Phenomenological limits on the QGP shear viscosity and what they imply for QGP thermalization*

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Probing the landscape of QCD matter: The future is now!



Probes:

- Collective flow
- Jet modification and quenching
- Thermal electromagnetic radiation
- Critical fluctuations
- . . .

The Big Bang



The Little Bang



Big Bang vs. Little Bang



Similarities: Hubble-like expansion, expansion-driven dynamical freeze-out chemical freeze-out (nucleo-/hadrosynthesis) before thermal freeze-out (CMB, hadron p_T -spectra) initial-state quantum fluctuations imprinted on final state

Differences: Expansion rates differ by 18 orders of magnitude

Expansion in 3d, not 4d; driven by pressure gradients, not gravity

Time scales measured in fm/c rather than billions of years

Distances measured in fm rather than light years

"Heavy-Ion Standard Model" still under construction \implies this talk

Big vs. Little Bang: The fluctuation power spectrum

Mishra, Mohapatra, Saumia, Srivastava, PRC77 (2008) 064902 and C81 (2010) 034903 Mocsy & Sorensen, NPA855 (2011) 241, PLB705 (2011) 71



Animation: P. Sorensen



Animation: P. Sorensen



Collision of two Lorentz contracted gold nuclei

Animation: P. Sorensen



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Produced fireball is ~ 10^{-14} meters across and lives for ~ $5x10^{-23}$ seconds

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Expansion of the Little Bang

distributions and correlations of produced particles



Azimuthal Distributions: x-space



Are particles emitted at random angles? No. They remember the initial geometry!

Azimuthal Distributions: p-space



Are particles emitted at random angles? No. They remember the initial geometry!

The Little Bang: The Movie

How anisotropic flow is measured:

Definition of flow coefficients:

$$\frac{dN^{(i)}}{dy p_T dp_T d\phi_p}(b) = \frac{dN^{(i)}}{dy p_T dp_T}(b) \left(1 + 2\sum_{n=1}^{\infty} \boldsymbol{v_n^{(i)}(y, p_T; b)} \cos(\phi_p - \Psi_n^{(i)})\right).$$

Define event average $\{\ldots\}$, ensemble average $\langle\ldots\rangle$

Flow coefficients v_n typically extracted from azimuthal correlations (k-particle cumulants). E.g. k = 2, 4:

$$c_{n}\{2\} = \langle \{e^{ni(\phi_{1}-\phi_{2})}\} \rangle = \langle \{e^{ni(\phi_{1}-\psi_{n})}\} \{e^{-ni(\phi_{2}-\psi_{n})}\} + \delta_{2} \rangle = \langle v_{n}^{2} + \delta_{2} \rangle$$

$$c_{n}\{4\} = \langle \{e^{ni(\phi_{1}+\phi_{2}-\phi_{3}-\phi_{4})}\} \rangle - 2\langle \{e^{ni(\phi_{1}-\phi_{2})}\} \rangle = \langle -v_{n}^{4} + \delta_{4} \rangle$$

 v_n is correlated with the event plane while δ_n is not ("non-flow"). $\delta_2 \sim 1/M$, $\delta_4 \sim 1/M^3$. 4th-order cumulant is free of 2-particle non-flow correlations.

These measures are affected by event-by-event flow fluctuations:

$$\langle v_2^2 \rangle = \langle v_2 \rangle^2 + \sigma^2, \qquad \langle v_2^4 \rangle = \langle v_2 \rangle^4 + 6\sigma^2 \langle v_2 \rangle^2$$

 $v_n\{k\}$ denotes the value of v_n extracted from the k^{th} -order cumulant:

 $v_2\{2\} = \sqrt{\langle v_2^2 \rangle}, \quad v_2\{4\} = \sqrt[4]{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle}$

Event-by-event shape and flow fluctuations rule!

