Shear and Bulk Viscosities of a Gluon Plasma in Perturbative QCD with Non-Collinear Radiations

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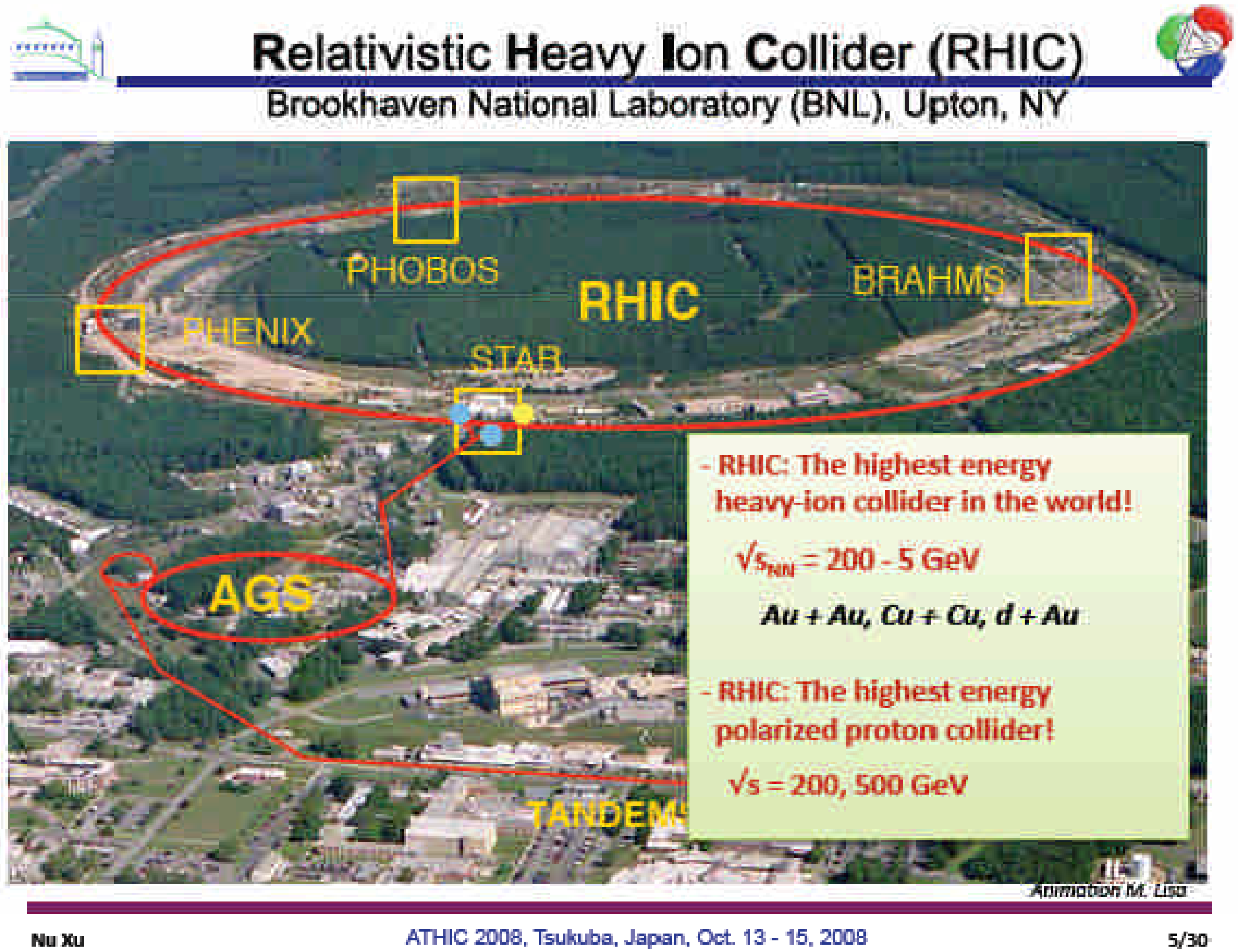
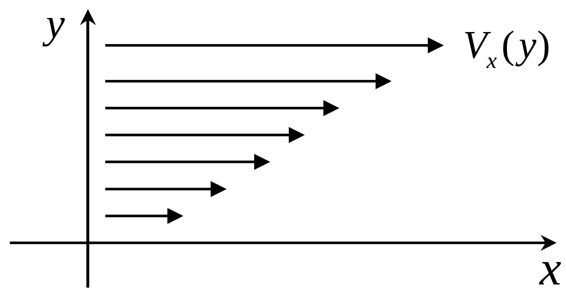
“RHIC Serves a Near-Perfect Fluid”



A Paradigm Shift!

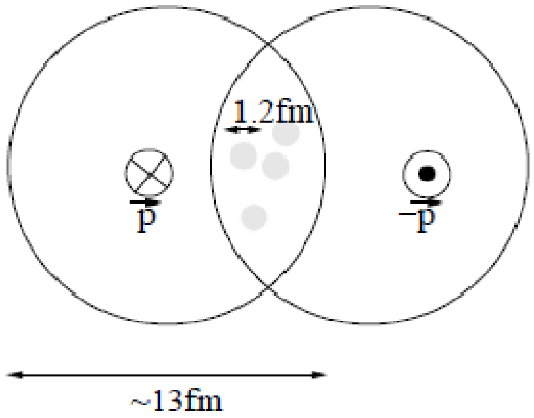
- Shear viscosity



Frictional force

$$T_{ij} = -\eta\left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \frac{1}{3}\delta_{ij}\nabla\cdot V(x)\right).$$

Elliptic Flow---measures shear viscosity



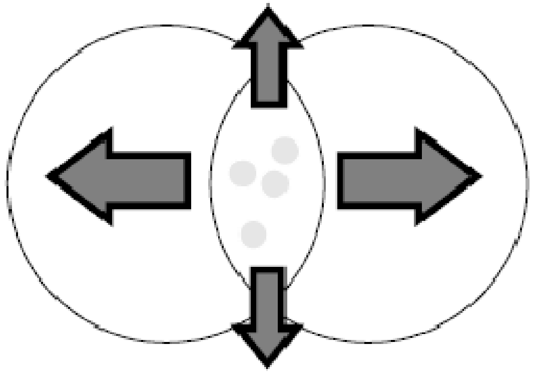


Figure: H. Meyer

Shear viscosity measures

how "perfect" a fluid is!

Smaller shear viscosity implies

larger particle interaction!

“Perfect Fluid”?

- Kovtun, Son, and Starinets (’05)

  Conjecture: Shear viscosity / entropy density

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

- Motivated by AdS/CFT

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

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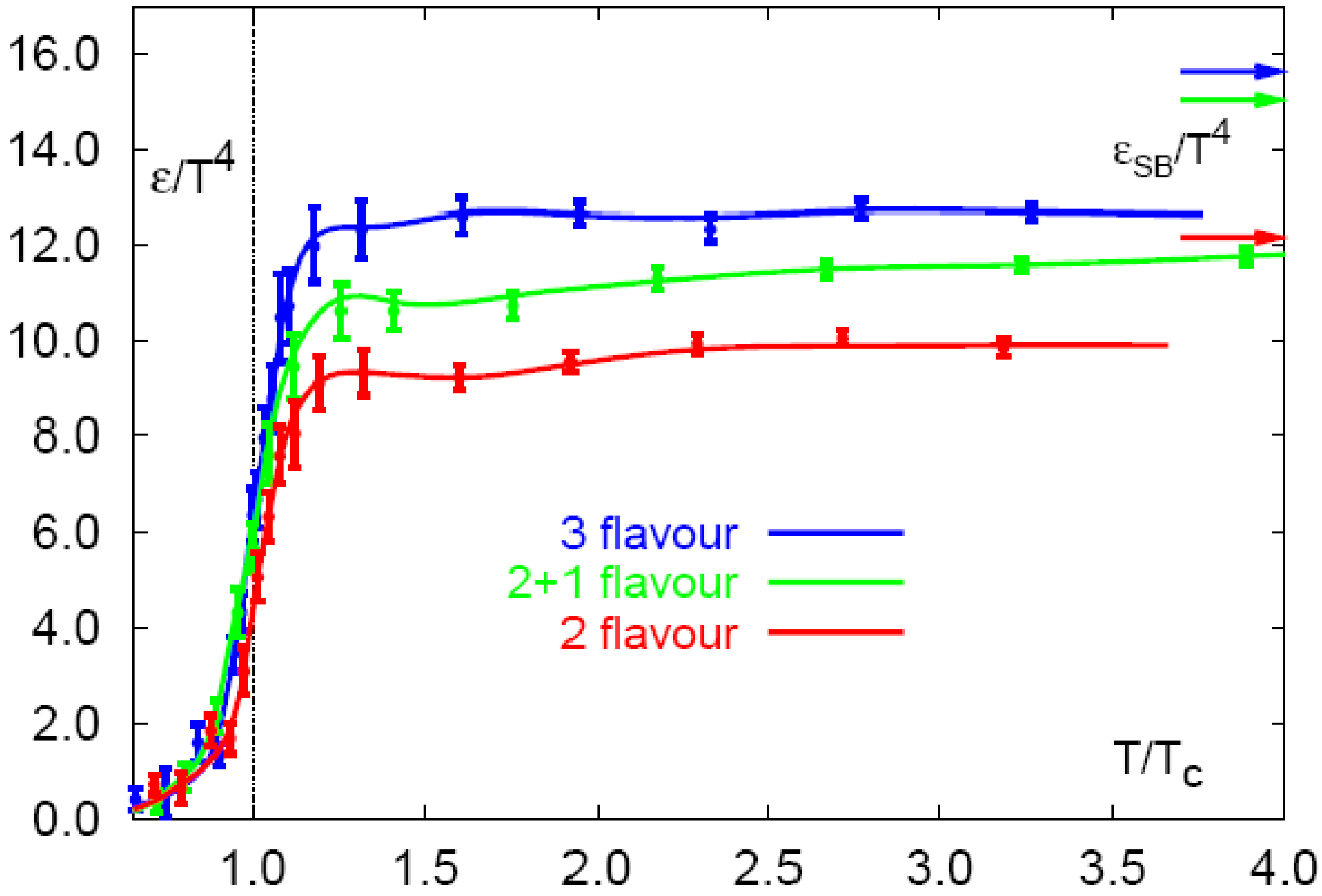
- “QGP” (quark gluon plasma) almost saturates the bound @ just above Tc (Teaney; Romatschke, Romatschke; Song, Heinz; Luzum)
- LQCD, gluon pasma (Karsch, Wyld; Nakamura, Sakai; Meyer)

