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Theory and Calculation of Two-nucleon Transfer Reactions

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Nuclear Theory and Modeling, L-414

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Ingredients for 2N transfer calculations

Consider reaction A(p,h)C   where h=p+2N and A=C+2N

- All nuclei have known spins, parities & energies

- Need overlaps (with radial shapes & phases!)
  - $\langle B|a_1a_2|A\rangle = \Phi(r_1,r_2)\,|ST\rangle = \phi(R,r)\,|S'T'\rangle$
  - $\langle p|a_1a_2|h\rangle = \Psi(r_1,r_2)\,|ST\rangle = \psi(R,r)\,|ST\rangle$
  - Both expanded in all partial waves of L($\underline{R}$) and ℓ($\underline{r}$).
- Intermediate states of d+B where d=p+N and B=C+N
  - All spins, parities, energies and 1N overlap functions
- Optical potentials for p+A, h+C and d+B scattering
- FRESCO code: www.fresco.org.uk

Example of overlap definition

TABLE II. Two-nucleon spectroscopic amplitudes for the $^{40}Ca(p,{}^3He)^{38}K$ reaction

E (MeV)	$J^\pi T$	transfer S	L	$(d_{5/2},d_{5/2})$	$(d_{5/2},2s_{1/2})$	$(d_{5/2},d_{3/2})$	$(2s_{1/2},2s_{1/2})$	$(2s_{1/2},d_{3/2})$	$(d_{3/2},d_{3/2})$
0.00	3^+,0	1	2,4	0.0193	−0.0421	0.5684	0.0	0.0	−2.5836
0.13	0^+,1	0	0	0.4810	0.0	0.0	0.2572	0.0	1.6439
0.46	1^+,0	1	0,2	−0.1137	0.0	0.9324	−0.4773	1.2344	−0.6052
1.70	1^+,0	1	0,2	−0.3501	0.0	0.7336	−0.3344	0.0160	1.4924
2.40	2^+,1	0	2	−0.2421	−0.3210	−0.1764	0.0	0.1877	−3.8434

e.g. from shell-model calculations (Oxbash, NCSM)
Do we have these for $<^{A-2}Sn|^{A}Sn>$?

The more components include here,
the more accurate are correlations described.
No limit here on what can be used in reaction calculations.

Simultaneous and Sequential Transfers

Simultaneous transfers:

- directly couple p+A to h+C channels.
- Transfer matrix element preserves internal 2N state:
  - $\underline{r}$ is fixed: both the magnitude r and partial wave $\mathbb{l}$.
  - Spin & isospin selective: S=S' and T=T'
- Cross sections behave like Bessels in L-transfer.

Sequential transfers:

- Couple p+A > d+B > h+C: 2-step DWBA (at least)
- Should include all (complete set of) possible states of d* and B* (at least in the range of the Q window) to avoid unwanted filtering of angular-momentum

Light-ion overlaps

Traditionally:

- Use zero-range 1-step transfers with D_0 constant
- Often use ratios of cross sections to different states
- Neglect 2-step sequential cross sections
- Large ‘unhappiness factor’ as expt >> theory.

Finite range transfers:

- We have excellent few-body wfs for ^{3}H and ^{3}He.
- Plausible wfs for ^{4}He and ^{6}Li, for $\langle d|\alpha\rangle$ and $\langle\alpha|^6\text{Li}\rangle$
- No need to use zero-range D_0 constant

Unbound intermediate states:

- Still problems for $(np)_{T=1}$, ^{5}He, ^{5}Li, ^{9}Li: use eg bin wfs

Two-nucleon Transfers to Probe 2N Correlations

- Nuclei like ^{6}Li, ^{11}Li have two ‘loose’ nucleons:
  - Need all two-body potentials & correlations.
  - Two-body systems NN and N+A have various virtual states, bound states & resonances to be elucidated.
- One-particle Transfers probe Spectroscopy
  - Magnitude: Spectroscopic Factors
  - Shape: Angular momentum Transfers
- Two-particle Transfers probe Correlations
  - Magnitude: Strength of correlation common to initial and final states.
  - Shapes: Angular momentum of NN pair w.r.t. core.

