Momentum diffusion of heavy quarks from Lattice QCD

in collaboration with

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Outline

Introduction

Lattice Setup

Results

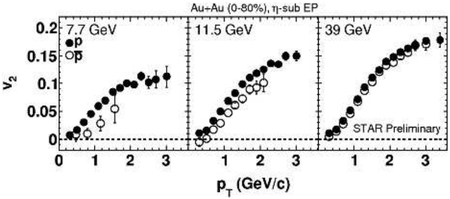
Discussion

Ways to study the plasma

- Charm and bottom quarks much heavier than RHIC and LHC temperatures
- Expect that they are produced in the early pre-equilibriated state of the collision and act as probe for the early time physics
- Perturbative arguments suggest energy loss mechanism to be very different for heavy quarks (HQ) from that of light quarks.
- Gluon bremstrahlung dominates for light quark jets Baier et. al. (1996); supressed for heavy quark jets Dokshitzer, Kharzeev (2001)
- For heavy quarks, collisional energy loss is at least as important as radiative energy loss for ~ 5 Gev, and more at lower momenta. Moore,Teaney (2005); Mustafa (2005)
- Comparative study of energy loss for the heavy quark and the light quark jets could offer crucial insights into the way the QGP plasma interacts.

What to expect?

- Collision with a thermal quark does not affect HQ energy much
- Weak coupling calculations relate thermalization time for heavy quarks (τ_R^H) and light quarks: τ_R^L: $\tau_R^H = \frac{M}{T}\tau_R^L$
  $M \to$ HQ mass and $T \to$ temperature of the medium.
  (For $T \sim 250$ MeV and charm $M \sim 1.5$ GeV, this is about a factor 6!)
- Early elliptic flow $\to$ azimuthal anistropy parameter v_2 is sensitive to this
- Expect mass ordering of the elliptic flow: $v_2^h \gg v_2^D \gg v_2^B$;
  Experimentally: $v_2^D \lesssim v_2^h$! Suggest early thermalization of charm quarks



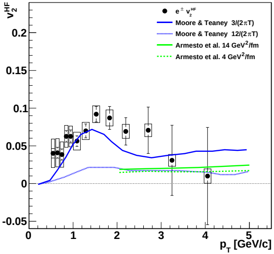


Figure: STAR, PHENIX

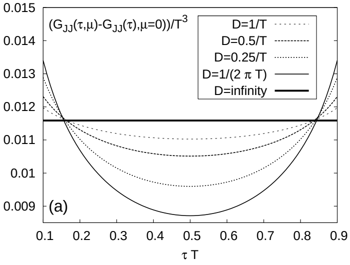
Input of non-perturbative physics

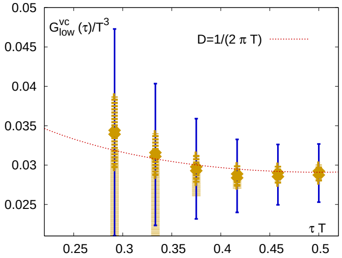
- $E_K \sim T$, $p \sim \sqrt{MT} \gg T \longrightarrow$ changes very little in a single collision; successive collisions are uncorrelated
- Langevin description for motion of HQ in the medium
  Svetitsky (1988), Moore Teaney (2005), Mustafa(2005)
- v_2 can be calculated in terms of the diffusion constant (D) of the heavy quark in the medium
- D is a parameter that can be tuned to match the experimental results
- v_2 of charmed mesons, and their p_T dependence well described, but requires small D Moore Teaney (2005) More
- An order of magnitude lower than leading order PT!
- Reliability of PT? Non-PT results clearly desirable for D

Non-perturbative calculations: Highs and Lows

- Lattice QCD: tool for first principles non-perturbative calculations
- Calculations in Euclidean space; extracting a real-time observable requires analytic continuation: very difficult in general!
- Operator of interest: correlator of HQ current $\bar{Q}\gamma^i Q$
- Euclidean correlator remarkably insenstive to D

Petreczky, Teaney (2006); Petreczky(2008)

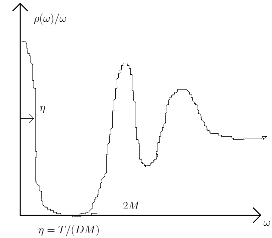


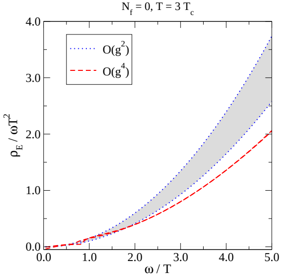


Other approaches?