⇨ QGP near Tc, a near perfect fluid, SQGP!!!

- PQGP: Asakawa, Bass, Müller; Xu, Greiner

QGP EOS (LQCD) ~ an ideal gas



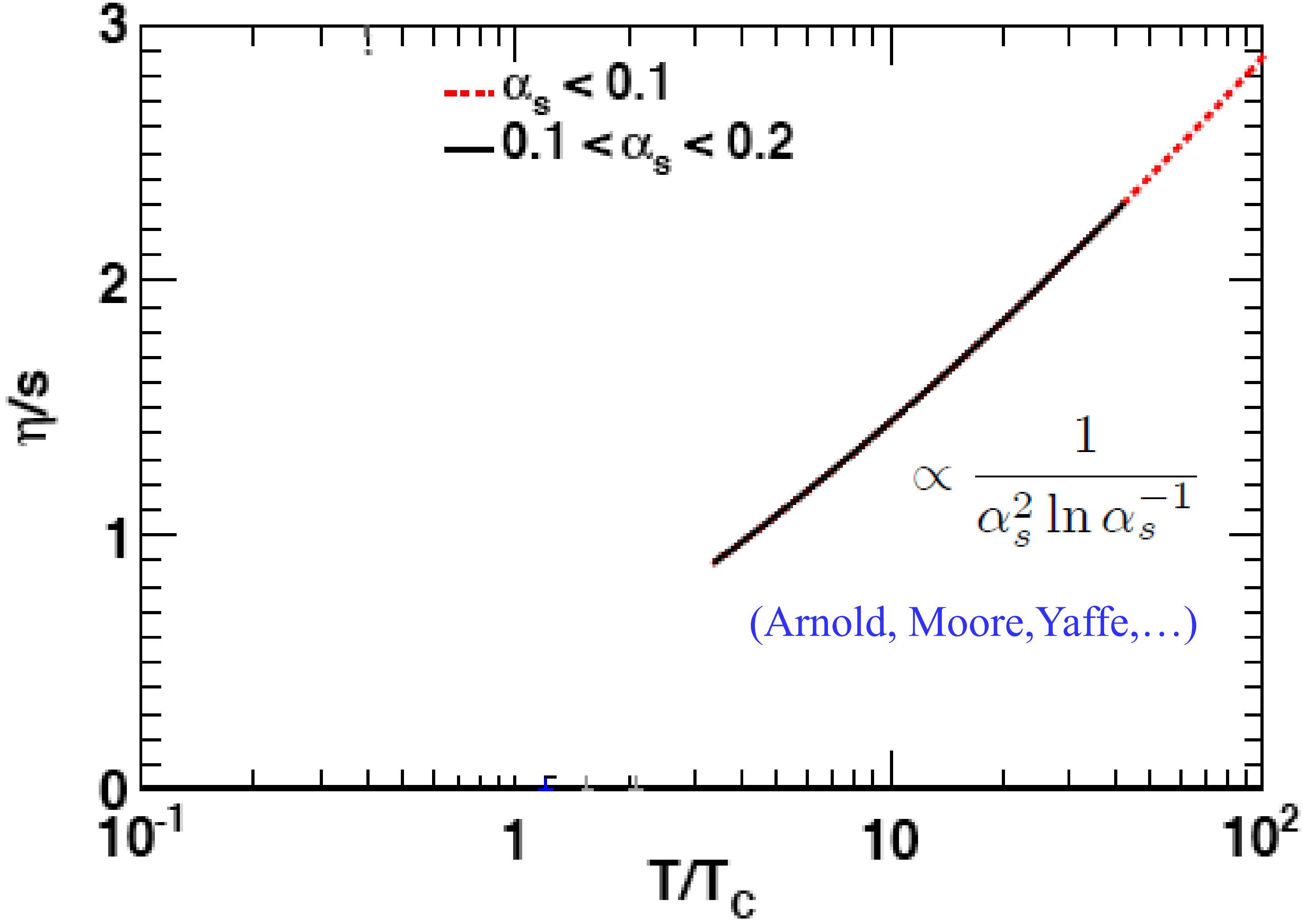
Karsch hep-lat/0106019

A closer look at QCD…

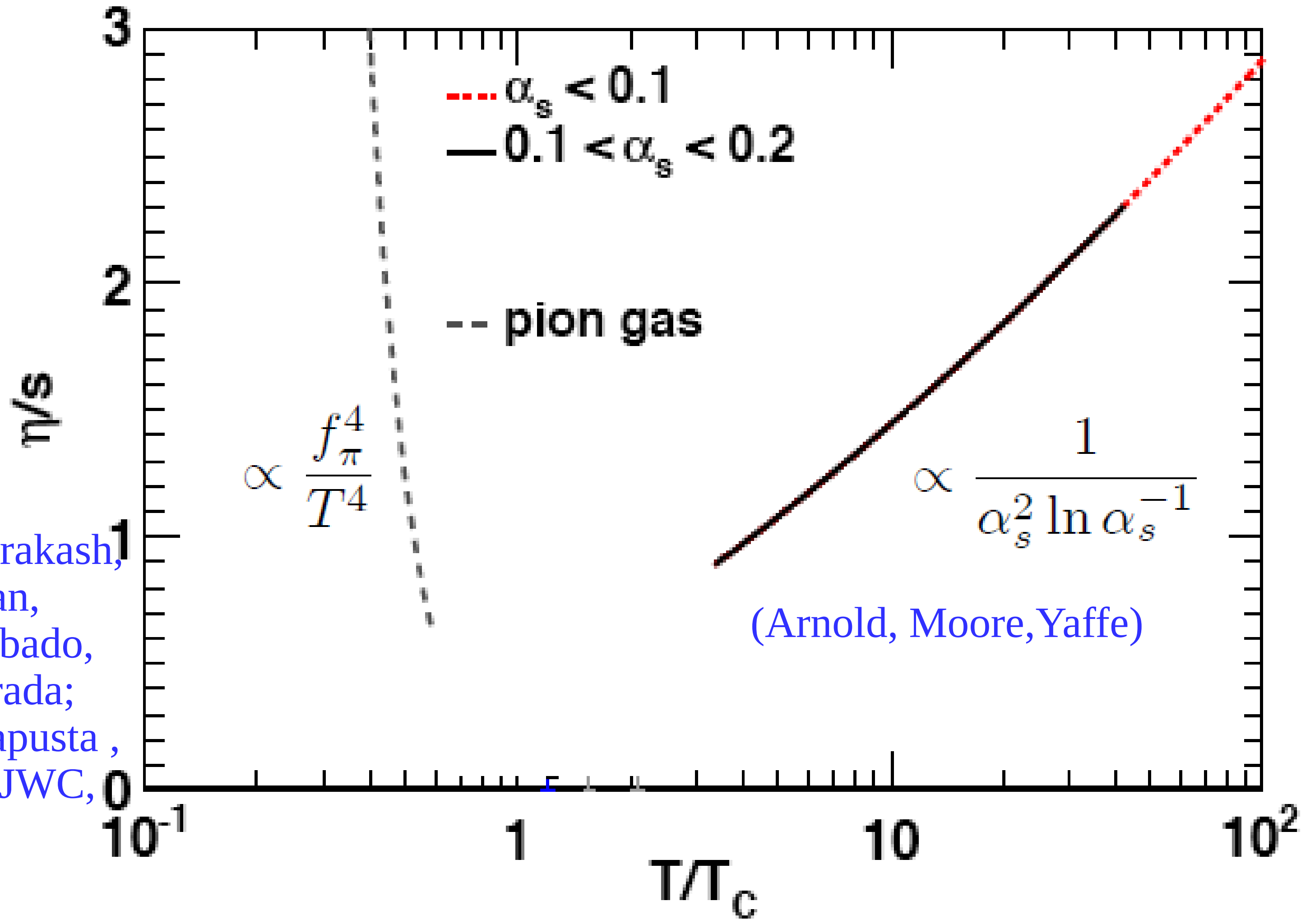
Boltzmann eq. = Kubo formula
@ LO weak expansion (Jeon, Yaffe)

