Heavy quarkonium and the renormalization of the cyclic Wilson loop

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in collaboration with
M. Berwein, N. Brambilla and A. Vairo

INT, April 2nd 2012

Outline

- Introduction to quarkonium in deconfined media
- Introduction to the cyclic Wilson loop
- Divergences in the cyclic Wilson loop
- Renormalization

Quarkonium in media: the long story (made short)

Volume 178, number 4 PHYSICS LETTERS B 9 October 1986

J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

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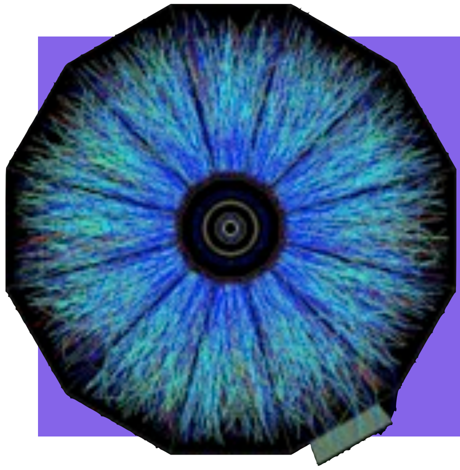
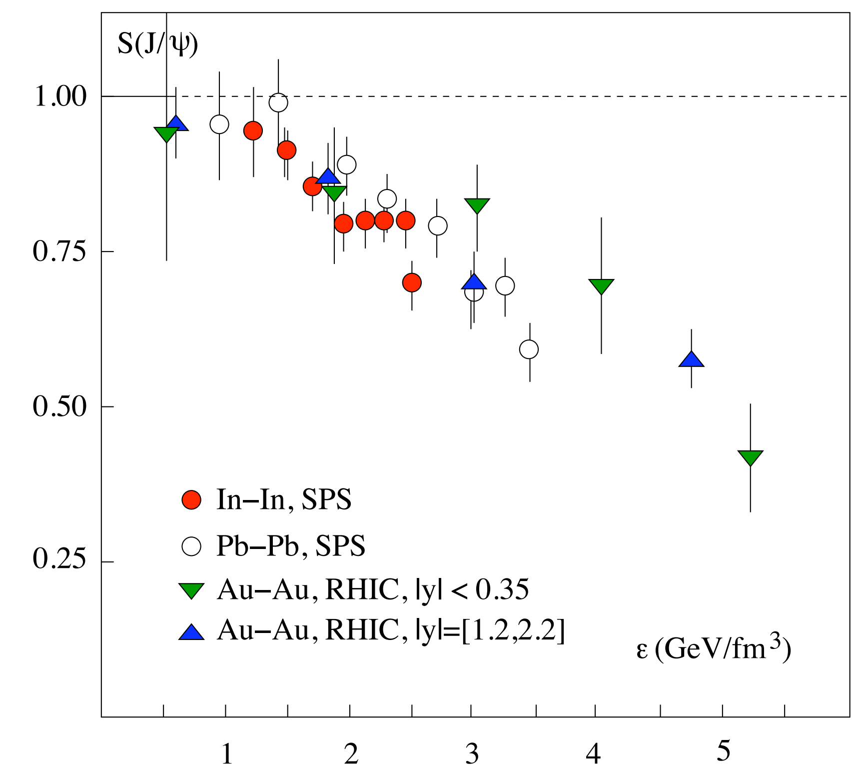
Received 17 July 1986

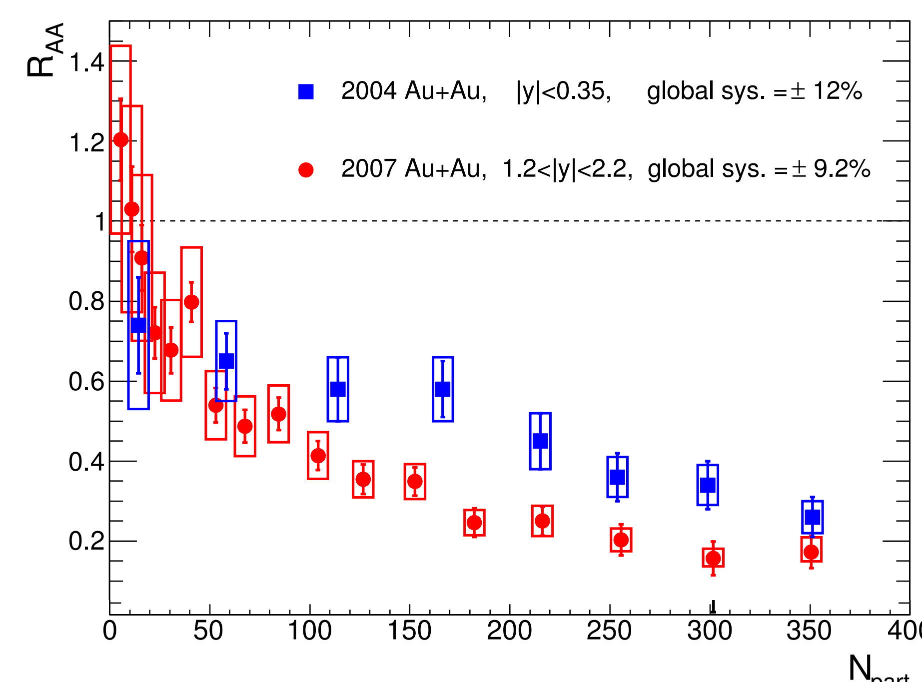
If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark–gluon plasma formation.



Quarkonium suppression

- Experimental data show a suppression pattern
- A good understanding of suppression requires understanding of
  - Production and cold nuclear matter effects
  - In-medium bound-state dynamics
  - Recombination effects
  - ...

