$\mathcal{N}$=4 super Yang-Mills Plasma

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based on:

A. Czajka & St. Mrówczyński, arXiv: 1203.1856 [hep-th]
A. Czajka & St. Mrówczyński, Physical Review **D83** (2011) 065021
A. Czajka & St. Mrówczyński, Physical Review **D84** (2011) 105020

Outline

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Motivation

Supersymmetry

symmetry of Nature

supersymmetric plasma
is an interesting
object of Nature

tool for AdS/CFT duality

Gravity in AdS

CFT: $\mathcal{N} = 4$ super Yang-Mills

QCD vs. super Yang-Mills ?

Motivation

Does rudimentary SUSY induce instabilities in fermionic sector?

QED PLASMA		SUSY QED PLASMA
There are unstable photon modes	⟺ SUSY ⟺	Are there unstable photino modes?

Lagrangian of $\mathcal{N}$=4 super Yang-Mills

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \frac{i}{2} \overline{\Psi}_i^a (\not{D}\Psi_i)^a + \frac{1}{2}(D_\mu \Phi_A)_a (D^\mu \Phi_A)_a$$

$$-\frac{1}{4} g^2 f^{abe} f^{cde} \Phi_A^a \Phi_B^b \Phi_A^c \Phi_B^d$$

$$-i \frac{g}{2} f^{abc} (\overline{\Psi}_i^a \alpha_{ij}^p X_p^b \Psi_j^c + i \overline{\Psi}_i^a \beta_{ij}^p \gamma_5 Y_p^b \Psi_j^c)$$

Type of the field	Range of the field's index	Spin	Number of degrees of freedom
A^μ - vector	$\mu, \nu = 0,1,2,3$	1	$2 \times (N_c^2 - 1)$
Φ_A - real (pseudo-)scalar	$A, B = 1,2,3,4,5,6$	0	$6 \times (N_c^2 - 1)$
λ_i - Majorana spinor	$i, j = 1,2,3,4$	$1/2$	$8 \times (N_c^2 - 1)$

Basic plasma characteristics

	QGP	$\mathcal{N}$=4 SYMP
energy density - $\boldsymbol{\varepsilon}$	$\frac{\pi^2 T^4}{60}\left[4(N_c^2-1)+7N_f N_c\right]$	$\frac{\pi^2 T^4}{2}(N_c^2-1)$
particle density - $\boldsymbol{n}$	$\frac{2\zeta(3)T^3}{\pi^2}\left[2(N_c^2-1)+3N_f N_c\right]$	$\frac{14\zeta(3)T^3}{\pi^2}(N_c^2-1)$
Debye mass - $\boldsymbol{m_D^2}$	$\frac{g^2 T^2}{6}\left(2N_c+N_f\right)$	$2g^2T^2N_c$
plasma parameter - $\boldsymbol{\lambda}$ $\left(\lambda \equiv \frac{1}{\frac{4}{3}\pi r_D^3 n}\right)$	$0.042g^3$	$0.257g^3$

All chemical potentials are assumed to vanish in both QGP and $\mathcal{N}$=4 SYMP.

Collective modes

Gluon dispersion equation

Equation of motion of gluon field $A^\mu(k)$

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

Collective modes - solutions: $\omega(\mathbf{k})$

$\Pi^{\mu\nu}(k)$
- retarded polarization tensor

Interaction of gluon with surrounding plasma

Fermion and scalar dispersion equations

Fermion field: $$\det[\hat{k} - \Sigma(k)] = 0$$

Scalar field: $$k^2 + P(k) = 0$$

$$\hat{k} \equiv k_\mu \gamma^\mu$$

Keldysh – Schwinger formalism

Description of non-equilibrium many-body systems

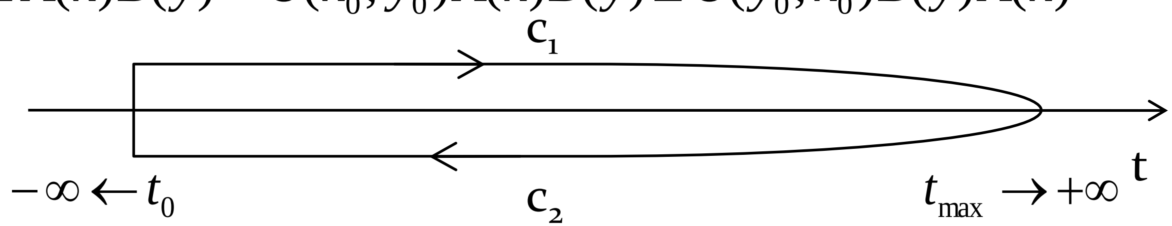
$$iG(x,y) \overset{\text{def}}{=} \left\langle \tilde{T}\phi(x)\phi(y) \right\rangle$$