- Problems with the heavy quark current correlator:
  ★ structure of the spectral function $\rho(\omega)$ affects extraction of the low-ω part
  ★ D needs to be extracted from the the width of the narrow transport peak at low-ω
- Alternative: look in the static limit?
- Propagation of heavy quarks replaced by Wilson lines Casladerrey-Solana, Teaney (2006)
- Can be reformulated as an correlation function of color electric fields

  Caron-Huot, Laine, Moore (2009)
- NLO-PT shows the corresponding $\rho(\omega)$ is smooth at low-ω $\to$ good news for lattice! Brunier, Laine, Langelage, Mether (2010)





Langevin formalism Moore and Teaney,2005

- For heavy quarks $M \gg T$ moving in the plasma, average thermal momentum $p \sim \sqrt{MT} \gg T$
- $\mathcal{O}(M/T)$ collisions by the quasiparticles of the plasma needed to change the motion of the quarks
- The motion can therefore be described by the Langevin equation

$$\frac{dp}{dt} = \xi(t) - \eta_D p; \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

  $\xi(t) \longrightarrow$ random force; $\eta_D \longrightarrow$ drag
  $\kappa \longrightarrow$ strength of the stochastic interaction: Property of the medium
  $p(t) = [p_0 + \int_0^t e^{\eta_D s}\xi(s)ds]e^{-\eta t}$
  Relaxation governed by η_D
  Related to the relaxation time $\tau_R = 1/\eta_D$

- The momentum diffusion coefficient,κ is

$$\kappa = \frac{1}{3}\int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t)\xi_i(0)\rangle$$

- Related to η_D by Fluctuation-Dissipation relation: $\eta_D = \frac{\kappa}{2MT}$

NR-QCD formulation Caron-Huot, Laine, Moore (2009)

- For field theoretic generalization, in terms of the heavy quark current $J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$:

$$\kappa \equiv \frac{1}{3T\chi}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty} M^2\int_{-\infty}^{\infty} dt\, e^{i\omega(t-t')}\int d^3x \left\langle \frac{1}{2}\left\{\frac{dJ^i(t,x)}{dt}, \frac{dJ^i(t',0)}{dt'}\right\}\right\rangle\right]$$

- In the static limit, the force on the heavy quark (HQ):

$$M\frac{dJ^i}{dt} = \{\phi^\dagger E^i\phi - \theta^\dagger E^i\theta\}$$

ϕ, θ: 2-component HQ and H$\bar{\text{Q}}$ operators; E^i: colour electric field

- In this limit, this is the only contribution
- Further simplifications can be done:

$$\langle\theta_a(\tau,\vec{x})\theta_b^\dagger(0,\vec{0})\rangle = \delta^3(\vec{x})\, U_{ab}(\tau,0)\ \exp(-M\tau)$$

- Final expression for infinitely heavy quarks:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}[U(\beta,\tau)gE_i(\tau)U(\tau,0)gE_i(0)]\rangle}{\langle \mathrm{Re\,Tr}[U(\beta,0)]\rangle}$$

From $G(\tau)$ to $\rho(\omega)$

Need to solve an integral equation to get κ from $G(\tau)$:

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh[(\frac{\beta}{2} - \tau)\omega]}{\sinh[\frac{\beta\omega}{2}]}$$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$$

- In general, the inversion problem is ill-defined
- Usually, some assumptions on $\rho(\omega)$ to get any meaningful output
- MEM has been used for this in literature $\rightarrow$ requires a large no of points, as well as per-mille errorbars on data
- Such accuracy much more difficult with gauge field observables than with meson correlation functions
- For our case, will parametrize $\rho(\omega)$ with small number of parameters, and subsequently extract them using fitting

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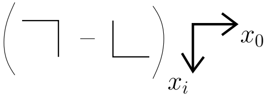
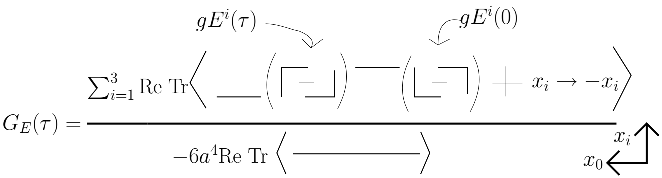
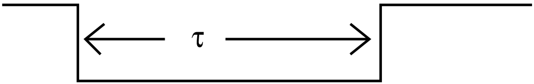
Lattice operators

- Note $g\,E_i \equiv [D_0, D_i] = D_0 D_i - D_i D_0$. Replace this by (suggested in Caron-Huot, Laine, Moore (2009))



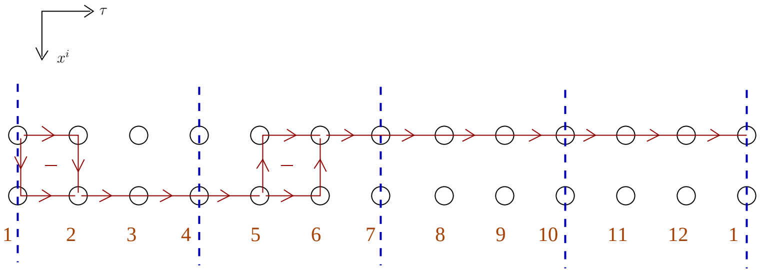




$$G_E(\tau) = 2C(\tau) - C(\tau + 1) - C(\tau - 1)$$

Need for the multilevel algorithm

- Known that the signal for the Polyakov loop becomes exponentially supressed for large N_t
- Reliable extraction of κ needs large N_t
- Use of Multilevel algorithm Lüscher Weisz (2001 & 2002) essential
- Downside: requires large memory



