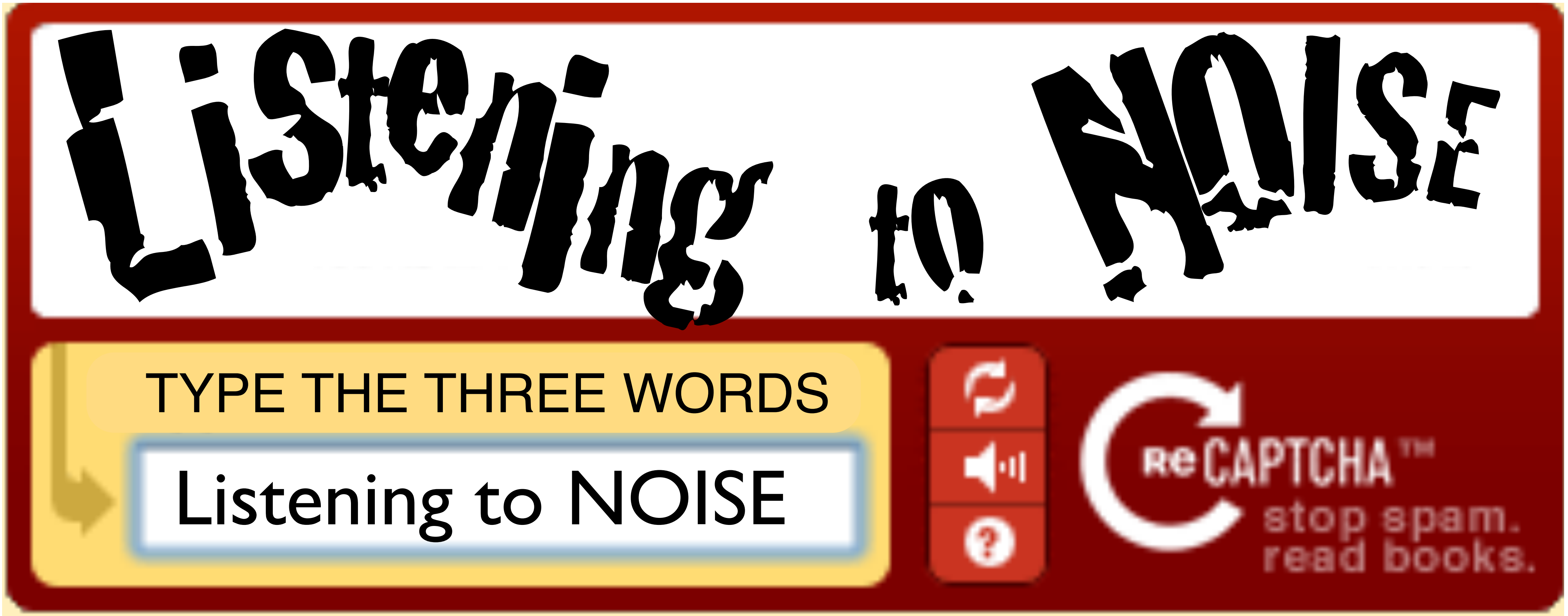


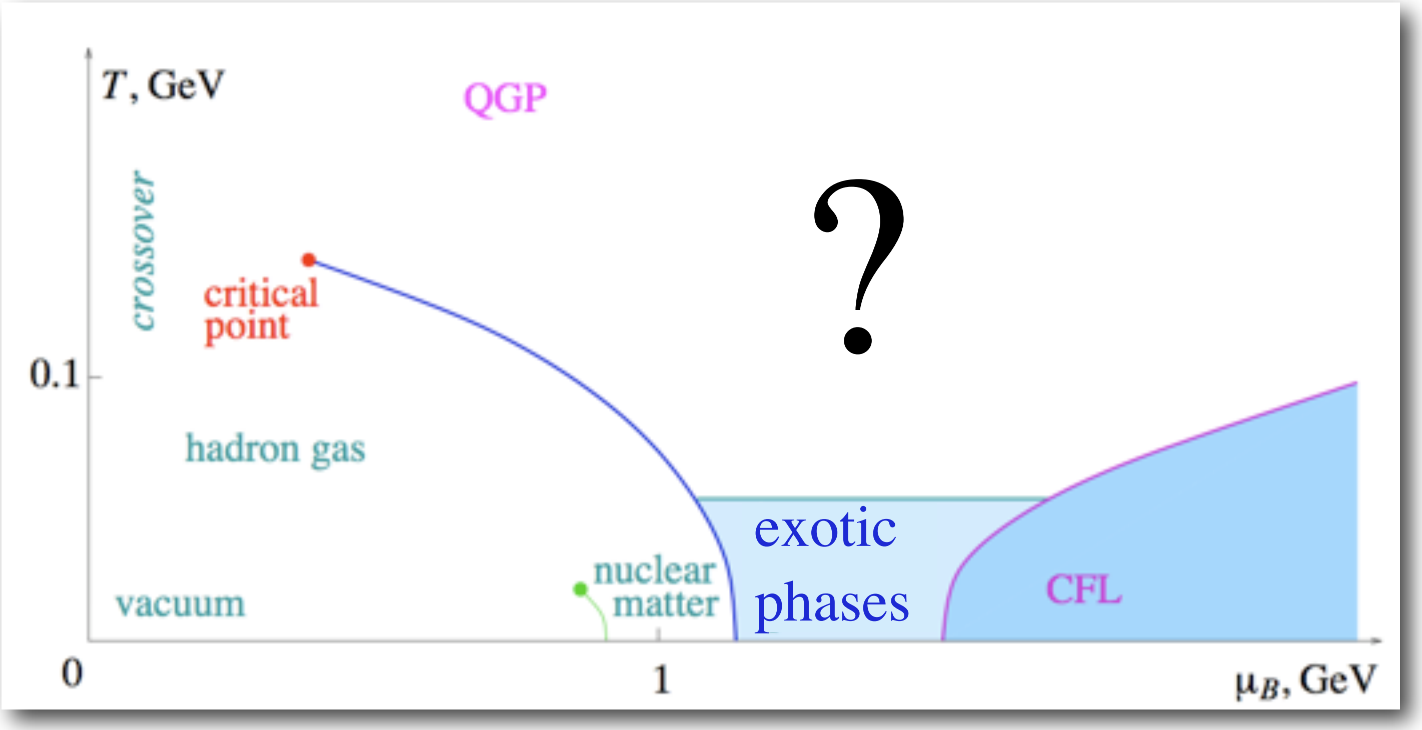
Noise, sign problems, and statistics

arXiv:1106.0073 [hep-lat]

Michael Endres, D.K., Jong-Wan Lee, Amy Nicholson

...& work in progress

Physics motivation: can't we get beyond this cartoon??



Sign problem!

This talk:

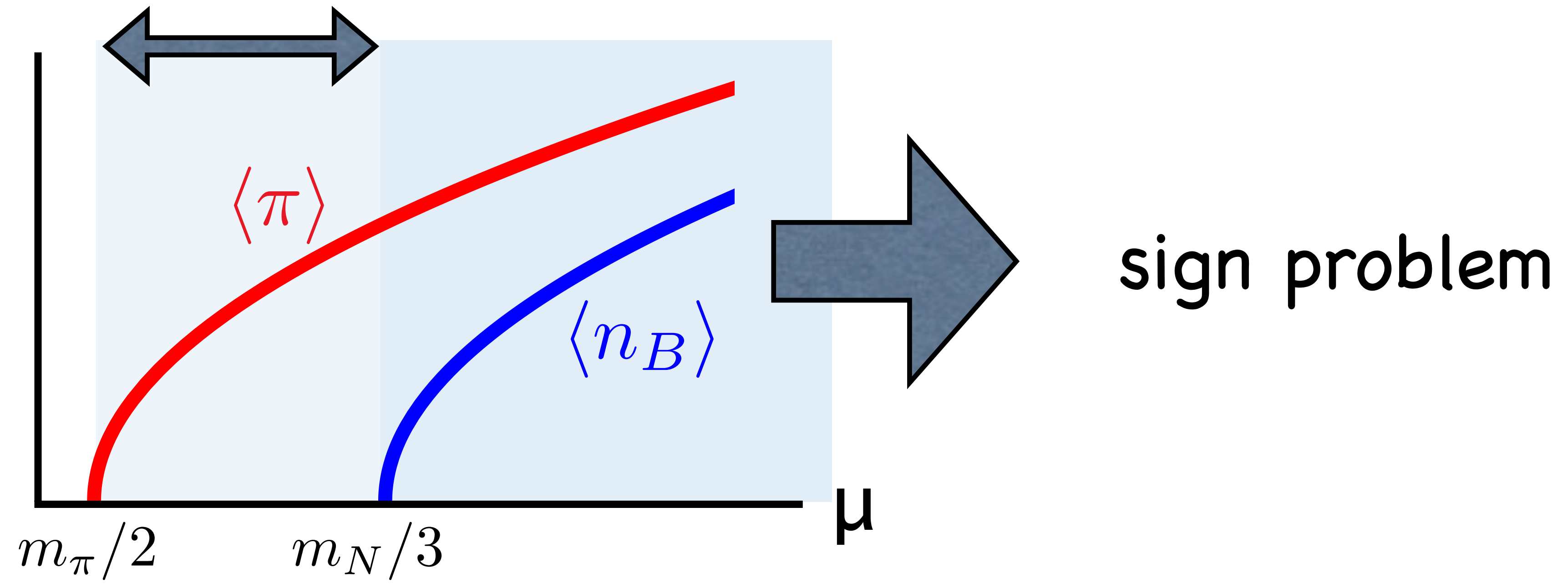
- From sign problem to noise
- Surprisingly universal features of noise
- Can these features be used to tame the noise?

The "sign" problem in the grand canonical approach:
Det($\not{D}+\mu\gamma^0$) complex

- physics happens for $\mu \geq m_N/3$...
- ...but sign problem starts at $\mu = m_\pi/2$!

P.E. Gibbs, 1986

$$\Delta\mu = \left(\frac{m_N}{3} - \frac{m_\pi}{2}\right)$$



Explanation (2-flavor QCD):

|Det($\not{D}+\mu\gamma^0$)| ≈ <u>isospin</u> chemical potential

K. Splittorff
J. Verbaarschot, 2006

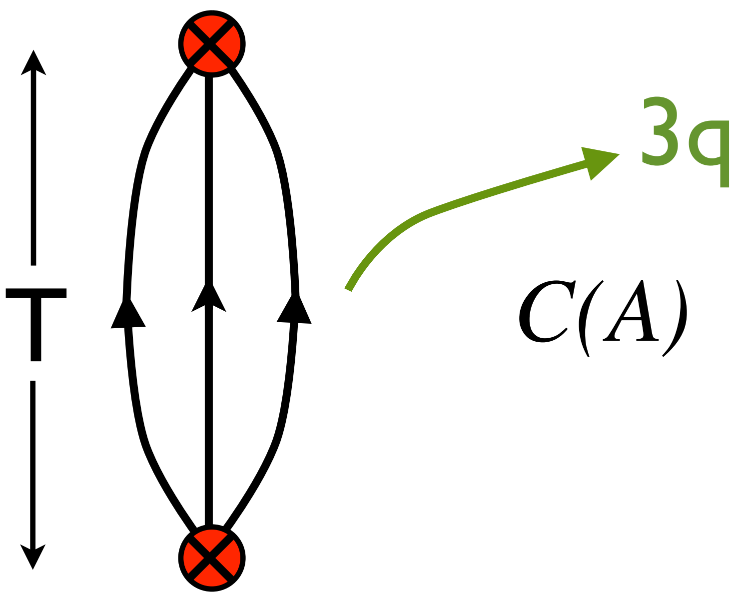
Role of phase: eliminate pion condensate for $\mu \geq m_\pi/2$!

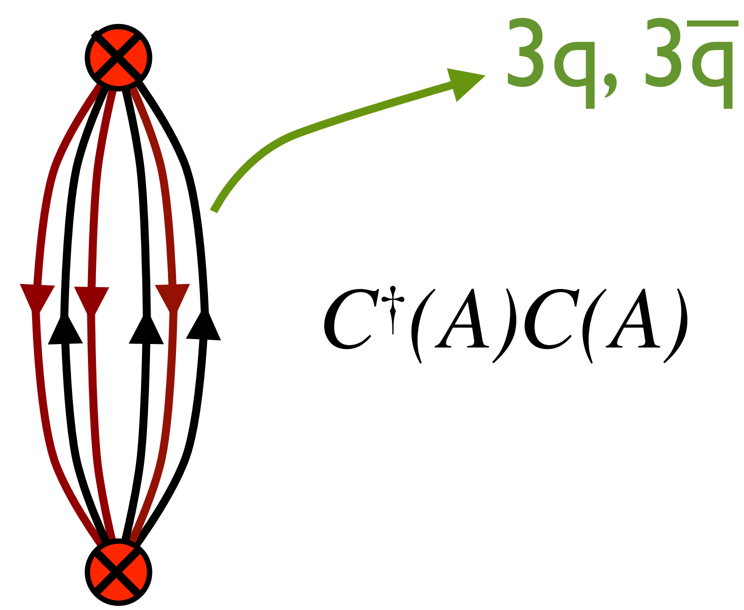
Canonical approach?

Compute correlator of N quarks with μ=0

No sign problem...but now a noise problem

Parisi, Lepage 1980's





nucleon correlator

signal: $\sim e^{-m_N T}$

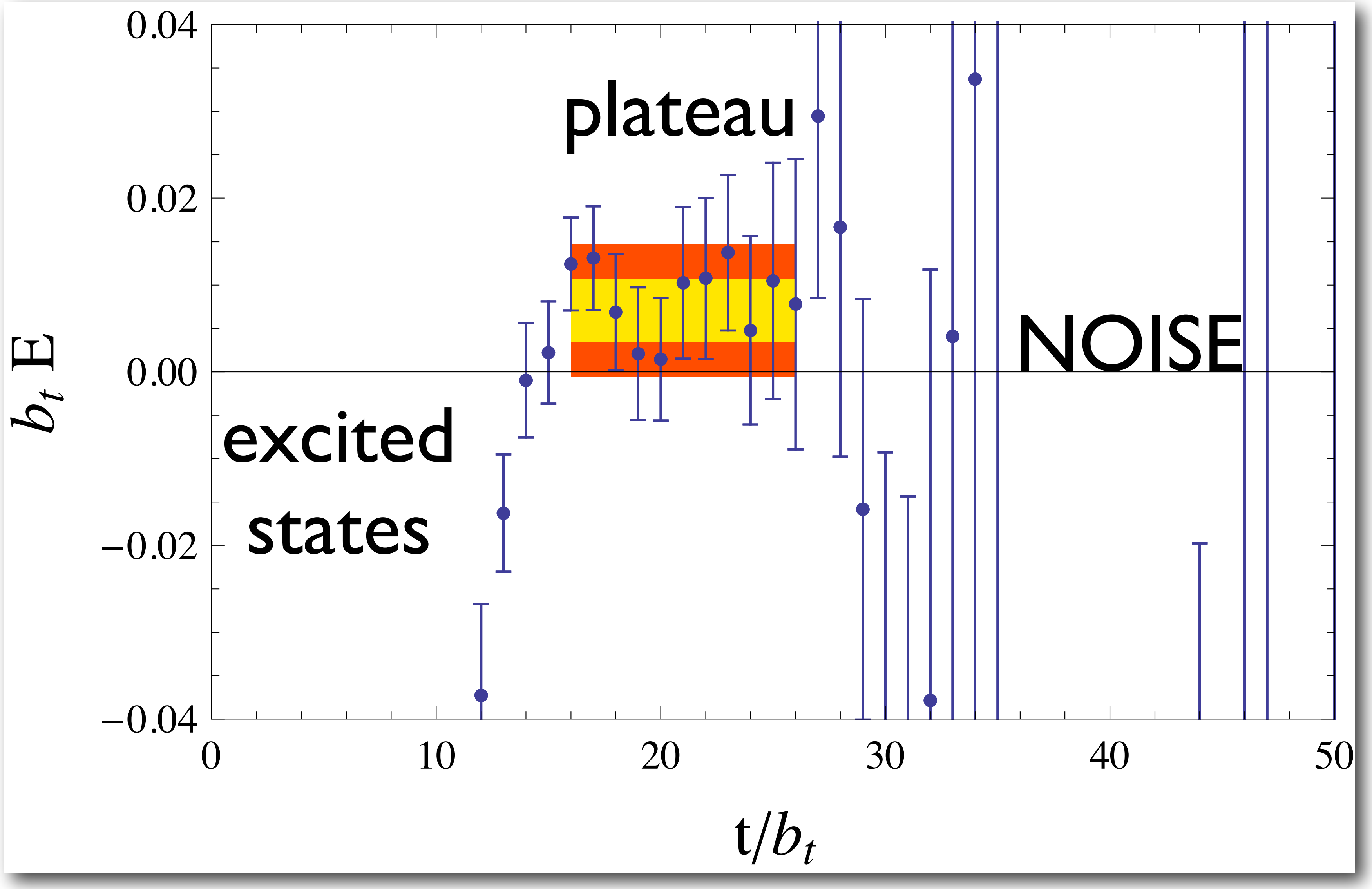
noise: $\sim \frac{1}{\sqrt{N_{\text{conf.}}}} e^{-\frac{3}{2} m_\pi T}$

$$\frac{\text{signal}}{\text{noise}} \sim \sqrt{N_{\text{conf.}}}\, e^{-3T\left(\frac{m_N}{3} - \frac{m_\pi}{2}\right)}$$

