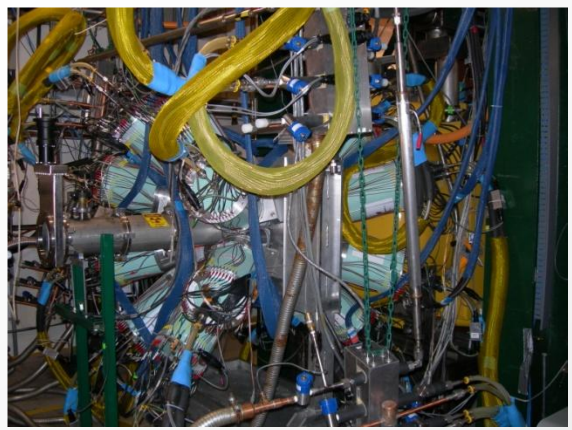
NSCL JINA MICHIGAN STATE UNIVERSITY

Theoretical challenges for charge-exchange experiments (with RI beams)

Remco Zegers

S800 spectrograph



SeGA

LENDA

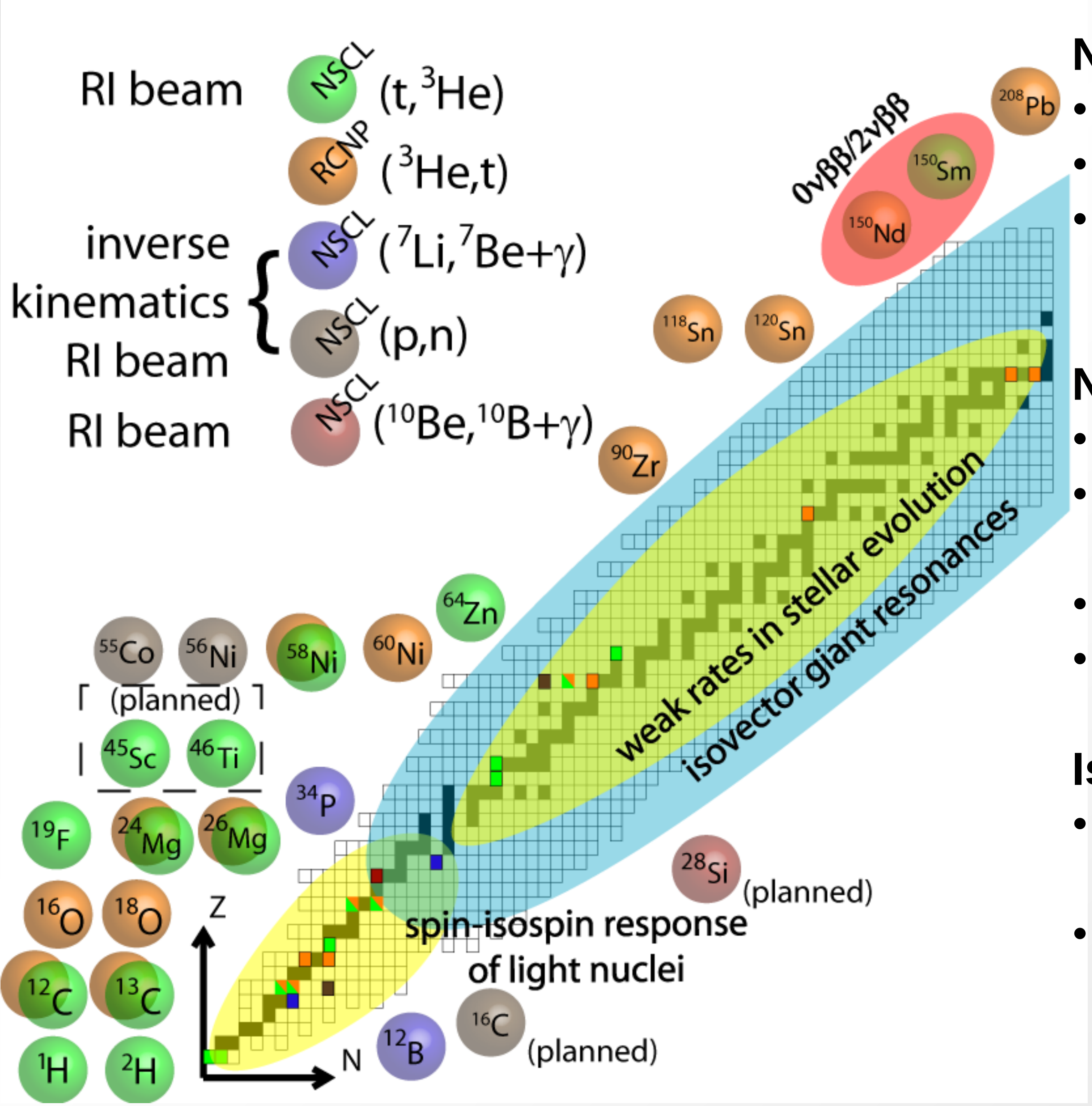
LH_2 target

contents

- CE reactions at intermediate energies: a tool for extracting Gamow-Teller strengths
- Two recent results of CE experiments with RI beams
  - $^{12}Be(^{7}Li,^{7}Be)^{12}B^{*}$ at 80 MeV/u
  - $^{56}Ni(p,n)$ reaction at 110 MeV/u (astrophysics)
- Theoretical challenges (focusing on GT strengths)
  - Reaction theory: study of (^{3}He,t) and (t,^{3}He) data
- Beyond Gamow-Teller strengths…

General comment: *RIBF/FRIB/FAIR generate RI beams at energies of 100's MeV/u. Highest intensities will be achieved near these energies. Developing/improving reaction theory for these energies is critical…*

NSCL Charge-Exchange group program



Nuclear Astrophysics

- weak rates for late stellar evolution
- neutrino processes
- specific weak interactions for novae

Nuclear structure

- Spin-isospin response
- test of structure models up to high excitation energies
- Shell evolution
- Double beta decay

Isovector giant resonances

- macroscopic properties of nuclear matter (neutron-skin, EOS)
- microscopic descriptions high in the continuum

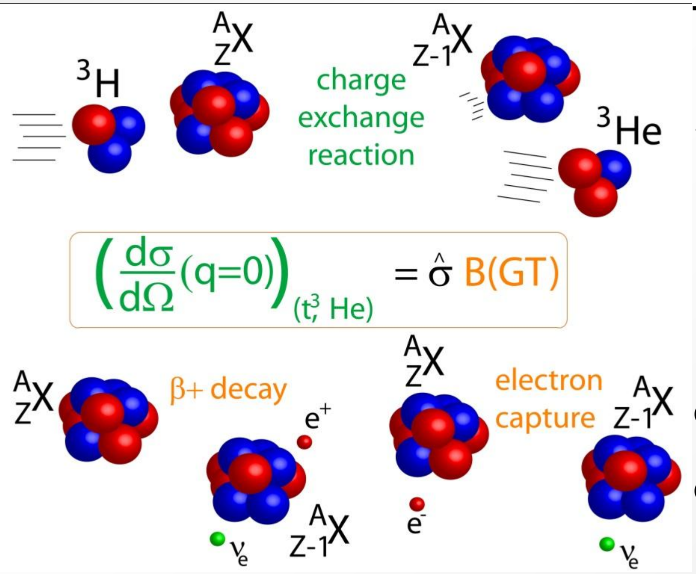
CE reactions at intermediate energies: extraction of Gamow-Teller strengths

$$\left.\frac{d\sigma}{d\Omega}\right|_{q=0} = KN\left|J\right|^2 B(GT) = \hat{\sigma}\cdot B(GT)$$

$\mathrm{E} \approx 100\,A\mathrm{MeV}$ and above

- single-step direct reaction
- governed by meson-exchange potentials

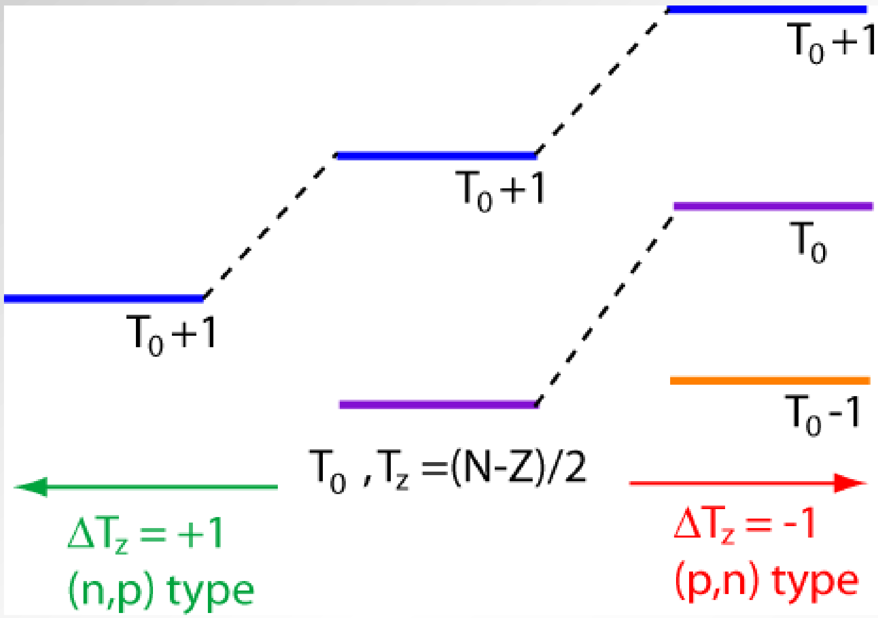
K: kinematic factor
N: distortion factor
J: volume integral of effective interaction
B(GT): GT strength

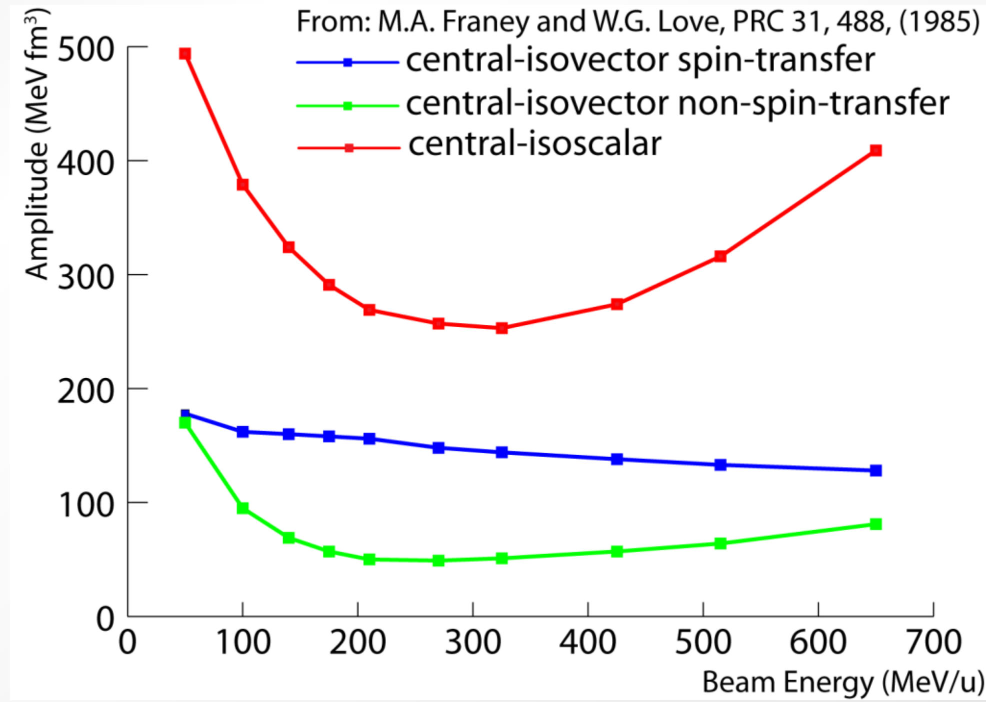


Calibrate unit cross section using transitions for which B(GT) is known from β-decay → apply excitations for which B(GT)s are not known.

In first order, there is no issue with absolute normalization of the strengths and the extracted strengths are model-independent.
Second order effects…later

Probes & Energy

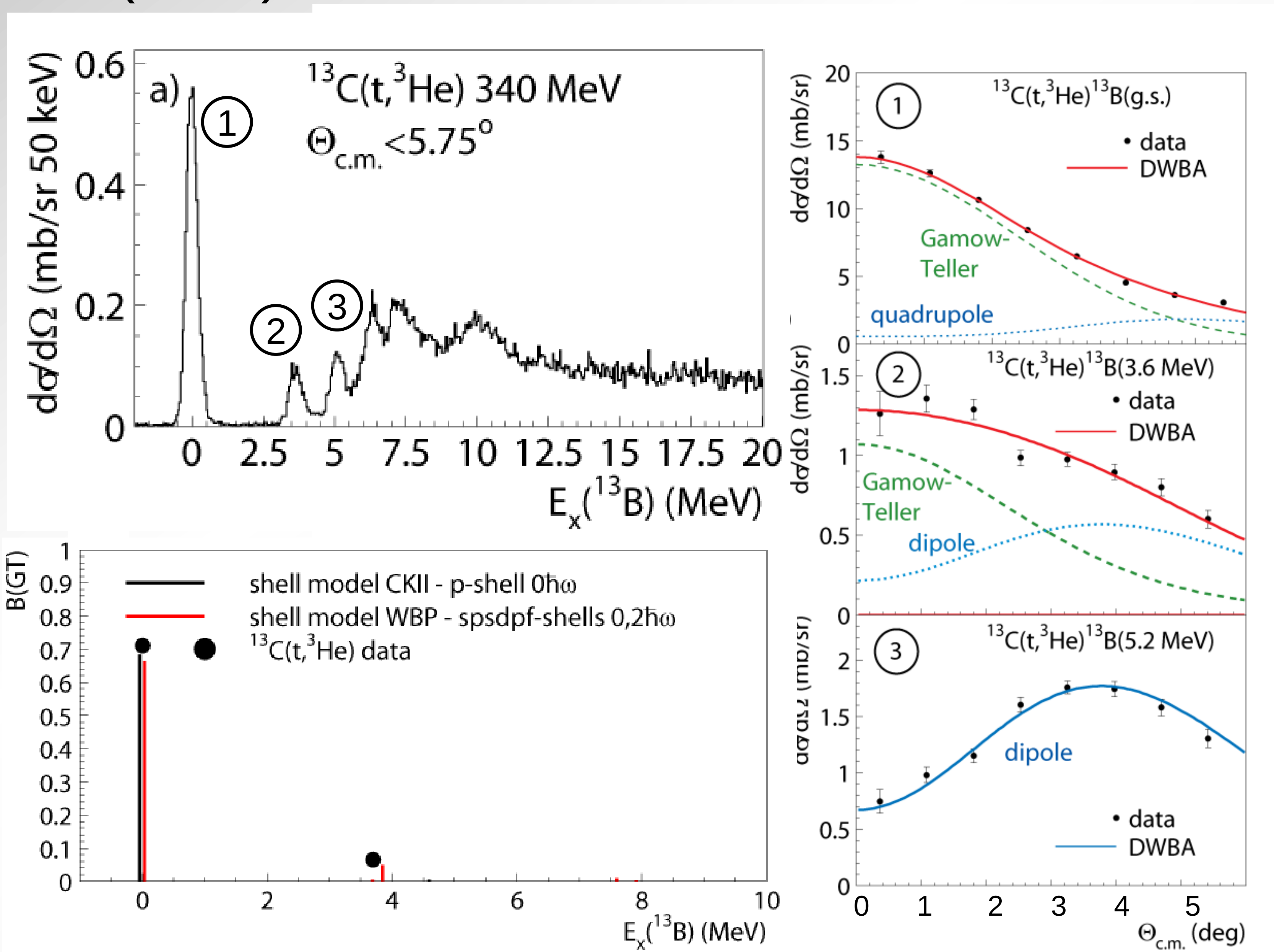




- Charge-exchange reactions are isospin-transfer reactions: ΔT=1 (isovector)
- charge-exchange probes (p,n) (n,p) (d,^{2}He) (^{3}He,t) (t,^{3}He) (^{7}Li,^{7}Be), HICE (π^+,π^o) (π^-,π^o)…

