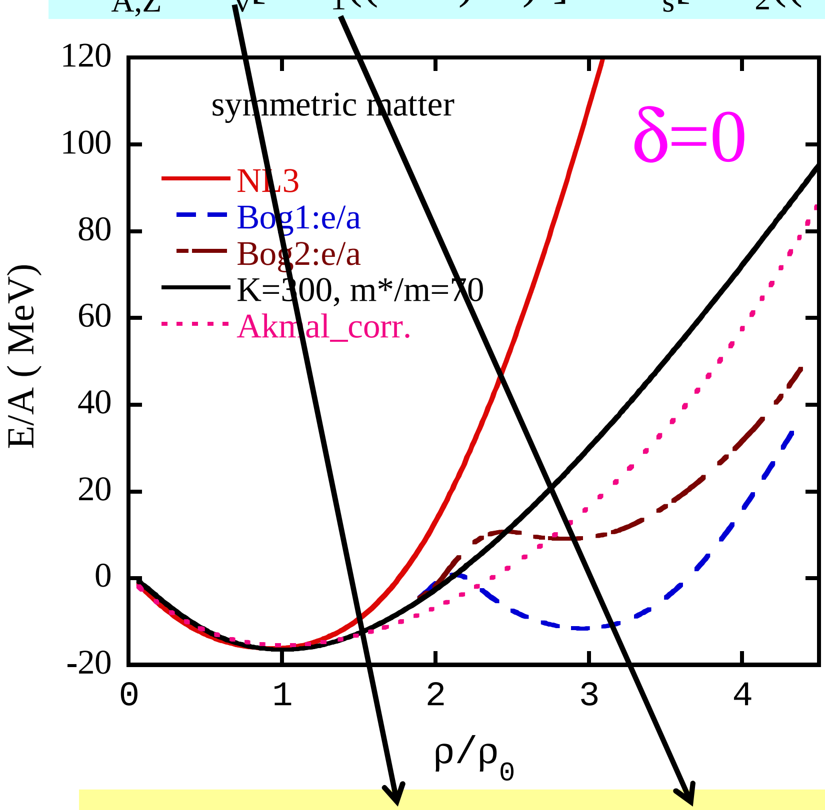
Probing the EoS of Asymmetric Matter

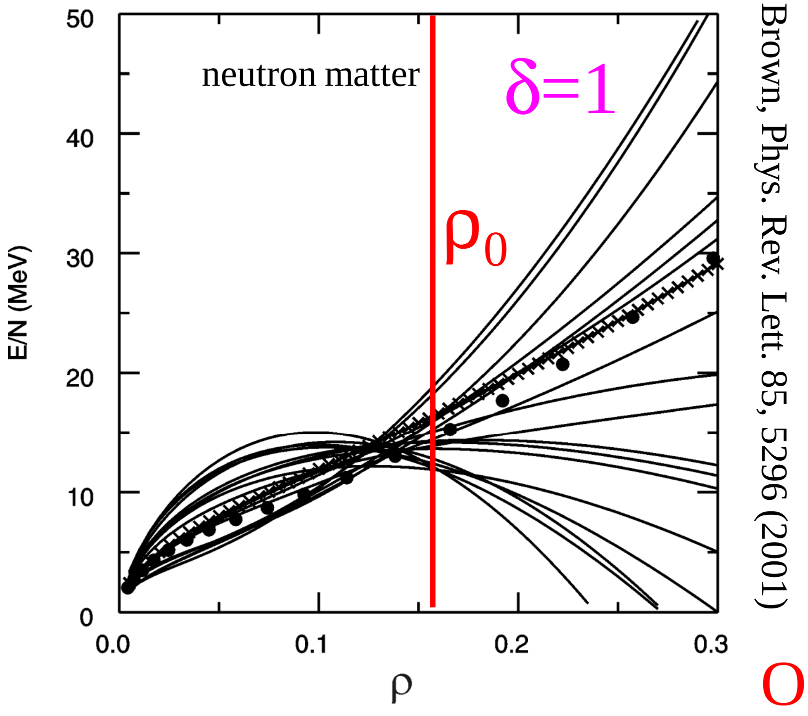
William Lynch, NSCL MSU

- Motivations
- Sources of constraints on the EOS and symmetry energy.
  - Astrophysics
  - Nuclear experiments
- Laboratory constraints from nuclear collisions
- What improvements in theory are needed.
- Summary and outlook

EoS: How does it depend on ρ and δ?

$$B_{A,Z} = a_v[1-b_1((N-Z)/A)^2]A - a_s[1-b_2((N-Z)/A)^2]A^{2/3} - a_c Z^2/A^{1/3} + \delta_{A,Z}A^{-1/2} + C_d Z^2/A,$$





Brown, Phys. Rev. Lett. 85, 5296 (2001)

$$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$$

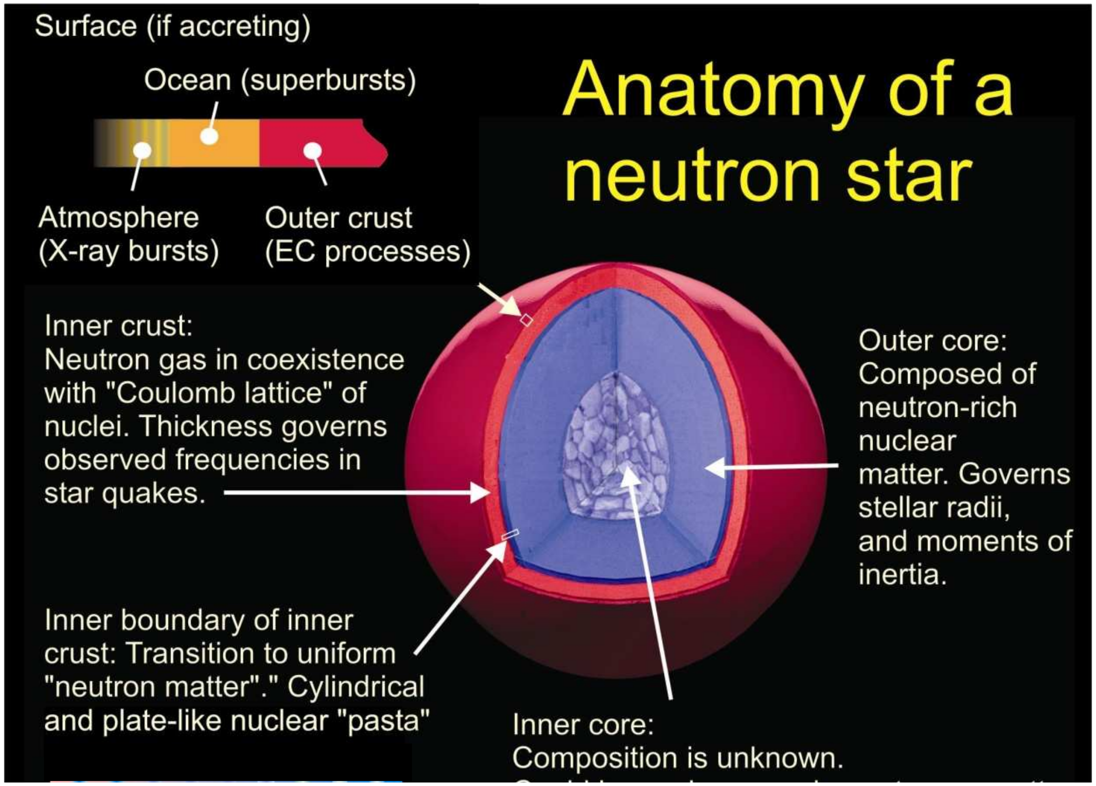
$$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$

$$P = \rho^2 \frac{\partial(E/A)}{\partial\rho}\bigg|_{s/a}$$

- Nuclear effective interactions do not constrain neutron matter,
- Main uncertainty is the density dependence of the symmetry energy

EOS, Symmetry Energy and Neutron Stars

- Influences neutron Star stability against gravitational collapse
- Stellar density profile
- Internal structure: occurrence of various phases.
- Observational consequences:
  - Cooling rates of proto-neutron stars and
  - Temperatures and luminosities of X-ray bursters .
  - Stellar masses, radii and moments of inertia.
    - Can be studied by X-ray observers.

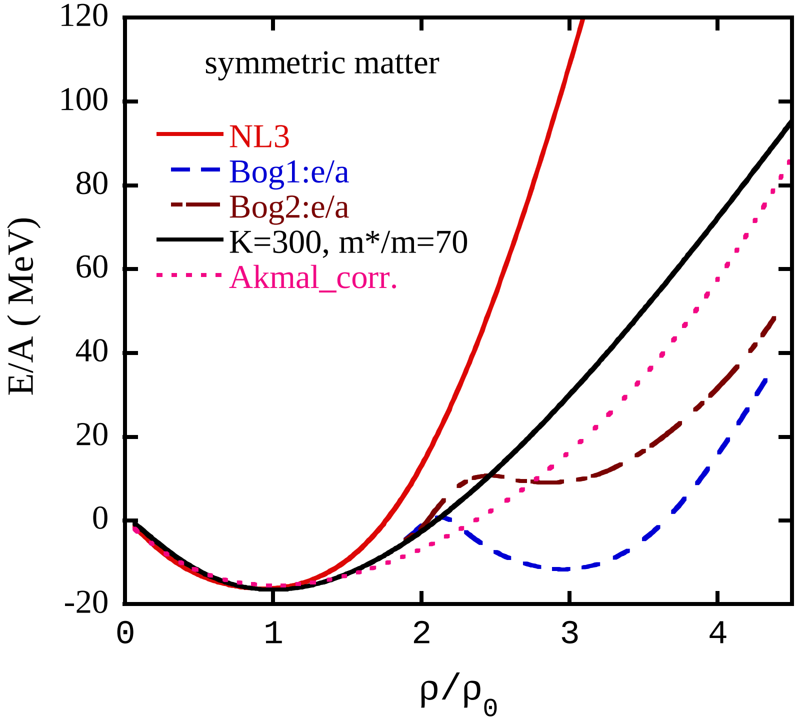


- Precise studies of low mass X-ray binaries could have been possible with the “International X-ray Observatory” (not in the budget).
- In the interim, scientists are working to extract constraints from available data on X-ray bursters and low mass X-ray binaries.

⇒ It is important to obtain laboratory constraints.

