Probing the density dependence of nuclear symmetry energy

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- **Relationship between the symmetry energy and nucleon optical potential**
- **Why is the density dependence of nuclear symmetry energy so uncertain ?**
- **Attempts to constrain the symmetry energy from transport model studies of heavy-ion reactions**

What is the Equation of State of neutron-rich nuclear matter?

$$E_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial \delta^2} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

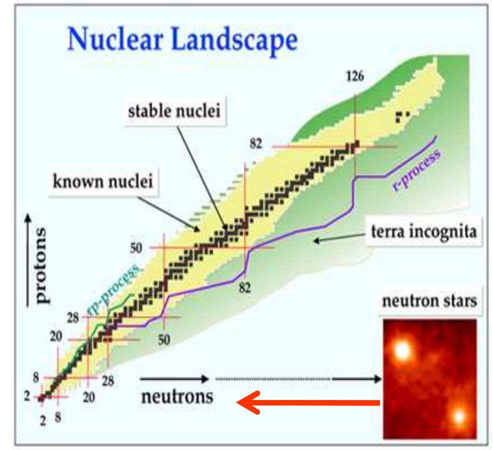
symmetry energy

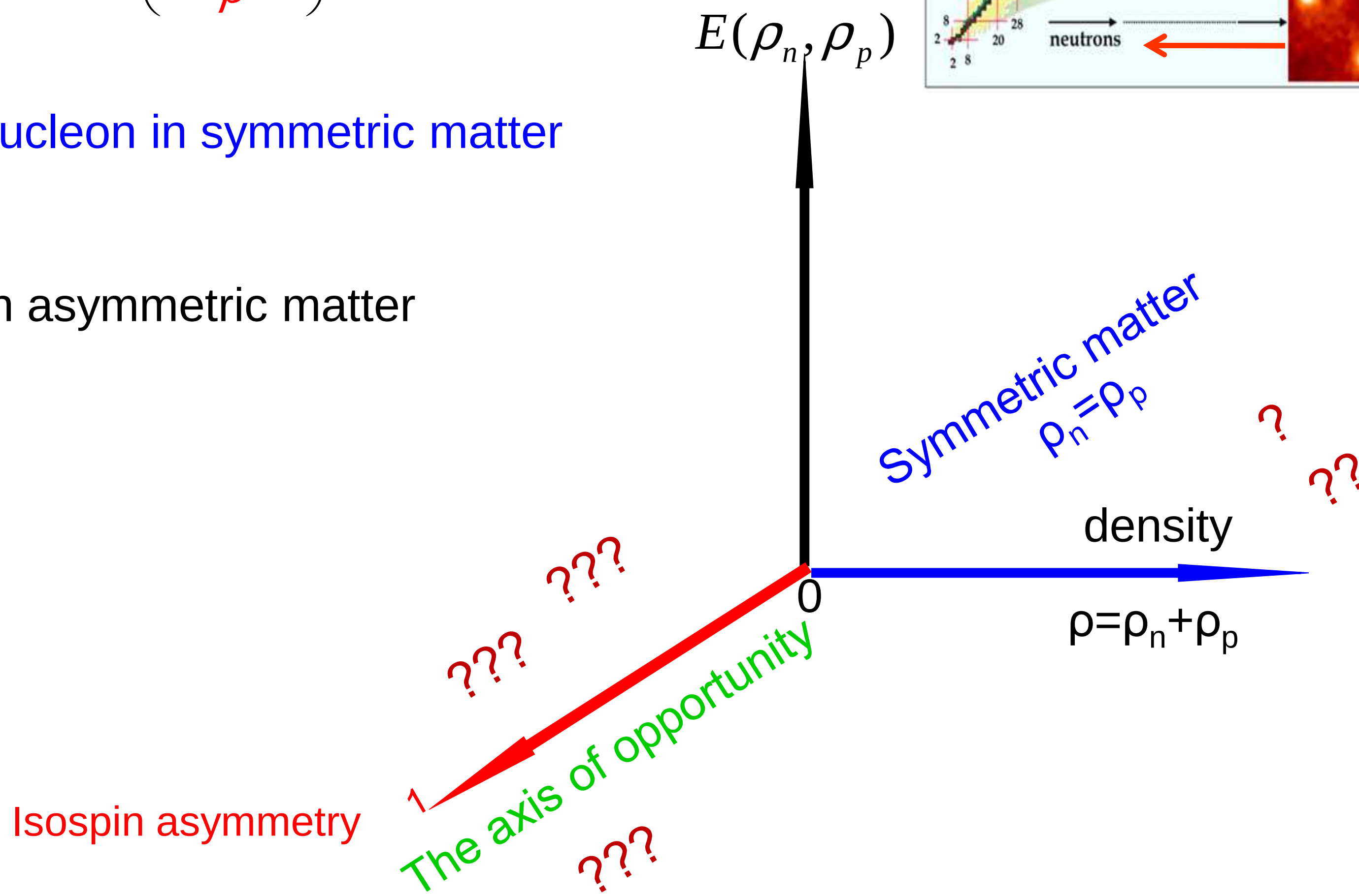
Isospin asymmetry δ

$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{sym}(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric matter

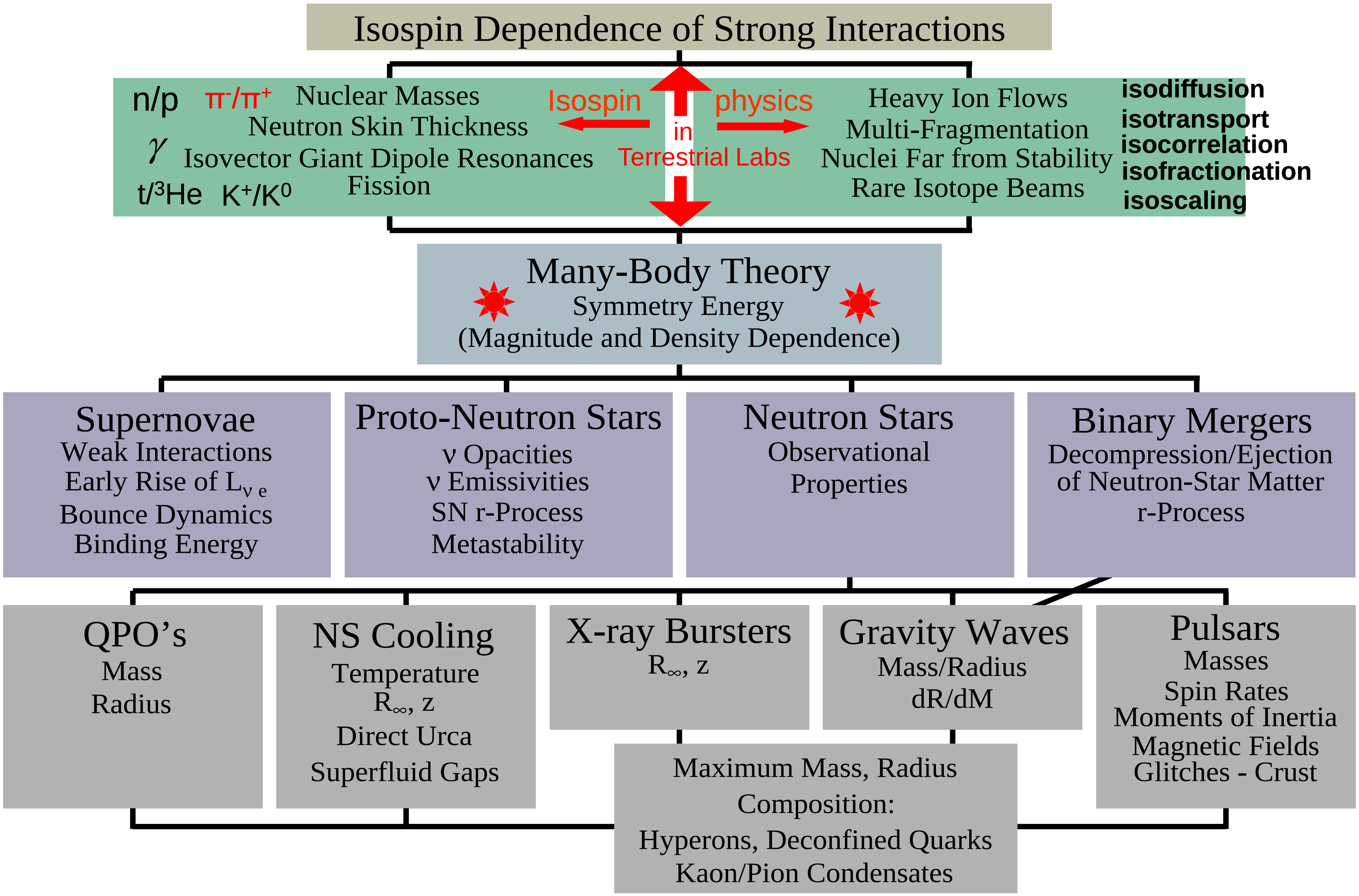
Energy per nucleon in asymmetric matter





The multifaceted influence of the isospin dependence of strong interaction and symmetry energy in nuclear physics and astrophysics

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, *Phys. Rep. 411, 325 (2005).*



Nuclear Structure Near the Limits of Stability (INT-05-3)
September 26 to December 2, 2005

Organizers: Erich Ormand, George Bertsch,
Jonathan Engel, Witek Nazarewicz

2) What are the constraints on the nuclear energy density functional?

Another goal is to understand connections between the symmetry energy and isoscalar and isovector mean fields, and in particular the influence of effective mass and pair correlations on symmetry energy versus the isospin. Such understanding will allow us to better determine isospin corrections to nuclear mean fields and energy density functionals.

http://www.int.washington.edu/PROGRAMS/dft

Relation between the symmetry energy and the mean-field potential

Lane potential $U_{n/p} = U_0 \pm U_{sym}\delta$

kinetic isoscalar isovector

Symmetry energy $E_{sym}(\rho) = \frac{1}{6}\frac{\partial(t+U_0)}{\partial k}|_{k_F} k_F + \frac{1}{2}U_{sym}(\rho, k_F)$

Isoscarlar effective mass $m^*/m = [1 + \frac{m}{k_F}\frac{\partial U_0}{\partial k}|_{k_F}]^{-1}$

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m^*} + \frac{1}{2}U_{sym}(\rho, k_F)$$

Using K-matrix theory

K. A. Brueckner and J. Dabrowski, Phys. Rev. **134**, B722 (1964); J. Dabrowski and P. Haensel, Phys. Lett. B **42**, 163 (1972); Phys. Rev. C **7**, 916 (1973); Can. J. Phys. **52**, 1768 (1974). J. Dabrowski, Physics Letters 8, 90 (1964)

Symmetry energy $E_{sym}(\rho)$ and its density slope $L(\rho)$ at arbitrary density based on the Hugenholtz-Van Hove (HVH) theorem

N. M. Hugenholtz, L. Van Hove, Physica **24**, 363 (1958)

Single-particle potential

Fermi momentum

Kinetic energy

Energy density

$$t(k_F^n) + U_n(\rho, \delta, k_F^n) = \frac{\partial \xi}{\partial \rho_n},$$

$$t(k_F^p) + U_p(\rho, \delta, k_F^p) = \frac{\partial \xi}{\partial \rho_p},$$

valid for any interactin[g]
is independent of the p[...]

