Heavy quarkonium and the renormalization of the cyclic Wilson loop

Jacopo Ghiglieri, McGill University in collaboration with M. Berwein, N. Brambilla and A. Vairo INT, April 2nd 2012

Outline

- Introduction to quarkonium in deconfined media
- Introduction to the cyclic Wilson loop
- Divergences in the cyclic Wilson loop
- Renormalization

Quarkonium in media: the long story (made short)

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J/w SUPPRESSION BY QUARK-GLUON PLASMA FORMATION $\dot{\bm{\pi}}$

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If high energy heavy ion collisions lead to the formation of a hot quark-gluon plasma, then colour screening prevents ce binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark-gluon plasma formation.

Quarkonium suppression and RHIC

- Experimental data show a suppression pattern
- A good understanding of suppression requires understanding of
	- Production and cold nuclear matter effects
	- In-medium bound-state dynamics
	- Recombination effects

 \bullet ...

Charmonium suppression in experiments

 $\overline{}$ $I(u)$ cunniscript has hear r reduced the tracking chambers in each station. The $\frac{1}{2}$ • *J/!* suppression has been measured at SPS, RHIC and now LHC. SPS~RHIC

a function of the collision of the collision centrality. A 4.4 efficiency loss in the most central bin α

Bottomonium suppression in experiments SMILIM CHAMMACCION IN AVISA *^µ*+*µ* pair with *[|]y[|] <* 2.4 with 6.5 *< ^p*^T *<* 30 GeV/*^c* and 0–100% centrality.

• First quality data on the Y family from CMS

20

 $m \circ f$ the $\mathcal{N}(\mathcal{DC})$ on $\mathcal{N}(\mathcal{DC})$ • Significant suppression of the *Y*(2S) and *Y*(3S) CMS, **PRL107** and CMS-PAS-HIN-10-006 (2011)

 \circledcirc

• Proposed in 1986 as a probe and "thermometer" of the medium produced by the collision Matsui Satz **PLB178** (1986)

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$$
V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}
$$

$$
r \sim \frac{1}{m_D} \longrightarrow \text{Bound state}
$$

 \circledcirc

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 $r \sim$ 1 *m^D* Bound state dissolves

• Studied with potential models, lattice spectral functions, AdS/CFT and now with EFTs

• *Assumption:* Schrödinger equation with all medium effects encoded in *T*-dependent potential

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- *Assumption:* Schrödinger equation with all medium effects encoded in *T*-dependent potential
- Potential extracted from lattice data of *ad-hoc* correlators
- Many different techniques and issues developed over the years: *U* vs *F*, gauge-dependent lattice correlators
- All models agree on a qualitative picture of sequential dissociation

The real-time potential

• Perturbative computation of the real-time potential between a static quark and antiquark for *T>>1/r*:

$$
V_{\text{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)
$$

When Laine Philipsen Romatschke Tassler **JHEP0703** (2007) $r \sim$ 1 $\frac{1}{m_D}$ **I**m*V* \gg Re*V*

The EFT approach

- Generalization of the successful framework of NR EFTs to finite temperature
- Rigorous definition of the potentials as Wilson coefficients of the EFT, with potential model picture as zeroth-order approximation
- Power counting and possibility of systematic improvement
- Potentials have real and imaginary parts. The real parts do not correspond to the thermodynamical free energies measured on the lattice Brambilla JG Petreczky Vairo 2008-10, Escobedo Soto 2008-10, Brambilla Escobedo JG Soto Vairo 2010, Brambilla Escobedo JG Vairo 2011, Escobedo Mannarelli Soto 2011

The cyclic Wilson loop

Thermodynamical free energies Tree energies

The calculations in this paper have been performed in static gauge \mathcal{C} as \mathcal{C} defined as \mathcal{C}

Perturbative: Burnier Laine Vepsäläinen **JHEP1001** (2010) Lattice: Kaczmarek Karsch Petreczky Zantow **PLB243** (2002) k ² $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ \overline{t} = 3.75 \overline{t} and large (right) distances. The band corresponds to band : Kaczmarek Karsch Petreczky Zantow PLB "unresummed" results can only be applied at short distances, "resummed" ones only at large distances.