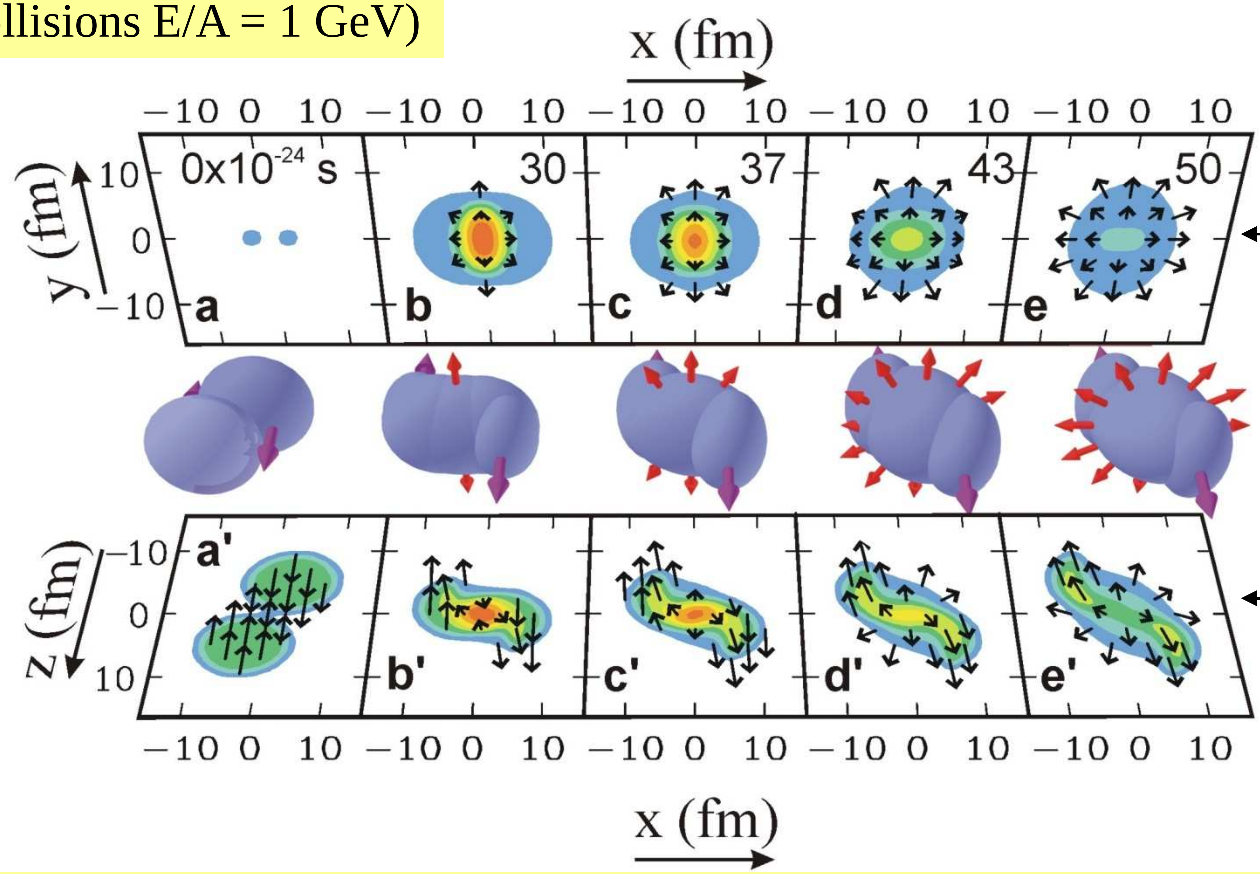
Need for laboratory probes sensitive to higher densities $\rho \geq 2\rho_0$

- In a Taylor series about ρ_0, the incompressibility, K_{nm} provides the term proportional to $(\rho-\rho_0)^2$.
- The solid black, dashed brown and dashed blue EoS's all have K_{nm}=300 MeV.
  - To probe the EoS at $3\rho_0$, you need to compress matter to $3\rho_0$ to determine the higher order terms. This is the primary motivation for probing the EoS with nucleus-nucleus collisions.



Constraining the EOS at high densities by laboratory collisions

Au+Au collisions E/A = 1 GeV)



pressure contours

density contours

- Two observable consequences of the high pressures that are formed:
  - Nucleons deflected sideways in the reaction plane.
  - Nucleons are “squeezed out” above and below the reaction plane. .

Flow studies of the symmetric matter EOS

- Theoretical tool: transport theory:
  - Example Boltzmann-Uehling-Uhlenbeck eq. (Bertsch Phys. Rep. 160, 189 (1988).) has derivation from TDHF:

$$\frac{\partial f_1}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla}_{\mathbf{r}} f_1 - \vec{\nabla}_{\mathbf{r}} U \cdot \vec{\nabla}_{\mathbf{p}} f_1 = \frac{4}{(2\pi)^3} \int d^3 k_2 \, d\Omega \, \frac{d\sigma_{nn}}{d\Omega} v_{12} \left[f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4) \right]$$

    - f is the Wigner transform of the one-body density matrix
    - semi-classically, = $f(\vec{r}, \vec{p}, t)$ (number of nucleons/d^3rd^3p at $\vec{r}$ and $\vec{\mathrm{p}}$).
    - BUU can describe nucleon flows, the nucleation of weakly bound light particles and the production of nucleon resonances.
  - The production of heavier fragments is a difficult problem. It have been calculated with Anti-Symmetrized Molecular Dynamics (AMD) and other molecular dynamics techniques with mixed success. Such observables are sensitive to fluctuations in the mean field that give rise to spinodal decomposition.
  - The most accurately predicted observables are those that can be calculated from $f(\vec{r}, \vec{p}, t)$ i.e. flows and other average properties of the events.

Some technical points

- Semi-classical: “time dependent Thomas-Fermi theory”
  - Respect of Pauli principle is assured by Liouville's theorem and by the blocking factors in the collision integral.
- Each nucleon is represented by ~1000 test particles that propogate classically under the influence of the self-consistent mean field U and subject to collisions due to the residual interaction.
- Mean field is momentum dependent:
  - Momentum dependence of N-N interaction
  - Fock term
- Nucleon-nucleon cross sections are modified in the medium

Procedure to study EOS using transport theory

- Measure collisions
- Simulate collisions with BUU or other transport theory
- Identify observables that are sensitive to EOS (see Danielewicz et al., Science 298,1592 (2002). for flow observables)
  - Directed transverse flow (in-plane) ▶
  - “Elliptical flow” out of plane, e.g. “squeeze-out”
  - Kaon production. (Schmah, PRC C **71**, 064907 (2005))

  symmetric matter EOS

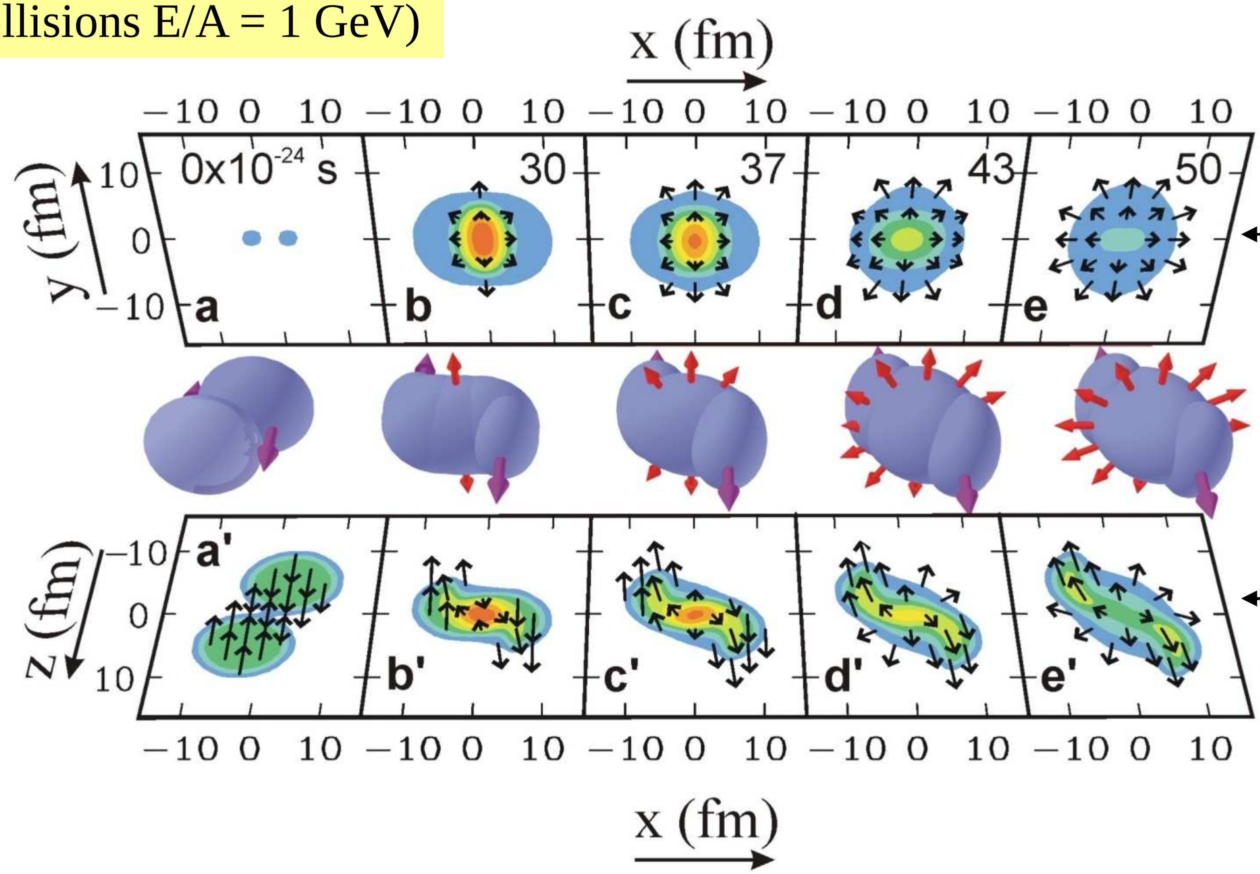
  - Isospin diffusion
  - Neutron vs. proton emission and flow.
  - Pion production.

  symmetry energy

- Find the mean field(s) that describes the data. If more than one mean field describes the data, resolve the ambiguity with additional data.
- Constrain the effective masses and in-medium cross sections by additional data.
- Use the mean field potentials to calculate the EOS.

Constraining the EOS at high densities by laboratory collisions

Au+Au collisions E/A = 1 GeV)



pressure contours

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- Two observable consequences of the high pressures that are formed:
  - Nucleons deflected sideways in the reaction plane.
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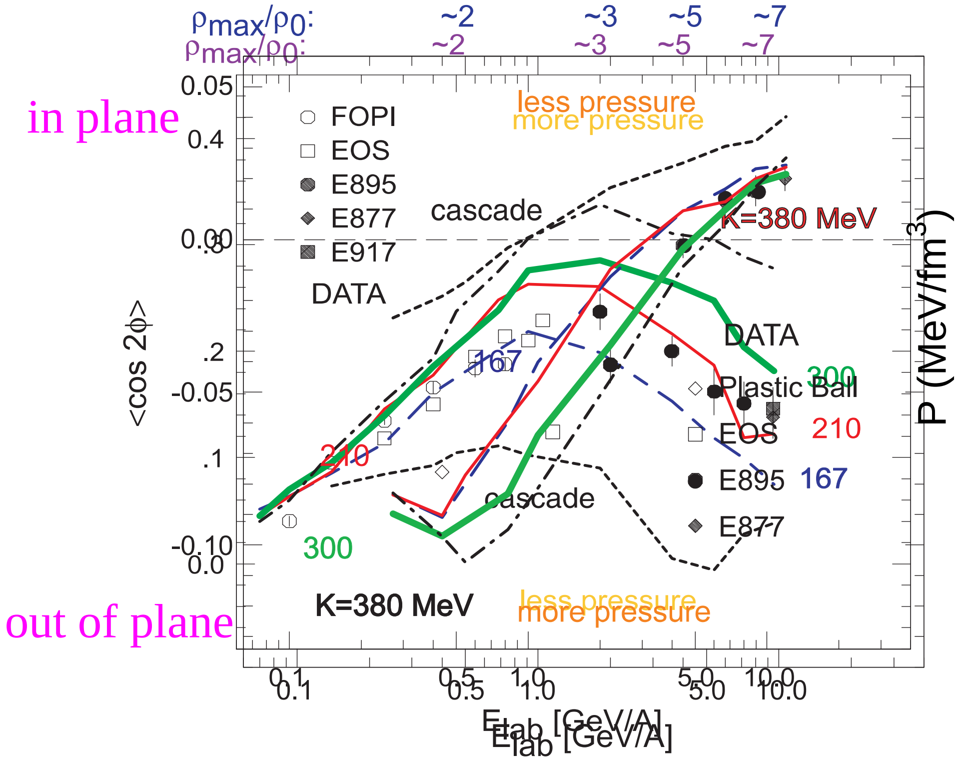
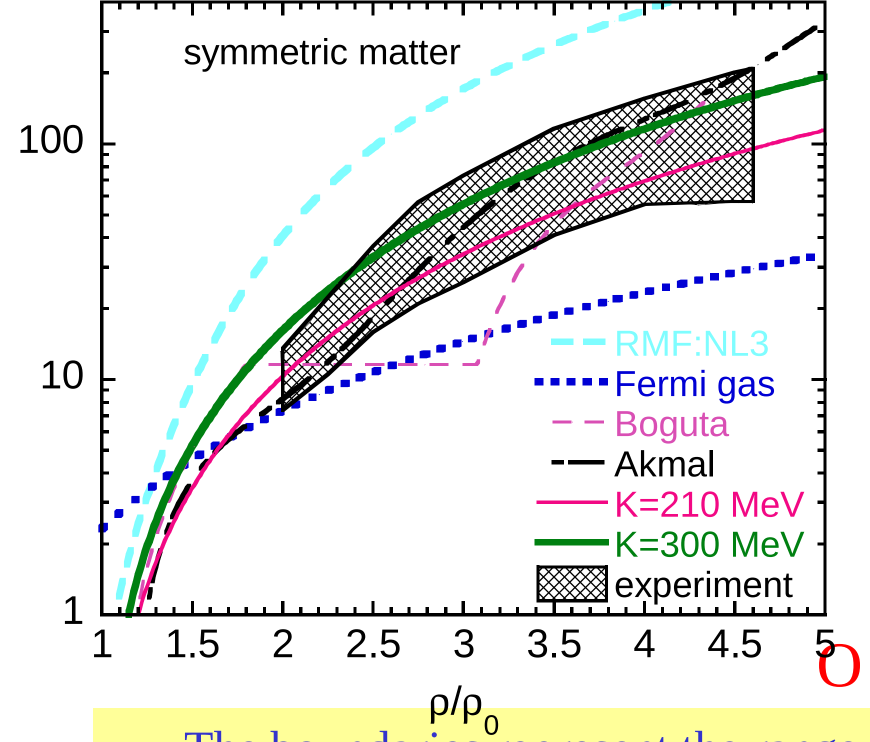
Determination of symmetric matter EOS from nucleus-nucleus collisions