Isovector effective mass

$$\frac{m_n^* - m_p^*}{m} = -2\delta \frac{m}{\hbar^2 k_F} \frac{dU_{\rm sym}}{dk}\bigg|_{k_F} \Big/ \left(1 + 2\frac{m}{\hbar^2 k_F}\frac{dU_0}{dk}\bigg|_{k_F}\right)$$

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m^*} + \frac{1}{2}U_{sym}(\rho, k_F)$$

Slope at arbitrary density ρ

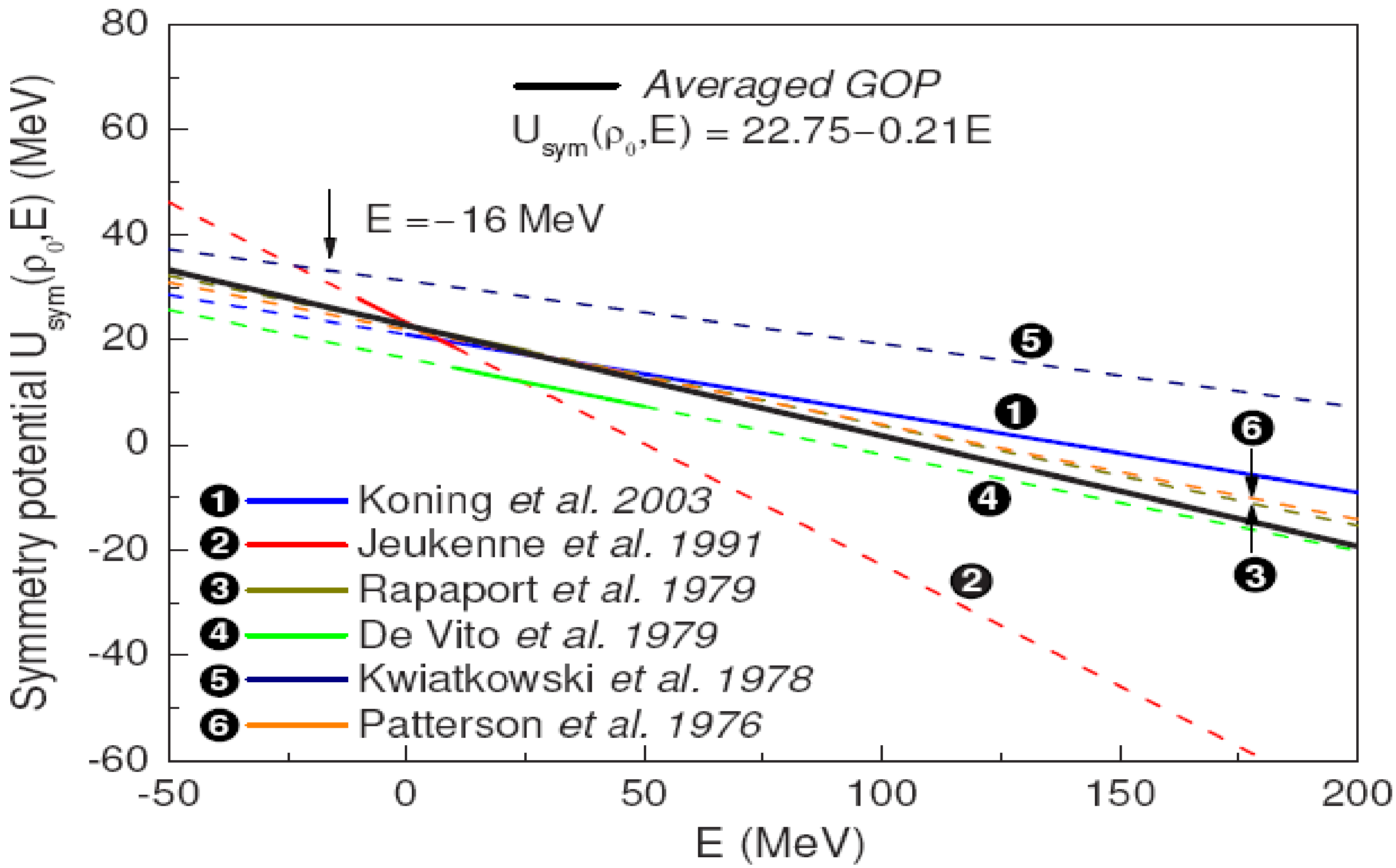
$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m^*} + \frac{3}{2}U_{sym}(\rho, k_F) + \frac{\partial U_{sym}}{\partial k}|_{k_F} k_F$$

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, arXiv:1004.4403, NPA 865, 1 (2011).

Symmetry potential at saturation density from global nucleon optical potentials

Systematics based on world data accumulated since 1969:

(1) Single particle energy levels from pick-up and stripping reactions
(2) Neutron and proton scattering on the same target at about the same energy
(3) Proton scattering on isotopes of the same element
(4) (p,n) charge exchange reactions



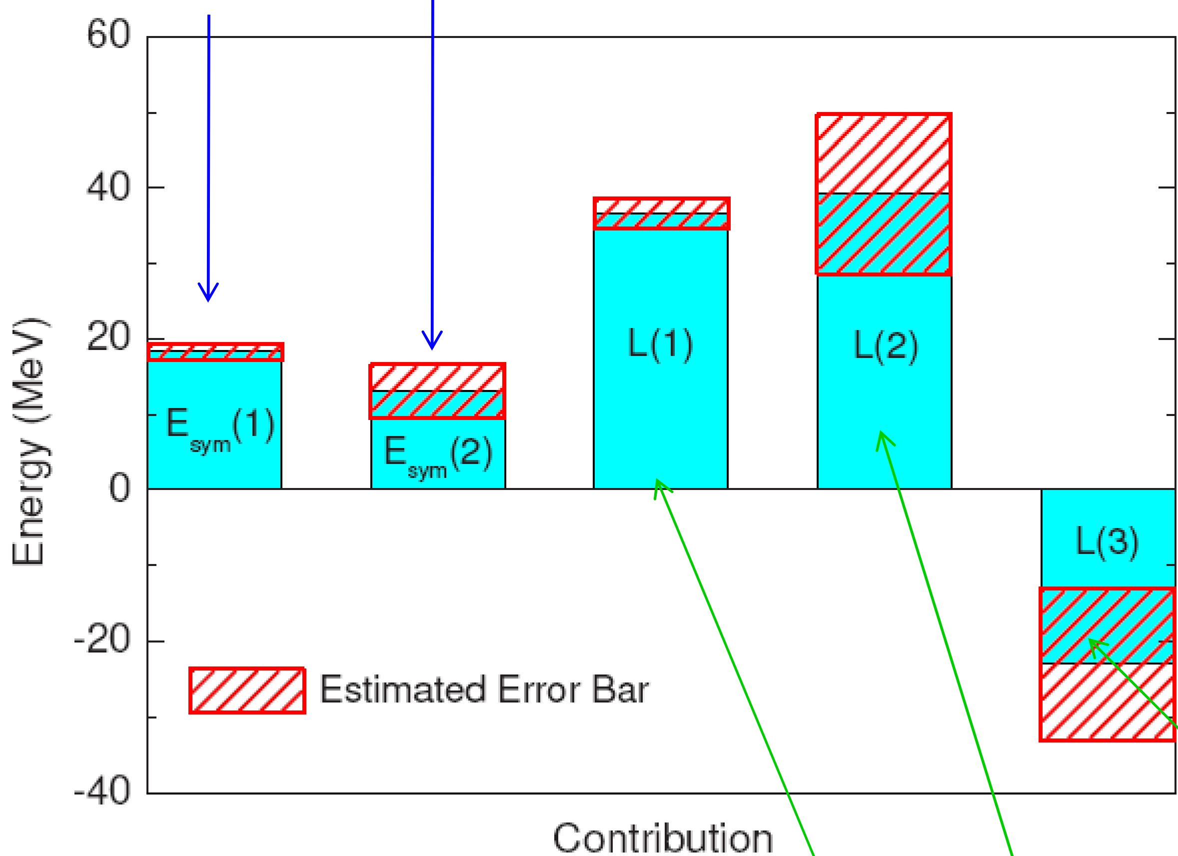
Constraining the symmetry energy near saturation density using global nucleon optical potentials

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L(\rho_0)}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + O\left(\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right)$$

C. Xu, B.A. Li and L.W. Chen, PRC 82, 054606 (2010).

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m^*} + \frac{1}{2}U_{sym}(\rho, k_F)$$

$$E_{sym}(\rho_0) = 31.3 \text{ MeV} \pm 4.5 \text{ MeV}$$



$$(m_n^* - m_p^*)/m = 0.32\delta \pm 0.15\delta$$

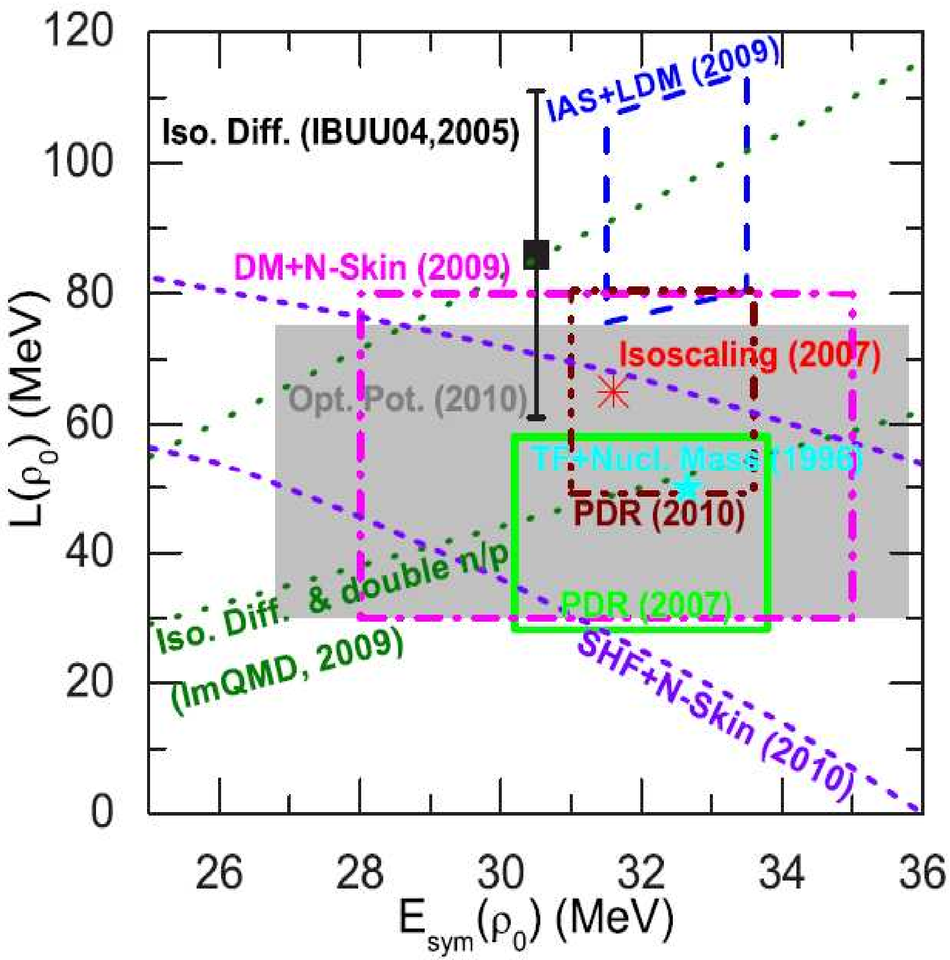
$$L(\rho_0) = 52.7 \text{ MeV} \pm 22.5 \text{ MeV}$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m^*} + \frac{3}{2}U_{sym}(\rho, k_F) + \frac{\partial U_{sym}}{\partial k}\Big|_{k_F} k_F$$

Constraints extracted from experimental data using various models

GOP: global optical potentials (Lane potentials)
C. Xu, B.A. Li and L.W. Chen, PRC 82, 054606 (2010)

Iso. Diff & double n/p (ImQMD, 2009),
M. B. Tsang et al., PRL92, 122701 (2009).



Iso Diff. (IBUU04, 2005),
L.W. Chen et al., PRL94, 32701 (2005)

IAS+LDM (2009),
Danielewicz and J. Lee, NPA818, 36 (2009)

PDR (2010) of ^{68}Ni and ^{132}Sn,
A. Carbone et al., PRC81, 041301 (2010).