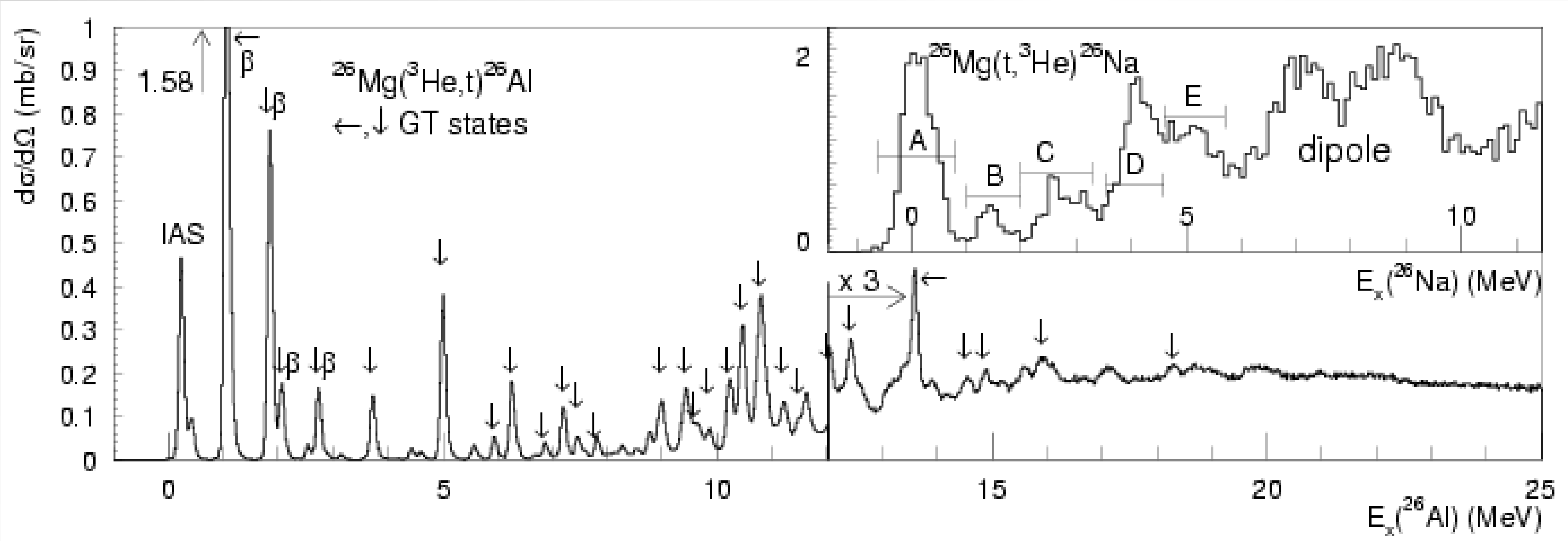
- E~100 AMeV and above
  - Distortions/rescattering minimized
  - Spin-flip transitions dominate over non-spin-flip transitions

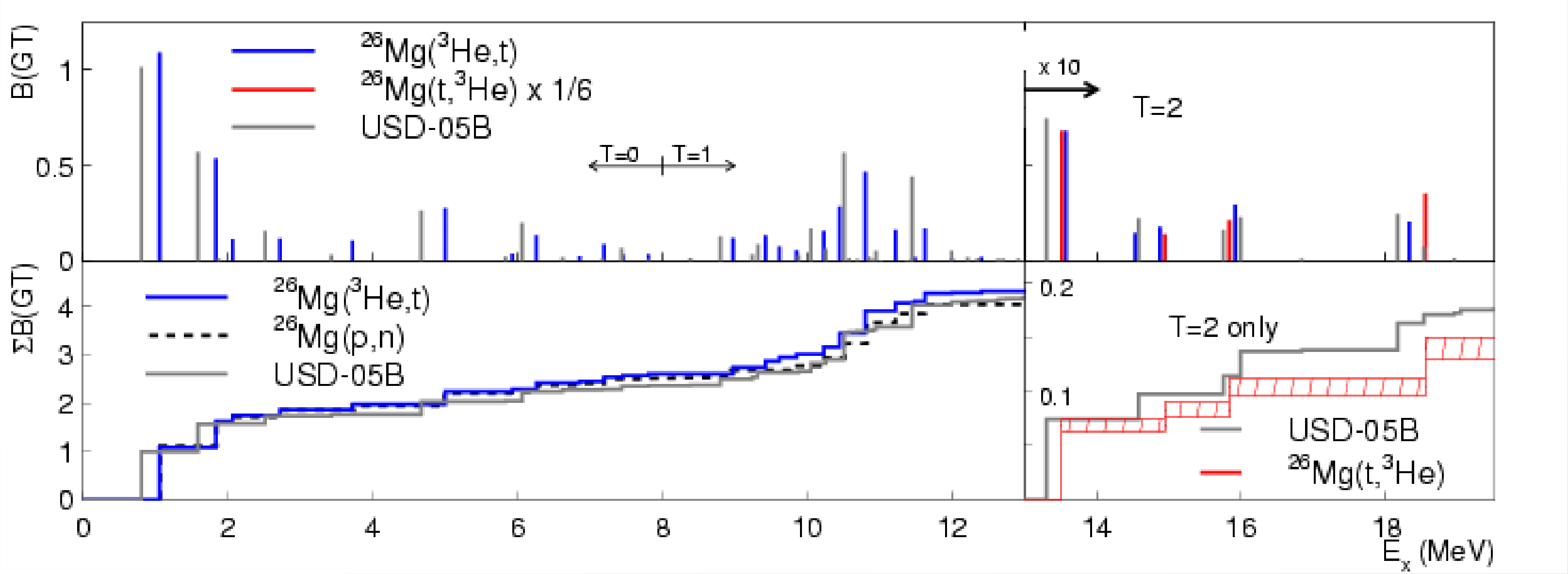
$^{13}C(t,^{3}He)^{13}B^{*}$ reaction: Gamow-Teller transitions



C. Guess et al. PRC 80, 024305 (2009)

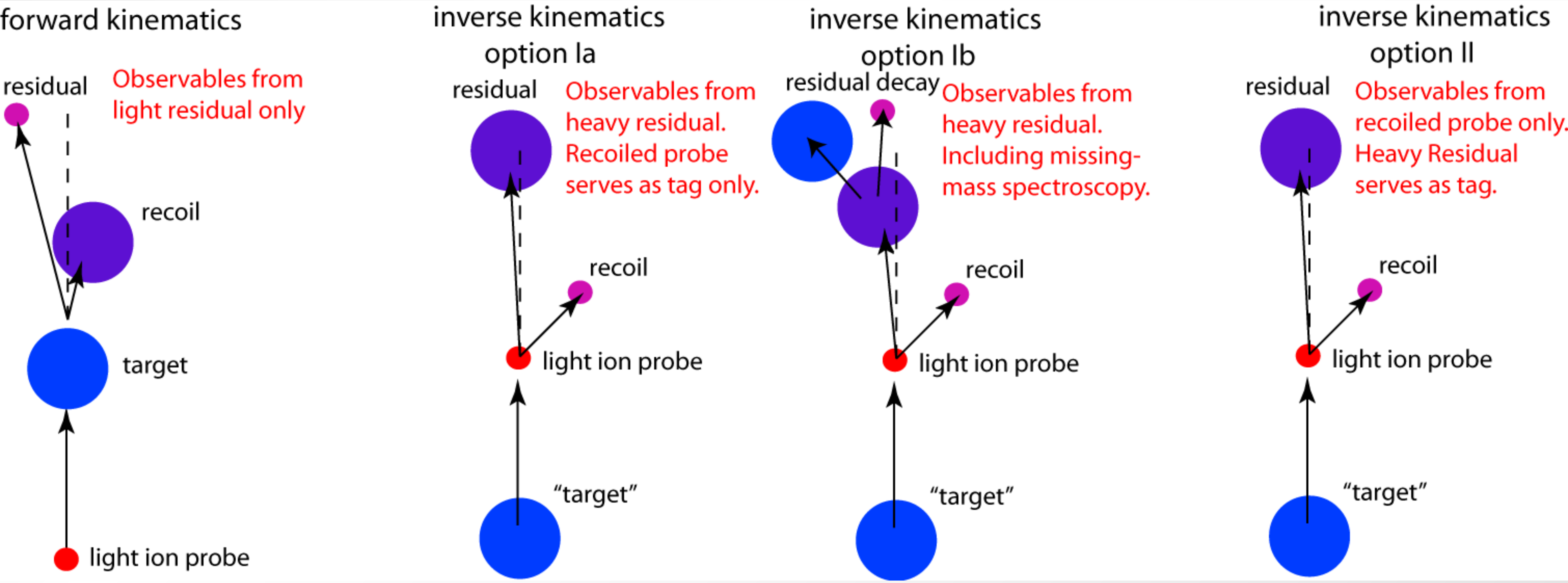
^{26}Mg(^{3}He,t) & ^{26}Mg(t,^{3}He)





CE in inverse kinematics with RI beams

- Required
  - Measure excitation energy over 'wide' range
  - Measure C.M. scattering angle
- Background free
- Ensure clean single-step CE reaction



Energy resolution?
Angle resolution?
Decay in flight?

(^{7}Li,^{7}Be) probe

Energy resolution?
Decay in flight?
Heavy nuclei?

various

Recoil escape target?
Angle resolution
Energy Resolution

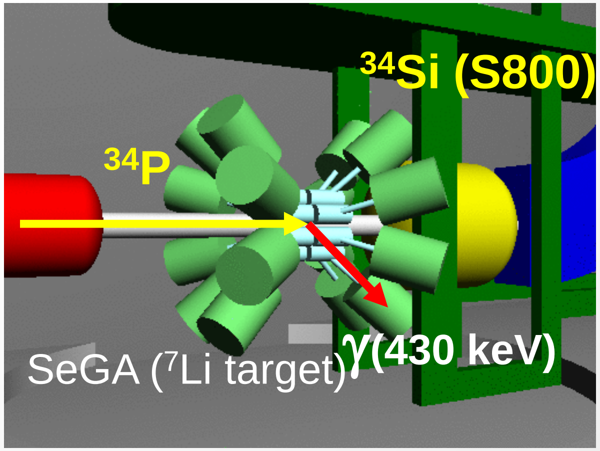
(p,n) probe (d,^{2}He)?

(^{7}Li,^{7}Be+γ) reaction in inverse kinematics

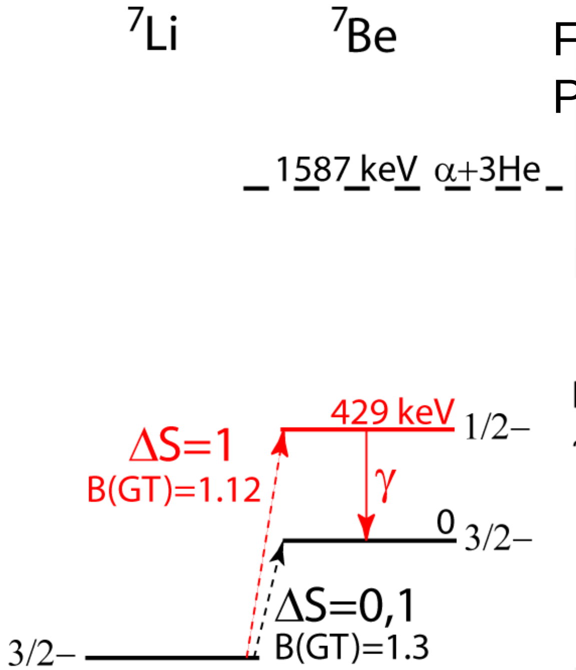
Detected and Momentum analyzed in Spectrometer
Decay in flight – Doppler broadened γ's detected in SeGA

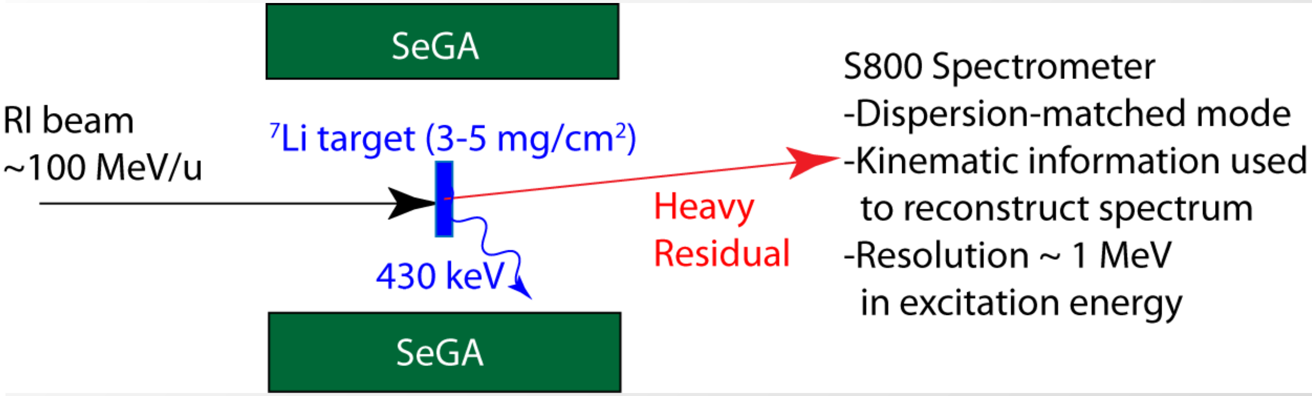
$$^{Z}X + ^{7}Li \rightarrow ^{Z-1}A + ^{7}Be^{*}$$