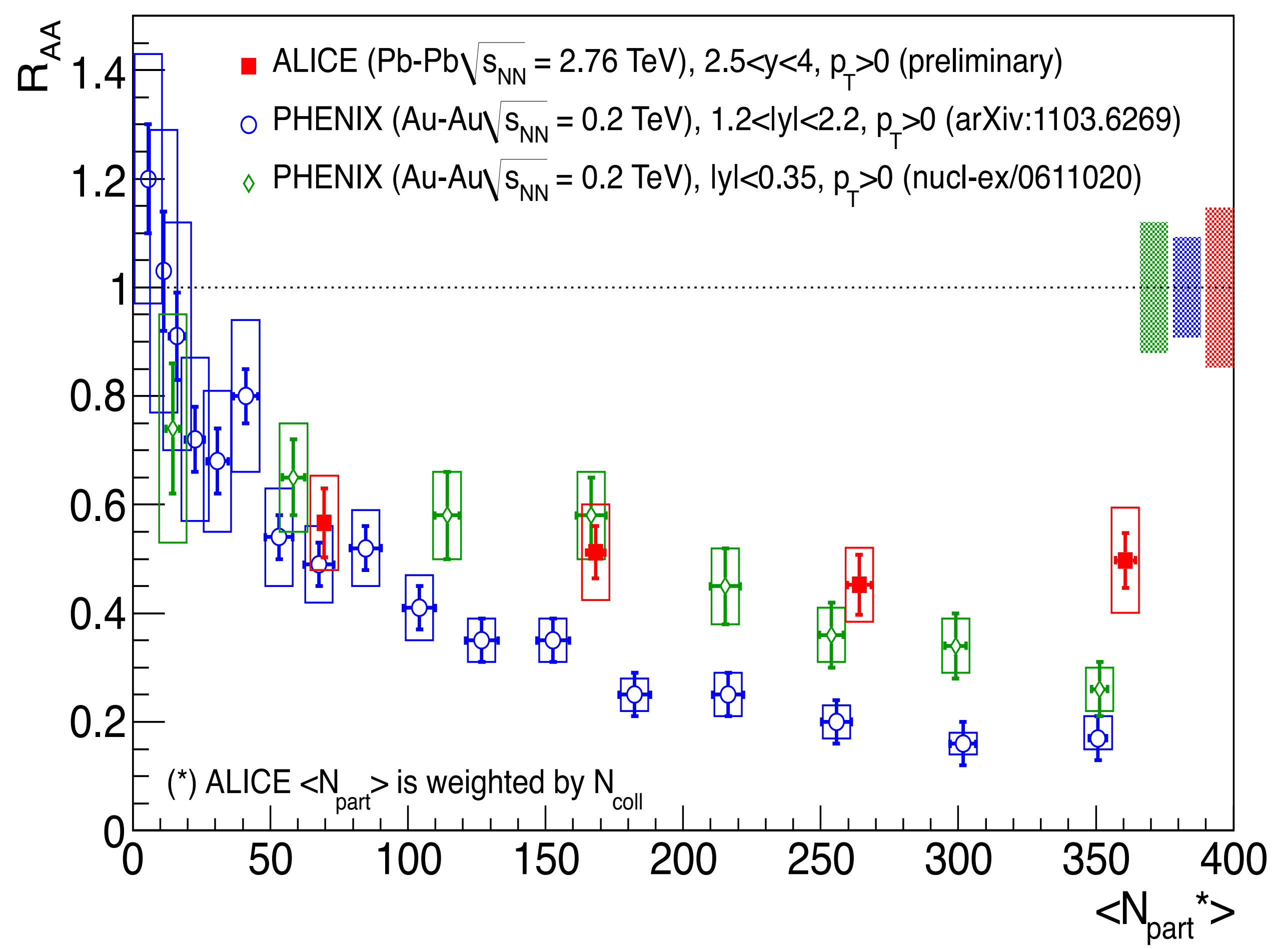


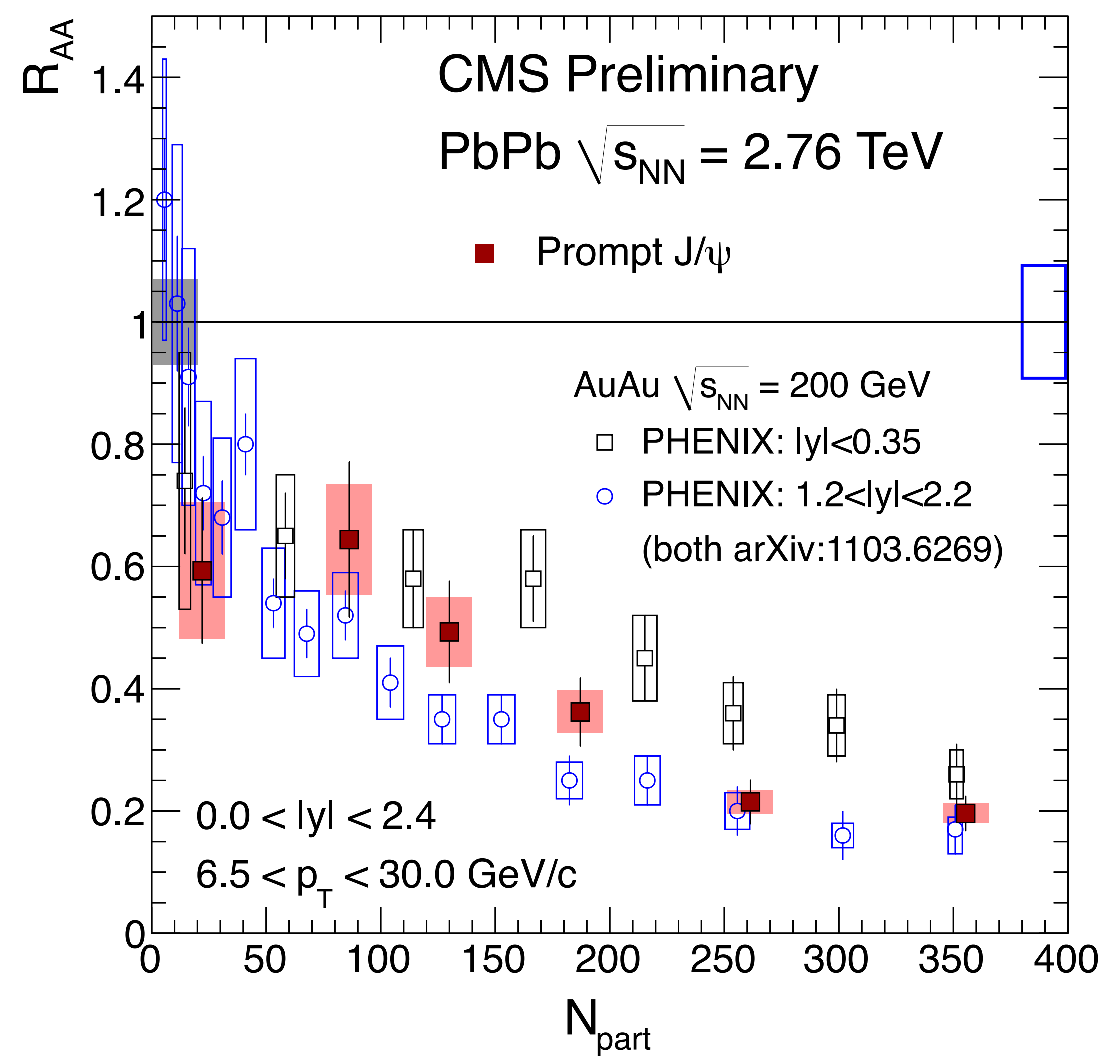


PHENIX **1103.6269**

Charmonium suppression in experiments

- J/ψ suppression has been measured at SPS, RHIC and now LHC. SPS~RHIC

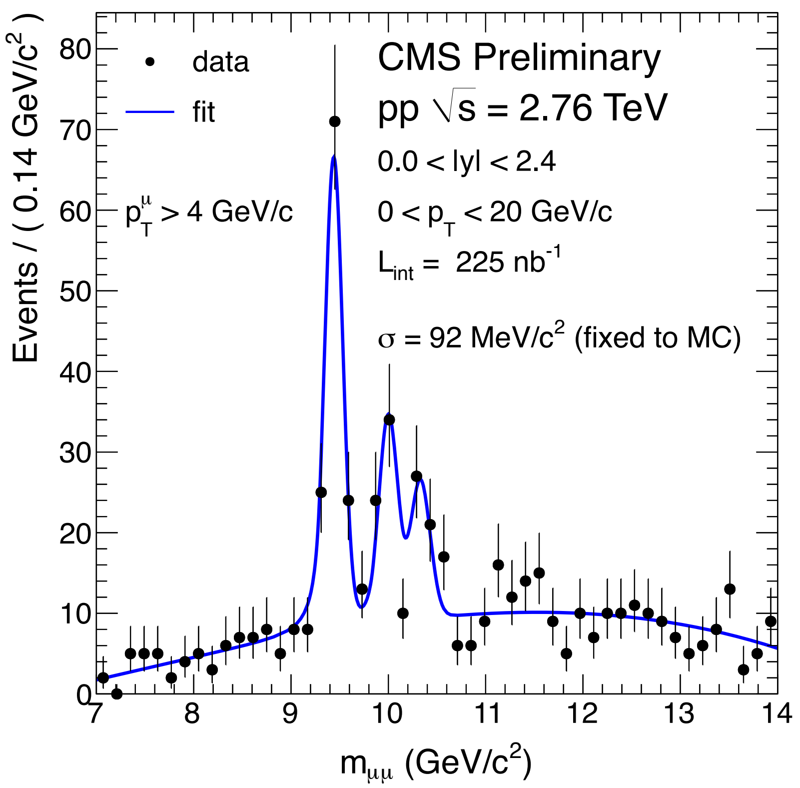


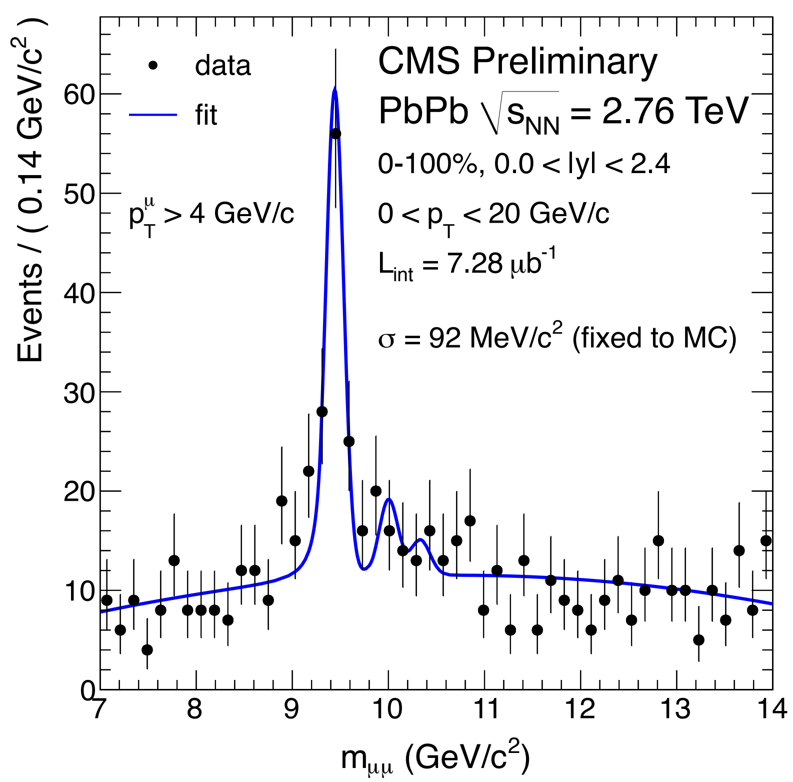


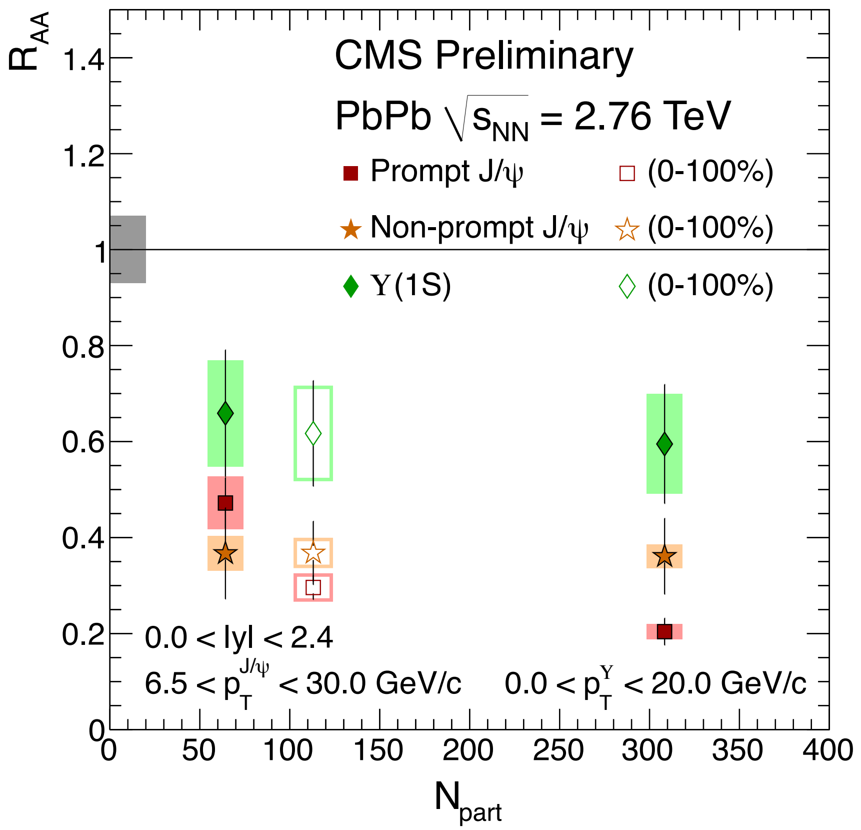
- Nuclear modification factor $R_{AA} \equiv \dfrac{\text{Yield}_{AA}}{\text{Yield}_{pp} \times N_{\text{bin}}}$

Bottomonium suppression in experiments

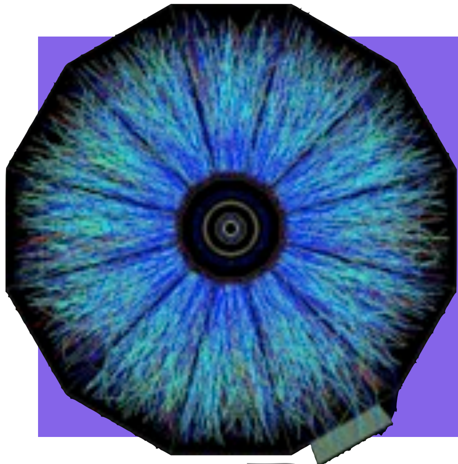
- First quality data on the Υ family from CMS





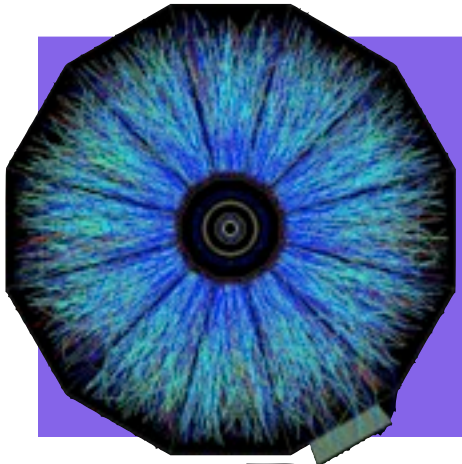


- Significant suppression of the Υ(2S) and Υ(3S)
  CMS, **PRL107** and CMS-PAS-HIN-10-006 (2011)



Quarkonium suppression

- Proposed in 1986 as a probe and "thermometer" of the medium produced by the collision
  Matsui Satz **PLB178** (1986)



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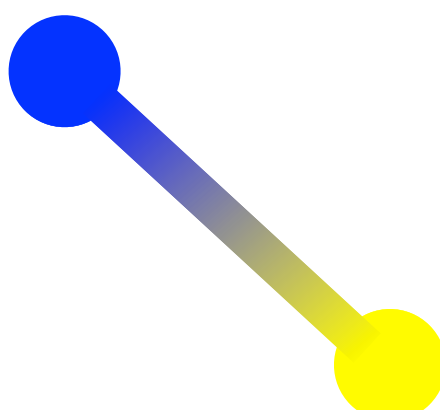