PDR (2007) in ^{208}Pb
Land/GSI, PRC76, 051603 (2007)

SHF+N-skin of Sn isotopes,
L.W. Chen et al., PRC 82, 024301 (2010)

Isoscaling (2007),
D.Shetty et al. PRC76, 024606 (2007)

DM+N-Skin (2009): M. Centelles et al., PRL102, 122502 (2009)

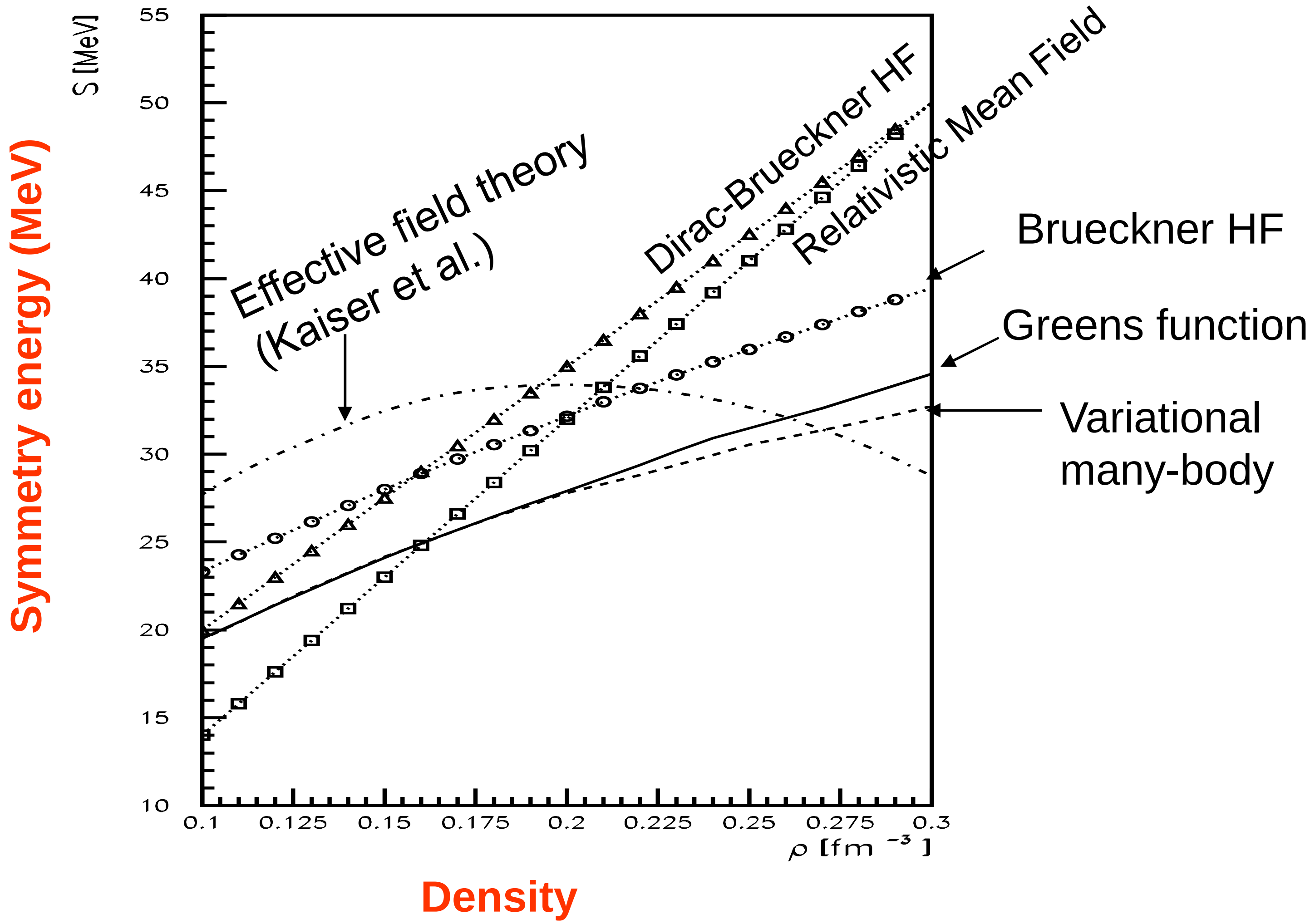
TF+Nucl. Mass (1996), Myers and Swiatecki, NPA601, 141 (1996)

$E_{sym}(\rho_0) \approx 31 \pm 4$ MeV

$L \approx 60 \pm 23$ MeV

$E_{sym}(\rho)$ predicted by microscopic many-body theories

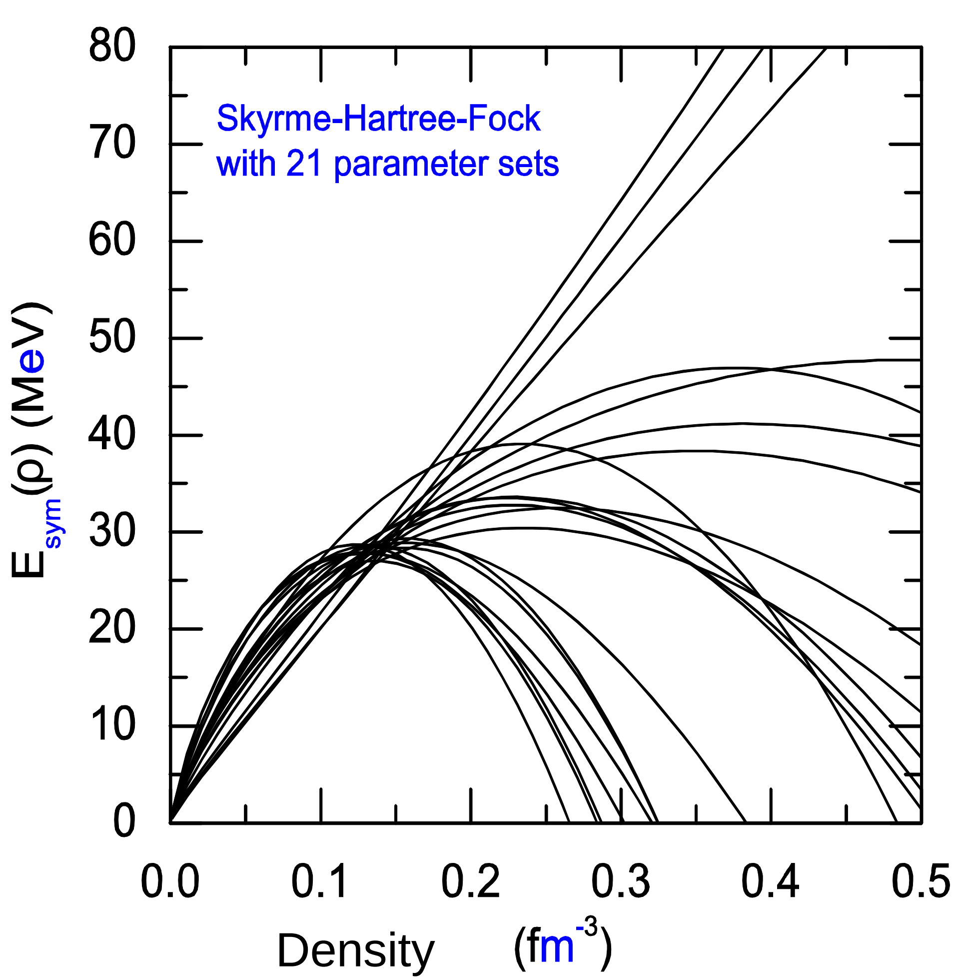
The extracted experimental constraints as of 2011 can't exclude any of these theories

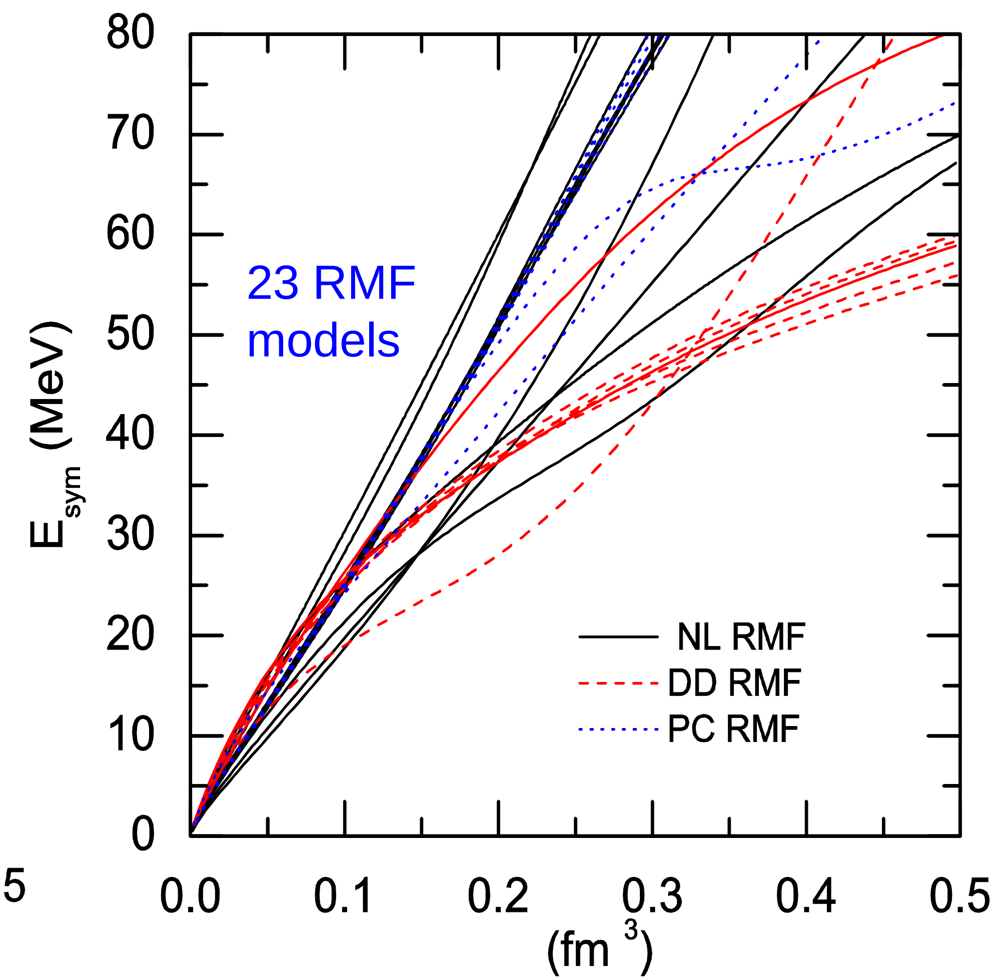


A.E. L. Dieperink et al., Phys. Rev. C68 (2003) 064307

The E_{sym} (ρ) from model predictions using popular effective interactions

Examples:





L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C72, 064309 (2005); C76, 054316 (2007).

Why is the symmetry potential/energy so uncertain?

- Short-range tensor force due to the rho meson exchange
- Isospin-dependence of nucleon-nucleon correlations
- Spin-isospin dependence of the 3-body force
- Isospin dependence of pairing and clustering at low densities

Is the kinetic part of the symmetry energy in nuclear matter really

$$E_{sym}^{kin} = E_{PNM}^{kin} - E_{SNM}^{kin} = (2^{\frac{2}{3}} - 1)(\frac{3}{5}\frac{\hbar^2 k_F^2}{2m}) \propto \rho^{2/3} \ ?$$

(Valid for free Fermi gas, widely used in nuclear and astrophysical applications)

Probably NOT, it may be very small and negative because of the high momentum tail of the single-nucleon momentum distribution in isospin symmetric matter induced by the tensor force existing only between n-p but not n-n and p-p

Tensor force induced isospin dependence of NN correlation and its effects on single-particle momentum distribution