Same factor as grand canonical

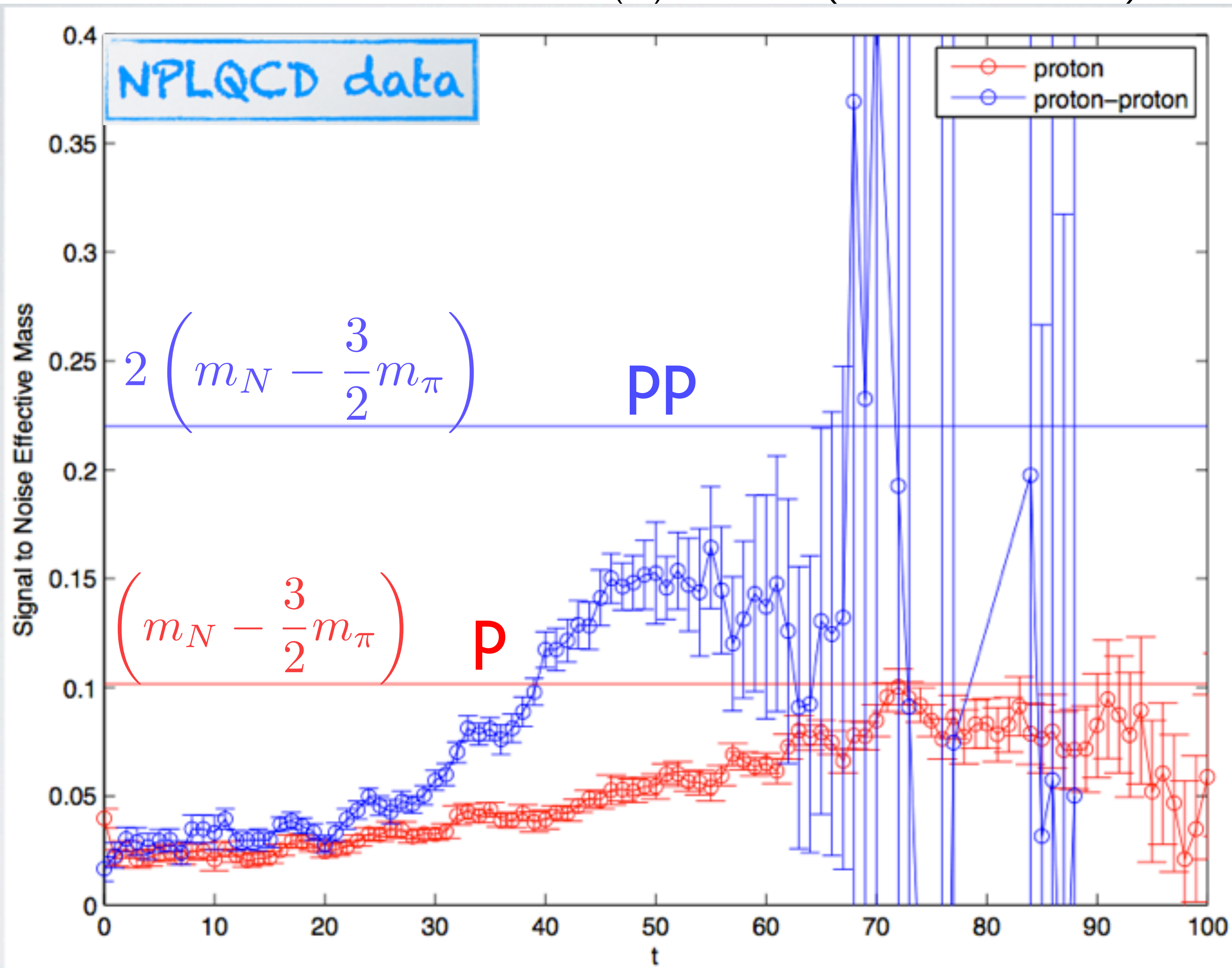
EXAMPLE:

Triton B.E. S. R. Beane et al. (NPLQCD), Phys. Rev. D 80, 074501 (2009) $(m_\pi = 390\ \mathrm{MeV})$



Actual QCD data

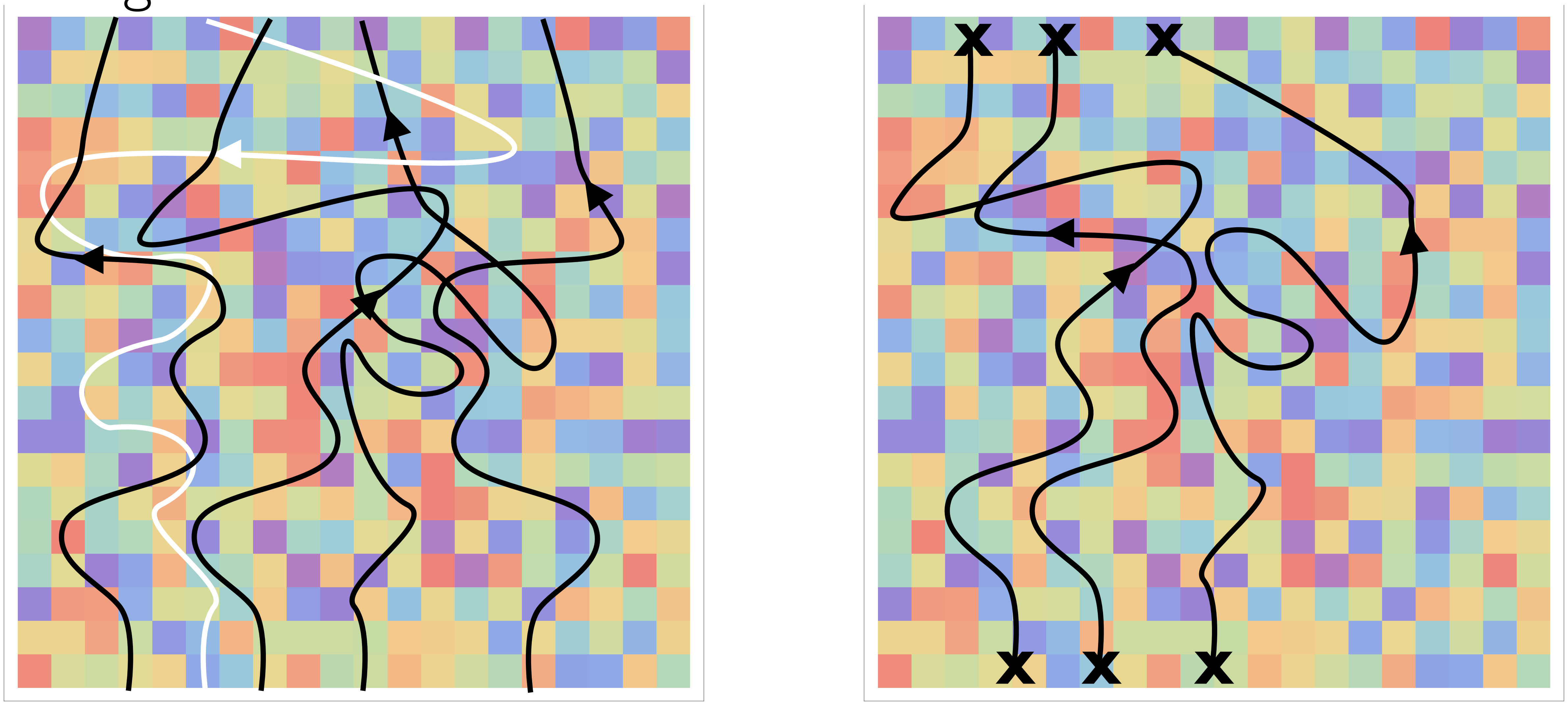
Plotted: $$-\frac{1}{t}\ln\frac{\sigma(t)}{\bar{x}(t)} \sim A\left(m_N - \frac{3}{2}m_\pi\right)$$



Conclusion:
Parisi/Lepage
= qualitative estimate of noise problem

Think like a quark in a single gauge configuration

grand canonical canonical



Am I going to be in a light pion? Or a heavy nucleon?

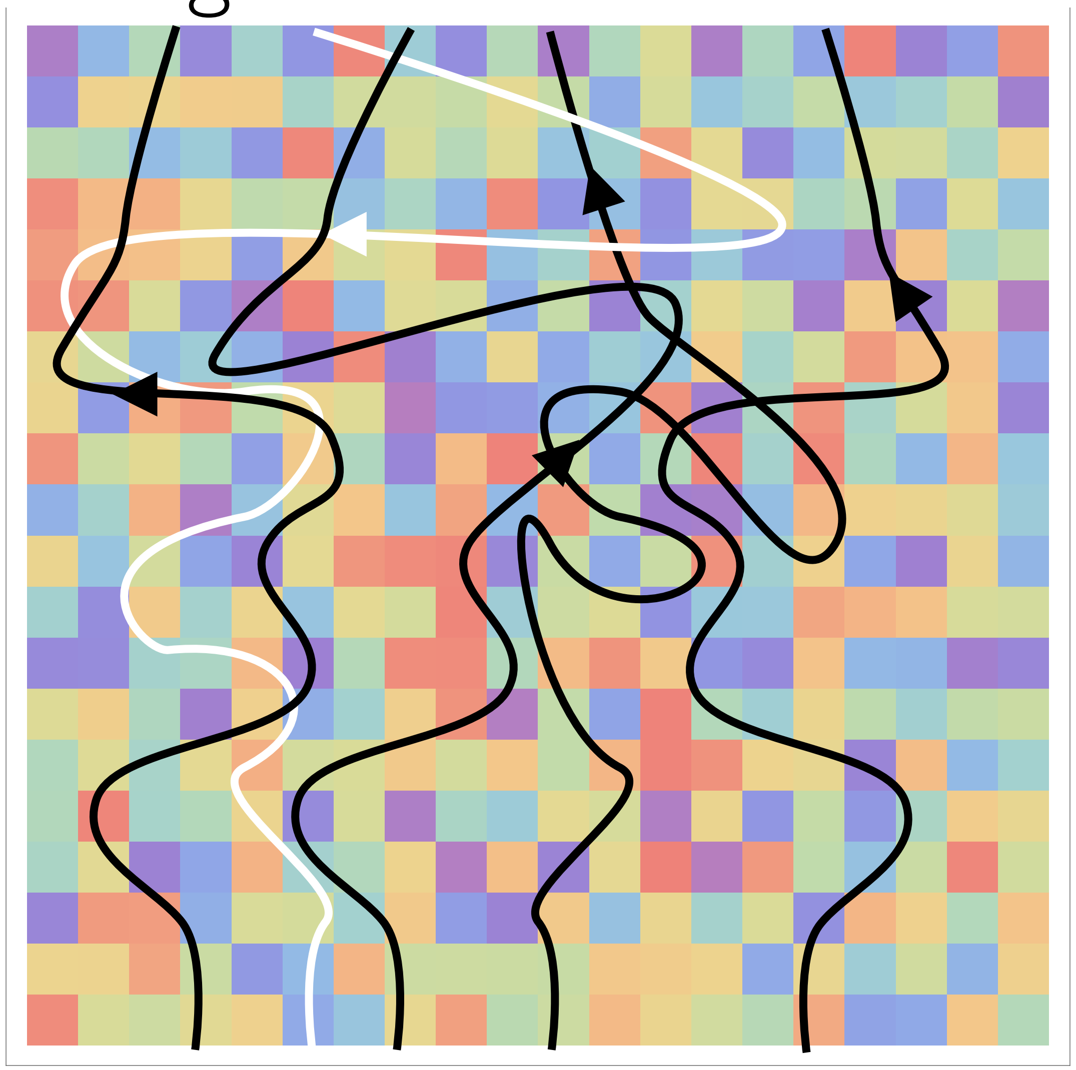
Don't know!

Play safe: *assume pion.* Propagator: $\sim e^{-(m_\pi/2)T}$

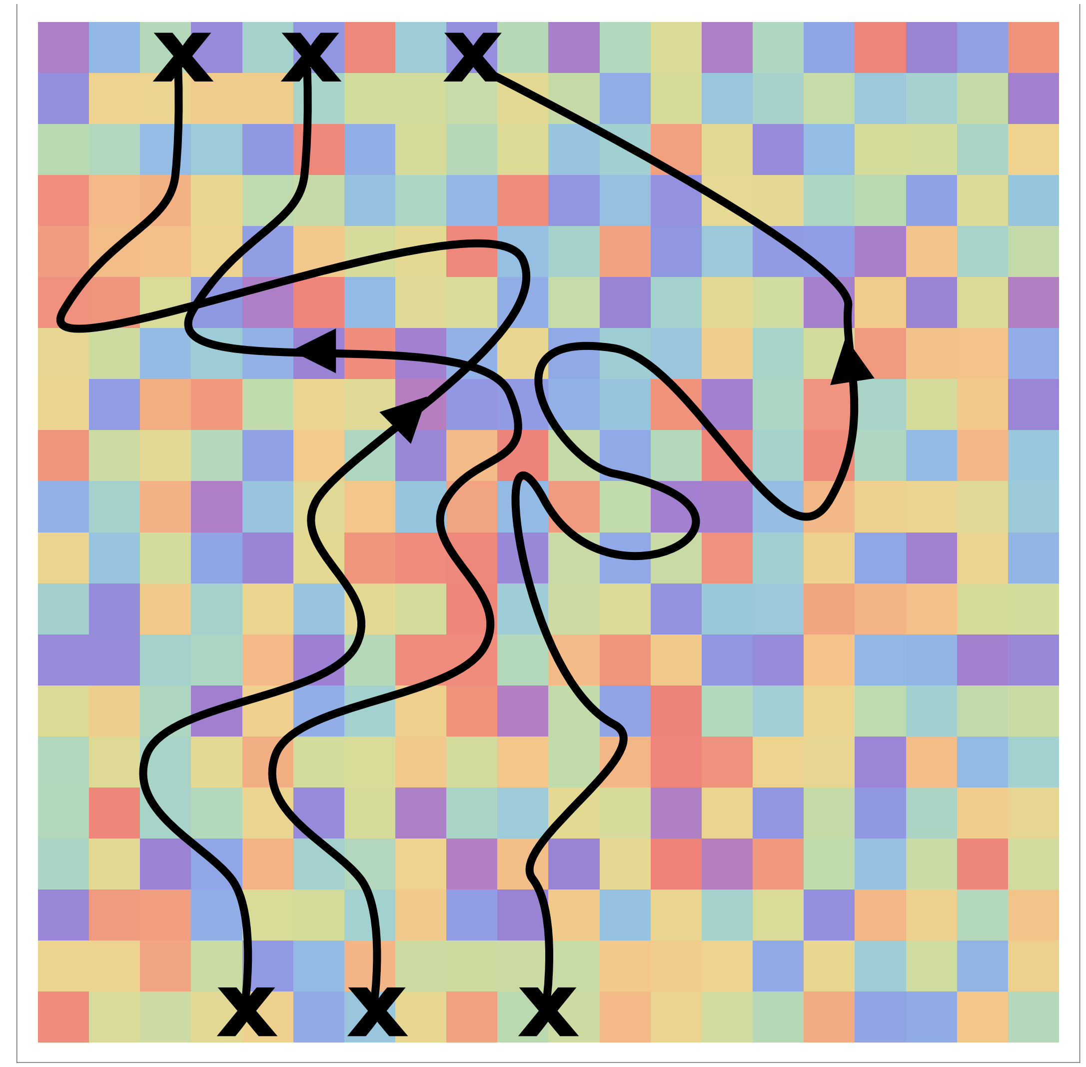
If nucleon, cancellations between configurations to: $\sim e^{-(m_N/3)T}$

Think like a quark in a single gauge configuration

grand canonical



canonical



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NOISE

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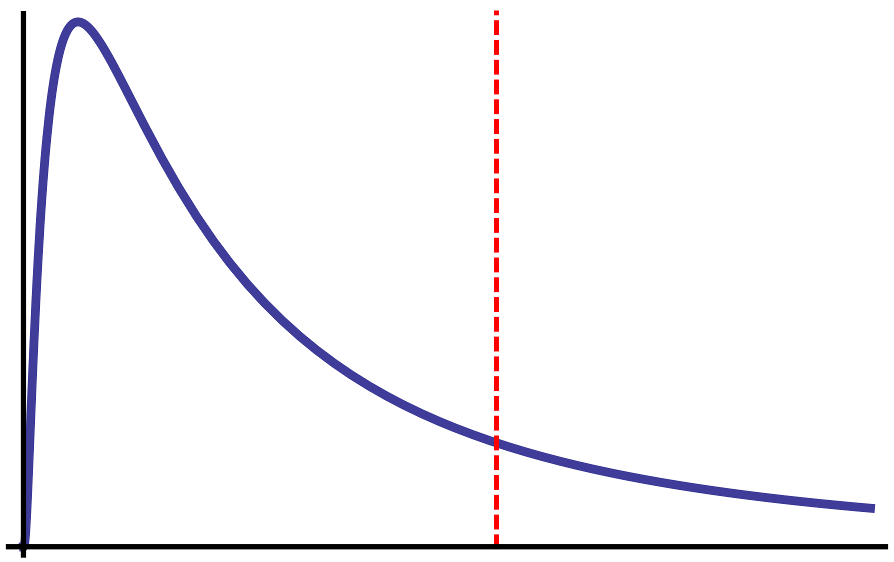

- *Long tail, small mean*
- *"overlap problem"*
  poor sampling

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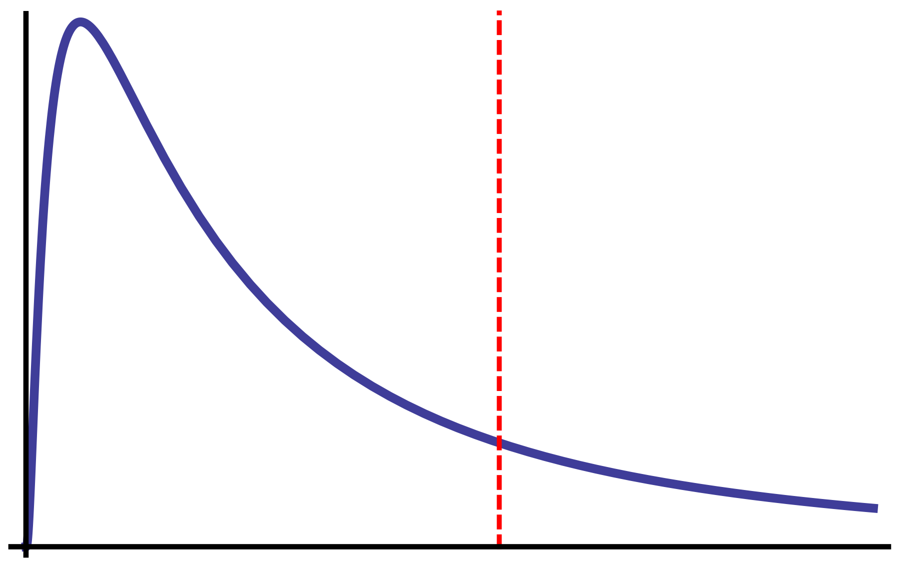