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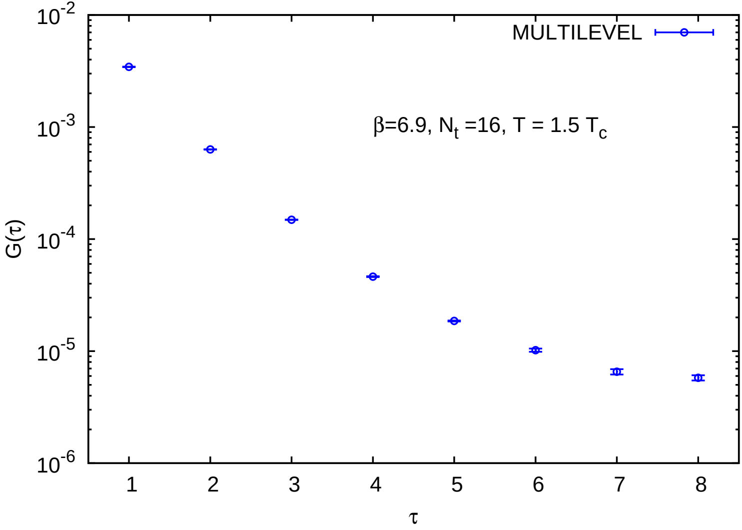

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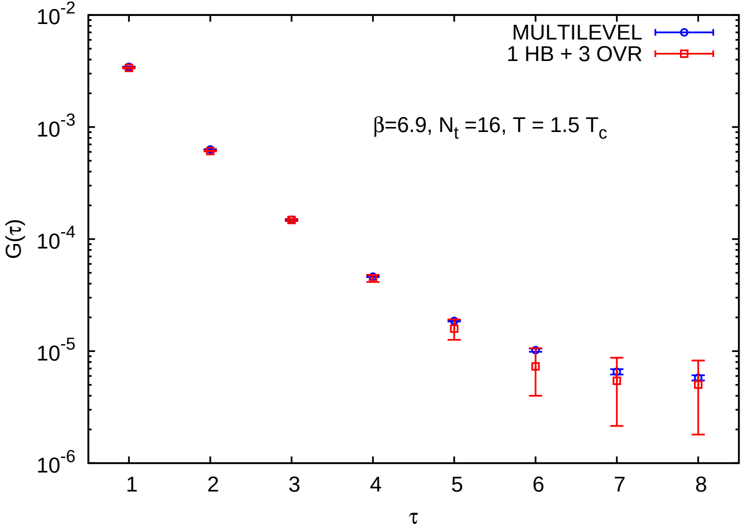

Lattice sizes

- Explored $N_t = 12 - 24$
- For finite volume analysis: $N_s/N_t = 2 - 4$
- Temperature range from just above T_c to $3T_c$
- Reliable extraction possible only for $N_t \geq 20$
- Typical stats: several hundred independent configs, each with several thousand multilevel updates
- Correlation function have a few % error-bars at the largest τ for $N_t \sim 20$
- Very fine lattices: typical lattice spacings 0.02 - 0.03 fm

β	6.76	6.80	6.90	7.192	7.255
N_t	20	20	20	24	20
T/T_c	1.04	1.09	1.24	1.5	1.96

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Parametrization strategy

- LO perturbative form of $\rho(\omega) \sim b\omega^3$
- In the $\omega \to 0$ limit need $\rho(\omega) \sim a\omega$ to see diffusion in $\mathcal{N} = 4$ SYM plasma Casalderrey-Solana, Teaney 2006
- Ansatz: $\rho_1(\omega) = a\omega\Theta(\Lambda - \omega) \;+\; b\omega^3$
- Calculations in classical lattice gauge theory suggest

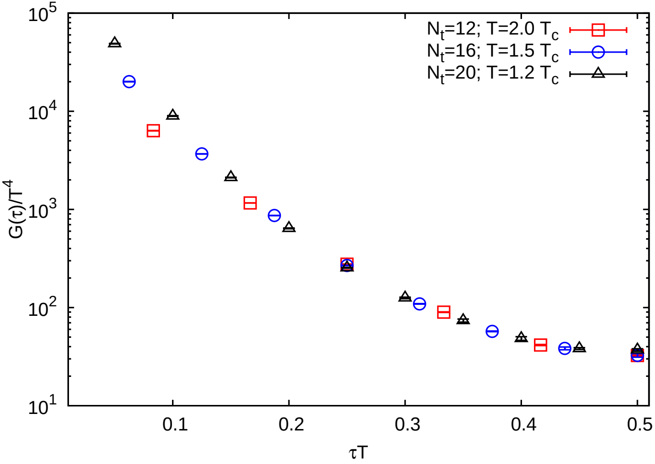
$$\rho(\omega) \;\sim\; c \tanh\frac{\omega\beta}{2} \qquad \text{for} \;\; \omega a \ll 1.$$

- Also used the following fit form to cross-check:

$$\rho_2(\omega) \;=\; c \tanh\frac{\omega\beta}{2}\,\Theta(\Lambda - \omega) \;+\; b\omega^3.$$