THEORY OF NUCLEAR MATTER

H. A. BETHE[1]
Cornell University, Ithaca, N. Y.

Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

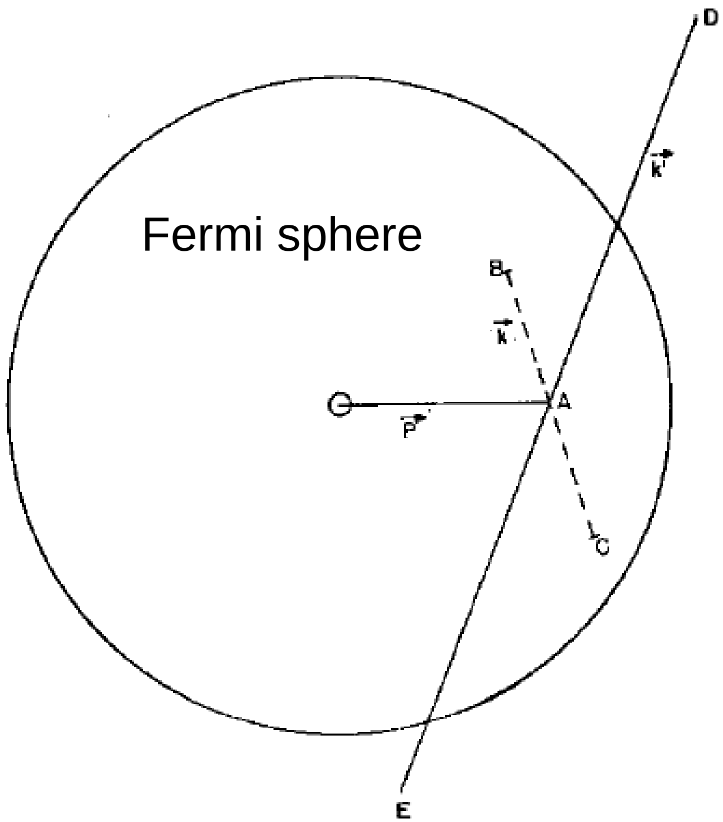
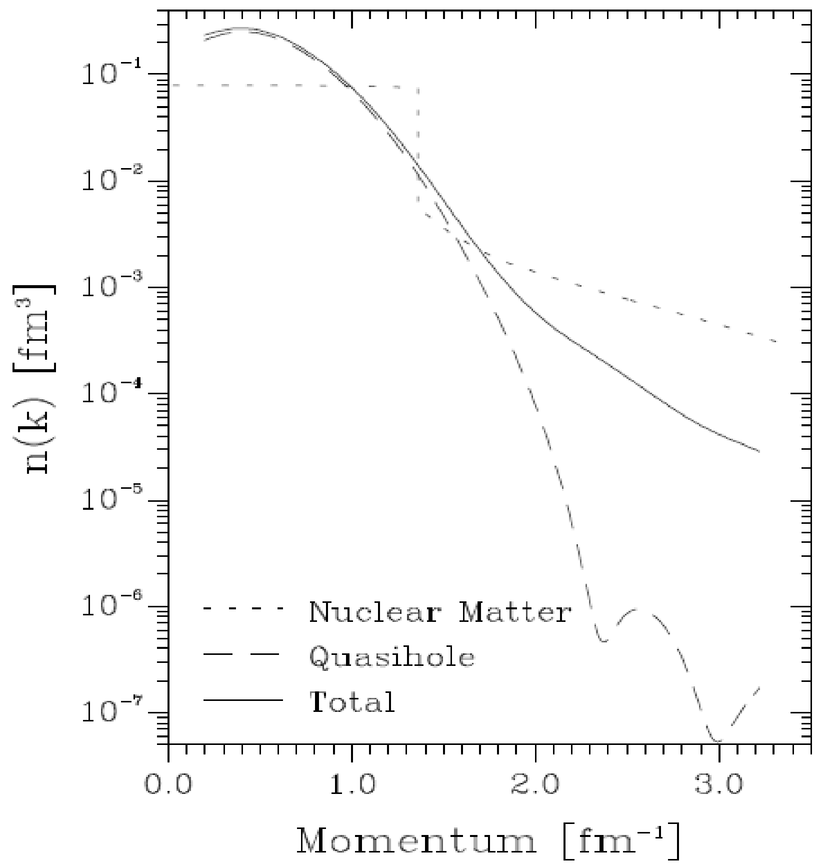


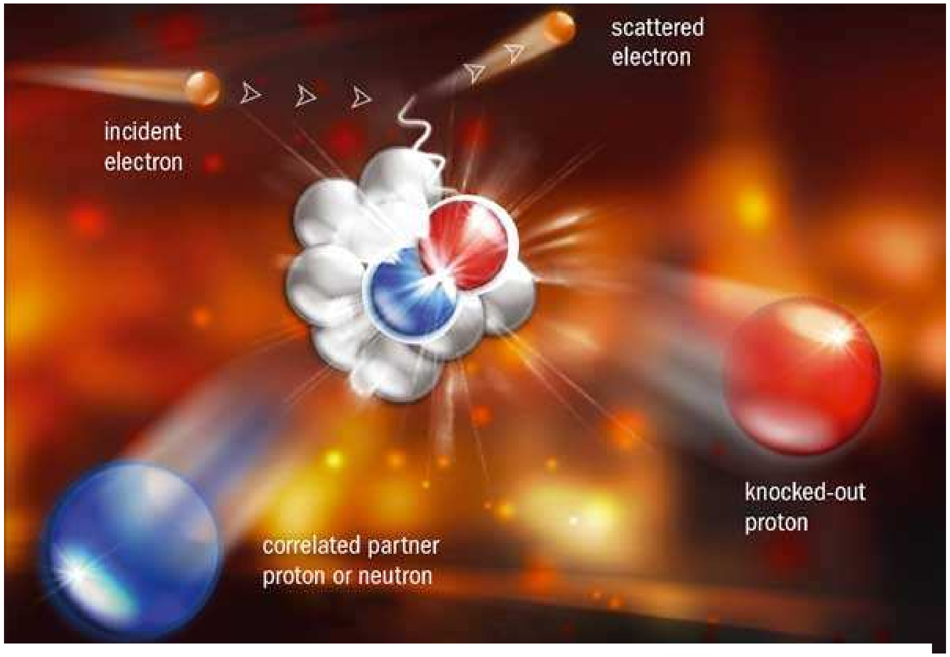
FIGURE 10. Two nucleons are initially in states B and C, having average momentum $\mathbf{P}$ and relative momentum $\mathbf{k}$. When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum $\mathbf{k}'$. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in DE.



Dickhoff, Muether, Neff, Polls, Rios et al

A. N. Antonov, P. E. Hodgson, and I. Zh. Petkov, Nucleon Momentum and Density Distributions in Nuclei (Clarendon Press, Oxford, 1988).

Isospin-dependence of Short Range NN Correlations and Tensor Force



Two-nucleon knockout by an electron

R Subedi et al. Science 320, 1475 (2008)

M. Strikman, CERN Courier Jan 27, 2009

H. Baghdasaryan *et al.* (CLAS ollaboration)
Phys. Rev. Lett. **105**, 222501 (2010)

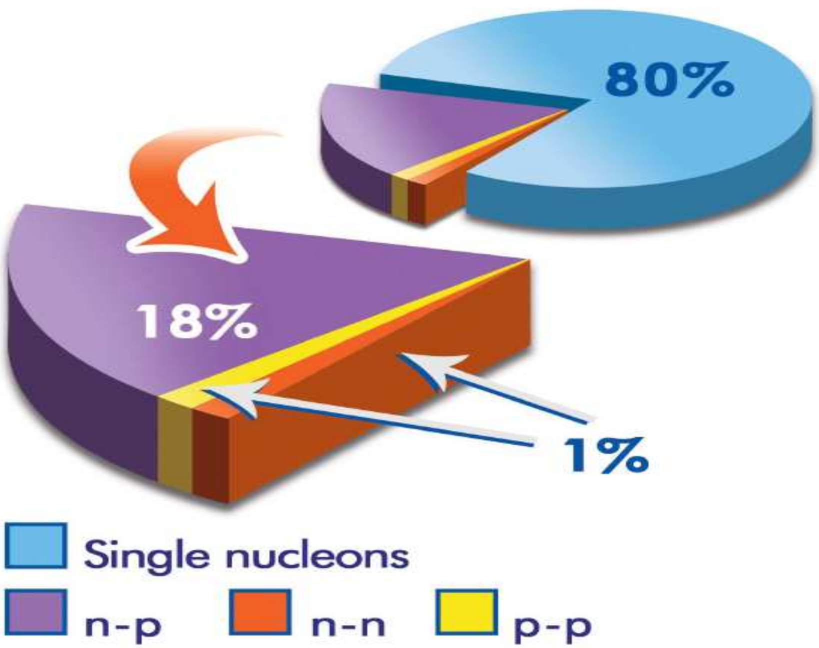


Figure 3: The average fraction of nucleons in the various initial state configurations of ^{12}C.

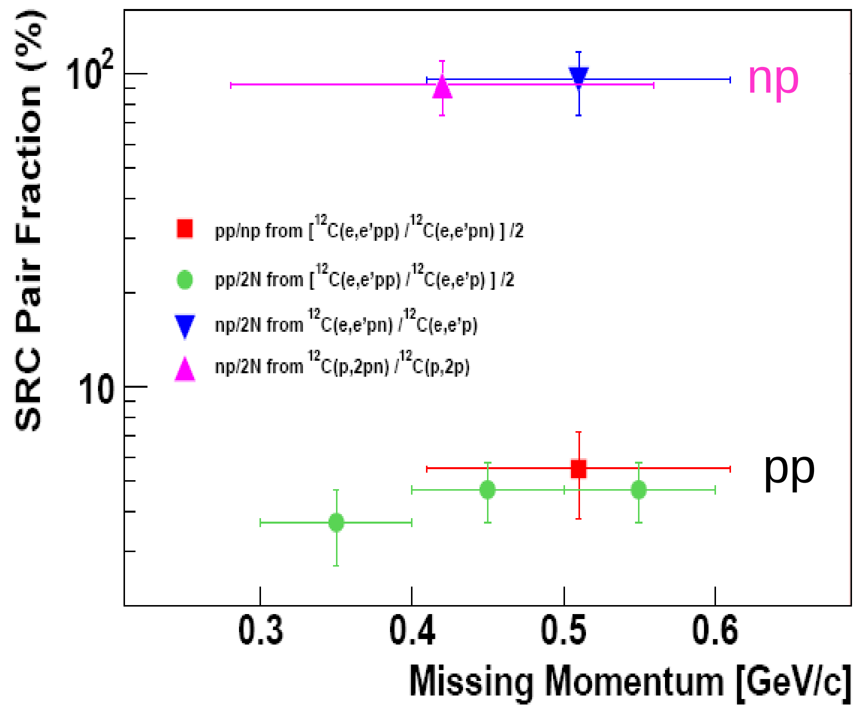


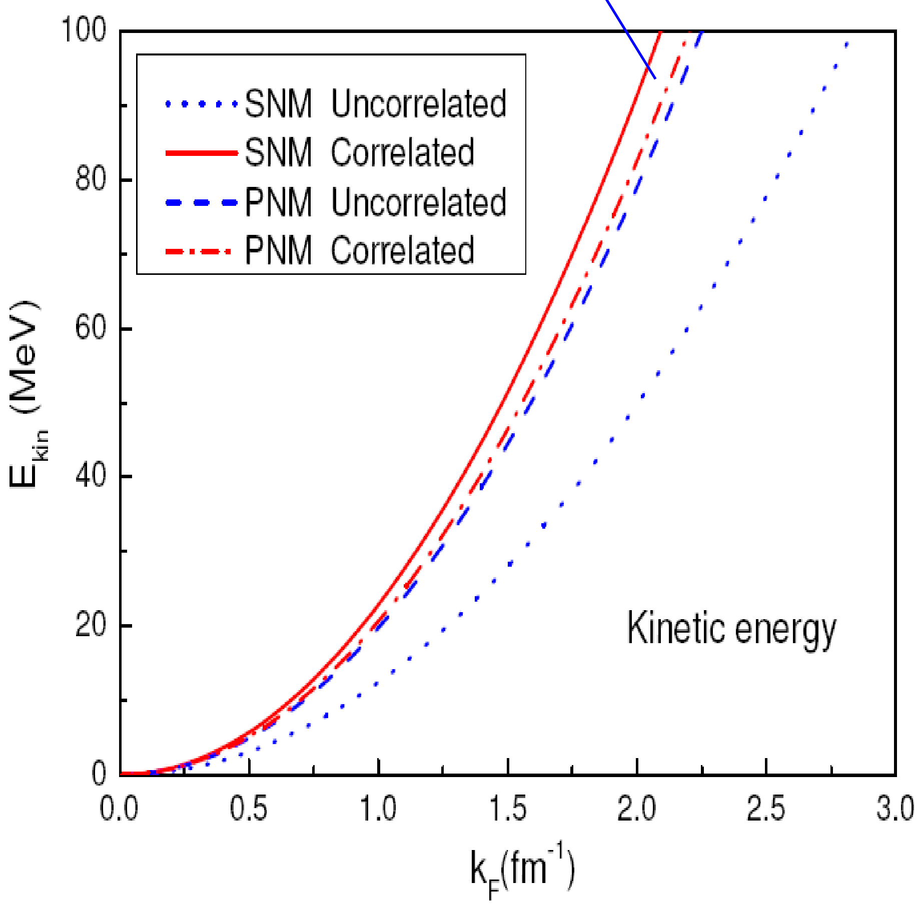
Figure 2: The fractions of correlated pair combinations in carbon as obtained from the (e,e'pp) and (e,e'pn) reactions, as well as from previous (p,2pn) data. The results and references are listed in Table 1.