Decay 'at rest' – 430 keV γ detected in SeGA
Tag for charge-exchange reaction



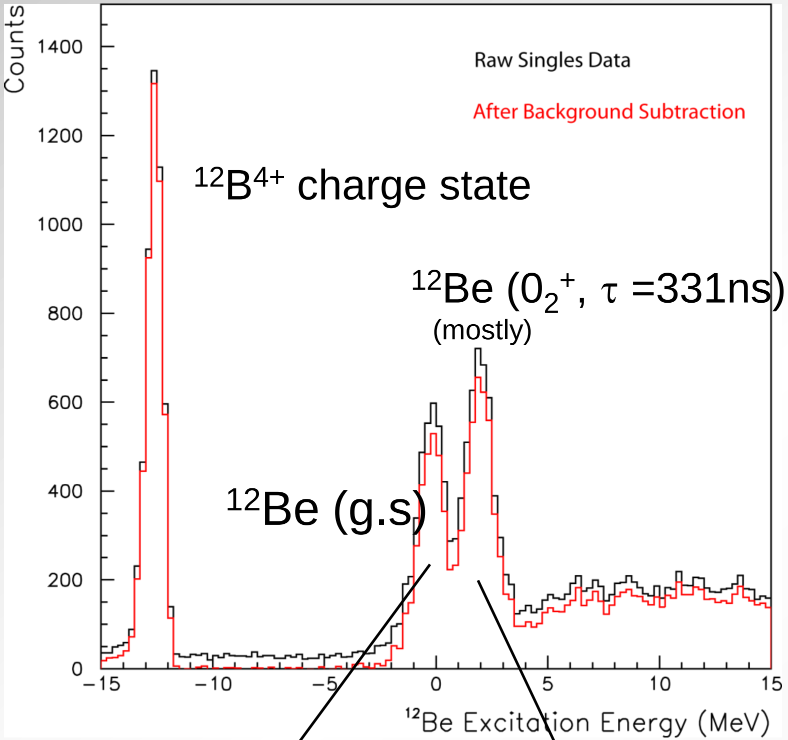
First application ^{34}P(^{7}Li,^{7}Be)^{34}Si*
PRL 104, 212504 (2010)

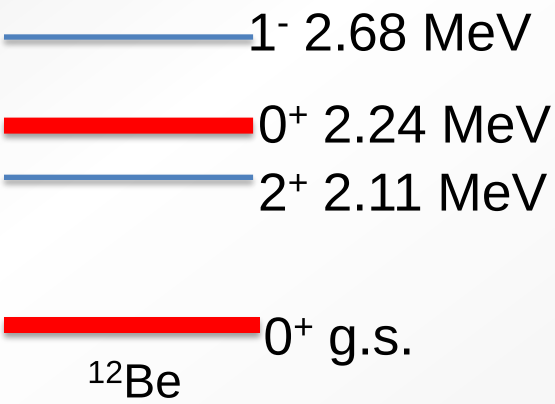




$^{12}B(^7Li,^7Be)^{12}Be^*$ in inverse kinematics

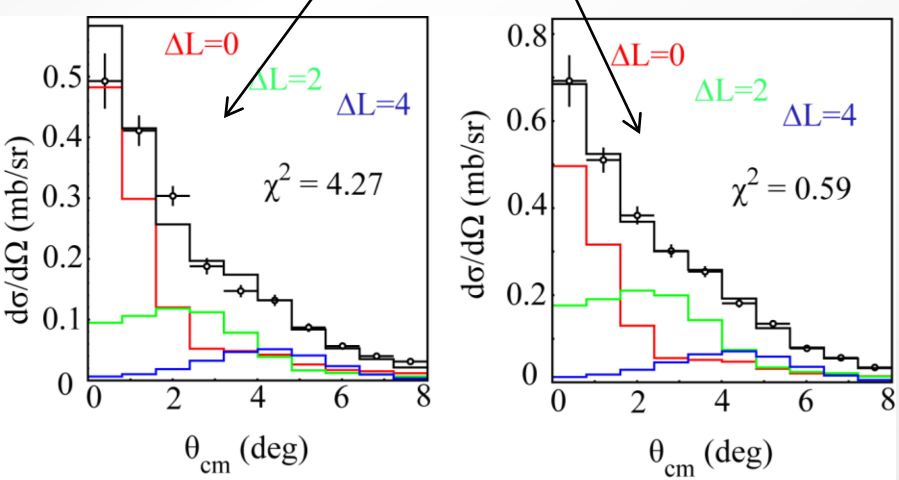
R. Meharchand et al.





From β-decay

$E_x(^{12}Be)$	J^π	B(GT)
0 MeV (g.s)	0_1^+	0.184 ± 0.008
2.24 MeV	0_2^+	0.214 ± 0.051



$$\frac{B(GT)_{0_2^+}}{B(GT)_{g.s.}} = 1.162 \pm 0.16$$

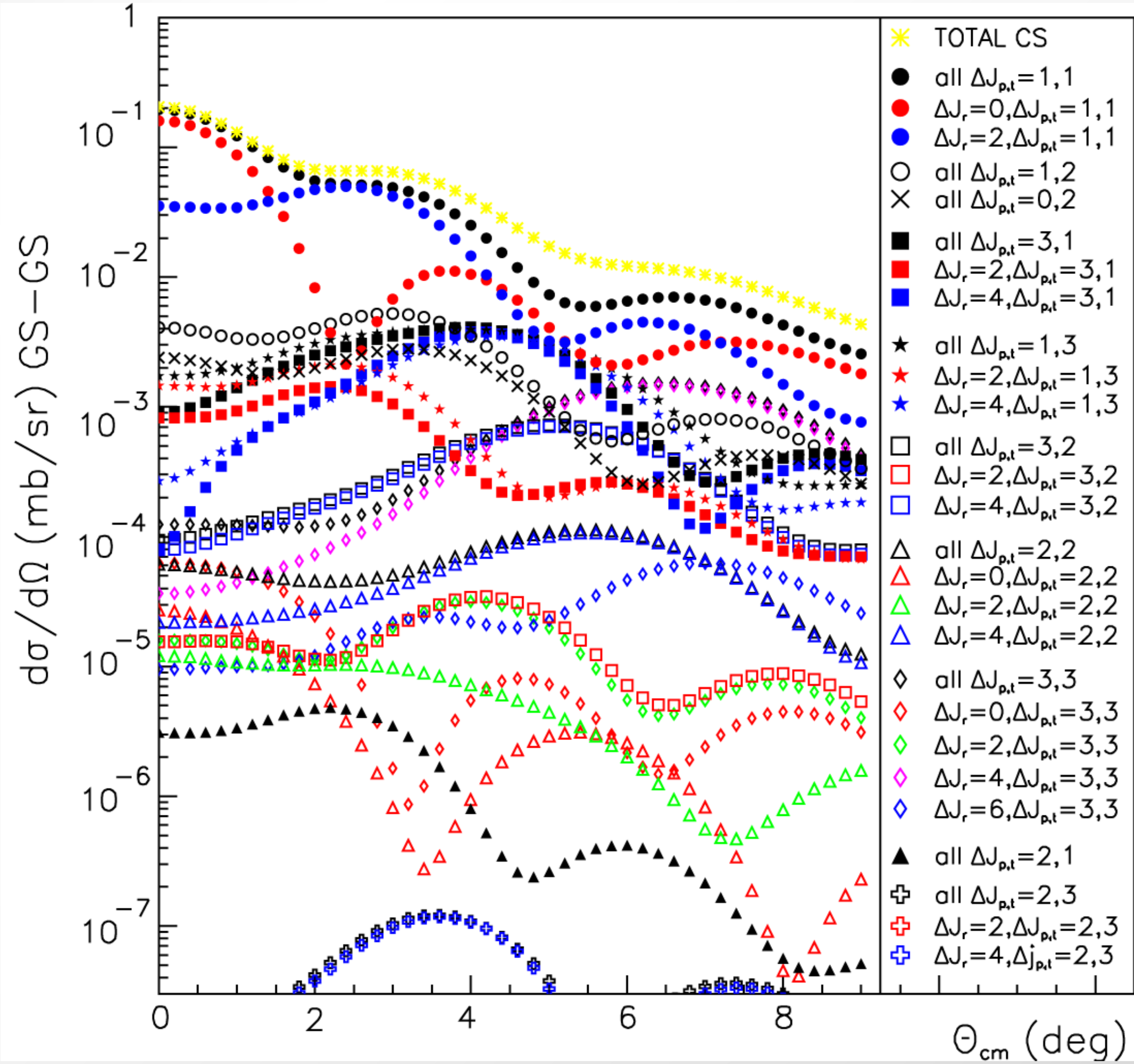
Including systematic errors

DWBA – Most Complicated Case

$^{12}B(g.s.,1^+) \rightarrow {}^{12}Be\ (2.11\ MeV,2^+)$, $^{7}Li(g.s.,3/2^+) \rightarrow {}^{7}Be(g.s.,3/2^-)$

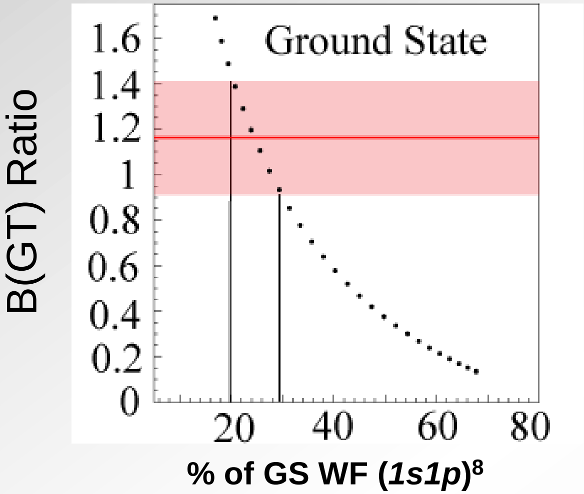
Both target and projectile have complex structures: many contributions to the total cross section.

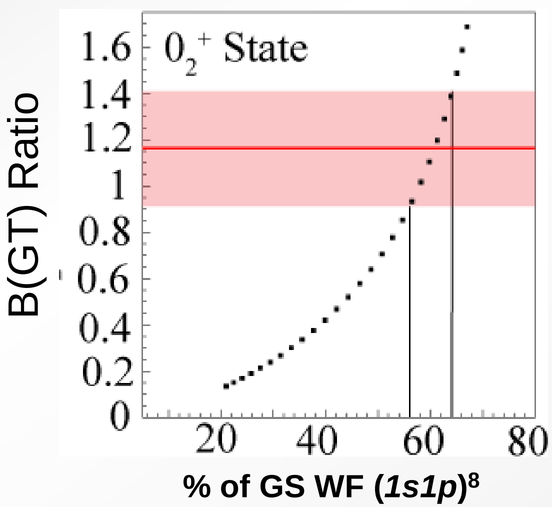
It works because under the experimental conditions, a few are dominant.



Structure studies

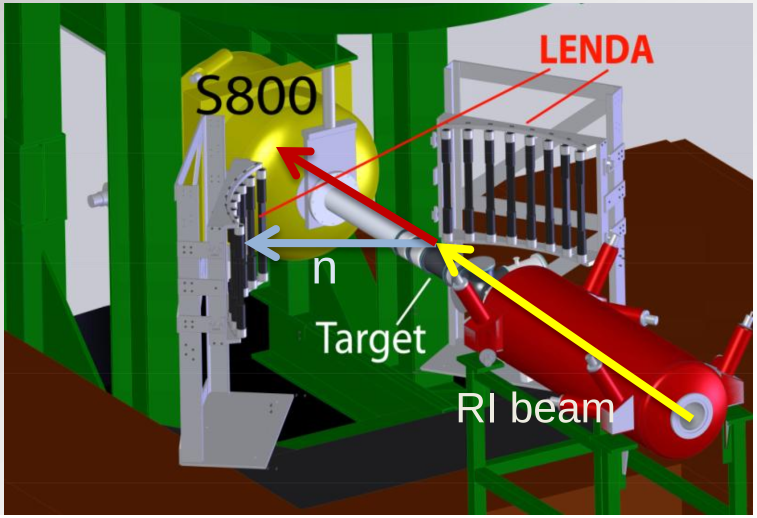
- The *ratio* of B(GT) for the 0^+ states is a sensitive probe of the *p*-component of wave-function (^{12}B is predominantly p-shell)
- The ratio of B(GT) is very sensitive to the p-sd shell gap in ^{12}Be