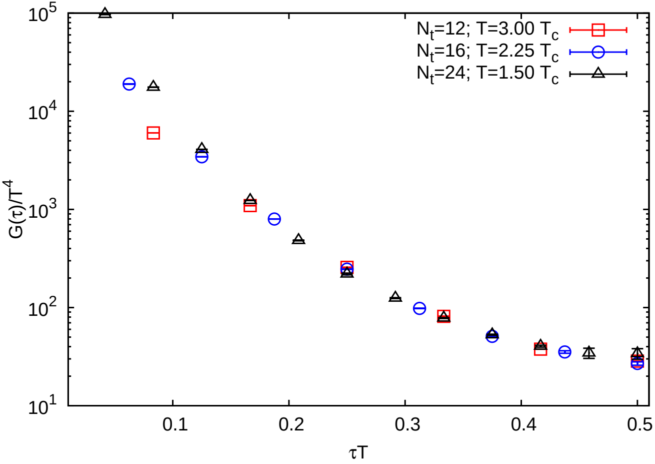
- Not feasable to do a 3-param fit. κ and Λ strongly correlated
- Keep Λ fixed, and do a full covariance matrix fit for $\tau a \in [N_t/4, N_t/2]$

Correlation functions



- Small-τ affected by lattice artefacts
- Large-τ region shows scaling: hint of continuum physics?

Correlation functions



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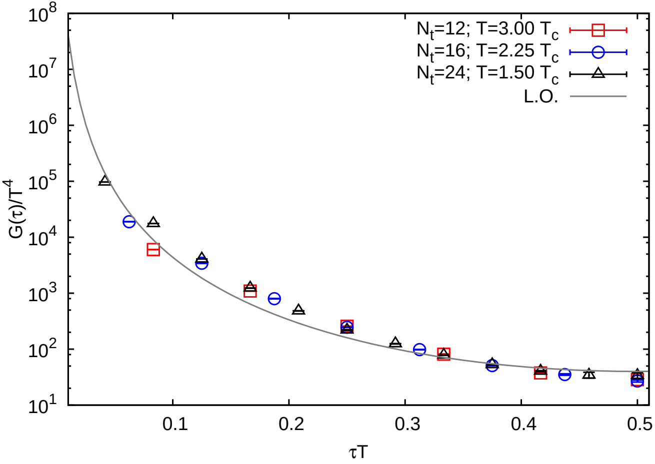
The LO contribution



Large N_t needed for reliable extraction!

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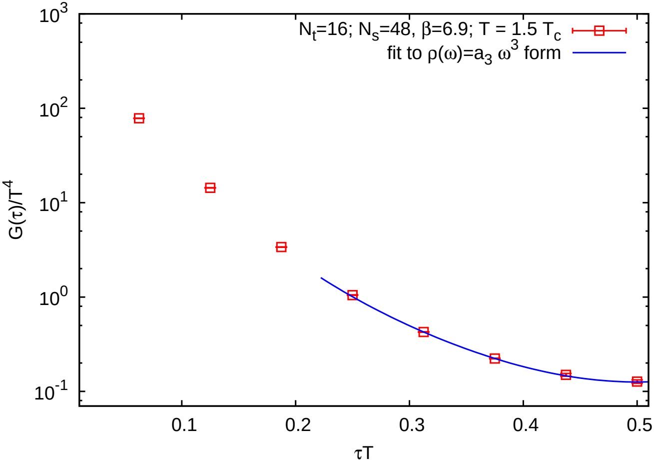

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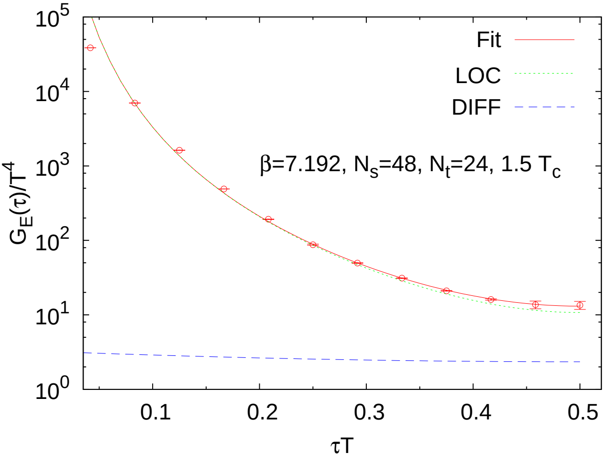
The diffusive part