(Alver and Roland, PRC81 (2010) 054905)



- Each event has a different initial shape and density distribution, characterized by different set of harmonic eccentricity coefficients ε_n
- Each event develops its individual hydrodynamic flow, characterized by a set of harmonic flow coefficients v_n and flow angles ψ_n
- At small impact parameters fluctuations ("hot spots") dominate over geometric overlap effects (Alver & Roland, PRC81 (2010) 054905; Qin, Petersen, Bass, Müller, PRC82 (2010) 064903)

Panta rhei: "soft ridge" = "Mach cone" = flow!



- anisotropic flow coefficients v_n and flow angles ψ_n correlated over large rapidity range! M. Luzum, PLB 696 (2011) 499: All long-range rapidity correlations seen at RHIC are consistent with being entirely generated by hydrodynamic flow.
- ullet in the 1% most central collisions $v_3>v_2$
 - \implies prominent "Mach cone"-like structure!
 - \implies event-by-event eccentricity fluctuations dominate!

Event-by-event shape and flow fluctuations rule!



• in the 1% most central collisions $v_3 > v_2 \implies$ prominent "Mach cone"-like structure!

- triangular flow angle uncorrelated with reaction plane and elliptic flow angles
 - \Longrightarrow due to event-by-event eccentricity fluctuations which dominate the anisotropic flows in the most central collisions

Fluctuation-driven anisotropic flow is indeed collective!



ALICE (J. Grosse-Oetringhaus) Quark Matter 2011

- Two-particle Fourier coefficients factorize $(v_{n\Delta}(p_{T1}, p_{T2}) = v_n(p_{T1})v_n(p_{T2}))$ as required
- Factorization shown to work for n = 2, 3, 4, 5 as long as both $p_{T1}, p_{T2} < 3 \text{ GeV}/c$ (bulk matter)

Converting initial shape fluctuations into final flow anisotropies the QGP shear viscosity $(\eta/s)_{
m QGP}$

How to use elliptic flow for measuring $(\eta/s)_{ m QGP}$

Hydrodynamics converts **spatial deformation of initial state** \implies **momentum anisotropy of final state**, through anisotropic pressure gradients

Shear viscosity degrades conversion efficiency

 $\varepsilon_x = \frac{\langle\!\langle y^2 - x^2 \rangle\!\rangle}{\langle\!\langle y^2 + x^2 \rangle\!\rangle} \Longrightarrow \varepsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$

of the fluid; the suppression of ε_p is monotonically related to η/s .



The observable that is most directly related to the total hydrodynamic momentum anisotropy ε_p is the total (p_T -integrated) charged hadron elliptic flow v_2^{ch} :

$$\varepsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle} \Longleftrightarrow \frac{\sum_i \int p_T dp_T \int d\phi_p \, p_T^2 \, \cos(2\phi_p) \, \frac{dN_i}{dy p_T dp_T d\phi_p}}{\sum_i \int p_T dp_T \int d\phi_p \, p_T^2 \, \frac{dN_i}{dy p_T dp_T d\phi_p}} \iff v_2^{\text{ch}}$$

How to use elliptic flow for measuring $(\eta/s)_{ m QGP}$ (contd.)

- If ε_p saturates before hadronization (e.g. in PbPb@LHC (?))
 - $\Rightarrow~v_2^{\rm ch}\approx$ not affected by details of hadronic rescattering below $T_{\rm c}$

but: $v_2^{(i)}(p_T)$, $\frac{dN_i}{dyd^2p_T}$ change during hadronic phase (addl. radial flow!), and these changes depend on details of the hadronic dynamics (chemical composition etc.)