Thermodynamical free energies -0.2 0.97 \blacktriangleleft \blacksquare 1.58

• Correlator of two Polyakov loops: (difference in) free energy of a quark-antiquark pair $\qquad \qquad i \longrightarrow i$ $\langle \text{Tr } L(\mathbf{x}) \text{Tr } L^{\dagger}(\mathbf{0}) \rangle$ *j* $i \longrightarrow i$ j COTTURIOI OT LIVO I OTJU $r = r$ 3.29 0. (UNICICIRE III) HEE CIRIE rT $f(x) = \frac{1}{2}$ $L(X)$ if $L'(\mathbf{U})$ as function of j and the solution of j

Gauge independent and well defined, but probes the octet sector as well

-0.6

0.4

- \mathbf{J} r (Figure 11001) and the control of t
Transition of the control of the c 1.58 VS^2 distances/EFT analysis 3.29 Brambilla JG Petreczky Vairo • Perturbation theory at short **PRD82** (2010)
- Intermediate distances *r~1/ mD* Nadkarni **PRD33** (1986)
- 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 Large distances *r*≫*1/mD* Petreczky 1001.5284

Braaten Nieto PRI 74 (1995) ECZNY 1001.020⁴ Braaten Nieto **PRL74** (1995).

Thermodynamical free energies

• The Cyclic Wilson loop: a gauge invariant completion of the singlet free energy

$$
W_c \equiv \frac{1}{N_c} \langle \text{Tr} \, U(\tau = 0; \mathbf{0}, \mathbf{r}) L(\mathbf{r}) U^{\dagger}(\tau = 0; \mathbf{0}, \mathbf{r}) L^{\dagger}(\mathbf{0}) \rangle
$$

- It corresponds to two Polyakov lines connected by an adjoint spacelike Wilson line One can see that the cusp points turn into intersection points. The contour for the Polyakov loop
- The restored gauge invariance comes at a price: no longer a simple QQbar free energy and additional divergences
- The renormalization of this object is our goal possible path ordering prescriptions for contours that occupy the same points in space-time this object is our coal $\sum_{i=1}^n$ for the simplest case of $\sum_{i=1}^n$ summer that is $\sum_{i=1}^n$ such that is not intersects with its line in the smooth curve that is not intersect with its line is not intersect with its line in the simple st

The real-time potential from the **lattice** 0 0.1 0.2 0.3 0.4 0.5 0.6 10-30 $\overline{}$ 10-20 11 I C ρ \overline{a} 0.0 0.3 70 16 11 17 17 1 FIG. 1. (Left) Typical results for the Euclidean-time Wilson loop and the corresponding MEM reconstruction data indicated by \mathbf{r} is purely real and not symmetric in the symmetric induction and their prior dependence. Positive with a different with a different separately with a different vertical scale. Positive with a different vertical scale. Plants are plants are plants are plants and the scale. Plants are

• Rothkopf Hatsuda Sasaki 1108.1579: determine the static potential on the lattice by extracting the spectral representation of the Wilson loop with the Maximum Entropy Metod composed of three parts, ρloop, ρstaple and ρhandle, acaloop with the Maximum Entr

$$
W_{\square}^{\mathcal{E}}(r,\tau) = \int_{-\infty}^{+\infty} d\omega e^{-\omega \tau} \rho_{\square}(r,\omega).
$$

The real-time potential from the **lattice** 0 0.1 0.2 0.3 0.4 0.5 0.6 10-30 $\overline{}$ 10-20 11 I C ρ \overline{a} 0.0 0.3 70 16 11 17 17 1 FIG. 1. (Left) Typical results for the Euclidean-time Wilson loop and the corresponding MEM reconstruction data indicated by \mathbf{r} is purely real and not symmetric in the symmetric induction and their prior dependence. Positive with a different with a different separately with a different vertical scale. Positive with a different vertical scale. Plants are plants are plants are plants and the scale. Plants are

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W_{\square}^{\mathcal{E}}(r,\tau)=\int_{-\infty}^{+\infty}d\omega e^{-\omega\tau}\rho_{\square}(r,\omega).
$$

Motivation

- Continue the program of comparison between perturbation theory and lattice for quarkoniumrelated quantities
- Relevance for the analytical continuation/MEM program (last point is the cyclic Wilson loop)
- Possible relevance for the null Wilson loop community?