^{11}Li Structure: Intruder States and Size?

- Normal Shell Structure: expect $(0p_{1/2})^2$ for neutrons
- s-wave Intruder State seen in ^{10}Li, and ^{11}Be:
  - Now expect superposition of $(0p_{1/2})^2$ and $(0s_{1/2})^2$
  - Use Three-Body models of ^{9}Li + n + n with n-n & n-Li potentials
  - Generate range of ^{11}Li models P0 -- P4, with various s^2 %:

	$1s$ $(0s)$ a_0 (fm)	$0p_{1/2}$ reson. (MeV)	E_{11} g.s. (MeV)	R_m rms (fm)	$(s_{1/2})^2$ weight (%)	$(p_{1/2})^2$ wt. (%)	$^1S_0$ (nn) wt. (%)	$^3P_1$ (nn) wt. (%)
P0	0.7	0.175	-0.33	3.05	3	94	38	59
P1	-11	0.22	-0.32	3.28	23	72	52	44
P2	-18	0.25	-0.32	3.39	31	64	53	37
P3	-27	0.30	-0.33	3.64	45	51	60	29
P4	-44	0.35	-0.31	3.73	64	30	67	16

Effect of low-lying 1*s* intruder states: Binding energies E_{11}, r.m.s. radii, and weights of selected channels.
The SSC *nn* potentials was used. All $0p_{3/2}$ eigenstates are at E = -4.1 MeV.
The matter r.m.s. radius of ^{9}Li is taken as 2.32 fm.

No core Excitation yet!

Two-Neutron Transfers (A) SIMULTANEOUS

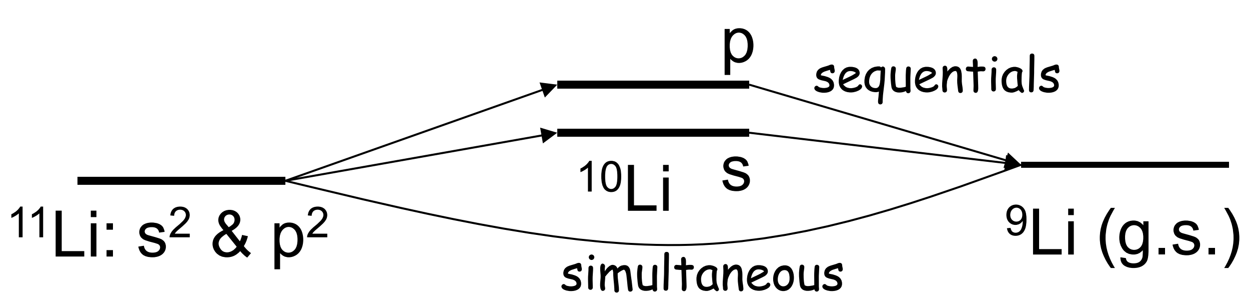
- Simultaneous Transfers
  - Use 3-body wave functions <p|t> and <^{9}Li|^{11}Li>
  - The relative neutron-neutron states must be equal
    - Since the <p|t> overlap is 99% $^{1}S_{0}$-waves, we only probe the $^{1}S_{0}$-wave component of ^{11}Li.
    - This $^{1}S_{0}$-part of ^{11}Li <u>increases</u> in P0→P4 models, so expect <u>increasing</u> cross sections
  - One Direct Step
    - Appears in First-order DWBA
  - Need p+^{11}Li and t+^{9}Li Optical Potentials

Two-Neutron Transfers (B) SEQUENTIAL

- Sequential Transfers
  - Use 2-body wave functions <p|d> & <d|t>, and <^{9}Li|^{10}Li> & <^{10}Li|^{11}Li>
  - <u>Should</u> have complete sets of d* and ^{10}Li* wfs:
    - d bound state
    - d* triplet continuum
    - d* singlet continuum (no bound state)
    - all ^{10}Li* s-wave and p-wave continua.
    - Overlaps of all above with triton and ^{11}Li(gs) respectively.
  - Need also all these d*+^{10}Li* Optical Potentials!
  - Two successive steps
    - appears in Second-order DWBA
    - Equally: by two iterations of coupled equations.