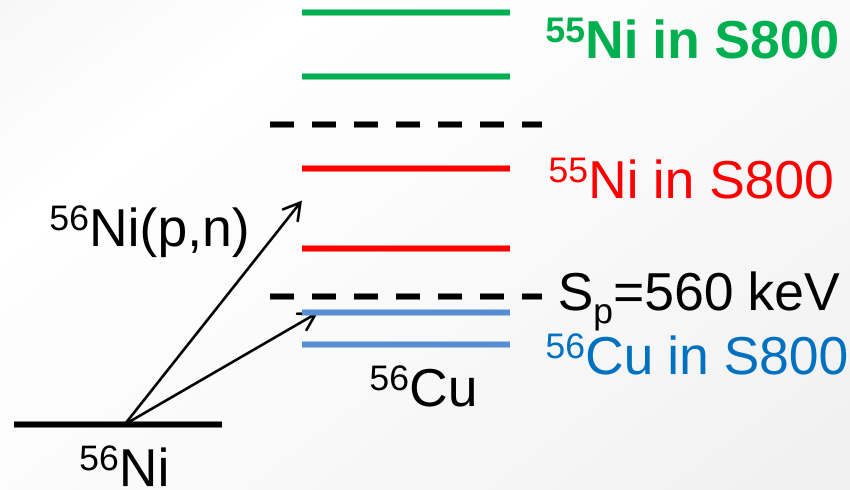


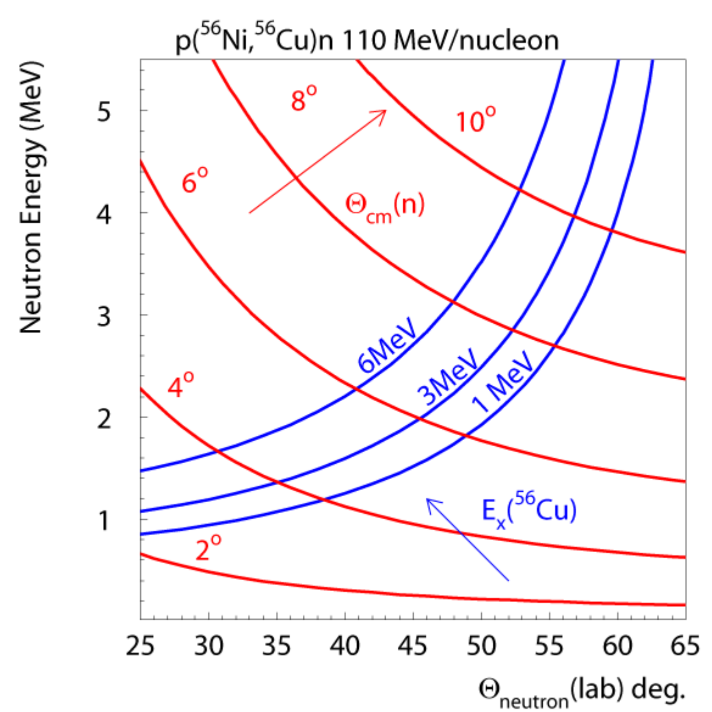
	Ground State			0_2^+ (2.24 MeV)		
Wavefunction Intensities	$(2s)^2$	$(1d)^2$	**$(1p)^2$**	$(2s)^2$	$(1d)^2$	**$(1p)^2$**
Barker (1976)	0.33	0.29	**0.38**	0.67	0.10	**0.23**
Fortune and Sherr (2006)	0.53	0.15	**0.32**	0.25	0.07	**0.68**
Romero-Redondo *et al.* (2008)	0.67-0.76	0.10-0.13	**0.13-0.19**	0.15-0.23	0.06-0.08	**0.71-0.78**
Barker (2009)	0.35	0.34	**0.31**	0.56	0.02	**0.42**
Blanchon *et al.* (2010)	0.25	0.185	**0.75**	0.73		**0.23**
Dufour *et al.* (2010)	0.16		**0.59**			
Navin et al.(2000) Pain *et al.* (2006)	0.38	0.30	**0.32**	0.32		**0.68**
Kanungo *et al.* (2010)	0.28			0.73		
THIS WORK			**0.25 ± 0.05**			**0.60 ± 0.04**

Gamow-Teller transition strengths from ^{56}Ni/^{55}Co via the ^{56}Ni,^{55}Co(p,n) reaction in inverse kinematics



M. Sasano, G. Perdikakis, R.G.T. Zegers et al.





Low Energy Neutron Detector Array (LENDA)

- Neutron->all necessary kinematic information

S800 spectrometer

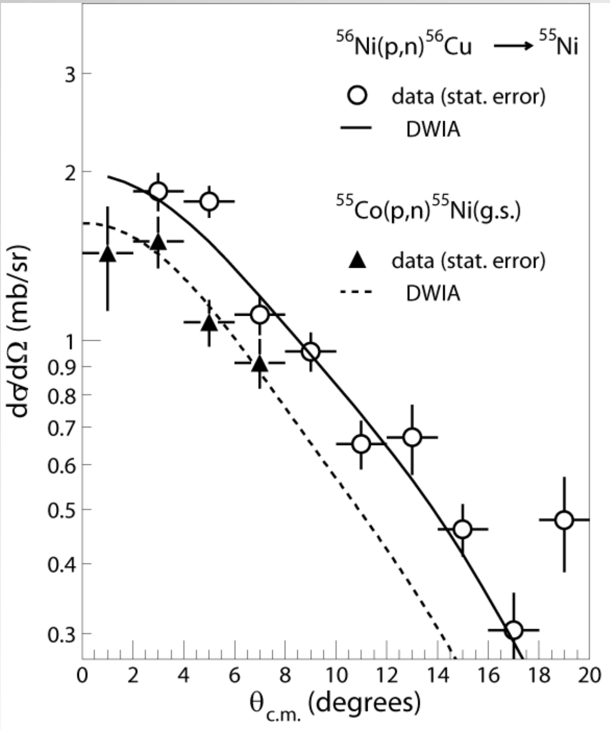
- Only used for tagging CE reaction

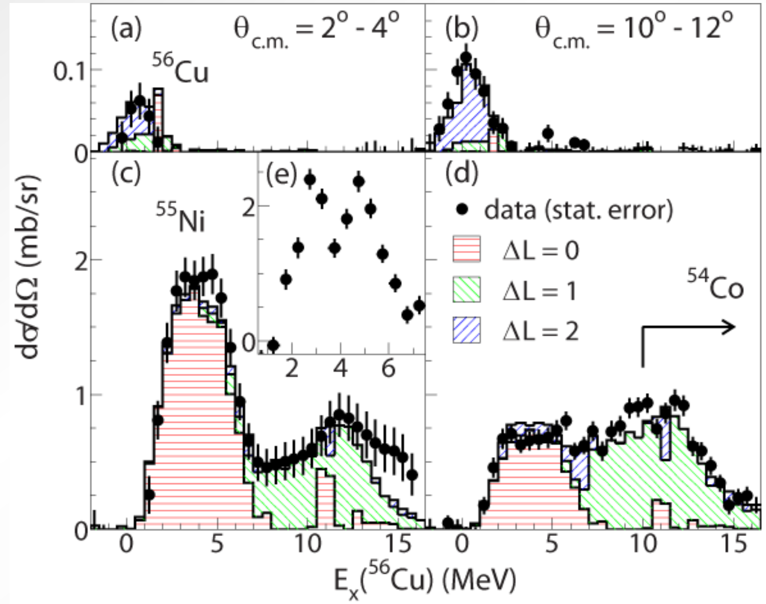
Liquid Hydrogen Target (Ursinus)

In-beam Diamond detector

- Neutron-TOF reference
- PID S800

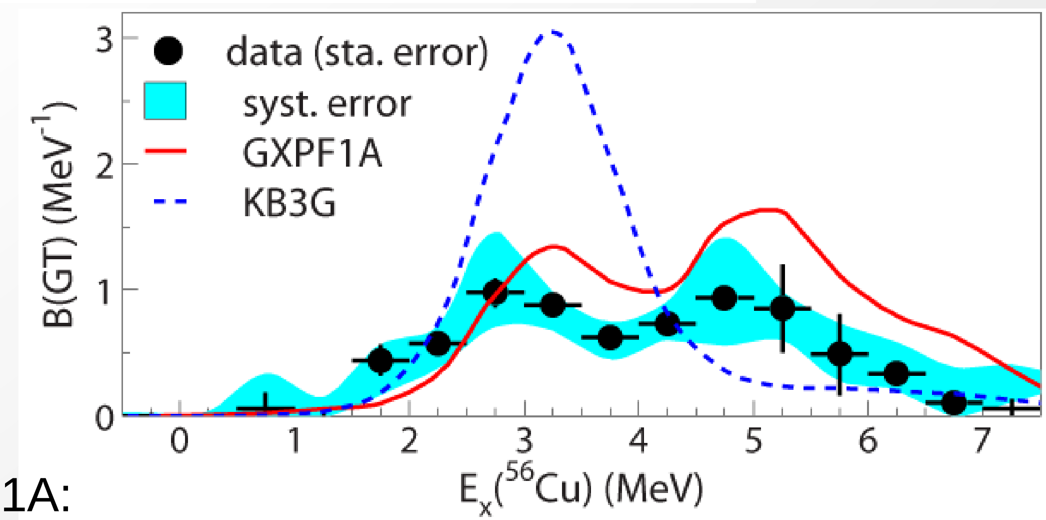
GT strengths from ^{56}Ni(p,n) at 110 MeV/u





Multipole Decomposition Analysis

Differential cross section measured for ΔL=0 excitations and the comparison with DWIA calculations. ^{55}Co(g.s.)(p,n)^{55}Ni(g.s.) reaction used to calibrate the unit cross section



Difference between KB3G and GXPF1A:

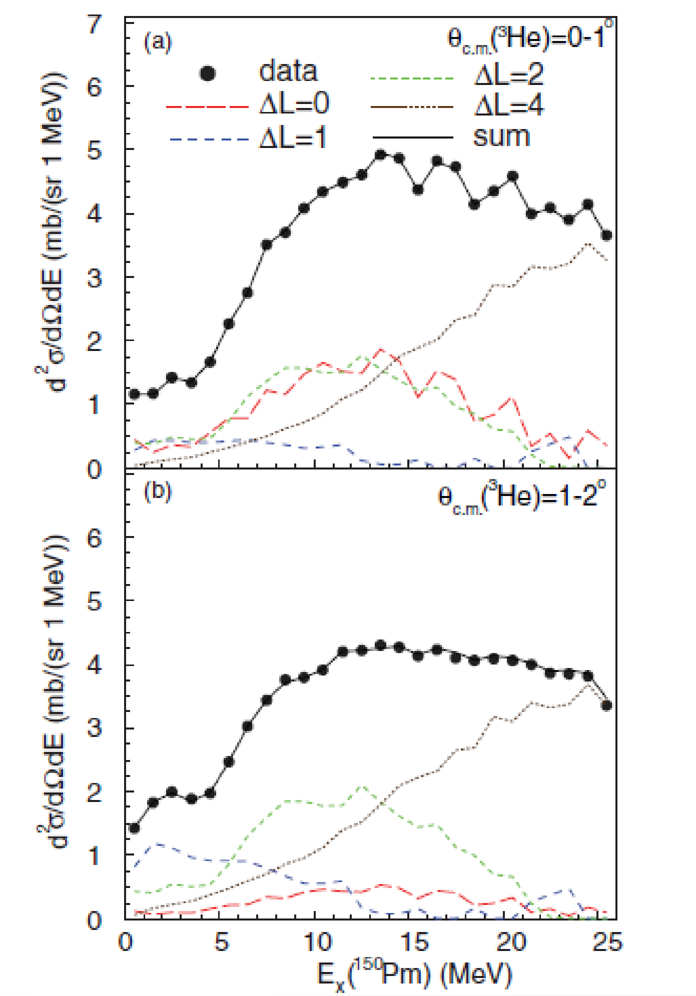
- KB3G weaker spin-orbit and pn-residual interactions
- KB3G lower level density

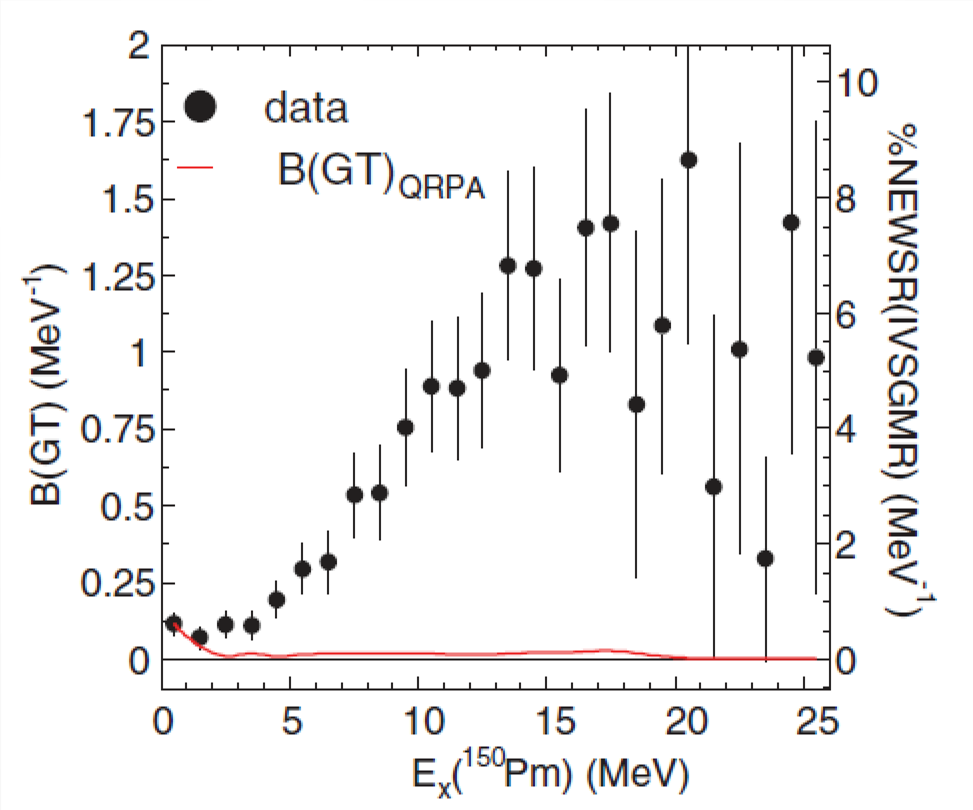
$^{150}Sm(t,^{3}He)$ at 115 MeV/u

IVSGMR

Carol Guess et al.