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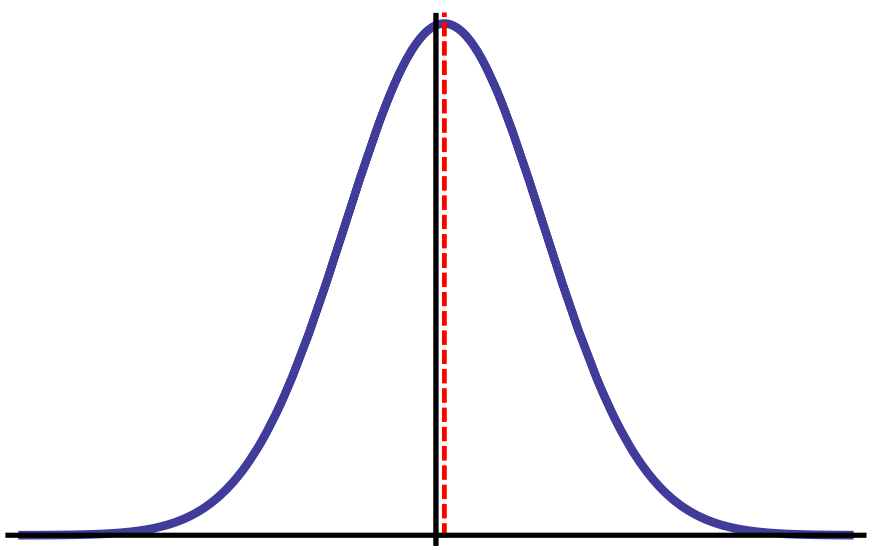
Possibility B:



- *Almost symmetric, small mean*
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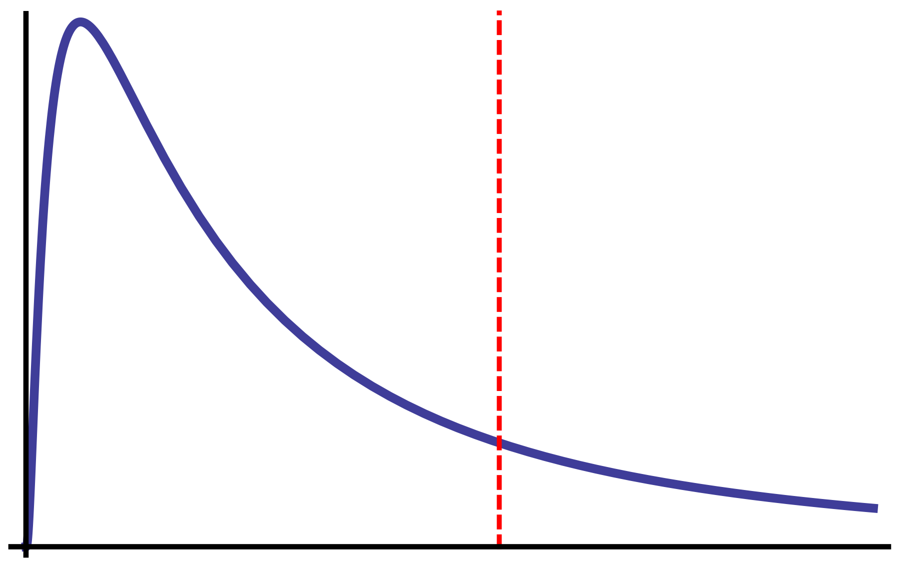

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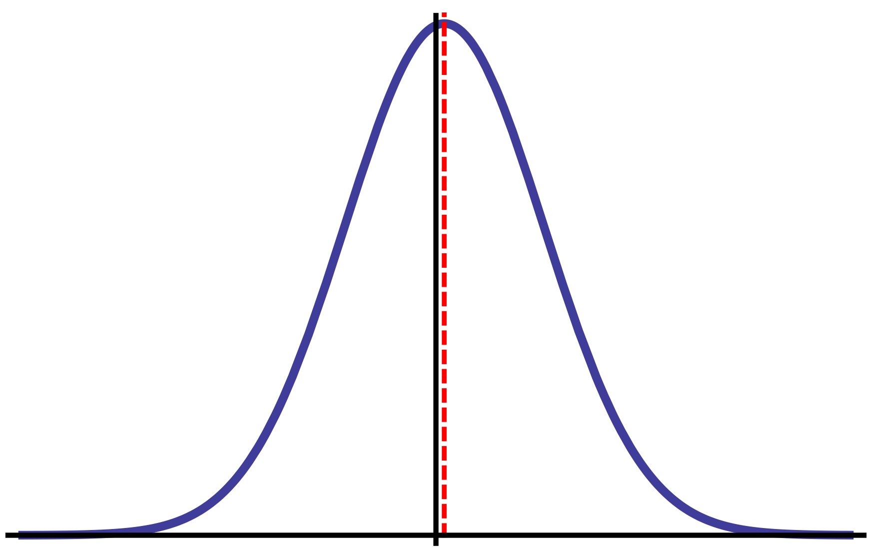

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Both possibilities could occur

Either could be related to a sign problem in grand canonical

Look at a simpler system: unitary fermions

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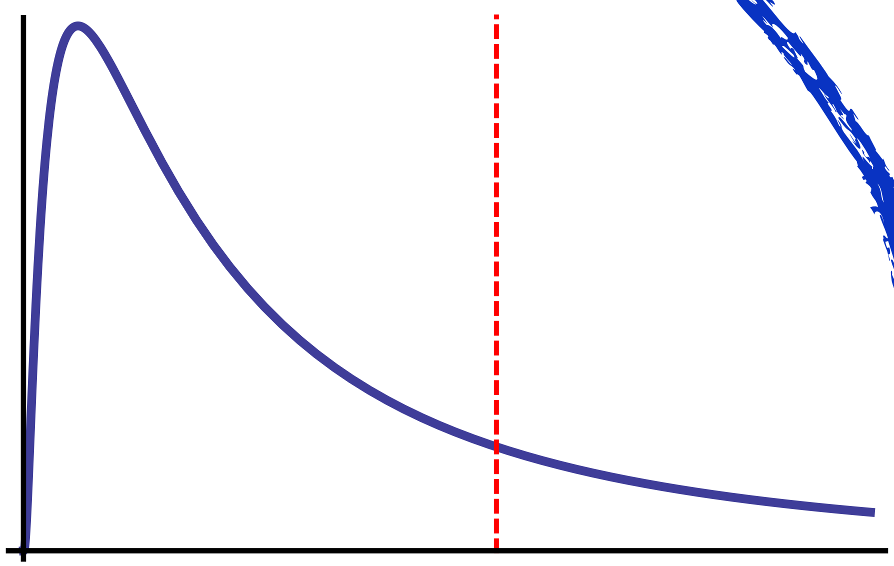

correlator

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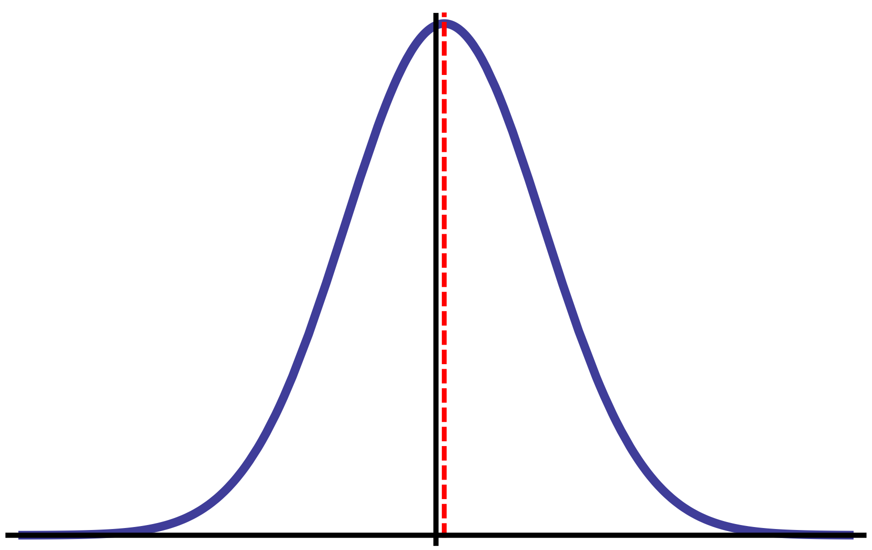

correlator

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Look at a simpler system: unitary fermions

Digression: what are unitary fermions?

(Lattice 2011 talks by J. Drut, J.-W. Lee)

Nonrelativistic 2-particle scattering:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

phase shift

"unitary" fermions: $p\cot\delta = 0$

$$\delta(p) = \frac{\pi}{2}$$

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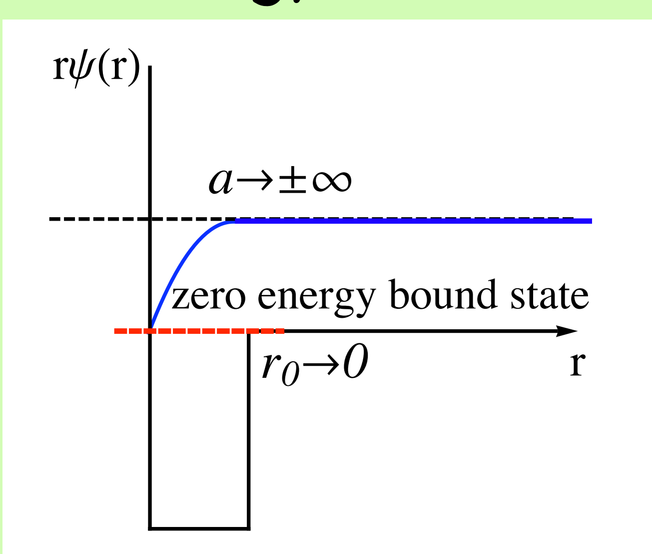
Zero-range potential
Zero-energy bound state



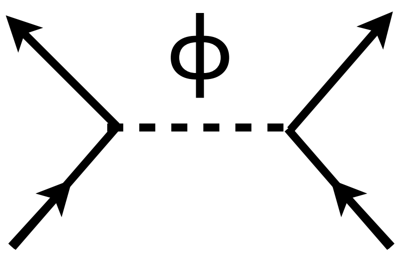
- A strongly-coupled conformal system
- Studied experimentally with dilute trapped atoms @ Feshbach resonance

  (JILA, MIT, Innsbruck)

- Exhibits superfluidity

Lattice model: M. Endres, D.K., J.W. Lee, A. Nicholson, arXiv:1106.5725 [hep-lat]

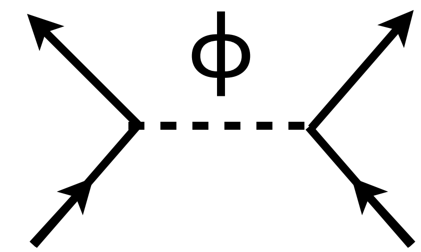
- Nonrelativistic fermions, μ=0
- Short-range momentum-dependent 4-fermion interaction induced by auxiliary scalar field
- Interaction tuned to conformal fixed pt.



- Less severe sign problem than QCD; no gauge symmetry; nonrelativistic (quenched)
- Have simulated up to N=70 fermions on 14^3 x 64 lattice
- ~1% accuracy in energies
- up to 2 billion configurations for auxiliary field

Simulation:

ϕ lives on time links
(couples to $\psi^*\psi$)

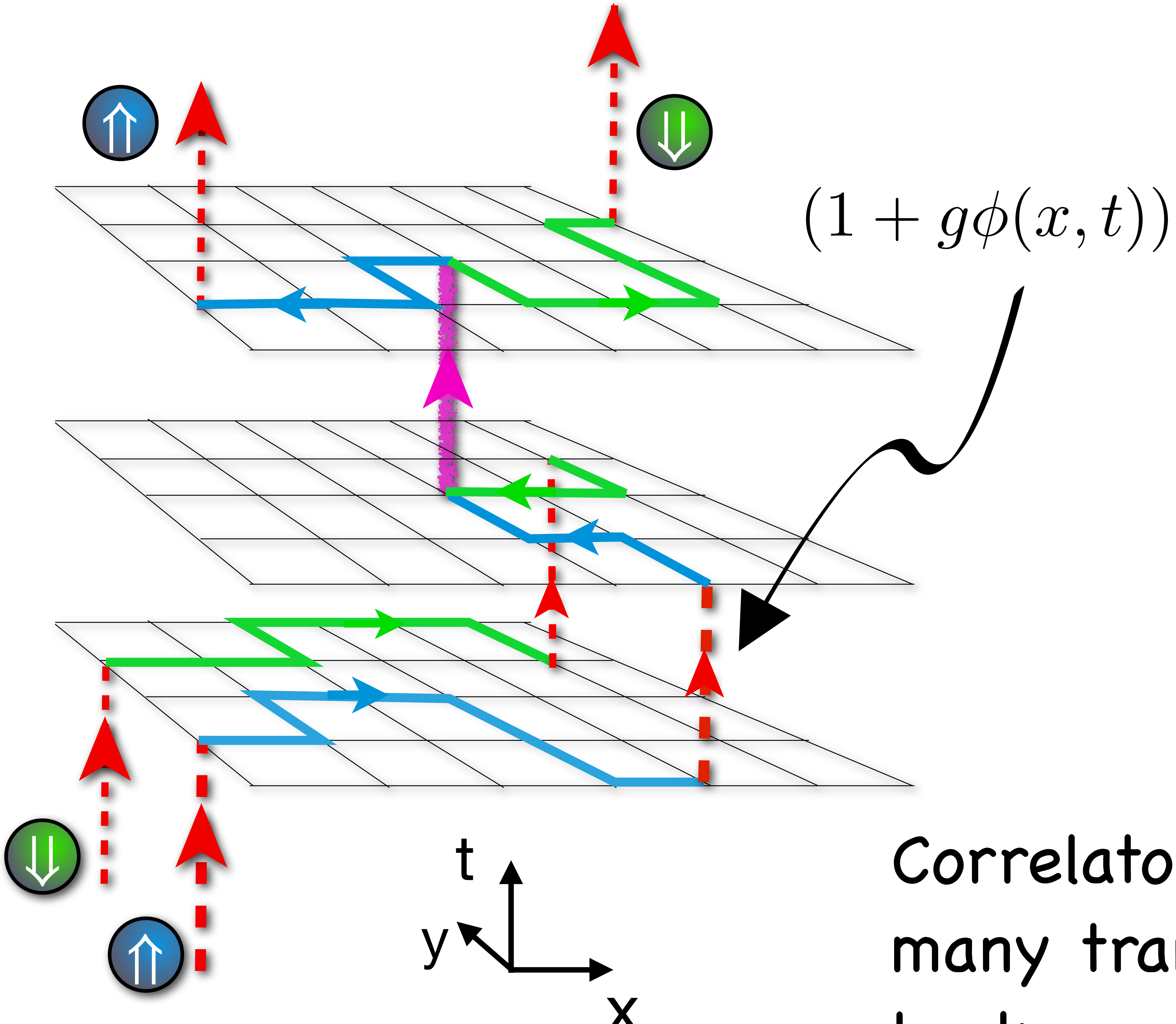


Generate an ensemble of random ϕ fields, compute average of an N-particle correlator $C_N(T;\phi)$ from t=0 to t=T

Extract ground state energy:

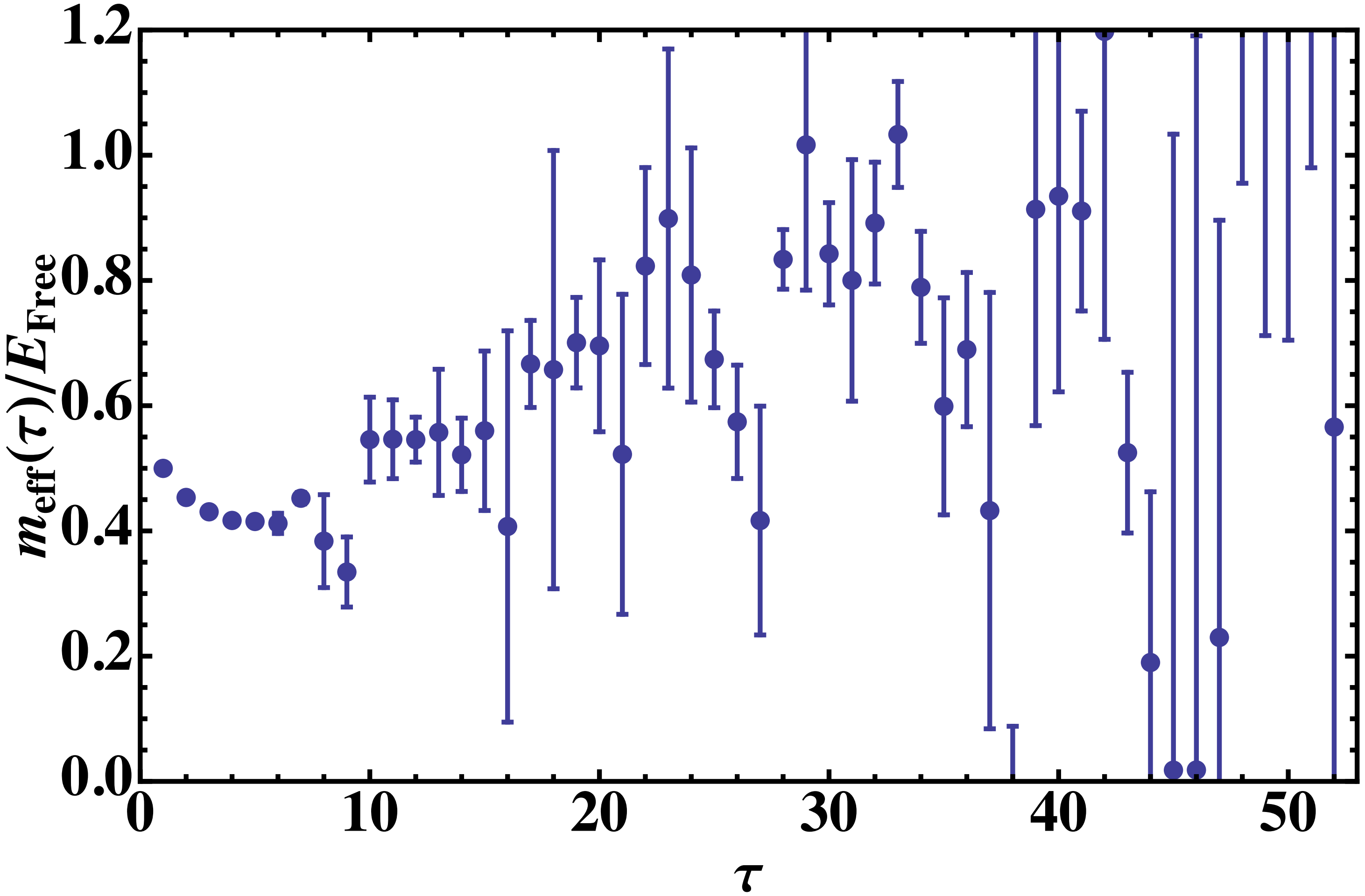
$$E_N = \lim_{T\to\infty}\left[-\frac{1}{T}\ln\langle C_N(T,\phi)\rangle_\phi\right]$$