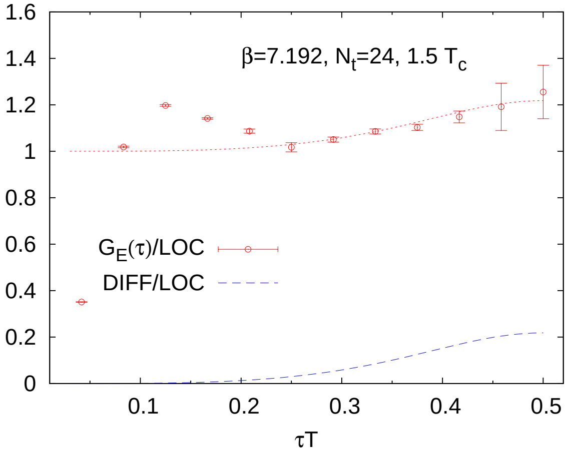


Results quoted with $\Lambda = 3T$

Diffusive contribution small: about $\sim 20\%$ for $\tau T = 0.5$

Not possible to see in $N_t = 12, 16$

The diffusive part



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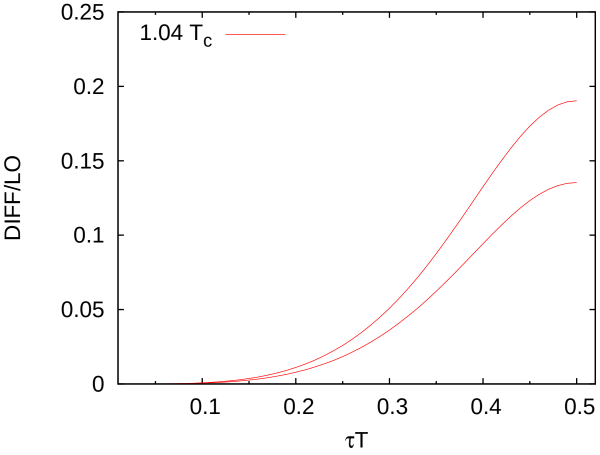
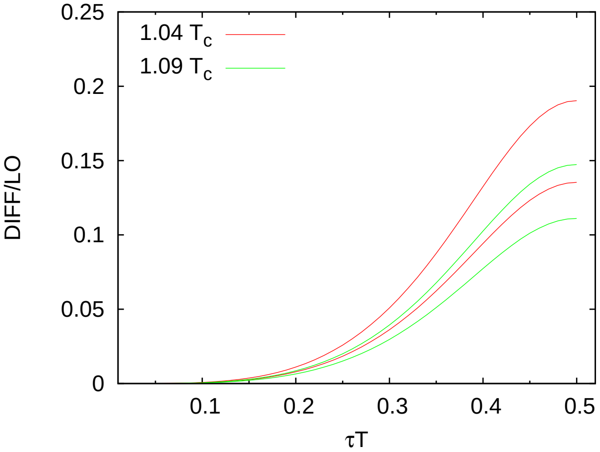
Diffusive part for various T



No significant temperature dependence! Diffusive part reaches to about 5% by $\tau T = 0.3$

Diffusive part for various T



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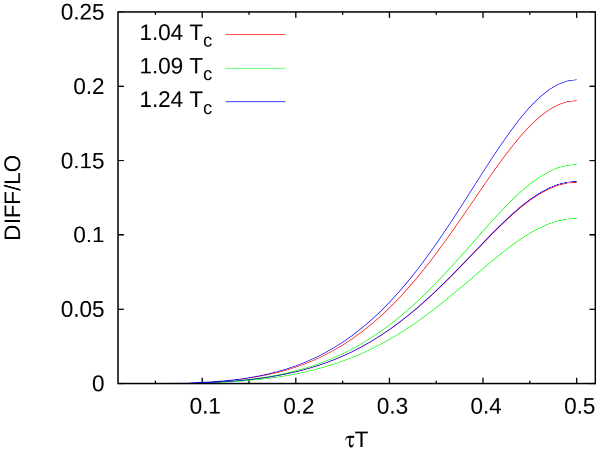

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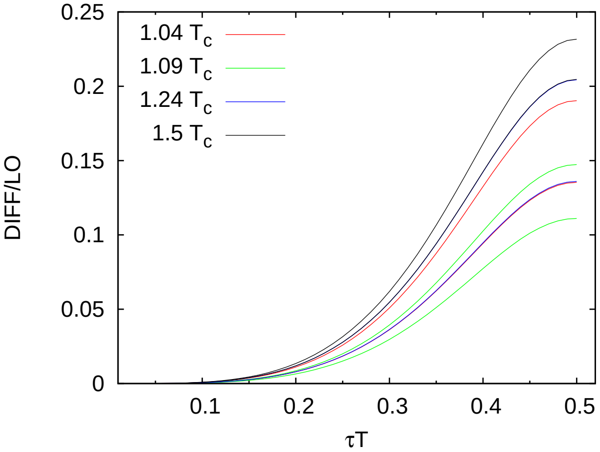

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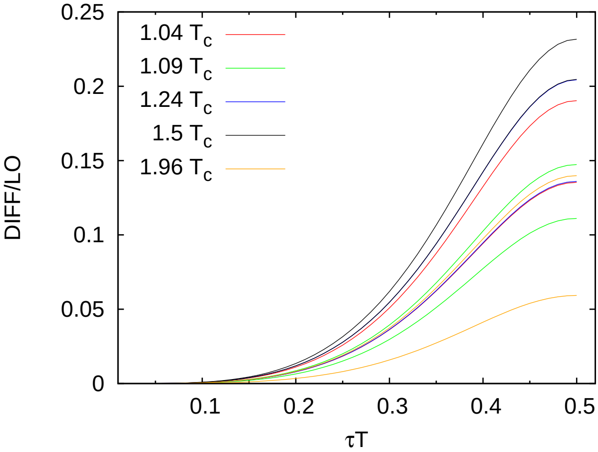
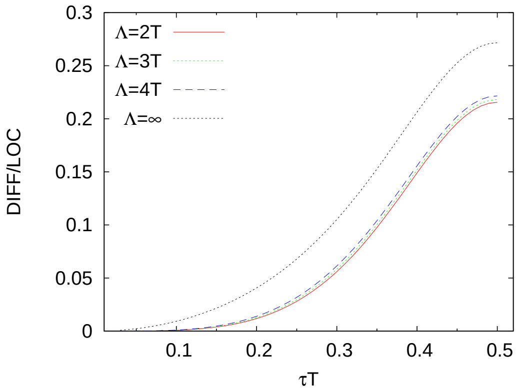