Divergences in the cyclic Wilson loop

Renormalization of Wilson loops

- A Wilson loop with a smooth, nonintersecting contour is finite in DR after charge renormalization
- Cusps in the contour introduce UV *cusp divergences,* renormalized multiplicatively through the *cusp anomalous dimension*, which only depends on the angle. Known in QCD to NLO \mathcal{F} for the non-cyclic (left) and the contours \mathcal{F} and the cyclic \mathcal{F} are shown. \Box can see that the cusp points turn into intersection points turn into intersection points. The \Box $\overline{}$ Cusp Divergences ² comes from the corners of dimensi

Polyakov **NPB84** (1980) Dotsenko Vergeles **NPB169** (1980) Brandt Neri Sato **PRD24** (1981) Korchemsky Radyushkin **NPB283** (1987) I VIYANOV INI DO. **Cusp divergence** ↵*sC^F* $M_{\rm H} = 1 - 1 - 1$ The C₁/1000

The divergence in the cyclic loop **l** liver den " , ce in the cy \blacksquare **STIC 100** div iverg len <u>|</u> ce in the c \mathbf{z} CIIC 10

particularly transparent, and may be ambiguous as just discussed, so we do not write it down

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• Burnier Laine Vepsäläinen computed the loop for $rT\sim1$ in **JHEP1001**. After charge renormalization the result was still UV divergent at order α_s^2 **IEP1001. Atter charge renorma** V divergent at order α^2

The function G in which the function G is which the complications are hidden, does have a simple expression in

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$$
\ln\left(\frac{\psi_{\rm W}(r)}{|\psi_{\rm P}|^2}\right) \approx \mathcal{G}_{\rm DR}\left(\frac{1}{\epsilon}\right)\frac{\bar{\mu}}{T}, rT\right) \frac{C_F \exp(-m_{\rm E}r)}{4\pi T r} - \frac{g^4 C_F N_{\rm c}}{(4\pi)^2} \frac{\exp(-2m_{\rm E}r)}{8T^2 r^2} \n+ \frac{g^4 C_F N_{\rm c}}{(4\pi)^2} \left\{ \frac{2 \text{Li}_2(e^{-2\pi T r}) + \text{Li}_2(e^{-4\pi T r})}{(2\pi T r)^2} + \frac{1}{\pi T r} \int_1^\infty dx \left[\frac{1}{x^2} \ln\left(1 - e^{-2\pi T r x}\right) + \left(\frac{1}{x^2} - \frac{1}{2x^4}\right) \ln\left(1 - e^{-4\pi T r x}\right) \right] \right\} \n+ \frac{g^4 C_F N_{\rm f}}{(4\pi)^2} \left[\frac{1}{2\pi T r} \int_1^\infty dx \left(\frac{1}{x^2} - \frac{1}{x^4} \right) \ln\frac{1 + e^{-2\pi T r x}}{1 - e^{-2\pi T r x}} \right] + \mathcal{O}(g^5) .
$$

$$
\mathcal{G}_{\text{DR}}\left(\frac{1}{\epsilon}, \frac{\bar{\mu}}{T}, rT\right) \stackrel{m_{\text{E}}r \ll 1}{\approx} g^2 \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[4N_c \left(\frac{1}{\epsilon} + \ln \frac{\bar{\mu}^2}{T^2} + \mathcal{O}(1) \right) \right] \right\}
$$
\n
$$
\mathcal{G}_{\text{DR}}\left(\frac{1}{\epsilon}, \frac{\bar{\mu}}{T}, rT\right) \stackrel{m_{\text{E}}r \gg 1}{\approx} g^2 \left\{ 1 - \frac{g^2 N_c T}{8\pi} \left[r \left(\frac{1}{\epsilon_{\text{UV}}} + \ln \frac{\bar{\mu}^2}{m_{\text{E}}^2} + \mathcal{O}(1) \right) \right] \right\}
$$

 $A_{\rm eff}$ is the dominant term of the dominant term of the coefficient function $G_{\rm eff}$ is the coefficient function $G_{\rm eff}$

The divergence in the cyclic loop

• We perform a calculation for *rT*≪*1*, focusing only on the UV aspects and on the contribution from the scale *1/r.*

$$
\ln W_c = \frac{C_F \alpha_s}{rT} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[\left(\frac{31}{9} C_A - \frac{20}{9} T_F n_f \right) + \beta_0 \left(\ln \mu^2 r^2 + 2\gamma_E \right) \right] \right\}
$$