Contour Green function of scalar field

$$\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho}(t)\ldots]$$

$\tilde{T}$ - ordering along the contour:

$$\tilde{T}A(x)B(y) = \Theta(x_0, y_0)A(x)B(y) \pm \Theta(y_0, x_0)B(y)A(x)$$



Polarization tensor

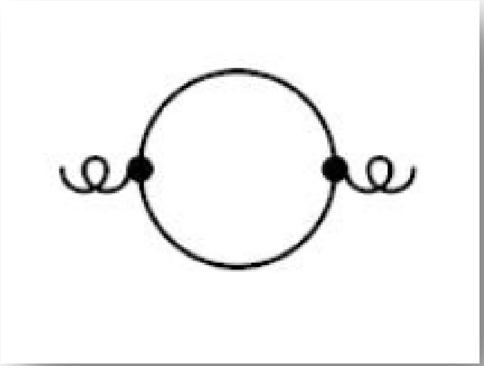
Dyson – Schwinger equation: $D = D_0 - D_0 \Pi D$

$D(x,y) = D(x-y)$ **Homogeneity, translational invariance**

Lowest order contributions to Π(x,y)

Contour–ordered Green functions have perturbative expansion similar to that of time-ordered Green functions.

Fermion loop



Contour polarization tensor

$$\Pi^{\mu\nu}_{ab}(x,y) = -ig^2 N_c \delta_{ab}\,\mathrm{Tr}[\gamma^\mu S(x,y)\gamma^\nu S(y,x)]$$

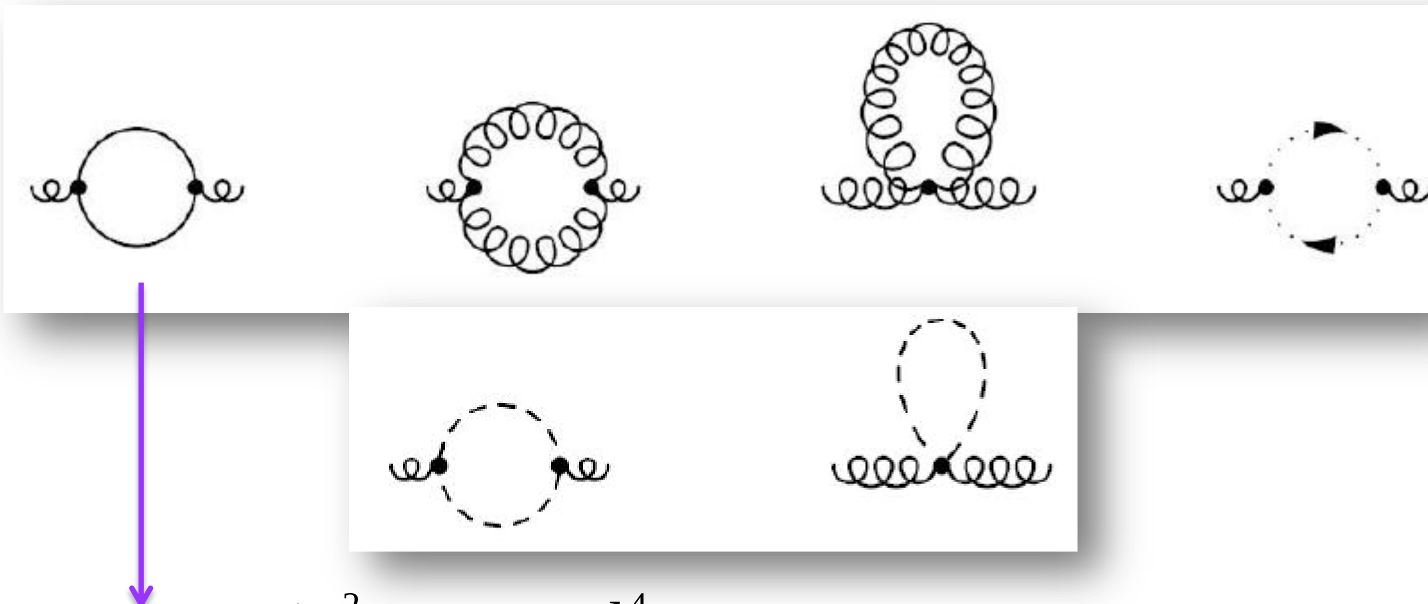
From contour to retarded polarization tensor