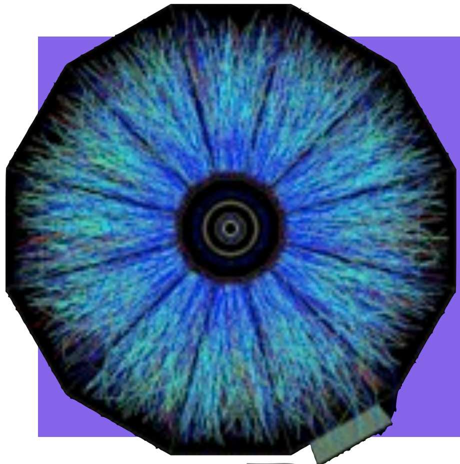

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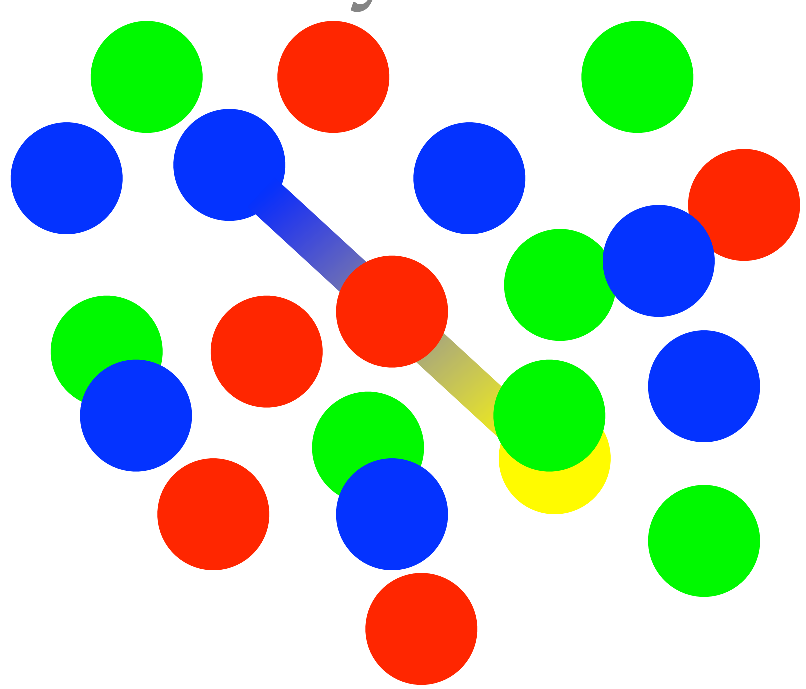

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \longrightarrow$$ Bound state dissolves

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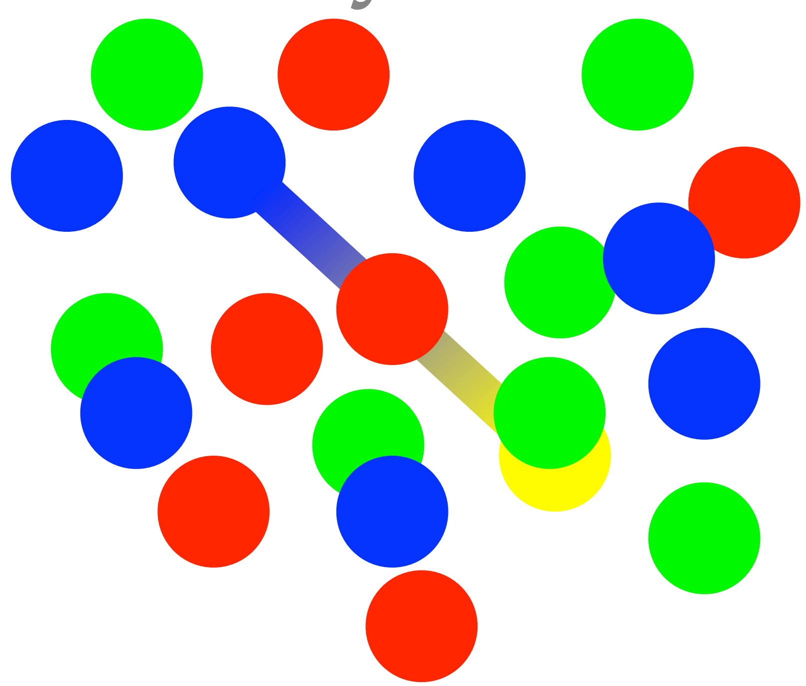

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- Studied with potential models, lattice spectral functions, AdS/CFT and now with EFTs

Potential models

- *Assumption:* Schrödinger equation with all medium effects encoded in T-dependent potential

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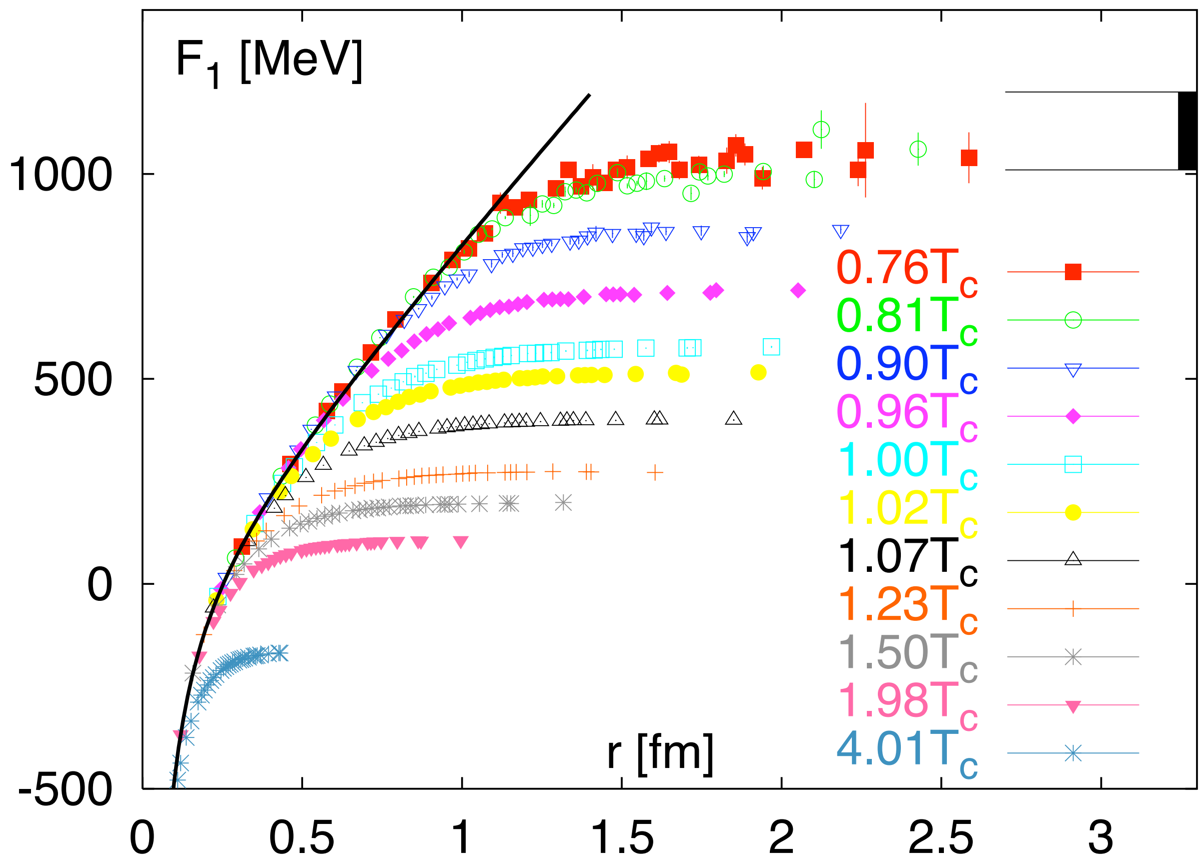

Kaczmarek Zantow 2005

Potential models

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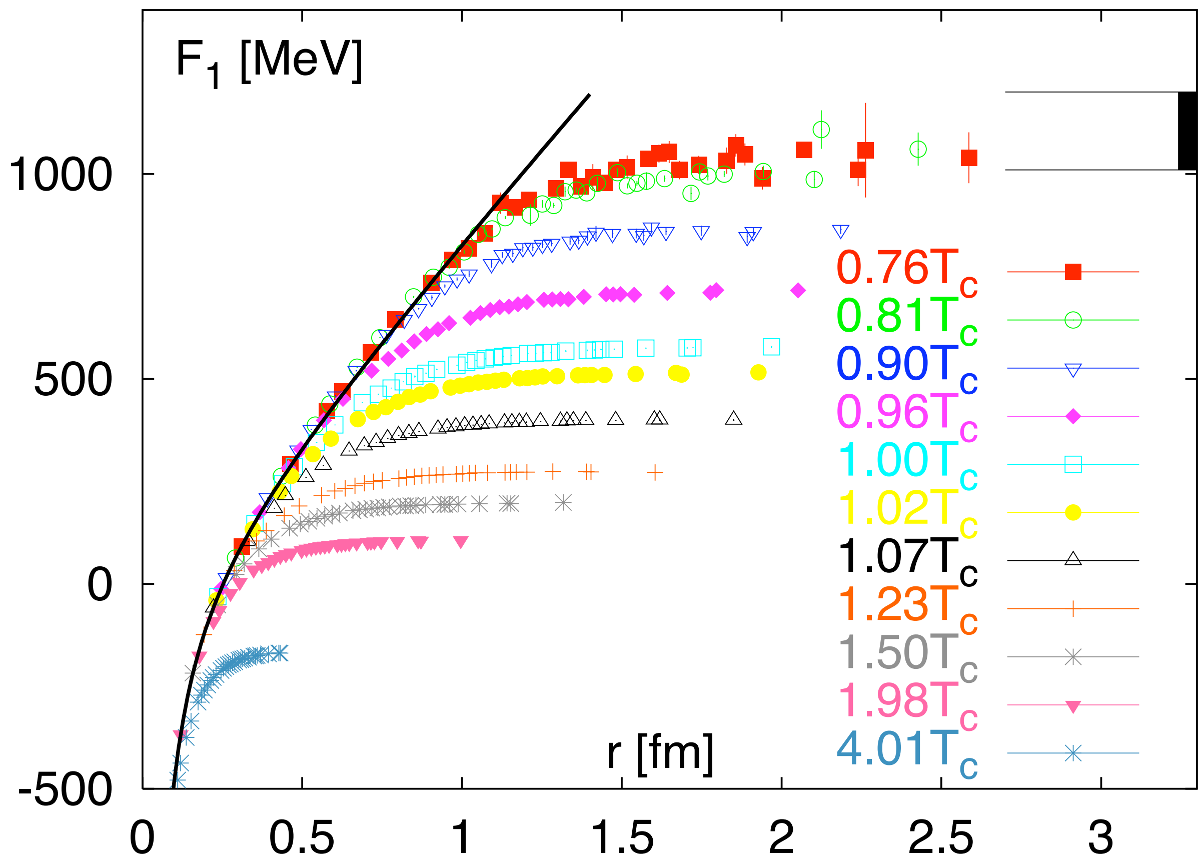
- Many different techniques and issues developed over the years: U vs F, gauge-dependent lattice correlators ....