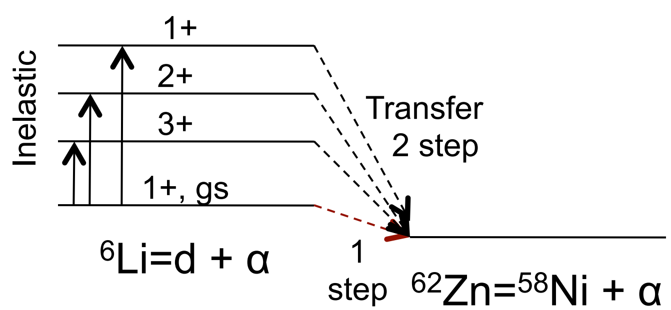
(C) COMBINED

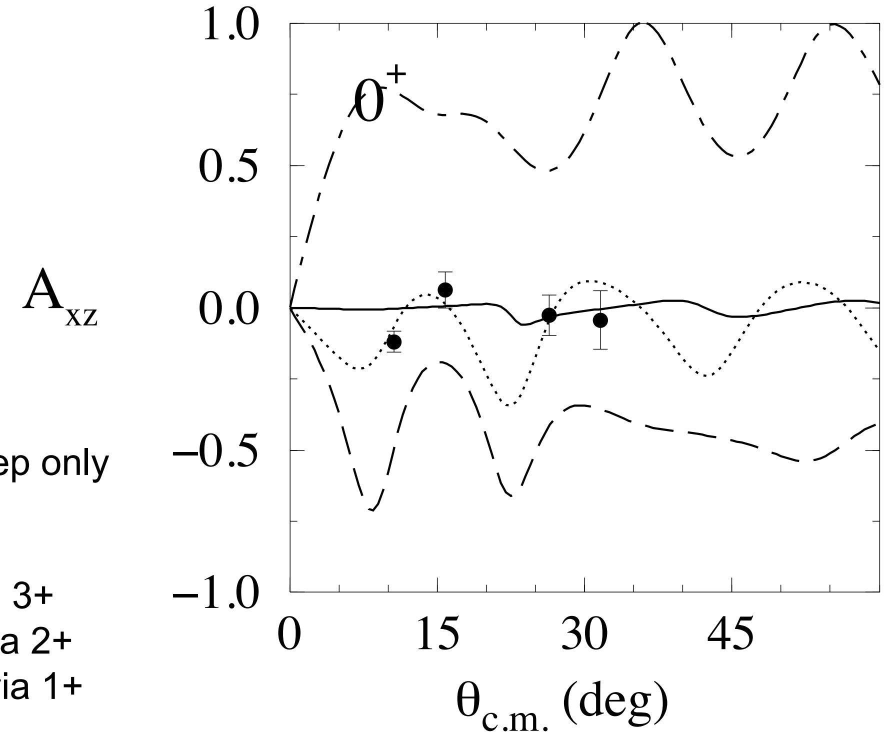
- Simultaneous + Sequential Transfers
  - Derive all Overlaps from triton and ^{11}Li 3-body wfns.
  - Need Combination of First- & Second-order DWBA
  - All routes contributes Amplitudes which Interfere
    - Interference between Sim + Seq
    - Interference between ^{10}Li s- and p-wave Routes.



(^{6}Li,d) Calculation: 1- and 2-step spin filters

- This is transfer of an alpha particle,
  but the principles of spin-filtering are similar.