$$\Pi^{contour}(x) \to \Pi^{\pm}(x)$$

$$\Pi^{+}(x,y) = \Theta(x_0 - y_0)\left(\Pi^{>}(x,y) - \Pi^{<}(x,y)\right)$$

$$\left(\Pi^{>,<}(x,y)\right)^{\mu\nu}_{ab} = -ig^2 N_c \delta_{ab}\, \mathrm{Tr}[\gamma^{\mu} S^{>,<}(x,y)\gamma^{\nu} S^{<,>}(y,x)]$$

Contributions to retarded polarization tensor in $\mathcal{N}$=4 SYMP

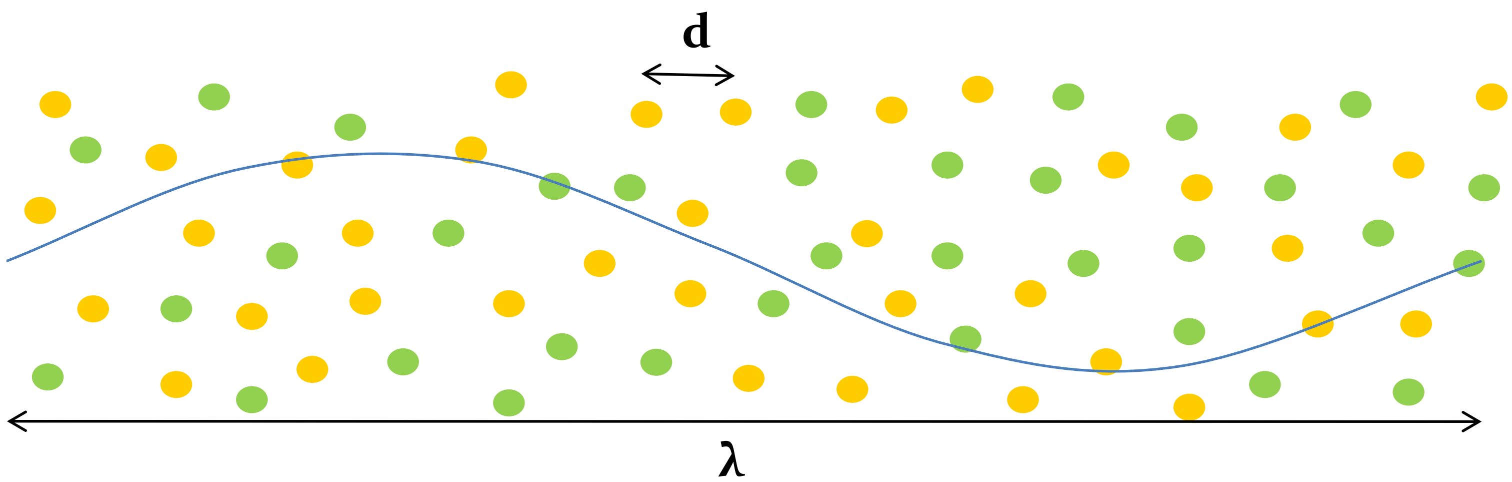


$$\left(\Pi^{+}(k)\right)^{\mu\nu}_{ab} = -\frac{ig^{2}}{2} N_{c}\delta_{ab}\int\frac{d^{4}p}{(2\pi)^{4}}\times$$

$$\times\mathrm{Tr}[\gamma^{\mu}S^{+}(p+k)\gamma^{\nu}S^{sym}(p)+\gamma^{\mu}S^{sym}(p)\gamma^{\nu}S^{-}(p-k)]$$

For every loop and tadpole there are analogical formulas

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma:

$$\lambda >> d$$

Hard Loop Approximation

The only dimensional parameter in free ultrarelativistic equilibrium plasma is temperature T.

$$\rho \sim \frac{1}{d^3} \sim T^3 \sim |\mathbf{p}|^3$$

$$\frac{1}{d} \sim |\mathbf{p}|$$ momentum of plasma constituent

$$\frac{1}{\lambda} \sim |\mathbf{k}|$$ wave vector of collective mode

COLLECTIVE MODES: $$k^\mu << p^\mu$$

Polarization tensor

$$\Pi^{\mu\nu}_{ab}(k) = g^2 N_c \delta_{ab} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

- the same structure as in QED and QCD
- symmetric

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$$

- transversal

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

Gauge independence!