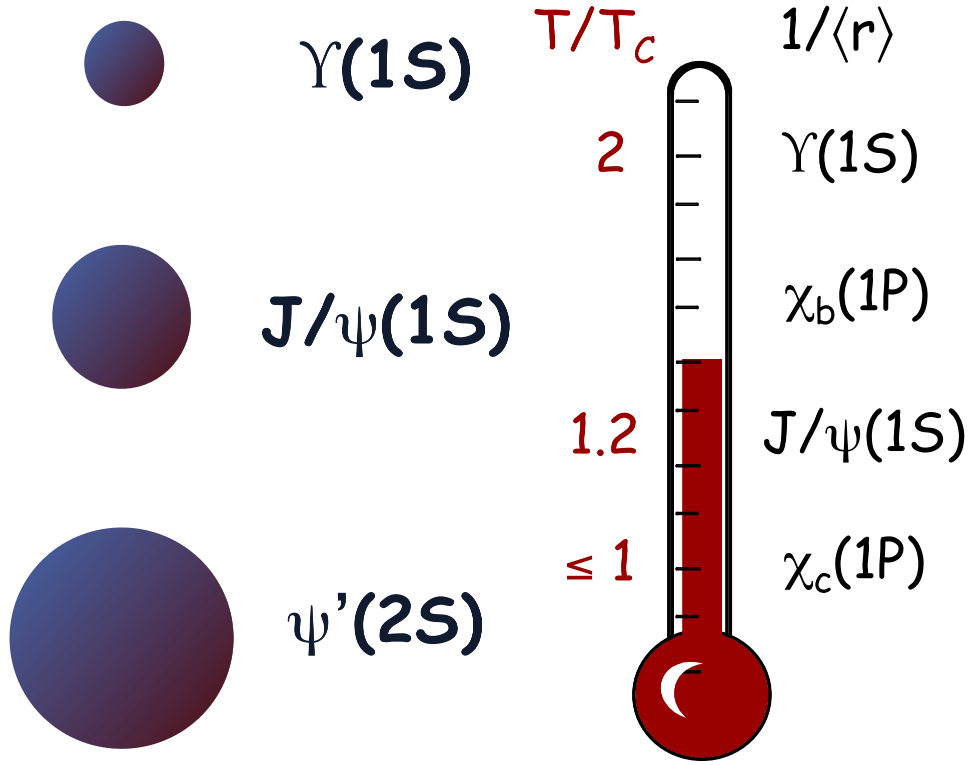


Kaczmarek Zantow 2005

Digal, Petreczky, Satz 01
Wong 05-07
Mannarelli, Rapp 05
Mocsy, Petreczky 05-08
Alberico, Beraudo, Molinari, de Pace 05-08
Cabrera, Rapp 2007
Wong, Crater 07
Dumitru, Guo, Mocsy, Strickland 09
Rapp, Riek 10

Potential models

- *Assumption:* Schrödinger equation with all medium effects encoded in T-dependent potential
- Potential extracted from lattice data of *ad-hoc* correlators
- Many different techniques and issues developed over the years: U vs F, gauge-dependent lattice correlators ....
- All models agree on a qualitative picture of sequential dissociation



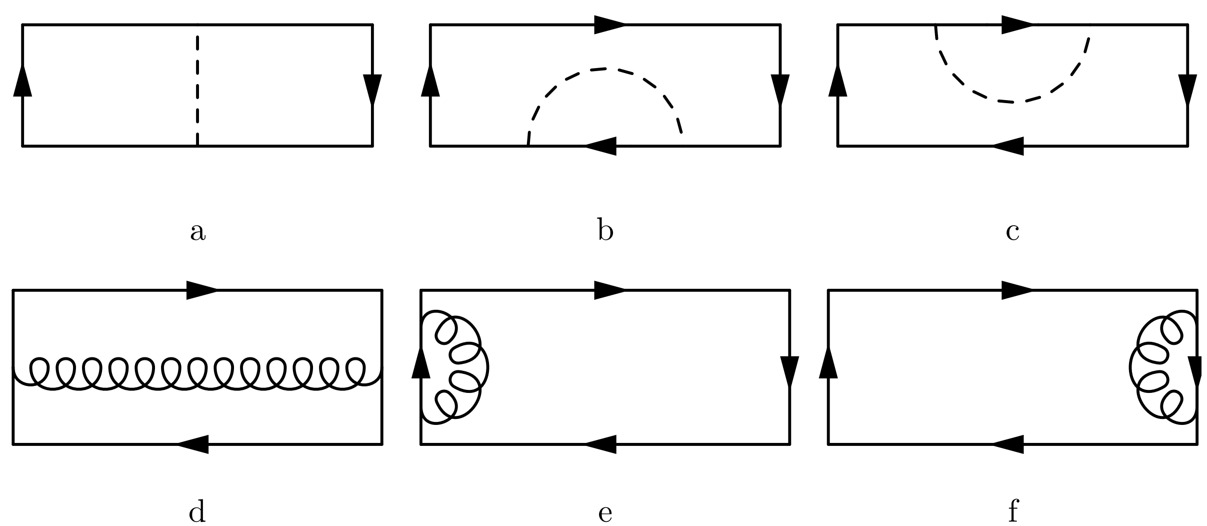
The real-time potential

- Perturbative computation of the real-time potential between a static quark and antiquark for *T>>1/r*:

$$V_{\text{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)$$

When $r \sim \frac{1}{m_D}$ $\text{Im}V \gg \text{Re}V$

Laine Philipsen Romatschke Tassler **JHEP0703** (2007)



$$W(\tau < 1/T) \to W(it), \quad t \to \infty$$

The EFT approach

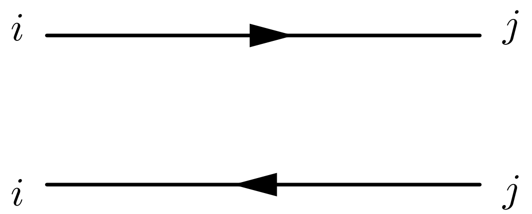
- Generalization of the successful framework of NR EFTs to finite temperature
- Rigorous definition of the potentials as Wilson coefficients of the EFT, with potential model picture as zeroth-order approximation
- Power counting and possibility of systematic improvement
- Potentials have real and imaginary parts. The real parts do not correspond to the thermodynamical free energies measured on the lattice
  Brambilla JG Petreczky Vairo 2008-10, Escobedo Soto 2008-10, Brambilla Escobedo JG Soto Vairo 2010, Brambilla Escobedo JG Vairo 2011, Escobedo Mannarelli Soto 2011

The cyclic Wilson loop

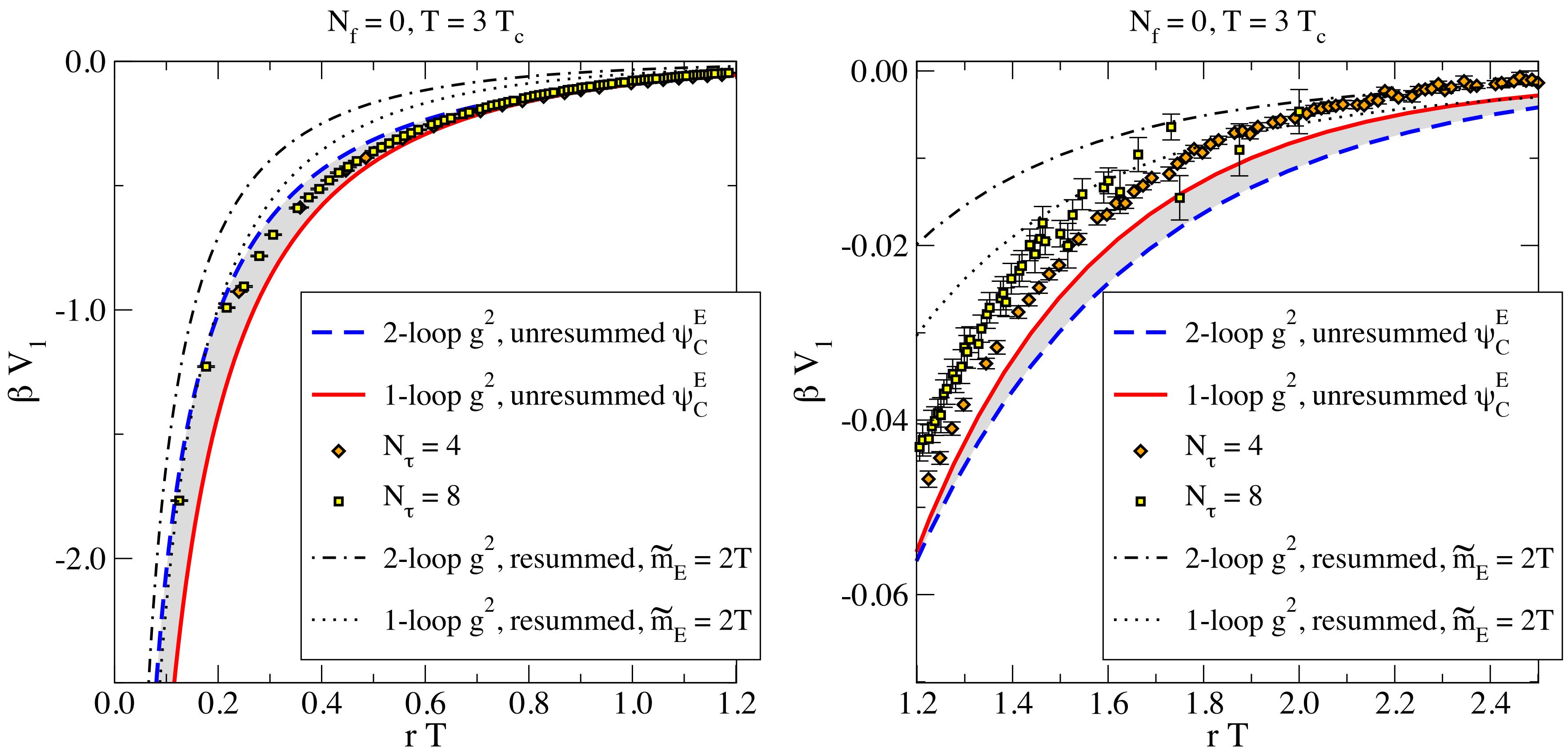
Thermodynamical free energies

- The "singlet free energy"

$$\langle \mathrm{Tr}\, L(\mathbf{x}) L^\dagger(\mathbf{0}) \rangle \qquad L = P \exp\left(ig \int_0^\beta d\tau A^0(\tau, \mathbf{x}) \right)$$