Plot $-\frac{1}{T}\ln\langle C_N(T,\phi)\rangle_\phi$ vs T, look for "plateau"



Correlators are products of many transfer matrices in background random Φ

Example of conventional effective mass plot



N= 46 fermions
L=12
40 M configs

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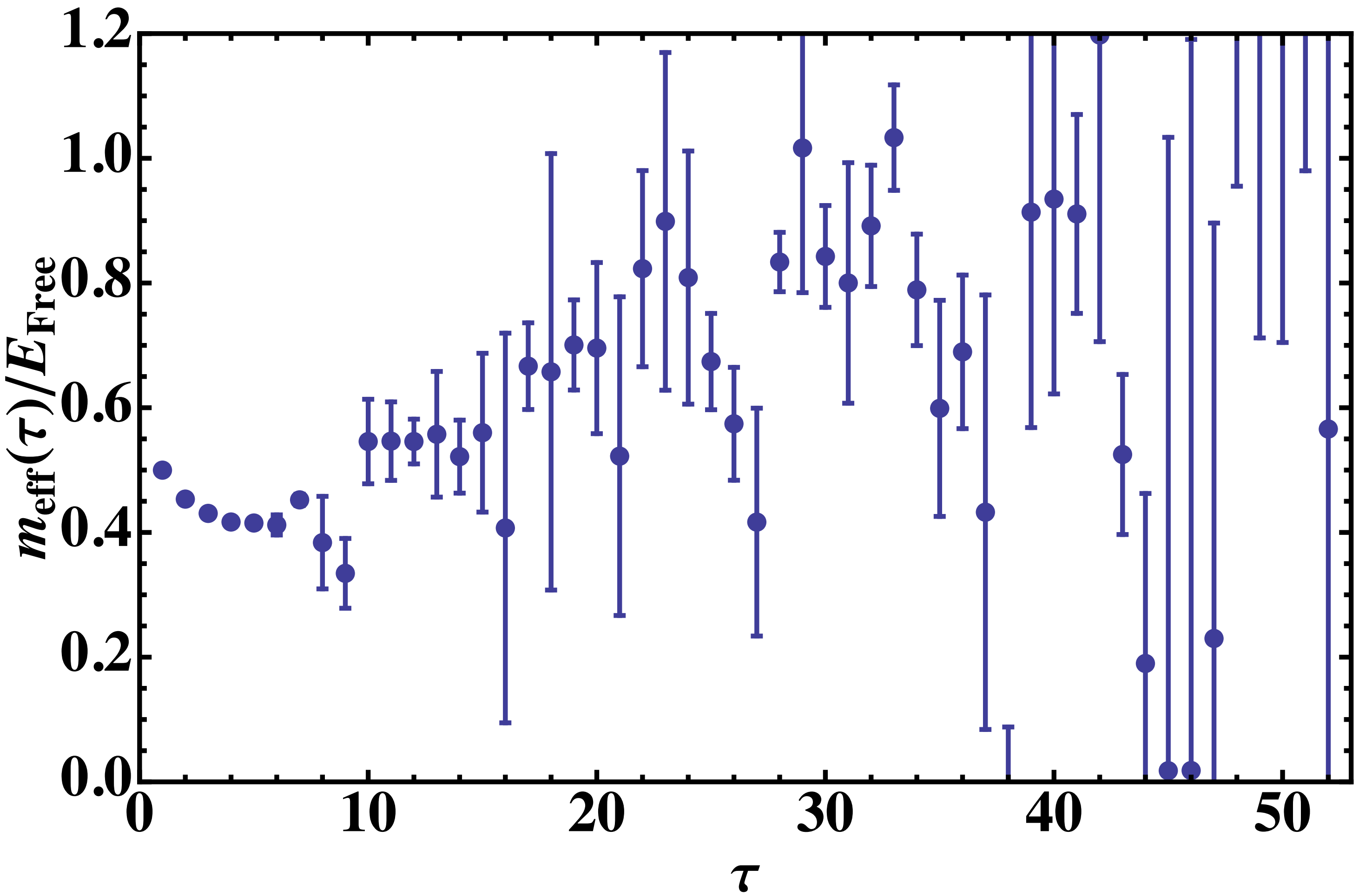

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- noise
- *drift*
- worse for larger N= # fermions

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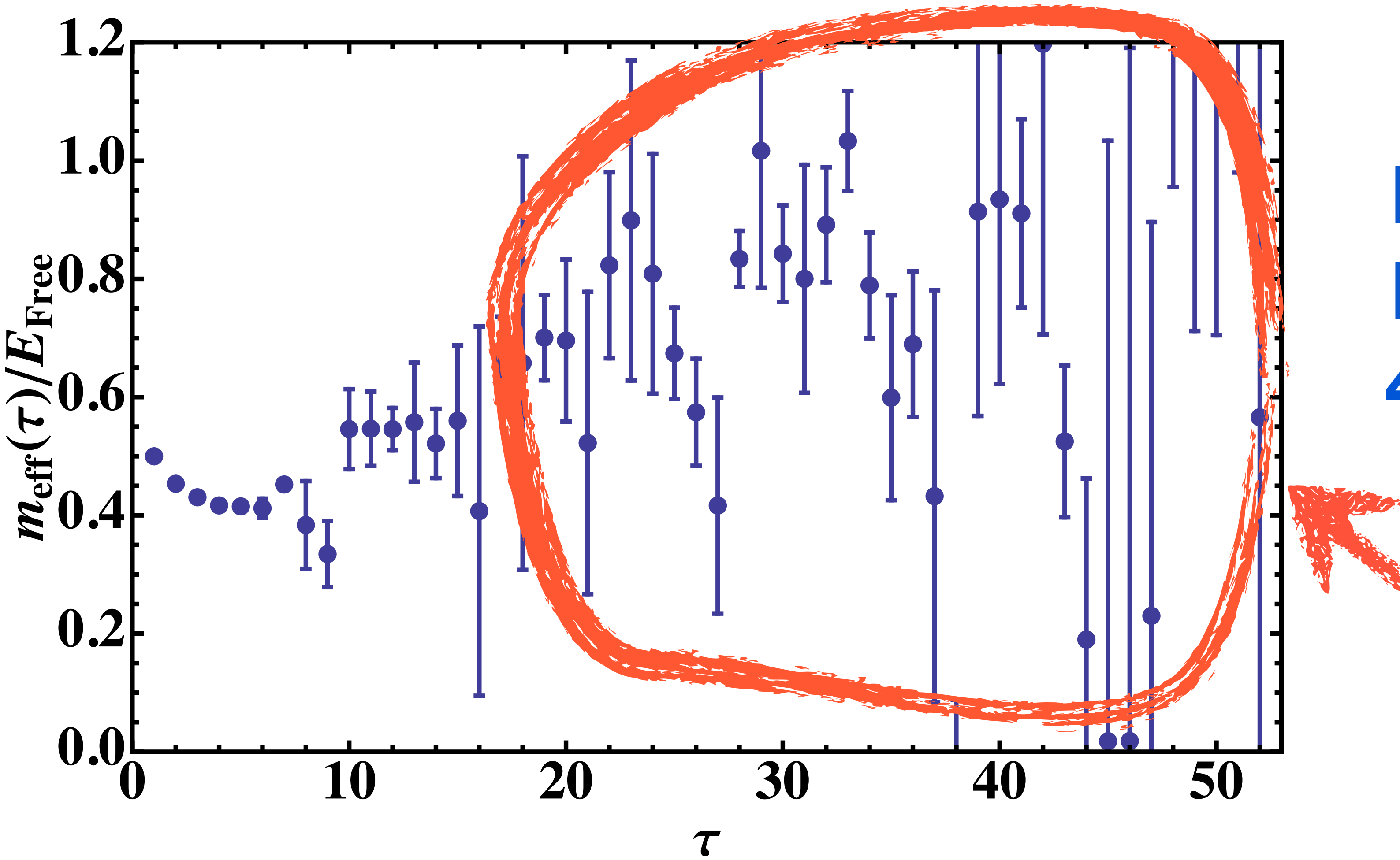

N= 46 fermions
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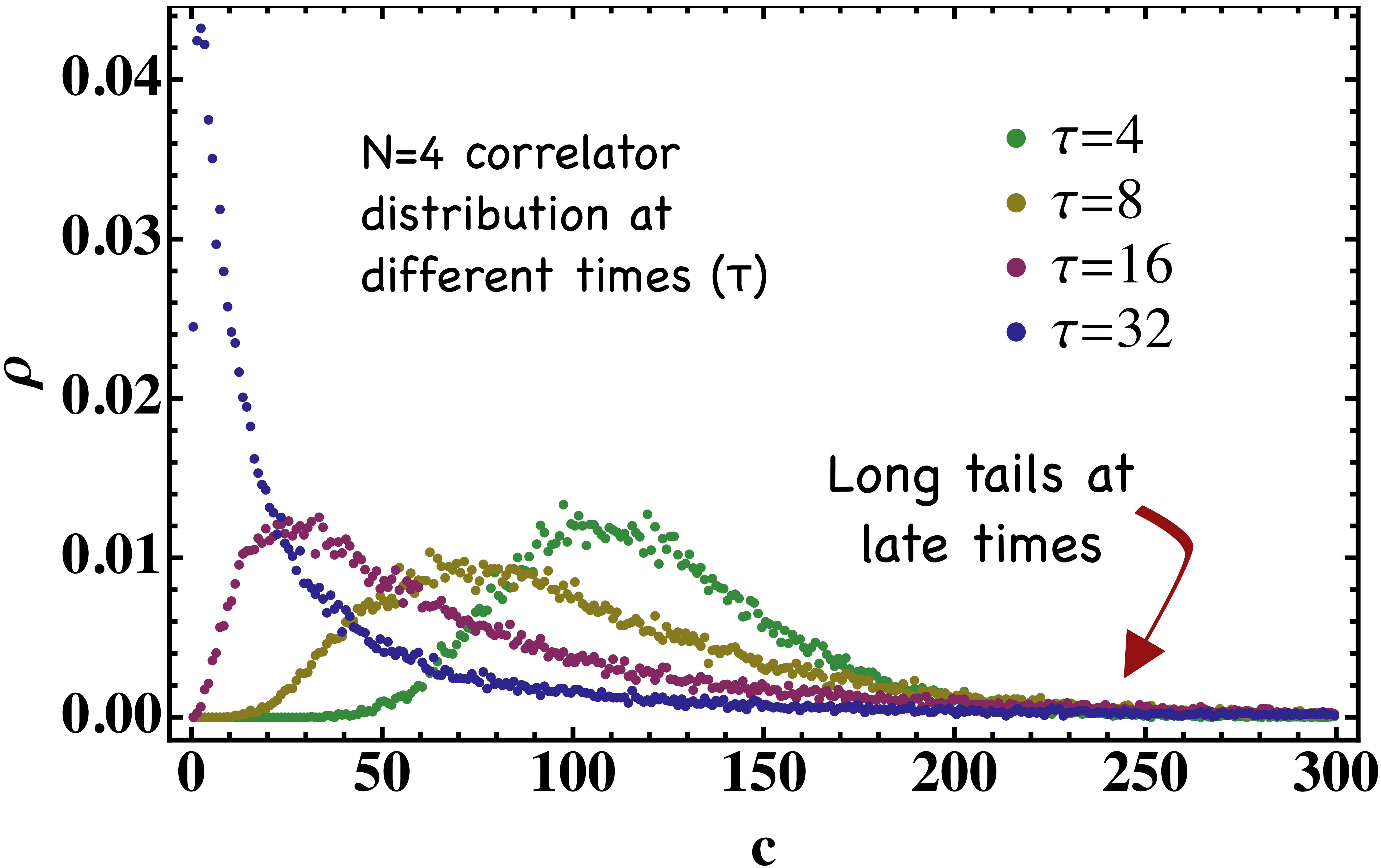
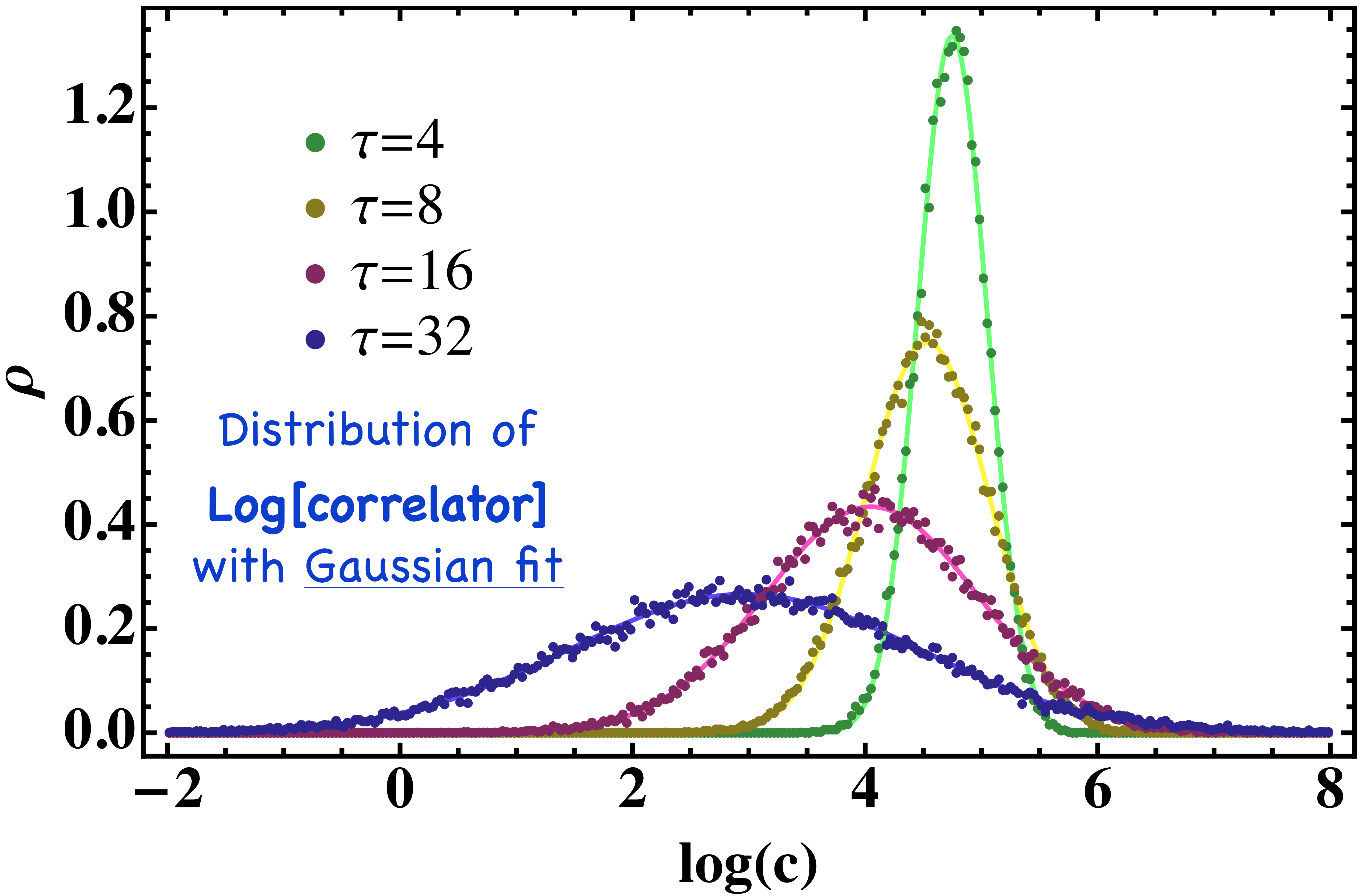
Is there any information here?