shear viscosity roughly proportional to the inverse collision rate

QCD w/ massless quarks



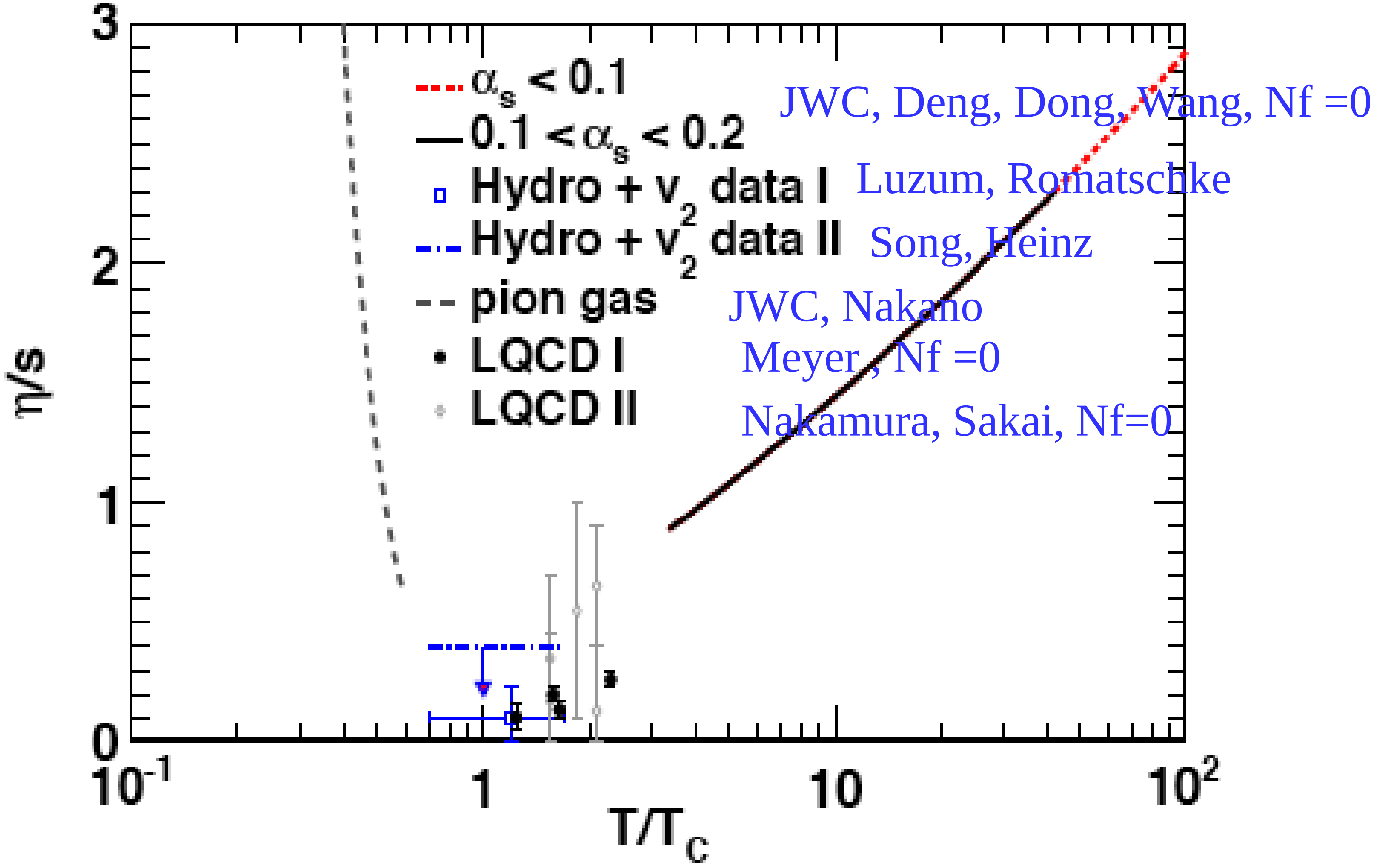
QCD w/ massless quarks



(Arnold, Moore, Yaffe)

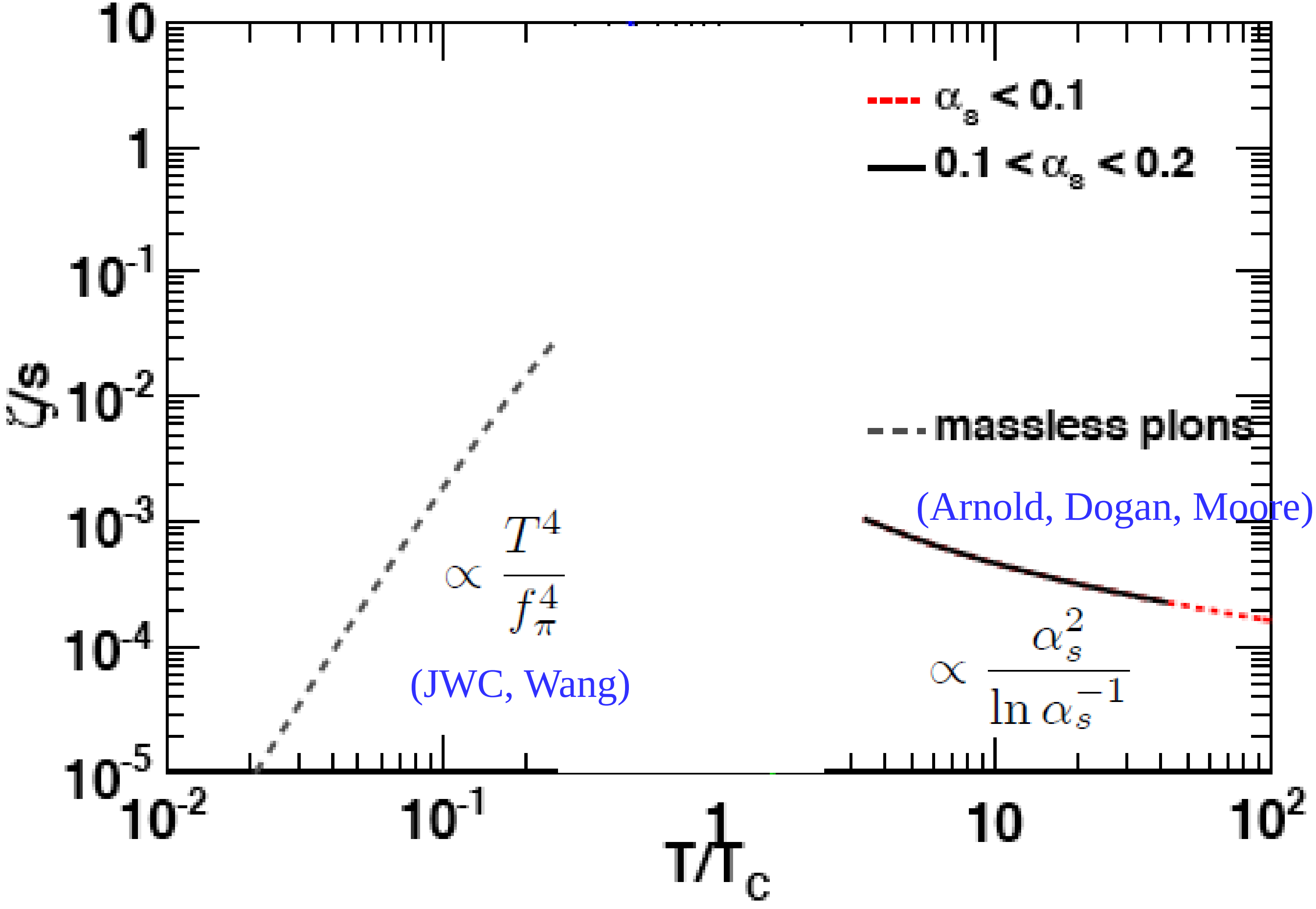
(Prakash, Prakash, Venugopalan, Welke; Dobado, Llanes-Estrada; Csernai, Kapusta , McLerran; JWC, Nakano)

“QCD”