PRC 83, 064318 (2011)

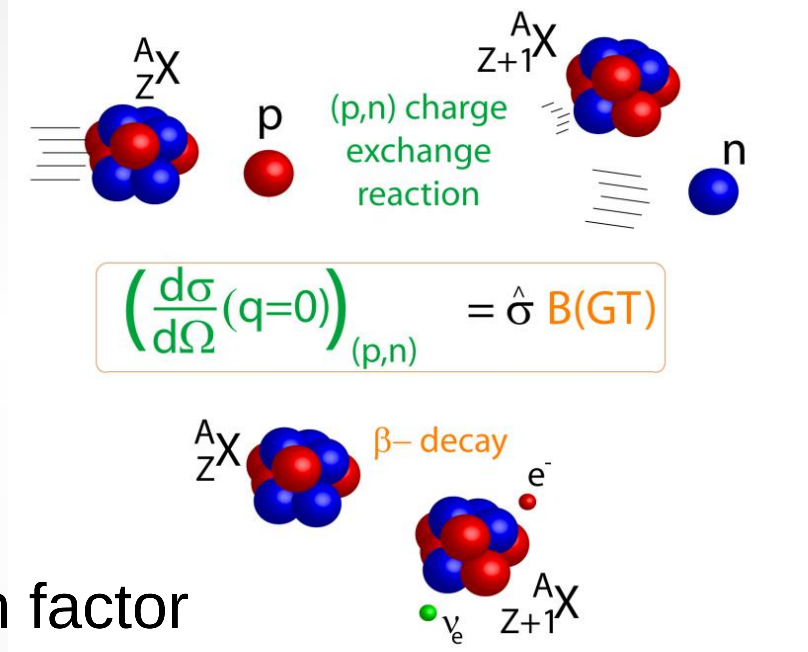




~100% of NEWSR for IVSGMR

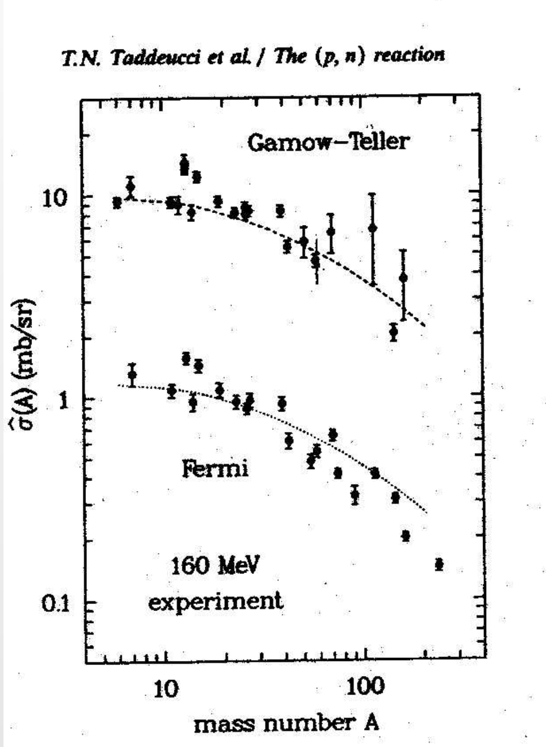
Reaction Theory

$$\left.\frac{d\sigma}{d\Omega}\right|_{q=0} = KN\left|J\right|^2 B(GT) = \hat{\sigma}\cdot B(GT)$$



- "Assumes" factorization is possible
- Eikonal approximation for distortion factor

- For (p,n)/(n,p) reactions:
  - Distorted-Wave Impulse Approximation -DW81
  - **Love-Franey NN interaction-1980's**
  - Exact treatment of exchange
  - Global optical potentials
  - Probes interior – less susceptible to surface effects



T.N. Taddeucci et al. NPA 469, 125 (1987)

Composite probes

- For studying stable nuclei: improved resolutions… (d,^{2}He), (^{3}He,t), (t,^{3}He)
- For studying unstable nuclei: (n,p) not available…composite probes must be used
- Heavy-Ion charge exchange: new unstable probes with specific selectivities (spin and/or isospin selectivity)

Commonly used codes (all freely available):

- ACCBA :(d,^{2}He) – (Okamura)
- **FOLD: other composite probes**

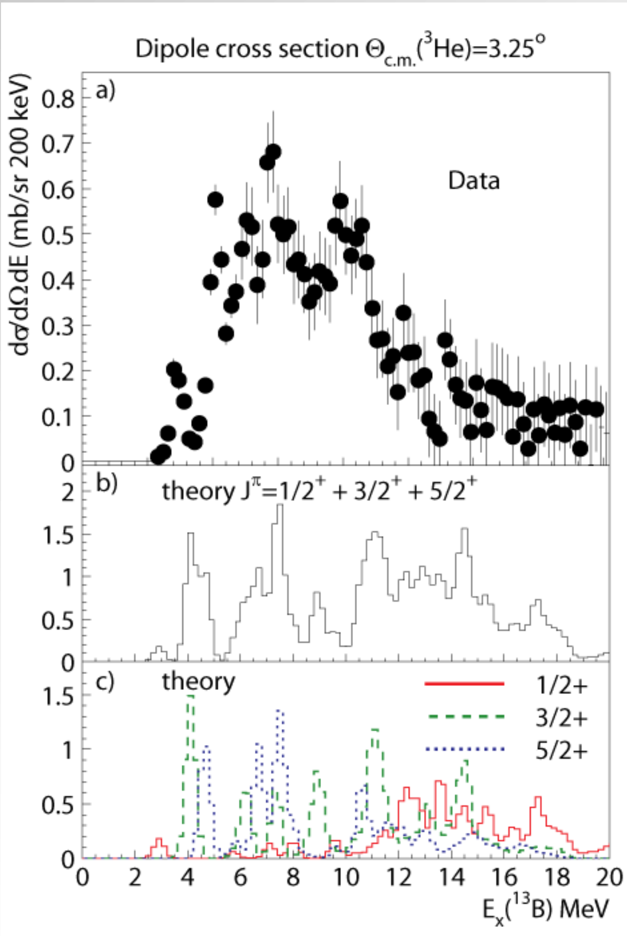
  F. Petrovich, J.Cook/J. Carr, Zegers/Fracasso/Colo
- DW81 [with simplified interaction], Raynal, Comfort.

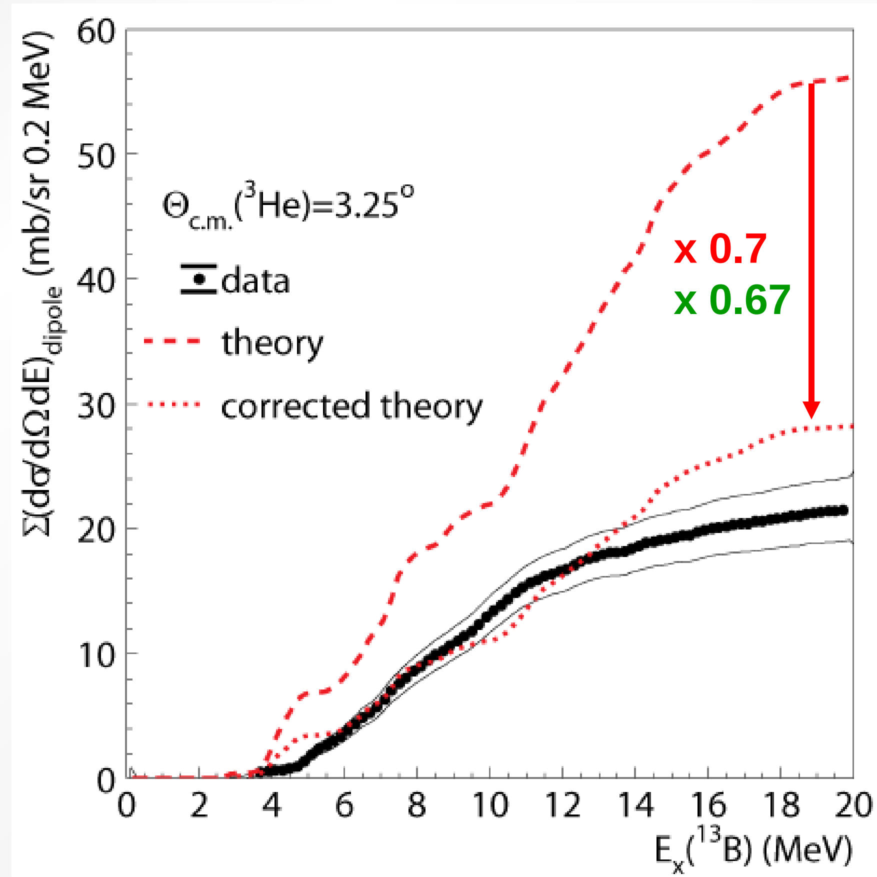
Note:

- We could extract much more information from the data if we could accurately calculate absolute cross section. This holds for GT as well as other excitations.

- forbidden (dipole) transitions
- giant resonances
- …

dipole transitions from ^{13}C to ^{13}B via $(t,^{3}He)$ at 115 MeV/u





- Cross sections for GT transitions are over-predicted by about 30%
- Quenching factor for GT strength for A=13 ~ 0.65
- **If** theoretical cross sections for dipole transitions are too high by 30% as well, the data suggest a similar quenching for dipole transitions as for GT transitions

Basic formalism

$$T_{fi} = < \chi_f^{+}(\vec{k_f}, \vec{R'}) \mid F(\vec{R'}) \mid \chi_i^{-}(\vec{k_i}, \vec{R'}) >$$

Structure part: 1p-1h one-body transition densities

$$F(\vec{R'}) = \sum_{\substack{j_1 j_2 m_1 m_2 t_{z_1} \\ t_{z_2} m'_{j_1} m'_{j_2} t'_{z_1} t'_{z_2}}} [< \Phi_{J_f}^{M_f} \phi_{J'_f T'_f}^{M'_f M_f^{T'}} \mid a^{\dagger}_{j_2 m_2 t_{z_2}} a_{j_1 m_1 t_{z_1}} c^{\dagger}_{m'_{j_2} t'_{z_2}} c_{m'_{j_1} t'_{z_1}} \mid \Phi_{J_i}^{M_i} \phi_{J'_i T'_i}^{M'_i M_i^{T'}} > \times$$

$$< \phi_{j_2 m_2 t_{z_2}}(\vec{r_1}) \phi_{m'_{j_2} t'_{z_2}} \mid V_{eff} \mid \phi_{j_1 m_1 t_{z_1}}(\vec{r_1}) \phi_{m'_{j_1} t'_{z_1}} >],$$

Double-folding of NN interaction over projectile & target transition densities

$$\begin{aligned} V_{12}(r) = \; & V_0(r) + V_\sigma(r)\vec{\sigma_1}\cdot\vec{\sigma_2} + V_\tau(r)\vec{\tau_1}\cdot\vec{\tau_2} + V_{\sigma\tau}(r)(\vec{\sigma_1}\cdot\vec{\sigma_2})\cdot(\vec{\tau_1}\cdot\vec{\tau_2}) + \\ & V_{LS}(r)(\vec{L}\cdot\vec{S}) + V_{LS\tau}(\vec{L}\cdot\vec{S})\cdot(\vec{\tau_1}\cdot\vec{\tau_2}) + \\ & V_T(r)S_{12} + V_{T\tau}(r)S_{12}(\vec{\tau_1}\cdot\vec{\tau_2}), \end{aligned}$$

Love-Franey interaction –energy dependent t -matrix

$$t_{NN} = \widetilde{V}(q) + \widetilde{V}(k_A)$$

Exchange contribution –destructive

Short-range approximation is used: known to overestimate Cross section (Udagawa et el.)

Study of (t,^{3}He) & (^{3}He,t) at 115-140 MeV/u

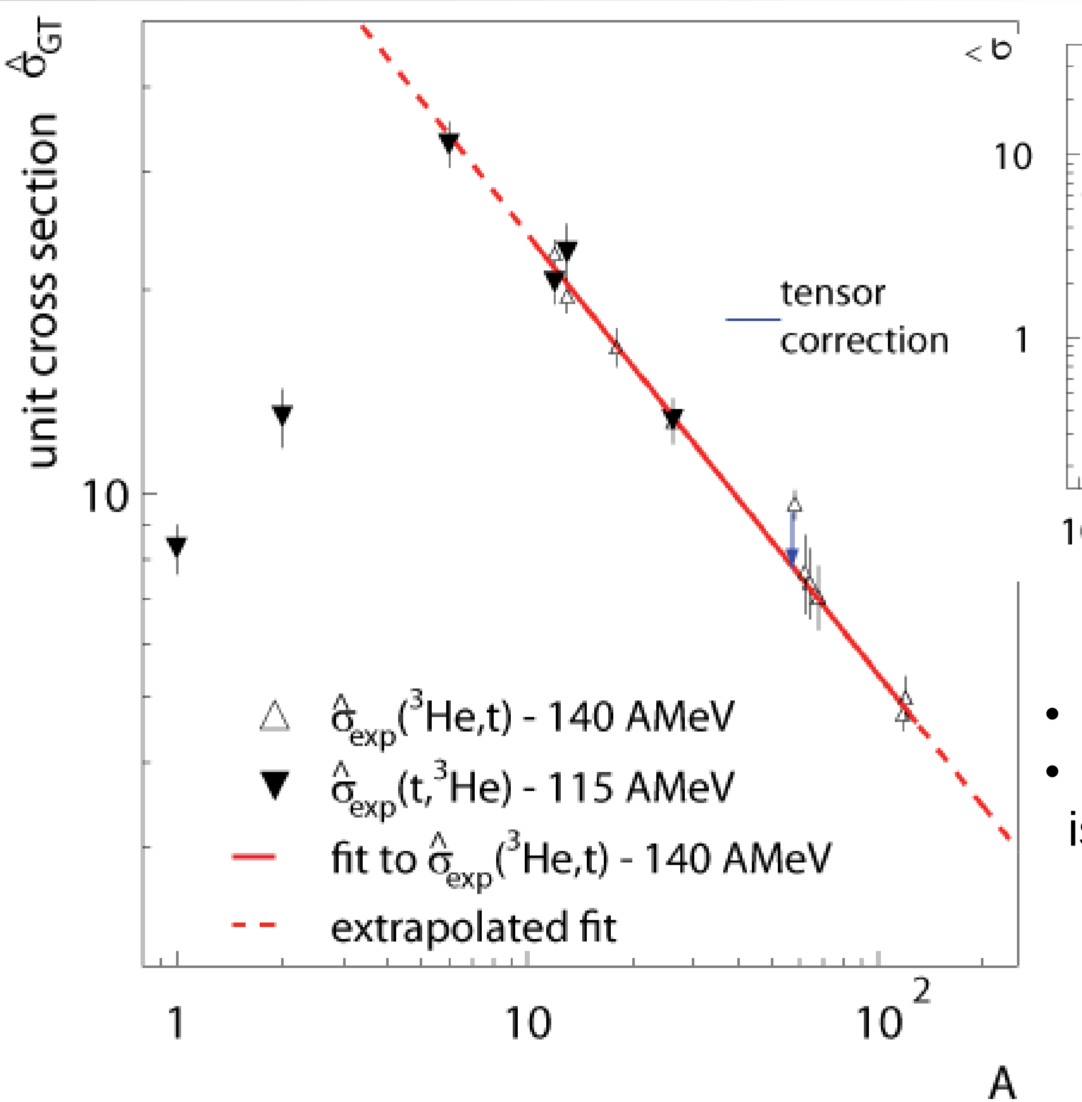
PRL 99, 202501 (2007) / Phys. Rev. C 83, 054614 (2011)