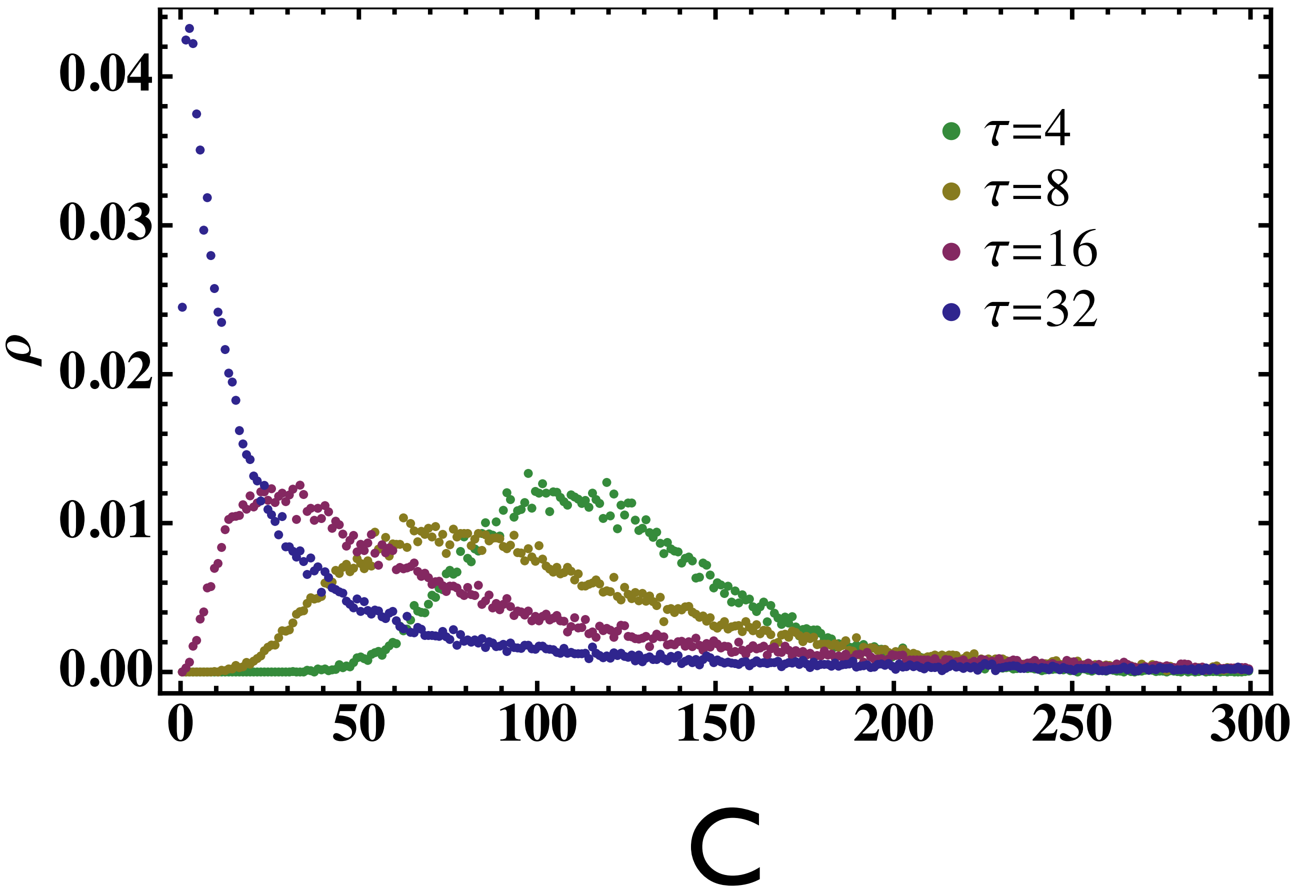
- **noise**
- *drift*
- worse for larger N= # fermions

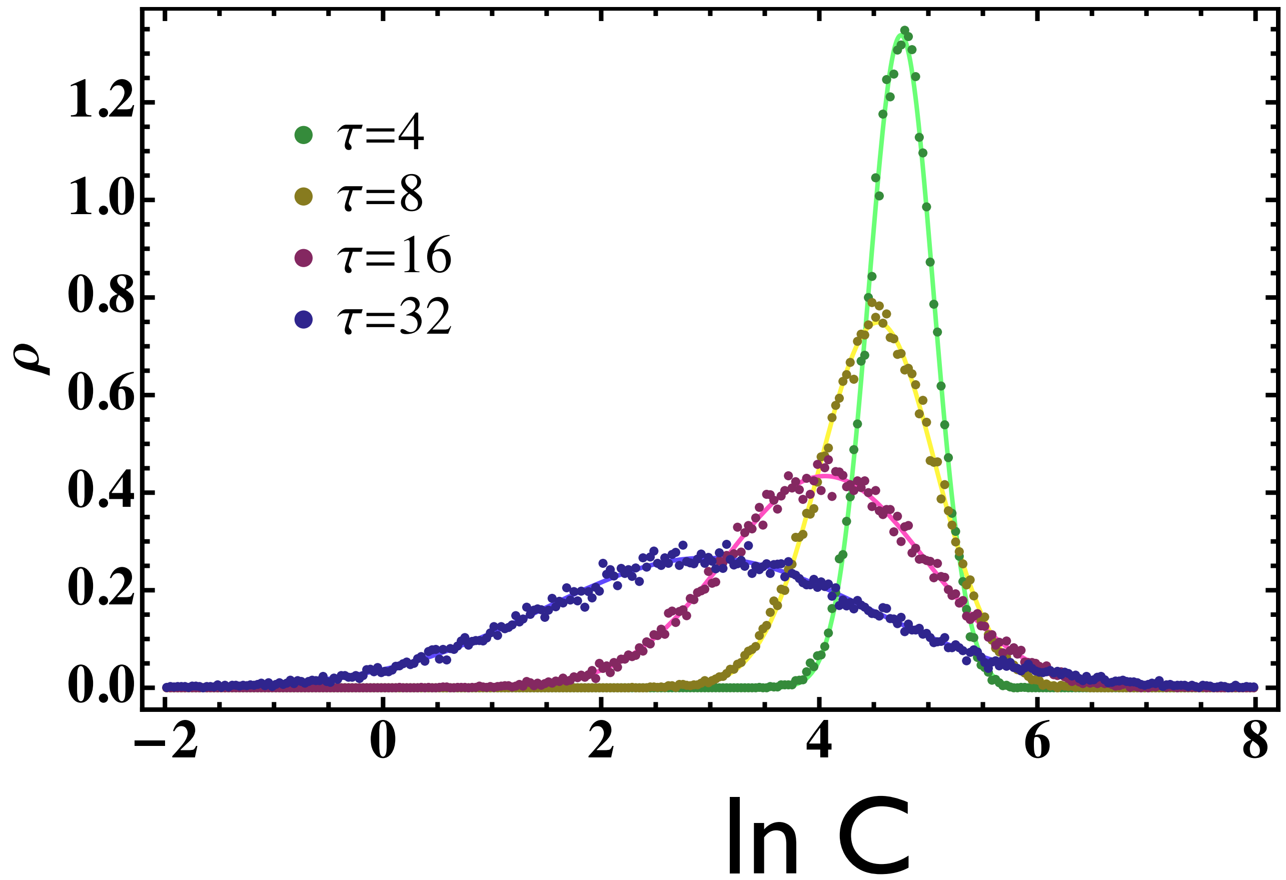
Look at raw correlator probability distributions:

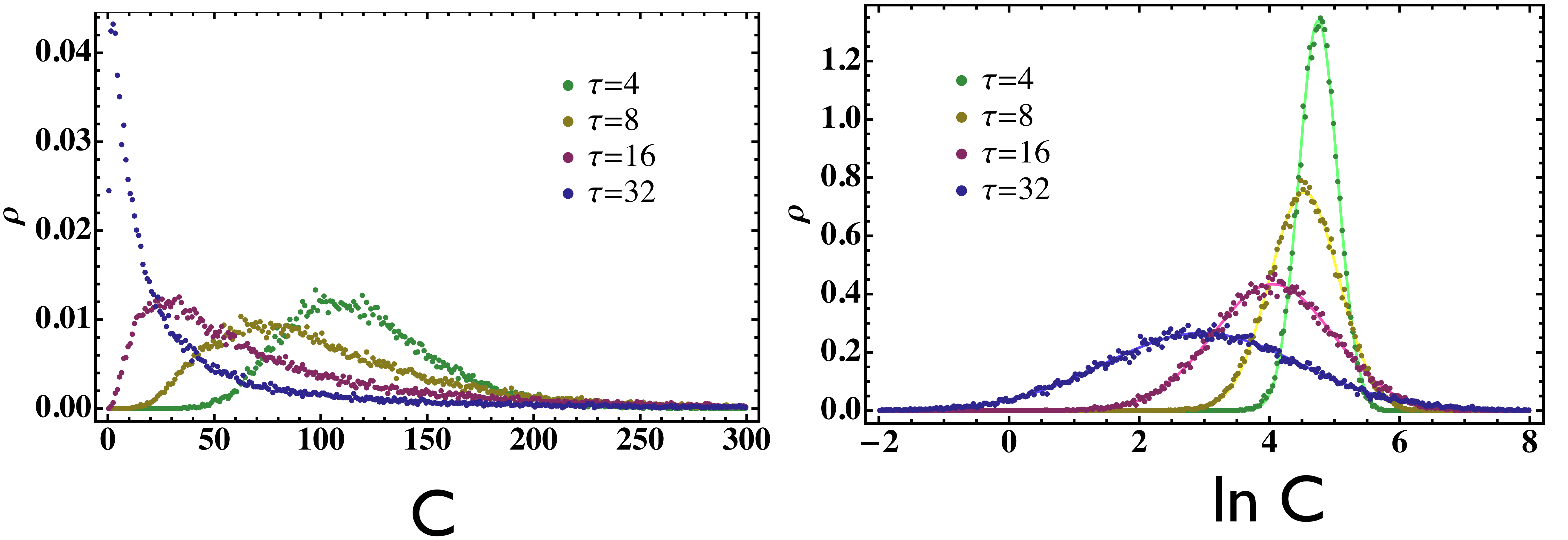


...but look at distribution for LOG of correlator:




0.04
0.03
0.02
0.01
0.00
ρ
τ=4
τ=8
τ=16
τ=32
0
50
100
150
200
250
300
C


1.2
1.0
0.8
0.6
0.4
0.2
0.0
ρ
τ=4
τ=8
τ=16
τ=32
−2
0
2
4
6
8
ln C



- Correlators seem to flow toward a log-normal distribution (which is described by only two parameters)
- Noise and drift in measurement due to problems sampling long tail for computing <C>
- "Universal" description, in RG sense?

Digression: statistics & the RG

Digression: statistics & the RG

Probability distribution:

$P(x)$

Digression: statistics & the RG

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$$P(x)$$

Moment generating function:

$$\phi(t) = \langle e^{-tx} \rangle = 1 - t\langle x \rangle + \frac{t^2}{2}\langle x^2 \rangle + \ldots$$

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Cumulant generating function:

$$\begin{aligned} -\ln \phi(t) &= t\langle x \rangle + \frac{t^2}{2}\left(\langle x^2 \rangle - \langle x \rangle^2\right) + \ldots \\ &= \sum_{n=1}^{\infty} \frac{t^n}{n!} \kappa_n \end{aligned}$$

Digression: statistics & the RG

Analogue with path integral

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Like partition function Z[J]

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Like effective action W[J]

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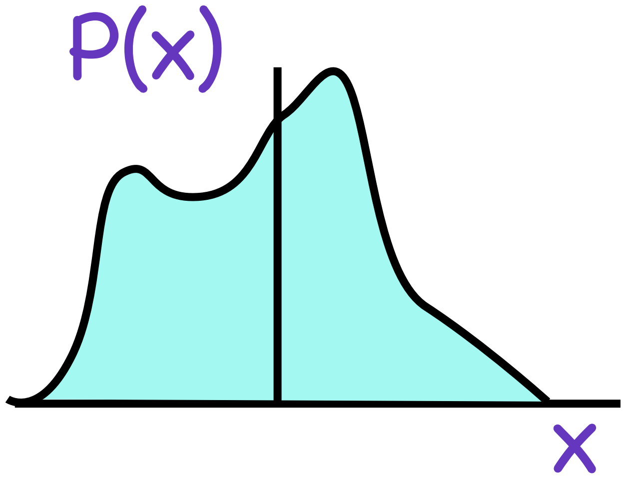
Like effective action W[J]

n^{th} cumulant - like *n*-pt. operators in effective action, increasing dimension

The Central Limit Theorem as RG flow

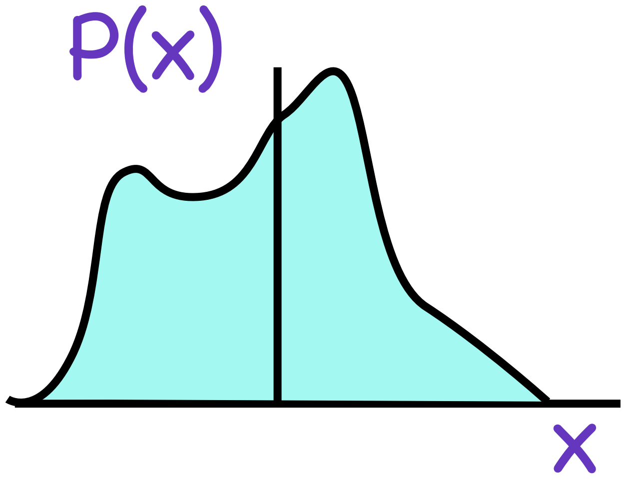
P(x) = some probability distribution with zero mean, unit variance.



Characterize by cumulants:

$$P(0, 1, \kappa_3, \kappa_4, \ldots; x)$$

The Central Limit Theorem as RG flow



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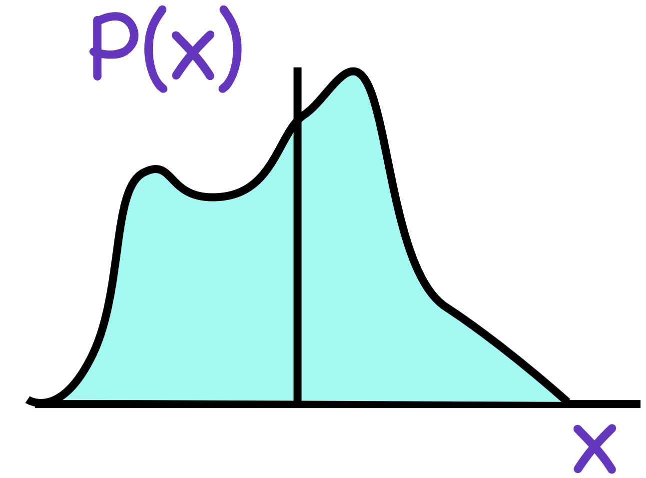
Average pairwise:
(and rescale)

$$y_1 = \frac{x_1 + x_2}{\sqrt{2}}, \quad y_2 = \frac{x_3 + x_4}{\sqrt{2}}, \ldots$$

The Central Limit Theorem as RG flow



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$$\kappa_n \to 2^{(1-n/2)} \kappa_n$$

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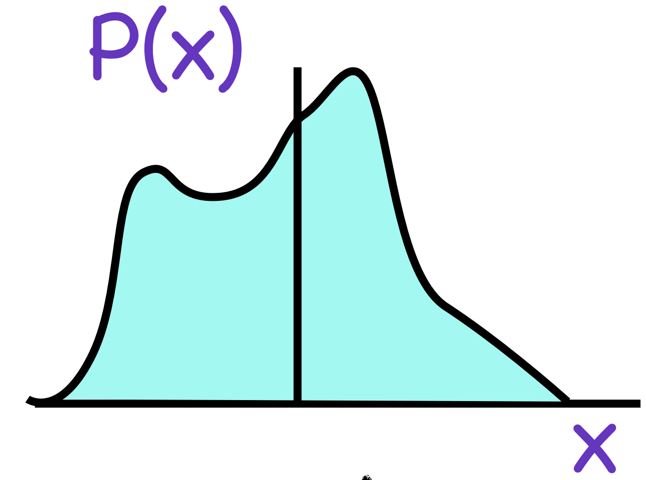

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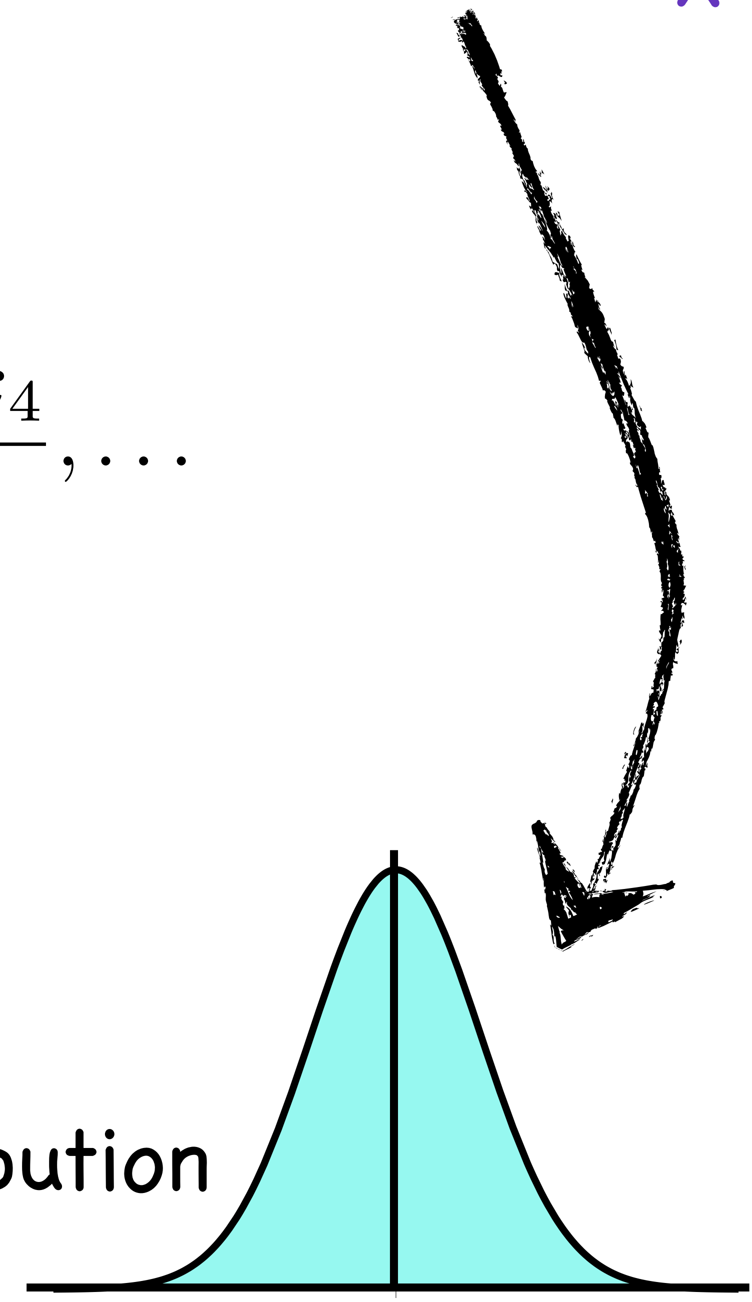
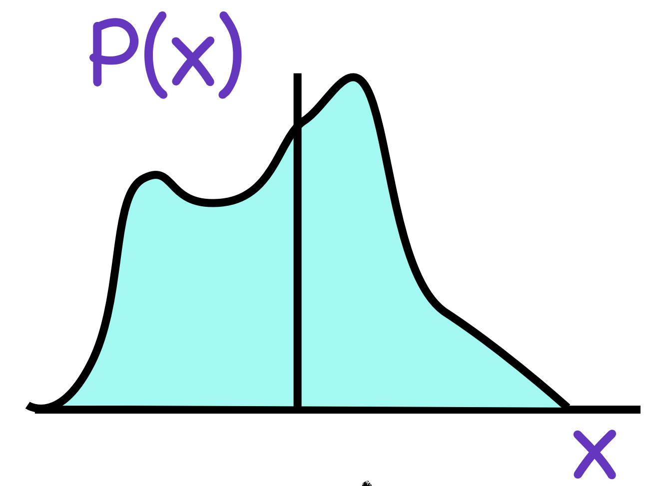
Repeat: $P \Rightarrow P(0,1,0,0,\ldots;x)$