Polarization tensor

$$\Pi_{ab}^{\mu\nu}(k) = g^2 N_c \delta_{ab} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

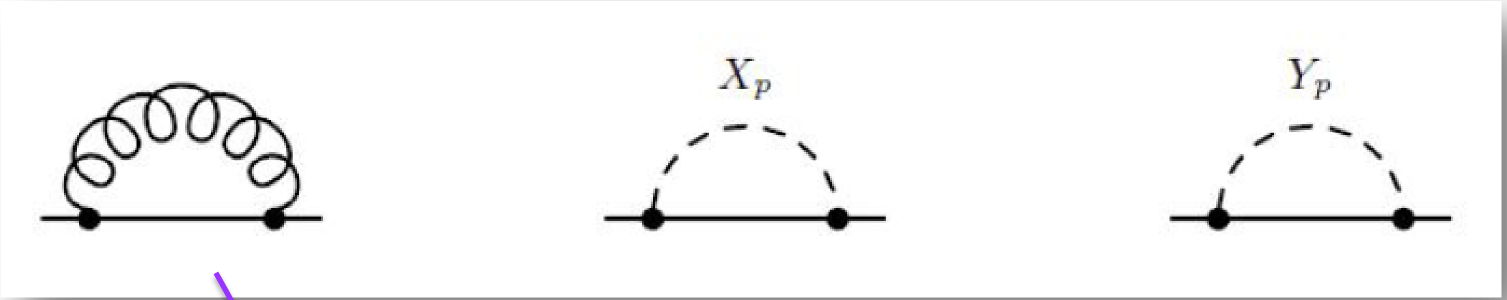
$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

Effect of SUSY:

- vacuum contribution vanishes
- the coefficients in front of the distribution functions are the numbers of degrees of freedom

$$f_{QGP}(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + \frac{N_f}{N_c}\left(n_q(\mathbf{p}) + n_{\bar{q}}(\mathbf{p})\right)$$

Fermion self-energy



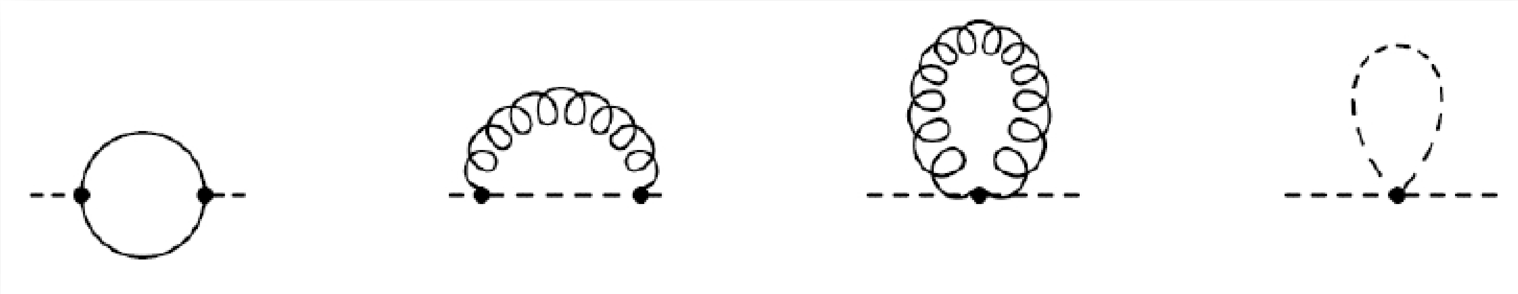
$$\Sigma_{QED}(k) = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{f_\gamma(\mathbf{p}) + f_e(\mathbf{p})}{E_p} \frac{\hat{p}}{k \cdot p + i0^+}$$

$$\Sigma_{ab}^{ij}(k) = \frac{g^2}{2} N_c \delta_{ab} \delta^{ij} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{\hat{p}}{k \cdot p + i0^+}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

The fermion self-energy has **the same structure** for the $\mathcal{N}$=4 SYM, SUSY QED and usual QED plasma.