+
$$
\frac{4\pi C_F \alpha_s}{T} \int_k \frac{e^{i\mathbf{r} \cdot \mathbf{k}} - 1}{(\mathbf{k}^2)^2} \left(-\Pi_{00\,CG}^{(T)}(0, \mathbf{k}) \right) + C_F C_A \alpha_s^2
$$

+
$$
\frac{4C_F C_A \alpha_s^2}{T} \int_k \frac{e^{i\mathbf{r} \cdot \mathbf{k}}}{\mathbf{k}^2} \left[\frac{1}{\epsilon} + 1 + \gamma_E + \ln \pi + \ln \mu^2 r^2 \right]
$$

+
$$
\frac{2C_F C_A \alpha_s^2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \zeta(2n)}{n(4n^2 - 1)} (rT)^{2n - 1}
$$

The divergent terms agree. The divergence is UV and cannot be renormalized multiplicatively

Origin of the divergence

• In Coulomb gauge the singlet free energy is finite $\ln \langle \text{Tr} L(\mathbf{r}) L^{\intercal}(\mathbf{0}) \rangle =$ $\frac{C_F \alpha_s}{rT}$ 1 + α_s 4π $\left[\left(\frac{31}{9}C_A-\frac{20}{9}\right)\right]$ $T_F n_f$ ◆ $+ \beta_0 \left(\ln \mu^2 r^2 + 2 \gamma_E \right)$ ן ן $+$ $4\pi C_F\alpha_s$ *T* Z *k* $e^{i\mathbf{r}\cdot\mathbf{k}}-1$ $({\bf k}^{2})^{2}$ $\left(-\Pi_{00\,\text{CG}}^{(T)}(0,\mathbf{k})\right)$ \setminus

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- Add the strings: a lot of diagrams cancel because of cyclicity (all those where the two strings are connected on at least one side by the singlet component of a Polyakov line)

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- Add the strings: a lot of diagrams cancel because of cyclicity (all those where the two strings are connected on at least one side by the singlet component of a Polyakov line)

• The divergence is then given by these diagrams

Thinking cylindrically

• The divergence is related to the cusp divergence, but not quite the same. Indeed, thinking cylindrically, the cyclic Wilson loop is topologically different from a regular one periodic boundary conditions: $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are identified by $\frac{1}{2}$ opologically differ

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- It does not have cusps, but a continuous set of intersections.
- Wilson loops with intersections are renormalized in matrix form, by considering all possible choices of paths at the intersection renormalization matrices at the 2 intersections must be identical \overline{C} form, by considering all f set of associated loops mix under renormalization

Brandt Neri Sato **PRD24** (1981) renormalization matrix depends only on intersection angles

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- Wilson loops with intersections are renormalized in matrix form, by considering all possible choices of paths at the intersection renormalization matrices at the 2 intersections must be identical \overline{C} *W* (*R*) form, by considering all f set of associated loops mix under renormalization

$$
\bigcirc \hspace{-7.75cm} \bigcirc \hspace{-7.75cm} \bigcirc \hspace{-7.75cm} \bigcirc \hspace{2.75cm} W_R^i = Z^{ij}(\theta)W^j
$$

Brandt Neri Sato **PRD24** (1981) renormalization matrix depends only on intersection angles

• The procedure is the same in the case of n intersections. In our case in principle *n=∞*, but in practice there are only two independent paths: periodic boundary conditions: $\frac{1}{2}$ only two independent

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• They are the cyclic loop (W_c) and the correlator of two Polyakov loops (*CPL*). The latter being finite, the renormalization matrix reads P_0 lyakov loops (C_{pr}) $\frac{1}{\sqrt{2}}$ renormalization matrix reads

$$
\left(\begin{array}{c}W_c^R \\ C_{PL}\end{array}\right)=\left(\begin{array}{cc}Z&(1-Z)\\0&1\end{array}\right)\left(\begin{array}{c}W_c \\ C_{PL}\end{array}\right)
$$

Intermediate summary

• We have obtained that the cyclic Wilson loop is not renormalized multiplicatively. Due to the periodic boundary conditions, it mixes with the Polyakov loop correlator under renormalization.