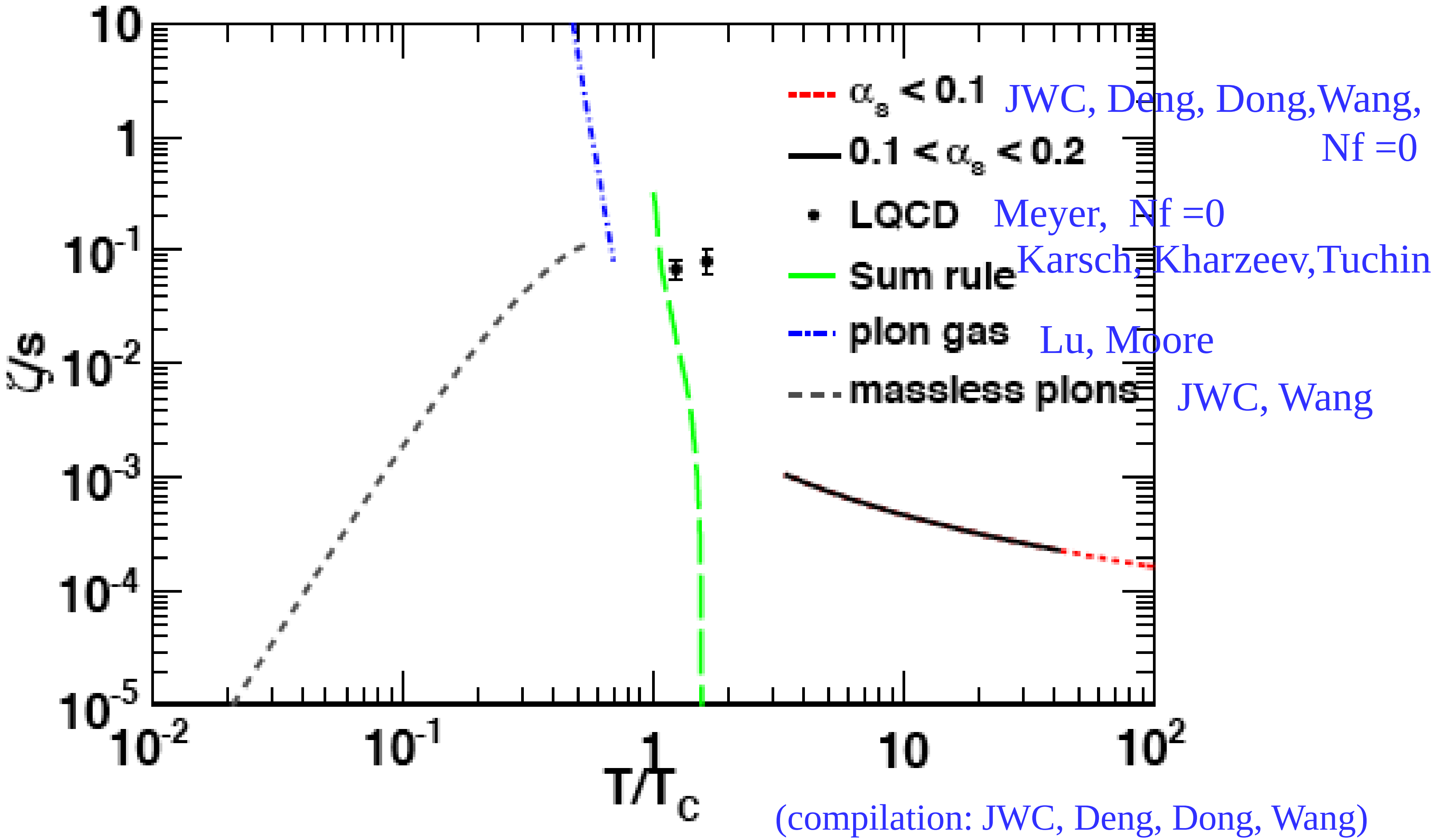


(compilation: JWC, Deng, Dong, Wang)

QCD w/ massless quarks



“QCD”



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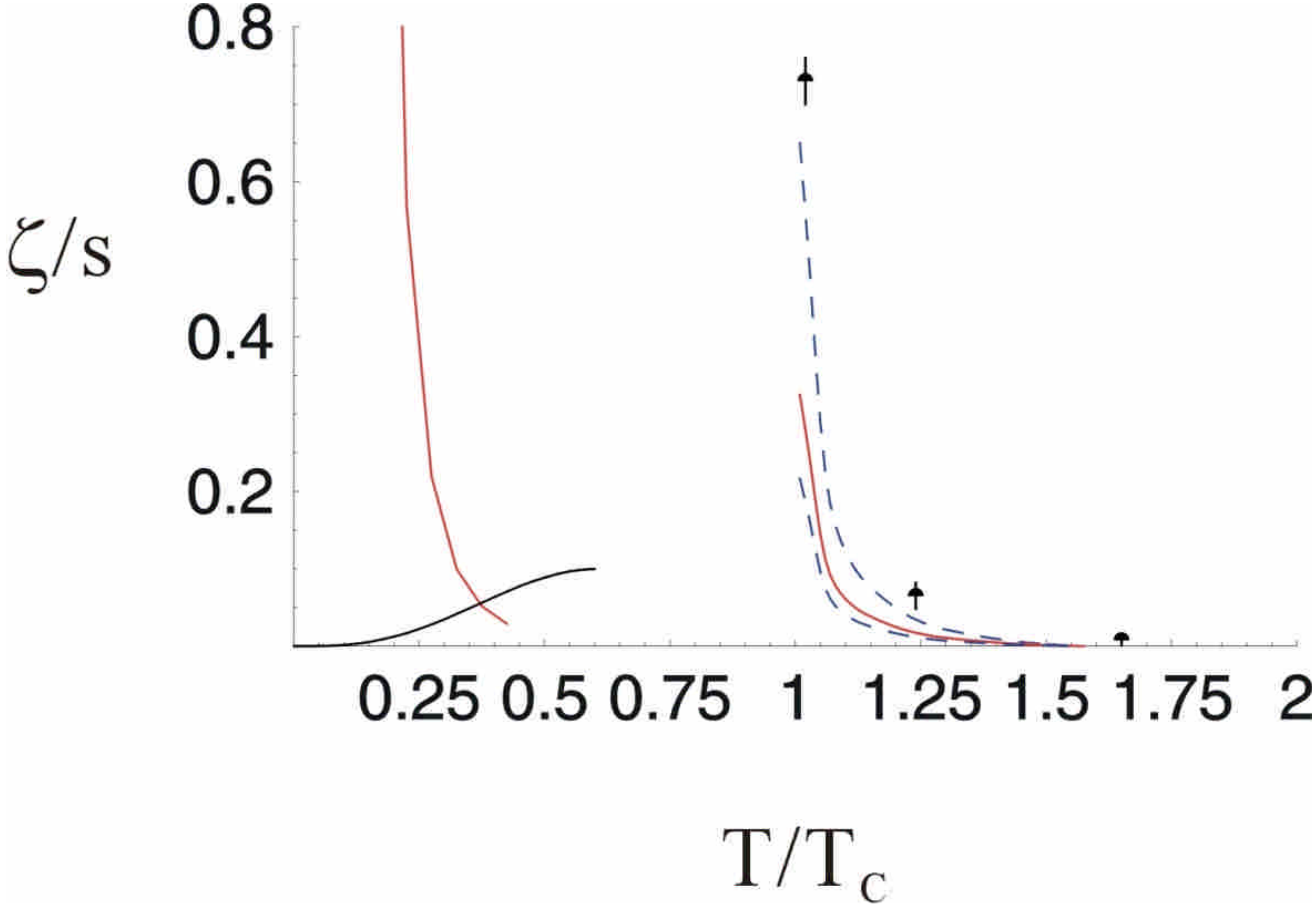
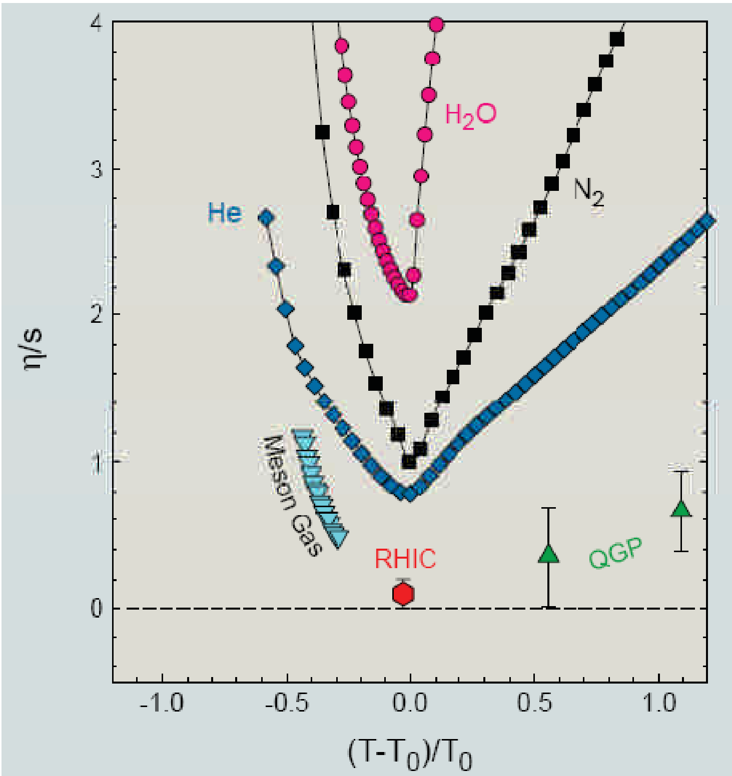
"QCD"