Scalar self-energy



$$P_{ab}^{AB}(k) = -2g^2 N_c \delta_{ab} \delta^{AB} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

Scalar self-energy:

- **independent of *k***
- **vanishes in the vacuum limit**

Hard loop effective action

Self-energy constrains the form of effective action

$$\mathcal{L}_2^{(\Psi)}(x) = \int d^4y\, \overline{\Psi}(x)\Sigma(x-y)\Psi(y)$$

$$\Sigma(x,y) = \frac{\delta^2 S[\Psi,\overline{\Psi}]}{\delta\overline{\Psi}(x)\delta\Psi(y)}$$

$$\mathcal{L}_{\mathrm{HL}} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{i}{2}\overline{\Psi}_i^a(\not{D}\Psi_i)^a + \frac{1}{2}(D_\mu\Phi_A)_a(D^\mu\Phi_A)_a$$
$$+\mathcal{L}_{HL}^{(A)} + \mathcal{L}_{HL}^{(\Psi)} + \mathcal{L}_{HL}^{(\Phi)}$$

From effective action to self-energies

$$\mathcal{L}_{\rm HL}^{(A)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} F_{\mu\nu}^a(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_\rho^{b\mu}(x)$$

$$\mathcal{L}_{\rm HL}^{(\Psi)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \overline{\Psi}_i^a(x) \left(\frac{p\cdot\gamma}{p\cdot D} \right)_{ab} \Psi_i^b(x)$$

$$\mathcal{L}_{\rm HL}^{(\Phi)}(x) = -2 g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \Phi_A^a(x) \Phi_A^a(x)$$

The structure of each term of the effective action appears to be unique

Structure of each self-energy is unique

Gauge bosons collective modes

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

Solutions: $\omega(\mathbf{k})$ $\qquad$ $k^\mu \equiv (\omega, \mathbf{k})$

The structure of $\Pi^{\mu\nu}(k)$

- coincides with the gluon polarization tensor of QCD plasma
- such as of QED and SUSY QED plasma

The **spectrum of collective excitations** of gauge bosons in $\mathcal{N}$=4 super Yang-Mills, QCD, QED and SUSY QED plasma is **the same**.

There is a whole variety of possible collective excitations, in particular there are **unstable modes**.

Fermion collective modes

The structure of fermion self-energy

- is such as of quark self-energy in QCD plasma
- is such as of QED and SUSY QED plasma

There are **identical spectra of collective excitations** of fermions in all systems.

No unstable modes found!

Scalar collective modes

The scalar self-energy is **independent of momentum**.

It is **negative and real**.

$$P = -m_{eff}^2$$

$\mathbf{m_{eff}}$ is the effective scalar mass

The solution of dispersion equation:

$$E_p = \pm\sqrt{m_{eff}^2 + \mathbf{p}^2}$$

Collisional characteristics

Elementary processes

Process	$\frac{1}{g^4}\frac{1}{N_c(N_c^2-1)}\sum\|M\|^2$
$GG \leftrightarrow GG$	$2\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$
$GF \leftrightarrow GF$	$32\left(\frac{s^2+u^2}{t^2}-\frac{u}{s}-\frac{s}{u}\right)$
$GG \leftrightarrow FF$	$32\left(\frac{t^2+u^2}{s^2}-\frac{u}{t}-\frac{t}{u}\right)$
$GS \leftrightarrow GS$	$24\left(\frac{s^2+u^2}{t^2}+1\right)$
$GG \leftrightarrow SS$	$24\left(\frac{t^2+u^2}{s^2}+1\right)$
$GF \leftrightarrow SF$	$-96\left(\frac{u}{s}+\frac{s}{u}+1\right)$
$GS \leftrightarrow FF$	$-96\left(\frac{u}{t}+\frac{t}{u}+1\right)$
$FS \leftrightarrow FS$	$-96\left[\frac{2us}{t^2}+3\left(\frac{u}{s}+\frac{s}{u}\right)+1\right]$
$SS \leftrightarrow FF$	$-96\left[\frac{2ut}{s^2}+3\left(\frac{u}{t}+\frac{t}{u}\right)+1\right]$
$SS \leftrightarrow SS$	$72\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$
$FF \leftrightarrow FF$	$128\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$

S. C. Huot, S. Jeon, and G. D. Moore, Phys. Rev. Lett. **98**, 172303 (2007)

Transport coefficients

Collisional processes

transport properties of ultrarelativistic plasma

Temperature is the only dimensional parameter.