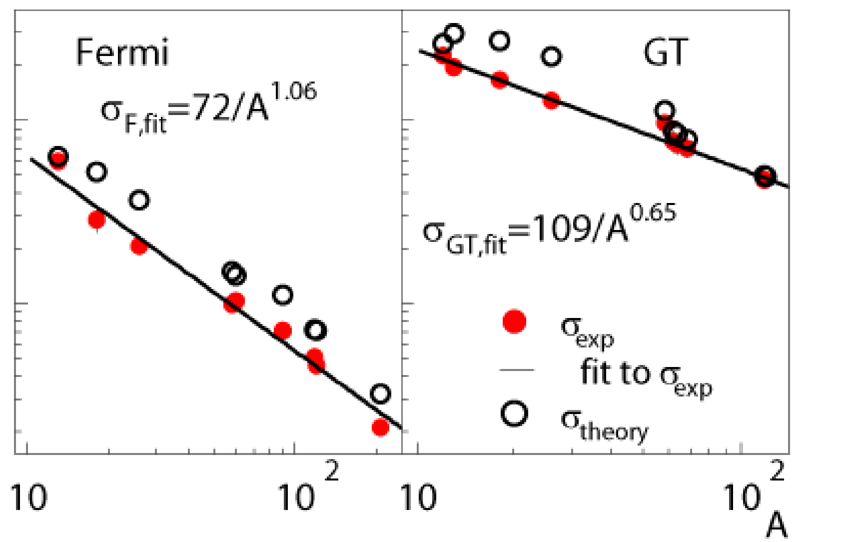
- Significant amount of data available from studies at RCNP (^{3}He,t) and NSCL (t,^{3}He)

i	f	$B(GT)$	$d\sigma/d\Omega_{c.m.}(0°)$ (mb/sr)	$d\sigma/d\Omega_{c.m.}(q=0)$ (mb/sr)	$\hat{\sigma}$ (mb/sr)	Ref.
^{12}C(0$^+$,g.s.)	^{12}N(1$^+$,g.s.)	0.88	16.1 ± 0.12	19.9 ± 1.0	22.6 ± 1.1	[25]
^{13}C(1/2$^-$,g.s.)	^{13}N(3/2$^-$,15.1 MeV)	0.23 ± 0.01	3.65 ± 0.10	4.51 ± 0.26	19.7 ± 1.1	[34]
^{18}O(0$^+$,g.s.)	^{18}F(1$^+$,g.s.)	3.11	51.2 ± 2.2	51.2 ± 3.4	16.5 ± 1.1	[25]
^{26}Mg(0$^+$,g.s.)	^{26}Al(1$^+$,1.06 MeV)	1.1	13.9 ± 0.3	14.1 ± 0.8	12.8 ± 0.7	[22]
^{58}Ni(0$^+$,g.s.)	^{58}Cu(1$^+$,g.s.)	0.155	1.5 ± 0.01	1.5 ± 0.08	9.65 ± 0.48[a]	[25]
^{62}Ni(0$^+$,g.s.)	^{62}Cu(1$^+$,g.s.)	0.073			7.7 ± 1.0[b]	[4,25]
^{64}Ni(0$^+$,g.s.)	^{64}Cu(1$^+$,g.s.)	0.123			7.4 ± 0.9[b]	[4,25]
^{68}Zn(0$^+$,g.s.)	^{68}Ga(1$^+$,g.s.)	0.073			7.0 ± 0.8[b]	[4,25]
^{118}Sn(0$^+$,g.s.)	^{118}Sb(1$^+$,g.s.)	0.344	1.71 ± 0.04	1.62 ± 0.09	4.72 ± 0.26	[25]
^{120}Sn(0$^+$,g.s.)	^{120}Sb(1$^+$,g.s.)	0.345	1.80 ± 0.10	1.72 ± 0.13	5.00 ± 0.37	[25]

i	f	$B(GT)$	$d\sigma/d\Omega_{c.m.}(0°)$ (mb/sr)	$d\sigma/d\Omega_{c.m.}(q=0)$ (mb/sr)	$\hat{\sigma}$ (mb/sr)	Ref.
^{1}H(1/2$^+$)	^{1}n(1/2$^+$)	3	25 ± 2	25 ± 2	8.3 ± 0.7	this work
^{2}H(1$^+$)	2n(0$^+$)				13.0 ± 1.3[a]	this work
^{6}Li(1$^+$,g.s.)	^{6}He(0$^+$,g.s.)	1.577	51 ± 4	52 ± 4	32.9 ± 2.6	[28], reevaluated
^{12}C(0$^+$,g.s.)	^{12}B(1$^+$,g.s)	0.99	16.6 ± 1.2	20.4 ± 1.5	20.5 ± 1.5	this work
^{13}C(1/2$^-$,g.s.)	^{13}B(3/2$^-$,g.s.)	0.711	13.1 ± 1.3	16.2 ± 1.6	22.8 ± 2.3	[32]
^{26}Mg(0$^+$,g.s.)	^{26}Mg(1$^+$,0.08 MeV)	0.41 ± 0.02[b]	4.1 ± 0.3	5.27 ± 0.4	12.8 ± 1.0	[22]

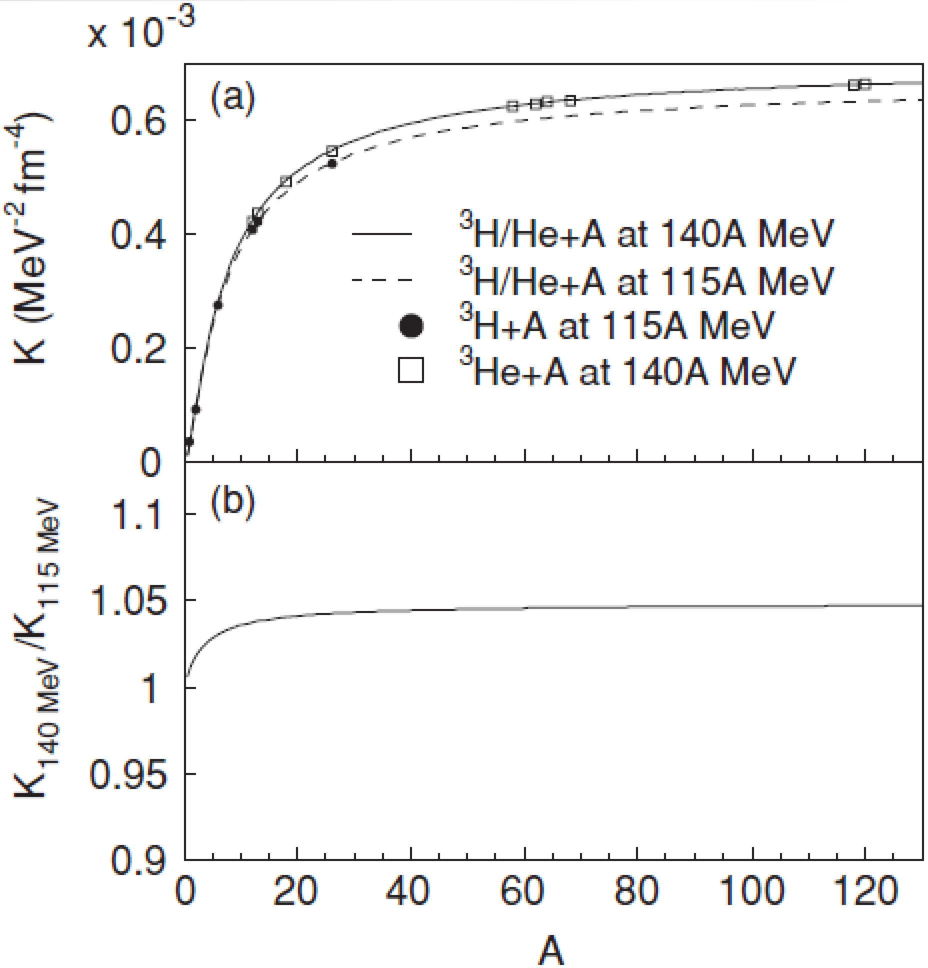
Unit cross section vs mass number





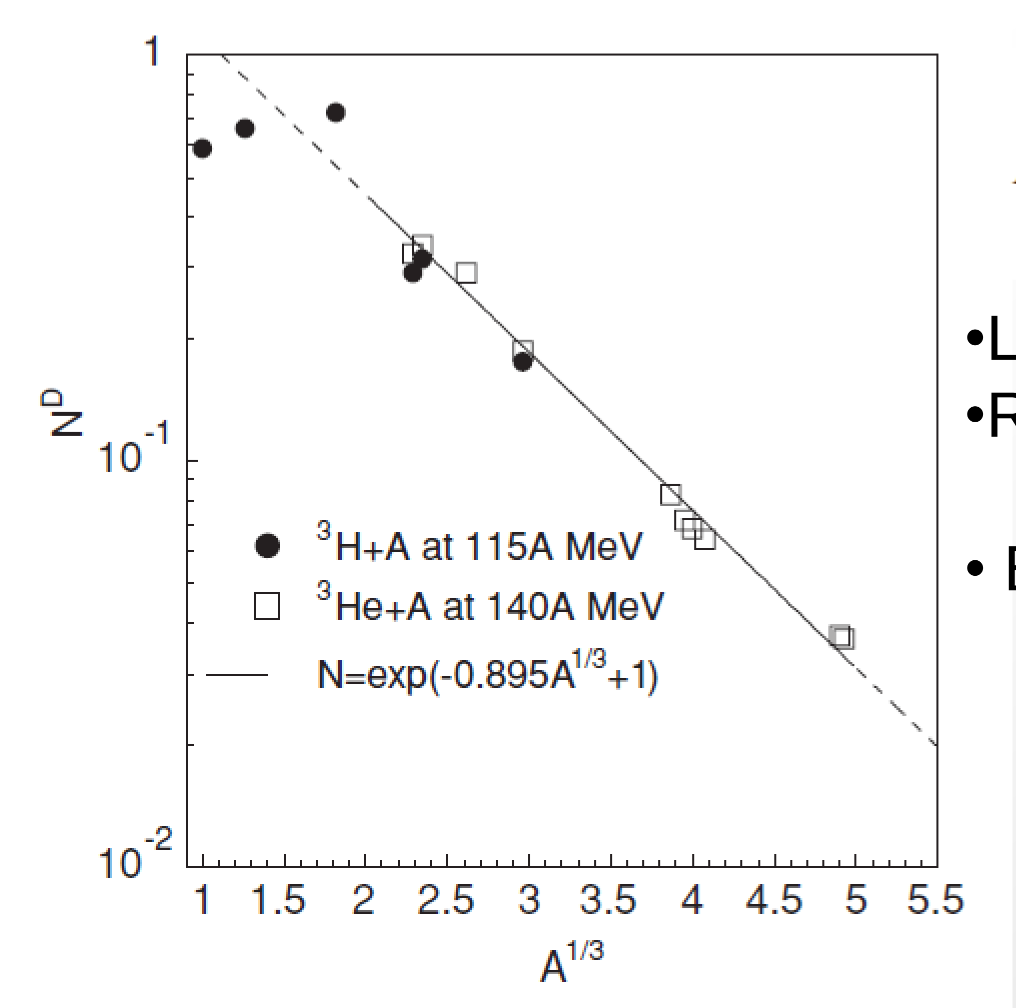
- Simple dependence on A
- Composite probe: surface of nucleus is probed!

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{GT} = K N^D |J_{\sigma\tau}|^2 B(GT)$$



$$K = \frac{E_i E_f}{(\pi\hbar^2 c^2)^2}\frac{k_f}{k_i}$$

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{GT} = K N^D |J_{\sigma\tau}|^2 B(GT)$$



$$N^D = \frac{\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{\mathrm{DWBA}}}{\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{\mathrm{PWBA}}}$$

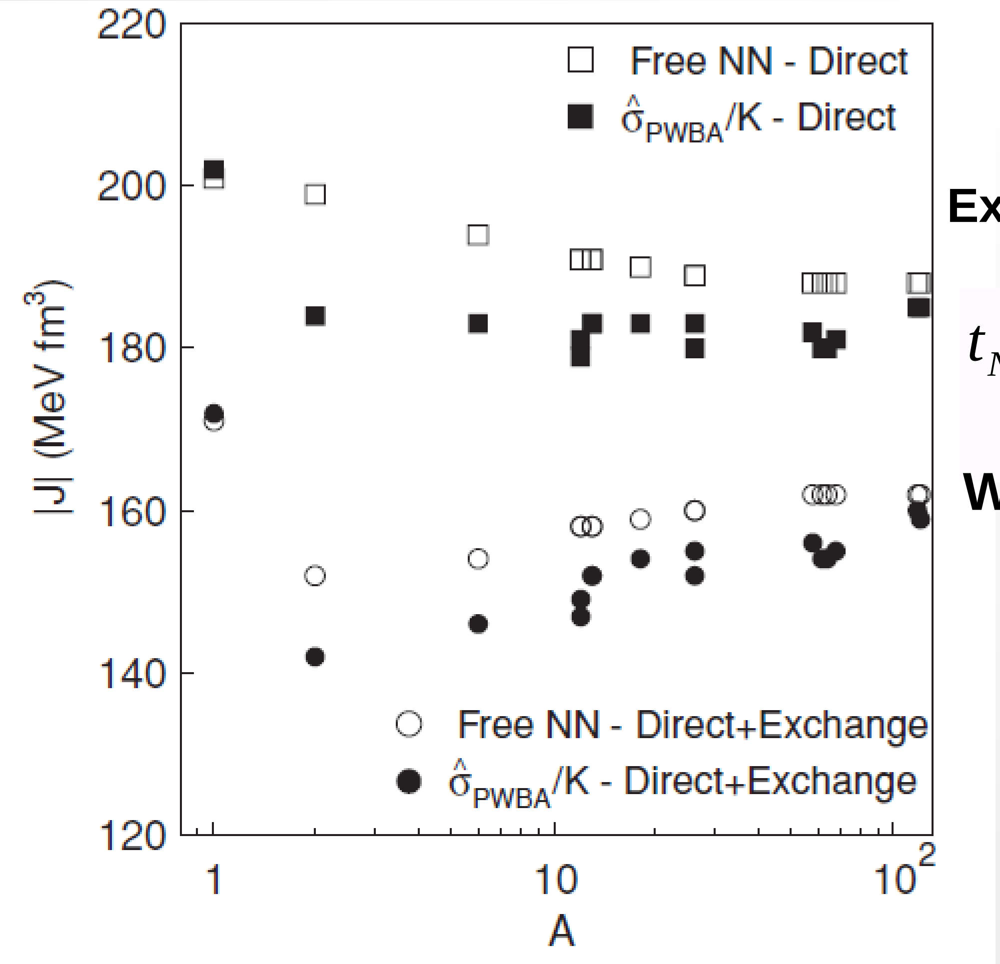
- Local deviations?
- Rare isotopes?