… P flows to normal distribution



The Central Limit Theorem as RG flow



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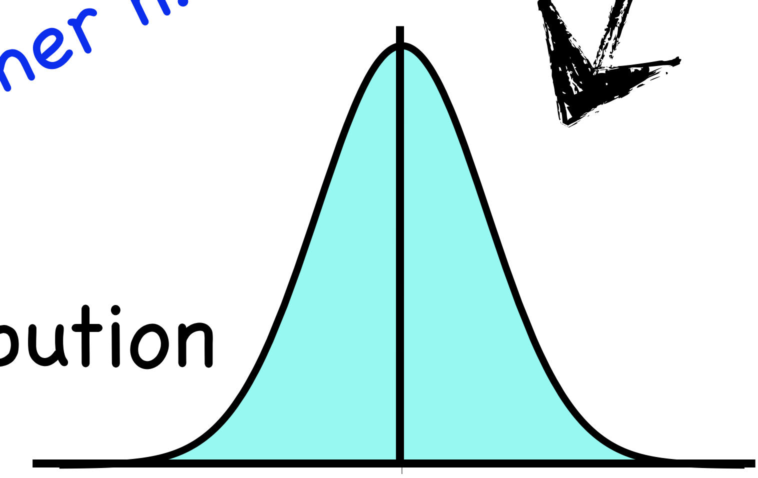
$$\kappa_n \to 2^{(1-n/2)} \kappa_n$$

mean is relevant
variance is marginal
higher n: irrelevant

Repeat: $P \Rightarrow P(0, 1, 0, 0, \ldots; x)$

... P flows to normal distribution



In our case: correlator $C(\Phi)$ driven toward log-normal distribution (BEFORE AVERAGING OVER Φ)

⇒ $\log[C(\Phi)]$ driven toward normal distribution

If cumulants κ_n of $\log[C(\Phi)]$ behave as irrelevant operators, is there the equivalent of an effective field theory approach?

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⇒ $\log[C(\Phi)]$ driven toward normal distribution

If cumulants κ_n of $\log[C(\Phi)]$ behave as irrelevant operators, is there the equivalent of an effective field theory approach?

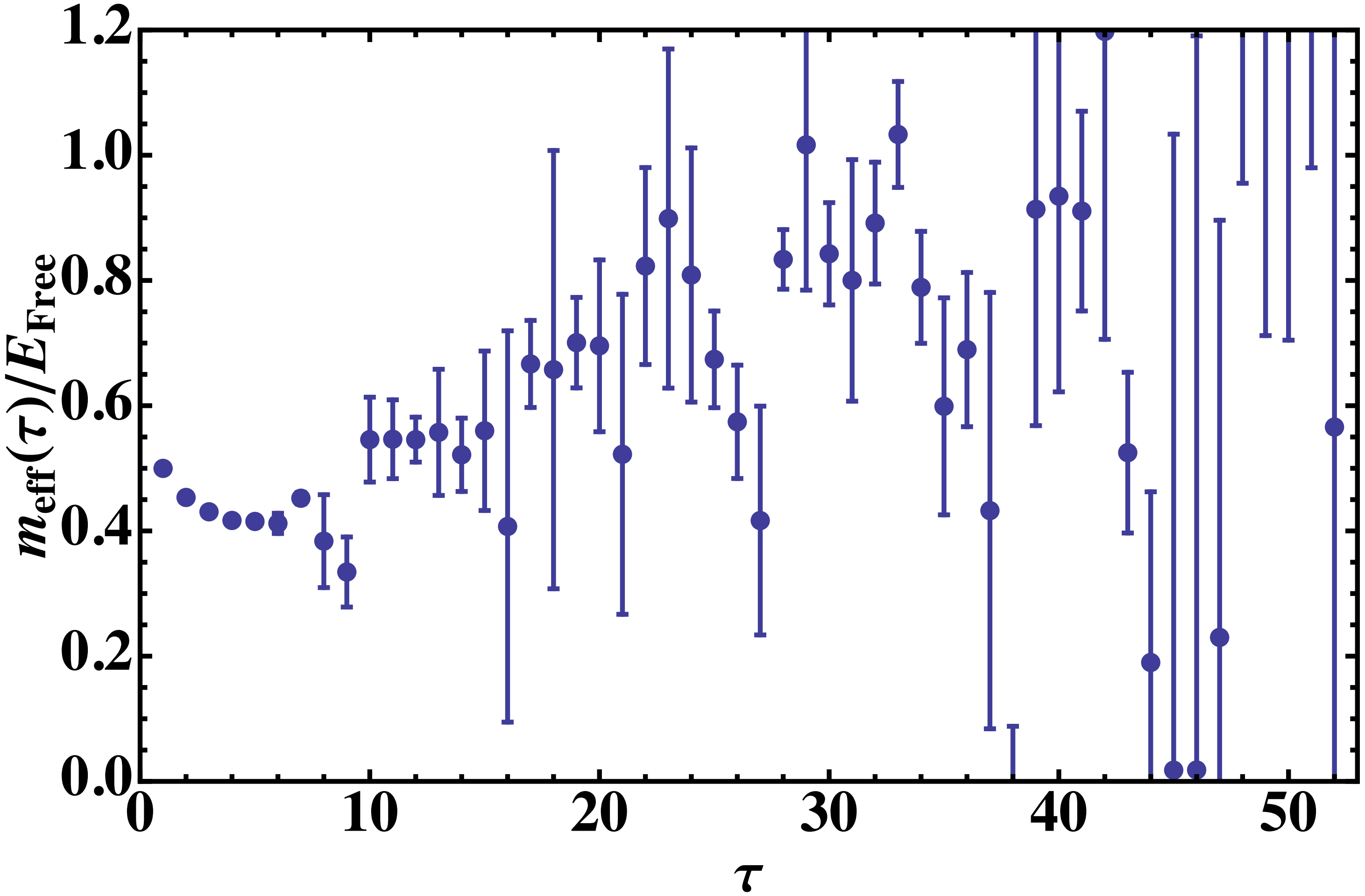
YES, truncate exact relation: $$\ln\langle C\rangle = \sum_{n=1}^{\infty}\frac{\kappa_n}{n!}$$

cumulants of lnC

κ_n are computed from finite sample.

Back to real data for N=46 unitary fermions (40 M configs.)

Conventional effective mass plot already shown:



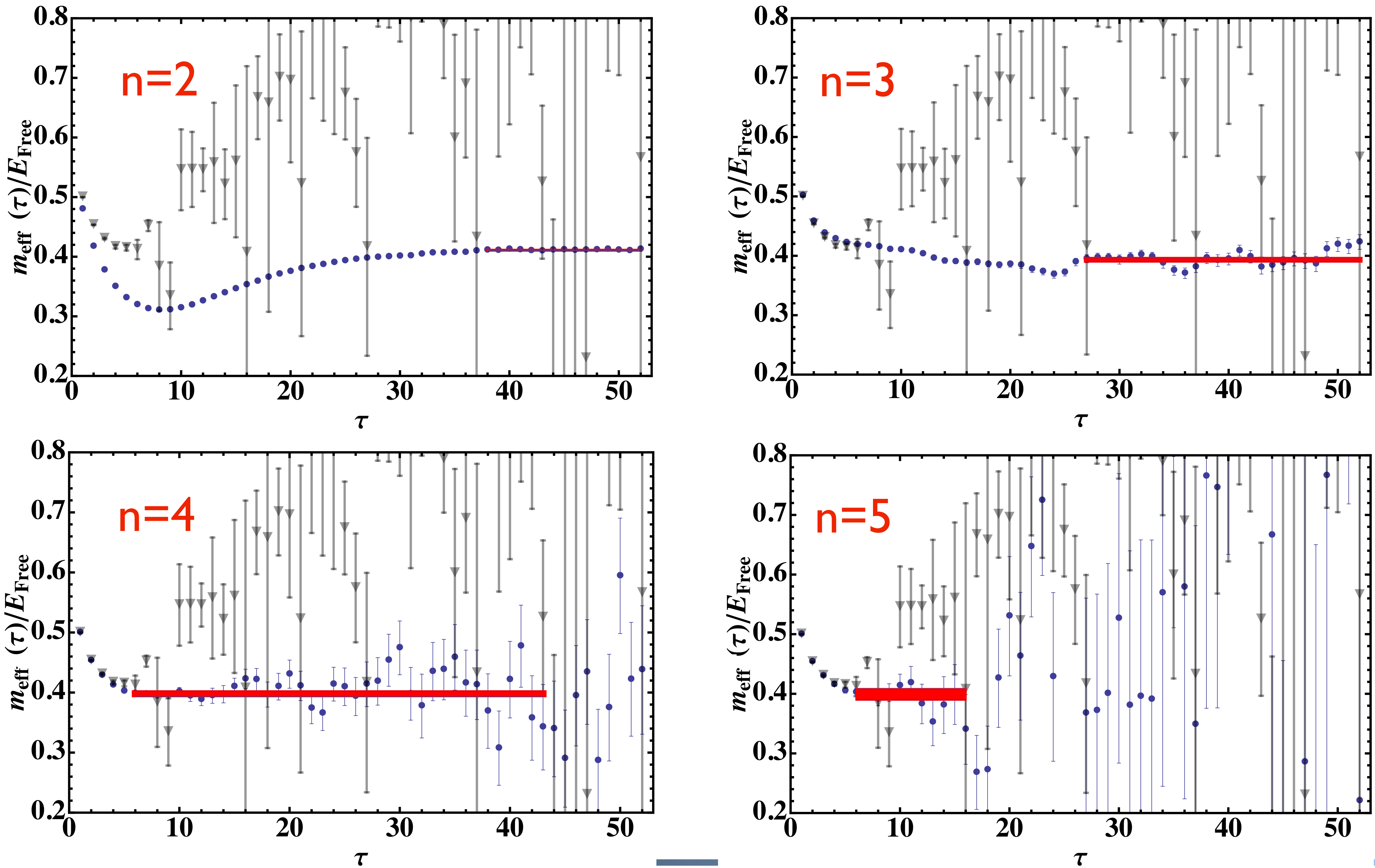
N= 46 fermions
L=12

40 M configs

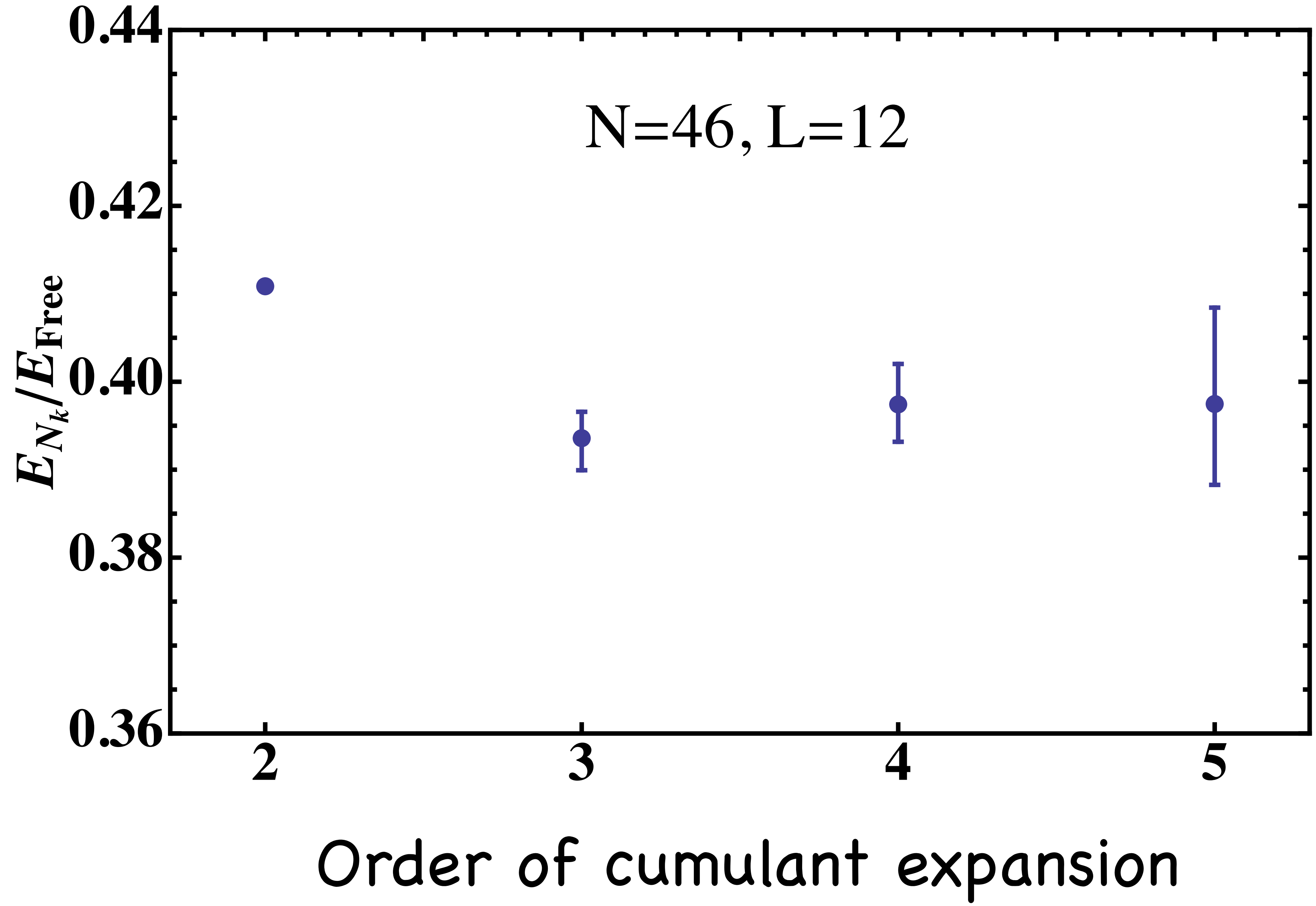
Applying the cumulant expansion for the same data

- conventional effective mass plot (gray)
- cumulant expansion (blue) to order n

$$\ln\langle C\rangle = \sum \frac{\kappa_n}{n!}$$



Determination of ground state energy for N=46 from cumulant expansion of log[C]



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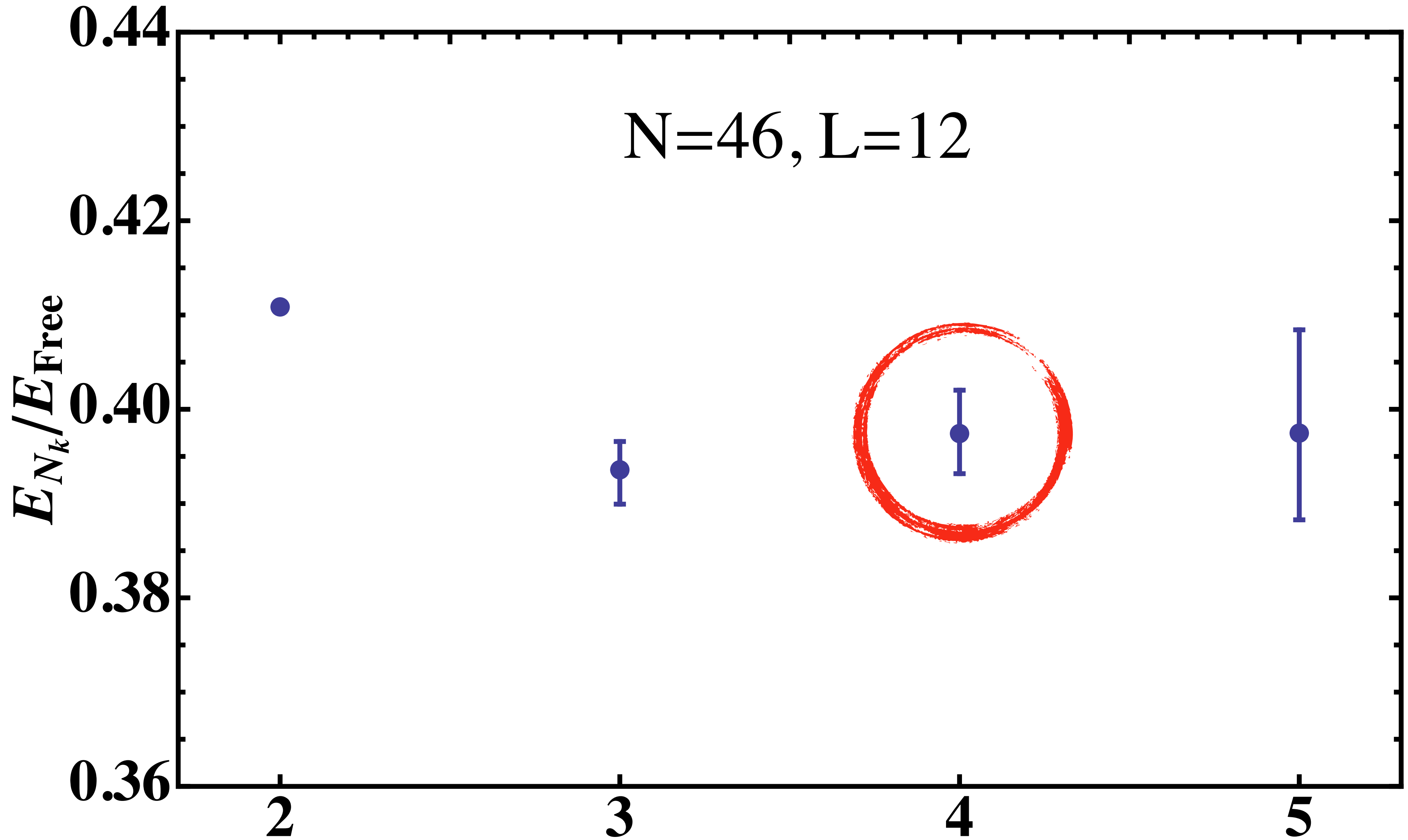

Order of cumulant expansion

Why do almost log-normal distributions arise?