Gauge dependent, Coulomb gauge popular

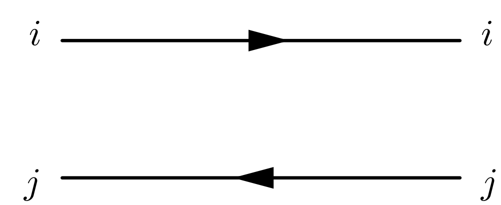


Perturbative: Burnier Laine Vepsäläinen **JHEP1001** (2010)

Lattice: Kaczmarek Karsch Petreczky Zantow **PLB243** (2002)

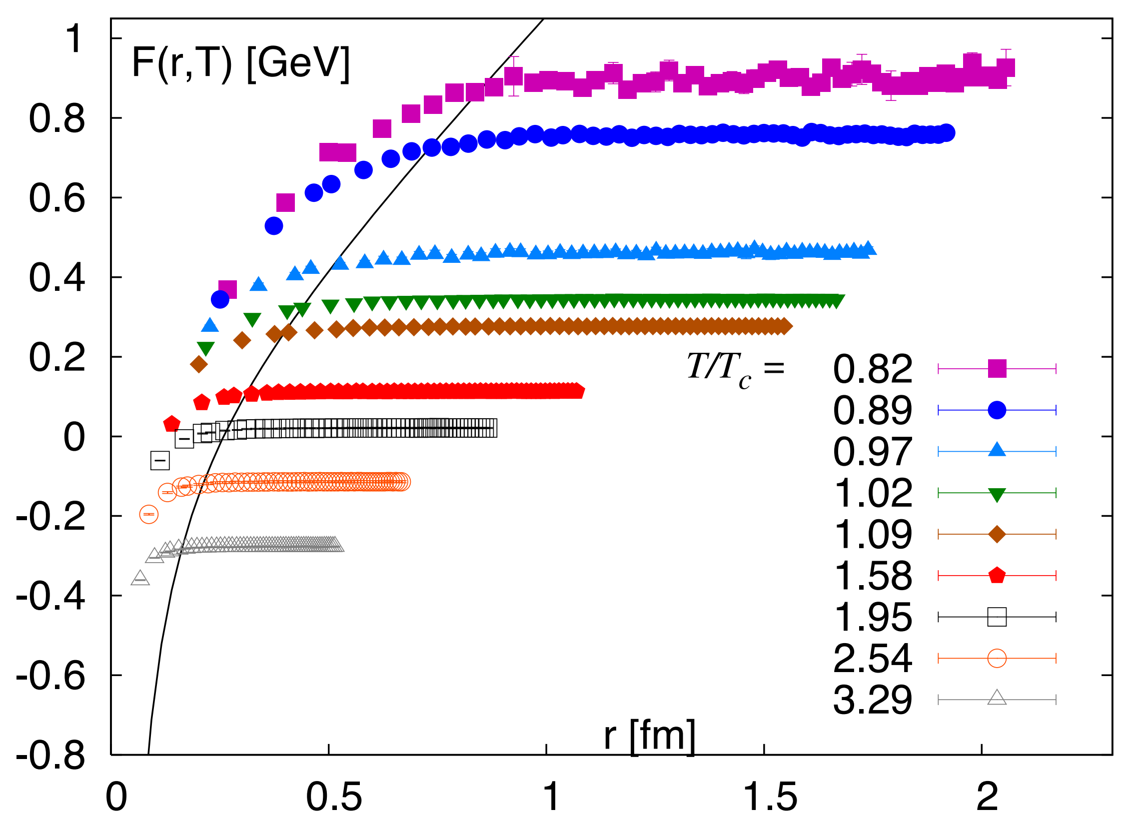
Thermodynamical free energies

- Correlator of two Polyakov loops: (difference in) free energy of a quark-antiquark pair



$$\langle \mathrm{Tr}\, L(\mathbf{x})\, \mathrm{Tr}\, L^\dagger(\mathbf{0}) \rangle$$

Gauge independent and well defined, but probes the octet sector as well



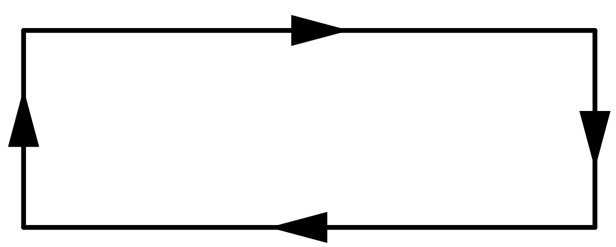
Petreczky 1001.5284

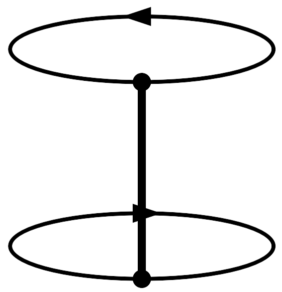
- Perturbation theory at short distances/EFT analysis Brambilla JG Petreczky Vairo **PRD82** (2010)
- Intermediate distances $r \sim 1/m_D$ Nadkarni **PRD33** (1986)
- Large distances $r \gg 1/m_D$ Braaten Nieto **PRL74** (1995)

Thermodynamical free energies

- The Cyclic Wilson loop: a gauge invariant completion of the singlet free energy

$$W_c \equiv \frac{1}{N_c}\langle \mathrm{Tr}\, U(\tau=0;\mathbf{0},\mathbf{r})L(\mathbf{r})U^\dagger(\tau=0;\mathbf{0},\mathbf{r})L^\dagger(\mathbf{0})\rangle$$



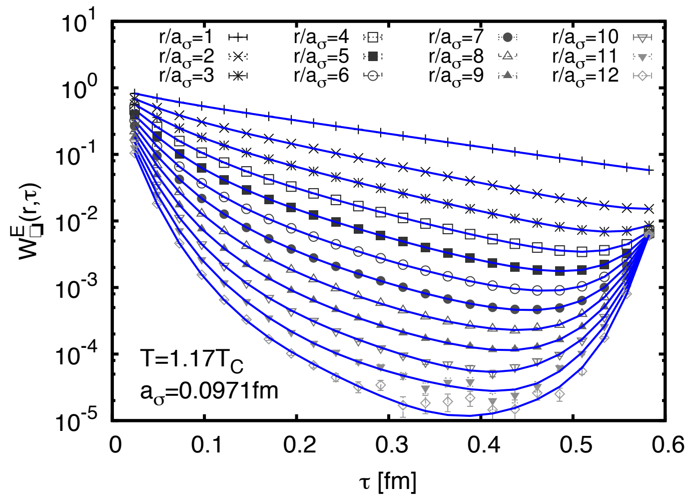


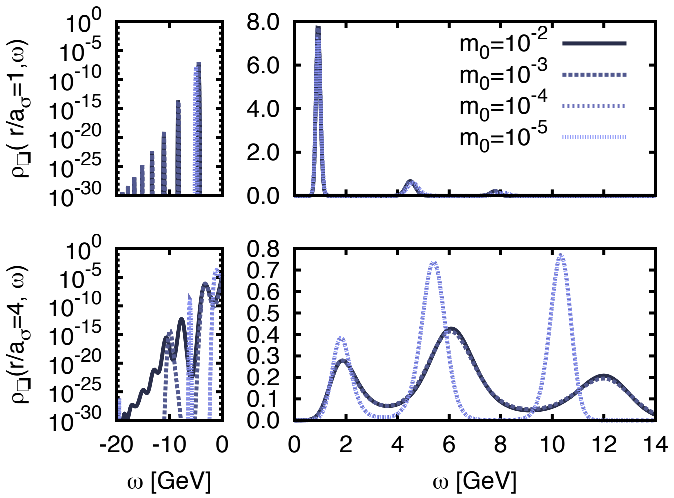
- It corresponds to two Polyakov lines connected by an adjoint spacelike Wilson line
- The restored gauge invariance comes at a price: no longer a simple QQbar free energy and additional divergences
- The renormalization of this object is our goal

The real-time potential from the lattice

- **Rothkopf Hatsuda Sasaki 1108.1579**: determine the static potential on the lattice by extracting the spectral representation of the Wilson loop with the Maximum Entropy Metod

$$W_{\square}^{\mathrm{E}}(r,\tau) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_{\square}(r,\omega).$$





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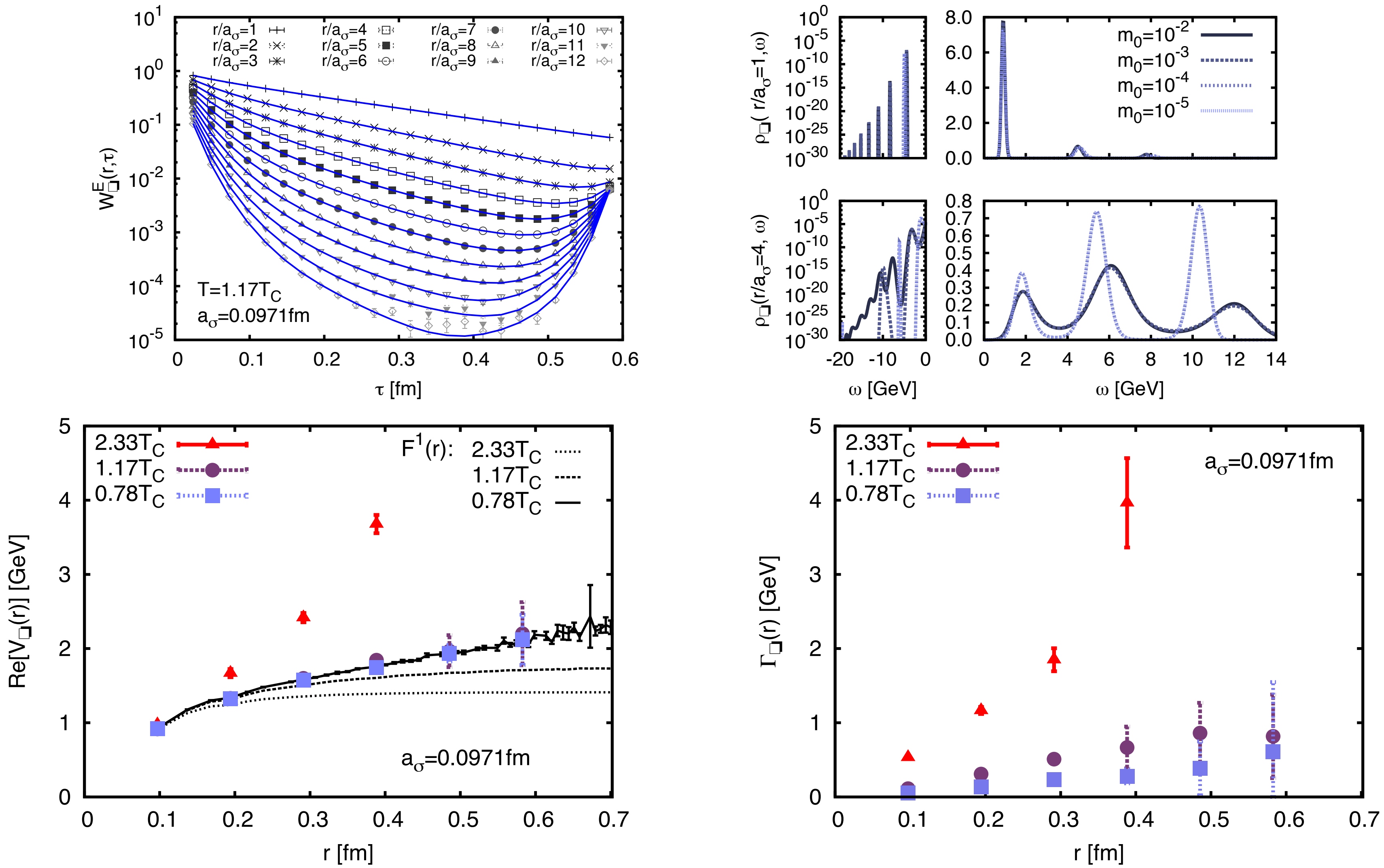


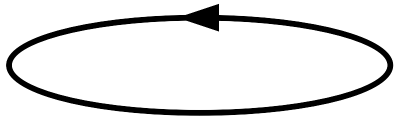
Motivation

- Continue the program of comparison between perturbation theory and lattice for quarkonium-related quantities
- Relevance for the analytical continuation/MEM program (last point is the cyclic Wilson loop)
- Possible relevance for the null Wilson loop community?