 $\Rightarrow v_2(p_T)$ of a single particle species **not** a good starting point for extracting η/s

- If ε_p does not saturate before hadronization (e.g. AuAu@RHIC), dissipative hadronic dynamics affects not only the distribution of ε_p over hadronic species and in p_T, but even the final value of ε_p itself (from which we want to get η/s)
 - ⇒ need hybrid code that couples viscous hydrodynamic evolution of QGP to realistic microscopic dynamics of late-stage hadron gas phase
 - ⇒ **VISHNU** ("Viscous Israel-Steward Hydrodynamics 'n' UrQMD")

(Song, Bass, UH, PRC83 (2011) 024912) Note: this paper shows that $UrQMD \neq viscous hydro!$

Extraction of $(\eta/s)_{ m QGP}$ from AuAu@RHIC



 $1 < 4\pi(\eta/s)_{
m QGP} < 2.5$

- All shown theoretical curves correspond to parameter sets that correctly describe centrality dependence of charged hadron production as well as p_T -spectra of charged hadrons, pions and protons at all centralities
- v_2^{ch}/ε_x vs. $(1/S)(dN_{ch}/dy)$ is "universal", i.e. depends only on η/s but (in good approximation) not on initial-state model (Glauber vs. KLN, optical vs. MC, RP vs. PP average, etc.)
- dominant source of uncertainty: $arepsilon_x^{
 m Gl}$ vs. $arepsilon_x^{
 m KLN}$
- smaller effects: early flow \rightarrow increases $\frac{v_2}{\varepsilon}$ by \sim few % \rightarrow larger η/s

bulk viscosity
$$ightarrow$$
 affects $v_2^{
m ch}(p_T)$, but $pprox$ not $v_2^{
m ch}$

Zhi Qiu, UH, PRC84 (2011) 024911



Extraction of $(\eta/s)_{ m QGP}$ from AuAu@RHIC

H. Song, S.A. Bass, UH, T. Hirano, C. Shen, PRL106 (2011) 192301



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bulk viscosity
$$\rightarrow$$
 affects $v_2^{ch}(p_T)$, but \approx not v_2^{ch}
e-by-e hydro \rightarrow decreases $\frac{v_2^{ch}}{\varepsilon}$ by $\lesssim 5\%$ (dep. on η/s)



Global description of AuAu@RHIC spectra and v_2



- $(\eta/s)_{QGP} = 0.08$ for MC-Glauber and $(\eta/s)_{QGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $v_2(p_T)$ at all collision centralities
- Note: $T_{\text{chem}} = 165 \text{ MeV}$ reproduces the proton spectra from STAR, but not from PHENIX! \implies Slightly incorrect chemical composition in hadronic phase? Not enough $p\bar{p}$ annihilation in UrQMD?

Global description of AuAu@RHIC spectra and v_2



VISHNU (H. Song, S.A. Bass, UH, T. Hirano, C. Shen, PRC83 (2011) 054910)

- $(\eta/s)_{\rm OGP} = 0.08$ for MC-Glauber and $(\eta/s)_{\rm OGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $v_2(p_T)$ at all collision centralities
- A purely hydrodynamic model (without UrQMD afterburner) with the same values of η/s does almost as well (except for centrality dependence of proton $v_2(p_T)$) (C. Shen et al., PRC84 (2011) 044903) Main difference: VISHNU develops more radial flow in the hadronic phase (larger shear viscosity), pure viscous hydro must start earlier than VISHNU ($\tau_0 = 0.6$ instead of 1.05 fm/c), otherwise proton spectra are too steep
- These η/s values agree with Luzum & Romatschke, PRC78 (2008), even though they used EOS with incorrect hadronic chemical composition \implies shows robustness of extracting η/s from total charged hadron v_2

Pre- and postdictions for PbPb@LHC



- After normalization in 0-5% centrality collisions, MC-KLN + VISHNU (w/o running coupling, but including viscous entropy production!) reproduces centrality dependence of $dN_{\rm ch}/d\eta$ well in both AuAu@RHIC and PbPb@LHC
- $(\eta/s)_{QGP} = 0.16$ for MC-KLN works well for charged hadron $v_2(p_T)$ and integrated v_2 in AuAu@RHIC, but overpredicts both by about 10-15% in PbPb@LHC
- Similar results from predictions based on pure viscous hydro (C. Shen et al., PRC84 (2011) 044903)
- but: At LHC significant sensitivity of v₂ to initialization of viscous pressure tensor π^{μν} (Navier-Stokes or zero) ⇒ need pre-equilibrium model.