Danielewicz et al., Science 298,1592 (2002).

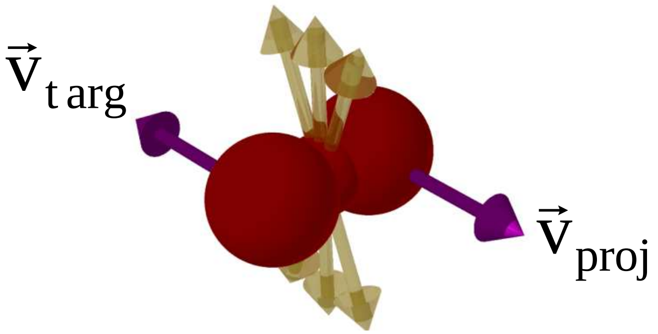


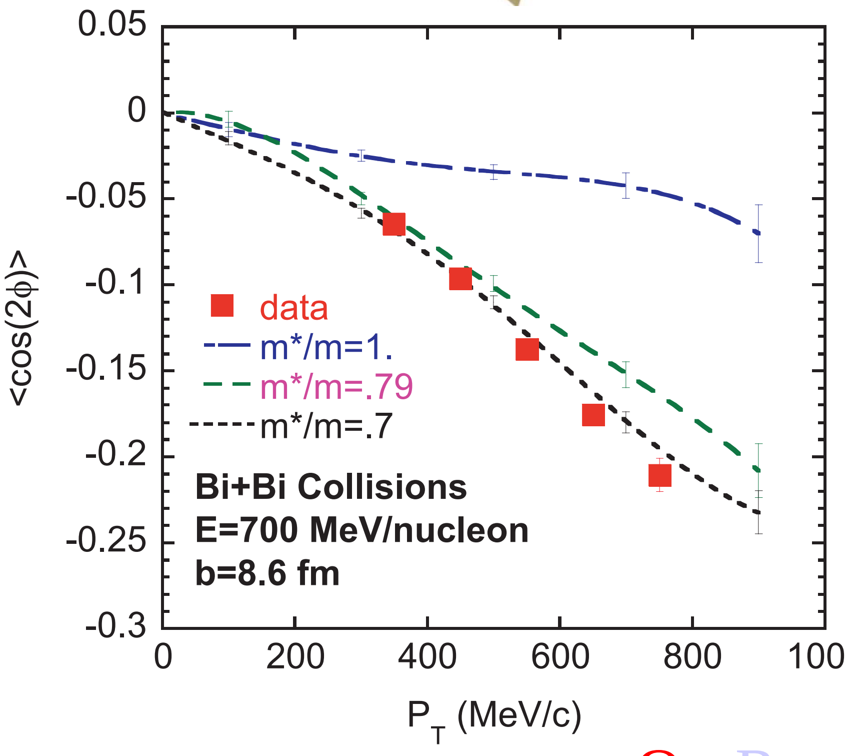
- The curves labeled by K represent calculations with parameterized Skyrme mean fields
  - They are adjusted to find the pressure that replicates the observed transverse flow.

- The boundaries represent the range of pressures obtained for the mean fields that reproduce the data.
- They also reflect the uncertainties from the effective masses ▶ and in-medium cross sections.

Theoretical problem: constraining the momentum dependence

- Momentum dependence, e.g. from meson exchange or from the Foch term, reduces the effective mass, increasing the acceleration and making the mean field potential appear "stiffer".
- Ancillary measurements are needed to constrain the momentum dependence
  - Out-of-plane enhancement in peripheral collisions.
  - Measurements of transverse flow in asymmetric systems.
- Such observables were discovered and analyzed long after the experimental program was completed.
  - This is really too late!



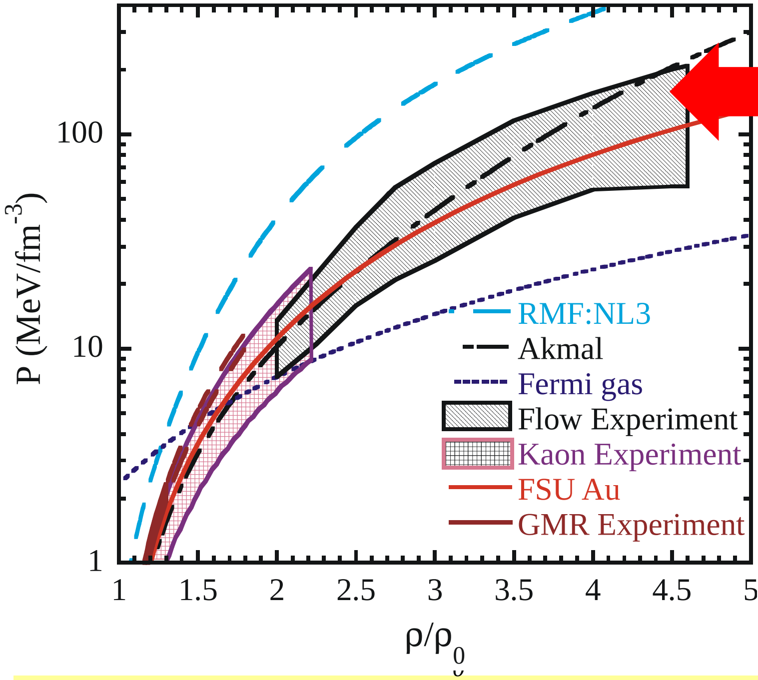


Danielewicz, *Nucl. Phys. A* **673**, 375.

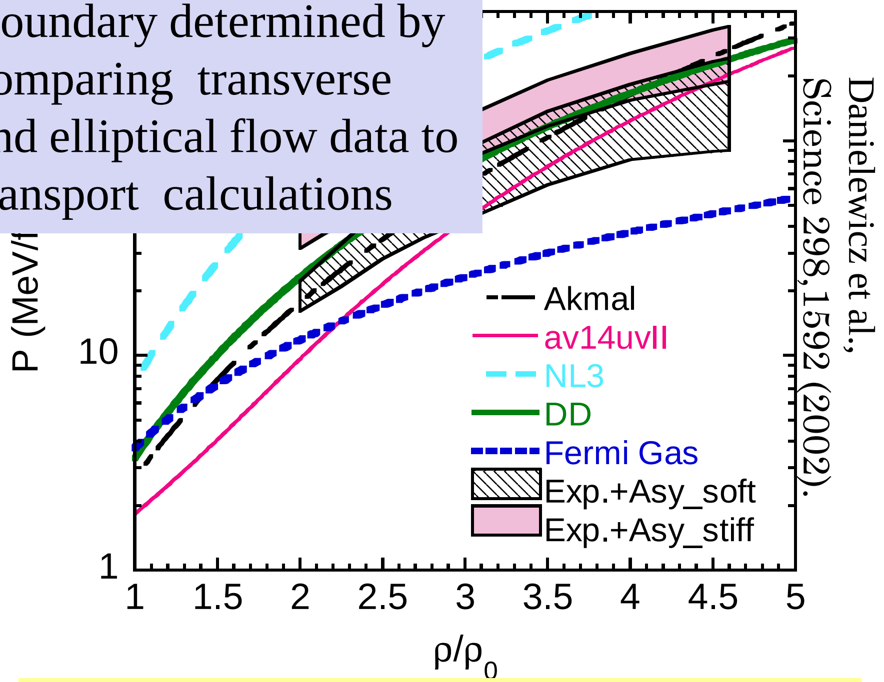
O R

Example: Flow Constraints on symmetric matter EOS at $\rho>2\rho_0$.

$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$ $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A \approx 1$



Boundary determined by comparing transverse and elliptical flow data to transport calculations



Danielewicz et al., Science 298,1592 (2002).

- Flow confirms the softening of the EOS at high density.
- Constraints from kaon production are consistent with the flow constraints and bridge gap to GMR constraints.
- Note: analysis requires additional constraints on m* and σ_{NN}.

- The symmetry energy dominates the uncertainty in the n-matter EOS.
- Both laboratory and astronomical constraints on the density dependence of the symmetry energy are urgently needed.

Probing the symmetry energy at sub-saturation densities: What influences the choice of reaction observables?

- The symmetry mean field potential energy has an opposite sign for neutrons and protons.
- ⇒ Desirable features for probes
  - Vary isospin of detected particle
  - Vary isospin asymmetry $\delta=(N-Z)/A$ of reaction.
- Also supra-saturation and sub-saturation densities are only achieved ***momentarily***
- Therefore, theoretical description must follow the reaction dynamics self-consistently from contact to detection.
- Isospin diffusion, n/p flows and pion production can be calculated using transport theories :
  - Depend to first order on the single particle distribution function, which can be more accurately calculated in BUU or QMD transport theory.
  - May be less sensitive to uncertainties in (1) the production mechanism for complex fragments and (2) secondary decay.