$$
W_c^R = Z W_c + (1 - Z) C_{PL}
$$

- This renormalization prescription is valid at weak and strong coupling
- We are now going to test it in perturbation theory

Cylindrical divergences

+ + = finite

• The standard cusp divergence arises when all vertices are contracted at the singular point at the cincular noint ac che o

• In the case of the intersection, one always has to think cylindrical. their contributions are already included in the substitution of the charge. However, in the

• Only the last diagram contributes to the intersection divergence uagram contributes to the c $- \sim$ and diagram and we get a first point, we get dst chagianit contributes to the mit α contraction at the singular point with α \mathbf{c} the intersection and thus give a divergence. This is illustrated in the second in the second in the second

Perturbative renormalization

Building blocks

• The Polyakov loop correlator at order α_s^2 at short distances $C_{PL} = 1 + C_F \alpha_s$ *m^D* $\frac{d^{2}D}{T} + C_F \alpha_s^2$ s $\sqrt{ }$ C_A $\left(\ln \frac{m_L^2}{\sigma^2}\right)$ $\frac{\mu_D}{T^2}$ + 1 2 $\Big\} - n_f \ln 2 \Big\] +$ N_c^2-1 8*N*² *c* $\alpha_{\rm s}^2$ $(rT)^2$ $C_{PL} = 1 + \mathcal{O}(g^3)$

McLerran Svetitsky **PRD24** (1981) Gross Pisarski Yaffe **PRD24** (1981) Brambilla JG Petreczky Vairo **PRD82** (2010) Burnier Laine Vepsäläinen **JHEP1001** (2009)

• Expansion of the renormalization constant

$$
Z \equiv 1 + Z_1 \alpha_s + Z_2 \alpha_s^2 + \dots
$$

• We now evaluate *Z1*

Leading-order renormalization

• The renormalization equation gives

$$
W_c^R = ZW_c + (1 - Z)C_{PL}
$$

= 1 + $\frac{C_F\alpha_s}{rT} + \frac{C_F^2\alpha_s^2}{2r^2T^2}$

$$
+ \frac{4\pi C_F\alpha_s}{T} \int_k \frac{e^{i\mathbf{r} \cdot \mathbf{k}}}{\mathbf{k}^2} \left(\frac{C_A\alpha_s}{\pi \epsilon} + Z_1\alpha_s + ...\right) + ...
$$

• This implies

$$
Z_1 = -\frac{C_A}{\pi} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)
$$

Leading-order renormalization

• The renormalization equation gives

 $W_c^R = ZW_c + (1 - Z)C_{PL}$ $=1+\frac{C_F\alpha_s}{T}$ $\frac{r}{rT}$ + $C_F^2\alpha_s^2$ $2r^2T^2$ $+$ $4\pi C_F\alpha_s$ *T* z
Z *k* $e^{i\mathbf{r}\cdot\mathbf{k}}$ \mathbf{k}^2 $\int C_A \alpha_s$ $\pi\epsilon$ $+ Z_1\alpha_s + \ldots + \ldots$ $C_{PL} = 1 + \mathcal{O}(g^3)$ $Z \equiv 1 + Z_1 \alpha_s + \dots$

 \overline{V}

• This implies $z_1 = -\frac{C_A}{\pi}$ $\sqrt{1}$ $\frac{2}{\epsilon} - \gamma_E + \ln 4\pi$

$$
\ln W_c^R = \frac{C_F \alpha_s}{rT} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[\left(\frac{31}{9} C_A - \frac{20}{9} T_F n_f \right) + \beta_0 \left(\ln \mu^2 r^2 + 2\gamma_E \right) \right] + \frac{\alpha_s C_A}{\pi} \left[1 + 2\gamma_E - \ln 4 + 2 \ln \mu^2 r^2 + \sum_{n=1}^{\infty} \frac{2(-1)^n \zeta(2n)}{n(4n^2 - 1)} (rT)^{2n} \right] \right\}
$$

+
$$
\frac{4\pi \alpha_s C_F}{T} \int_k \frac{e^{i\mathbf{r} \cdot \mathbf{k}} - 1}{(\mathbf{k}^2)^2} \left(-\Pi_{00\,CG}^{(T)}(0, \mathbf{k}) \right) + C_F C_A \alpha_s^2
$$