$$\eta \sim \frac{T^3}{g^4 \ln g^{-1}}$$

S. C. Huot, S. Jeon, and G. D. Moore, Phys. Rev. Lett. **98**, 172303 (2007)

Energy loss and momentum broadening

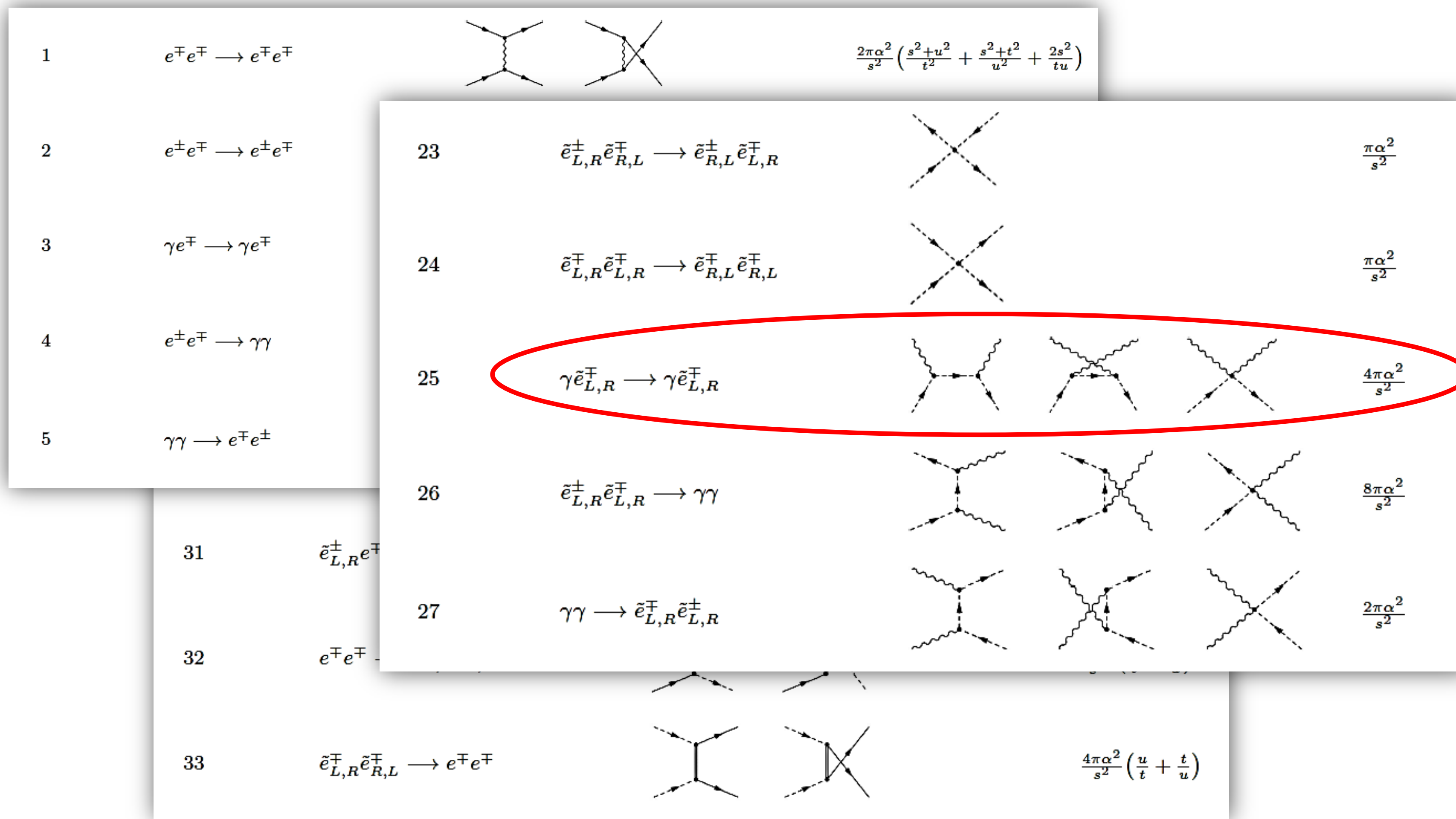
- **are not constrained by dimensional arguments**

$$\frac{dE}{dx} \sim T^2, ET, E^2, \ldots$$

$$\hat{q} \sim T^3, ET^2, E^2T, E^3, \ldots$$

- **depend on a specific scattering process under consideration**

Cross sections of binary interactions in SUSY QED



1	$e^{\mp}e^{\mp} \longrightarrow e^{\mp}e^{\mp}$	$\frac{2\pi\alpha^2}{s^2}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}+\frac{2s^2}{tu}\right)$
2	$e^{\pm}e^{\mp} \longrightarrow e^{\pm}e^{\mp}$	
3	$\gamma e^{\mp} \longrightarrow \gamma e^{\mp}$	
4	$e^{\pm}e^{\mp} \longrightarrow \gamma\gamma$	
5	$\gamma\gamma \longrightarrow e^{\mp}e^{\pm}$	