η/s goes to a local minimum near a phase transition

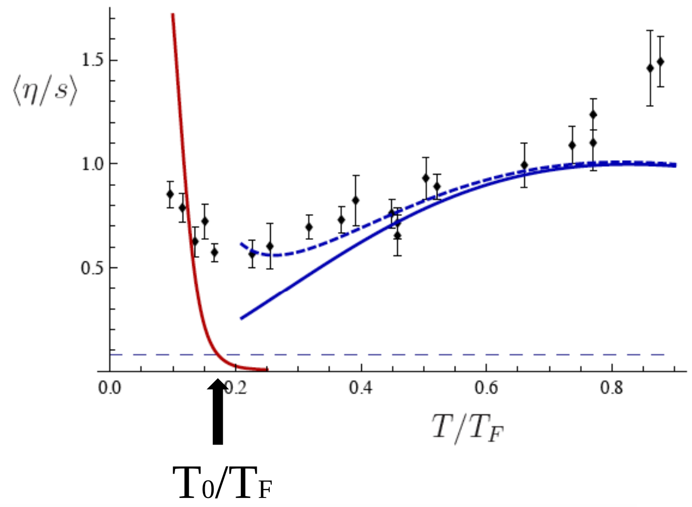
in more than 30 systems

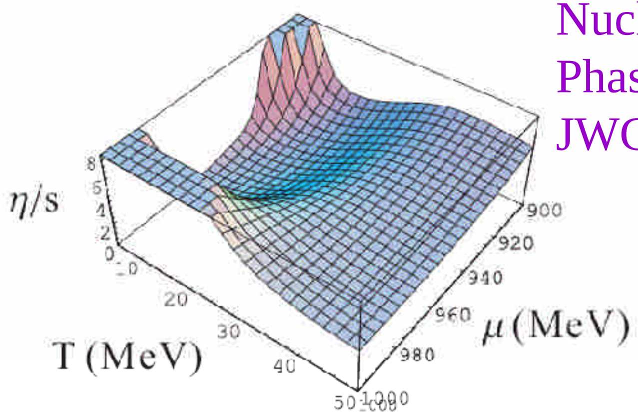


Lacey et al., PRL 98:092301,2007;
2007 US Nuclear Science Long Range Plan

Cold Unitary Atoms

Schafer, Chafin; Rupak & Schafer





Nuclear liquid-gas Phase transition
JWC et al. 2007

Universality?

Universal η/s and ζ/s behaviors?

(η/s reaches local minimum near p.t.

ζ/s reaches local maximum near p.t.)

If yes, mapping out the QCD phase diagrams (including critical end point(s) [Lacey]) by η/s ?

Mapping QCD phase diagram by η/s ?

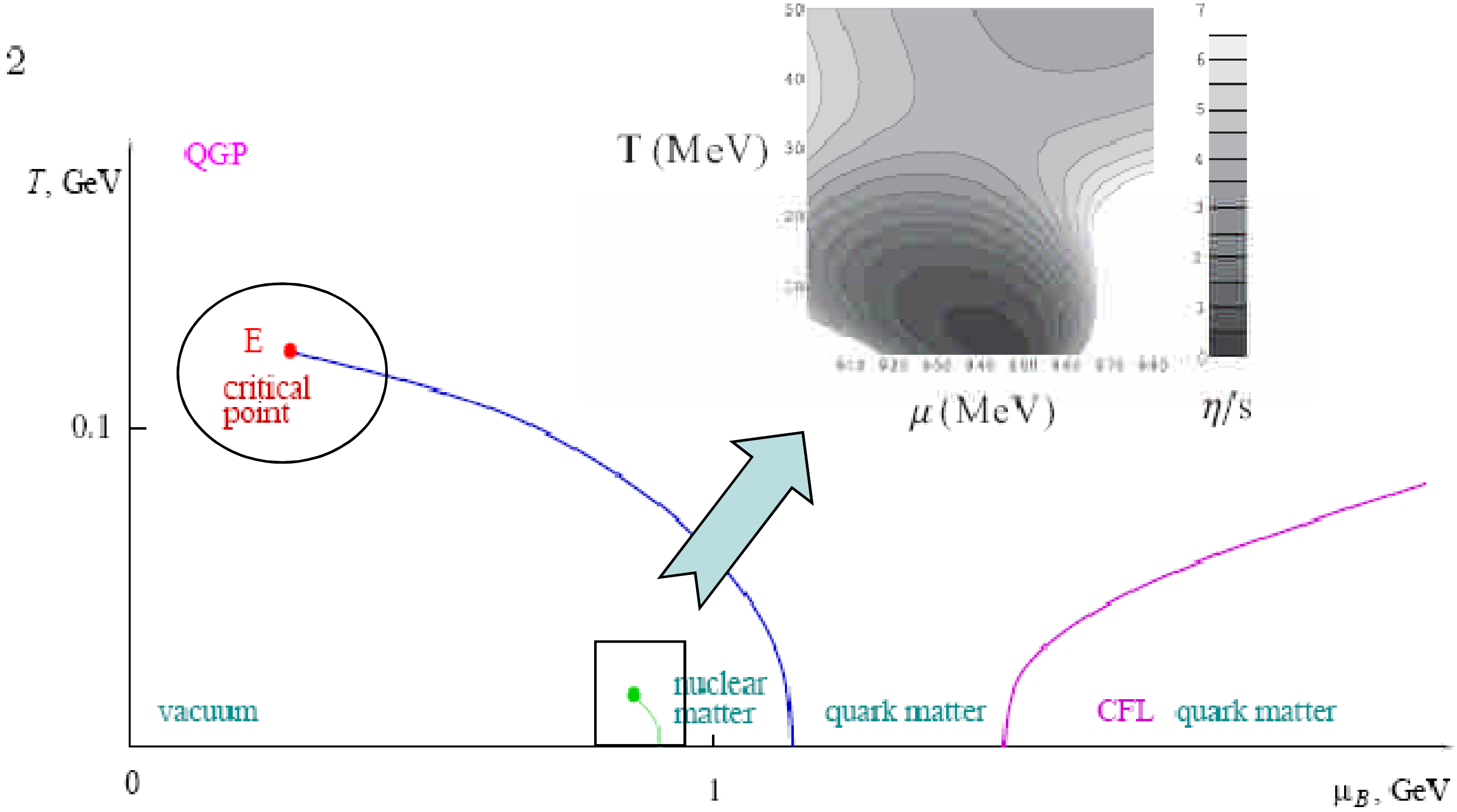


Fig. 1. QCD phase diagram

Locating critical end point (Lacey)

No Universality.

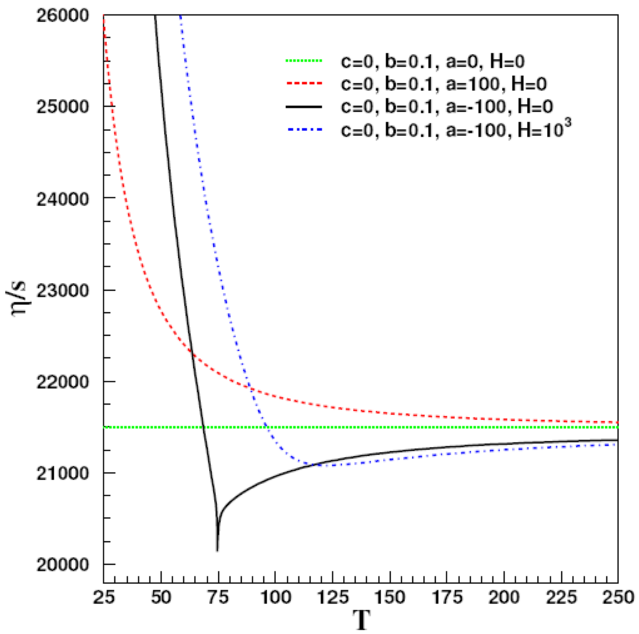
No Universality.

Counter-examples Found!