Probe: Isospin diffusion in peripheral collisions

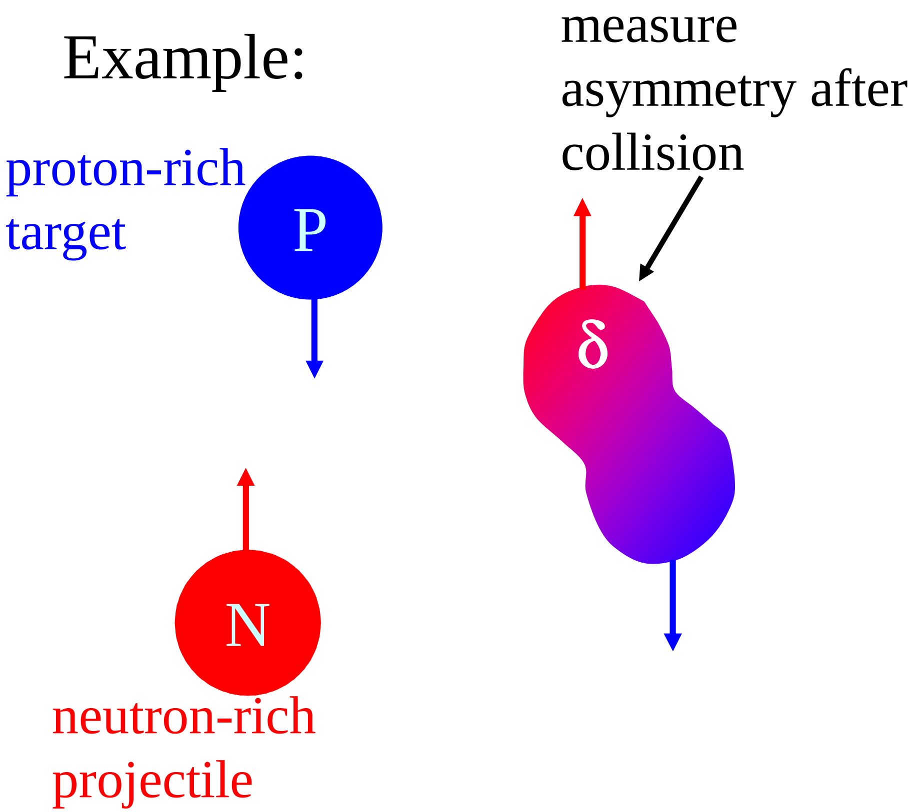
- Collide projectiles and targets of differing isospin asymmetry
- Probe the asymmetry $\delta=(N-Z)/(N+Z)$ of the projectile spectator during the collision.
- The use of the isospin transport ratio $R_i(\delta)$ isolates the diffusion effects:

$$R_i(\delta) = 2 \cdot \frac{\delta - (\delta_{both\_neut.-rich} + \delta_{both\_prot.-rich})/2}{\delta_{both\_neut.-rich} - \delta_{both\_prot.-rich}}$$

- Useful limits for R_i for $^{124}Sn+^{112}Sn$ collisions:
  - $R_i = \pm 1$: no diffusion
  - $R_i \approx 0$: Isospin equilibrium
- Softer symmetry energy enhances diffusion more.

Systems {
mixed $^{124}Sn+^{112}Sn$
n-rich $^{124}Sn+^{124}Sn$
p-rich $^{112}Sn+^{112}Sn$

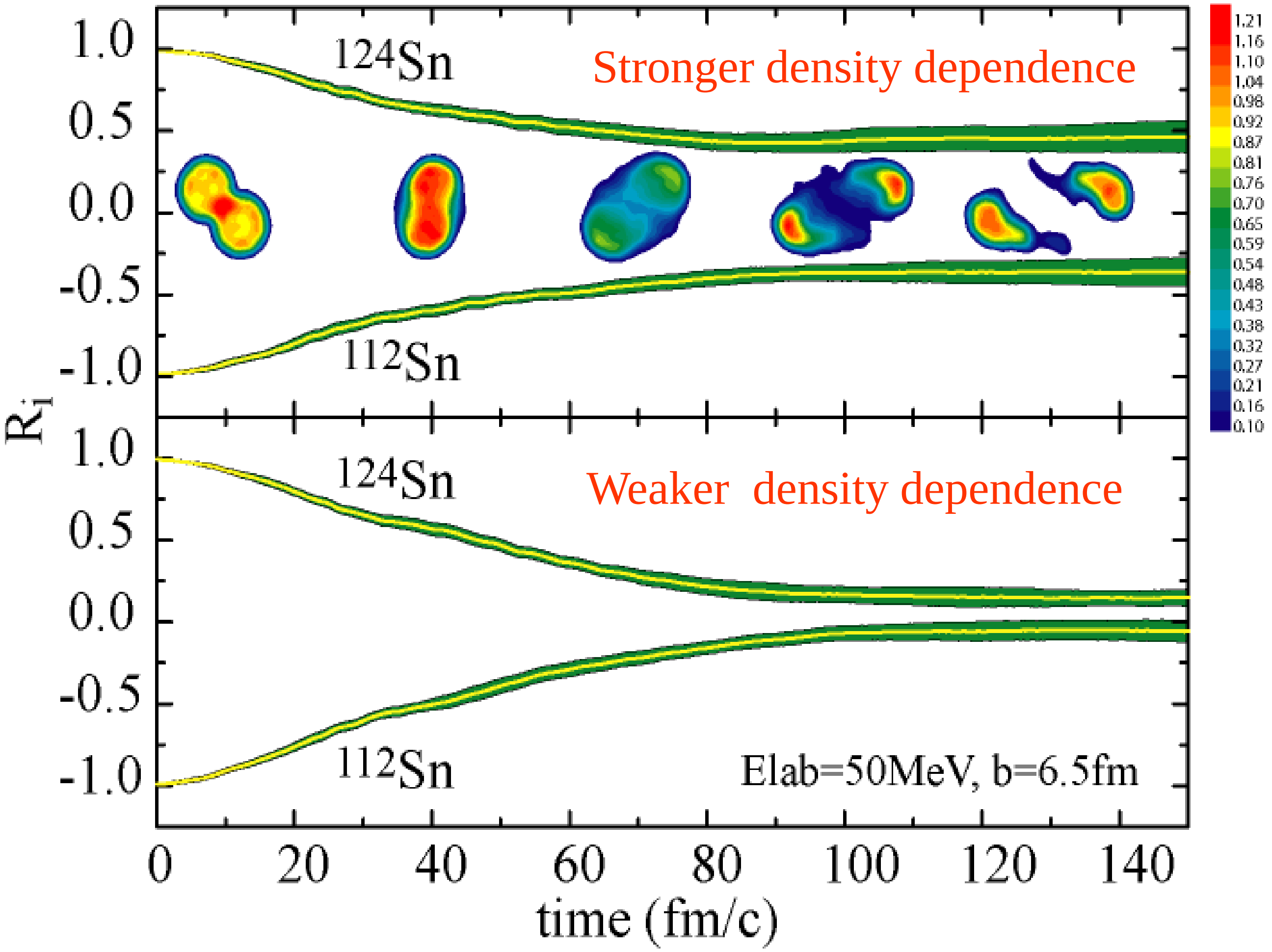
Example:



Sensitivity to symmetry energy

$$R_i(\delta) = 2 \cdot \frac{\delta - (\delta_{Neutron-rich} + \delta_{Proton-rich})/2}{\delta_{Neutron-rich} - \delta_{Proton-rich}}$$

- The asymmetry of the spectators can change due to diffusion, but it also can changed due to pre-equilibrium emission.
- The use of the isospin transport ratio $R_i(\delta)$ isolates the diffusion effects:



Lijun Shi, thesis

Tsang et al., PRL92(2004)

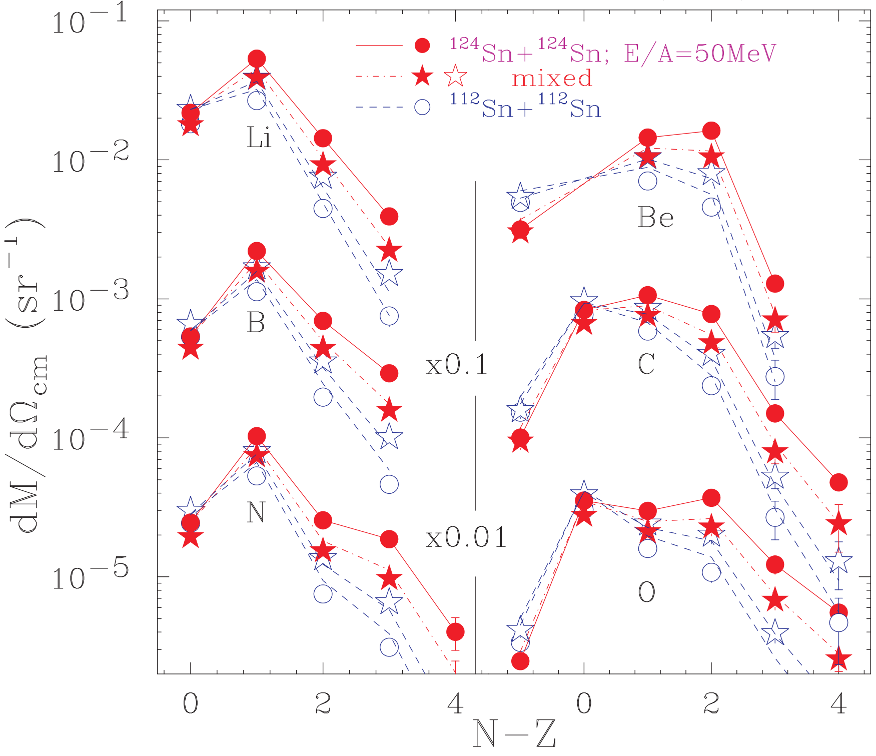
Probing the asymmetry of the Spectators

- The main effect of changing the asymmetry of the projectile spectator remnant is to shift the isotopic distributions of the products of its decay

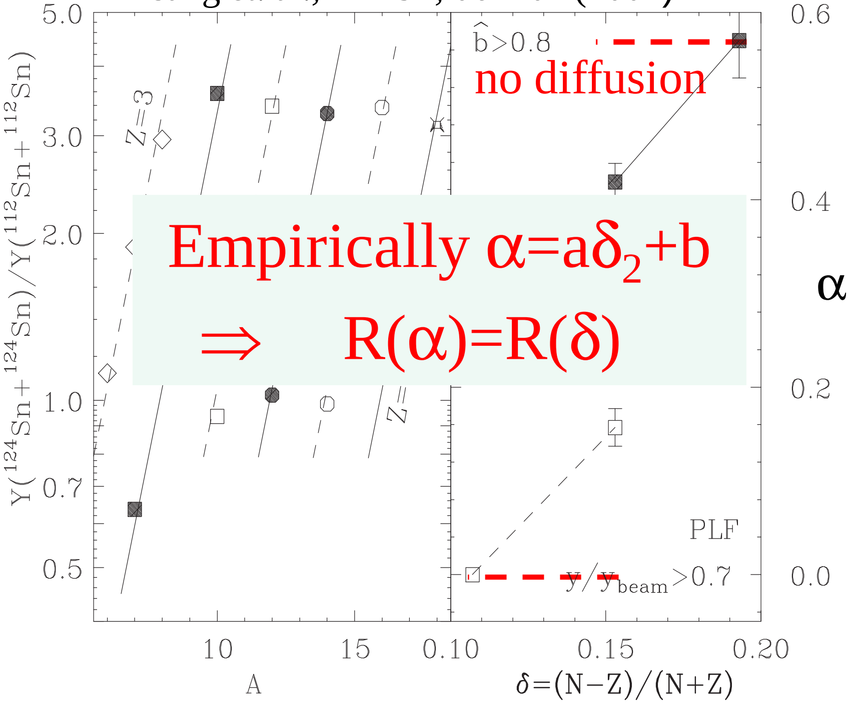
- This can be described by the isoscaling parameters α and β:

$$\frac{Y_2(N,Z)}{Y_1(N,Z)} = C\exp(\alpha N + \beta Z)$$

Liu et al.PRC 76, 034603 (2007).