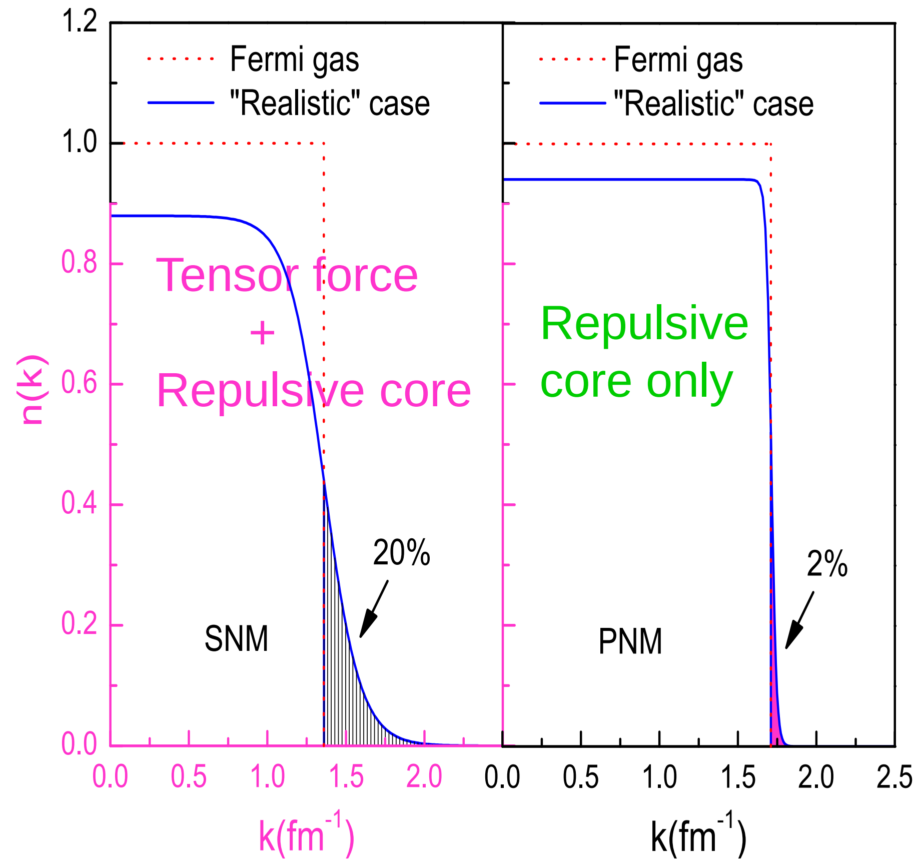
How can the kinetic contribution to the symmetry energy be negative?

Chang Xu and Bao-An Li, arXiv:1104.2075

$$E_{sym}^{kin} = E_{PNM}^{kin} - E_{SNM}^{kin} < 0$$

$$E_{kin} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk,$$





arXiv:1107.5412v1 [nucl-th] 27 Jul 2011
arXiv:1107.5412

Nuclear symmetry energy and the role of the tensor force

Isaac Vidaña[1], Artur Polls[2] and Constança Providência[1]

Using the Hellmann–Feynman theorem
Brueckner–Hartree–Fock approach
V18 potential plus the Urbana IX three-body force.

	E_{NM}	E_{SM}	E_{sym}	L
$\langle T \rangle$	53.321	54.294	−0.973	14.896
$\langle V \rangle$	−34.251	−69.524	35.273	51.604
Total	19.070	−15.230	34.300	66.500

in our calculation). We note that the kinetic contribution to E_{sym} is very small and negative. This is in contrast with the result for a free Fermi gas (FFG), whose contribution at ρ_0 is ~ 14.4 MeV. A similar result has been recently found by Xu and Li [41]. According to these au-

Potential contribution to the symmetry potential/energy is more uncertain

Within an interacting Fermi gas model:
Structure of the nucleus, M.A. Preston and R.K. Bhaduri (1975)

NN correlation functions

$$U_{sym}(k_F,\rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij}) f^{T1}(r_{ij}) - V_{T0}(r_{ij}) f^{T0}(r_{ij})] d^3 r_{ij}$$

Isospin-dependence of strong interaction:

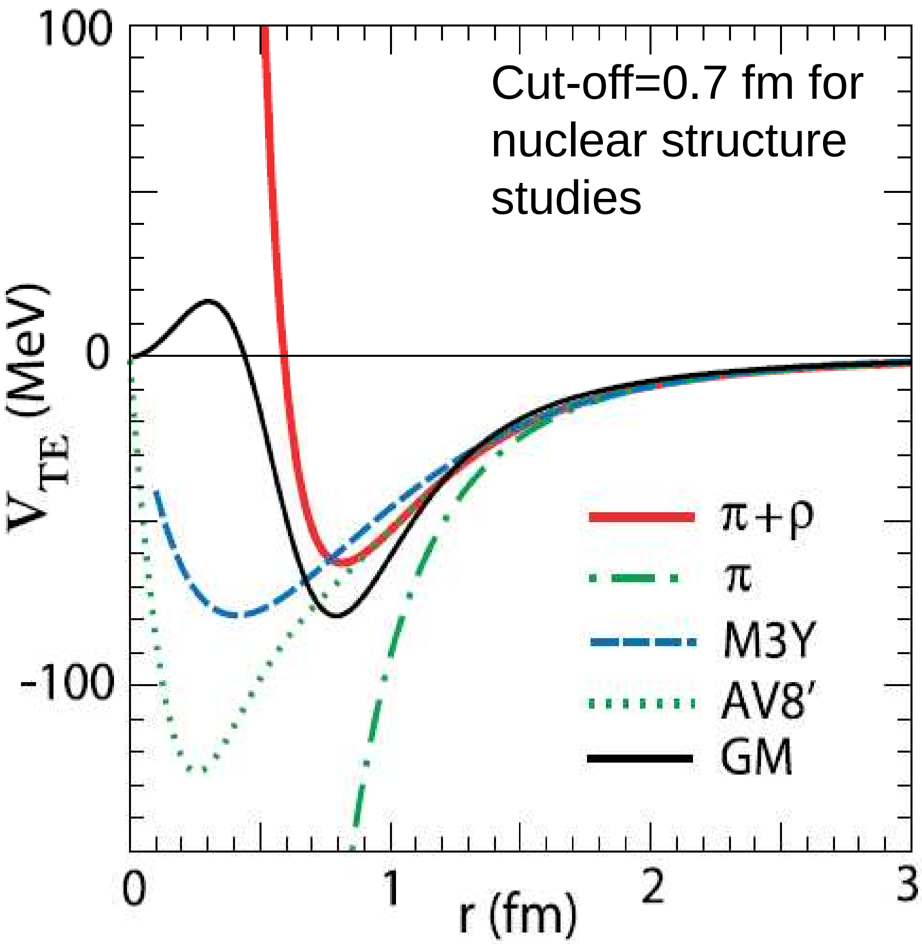
$$V_{T0} = V'_{np} \quad \text{(n-p pair in the T=0 state)}$$

$$V_{T1} = V_{nn} = V_{pp} = V_{np} \quad \text{(charge independence in the T=1 state)}$$

Nucleons having isospin t=1/2
t_3=1/2 for neutrons
t_3=-1/2 for protons
T=0 or 1 for NN pairs

Uncertainty of the tensor force at short distance

Takaharu Otsuka et al., PRL 95, 232502 (2005); PRL 97, 162501 (2006)



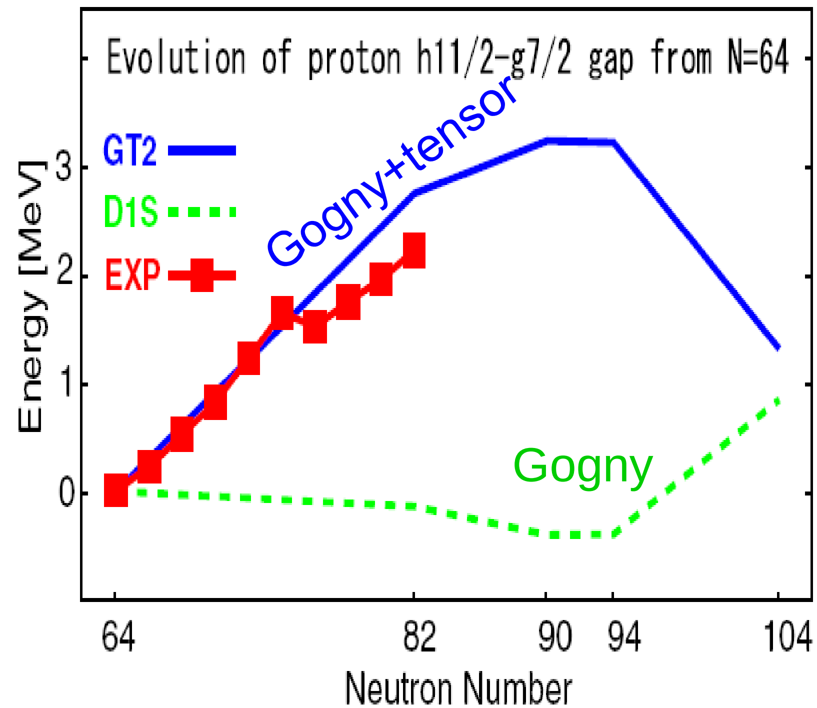


FIG. 4 (color online). Evolution of $1h_{11/2}$-$1g_{7/2}$ energy gap. The difference from the value of $N = 64$ is plotted for experimental data [21] and calculated results with GT2 and D1S interactions.

[21] J. P. Schiffer *et al.*, Phys. Rev. Lett. **92**, 162501 (2004).

Tensor force contribution to the potential part of the symmetry energy within a simple model

G.E. Brown and R. Machleidt, Phys. Rev. C50, 1731 (1994).

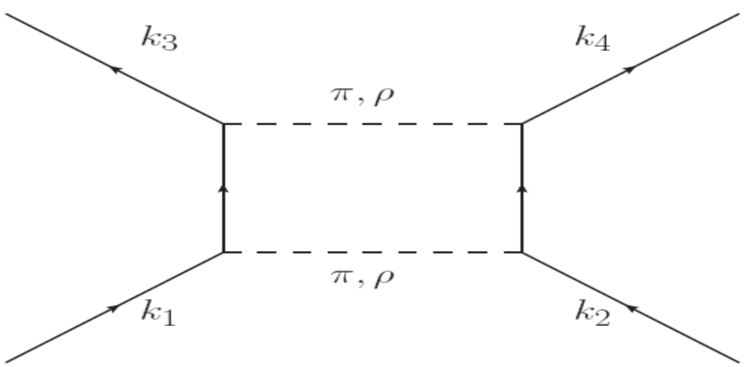
S.-O. Bacnman, G.E. Brown and J.A. Niskanen, Phys. Rep. 124, 1 (1985).

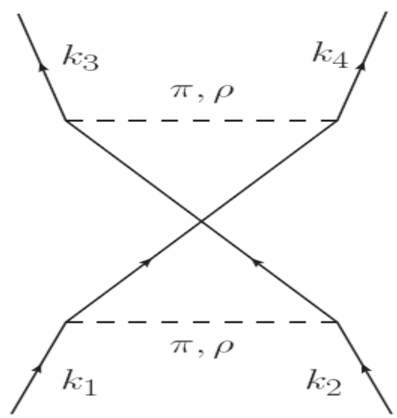
EFFECTIVE TENSOR INTERACTION IN NUCLEI *