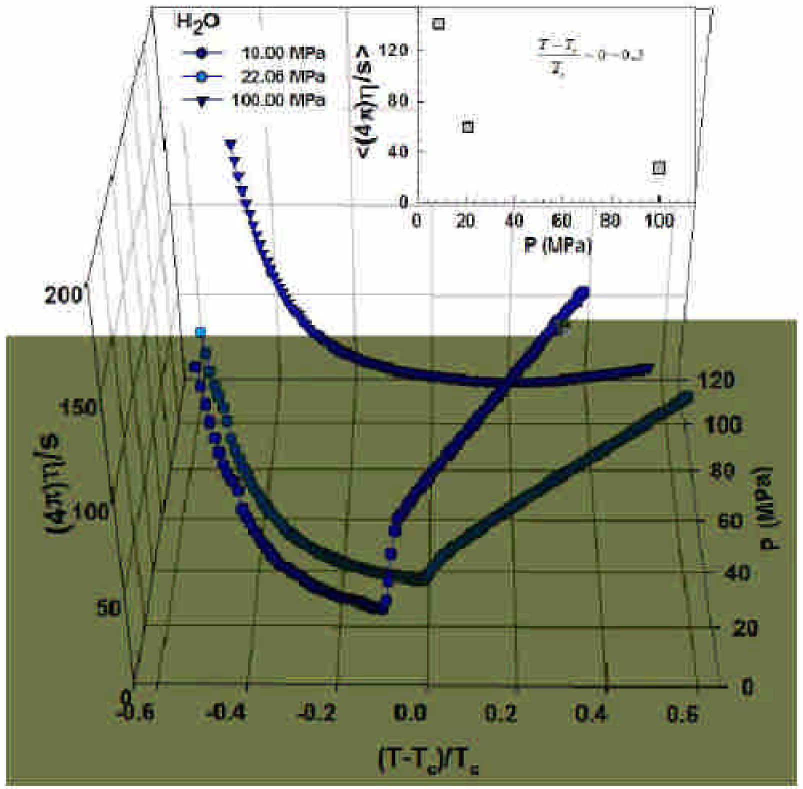
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}a\phi^2 - \frac{1}{4}b\phi^4 - \frac{1}{6}c\phi^6$$

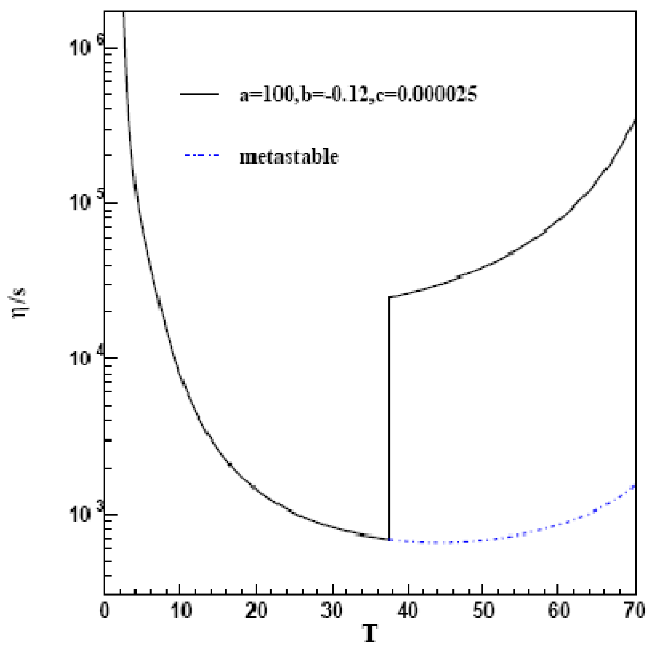
(JWC, M. Huang, Y.H. Li, E. Nakana, D.L. Yang)

- Weak coupling, Boltzmann eq.
- Mean field calculation
- CJT formulism (Cornwall, Jackiw, Tomboulis)



η/s of Water

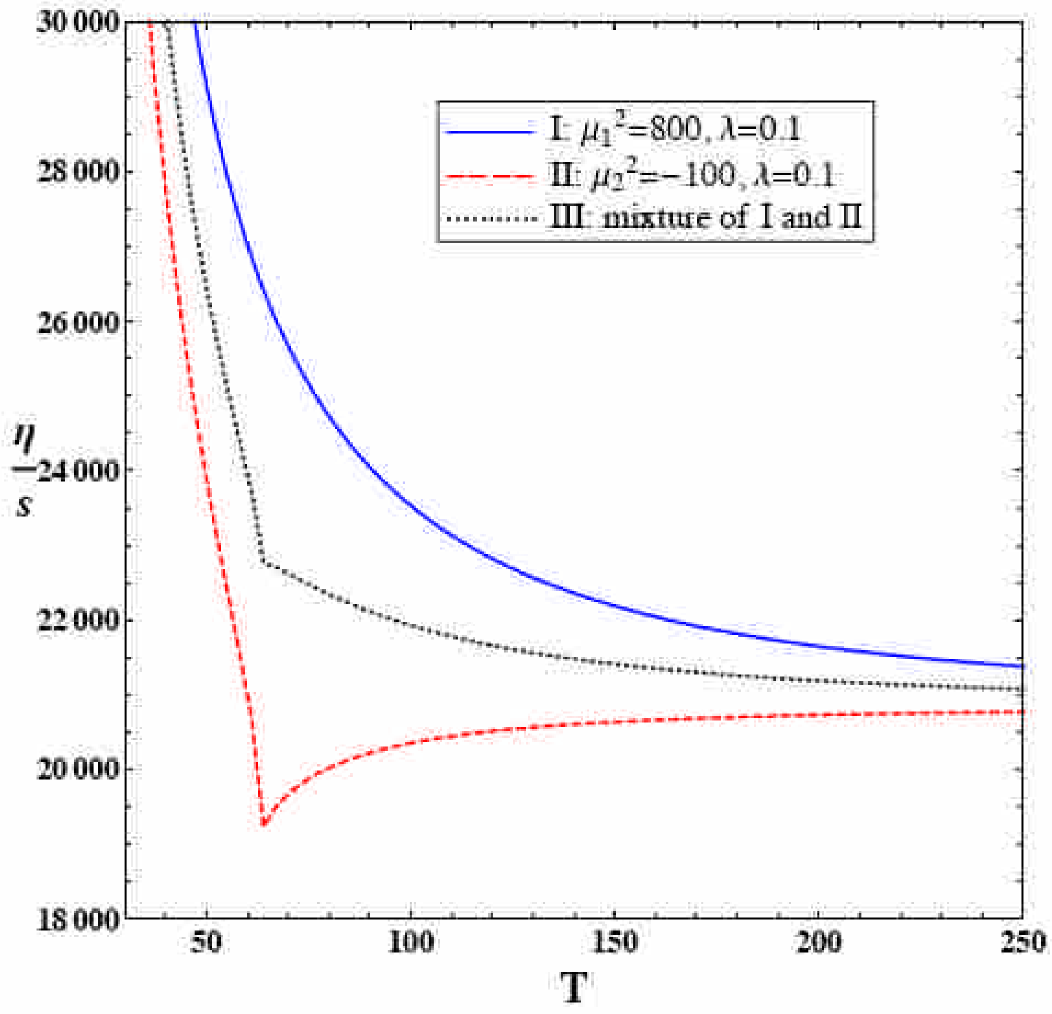




(Lacey et al.)

Simplest counter-example

(JWC, Hsieh, Lin)



So…

Universal η/s and ζ/s behaviors

(η/s reaches local minimum near p.t.

ζ/s reaches local maximum near p.t.)

do not exist.

However, it is still intriguing why it appears in so many systems. It is still a mystry to be understood.

Gluon Viscosity @ Weak Coupling

Gluon viscosity: gg → gg (22), gg → ggg (23) both LO (22 suppressed, forward scattering cannot redistribute momentum) 22 larger by a log (LL)

- Xu & Greiner (XG):

R23 ~ 7 R22 (PQGP!!!)

- Arnold, Moore,Yaffe (AMY):

R23 ~0.1 R22 (SQGP)

Who is right?

- XG:

(1) parton cascade (classical particles)

(2) soft bremsstrahlung

- AMY:

(1) Boson statistics

(2) collinear splitting g $\rightarrow$ gg (12) massless g

- JWC, Deng, Dong, Ohnishi, Wang:

(1) statistics 80% difference @ LL

(2) "exact" matrix element in vaccum, can be reduced to either soft or collinear limit

Thermal mass insertion in 23

$$|M_{0\to 12345}|^2$$
$$= \frac{1}{10} N_g (4\pi\alpha_s N_c)^3 \Big[(12)^4 + (13)^4 + (14)^4 + (15)^4 + (23)^4$$
$$+ (24)^4 + (25)^4 + (34)^4 + (35)^4 + (45)^4\Big]$$
$$\times \sum_{\text{perm}\{1,2,3,4,5\}} \frac{1}{(12)\,(23)\,(34)\,(45)\,(51)},$$

$$(ij) \equiv p_i \cdot p_j \qquad \sum_{i=1}^{5} p_i = 0.$$

$$(ij) \Rightarrow \frac{1}{2}[(p_i + p_j)^2 - m_D^2]$$

Soft Bremm. Limit

$p_1 = p,\, p_2 = p',\, p_3 = p + q - k,\, p_4 = p' - q$

$p_5 = k$

p_{1-4} hard q and $p_5 = k$ soft

CM frame

$$\begin{aligned} p &= \left(\sqrt{s}, 0, 0, 0\right), \\ p' &= \left(0, \sqrt{s}, 0, 0\right), \\ k &= \left(y\sqrt{s}, \left(k_T^2 + m_\infty^2\right)/y\sqrt{s}, k_T, 0\right), \\ q &= \left(0, \left(k_T^2 + m_\infty^2\right)/y\sqrt{s}, \mathbf{q}_T\right). \end{aligned}$$

$s \to \infty,\ y \to 0,\ y\sqrt{s}$ fixed

GB formula reproduced!