Tsang et. al.,PRL 92, 062701 (2004)



no diffusion

Empirically $\alpha = a\delta_2 + b$

$\Rightarrow$ R(α)=R(δ)

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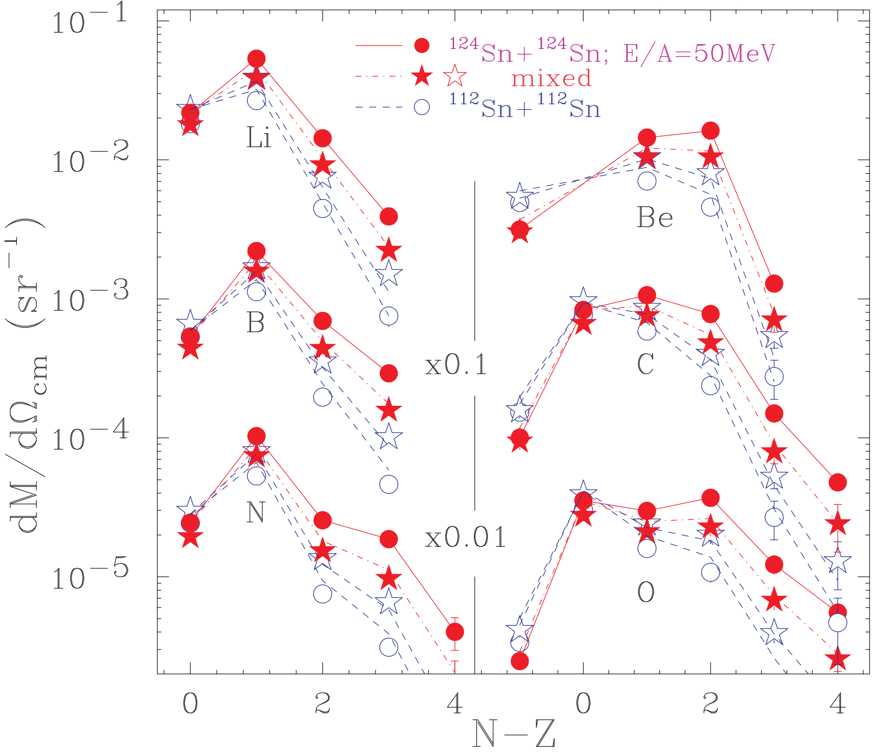
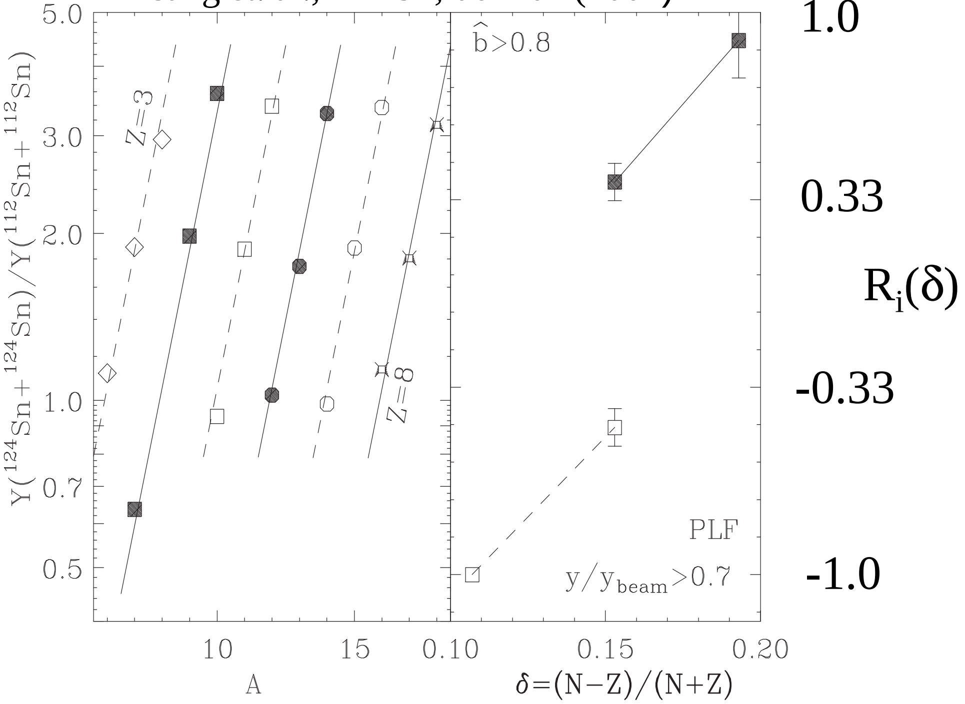


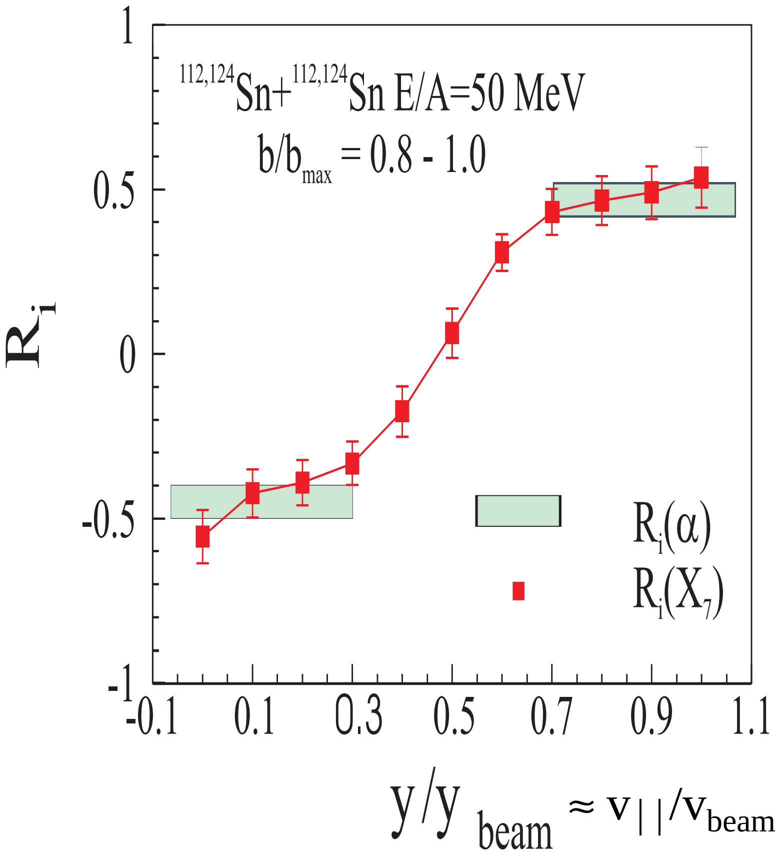
Tsang et. al.,PRL 92, 062701 (2004)



Quantitative values

- Gates were set on the values for $R_i(\alpha)$ near beam rapidity.
  - $R_i(\alpha) \approx 0.47 \pm 0.05$ for $^{124}Sn+^{112}Sn$
  - $R_i(\alpha) \approx -0.44 \pm 0.05$ for $^{112}Sn+^{124}Sn$
- Obtained similar values for $R_i(\ln(Y(^7Li)/Y(^7Be))$
  - Allows exploration of dependence on rapidity and transverse momentum.
  - Observe very little p_T dependence

Liu et al.PRC 76, 034603 (2007). c



Comparison to ImQMD calculations

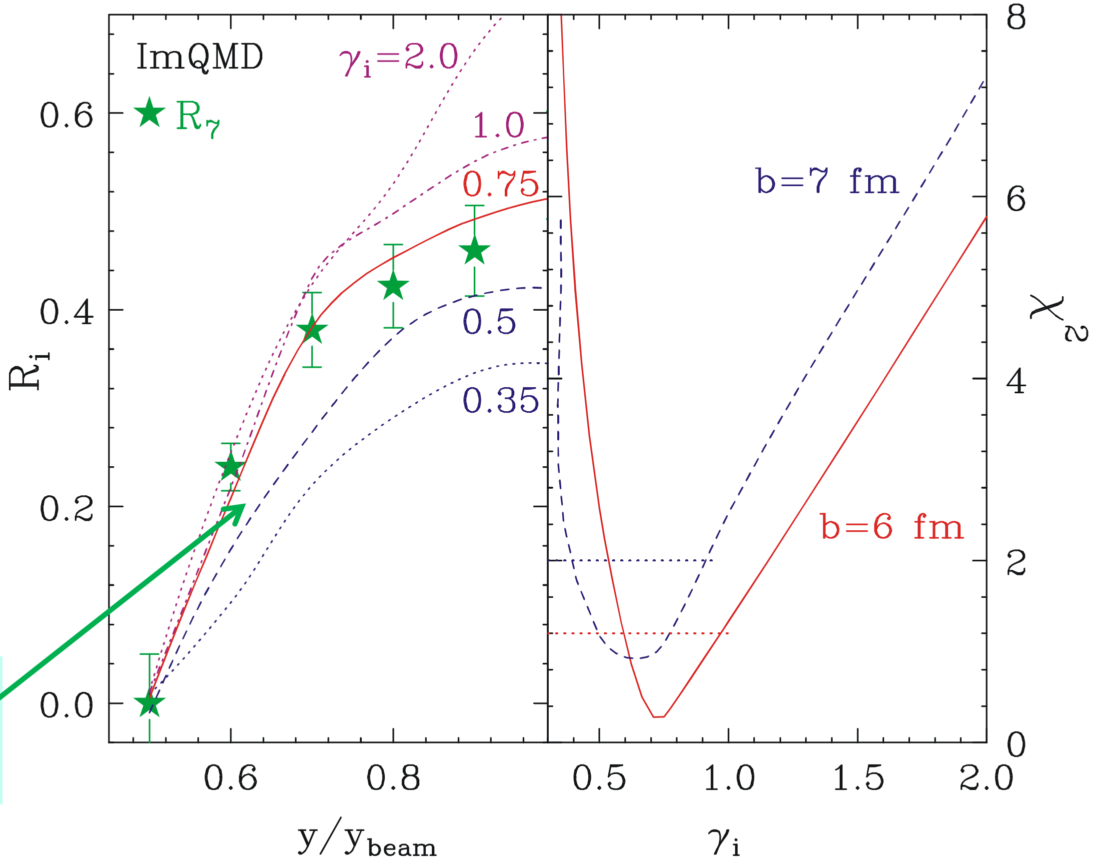
(Yinxun Zhang and Zhuxia Li)

- ImQMD calculations were performed for γ_i=0.35-2.0, S_{int}=17.6 MeV.
- Momentum dependent mean fields with $m_n^*/m_n = m_p^*/m_p = 0.7$ were used. Symmetry energies: $S(\rho) \approx 12.3\cdot(\rho/\rho_0)^{2/3} + 17.6\cdot(\rho/\rho_0)^{\gamma_i}$

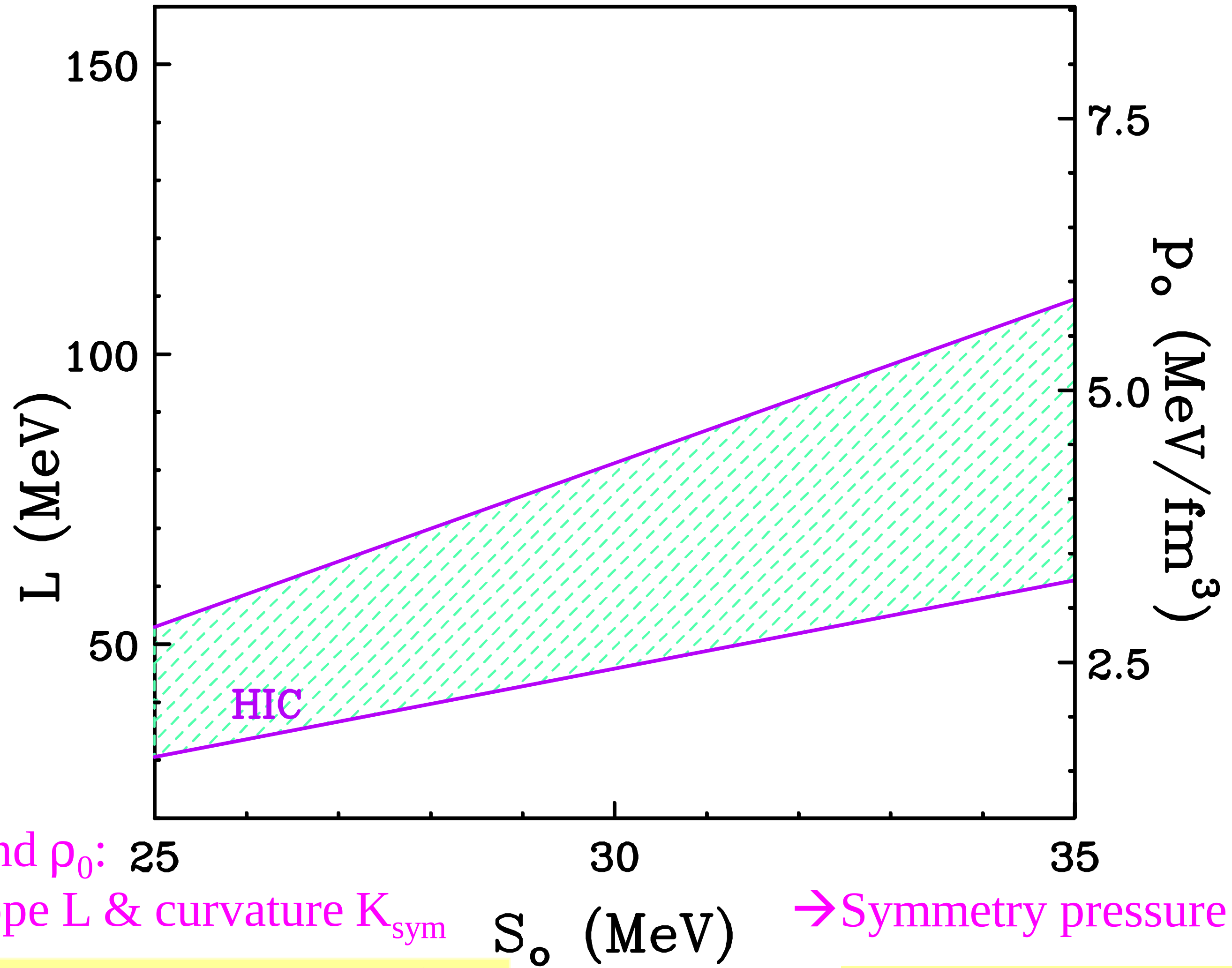
- Experiment samples a range of impact parameters
  - b≈5.8-7.2 fm.
  - larger b, smaller γ_i
  - smaller b, larger γ_i