- Error is estimated at approximately 10%

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{GT} = K N^D |J_{\sigma\tau}|^2 B(GT)$$

THEORY ONLY

J: volume integral of NN-interaction



$$\hat{\sigma}_{GT,PWBA} = K|J_{\sigma\tau}|^2$$

Exchange: short-range approximation

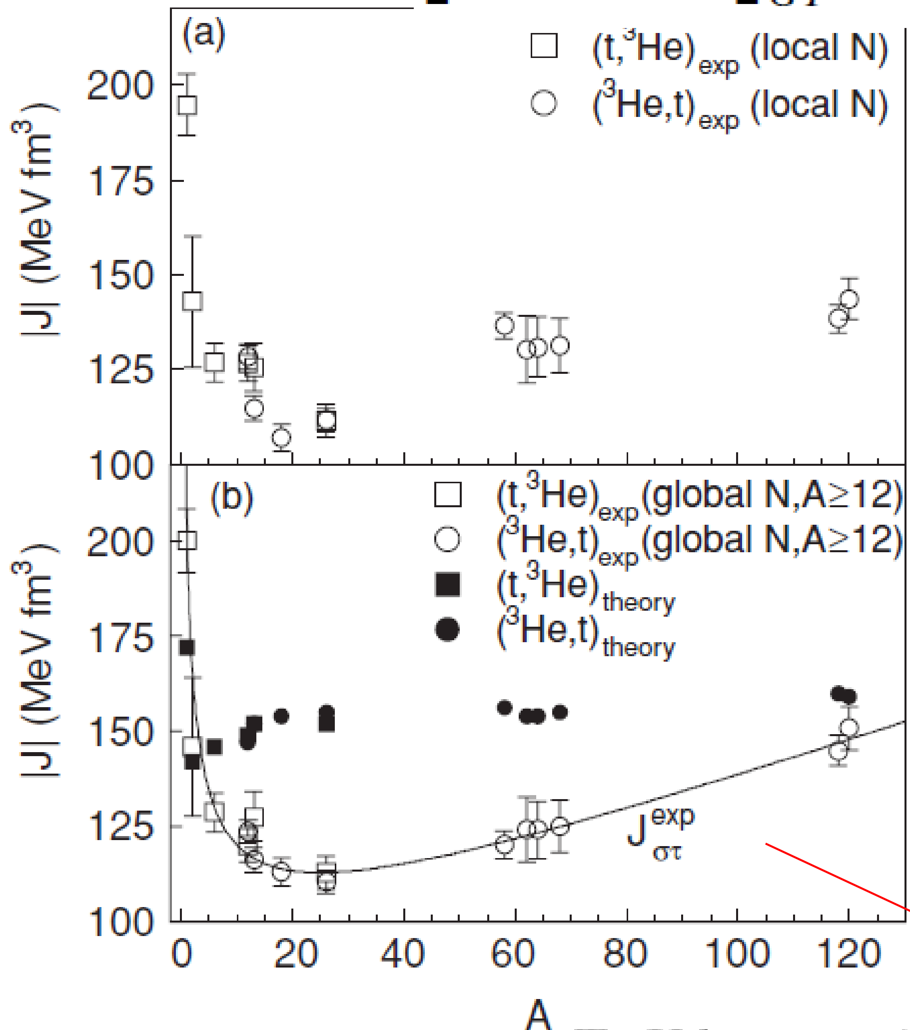
$$t_{NN} = \tilde{V}(q) + \tilde{V}(k_A)$$

Which value for k_A?

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right]_{GT} = K N^D |J_{\sigma\tau}|^2 B(GT)$$

COMPARISON WITH EXPERIMENT

theory too high…
effect of exchange terms?
(Udagawa et al.)



$$\left|J_{\sigma\tau}^{\mathrm{exp}}\right| = \frac{128.5}{\sqrt{A}} + 0.515A + 74.3 \quad \text{for} \quad A \leqslant 120.$$

T. Udagawa, A. Schulte, and F. Osterfeld, Nucl. Phys. **A474**, 131 (1987).
B. T. Kim, D. P. Knobles, S. A. Stotts, and T. Udagawa, Phys. Rev. C **61**, 044611 (2000).

Effect of tensor interaction

$$V_{12}(r) = V_0(r) + V_\sigma(r)\vec{\sigma_1}\cdot\vec{\sigma_2} + V_\tau(r)\vec{\tau_1}\cdot\vec{\tau_2} + V_{\sigma\tau}(r)(\vec{\sigma_1}\cdot\vec{\sigma_2})\cdot(\vec{\tau_1}\cdot\vec{\tau_2}) + V_{LS}(r)(\vec{L}\cdot\vec{S}) + V_{LS\tau}(\vec{L}\cdot\vec{S})\cdot(\vec{\tau_1}\cdot\vec{\tau_2}) + V_T(r)S_{12} + V_{T\tau}(r)S_{12}(\vec{\tau_1}\cdot\vec{\tau_2}),$$

central term

non-central term

$$S_{12} = \frac{(\vec{\sigma_1}\cdot\vec{r})(\vec{\sigma_2}\cdot\vec{r})}{r^2} - \vec{\sigma_1}\cdot\vec{\sigma_2}.$$

$0^+ \rightarrow 1^+$ transition

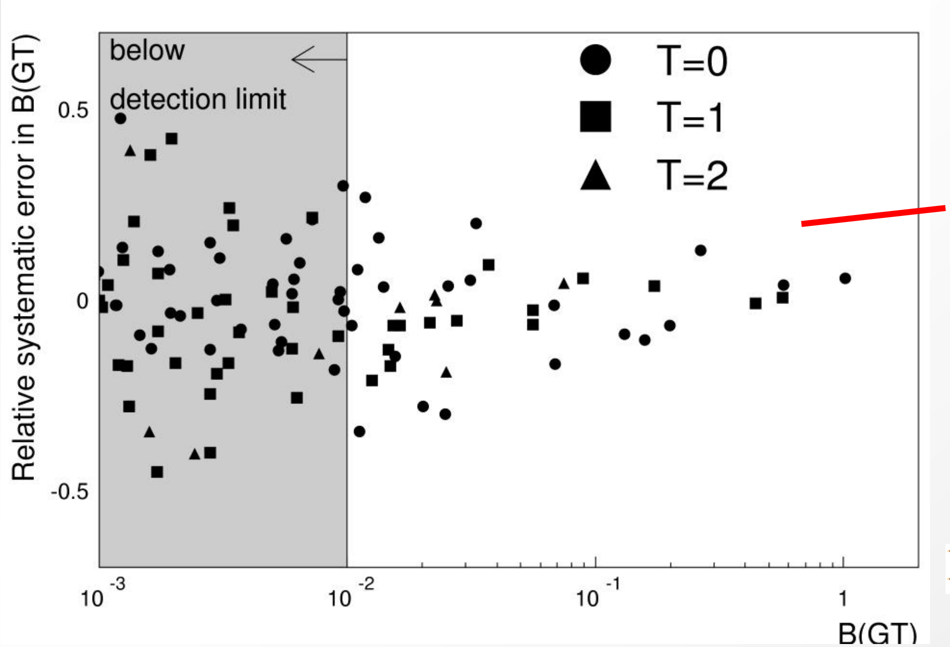
$\Delta L=0$ $\Delta S=1$ $\Delta J=1$ Gamow-Teller component: formfactor 1

$\Delta L=2$ $\Delta S=1$ $\Delta J=1$ Quadrupole component: formfactor 2

Interference through tensor interaction: the $\Delta L=0$ component can be modified significantly without changing the angular distributions at forward angles strongly.

Data from ^{26}Mg(^{3}He,t) – 4 transitions with known β-decay strengths

$E_x(^{26}Al)$	$B(GT)_\beta$	$d\sigma/d\Omega(0^o)(^3He,t)$	$d\sigma/d\Omega(0^o)/B(GT)_\beta$
1.06 MeV	1.098	13.9±0.3	12.7±0.3
1.85 MeV	0.536	6.7±0.2	12.5±0.4
2.07 MeV	0.091	1.45±0.03	15.9±0.3
2.74 MeV	0.113	1.5±0.03	13.27±0.3



theoretical study in DWBA in which the theoretical cross section is treated as data

Effects hard to determine on a state-by-state basis: requires Accurate structure input

Phys. Rev. C 74, 024309 (2006).

A problem if the transition used for calibrating the unit cross section is strongly affected by the tensor, or if high (<10%) is required for a particular transitions.

Beyond GT transitions

- Proportionality for non-GT excitations?
- If proportionality is not generally valid for non-GT transitions, we need to be able to calculate accurate cross sections so we can draw conclusions on strength exhaustion etc.
- Transitions in the continuum? (DWBA requires bound single particle orbitals)

->input transition densities directly

Survey of GT strengths in pf-shell (experimental and theoretical)

i	f	β−decay	(n,p)	$(d,{}^2\mathrm{He})$	$(t,{}^3\mathrm{He})$	$(p,n)^{\mathrm{a}}$	QRPA	KB3G	GXPF1a
$^{45}\mathrm{Sc}(\frac{7}{2}^-)$	$^{45}\mathrm{Ca}(\frac{5}{2}^-,\frac{7}{2}^-,\frac{9}{2}^-)$	x	x				x	x	x
$^{48}\mathrm{Ti}(0^+)$	$^{48}\mathrm{Sc}(1^+)$		x	x			x	x	x
$^{50}\mathrm{V}(6^+)$	$^{50}\mathrm{Ti}(5^+,6^+,7^+)$			x			x	x	x
$^{51}\mathrm{V}(\frac{7}{2}^-)$	$^{51}\mathrm{Ti}(\frac{5}{2}^-,\frac{7}{2}^-,\frac{9}{2}^-)$		x	x			x	x	x
$^{54}\mathrm{Fe}(0^+)$	$^{54}\mathrm{Mn}(1^+)$		x				x	x	x
$^{55}\mathrm{Mn}(\frac{5}{2}^-)$	$^{55}\mathrm{Cr}(\frac{3}{2}^-,\frac{5}{2}^-,\frac{7}{2}^-)$	x	x				x	x	x
$^{56}\mathrm{Fe}(0^+)$	$^{56}\mathrm{Mn}(1^+)$		x				x	x	x
$^{58}\mathrm{Ni}(0^+)$	$^{58}\mathrm{Co}(1^+)$		x	x	x		x	x[b]	x[b]
$^{59}\mathrm{Co}(\frac{7}{2}^-)$	$^{59}\mathrm{Fe}(\frac{5}{2}^-,\frac{7}{2}^-,\frac{9}{2}^-)$		x				x	x	x
$^{60}\mathrm{Ni}(0^+)$	$^{60}\mathrm{Co}(1^+)$		x			x	x	x[b]	x[b]
$^{62}\mathrm{Ni}(0^+)$	$^{62}\mathrm{Co}(1^+)$		x			x	x	x	x
$^{64}\mathrm{Ni}(0^+)$	$^{64}\mathrm{Co}(1^+)$	x	x	x			x	x	x
$^{64}\mathrm{Zn}(0^+)$	$^{64}\mathrm{Cu}(1^+)$	x		x	x		x	x	x

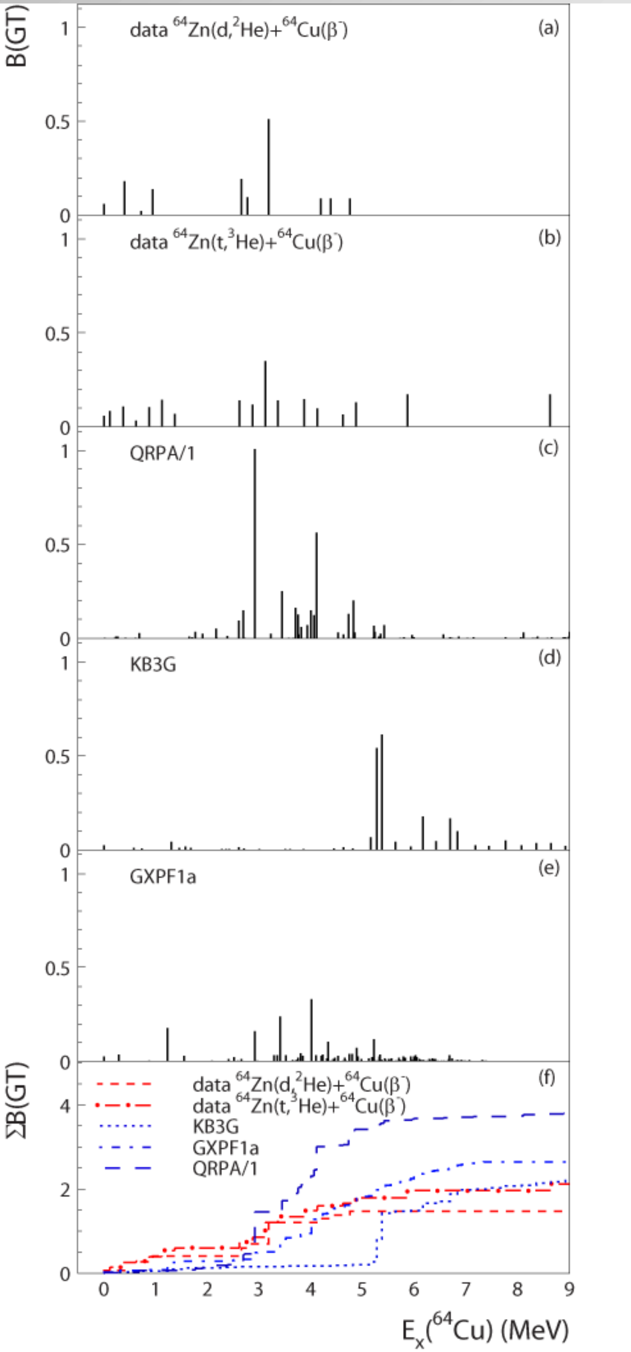
[a] Using $T_>$ transitions and applying isospin symmetry (see text)
[b] Shell-model calculations were performed in truncated model space (see text).

