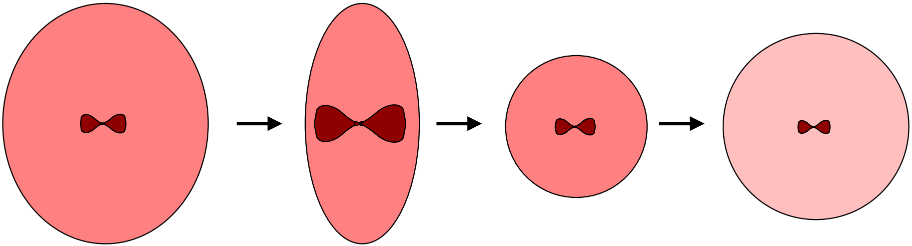


$$f(p) \sim \alpha^{-c}(1 + \alpha^{-d} F(\hat{p}))$$

Second attractor?

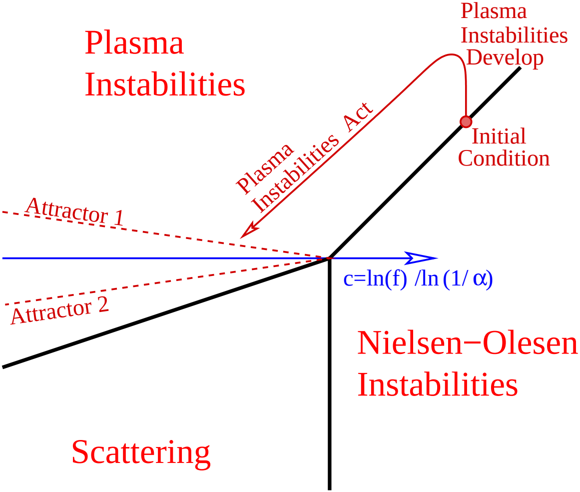
Assume nearly isotropic distribution:

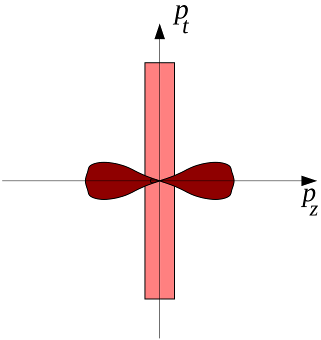
- Longitudinal expansion reduces energy: $\varepsilon \propto t^{-4/3}$
  - p_z reduces.
- Plasma-instabilities keep system nearly isotropic $\epsilon \sim t_{\mathrm{iso}}/t \sim \frac{Q^2}{\hat{q}_\epsilon t}$
  - Q reduces
- Instability induced particle joining at hard scale
  - Q increases



$$\epsilon \sim \alpha^{-d} \sim (Q_s t)^{-\frac{8}{135}}, \quad f(Q) \sim \alpha^{-1}(Q_s t)^{\frac{56}{135}}, Q \sim Q_s (Q_s t)^{-\frac{31}{135}}$$

Why do we think the first attractor is the relevant one?





- Initially, unstable modes in unpopulated part of phase space
- Growth from vacuum fluctuations, slowed by a log:
  $t_{growth} \sim Q_s^{-1} \ln^2(1/\alpha)$