$$|M_{12\to 345}|^2_{CM}$$

$$\approx 32(4\pi\alpha_s N_c)^3 N_g$$

$$\frac{s^2}{\left(k_T^2+m_\infty^2\right)\left(q_T^2+m_D^2\right)\left[\left(\mathbf{k}_T-\mathbf{q}_T\right)^2+m_D^2\right]}$$

Symmetry factor confusion…

$$|M_{12\to34}|^2 \approx 8N_g(4\pi\alpha_s N_c s)^2\left[\frac{1}{(t-m_D^2)^2}+\frac{1}{(u-m_D^2)^2}\right]$$

$$|M_{12\to34}|^2_{CM} \underset{\mathbf{q}^2\approx\mathbf{q}_T^2\approx0}{\approx} 16N_g(4\pi\alpha_s N_c)^2\frac{s^2}{(\mathbf{q}_T^2+m_D^2)^2}$$

⇧ need to cut the phase space!

$$|M_{12\to34}|^2_{CM} \underset{\mathbf{q}_T^2\approx0}{\approx} 8N_g(4\pi\alpha_s N_c)^2\frac{s^2}{(\mathbf{q}_T^2+m_D^2)^2}$$

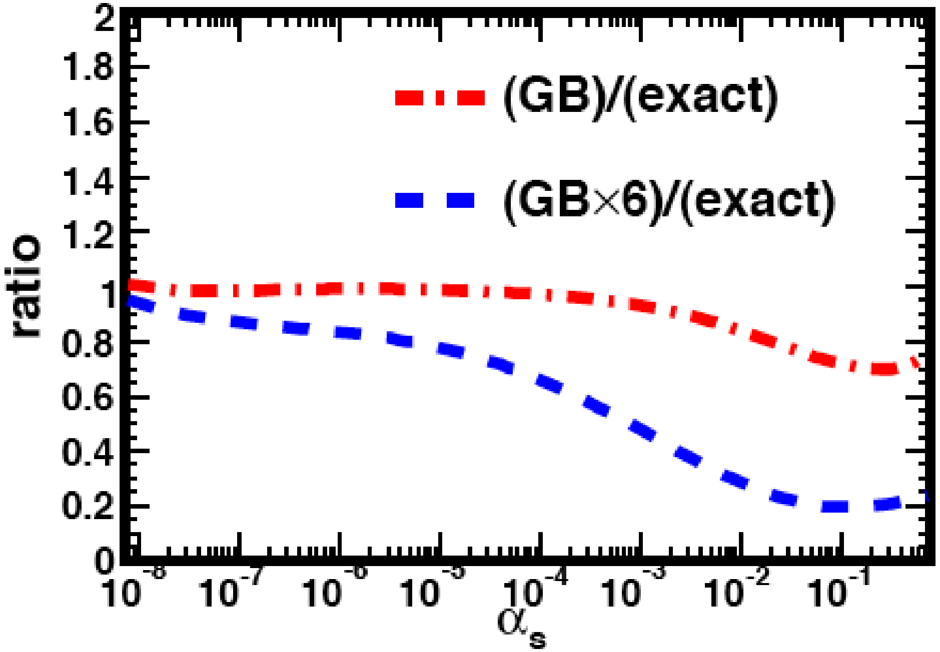
⇧ no phase space cut,
s and t included already

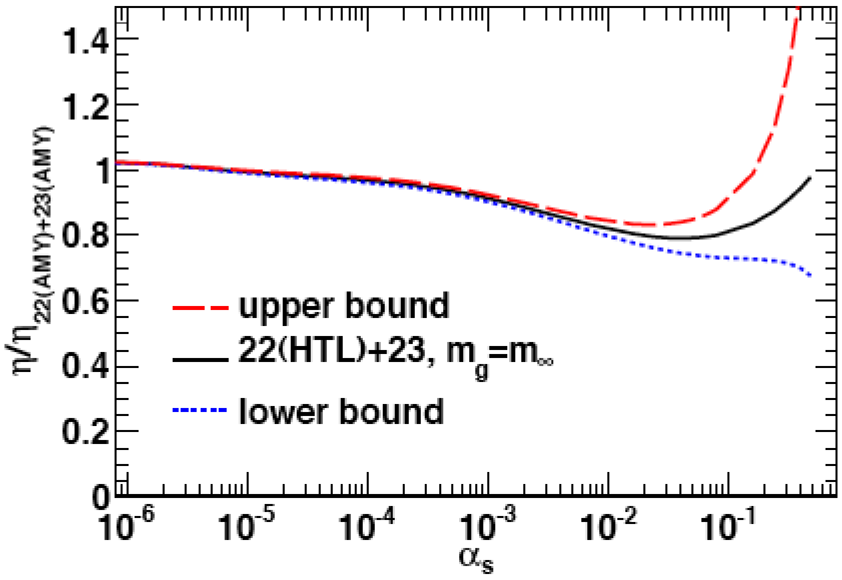
- Same thing in 23, multiply 6 to the rate with a cut phase space (3! momentum permutation) or just take the full phase space. Cannot do both!

23 Collision Rate:

GB(soft) > "Exact" > AMY(collinear)