Solid: 1-step only

Two-step:
Dotted: via 3+
Dashed: via 2+
Dotdash: via 1+

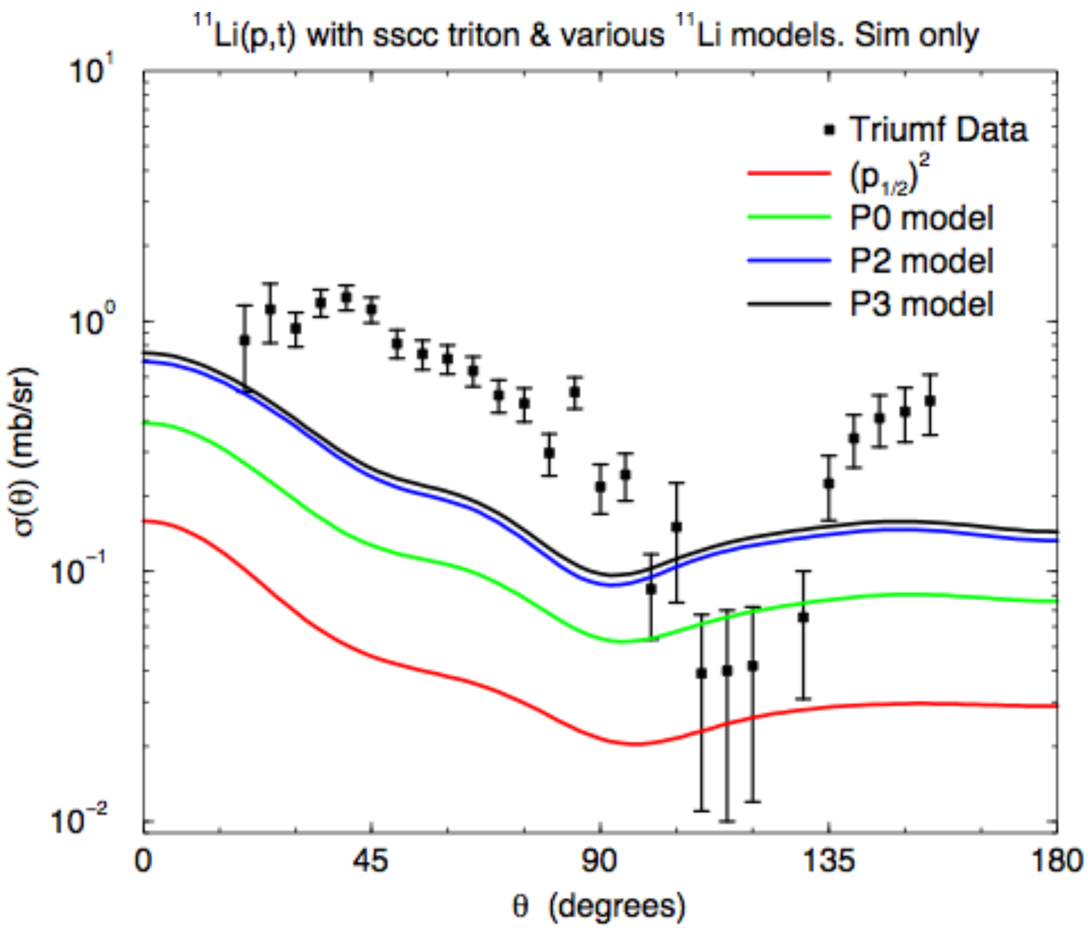
Conclusion:
Must include all allowed spin-orientations in intermediate states!

K.D. Veal, et al, Phys. Rev. C**60**, 064003 (1999)

Preliminary ^{11}Li Calculation

- My Preliminary Calculations:
  - d*: Use only Deuteron ground state
  - ^{10}Li*: Use both ^{10}Li s- and p-waves, with single 'Weak Binding Approximation' wfs chosen to have rms radii similar to ^{11}Li gs.
  - ^{11}Li: No core excitation.
- Optical potentials. Use 'global' parameterisations applied (approx.) to these very light systems:
  - p+^{11}Li: Becchetti & Greenlees
  - d+^{10}Li: Daehnick et al
  - t + ^{9}Li: Becchetti & Greenlees
- (room for improvement in all respects!)