No significant temperature dependence! Diffusive part reaches to about 5% by $\tau T = 0.3$

Λ dependence

- Quality of the fit rather insenstive to Λ
- Different $\Lambda \rightarrow$ different κ without affecting χ^2

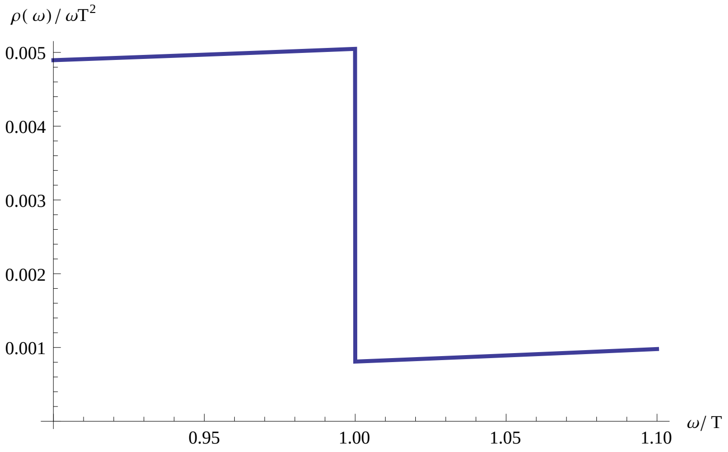


- Typically a 30-50% variation in κ

Λ dependence: Which value to quote?

- Why cutoff? Large-ω does not have diffusion
- The Λ dependence in the fit represents a “flat” direction
- Cut-off’s are approximation; no change expected for a smooth variation
- The “flat” direction has a more general nature
- Follow the conservative estimate of letting Λ vary $[2T, \infty]$
- Use systematic error-band
- To quote central value: Determine when the diffusive contribution starts competing with the LO contribution
- This happens around $\Lambda \sim 3T$ for our values
- Alternatively, jump in $\rho(\omega)$ is less when $\Lambda \sim 3T$

Λ dependence: Which value to quote?



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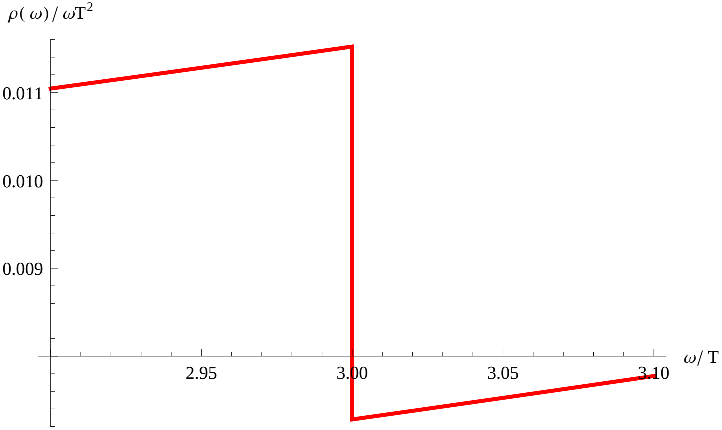
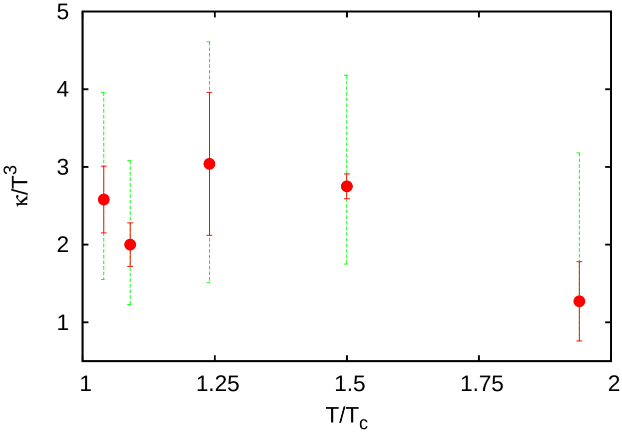
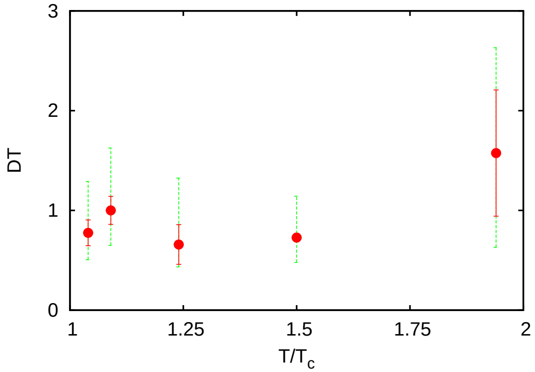