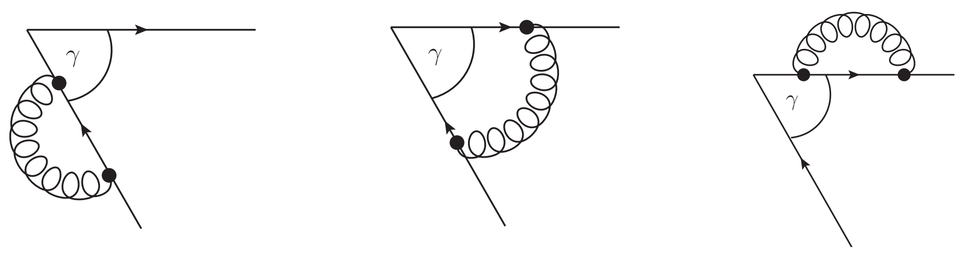
Divergences in the cyclic Wilson loop

Renormalization of Wilson loops

- A Wilson loop with a smooth, nonintersecting contour is finite in DR after charge renormalization



- Cusps in the contour introduce UV *cusp divergences,* renormalized multiplicatively through the *cusp anomalous dimension*, which only depends on the angle. Known in QCD to NLO



$$\frac{\alpha_s C_F}{2\pi\epsilon}\left(1+(\pi-\gamma)\cot\gamma\right)$$

Polyakov **NPB84** (1980) Dotsenko Vergeles **NPB169** (1980) Brandt Neri Sato **PRD24** (1981) Korchemsky Radyushkin **NPB283** (1987)

The divergence in the cyclic loop

- Burnier Laine Vepsäläinen computed the loop for *rT~1* in **JHEP1001**. After charge renormalization the result was still UV divergent at order α_s^2

$$\ln\left(\frac{\psi_W(r)}{|\psi_P|^2}\right) \approx \mathcal{G}_{DR}\left(\frac{1}{\epsilon},\frac{\bar\mu}{T},rT\right)\frac{C_F\exp(-m_E r)}{4\pi T r} - \frac{g^4 C_F N_c}{(4\pi)^2}\frac{\exp(-2m_E r)}{8T^2r^2}$$
$$+\frac{g^4 C_F N_c}{(4\pi)^2}\left\{\frac{2\mathrm{Li}_2(e^{-2\pi Tr})+\mathrm{Li}_2(e^{-4\pi Tr})}{(2\pi Tr)^2}\right.$$
$$\left.+\frac{1}{\pi Tr}\int_1^\infty \mathrm{d}x\left[\frac{1}{x^2}\ln\left(1-e^{-2\pi Trx}\right)+\left(\frac{1}{x^2}-\frac{1}{2x^4}\right)\ln\left(1-e^{-4\pi Trx}\right)\right]\right\}$$
$$+\frac{g^4 C_F N_f}{(4\pi)^2}\left[\frac{1}{2\pi Tr}\int_1^\infty \mathrm{d}x\left(\frac{1}{x^2}-\frac{1}{x^4}\right)\ln\frac{1+e^{-2\pi Trx}}{1-e^{-2\pi Trx}}\right]+\mathcal{O}(g^5)\ .$$

$$\mathcal{G}_{DR}\left(\frac{1}{\epsilon},\frac{\bar\mu}{T},rT\right)\overset{m_E r\ll 1}{\approx} g^2\left\{1+\frac{g^2}{(4\pi)^2}\left[4N_c\left(\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{T^2}+\mathcal{O}(1)\right)\right]\right\}$$

$$\mathcal{G}_{DR}\left(\frac{1}{\epsilon},\frac{\bar\mu}{T},rT\right)\overset{m_E r\gg 1}{\approx} g^2\left\{1-\frac{g^2 N_c T}{8\pi}\left[r\left(\frac{1}{\epsilon_{UV}}+\ln\frac{\bar\mu^2}{m_E^2}+\mathcal{O}(1)\right)\right]\right\}$$

The divergence in the cyclic loop

- We perform a calculation for $rT \ll 1$, focusing only on the UV aspects and on the contribution from the scale $1/r$.

$$\ln W_c = \frac{C_F \alpha_s}{rT}\left\{1 + \frac{\alpha_s}{4\pi}\left[\left(\frac{31}{9}C_A - \frac{20}{9}T_F n_f\right) + \beta_0\left(\ln\mu^2 r^2 + 2\gamma_E\right)\right]\right\}$$
$$+ \frac{4\pi C_F \alpha_s}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}} - 1}{(\mathbf{k}^2)^2}\left(-\Pi^{(T)}_{00\,\mathrm{CG}}(0,\mathbf{k})\right) + C_F C_A \alpha_s^2$$
$$+ \frac{4C_F C_A \alpha_s^2}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}}{\mathbf{k}^2}\left[\frac{1}{\epsilon} + 1 + \gamma_E + \ln\pi + \ln\mu^2 r^2\right]$$
$$+ \frac{2C_F C_A \alpha_s^2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \zeta(2n)}{n(4n^2-1)}(rT)^{2n-1}$$

The divergent terms agree. The divergence is UV and cannot be renormalized multiplicatively

Origin of the divergence

- In Coulomb gauge the singlet free energy is finite

$$\ln\langle \mathrm{Tr} L(\mathbf{r}) L^\dagger(\mathbf{0})\rangle = \frac{C_F\alpha_s}{rT}\left\{1+\frac{\alpha_s}{4\pi}\left[\left(\frac{31}{9}C_A-\frac{20}{9}T_F n_f\right)+\beta_0\left(\ln\mu^2 r^2+2\gamma_E\right)\right]\right\}$$

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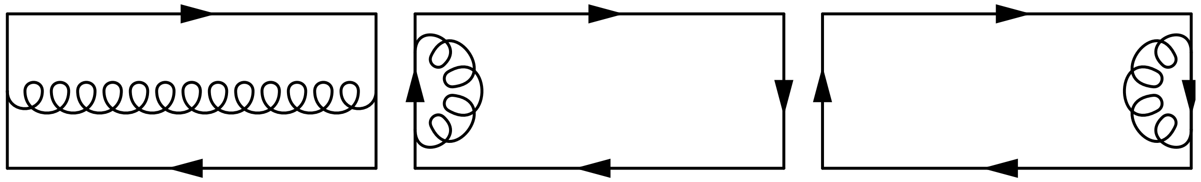
- Add the strings: a lot of diagrams cancel because of cyclicity (all those where the two strings are connected on at least one side by the singlet component of a Polyakov line)



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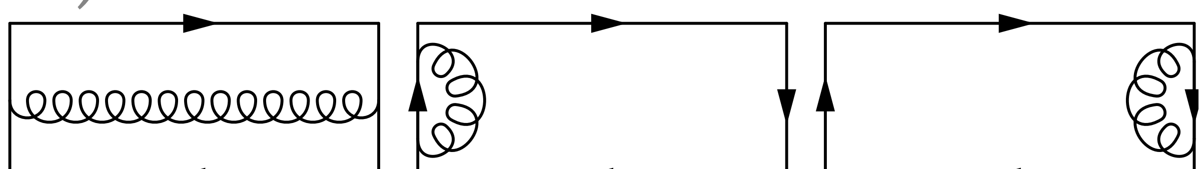
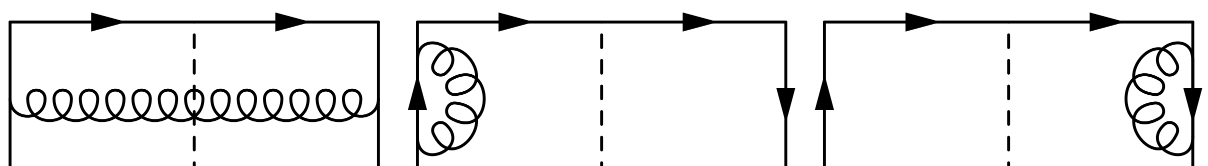


- The divergence is then given by these diagrams

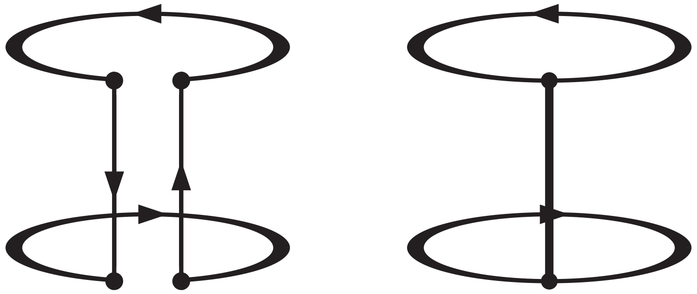


$C_F^2 - C_F C_A/2$ $\quad$ C_F^2 $\quad$ C_F^2

Thinking cylindrically

Renormalization

- The divergence is related to the cusp divergence, but not quite the same. Indeed, thinking cylindrically, the cyclic Wilson loop is topologically different from a regular one



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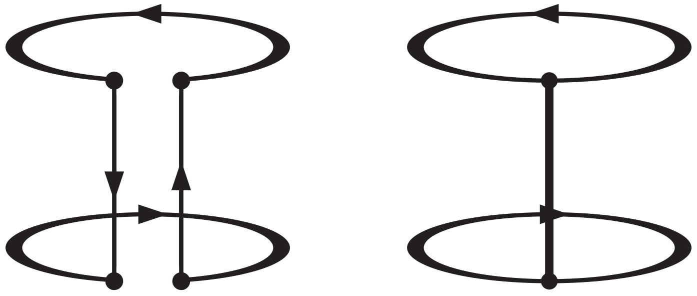

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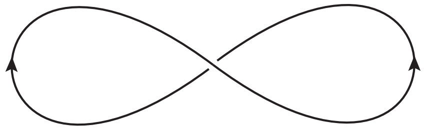
- Wilson loops with intersections are renormalized in matrix form, by considering all possible choices of paths at the intersection