\implies QGP at LHC definitely not much more viscous than at RHIC!

Why is $v_2^{ch}(p_T)$ the same at RHIC and LHC?

Answer: Pure accident! (Kestin & Heinz EPJC61 (2009) 545)



 $v_2^{\pi}(p_T)$ increases a bit from RHIC to LHC, for heavier hadrons $v_2(p_T)$ at fixed p_T decreases (radial flow pushes momentum anisotropy of heavy hadrons to larger p_T)

This is a hard prediction of hydrodynamics! (See also Nagle, Bearden, Zajc, NJP13 (2011) 075004)

Confirmation of increased mass splitting at LHC



- Qualitative features of data agree with VISH2+1 predictions
- VISH2+1 does not push proton v_2 strongly enough to higher p_T , both at RHIC and LHC
- At RHIC we know that this is fixed when using VISHNU is the same true at LHC?

Successful prediction of $v_2(p_T)$ for identified hadrons in PbPb@LHC



Perfect fit in semi-peripheral collisions!

The problem with insufficient proton radial flow exists only in more central collisions Adding the hadronic cascade (VISHNU) helps:

$v_2(p_T)$ in PbPb@LHC: ALICE vs. VISH2+1 & VISHNU

Data: ALICE, preliminary (Snellings, Krzewicki, Quark Matter 2011) Dashed lines: Shen et al., PRC84 (2011) 044903 (VISH2+1, MC-KLN, $(\eta/s)_{QGP}=0.2$) Solid lines: Song, Shen, UH 2011 (VISHNU, MC-KLN, $(\eta/s)_{QGP}=0.16$)



VISHNU yields correct magnitude and centrality dependence of $v_2(p_T)$ for pions, kaons and protons! Same $(\eta/s)_{QGP} = 0.16$ (for MC-KLN) at RHIC and LHC!

Back to the "elephant in the room": How to eliminate the large model uncertainty in the initial eccentricity?

Two observations:

I. Shear viscosity suppresses higher flow harmonics more strongly



⇒ Idea: Use simultaneous analysis of elliptic and triangular flow to constrain initial state models (see also Bhalerao, Luzum Ollitrault, PRC 84 (2011) 034910)

Two observations: II. ε_3 is \approx model independent



Initial eccentricities ε_n and angles ψ_n :

$$\varepsilon_{n}e^{in\psi_{n}} = -\frac{\int r dr d\phi r^{2}e^{in\phi} e(r,\phi)}{\int r dr d\phi r^{2} e(r,\phi)}$$

- MC-KLN has larger ε_2 and ε_4 , but similar ε_5 and almost identical ε_3 as MC-Glauber
- Angles of ε_2 and ε_4 are correlated with reaction plane by geometry, whereas those of ε_3 and ε_5 are random (purely fluctuation-driven)
- While v_4 and v_5 have mode-coupling contributions from ε_2 , v_3 is almost pure response to ε_3 and $v_3/\varepsilon_3 \approx \text{const.}$ over a wide range of centralities
- \implies Idea: Use total charged hadron v_3^{ch} to determine $(\eta/s)_{QGP}$, then check v_2^{ch} to distinguish between MC-KLN and MC-Glauber!

Large measured v_3 requires small $(\eta/s)_{QGP} \simeq 1/(4\pi)!$

Zhi Qiu, Chun Shen, UH, PLB707 (2012) 151 (VISH2+1)



- Both MC-KLN with $\eta/s = 0.2$ and MC-Glauber with $\eta/s = 0.08$ give very good description of v_2/ε_2 at all centralities.
- Only MC-Glauber initial conditions with $\eta/s = 0.08$ describe v_3/ε_3 PHENIX, comparing to calculations by Alver et al. (PRC82 (2010) 034913), come to similar conclusions at RHIC energies (PHENIX Coll., PRL107 (2011) 252301, and Lacey et al., arXiv:1108.0457 (QM11))
- Large v_3 measured at RHIC and LHC requires small $(\eta/s)_{QGP} \simeq 1/(4\pi)$ unless the fluctuations predicted by both models are completely wrong and ε_3 is really 50% larger than we presently believe!