κ estimates



Errors both systematic and statistical; for both kinds of $\rho(\omega)$ considered

DT estimates



- Using Einstein relations, $D = T/(M\eta_D) = 2T^2/\kappa$
- Lower than Meyer,2010 (same formulation, different operators & analysis)
- Agree with preliminary estimates of Francis et. al.,2011 (Same formulation, operators, different analysis); Ding et. al.,2011 (charm correlators, MEM) More

Systematics: finite volume effects

- Known that appreciable finite volume effects can arise if the spatial size causes deconfinement
- Our lattices ($LT \gtrsim 2$) always satisfy this condition
- Further, low-ω part can have a non-trivial volume dependence
- Results show lack of any significant volume dependence

$$\chi^2/d.o.f. = \frac{1}{N_t/4} \sum_{\tau=\frac{N_t}{2}+1}^{N_t/2} \frac{|G_1(\tau) - G_2(\tau)|}{\sqrt{\sigma_1(\tau)^2 + \sigma_2(\tau)^2}}.$$

β	N_t	$(LT\|_1, LT\|_2)$	χ^2/d.o.f.
6.4	12	(2, 4)	0.34
6.65	12	(2, 4)	0.75
	16	(2.25, 3)	1.12
6.9	12	(3, 4)	0.24
	16	(2.25, 3)	0.51
	20	(1.8, 2.4)	1.58
7.192	24	(2, 2.33)	0.29

Systematics: Renormalization

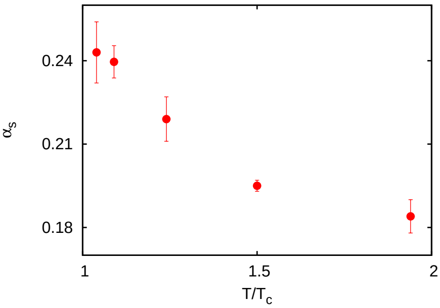
- Need to get physical correlator of electric fields: $G_E(\tau) = Z(a) G_E^{Lat}(\tau)$
- Non-pert renormalization not available for these operators
- Expected to be dominated by self-energy correction
- Can be taken care of using the tadpole factor: $Z_E^{-1} = \left(\frac{1}{N}\langle \mathrm{Tr} U_p \rangle\right)^{\frac{1}{4}}$
- Simplification: $\langle L \rangle$ cancels most of the straight line part; $Z(a) = Z_E^2$

Using the tadpole factor for renormalization gives values very close to those obtained by non-pert renorm. for other discretizations at smaller β

Koma, Koma, Wittig (2006), Koma Koma (2007)

Strong coupling constant α_S

- In LO, $\rho^{LO}(\omega) = \frac{8\alpha_S}{9}\omega^3$
- Use the fit coefficient of ω^3 term to define α_S using the scheme
- Can be related to $\alpha_S^{\overline{MS}}$ using the NLO calculation of Brunier et. al. (2010)



Agrees with a similar calculation of α_S from vector current correlators (Ding et. al., 2010) and other estimates of α_S from static observables

Kaczmarek,Zantow; 2005

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Other estimates: PT and AdS/CFT

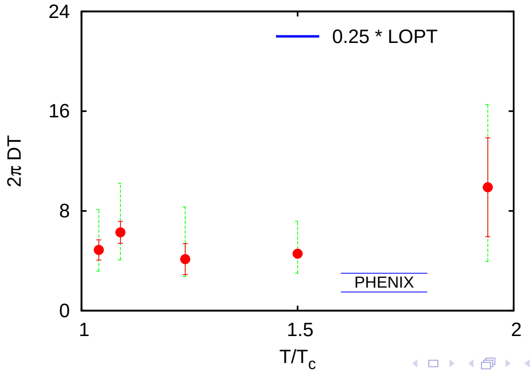
- At very high T, $DT\ 1/\alpha_S^2$
- LO PT gives a large value for DT Moore,Teaney; (2005) Brunier et. al. (2010)
  At 1.5 T_c, $\alpha_S^{\overline{MS}}(3T) \sim 0.23$; $m_D/T \sim 2.345$ giving $DT \sim 14$
  Not large change for $N_f \neq 0$; order of magnitude greater than the non-pert estimate!
- NLO corrections to κ start at $O(g)$. Calculated for $N_f = 3$ Caron-Huot, Moore (2007): with $\alpha_S \sim 0.2$, $DT \sim 8.4/2\pi$
- While a similar change will bring it close to the non-perturbative estimate of $N_f = 0$, issues with convergence need to be clarified
- On the other hand, computation in AdS/CFT available
  Casalderrey-Solana, Teaney, 2006

$$DT \simeq \frac{0.9}{2\pi}\left(\frac{1.5}{\lambda_{tH}}\right)^{\frac{1}{2}} ; \lambda_{tH} = \alpha_S N_c$$

- Note: parametric dependence on α_S different
- Putting $\alpha_S \approx 0.23$ and $N_c = 3$, $DT \approx 0.2$
  Lower than, but in the same ballpark as the non-pt estimate