T. T. S. KUO and G. E. BROWN **PLB 18, 54 (1965)**
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

Received 25 June 1965

$$< V_{\rm sym} >= \frac{12}{e_{\rm eff}} < [V_t(\mathbf{r})]^2 >$$

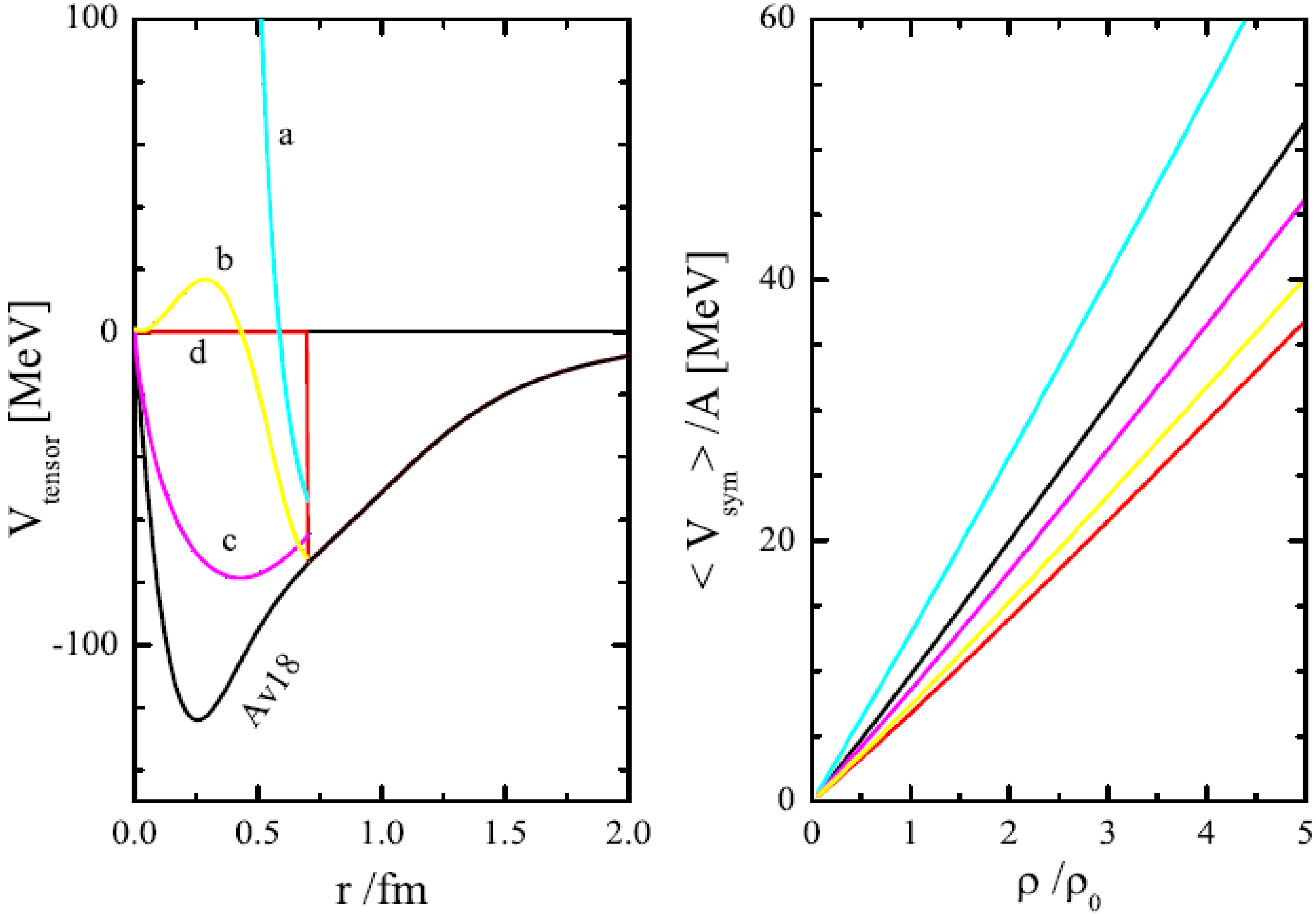




$$\frac{< V_{\rm sym} >}{A} = \frac{12}{e_{eff}} \cdot \frac{k_F^3}{12\pi^2} \{ \frac{1}{4} \int V_t^2(r) d^3 r - \frac{1}{16} \int [\frac{3 j_1(k_F r)}{k_F r}]^2 V_t^2(r) d^3 r \}$$

Effects of the short-range tensor force on the high-density symmetry energy

Ang Li and Bao-An Li, arXiv:1107.0496



Uncertainty of symmetry energy due to the 3-body force

α controls the in-medium many-body effects

(1) Isospin-independent

$$t_0(1+x_0P_\sigma)\rho^\alpha\left(\frac{r_1+r_2}{2}\right)\delta(r_1-r_2)$$

x_0 controls the mixing of different spin-isospin channels

X_0 from -1.56 to 1.92 are used in the over 160 Skyrme/Gogny effective interactions

isospin singlet channel $[\propto(1+x_0)\rho^{\alpha+1}]$

triplet channel $[\propto(1-x_0)\rho^{\alpha+1}]$

(2) Taking into account the isospin of the interacting nucleons

$$V_D = t_0(1+x_0P_\sigma)[\rho_{\tau_i}(\mathbf{r}_i)+\rho_{\tau_j}(\mathbf{r}_j)]^\alpha\delta(\mathbf{r}_{ij})$$

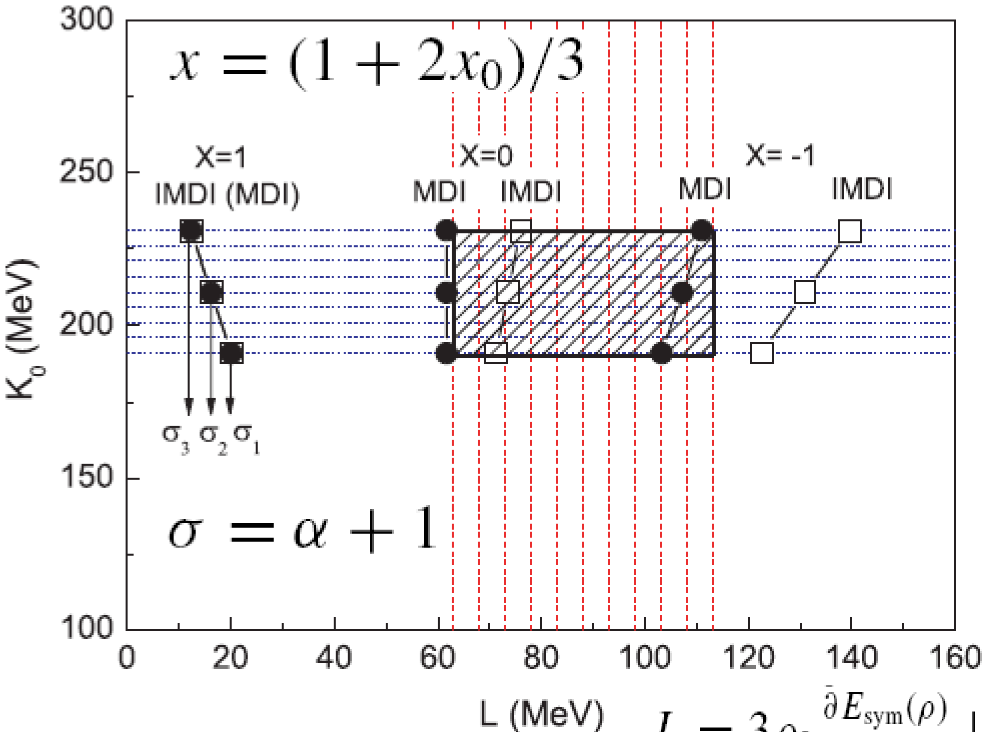
The potential energy density

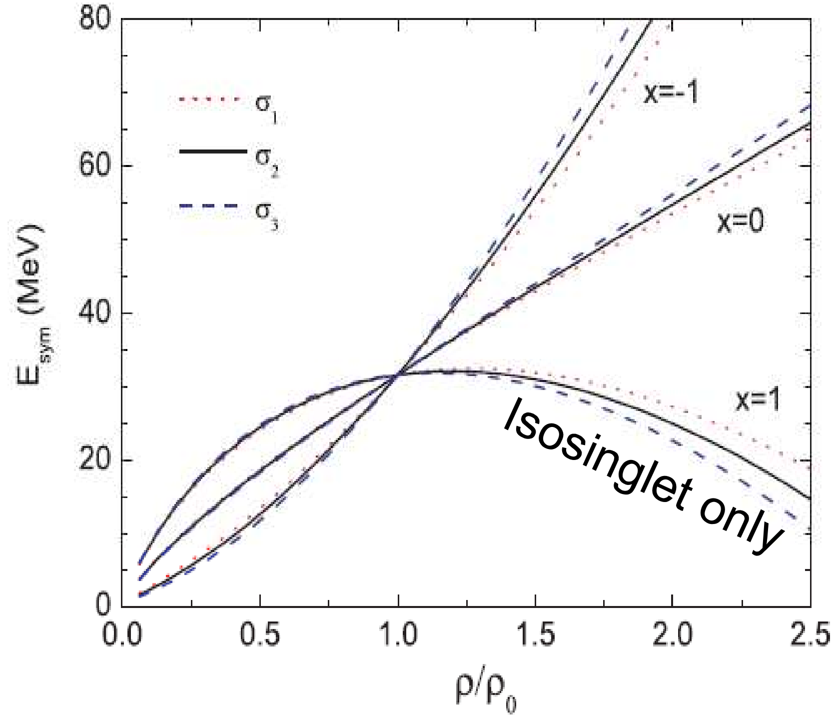
$$\xi(\rho) = t_0\left\{\left(1+\frac{x_0}{2}\right)\rho^\alpha\rho_n\rho_p+\frac{1}{16}(1-x_0)\times[(2\rho_n)^{\alpha+2}+(2\rho_p)^{\alpha+2}]\right\}.$$

M. Farine, J. M. Pearson, and E. Tondeur, Nucl. Phys. A **615**, 135 (1997).

Effects of the isospin-dependence of the 3-body force on symmetry energy

C. Xu and B.A. Li, PRC81, 044603 (2010)





$$L = 3\rho_0 \frac{\partial E_{\rm sym}(\rho)}{\partial \rho} |_{\rho_0}$$

$$K_0 = 210 \pm 20 \text{ MeV},$$

$$\sigma_{1,2,3} \text{ of } \frac{4}{3} - \frac{1}{30}, \frac{4}{3}, \text{ and } \frac{4}{3} + \frac{1}{30}$$

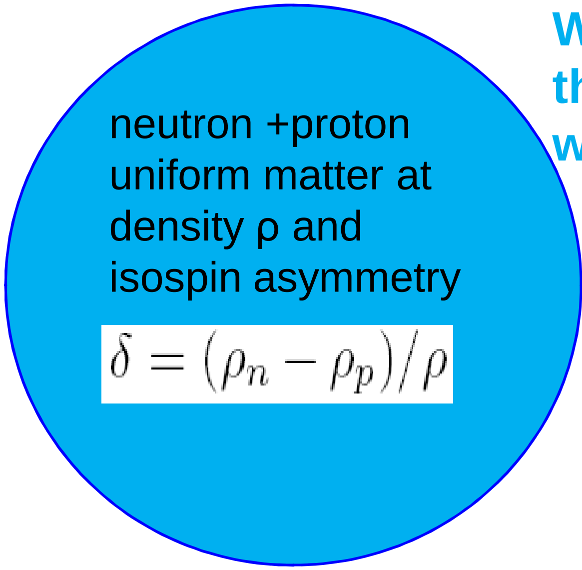
At low densities, σ=4/3

Alexandros Gezerlis, G.F. Bertsch, PRL 105, 212501 (2010)

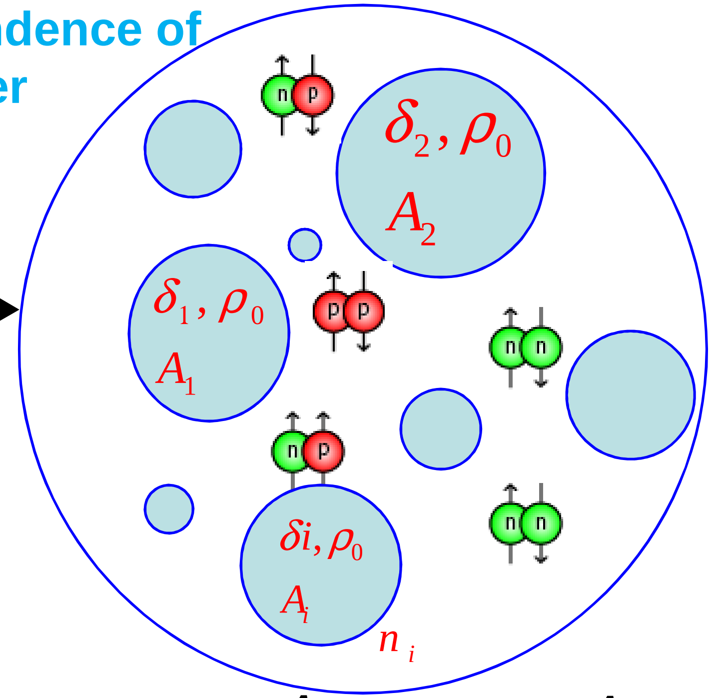
T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1119 (1957)

Some basic issues on symmetry energy at low densities

What is the isospin-dependence of the EOS of clustered matter with pairing?



as density decreases



Invariance of nuclear interaction under n-p exchange, for uniform matter

$$E(\rho,\delta) = E_0(\rho,\delta=0) + E_{sym}(\rho)\times\delta^2 + o(\delta^4)$$

$$\delta = \sum_i n_i \frac{A_i}{A}\delta_i \qquad \langle\rho\rangle = \frac{A}{V} \le \rho_0$$

$$E_{sym}(\rho) = \left(\frac{1}{2}\frac{\partial^2 E}{\partial\delta^2}\right)_{\delta=0} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