The "Little Bang Standard Model": Status 2012

We have come a long way over the last couple of years:
 I believe that the issue of the QGP shear viscosity at RHIC and LHC energies is now settled:

$$(\eta/s)_{
m QGP}(T_{
m c}{<}T{<}2T_{
m c})=rac{1}{4\pi}\pm 50\%$$

A moderate increase between $2T_{\rm c}$ and $3T_{\rm c}$ can at present not be excluded but is not mandated by the data.

- Ingredients that matter at the 50% level and are under control:
 - relativistic viscous fluid dynamics
 - realistic EOS with correct non-equilibrium composition in HG phase
 - microscopic description of the highly dissipative hadronic stage, including all resonance decays
 - fluctuating initial conditions, simultaneous study of v_2 and v_3
- Ingredients that matter at the <25% level and require more work:
 - bulk viscosity
 - temperature dependence of $(\eta/s)_{
 m QGP}$
 - pre-equilibrium flow
 - event-by-event hydro + cascade evolution
 - (3+1)-d vs. (2+1)-d evolution
 - study of higher harmonics; influence of nucleon growth with \sqrt{s} on fluctuations
 - flow fluctuations and flow angle correlations for different harmonics

The ultimate theoretical question:

Why is $(\eta/s)_{ m QGP}$ as small as it is?



Outlook: The fluctuation power spectrum



We need theory curves through the right plot!

The fluctuation powe ectrum of the Little Bang: a first try Staig & Shuryak, Quark Matter 2011



- Collisions between different species, at different collision centralities, and at different \sqrt{s} create Little Bangs with characteristically different power spectra
- The final flow power spectra depend on p_T rich experimental information!
- Relating the measured "anisotropic flow power spectrum" (i.e. v_n vs. n) to the "initial fluctuation power spectrum" (i.e. ε_n vs. n) provides access to the QGP transport coefficients (likely not just η/s, but also ζ/s, τ_π, τ_Π...)
- Power spectrum of initial fluctuations (in particular its √s dependence) can (probably) be calculated from first principles via CGC effective theory (Dusling, Gelis, Venugopalan, NPA872 (2011) 161; Schenke, Tribedy, Venugopalan, arXiv:1202.6646)

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Supplements

Eccentricity definitions:

Define event average $\{\ldots\}$, ensemble average $\langle\ldots\rangle$

Two choices for weight function in event average: (i) Energy density $e(x_{\perp}; b)$ (ii) Entropy density $s(x_{\perp}; b)$

Define $\sigma_x^2 = \{x^2\} - \{x\}^2$, $\sigma_{xy} = \{xy\} - \{x\}\{y\}$, etc., where x, y are reaction-plane coordinates $(e_x \parallel b)$

1. Standard eccentricity: $\varepsilon_s \equiv \overline{\varepsilon}_{RP} = \frac{\langle \sigma_y^2 - \sigma_x^2 \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle}$ (calculated from RP-averaged $\langle e \rangle$ or $\langle s \rangle$)

2. Average reaction-plane eccentricity: $\langle \varepsilon_{\rm RP} \rangle = \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2} \right\rangle$

3. Eccentricity of the participant-plane averaged source: $\bar{\varepsilon}_{part} = \frac{\langle \sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2} \rangle}{\langle \sigma_y^2 + \sigma_x^2 \rangle}$

4. Average participant-plane eccentricity: $\langle \varepsilon_{\text{part}} \rangle = \left\langle \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2} \right\rangle$