The Larger picture: Experiments and Theory

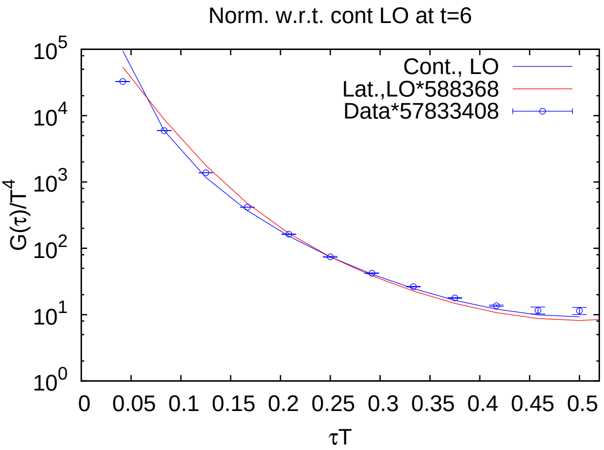
- Non-perturbative results different from PT.
- However, no thermal quarks in the calculation
- Expect scaling with full QCD as function of T/T_c?
- Values in the right ballpark to explain v_2 results from PHENIX in the Langevin formulation



Summary

- Calculated momentum diffusion coefficient of heavy quarks in the gluon plasma
- Multilevel algorithm essential for obtaining accurate data
- Essentially used fit ansatz to extract the diffusion constant
- Reasonably close estimates to explain experimental values using the Langevin formulation
- Significanly different from PT. Agreement with estimates of other groups Ding et. al. (2011), Francis et. al. (2011)
- Model independent estimates using subtracted correlation function? Brunier Laine (2012)
- More theoretical control over the renormalization constant desirable
- Finer lattices
- Improved discretizations of electric field?

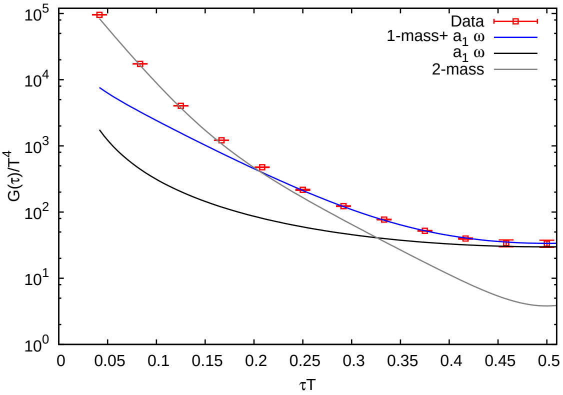
Extra: Lattice PT



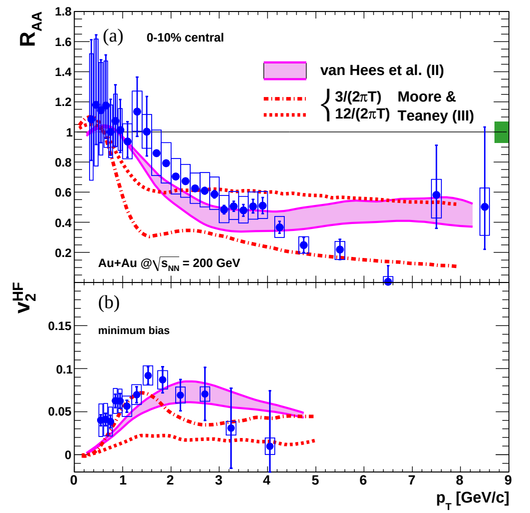
cont LO captures short distance effects better than lattice LO PT!

Extra: Other functional forms

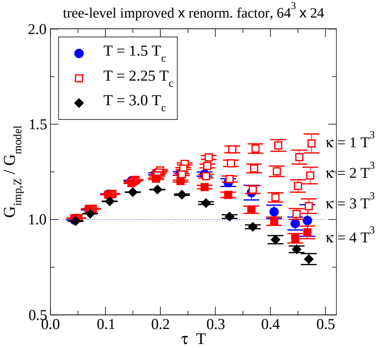
$$G(\tau) = \frac{a_1 \pi}{N_t^2} \frac{1}{\sin^2(\pi\tau/N_t)} + A_1 \cosh(M_1(\tau - 1/2T)) + A_2 \cosh(M_2(\tau - 1/2T))$$

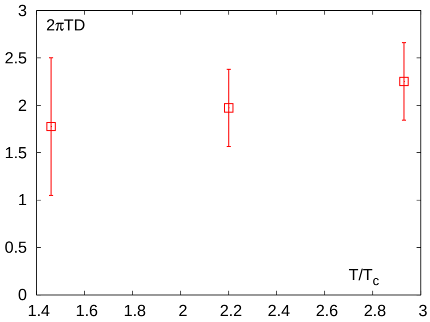


Different behaviour of M_1 and M_2; M_2 does not change with N_t, but $M_1/T \sim 16.5$ between 1.5 - 3 T_c



A. Adare et. al. (PHENIX) 2010





Left: Francis et. al. (2011); Right: Ding et. al. (2011) Back