Typically, multiplicative stochastic processes.

- Fracturing of materials
- Flow of oil through porous rock
- ...

Similar physics in electron propagation in random media

Try mean field treatment for probability distribution (inspired by Smolyarenko, Altschuler, 1997)

Mean field argument: distribution for y = Log[$C_N(\Phi,T)$]

N particle correlator

time separation T

$$P(y) = \int [D\phi]\, e^{-\int d^4x\, \frac{1}{2}m^2\phi^2}\, \delta(\ln C_N[\phi,T] - y)$$

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Perform semiclassical expansion in Φ.

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Perform semiclassical expansion in Φ.

Leading order result:

- Log Normal distribution (with corrections at higher order);
- μ, σ^2 scale with N and T as seen in data

A wild west computation of P(y)

$$P(y) = \int [D\phi]\, e^{-\int d^4x\, \frac{1}{2} m^2 \phi^2}\, \delta(\ln C_N[\phi, T] - y)$$

$$= \int [D\phi] \int \frac{dt}{2\pi} e^{it \ln[C_N[\phi,T]-y] - \int d^4x \frac{m^2}{2} \phi^2}$$

Find stationary point w.r.t. {T, Φ}, assume constant Φ

- What is m? renormalized!
  - Power divergent subtraction scheme (λ=ren. scale):

$$m^2 = \frac{M\lambda}{4\pi} \longrightarrow \frac{M k_F}{4\pi} \qquad\qquad \frac{N}{V} \equiv \frac{k_F^3}{6\pi^2}$$

- What is variation of ln[C] wrt Φ?

$$P(y) = \int [D\phi] \int \frac{dt}{2\pi} e^{it \ln[C_N[\phi,T]-y] - \int d^4x \frac{m^2}{2}\phi^2}$$

Need: $$\left.\frac{\partial \ln[C_N[\phi,T] - y]}{\partial \phi}\right|_{\phi(x)=\phi_0}$$

$$C_N[\phi_0, T] \sim \langle 0|[\psi(T)]^N [\psi(0)]^N |0\rangle_{\phi_0} \sim Z e^{-E_0(\phi_0,N)T}$$

Φ couples to $\Psi^\dagger\Psi$ on time link = current density...so Φ_0 looks like a constant vector potential.

$$E_0(\phi_0, N) = 2(E_N + N\phi_0)\ , \qquad E_N = \frac{3Nk_F^2}{10M}$$

So easy to find stationary point eqs:

$$t \to t_0 = -i\frac{Vm^2\phi_0}{N}$$

$$\phi \to \phi_0 = \frac{y - \ln Z + TE_0(N)}{NT}$$

Plug back in and find the probability distribution for y = ln[C_N]:

$$P(y) \propto e^{-(y-\bar{y})^2} 2\sigma^2 \ ,$$

$$\bar{y} = \ln Z - TE_0(N)$$

$$\sigma^2 = \frac{40}{9\pi} TE_0(N)$$

Log normal dist. for C_N!

Mean grows linearly in T

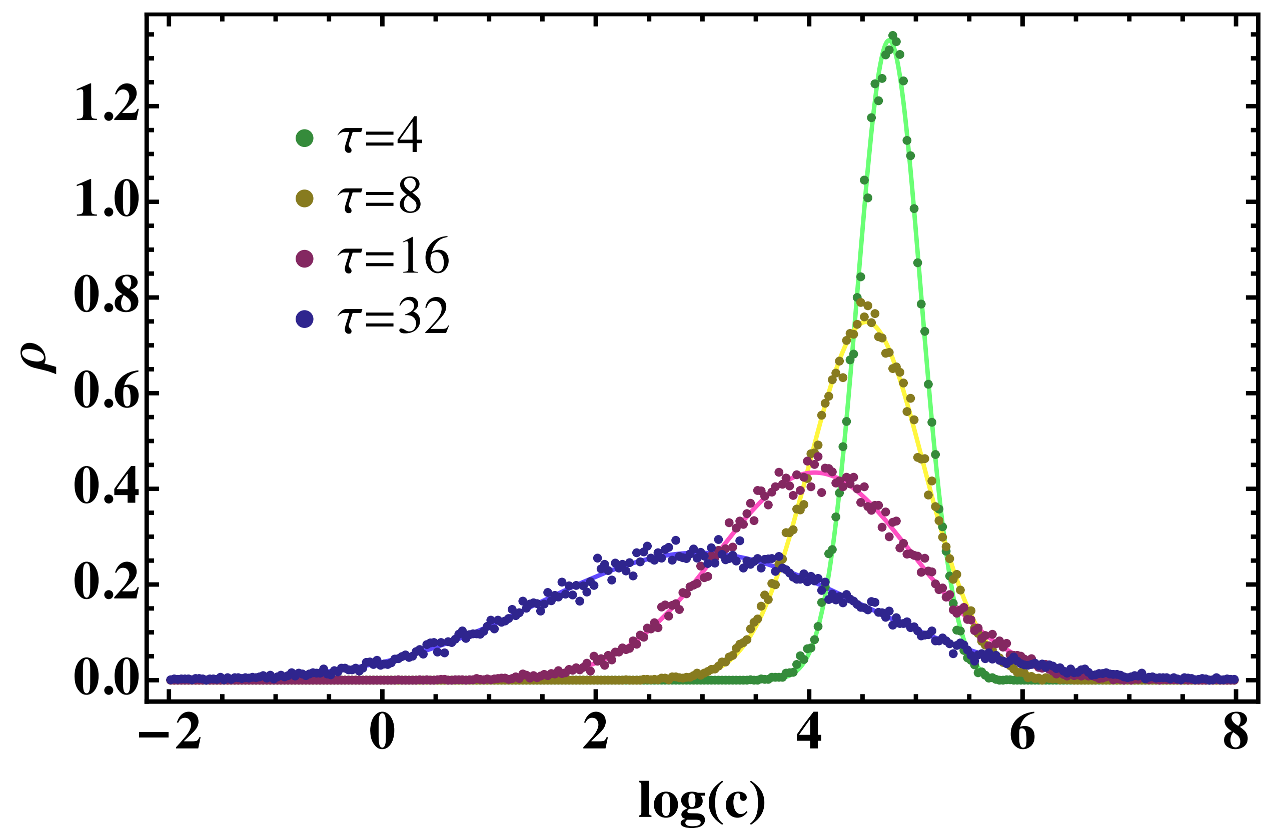
Variance grows linearly in T

Didn't compute fluctuations...so not really renormalized

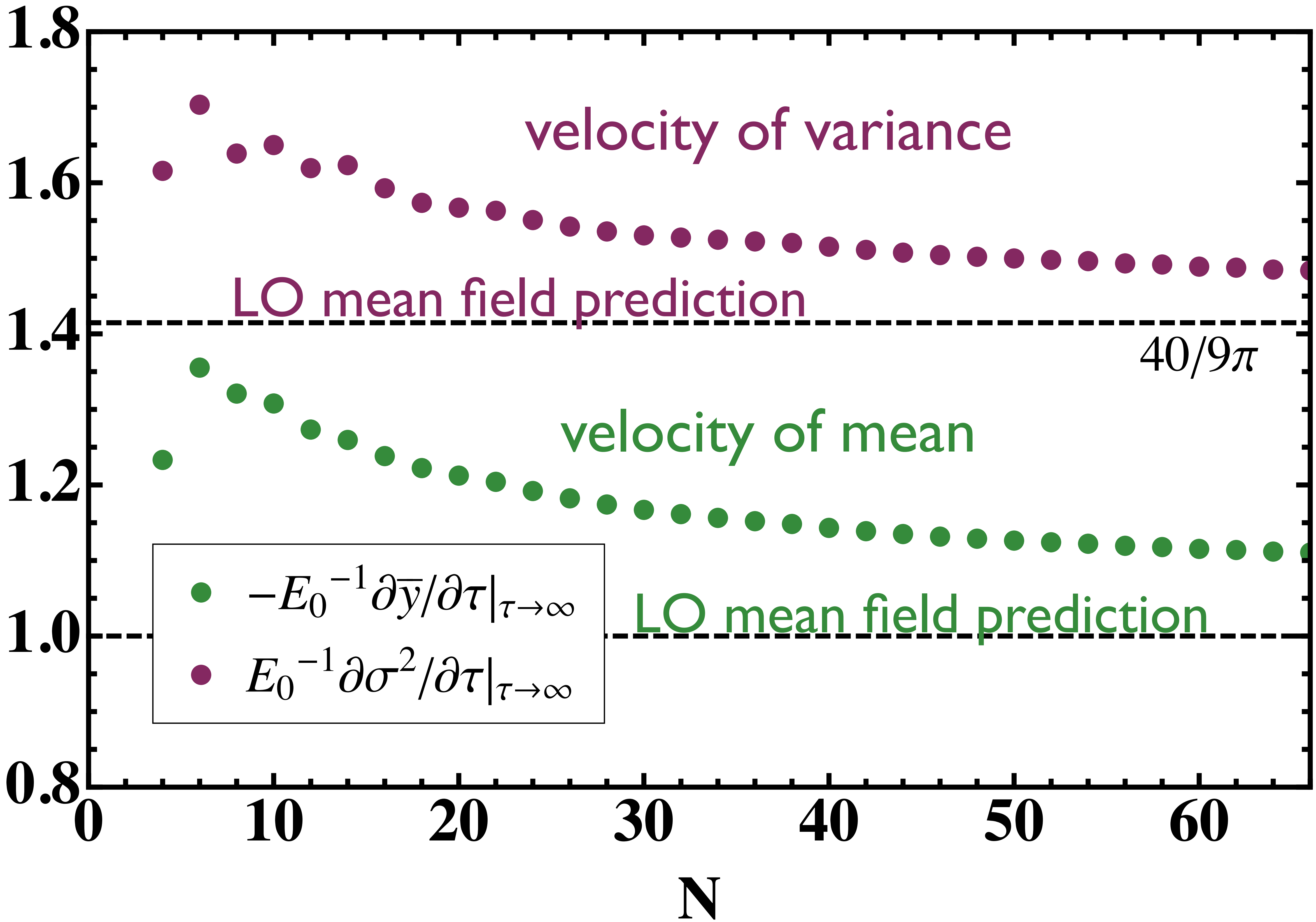
No obvious justification for semiclassical "expansion".

Result fits qualitatively:

- Explains log normal
- Mean and variance do grow linearly with time



Is there quantitative agreement? Yes...



...also suggests mean field might become exact for large N?

Directions to go:

- Understand phenomenon better

  Implications for spectrum? see:

  Amy Nicholson

  N-body Efimov states from two-particle noise

  arXiv:1202.4402
- Study toy model
- Does understanding = better approach to noise in unitary fermion calculations?
- Can we make a leap to QCD?

Useful to have an analytically soluble toy model:

1 particle, one spatial site

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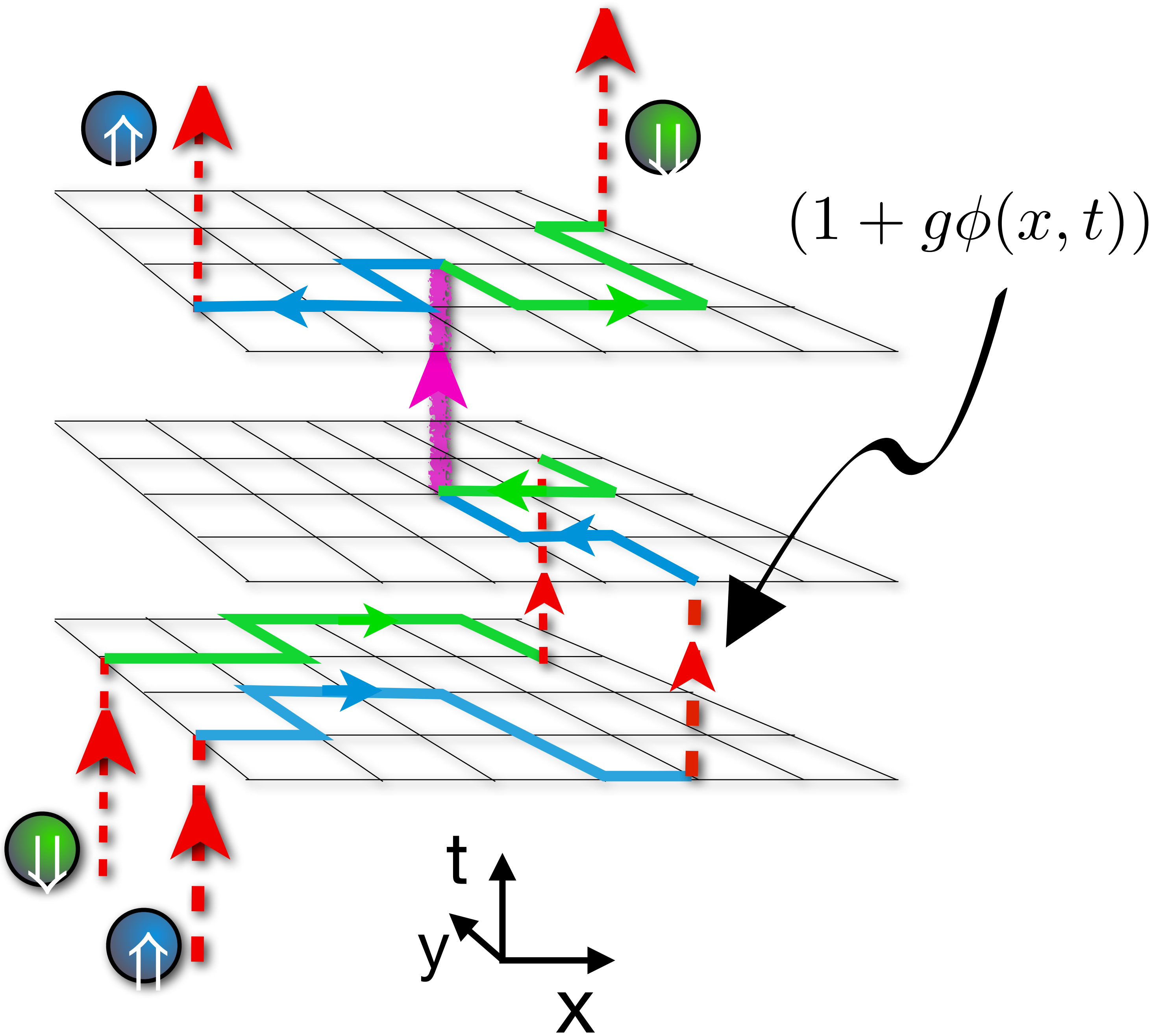

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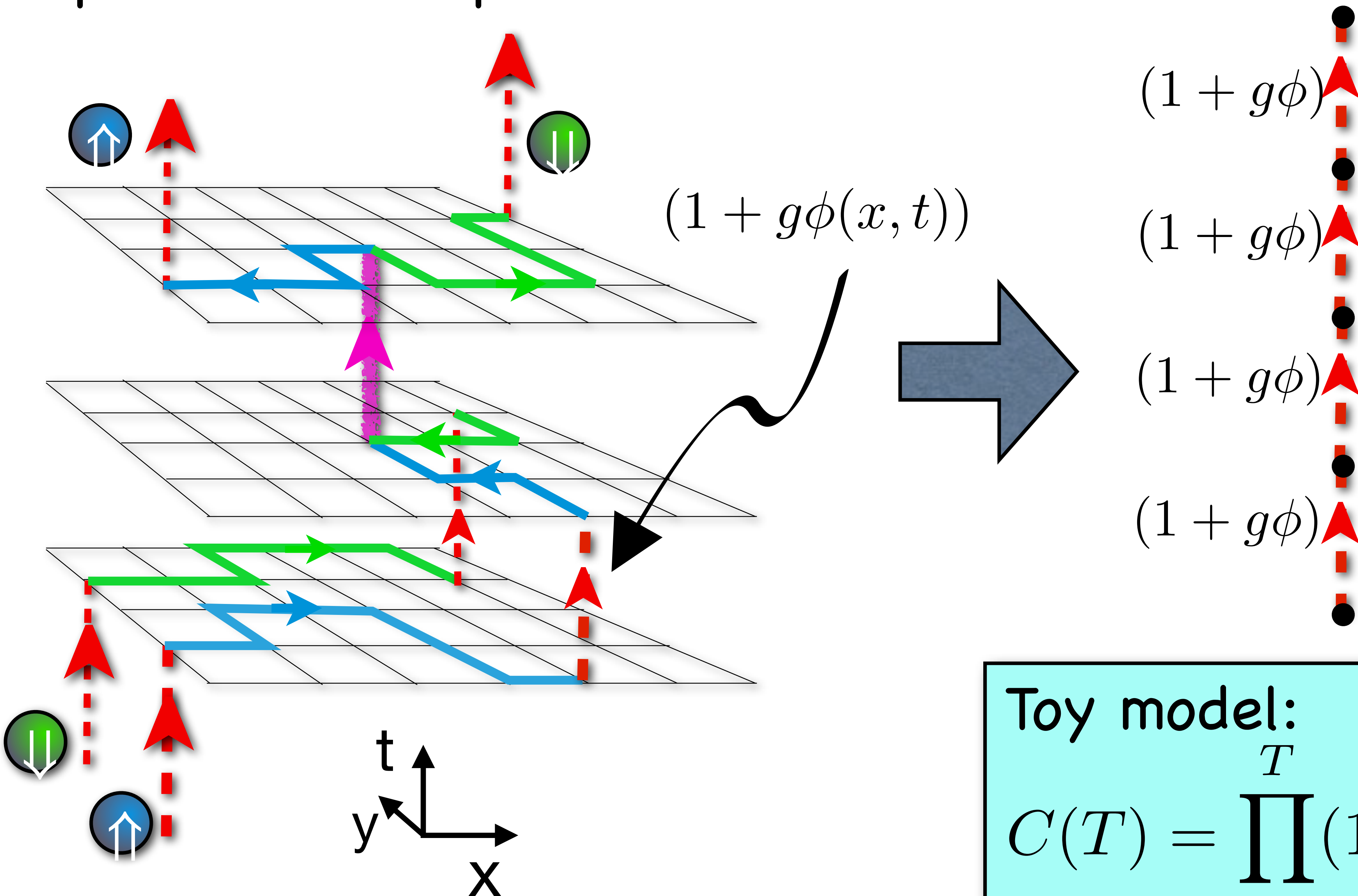

Toy model:

$$C(T) = \prod_{i=1}^{T} (1+g\phi_i)$$

$\phi_i \in [-1,1]$ uniform dist.