Results: Simultaneous Transfers



Shapes similar

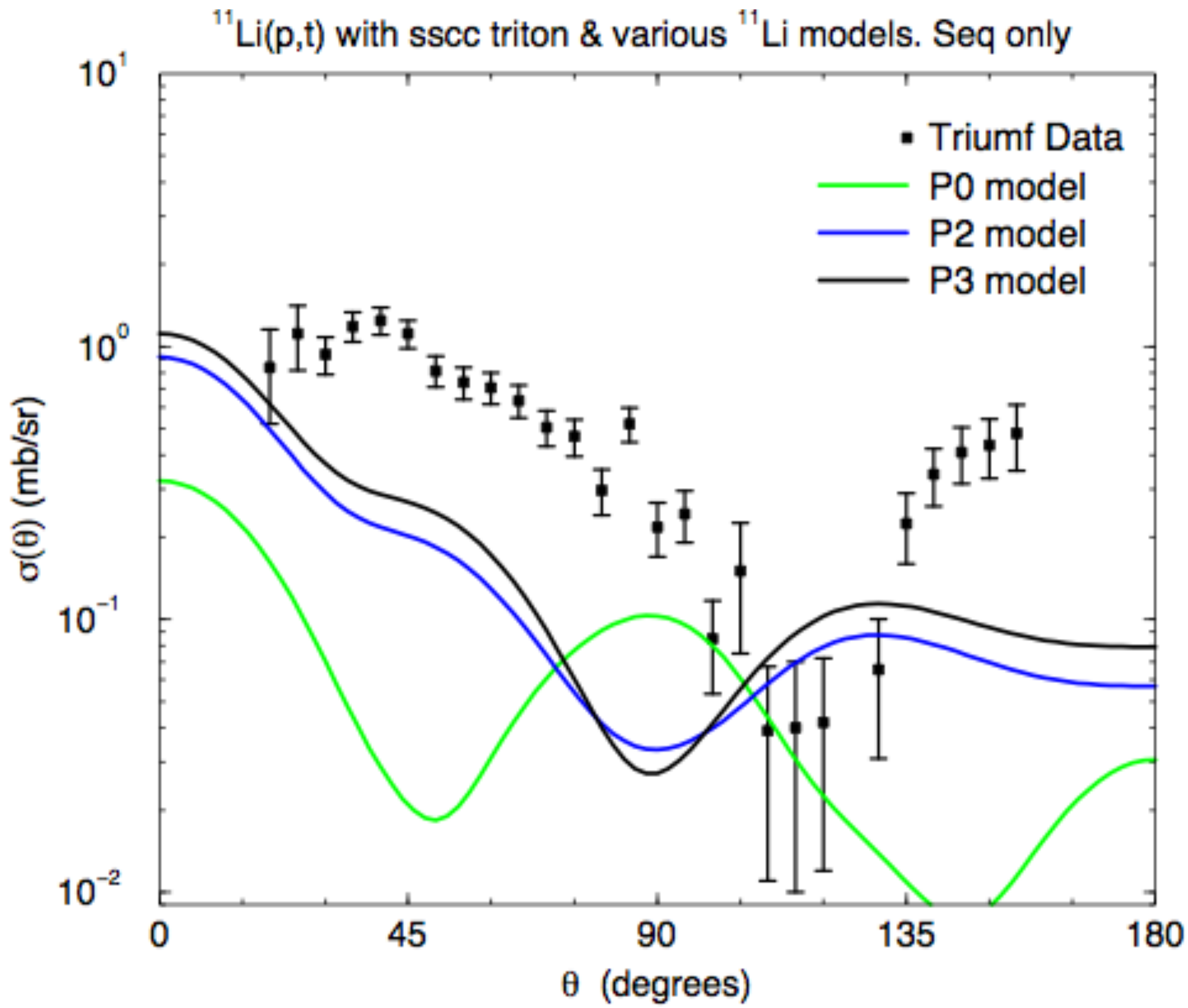
- shows s^2 filter from triton wave function

Magnitude varies

- shows s^2 strengths in the ^{11}Li wave functions

3 MeV/u

Results: Sequential Transfers



Shapes vary

- Shows interference between s- and p-wave parts of ^{10}Li.