23	$\tilde{e}^{\pm}_{L,R}\tilde{e}^{\mp}_{R,L} \longrightarrow \tilde{e}^{\pm}_{R,L}\tilde{e}^{\mp}_{L,R}$	$\frac{\pi\alpha^2}{s^2}$
24	$\tilde{e}^{\mp}_{L,R}\tilde{e}^{\mp}_{L,R} \longrightarrow \tilde{e}^{\mp}_{R,L}\tilde{e}^{\mp}_{R,L}$	$\frac{\pi\alpha^2}{s^2}$
25	$\gamma\tilde{e}^{\mp}_{L,R} \longrightarrow \gamma\tilde{e}^{\mp}_{L,R}$	$\frac{4\pi\alpha^2}{s^2}$
26	$\tilde{e}^{\pm}_{L,R}\tilde{e}^{\mp}_{L,R} \longrightarrow \gamma\gamma$	$\frac{8\pi\alpha^2}{s^2}$
27	$\gamma\gamma \longrightarrow \tilde{e}^{\mp}_{L,R}\tilde{e}^{\pm}_{L,R}$	$\frac{2\pi\alpha^2}{s^2}$

31	$\tilde{e}^{\pm}_{L,R}e^{\mp}$	
32	$e^{\mp}e^{\mp}$	
33	$\tilde{e}^{\mp}_{L,R}\tilde{e}^{\mp}_{R,L} \longrightarrow e^{\mp}e^{\mp}$	$\frac{4\pi\alpha^2}{s^2}\left(\frac{u}{t}+\frac{t}{u}\right)$

Energy loss and momentum broadening in SUSY QED plasma

A selectron is traversing an equilibrium photon gas.

$$|\mathcal{M}|^2 = 4e^4$$

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2 \left[1 - \frac{12\zeta(3)}{\pi^2}\frac{T}{E}\right] \underset{E>>T}{\approx} -\frac{e^4}{2^5 3\pi} T^2$$

$$\hat{q} = \frac{e^4}{12\pi^3} T^3 \left[\zeta(3) + \frac{\pi^4}{45}\frac{T}{E}\right] \underset{E>>T}{\approx} \frac{e^4 \zeta(3)}{12\pi^3} T^3$$

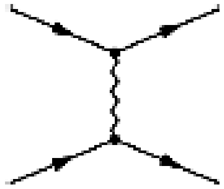
Comparison with Coulomb-like interaction

Energy loss for contact interaction

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2$$

Energy loss for Coulomb-like interaction

$$\frac{dE}{dx} = -\frac{e^4}{48\pi^3} T^2 \left(\ln \frac{E}{eT} + 2.031 \right)$$



E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991)

Energy loss

		Contact $\lvert\mathcal{M}\rvert^2 \sim e^4$	Coulomb $\lvert\mathcal{M}\rvert^2 \sim e^4 \frac{s^2}{t^2}$
energy change in single collision	ΔE	$\sim E$	$\sim e^2 T$
cross section	σ	$\sim \frac{e^4}{ET}$	$\sim \frac{e^2}{T^2}$
density	ρ	$\sim T^3$	$\sim T^3$
inverse mean path	$\lambda^{-1} = \sigma\rho$	$\sim \frac{e^4 T^2}{E}$	$\sim e^2 T$
energy loss	$\frac{dE}{dx} \sim \frac{\Delta E}{\lambda}$	$\sim e^4 T^2$	$\sim e^4 T^2$

Different interactions lead to the same energy loss!

Conclusions

- The collective modes of $\mathcal{N}$=4 super Yang-Mills plasma are the same as those of QGP.
  - There are no unstable fermion modes.
  - The structures of self-energies appear to be unique.

- The transport characteristics of SUSY plasma are similar to those of QGP.

Both systems are very similar to each other in the weak coupling regime!