Brandt Neri Sato **PRD24** (1981)

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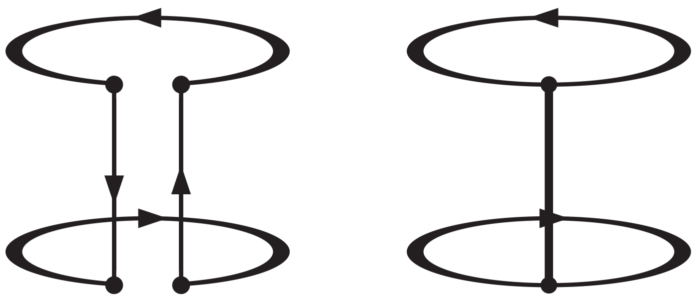
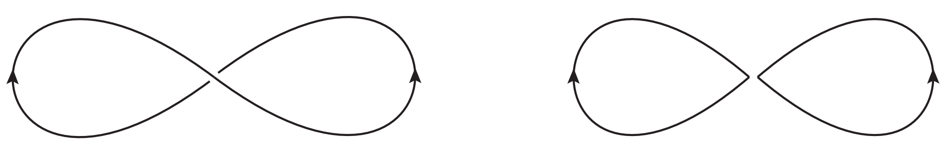


- It does not have cusps, but a continuous set of intersections.
- Wilson loops with intersections are renormalized in matrix form, by considering all possible choices of paths at the intersection

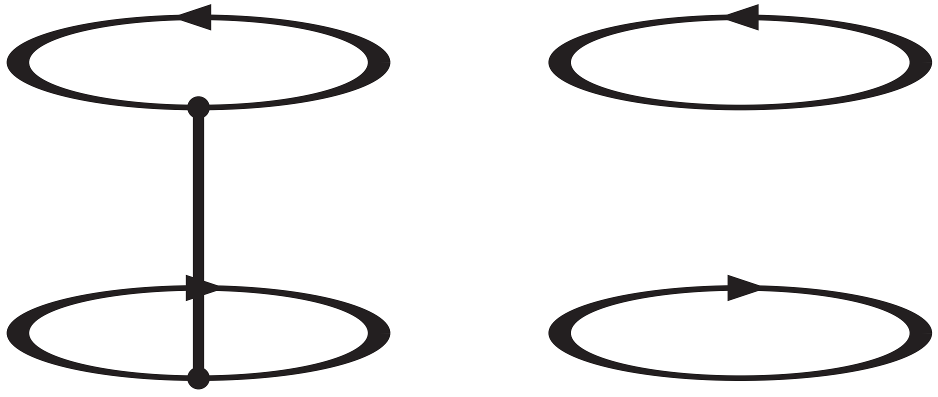


$$W^i_R = Z^{ij}(\theta) W^j$$

Brandt Neri Sato **PRD24** (1981)

Renormalization

- The procedure is the same in the case of n intersections. In our case in principle $n=\infty$, but in practice there are only two independent paths:



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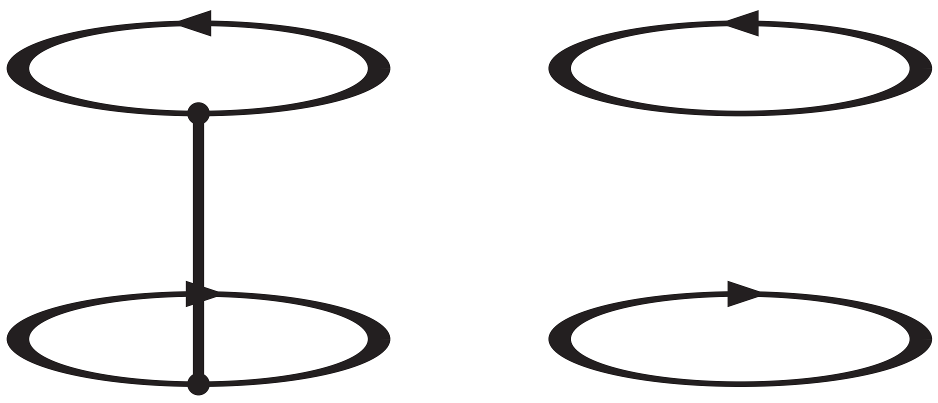


- They are the cyclic loop (W_c) and the correlator of two Polyakov loops (C_{PL}). The latter being finite, the renormalization matrix reads

$$\begin{pmatrix} W_c^R \\ C_{PL} \end{pmatrix} = \begin{pmatrix} Z & (1-Z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} W_c \\ C_{PL} \end{pmatrix}$$

Intermediate summary

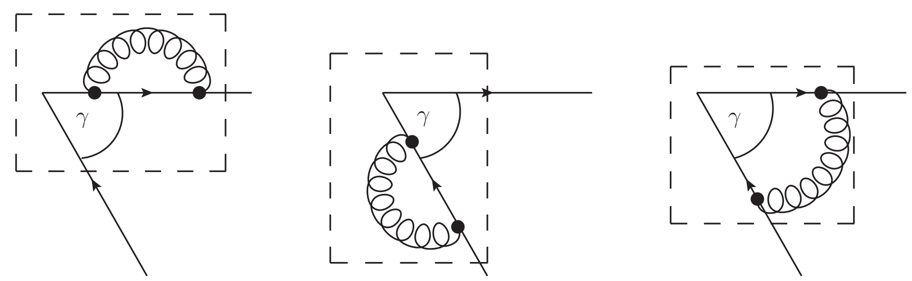
- We have obtained that the cyclic Wilson loop is not renormalized multiplicatively. Due to the periodic boundary conditions, it mixes with the Polyakov loop correlator under renormalization.

$$W_c^R = Z\, W_c + (1 - Z)\, C_{PL}$$

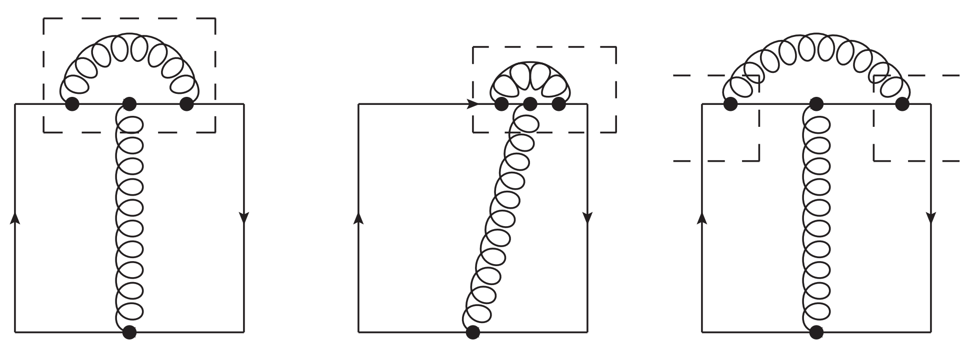
- This renormalization prescription is valid at weak and strong coupling
- We are now going to test it in perturbation theory

Cylindrical divergences

- The standard cusp divergence arises when all vertices are contracted at the singular point



- In the case of the intersection, one always has to think cylindrical.



- Only the last diagram contributes to the intersection divergence

Perturbative renormalization

Building blocks

- The Polyakov loop correlator at order α_s^2 at short distances

$$C_{PL} = 1 + C_F \alpha_s \frac{m_D}{T} + C_F \alpha_s^2 \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right] + \frac{N_c^2 - 1}{8 N_c^2} \frac{\alpha_s^2}{(rT)^2}$$

$$C_{PL} = 1 + \mathcal{O}\left(g^3\right)$$

McLerran Svetitsky **PRD24** (1981) Gross Pisarski Yaffe **PRD24** (1981) Brambilla JG Petreczky Vairo **PRD82** (2010) Burnier Laine Vepsäläinen **JHEP1001** (2009)

- Expansion of the renormalization constant

$$Z \equiv 1 + Z_1 \alpha_s + Z_2 \alpha_s^2 + \ldots$$

- We now evaluate Z_1

Leading-order renormalization

- The renormalization equation gives

$$W_c^R = ZW_c + (1-Z)C_{PL}$$
$$= 1 + \frac{C_F\alpha_s}{rT} + \frac{C_F^2\alpha_s^2}{2r^2T^2}$$
$$+ \frac{4\pi C_F\alpha_s}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}}{\mathbf{k}^2}\left(\frac{C_A\alpha_s}{\pi\epsilon} + Z_1\alpha_s + \ldots\right) + \ldots$$

$$C_{PL} = 1 + \mathcal{O}(g^3)$$
$$Z \equiv 1 + Z_1\alpha_s + \ldots$$

- This implies

$$Z_1 = -\frac{C_A}{\pi}\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

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\begin{aligned}
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$$

$$
\begin{aligned}
\ln W_c^R = \frac{C_F\alpha_s}{rT}\Bigg\{1 &+ \frac{\alpha_s}{4\pi}\left[\left(\frac{31}{9}C_A - \frac{20}{9}T_Fn_f\right) + \beta_0\left(\ln\mu^2r^2 + 2\gamma_E\right)\right] \\
&+ \frac{\alpha_s C_A}{\pi}\left[1 + 2\gamma_E - \ln 4 + 2\ln\mu^2r^2 + \sum_{n=1}^{\infty}\frac{2(-1)^n\zeta(2n)}{n(4n^2-1)}(rT)^{2n}\right]\Bigg\} \\
&+ \frac{4\pi\alpha_s C_F}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}-1}{(\mathbf{k}^2)^2}\left(-\Pi_{00\,CG}^{(T)}(0,\mathbf{k})\right) + C_FC_A\alpha_s^2
\end{aligned}
$$

Going to higher orders

$$C_{PL} = 1 + C_F\alpha_s\frac{m_D}{T} + C_F\alpha_s^2\left[C_A\left(\ln\frac{m_D^2}{T^2}+\frac{1}{2}\right) - n_f\ln 2\right] + \frac{N_c^2-1}{8N_c^2}\frac{\alpha_s^2}{(rT)^2}$$

- Non-trivial check of the renormalization equation by considering higher-order divergences

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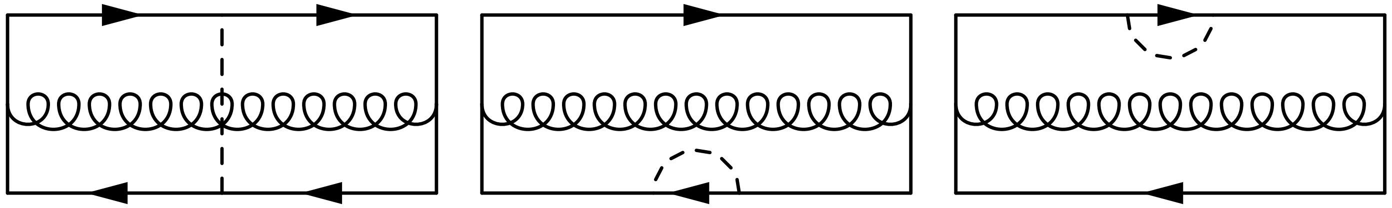