Going to higher orders

$$
C_{PL} = \left[1 + C_F \alpha_s \frac{m_D}{T} + C_F \alpha_s^2 \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2}\right) - n_f \ln 2\right] + \frac{N_c^2 - 1}{8N_c^2} \frac{\alpha_s^2}{(rT)^2}\right]
$$

• Non-trivial check of the renormalization equation by considering higher-order divergences

Going to higher orders

$$
C_{PL} = 1 + C_F \alpha_s \frac{m_D}{T} + C_F \alpha_s^2 \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right] + \frac{N_c^2 - 1}{8N_c^2} \frac{\alpha_s^2}{(rT)^2}
$$

• Non-trivial check of the renormalization equation by considering higher-order divergences

• In these diagrams the IR cancellation breaks down in the non-Abelian term

$$
C_F(e^{i\mathbf{k}\cdot\mathbf{r}}-1)\to C_F^2(e^{i\mathbf{k}\cdot\mathbf{r}}-1)-\frac{C_F C_A}{2}e^{i\mathbf{k}\cdot\mathbf{r}}
$$

Going to higher orders

$$
C_{PL} = 1 + \left[C_F \alpha_s \frac{m_D}{T} \right] + C_F \alpha_s^2 \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right] + \frac{N_c^2 - 1}{8N_c^2} \frac{\alpha_s^2}{(rT)^2}
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C_F(e^{i\mathbf{k}\cdot\mathbf{r}}-1)\to C_F^2(e^{i\mathbf{k}\cdot\mathbf{r}}-1)-\frac{C_F C_A}{2}e^{i\mathbf{k}\cdot\mathbf{r}}
$$

• This results in a UV-divergent contribution $C_F C_A \alpha_{\rm s}^2$ *T* Z *k* 1 $k^2 + m_D^2$ (1) ϵ $+ \ldots$ = $Z_1 C_F \alpha_s^2$ *m^D T*

Divergences at order *g6*

- We have carried out a full study of the cancellation of divergences at order *g6*
- There one has again iterations of the leading-order divergence, cancelled by *Z1*, and new divergences, cancelled by Z_2 (undetermined)
- The analysis is based on the topological classification of divergent graphs
- The analysis is gauge-invariant. For illustration let me show some examples in Coulomb gauge

Divergences at order *g6*

• The cancellation shown before at order g^5 carries through to all orders hown before at order ϱ^3 ca

Divergences at order *g6*

• The cancellation shown before at order g^5 carries through to all orders hown before at order ϱ^3 ca

• New divergences combining divergent and finite diagrams the same way of the same way of the same way of the same way of the \overline{O} *^s*) diagrams without the selfenergy $\mathbf{t}_\mathbf{S}$ two diagrams on the right are exactly equal, so they cancel. JHIDHHIL CONVETTE C Cancellations at *^O*(↵³

Divergences at order *g6* Cancellations at *^O*(↵³ *s*) proportional to (*rT*)² Cancellations at *^O*(↵³ *s*) proportional to (*rT*)²

The Cyclic Wilson Loop Check of the Renormalization Procedure at *^O*(↵³

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s)

s)

• The two-gluon exchange term in *C_{PL}* enters in 1
1
1 ϵ change term in C_{DI} enters in 1
1
1 Γ - ontoyo in

Divergences at order *g6* Cancellations at *^O*(↵³ *s*) proportional to (*rT*)² Cancellations at *^O*(↵³ *s*) proportional to (*rT*)² The Cyclic Wilson Loop Check of the Renormalization Procedure at *^O*(↵³

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s)

s) proportional to (*rT*)¹

s)

• The two-gluon exchange term in *C_{PL}* enters in 1
1
1 ϵ change term in C_{DI} enters in 1
1
1 $\overline{C_{\text{DL}}}$ ontarc in

• *Z*₂ has to be determined from All of these divergences together must be canceled by *Z*2↵²

completing the renormalization procedure to order *g6* tetting the renormalization procedure to order

Conclusions

- We have derived a generic renormalization equation for the cyclic Wilson loop, showing how it mixes with the Polyakov loop correlator
- We have tested this procedure in perturbation theory, determining the leading-order renormalization constant
- In order to match perturbative and lattice data, a non-trivial matching of the renormalization schemes in the two cases needs to be performed