5. r.m.s. part.-plane eccentricity: $\varepsilon_{\text{part}} \{2\} \equiv \sqrt{\langle \varepsilon_{\text{part}}^2 \rangle} \quad (=\sqrt{\langle \varepsilon_{\text{part}} \rangle^2 + \sigma_{\varepsilon}^2/2} \text{ for Gauss. fl.})$

6. 4th cumulant eccentricity: $\varepsilon_{\text{part}}\{4\} \equiv \left[\langle \varepsilon_{\text{part}}^2 \rangle^2 - (\langle \varepsilon_{\text{part}}^4 \rangle - \langle \varepsilon_{\text{part}}^2 \rangle^2)\right]^{1/4}$ $\left(=\sqrt{\langle \varepsilon_{\text{part}} \rangle^2 - \sigma_{\varepsilon}^2/2} \text{ for Gauss. fl.}\right)$

MC-Glauber eccentricities (*e***-weighted)**:



MC-KLN eccentricities (*e***-weighted)**:



Initial eccentricities $\varepsilon_n(n=2-5)$ vs. impact parameter

Zhi Qiu, UH, PRC84 (2011) 024911



INT, 03/19/2012 48(43)

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Comparison of ALICE PbPb@LHC v_2 data with VISH2+1



PbPb@LHC p_T -spectra: **ALICE** vs. **VISH2**+1 and **VISHNU**:



- \bullet Good description also of identified hadron spectra for centralities <50%
- VISHNU better than VISH2+1 in central collisions (more radial flow)
- Both models give too much radial flow in peripheral collisions \implies initial conditions?
- Both models overpredict proton yield by 50-70%!?

The new "proton anomaly": disagreement with the thermal model

Data: ALICE, preliminary (A. Kalweit, Strange Quark Matter 2011)

Model: A. Andronic et al., PLB673 (2009) 142; similar: S. Wheaton et al. (THERMUS), Comp. Phys. Comm. 180 (2009) 84



• "Standard" $T_{\rm chem} = 164 \,\text{MeV}$ reproduces strange hadrons but overpredicts (anti-)protons by 50%!

- $\bullet \ p \bar{p}$ annihilation in UrQMD not strong enough to repair this
- Similar problem already seen at RHIC but not taken seriously (STAR/PHENIX disagreement)

PbPb@LHC p_T -spectra: Glauber vs. KLN



- In central collisions no difference between the models.
- In peripheral collisions p_T -spectra from MC-Glauber IC too steep!

This is an artifact of single-shot hydro with averaged initial profile; for small $\eta/s = 0.08$ (but not

for $\eta/s = 0.2!$), e-by-e hydro gives flatter p_T -spectra in peripheral collisions, due to hot spots

Smearing effects from nucleon growth at high energies



s95p-PCE: A realistic, lattice-QCD-based EOS



High T: Lattice QCD (latest hotQCD results)

Low T: Chemically frozen HRG $(T_{\rm chem} = 165 \,{\rm MeV})$

No softest point!

Event-by-event vs. single-shot hydro

Eccentricity-scaled $v_{2,3}$ flow from e-by-e and single-shot hydro



- For most centralities, eccentricity-scaled $v_{2,3}$ from e-by-e and single-shot hydro agree within 5-10%
- Agreement between $\langle v_{2,3} \rangle / \langle \varepsilon_{2,3} \rangle$ and $v_{2,3} \{2\} / \varepsilon_{2,3} \{2\}$ is excellent at all centralities
- Agreement between $v_2\{2\}/\varepsilon_2\{2\}$ and $v_2\{4\}/\varepsilon_2\{4\}$ is good over most of the centrality range, but the analog relation for triangular flow does not work (note, however, limited statistics)
- \implies Can use single-shot hydro to compute $\langle v_{2,3} \rangle / \langle arepsilon_{2,3} \rangle = v_{2,3} \{2\} / arepsilon_{2,3} \{2\}$