mirror nuclei requires ability to calculate "fragments"

Tsang et al., PRL 102, 122701 (2009).



Diffusion is sensitive to S(0.4ρ), which corresponds to a contour in the (S_0, L) plane.



Expansion around ρ_0:

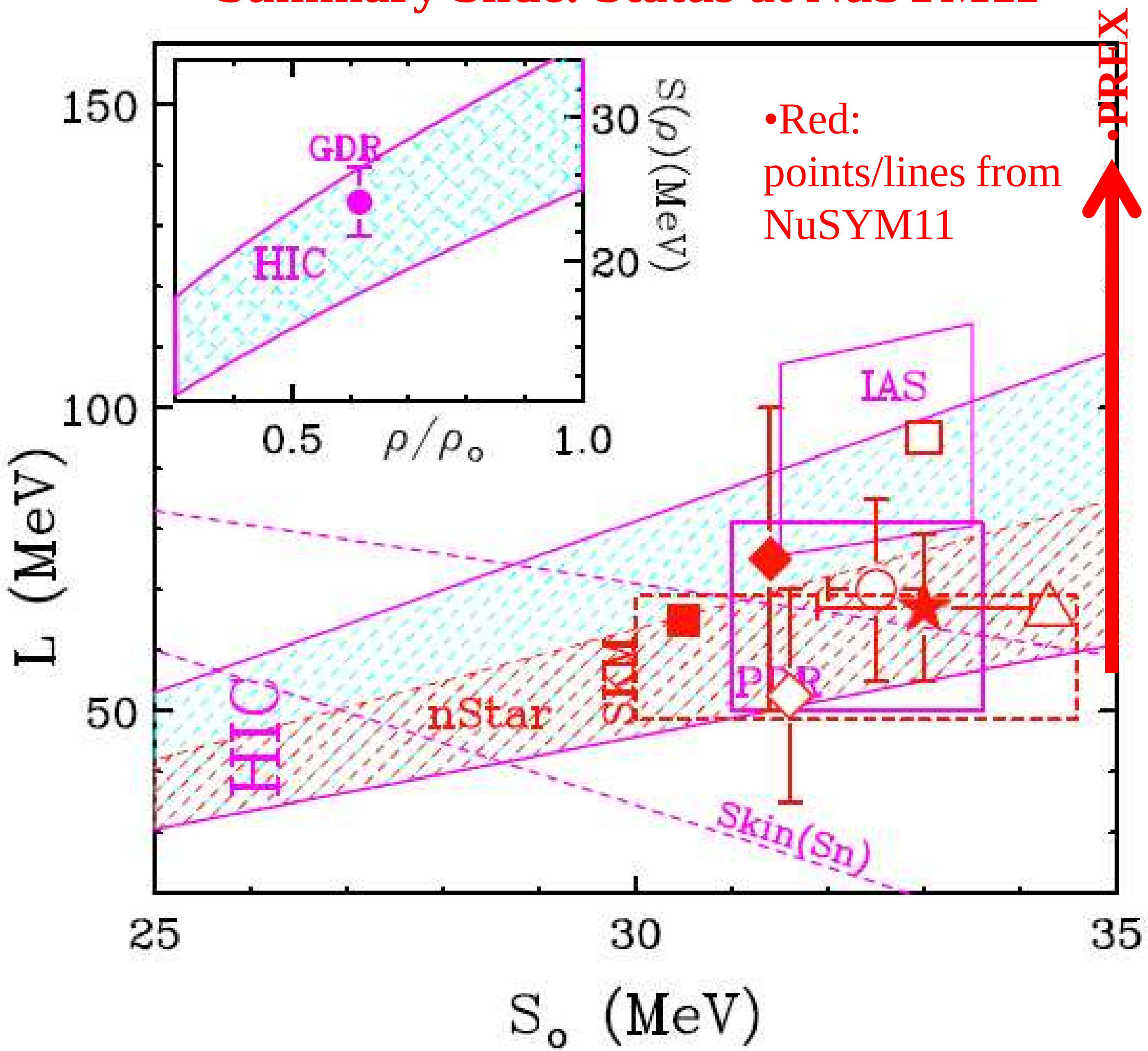
→Symmetry slope L & curvature K_{sym}

$$E_{sym} = S_o + \frac{L}{3}\left(\frac{\rho_B - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho_B - \rho_0}{\rho_0}\right)^2 + \ldots$$

→Symmetry pressure P_{sym}

$$L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho_B}\bigg|_{\rho_B=\rho_0} = \frac{3}{\rho_0} P_{sym}$$

•Summary Slide: Status at NuSYM11



•Red: points/lines from NuSYM11

- •Open circle (Moller):
- •FRDM
- •Star (Murakami):
- • ^{204}Pb, ^{206}Pb, ^{208}Pb
- •Squares (Kohley):
- •t/3He flow
- •nStar (Steiner):
- •Gamma<0.65
- •Diamonds (Roca-Maza): n-skin from antiproton probes
- •SKM (Stone):
- •Skyme Interactions
- •Triangle (Vidana): BHF

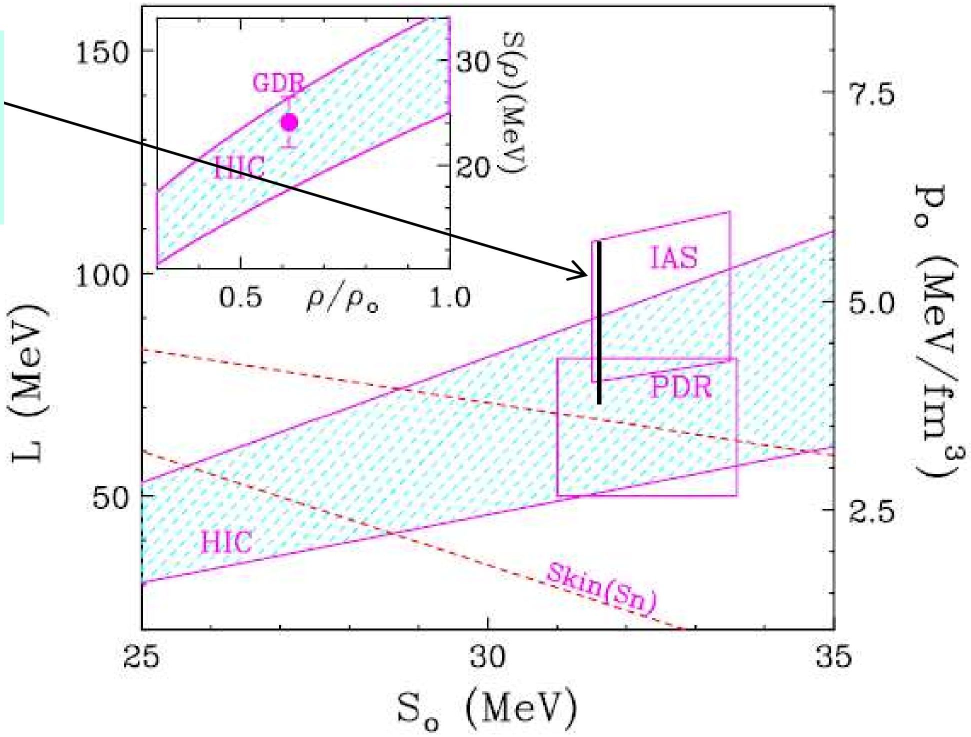
Intermediate Summary

- We have some significant constraints on the symmetry energy at subsaturation density.
  - Can expect these constraints to become more stringent.
- How can measurements make the reactions constraints more stringent?
  - Do better experiments and measure more observables.
- What do we need from theory?
  - We need the various transport models to have minimum set of standard options: standard mean field parameterizations –including momentum dependence, in-medium cross sections, output files.
    - Important for code verification.
    - Important to understand the origins of the calculated effects.
  - Need to constrain in-medium cross sections.
  - Need to improve the treatment of cluster production or find observables that are insensitive to it.
  - Need timely response to discrepancies between calculations.

Example: Constraints at $\rho<\rho_0$

M.B. Tsang, Prog. Part. Nucl. Phys 66, 400 (2011)

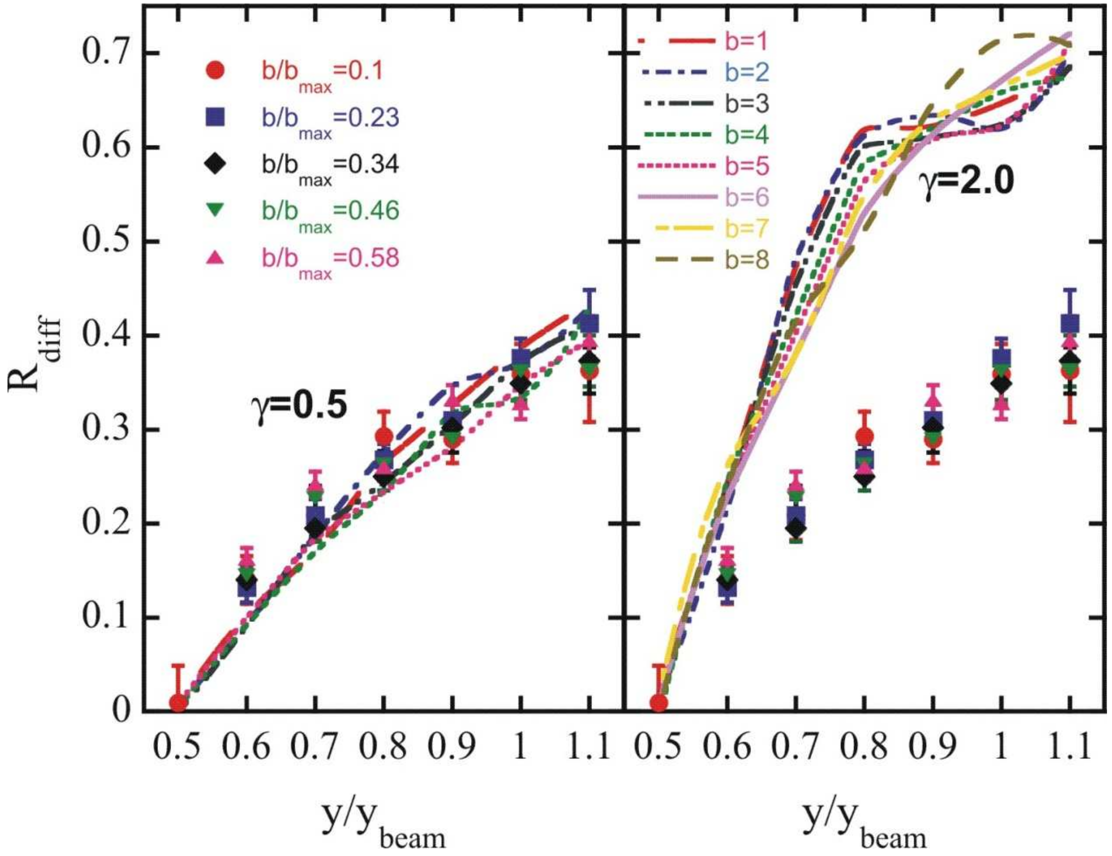
Isospin diffusion constraints by Li and Chen



- What additional observables can we measure?
- How can we improve the experiments?
- Can we understand the model dependencies?