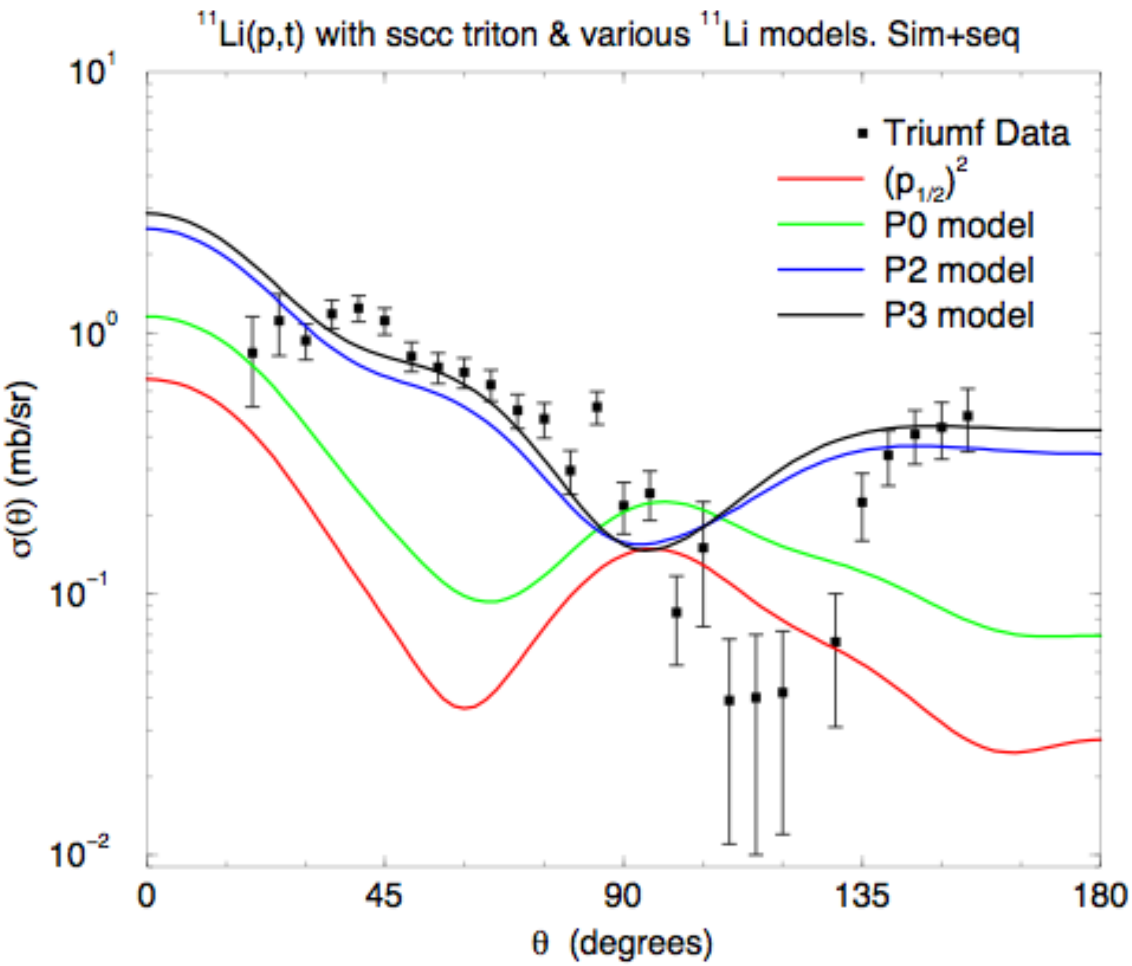
Note: this interference will diminish if a complete set of ^{10}Li states included at same energies.

(May reappear when energies in ^{10}Li* included properly)

Need further research.