O(10%) differences

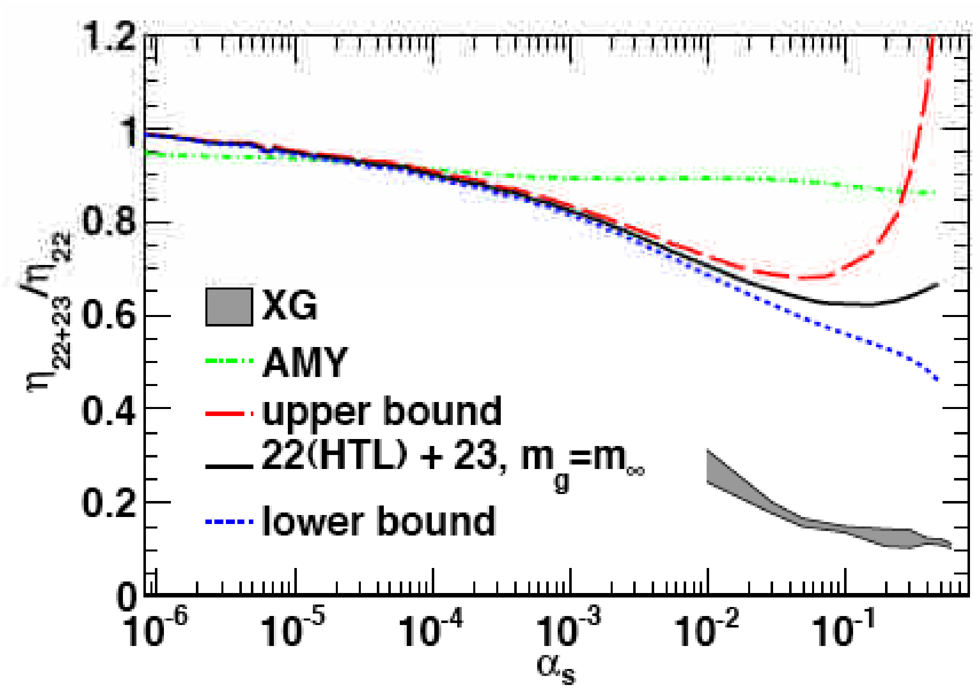


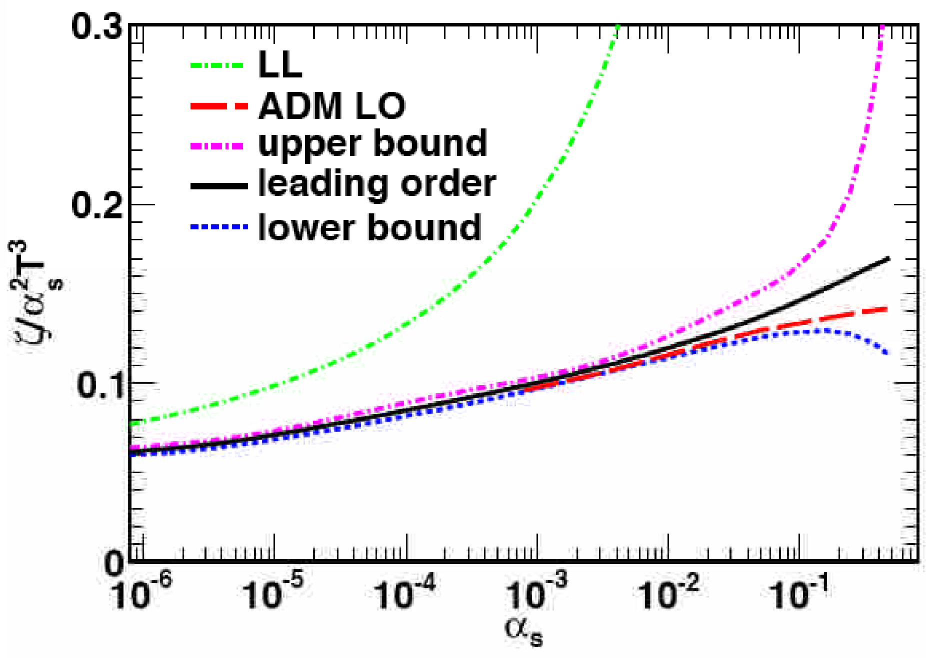


O(10%) differences

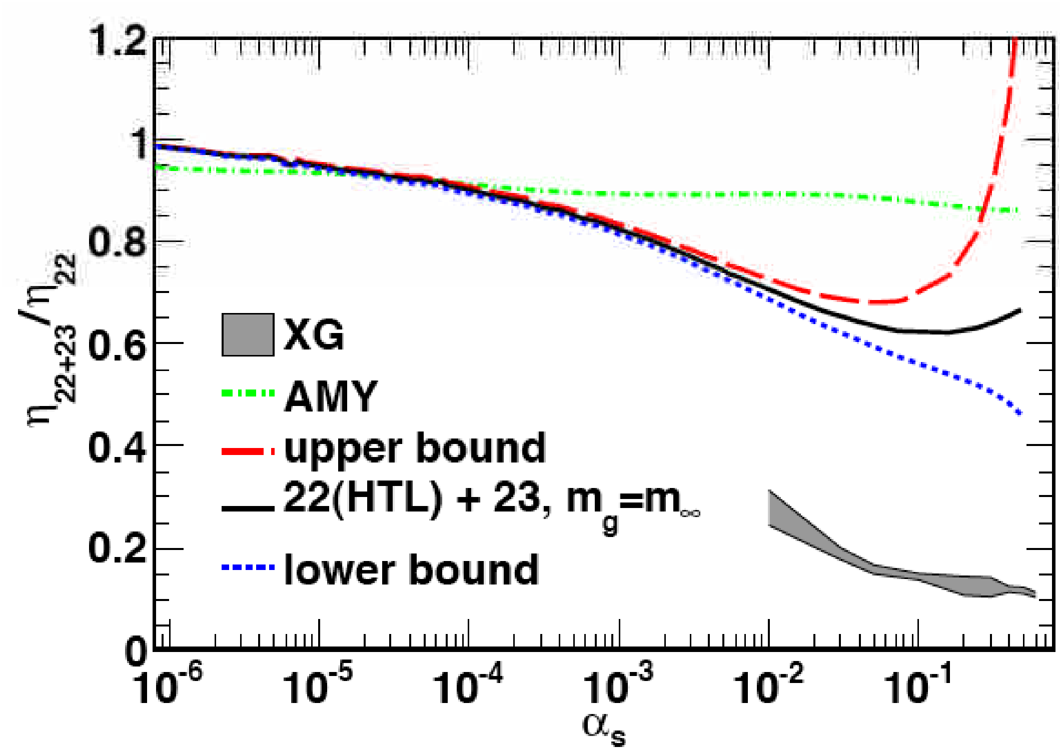
Why so small?

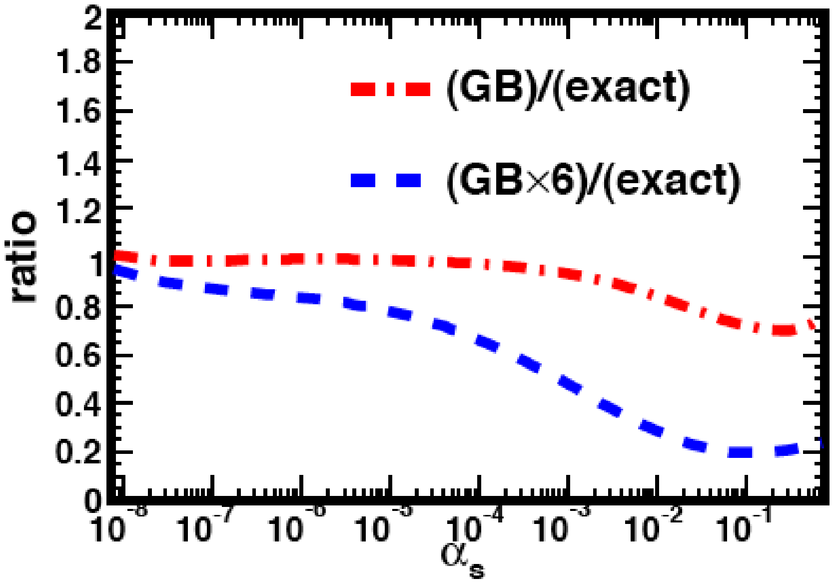
Consistent with AMY





A factor 6 mistake for XG?





Other cool things

- Power counting to other diagrams. 24, 33 subleading
- Go beyond variation, systematically approaching the true answer, independent of basis
- Nc dependence

$$\frac{\zeta}{s} = h_1\left(\alpha_s N_c\right), \quad \frac{\eta}{s} = h_2\left(\alpha_s N_c\right)$$

Conclusions and Outlook

Universal η/s and ζ/s behaviors

(η/s reaches local minimum near p.t.

ζ/s reaches local maximum near p.t.)

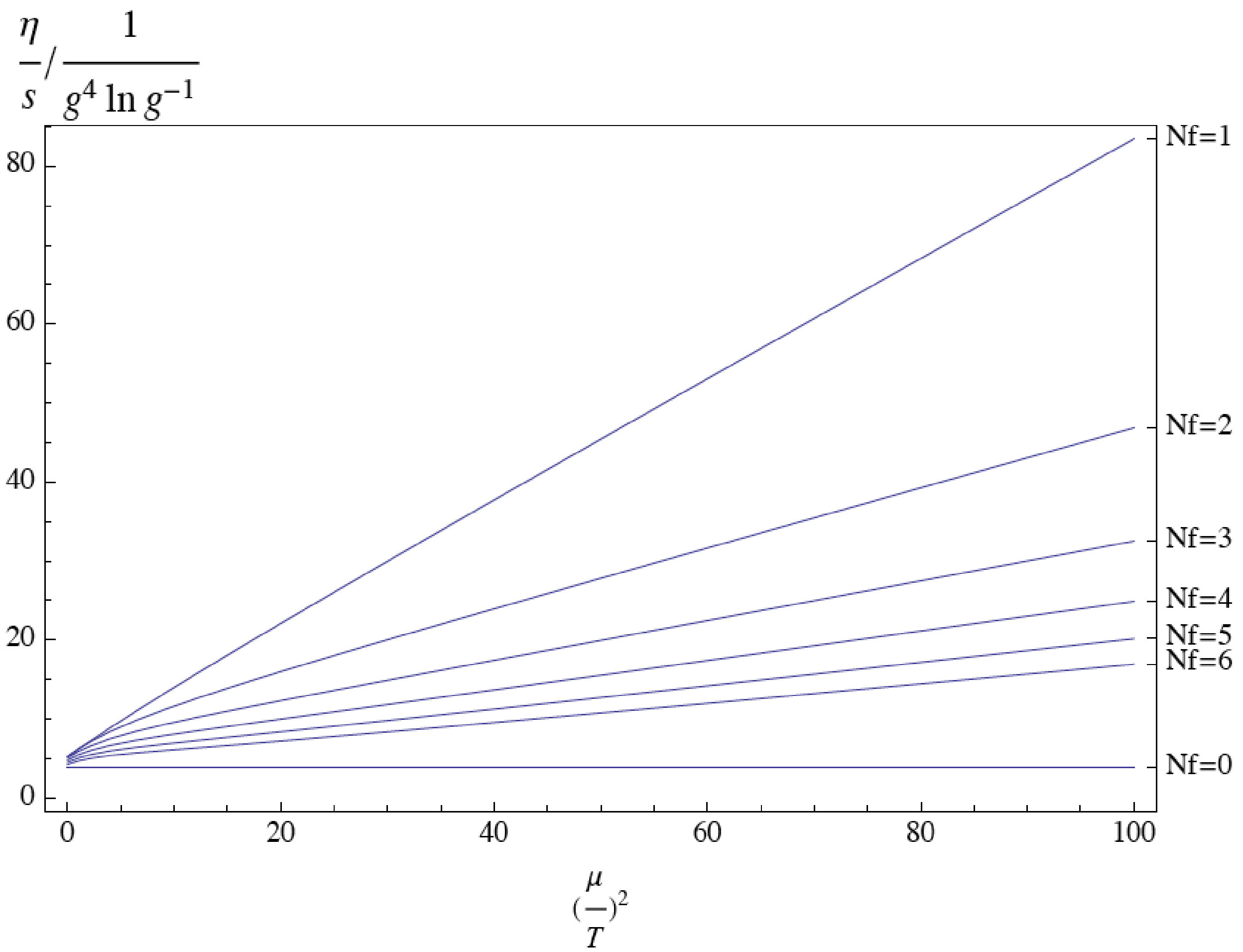
do not exist.

AMY vs. XG: agree with AMY

Work in progress: finite density transport coefficients in PQCD (Yen-Fu Liu)

Leading-Log results

(Yen-Fu Liu, preliminary)



Leading-Log results
(Yen-Fu Liu, preliminary)