Recent isospin diffusion measurements of Sn+Sn collisions at E/A =35 MeV and comparisons of ImQMD calculations

Z.Y. Sun **Phys. Rev. C 82, 051603(R) (2010)**



No complete stopping or isospin equilibration in central collisions. Greater stopping occurs at E/A=50 MeV

Data are in reasonable agreement with γ_i~0.5, consistent with E/A=50 MeV data.

Part of a program to improve the constraints on the symmetry energy at ρ≈1/2ρ involving new measurements of isospin diffusions and n/p ratios, as well as new transport calculations to explore the model dependence of such constraints.

Understanding theoretical predictions and their limitations: Model types and codes

Boltzmann-Uehling-Uhlenbeck	**Molecular Dynamics**
Many test particles / nucleon	One particle / nucleon, with finite width
Fragments from mean-field instabilities → suppressed for many test particles / nucleon	Fragments from N-body correlations
Collision rearranges test particles → smaller fluctuations	Collision rearranges whole nucleon → larger fluctuations
Partial Pauli blocking of test particles → less restrictive	Pauli blocking of whole nucleons → more restrictive

	Light clusters	**Isovector Momentum Dependence**
ImQMD05	N-body correlations	No
pBUU	A < 4	No
IBUU04	No	Yes

Example: In-Medium NN Cross Sections

- “Screened” geometric cross section
  - Danielewicz (pBUU)

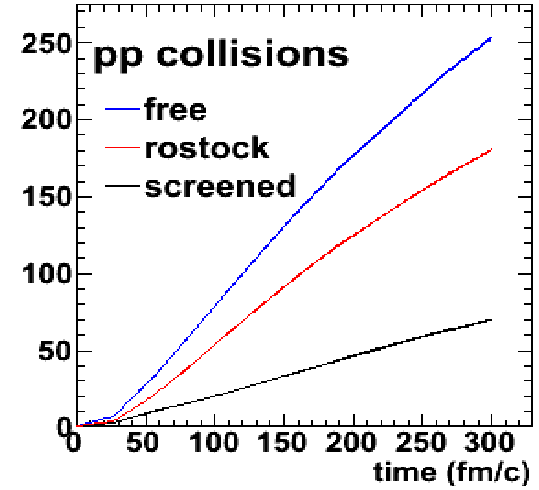
$$\sigma = \sigma_0 \tanh\left(\sigma_{\text{free}} / \sigma_0\right)$$

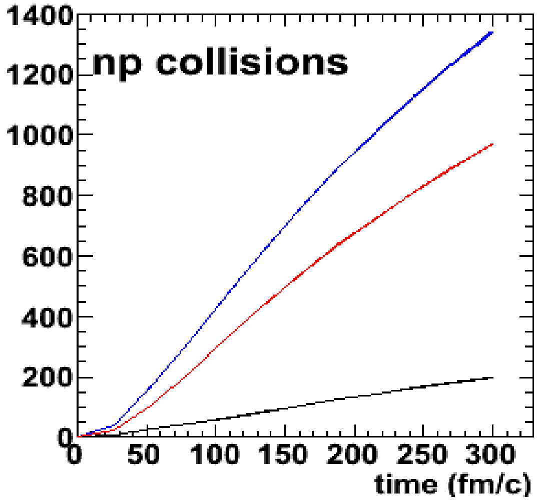
$$\sigma_0 = y\rho^{=2/3}, y = 0.85$$

- “Rostock”: parameterized results of BHF calc.
  - Similar to reductions in IBUU04, IMQMD05.

$$\sigma = \sigma_{\text{free}} \exp \quad -0.6\frac{\rho}{\rho_0}\frac{1}{1+\left(T_{\text{C.M.}} / 150\text{MeV}\right)^2}$$

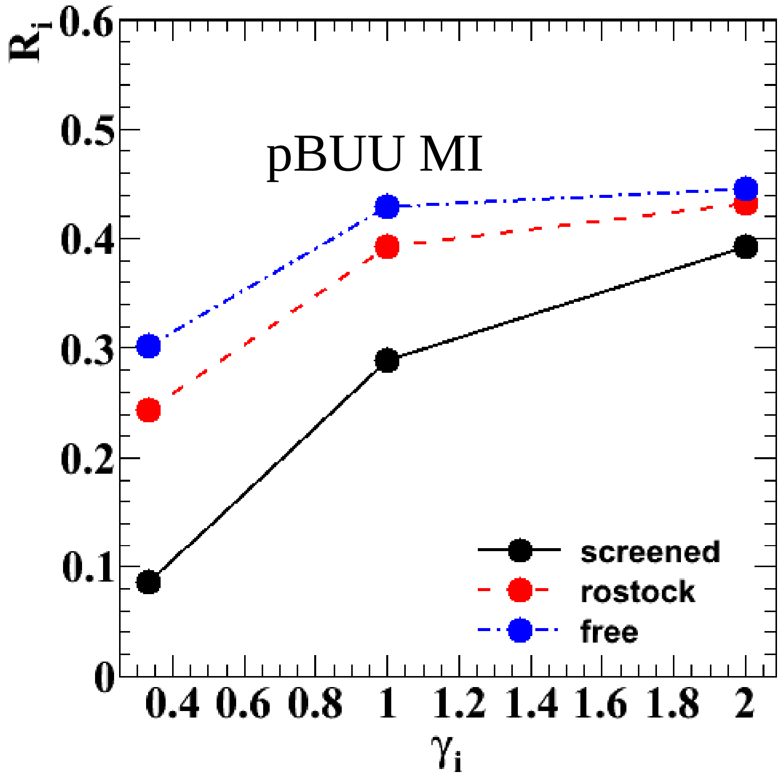
- Both give similar viscosity (similar effects) at E/A>400 MeV



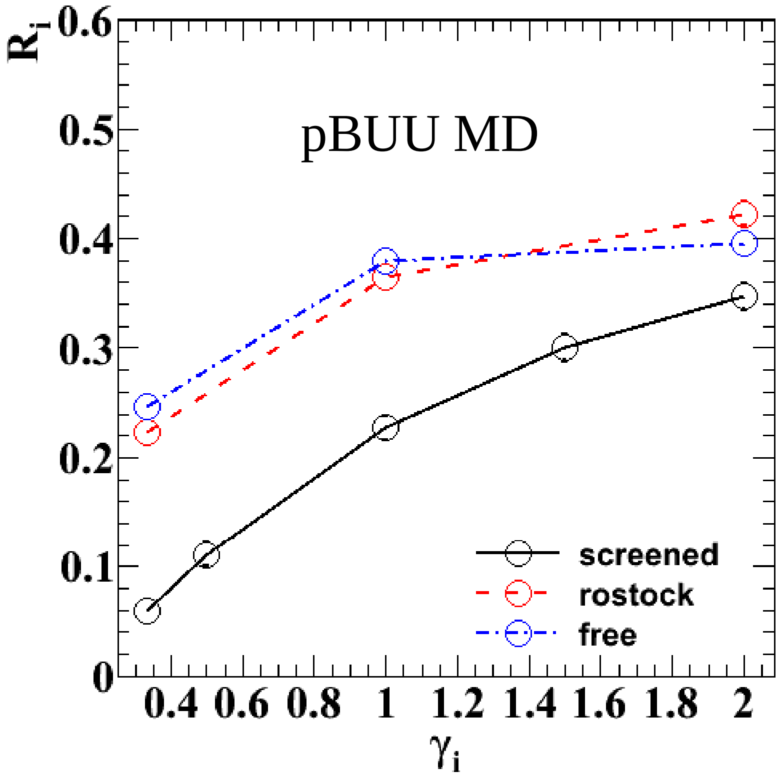


D.D.S.Coupland, et al.,arXiv:1107.3709

Cross section dependencies for different models



D.D.S.Coupland, et al.,arXiv:1107.3709



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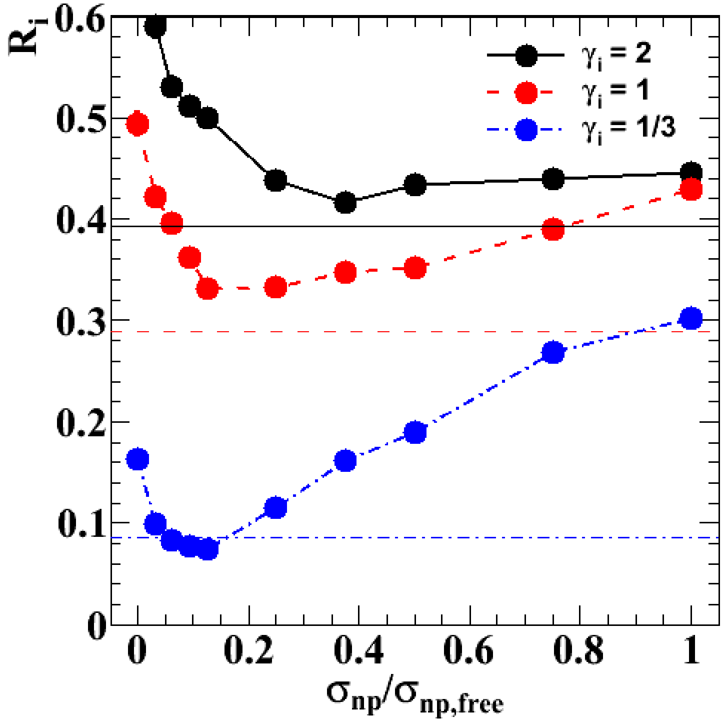
- pBUU – Strong dependence on cross section, influenced by momentum dependence. Screened reduces all σ_{nn}, σ_{pp} and σ_{np} to a small and similar value at moderate density.
- IBUU04 – Similar to pBUU Rostock
- ImQMD –studies done with different constrains on cross section dependence
  - Existing study with cross sections fixed to maintain constant collision rate ⇒ no observed dependence on isospin dependence of cross sections

Influence of Collisions on Diffusion



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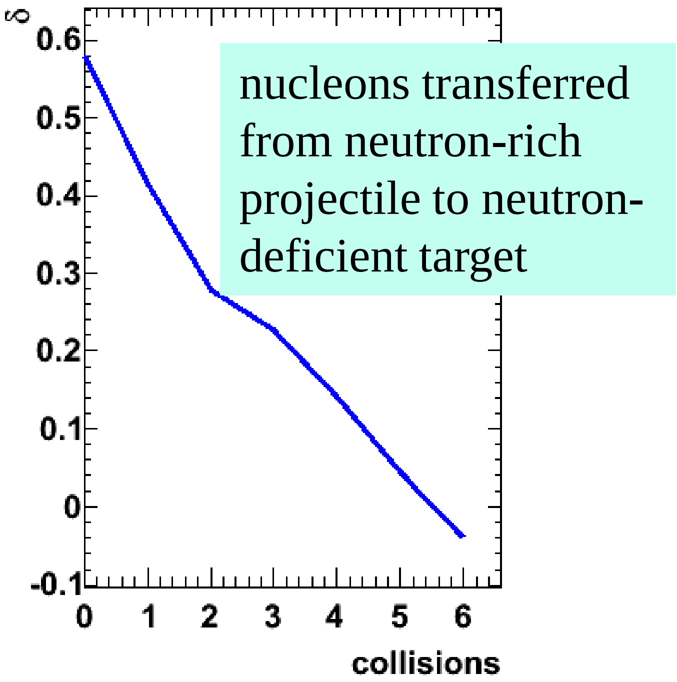