P0 and $(p_{1/2})^2$ models have the same sequential steps

Results: Simultaneous + Sequential Transfers



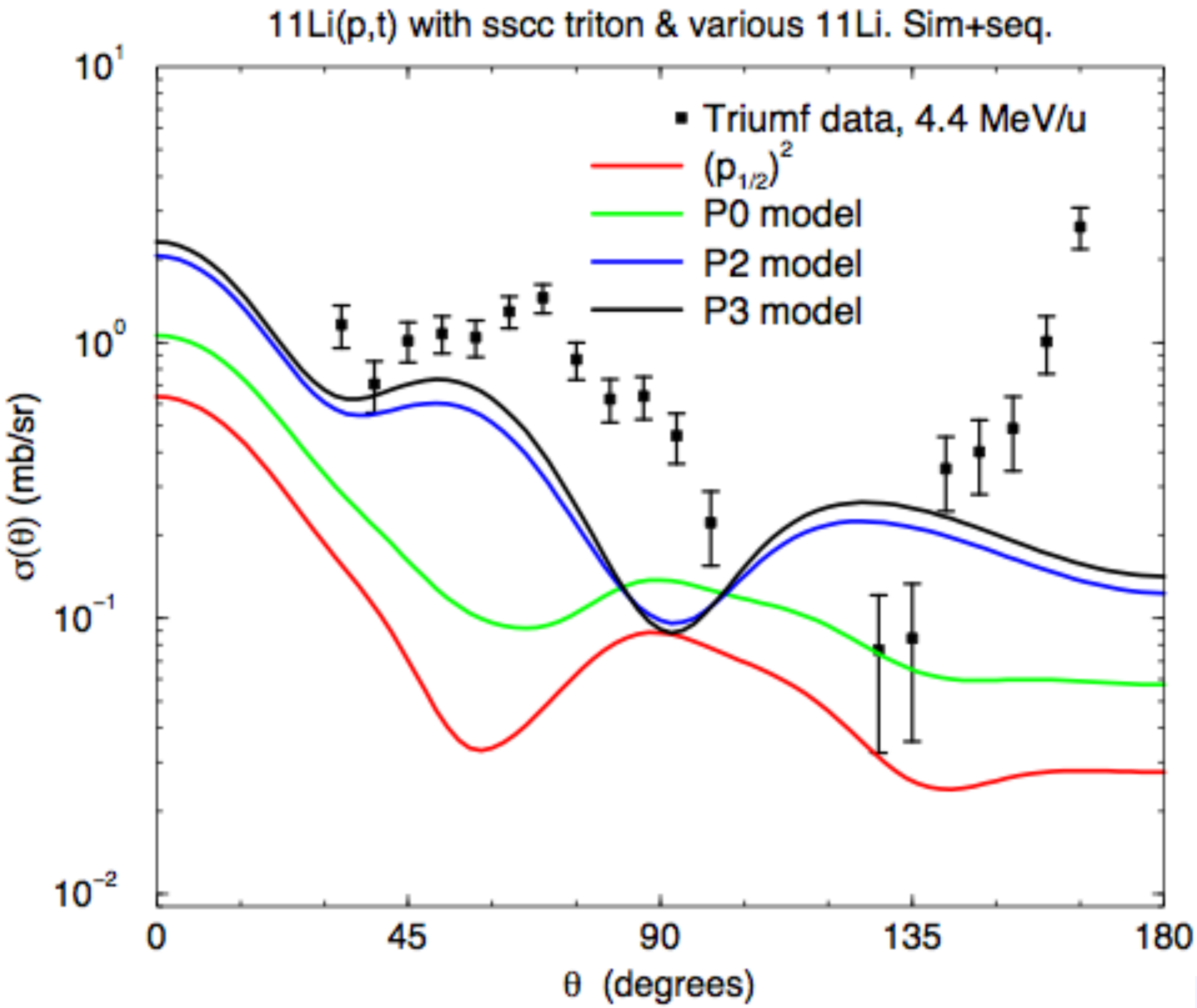
Shapes and magnitudes both vary

- Shows **both** s^2 strengths and interferences via ^{10}Li* states.