A.L. Cole, R.G.T. Zegers et al., to be published

(n,p) data – from TRIUMF and RCNP experiments
(d,^{2}He) data – from KVI experiments
(t,^{3}He) data – from NSCL experiment
(p,n) data – IUCF experiments

QRPA: S. Gupta/ P. Möller
KB3G: A. Poves et al.
GXPF1a: Honma et al.
Shell-model calculations with NuShellx
Mostly in full fp model space

^{64}Zn



(d,^{2}He)

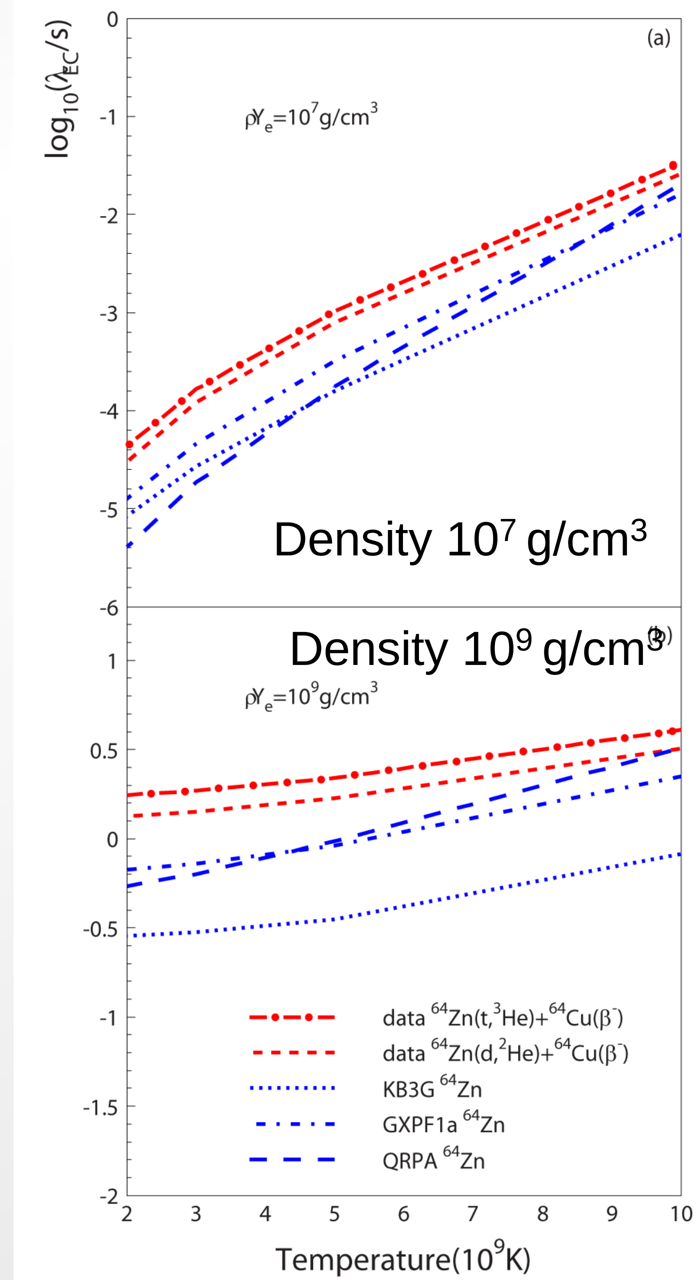
(t,^{3}He)

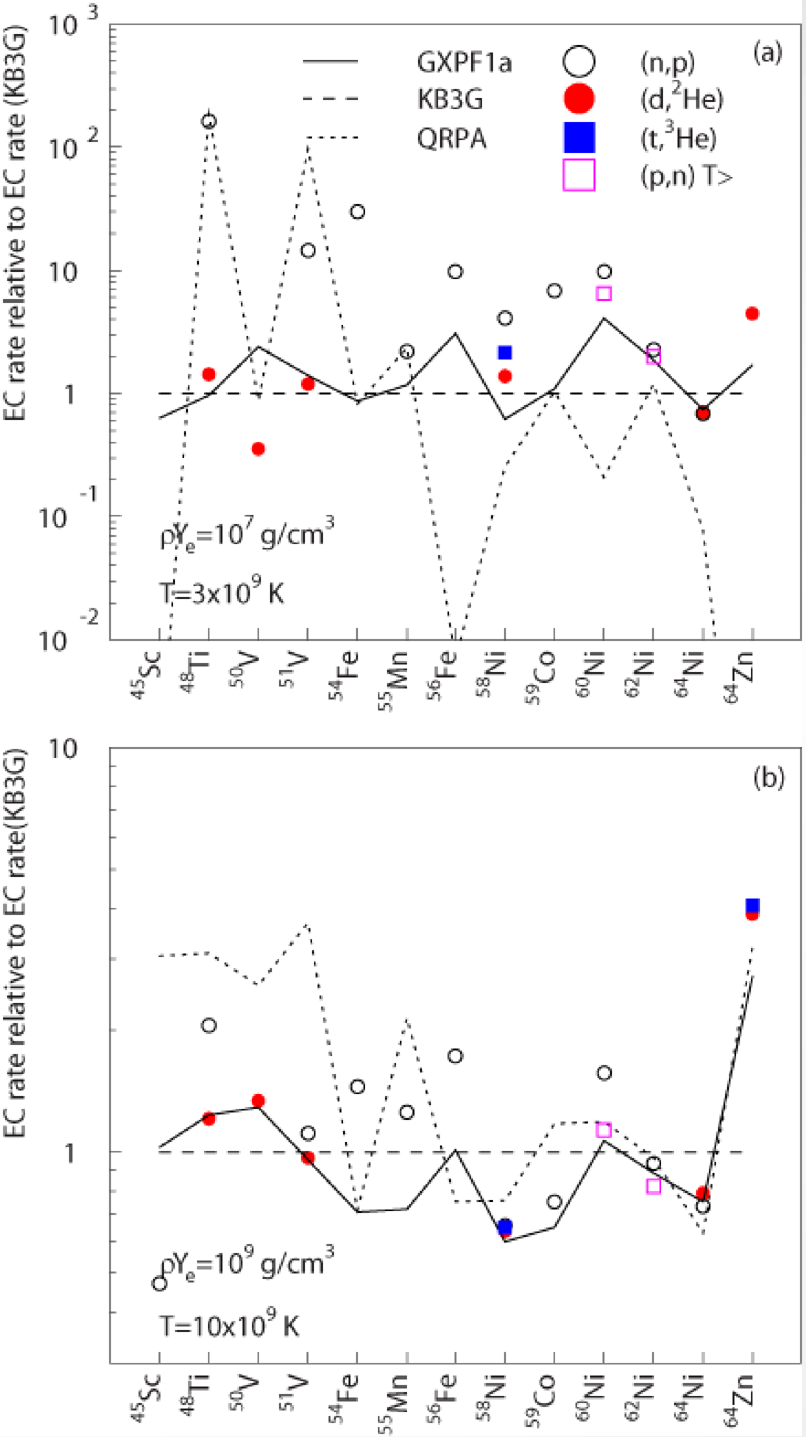
QRPA/1

KB3G

GXPF1a

EC RATES





Systematic comparison between experimental and theoretical weak reaction rates in stellar evolution

- Low-resolution data is suitable for testing Gamow-Teller strengths but not for extracting EC rates

- Both shell-model calculation do about equally well: KB3G (GXPF1a) better at low (high) density.
- QRPA calculations gives large deviations

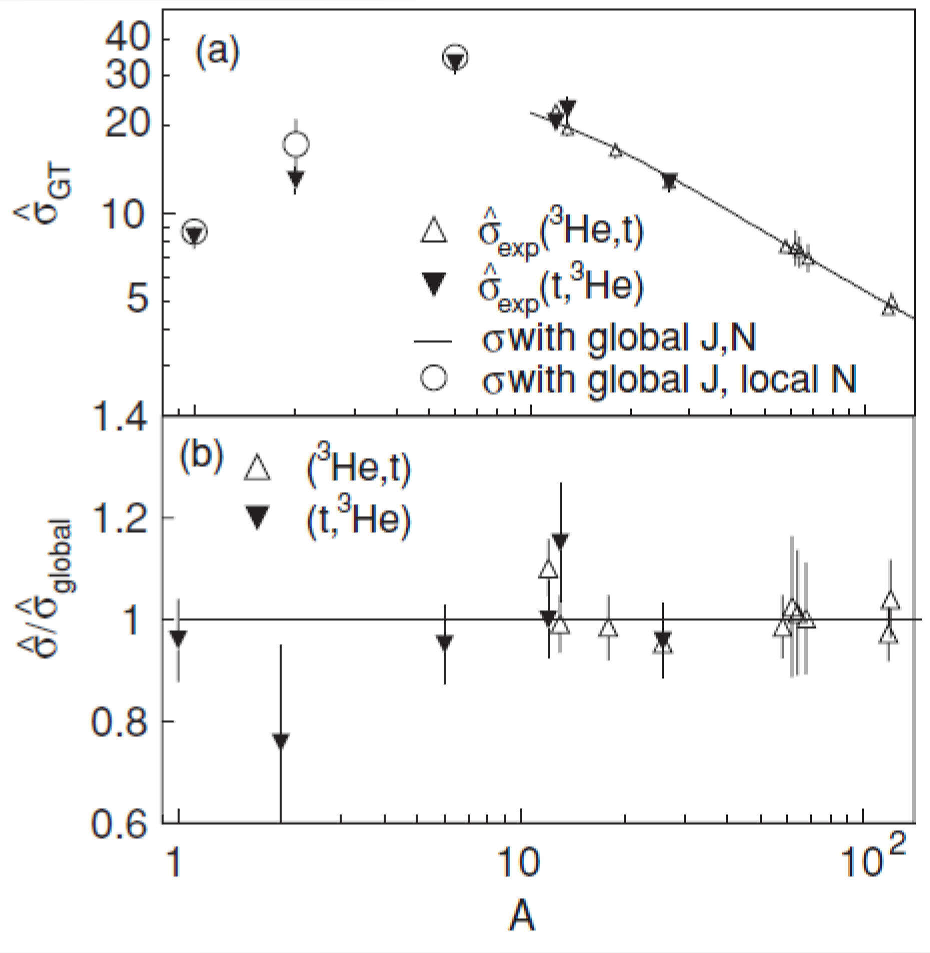
Stellar density (g/cm^3)	Average absolute factor of deviation from EC rates determined from CE experiments		
	GXPF1a	KB3G	QRPA
10^7	0.99	0.64	27
10^9	0.08	0.3	0.74

Wish list for reaction theory

- Improved/up-to-date NN interaction that can be used in DWBA codes
- Improved DWBA codes that can deal with exchange contributions for composite probes
- Theoretical optical potentials – benchmark with few experimental data
- Improved structure input for nuclei beyond pf-shell and for non-GT interactions: important for weak reaction rates in astrophysics, double beta decay, giant resonances
- …

Backup slides

Unit cross sections



Love-Franey interaction

t-matrix interaction strengths at 140 MeV

	Real					Imag			
Range	SE	TE	SO	TO	Range	SE	TE	SO	TO
0.25	9.473 99*E* +03	5.957 51*E* +03	−2.541 98*E* +03	7.864 82*E* +03	0.25	1.128 01*E* +03	9.742 99*E* +03	2.871 99*E* +03	4.689 91*E* +02
0.40	−2.920 63*E* +03	−1.978 48*E* +03	7.824 29*E* +02	−1.568 81*E* +03	0.40	−4.061 83*E* +02	−3.135 50*E* +03	−9.833 01*E* +02	−4.404 72*E* +02
1.40	−1.050 00*E* +01	−1.050 00*E* +01	3.150 00*E* +01	3.500 00*E* +00	1.40				
Range	LSE	LSO	TNE	TNO	Range	LSE	LSO	TNE	TNO
0.25	−7.541 22*E* +03	−3.096 27*E* +03	3.843 43*E* +04	1.658 08*E* +03	0.25	−7.252 38*E* +03	−9.735 18*E* +02	1.392 75*E* +04	−8.646 91*E* +03
0.40	−4.643 20*E* +02	−3.947 80*E* +02	−6.945 77*E* +03	−1.183 09*E* +02	0.40	1.319 36*E* +03	1.416 41*E* +02	−2.547 52*E* +03	1.357 98*E* +03
0.55			1.255 03*E* +03	4.660 86*E* +01	0.55			4.228 19*E* +02	−2.252 13*E* +02
0.70			−2.023 90*E* +02	1.407 32*E* +01	0.70			−4.279 37*E* +01	2.330 58*E* +01

$$t_\tau = \frac{1}{16}\left(t^{SE} - 3t^{TE} - t^{SO} + 3t^{TO}\right)$$

short-range arising from ρ-meson and 2π exchange

$$t_{\sigma\tau} = \frac{1}{16}\left(-t^{SE} - t^{TE} + t^{SO} + t^{TO}\right)$$

long-range arising from π exchange (OPEP)

$$t_{T\tau} = \frac{1}{4}\left\{-t^{TNE} + t^{TNO}\right\}$$

mediates Fermi

mediates Gamow-Teller

$$V_{eff}(r) = [V_\tau Y(r/R_\tau) + V_{\sigma\tau} Y(r/R_{\sigma\tau})(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS\tau} Y(r/R_{LS\tau}) \vec{L} \cdot \vec{S} + V_{T\tau} r^2 Y(r/R_{T\tau}) S_{12}] \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$Y(r/R) = e^{-r/R}/(r/R)$$ **Yukawa**

exchange terms

$$t_{NN} = \tilde{V}(q) + \tilde{V}(k_A)$$

- $\tilde{V}(q)$ → part of nn-interaction we have seen before
- $\tilde{V}(k_A)$ → exchange term

- The exchange term is due to antisymmetrization of the DWBA formalism, I.e. taking into account we can exchange particles between target and projectile
- k_A: momentum transfer needed to stop the projectile nucleon
- the exchange terms oppose the normal terms and lead to reduction of the amplitude
- calculation is complex and usually a short-range pseudo potential is used instead of doing the full calculation
- this approximation tends to lead to overprediction of the cross section