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- Conventional: $$E \to -\frac{1}{T} \ln \left[\frac{1}{N} \sum_{i=1}^{N} C(T, \phi_i) \right]$$

- New "EFT" approach: use identity

$$\ln \langle C \rangle = \sum_n \frac{\kappa_n}{n!} \qquad \kappa_n = \text{ cumulants of } \ln C(T, \phi)$$

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κ_n = 0 for n>2 if distribution is exactly log normal

Toy model:

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Can compute cumulants of (ln C) analytically:

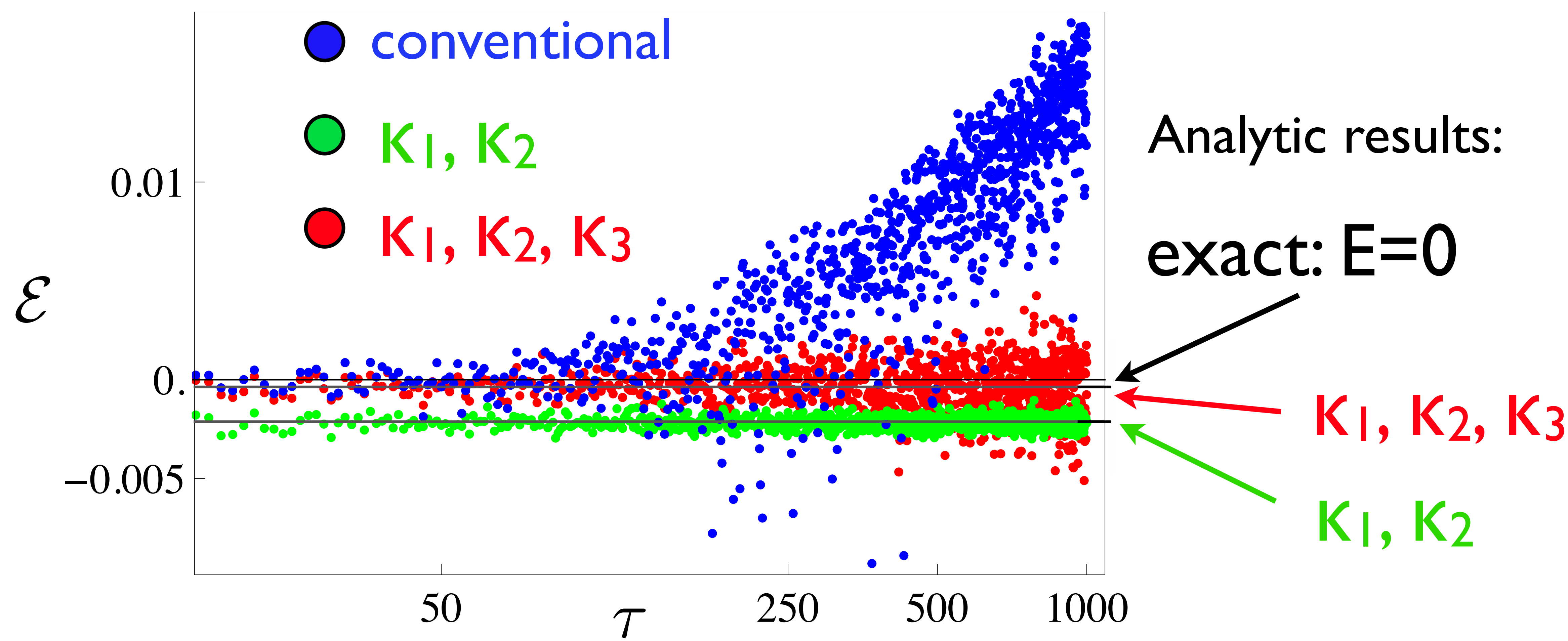
$$\kappa_1 = \tau \left[\tfrac{1}{2} \log\left(1 - g^2\right) + \frac{\tanh^{-1}(g)}{g} - 1 \right] ,$$

$$\frac{\kappa_n}{n!} = \tau \left(\frac{(-1)^n}{n} - \mathrm{Li}_{1-n}\left(\tfrac{1+g}{1-g}\right) \frac{\left(2\tanh^{-1}(g)\right)^n}{n!} \right) \qquad n > 1$$

Effective mass plot for toy model

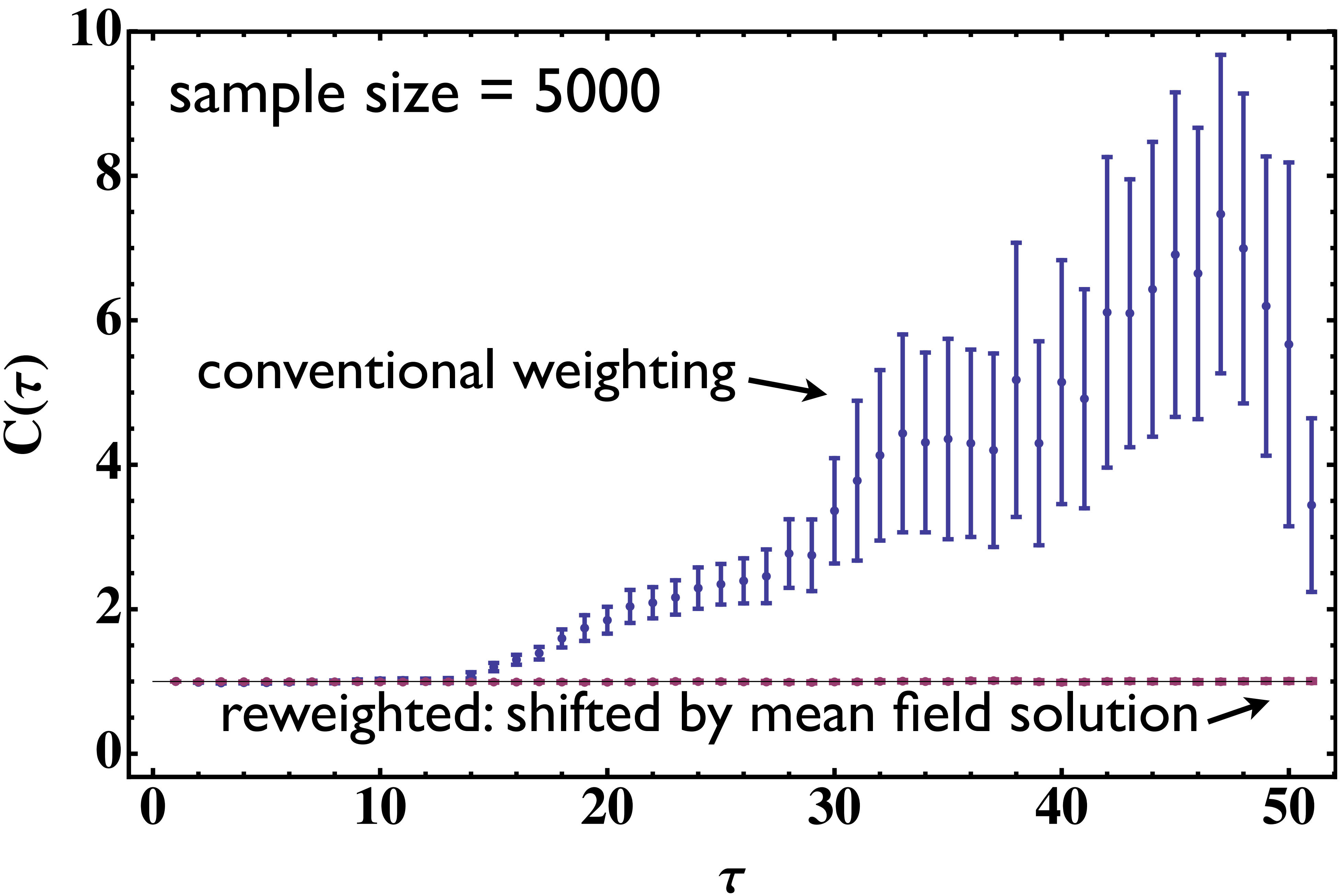
Simulation: sample size N=50,000
(each dot)

Spread & drift
= simulation errors



We see same phenomenon as in real simulation, but here have analytic results to compare with

Intriguing observation about toy model: improvement if reweighted by mean field solution (Endres)



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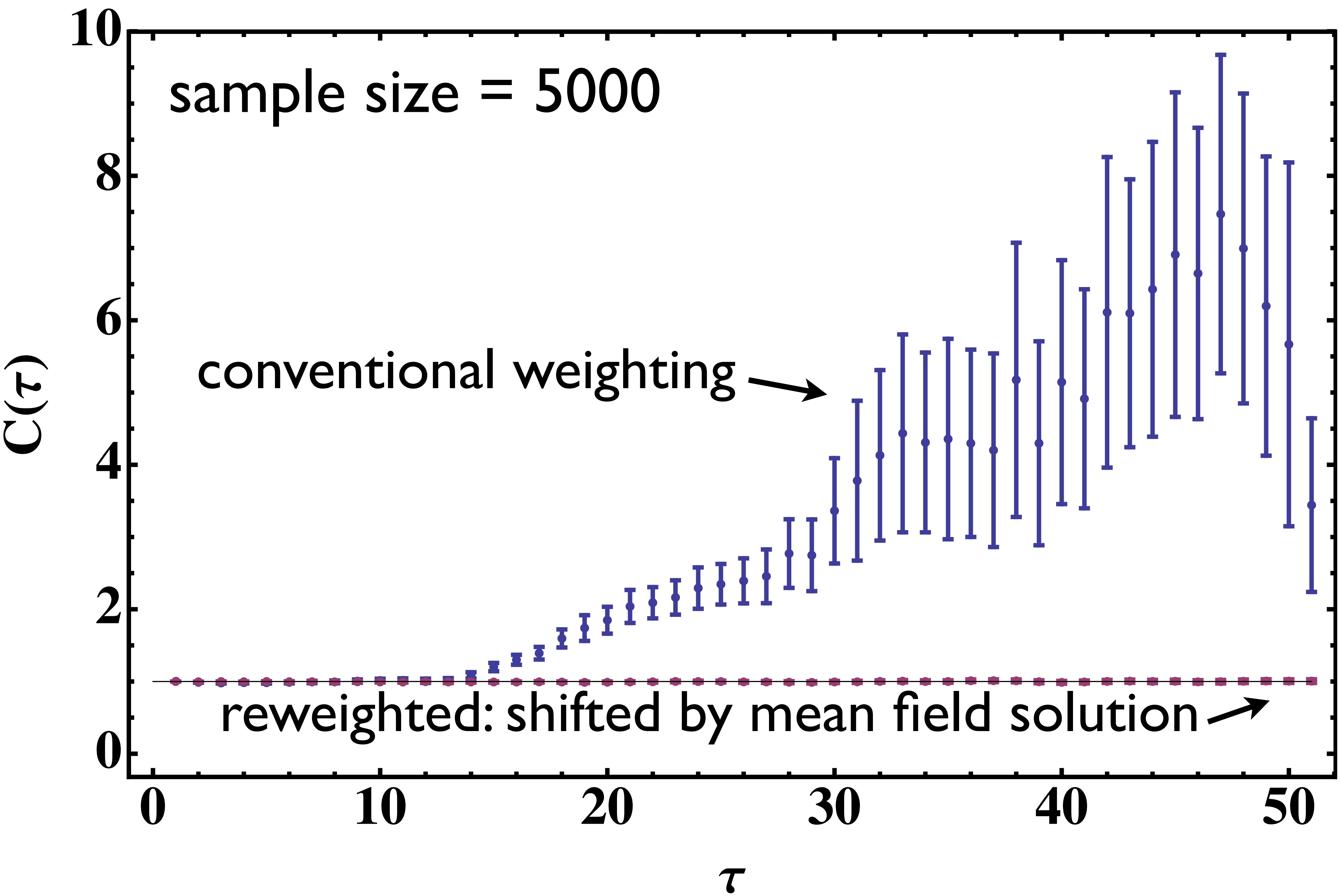


Model too simplistic

...but is there some way to use m.f.t. to reweight a real field theory?

Are distributions approaching log-normal appearing in QCD?

Apparently yes, at early time, although time dependence seems to be different

NPLQCD data

distribution of log $C_{\Lambda\Lambda}$

Each curve: 100,000 samples

SORRY - THIS FIGURE IS LOST IN PDF VERSION

...But at late time we do not expect log normal for baryon propagators in QCD

Lepage (& Savage):

$$x \equiv Re[C_{A\times 3q}(T)]$$

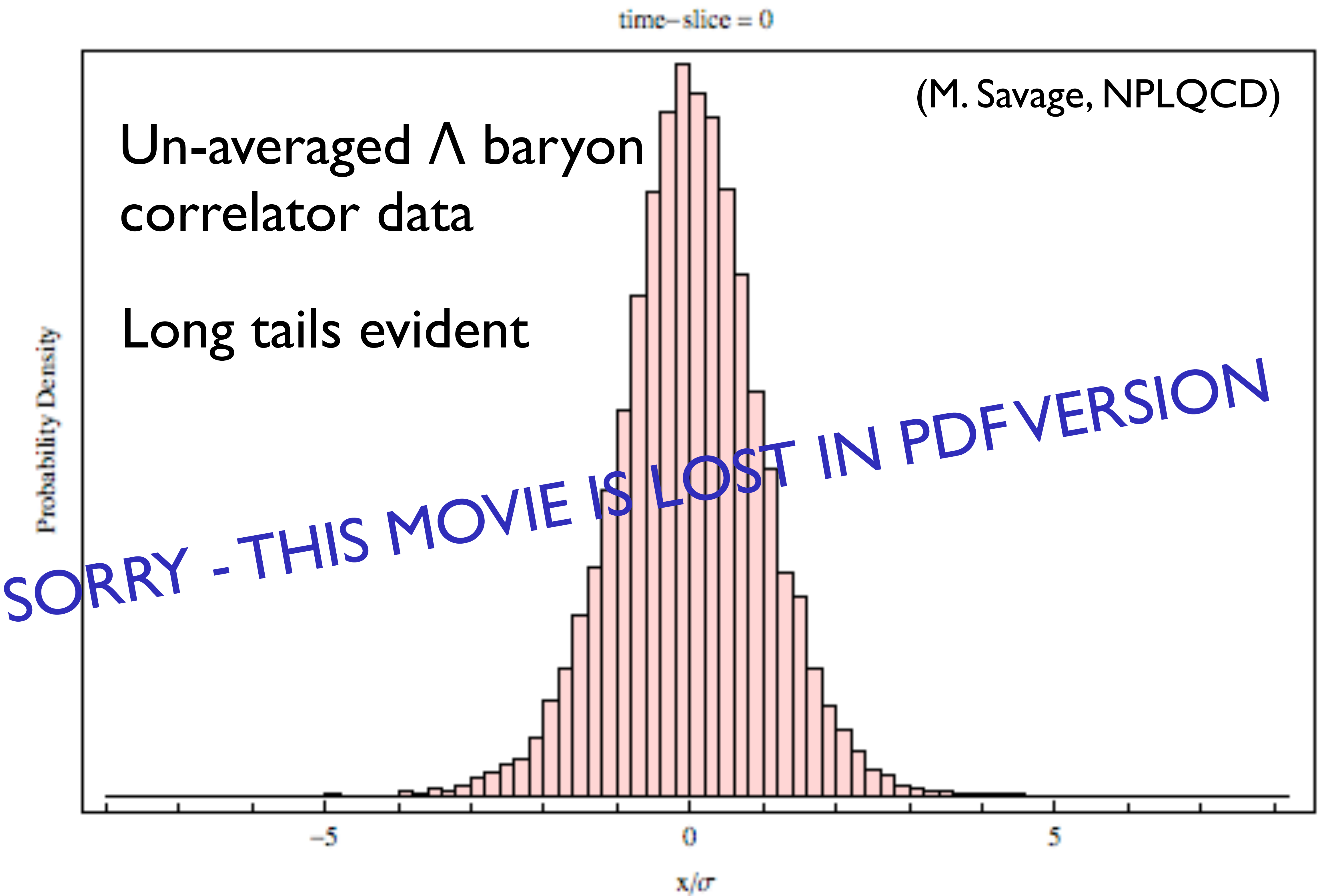
Real part of Euclidian correlator for A baryons

$$\langle x^{2k}\rangle \sim e^{-A3kM_\pi T}$$

$$\langle x^{2k+1}\rangle \sim \underline{e^{-AM_N T}} e^{-A3km_\pi T}$$

Odd moments die out faster...expect almost symmetric distribution at late time

Do heavy-tailed non-gaussian distributions occur in lattice QCD? Probably, especially large baryon number



Evidence suggests:

- QCD baryon correlators exhibit log normal distribution at short time...but in this window, conventional plateau analysis works as well as cumulant expansion
- later time, non log normal, nearly symmetric distribution appears..."sign problem" manifestation

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Questions instead of conclusions:

- Does QCD correlator distribution have some universal structure, more complex than log normal?
- Would understanding such a distribution aid in extracting spectrum masses from the noise? (eg "EFT" analysis of noise)
- Is there a mean field approach in QCD that could shed light on what is going on?