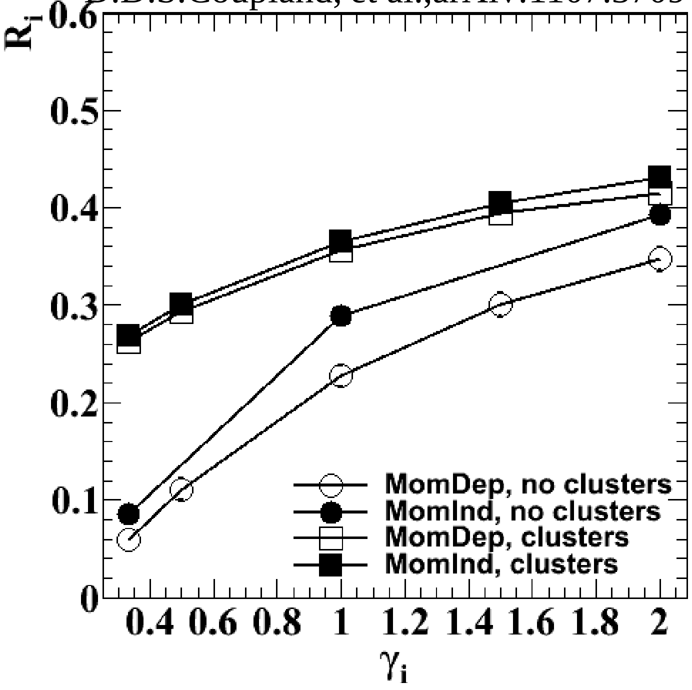
nucleons transferred from neutron-rich projectile to neutron-deficient target

- Behavior at large cross section consistent with mean – free path.
- Only np cross section causes a significant change in the diffusion
- Collisions reduce the diffusion caused by the symmetry energy
- Collisions reduce the asymmetry of the exchanged nucleons

Cluster production

- Test particles can undergo inelastic collisions and form clusters.
    - E.g. Three nucleons collide. Two fuse to form deuteron and the remaining one escapes, conserving 4 momentum.
- Not a native feature of BUU models
- Included in the pBUU code as inverse of breakup up through mass 3

D.D.S.Coupland, et al.,arXiv:1107.3709



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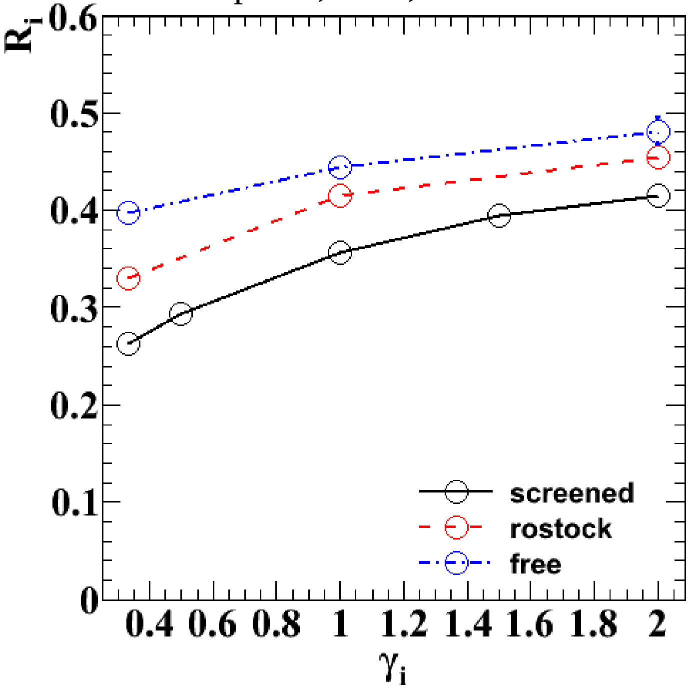
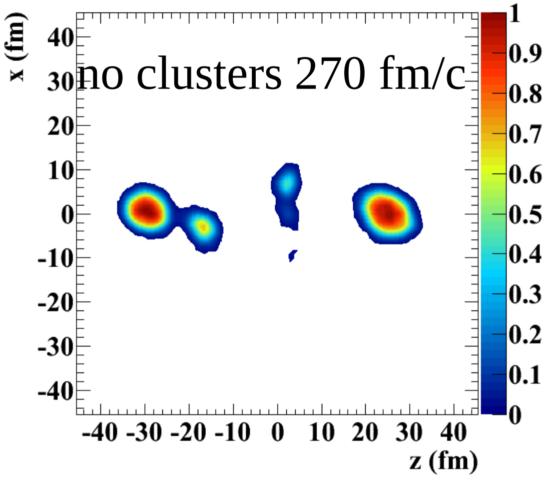
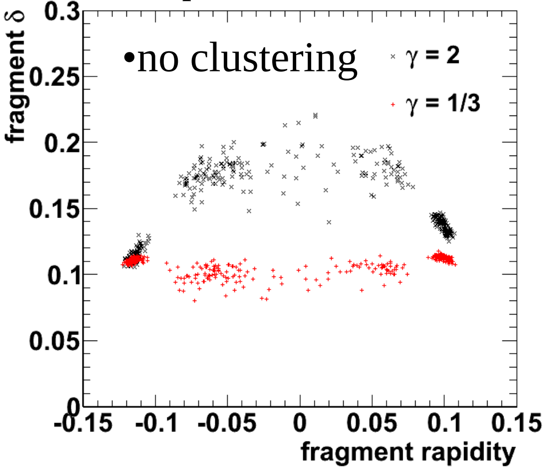


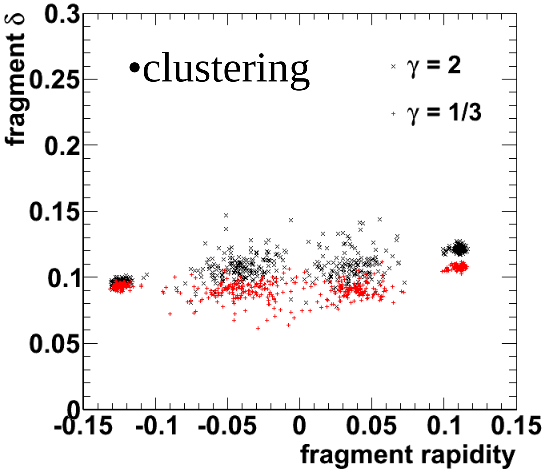
Clustering effects on dynamics

- Increases mean field instabilities → more violent neck breakup
- Additional NN collision phase space – larger cross section
- Without clusters, neck tends to be much more asymmetric than large residues. With clusters, neck is roughly the same asymmetry.
  - Important experimental objective: Compare heavy residue asymmetry with that of neck fragments.
- Mass 3 clusters are overproduced by factor of 3 relative to experiment.
- We need to include the alpha particle to understand this better.
- Cluster effects are also very pronounced in central collisions (not shown).



D.D.S.Coupland, et al.,arXiv:1107.3709

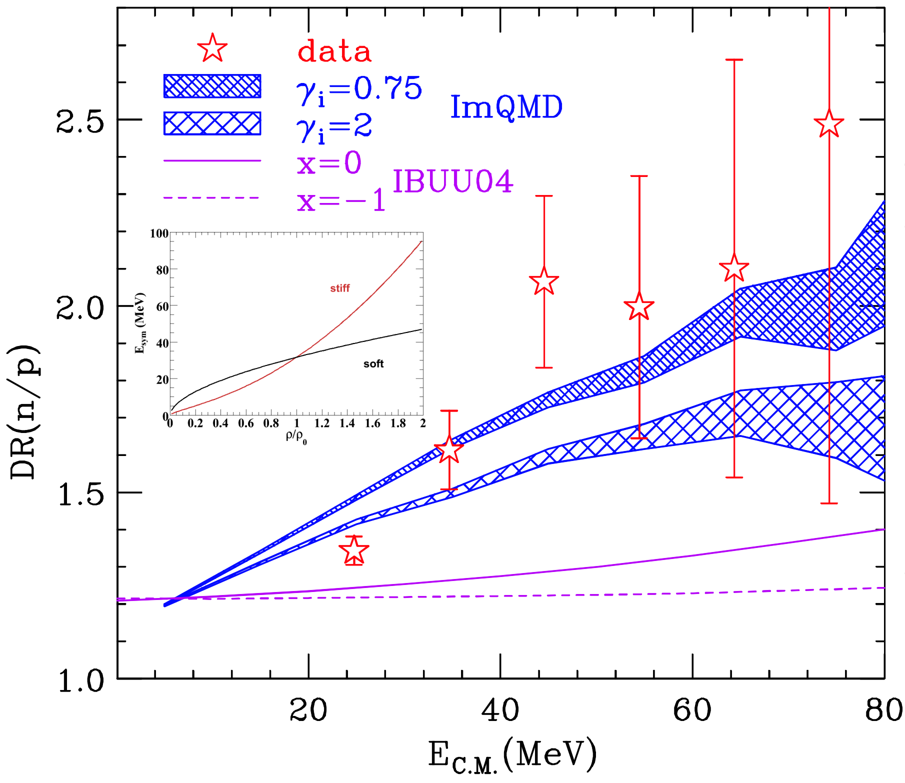




Another observable: Neutron/Proton Double Ratios

- How it works: Symmetry energy expels neutron excess;
  - Soft symmetry energy larger during expansion.
- The ImQMD and IBUU04 calculations don’t agree
  - IBUU04 assumes $m_n^* > m_p^*$. Is this why?
  - What are m_n^*, m_p^* from DOM; with uncertainties?
  - Need the ability to set m_n^* and m_p^*; as a standard option.
- Data have large error bars
  - New data are being analyzed

$$DR(n/p) = \frac{Y(n)/Y(p);\ {}^{124}Sn + {}^{124}Sn}{Y(n)/Y(p);\ {}^{112}Sn + {}^{112}Sn}$$



•Y. Zhang, Phys. Lett. B 664, 145 (2008)y

Summary and Outlook

- The density dependence of the symmetry energy is of fundamental importance to neutron stars.
- Heavy ion collisions provide unique possibilities to probe the EOS of dense asymmetric matter.
- Calculations suggest a number of promising observables that can probe the density dependence of the symmetry energy.
  - Isospin diffusion, isotope ratios, n/p spectral ratios, GMR, Pigmy and Giant Dipole provide some constraints at $\rho \leq \rho_{0,}$.
  - π^+ vs. π^- production, neutron/proton spectra and flows may provide constraints at $\rho \approx 2\rho_0$ and above. This is the key motivation for using nucleus-nucleus collisions.
- The availability of fast stable and rare isotope beams at a variety of energies at MSU, RIKEN and GSI allows the exploration of the symmetry energy at a range of densities.

What do we need from theory?

- We need the various transport models to have minimum set of standard options: standard mean field parameterizations –including momentum dependence, in-medium cross sections, output files.
  - Important for code verification.
  - Important to understand the origins of the calculated effects.
- Need to constrain in-medium cross sections.
- Need to improve the treatment of cluster production or find observables that are insensitive to it.
- Need timely response to discrepancies between calculations.
  - There are big discrepancies between the predictions for the symmetry energy dependence of pion production for example.
  - These important questions need resolution.