For clustered matter at averge density $\overline{\rho}$, isospin asymmetry δ, there is no more n $\leftrightarrow$ p invariance because of the Coulmb term in the binding energy, interactions among clusters and the asymmetry between proton and neutron driplines, the EOS can have odd terms in δ

$$E(\overline{\rho},\delta) = E_0(\overline{\rho},\delta=0) + E_{s1}(\overline{\rho})\times\delta + E_{s2}(\overline{\rho})\times\delta^2 + E_{s3}(\overline{\rho})\times\delta^3 + o(\delta^4)$$

What "symmetry energies" are we talking about for clustered matter? Even if we know the answer, at what densities? Normal or really low?

$$E(\overline{\rho},\delta) \approx E_0(\overline{\rho},\delta=0) + E_{s1}(\overline{\rho})\times\delta + E_{s2}(\overline{\rho})\times\delta^2 + E_{s3}(\overline{\rho})\times\delta^3 + o(\delta^4)$$

(1) Isospin-independent "symmetry energy-1"

For the sake of getting the traditionally defined symmetry energy and compare it with the one for uniform matter,

$$E_{sym}(\overline{\rho}) = \frac{1}{2}\left[E\,(\text{pure n-matter}) + E\,(\text{pure p-matter}) - 2\,\mathrm{E}\,(\text{symmetric matter})\right]$$

$$= E_{s2}(\overline{\rho}) = \frac{1}{2}\left(\frac{\partial^2 E}{\partial\delta^2}\right)_{\delta=0}$$

(2) Isospin-independent "symmetry energy-2"

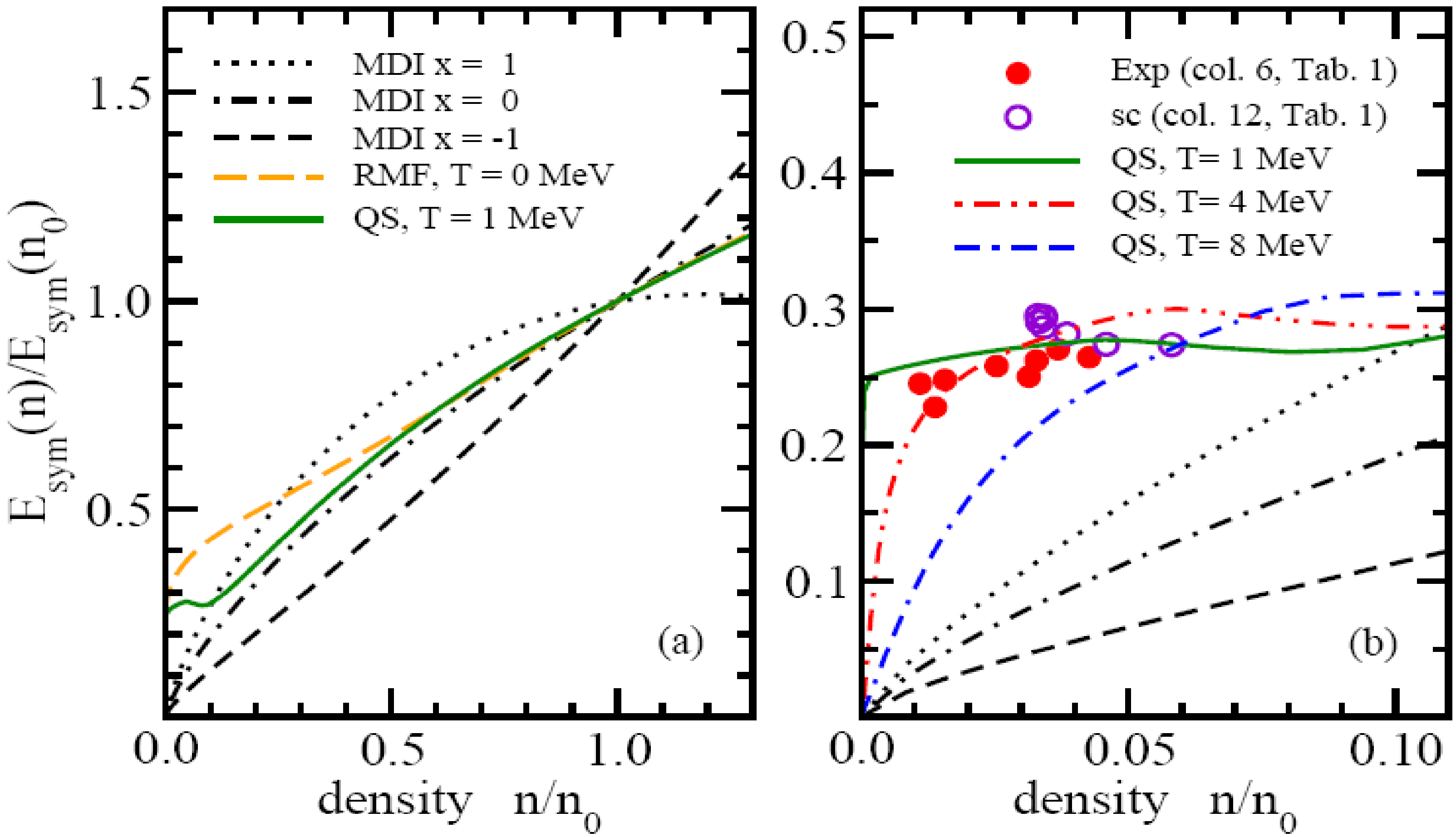
$$\mathrm{E}_{\mathrm{sym}}(\overline{\rho}) = E(\text{pure neutron matter})\text{-E(symmetric matter)} = E_{s1}(\overline{\rho}) + E_{s2}(\overline{\rho}) + E_{s3}(\overline{\rho})$$

(3) Isospin-dependent "symmetry energy" by forcing the EOS to be quadratic

$$E(\overline{\rho},\delta) = E_0(\overline{\rho},\delta=0) + E_{sym}(\overline{\rho},\delta)\times\delta^2$$

$$E_{sym}(\overline{\rho},\delta) = E_{s2}(\overline{\rho}) + E_{s1}(\overline{\rho})/\delta + E_{s3}(\overline{\rho})\times\delta + o(\delta^2)$$

Experimental extraction of the symmetry energy of clustered matter at very low densities



J.B. Natowitz, G. Ropke, S. Typel, D. Blaschke, A. Bonasera, K. Hagel, T. Klahn, S. Kowalski, L. Qin, S. Shlomo, R. Wada, H.H. Wolter
Phys.Rev.Lett.104:202501,2010

Isospin dependence of the EOS of clustered matter

$$E(\bar{\rho},\delta) \approx E_0(\bar{\rho},\delta=0) + E_{s1}(\bar{\rho})\times\delta + E_{s2}(\bar{\rho})\times\delta^2 + E_{s3}(\bar{\rho})\times\delta^3 + o(\delta^4)$$

Strong indication of linear dependence on δ for both n-rich and p-rich matter

Quantum statistical model

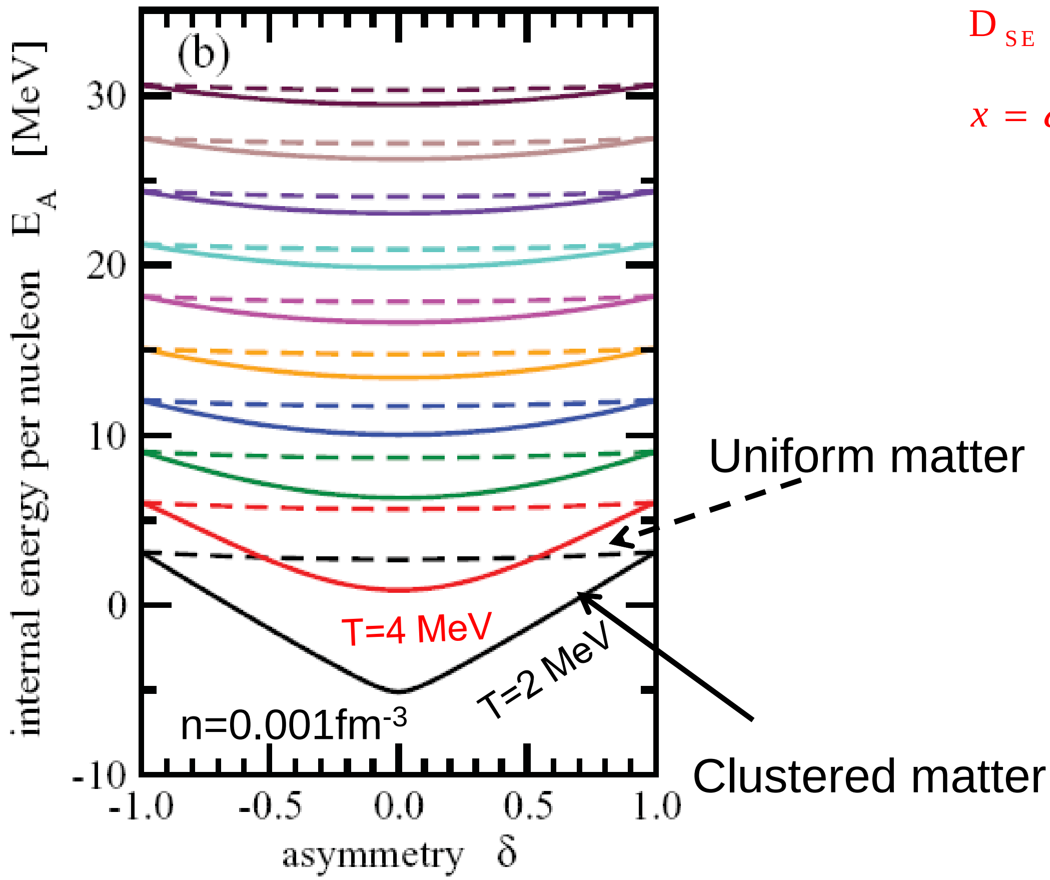
S. Typel, G. Ropke, T. Klahn,
D. Blaschke and H.H. Wolter,
PRC 81, 015803 (2010)

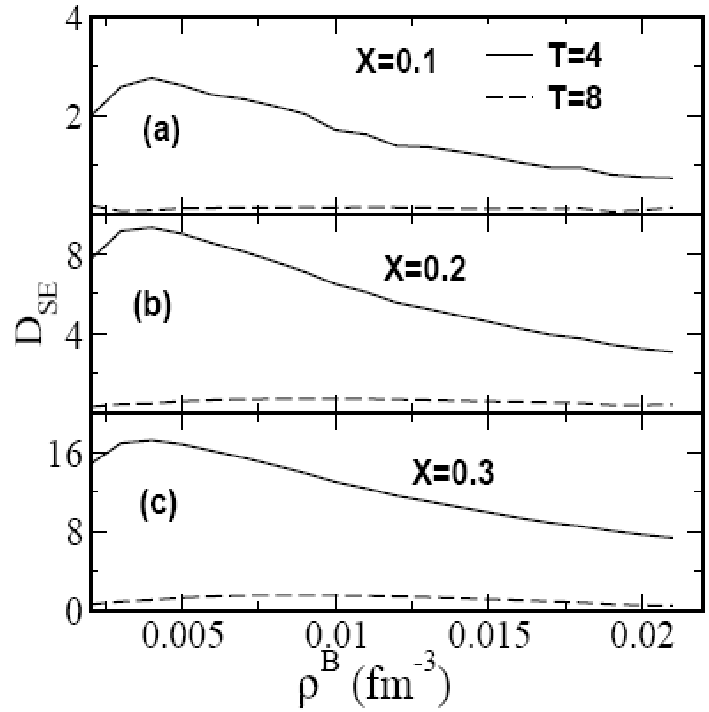
S-matrix approach

J. N. De, S. K. Samaddar, PRC 78, 065204 (2008)
S.K. Samaddar, J.N. De, X. Vinas and M. Centelles,
PRC 80, 035803 (2009)

$$D_{SE} = \frac{E_0 + E_{s2}x^2 - E(\rho,x)}{E_{s2}x^2}\times 100 \approx \frac{-E_{s1}}{E_{s2}x}\times 100$$