Constructive interference gives good forward angle strength.

Still miss minimum around 120 deg.

Results: Simultaneous + Sequential Transfers



Higher beam energy

Still miss minimum
around 120 deg.
(shift of ~ 20 deg?)

Poor potential radius?

4.4 MeV/u

^{11}Li Conclusions

Quantum Calculations of two-neutron transfers performed for the reaction ^{11}Li(p,t)^{9}Li.

- Used three-body models of triton and ^{11}Li, including all pairwise potentials and correlations
- Tried a range of models of ^{11}Li with different s^2 strength (no core excitation)
- Included (coherently) both simultaneous and (approx.) sequential transfer mechanisms

Results:

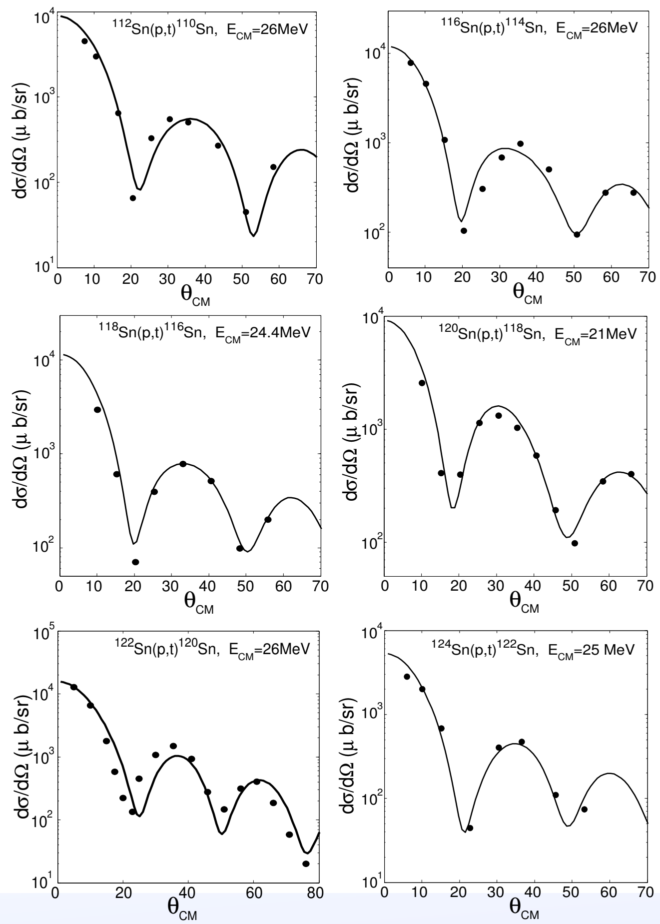
- Simultaneous transfers have fixed shape, magnitude shows s^2 strength
- Sequential transfers show some interferences.
- Coherent combination reproduces forward-angle experimental cross sections, but not minimum at 120 deg.

To Do:

- Continuum Bins for ^{10}Li*, and derived overlaps < ^{10}Li*|^{11}Li>
- Core excitation in ^{9}Li*, and hence in ^{10}Li* and ^{11}Li

New theory reported for $^{A}Sn(p,t)\,^{A-2}Sn$

- Results of Potel, et al and Broglia: arXiv:1105.6250
- Use BCS neutron pairing
- Include all simultaneous & sequential transfer terms
- Agrees with data within 15%
- Needs to be replicated using eg shell-model wfs.



Lessons Learned

Overlap functions

- Can use 'best' wave functions for projectile & target
- Perform calculations for multiple models,
- Compare with experiment, and see which is best.

Simultaneous transfers

- Shape the same for the multiple models
- Physics is in the magnitudes

Sequential transfers

- Need two-step transfers, especially at low energies
- Need all intermediate spin states, even if unbound

Will use test cases

- For example $^{12}C(p,t)$, $^{12}C(p,{}^{3}He)$, $\langle {}^{A-2}Sn | {}^{A}Sn \rangle$

Should make front-end codes, to use global optical potls, etc