- In these diagrams the IR cancellation breaks down in the non-Abelian term

$$C_F (e^{i\mathbf{k}\cdot\mathbf{r}} - 1) \to C_F^2 (e^{i\mathbf{k}\cdot\mathbf{r}} - 1) - \frac{C_F C_A}{2} e^{i\mathbf{k}\cdot\mathbf{r}}$$

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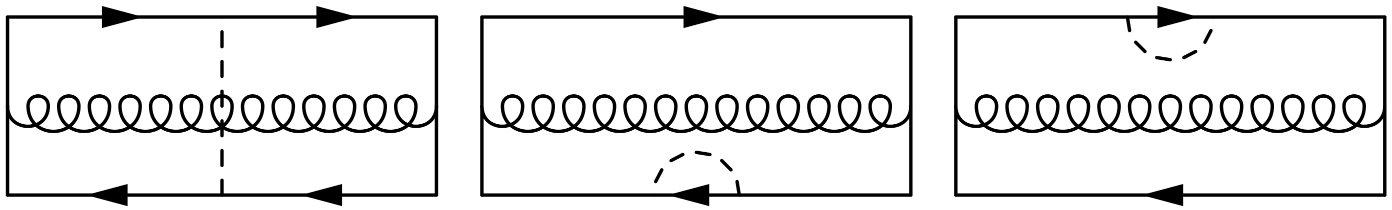

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- This results in a UV-divergent contribution

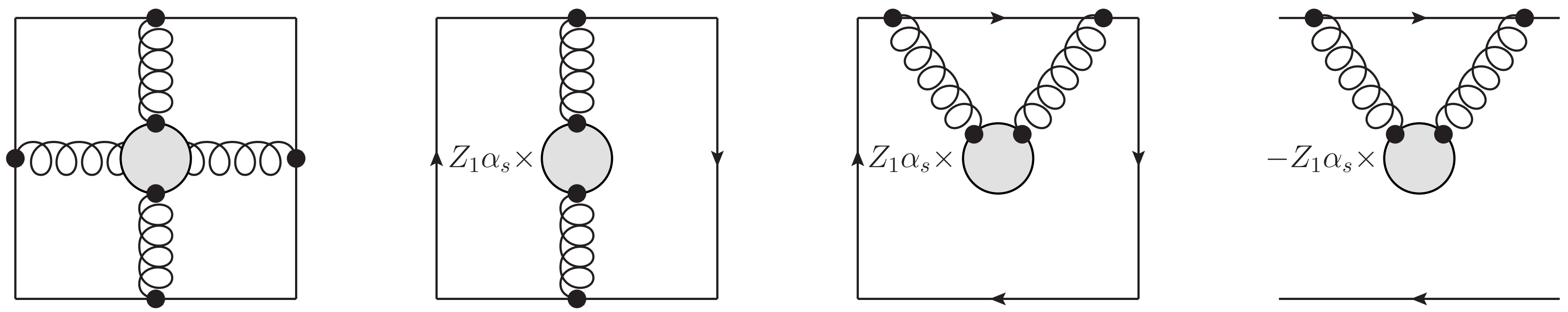
$$\frac{C_F C_A \alpha_s^2}{T} \int_k \frac{1}{k^2 + m_D^2} \left(\frac{1}{\epsilon} + \ldots \right) = Z_1 C_F \alpha_s^2 \frac{m_D}{T}$$

Divergences at order g^6

- We have carried out a full study of the cancellation of divergences at order g^6
- There one has again iterations of the leading-order divergence, cancelled by Z_1, and new divergences, cancelled by Z_2 (undetermined)
- The analysis is based on the topological classification of divergent graphs
- The analysis is gauge-invariant. For illustration let me show some examples in Coulomb gauge

Divergences at order g^6

- The cancellation shown before at order g^5 carries through to all orders



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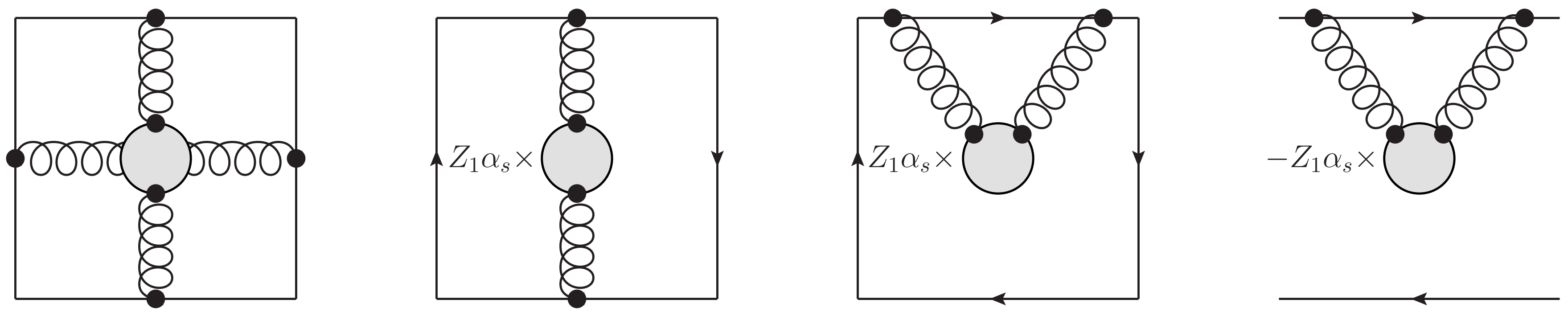
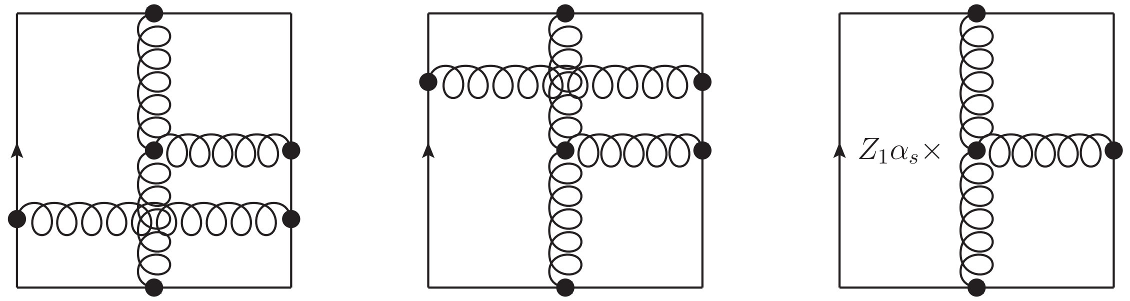


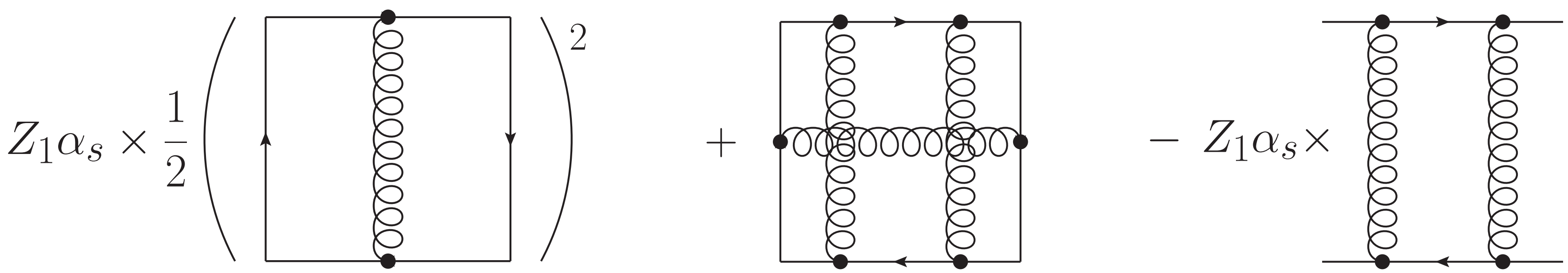
- New divergences combining divergent and finite diagrams



$$-\frac{C_F C_A \alpha_\mathrm{s}^3}{2\pi\epsilon} = -Z_1\alpha_\mathrm{s}\left(-\frac{1}{2}C_F C_A \alpha_\mathrm{s}^2\right)$$

Divergences at order g^6

- The two-gluon exchange term in C_{PL} enters in



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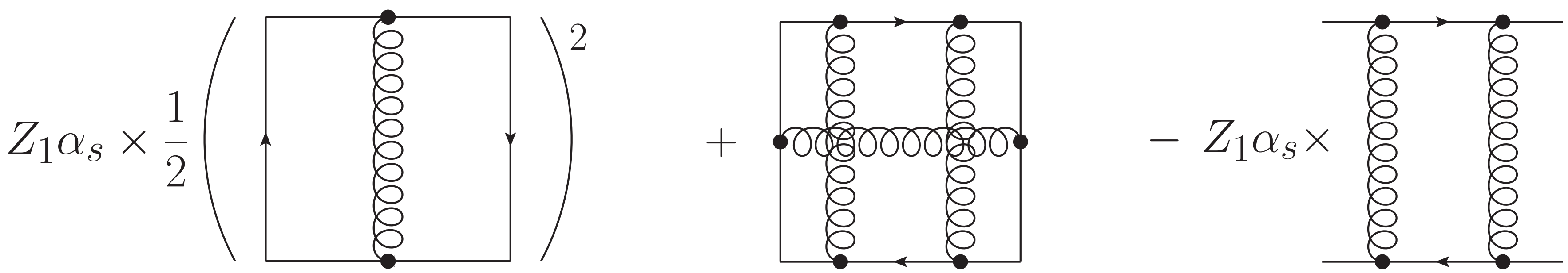
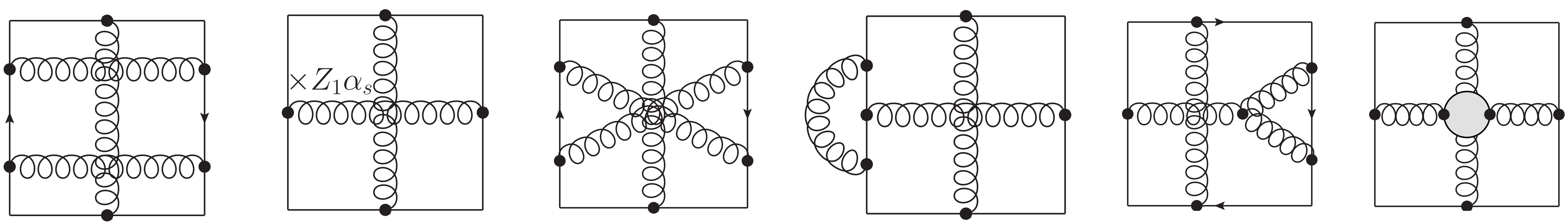


- Z_2 has to be determined from



completing the renormalization procedure to order g^6

Conclusions

- We have derived a generic renormalization equation for the cyclic Wilson loop, showing how it mixes with the Polyakov loop correlator
- We have tested this procedure in perturbation theory, determining the leading-order renormalization constant
- In order to match perturbative and lattice data, a non-trivial matching of the renormalization schemes in the two cases needs to be performed