$x = \delta =$ isospin asymmetry





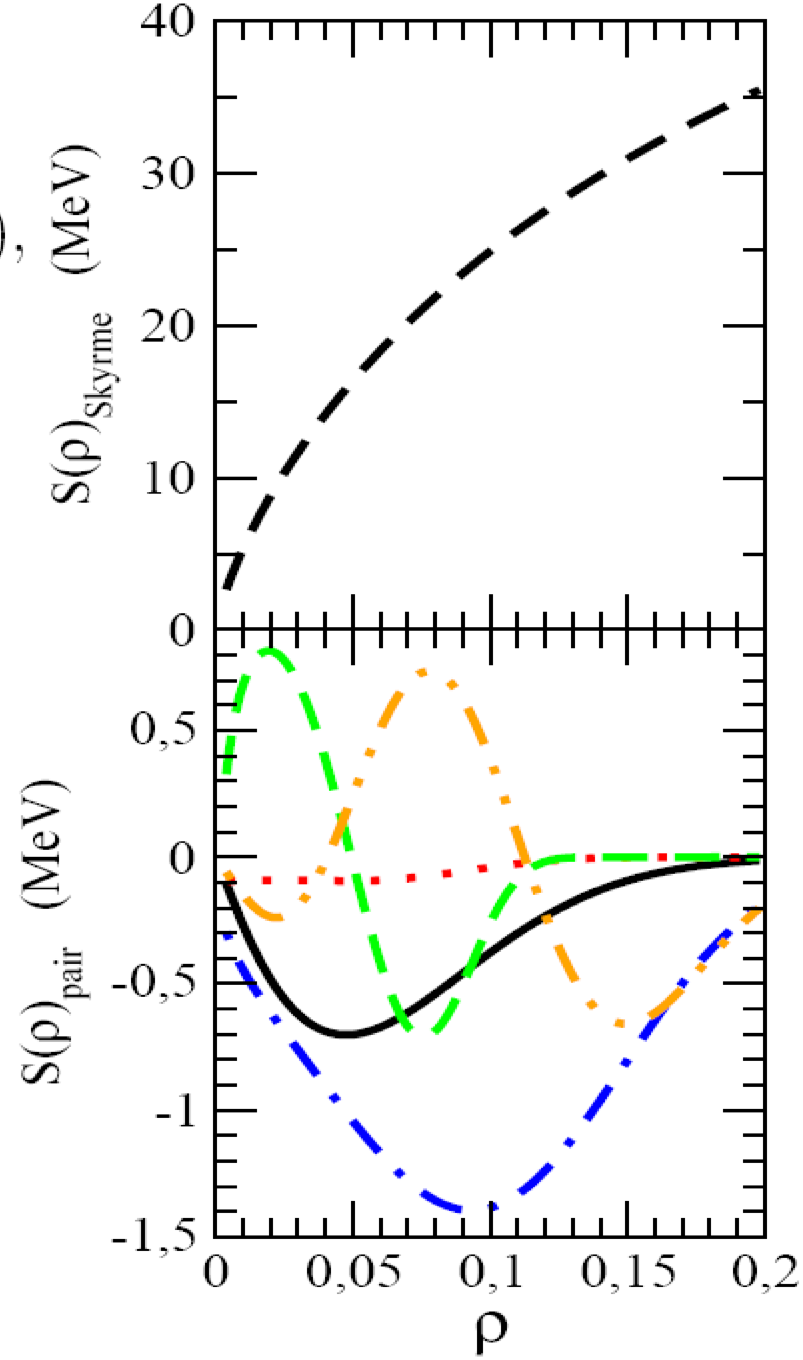
The anharmonic behavior depends on whether all mirror nuclei are included in pairs

Pairing effects on symmetry e

$$v_{\mathrm{pair}}^{\mathrm{IS}}(\vec{r},\vec{r}') = v_0 \left(1 - \eta \left(\frac{\rho}{\rho_0}\right)^{\alpha}\right) \delta(\vec{r}-\vec{r}'),$$

$$v_{\mathrm{pair}}^{\mathrm{MSH}}(\vec{r},\vec{r}') = v_0 \left[1 - (1-\delta)\eta_s \left(\frac{\rho}{\rho_0}\right)^{\alpha_s} - \delta\eta_n \left(\frac{\rho}{\rho_0}\right)^{\alpha_n}\right] \delta(\vec{r}-\vec{r}'),$$

$$v_{\mathrm{pair}}^{\mathrm{YS}}(\vec{r},\vec{r}') = v_0 \left[1 - (\eta_0 + \eta_1 \tau_3 \delta) \frac{\rho}{\rho_0} - \eta_2 \left(\delta \frac{\rho}{\rho_0}\right)^2\right] \delta(\vec{r}-\vec{r}'),$$



Considering nn and pp pairing in T=1 only
using 3 kinds of pairing intereactions

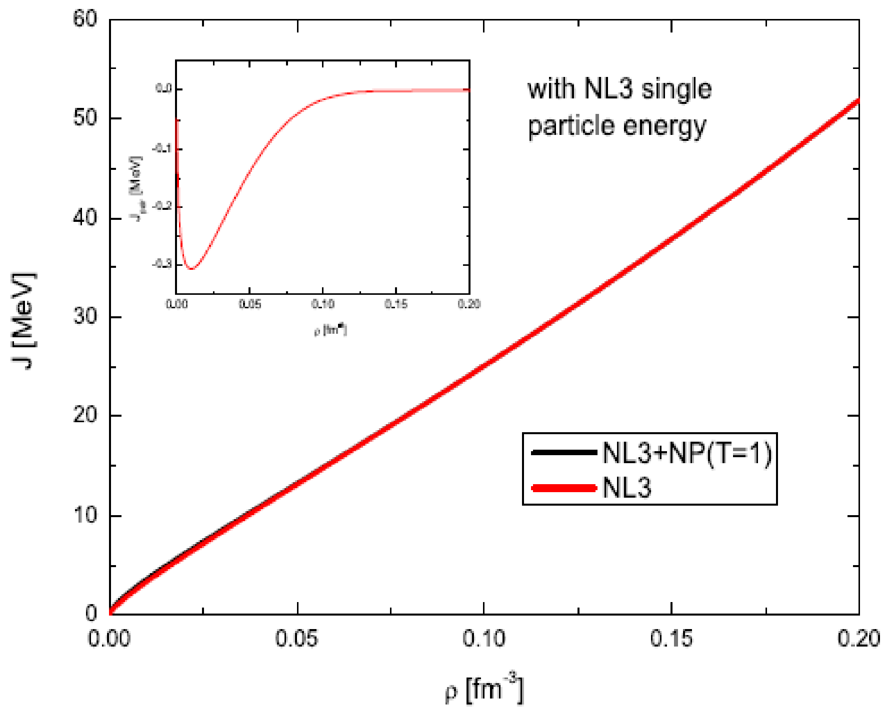
E. Khan, J. Margueron, G. Colò, K. Hagino,
and H. Sagawa, PRC **82**, 024322 (2010)

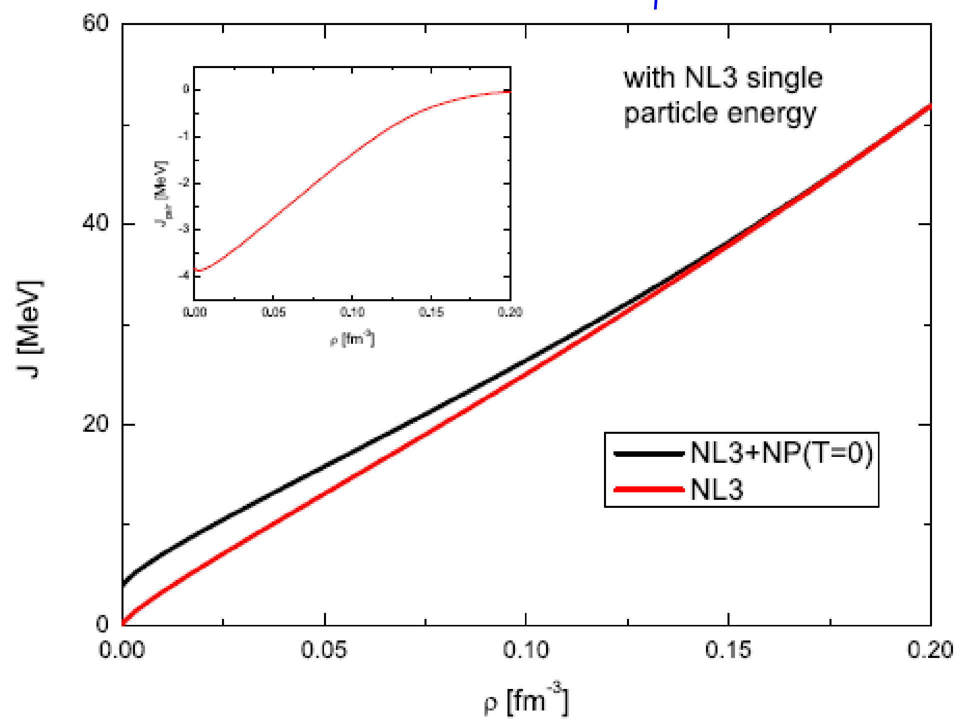
Effects of n-p pairing on symmetry energy at low densities

Yuan Tian, Bao-An Li and Zhongyu Ma (2011) using separable Paris potential

$$J_{sym}^{np} = -E_{pair}^{np}\text{(symmetric matter)}$$

$$\Delta(k) = -\int k'^2 dk' \frac{V(k,k')\Delta(k')}{2\sqrt{\epsilon(k')^2 + \Delta(k')^2}}$$





To my best knowledge,
Nobody has considered both clusters and pairing at low densities simultaneously yet

Promising Probes of the $E_{sym}(\rho)$ in Nuclear Reactions

At sub-saturation densities

- Global nucleon optical potentials from n/p-nucleus and (p,n) reactions
- **Sizes of n-skins of unstable nuclei from total reaction cross sections**
- **Parity violating electron scattering studies of the n-skin in ^{208}Pb at JLab**
- **n/p ratio of FAST, pre-equilibrium nucleons**
- **Isospin fractionation and isoscaling in nuclear multifragmentation**
- **Isospin diffusion/transport**
- **Neutron-proton differential flow**
- **Neutron-proton correlation functions at low relative momenta**
- **t/^{3}He ratio**

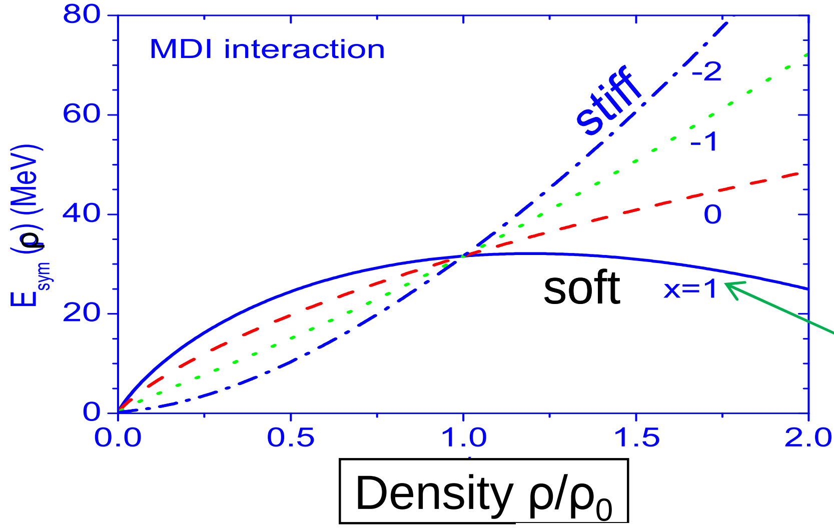
Towards supra-saturation densities

- **π^-/π^+ ratio, K^+/K^0 ?**
- **Neutron-proton differential transverse flow**
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- **Nucleon elliptical flow at high transverse momentum**
- **t-^{3}He differential and difference transverse flow**

(1) Correlations of multi-observable are important
(2) Detecting neutrons simultaneously with charged particles is critical

B.A. Li, L.W. Chen and C.M. Ko, *Physics Reports 464, 113 (2008)*

Symmetry energy and single nucleon potential used in the IBUU04 transport model



$$x = (1 + 2x_0)/3$$

X_0 is the coefficient of the spin exchange operator in the effective 3-body force term

Default: Gogny force

Potential energy density

$$V(\rho,\delta) = \frac{A_1}{2\rho_0}\rho^2 + \frac{A_2}{2\rho_0}\rho^2\delta^2 + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma}(1 - x\delta^2) + \frac{1}{\rho_0}\sum_{\tau,\tau'} C_{\tau,\tau'} \int\int d^3p\, d^3p' \frac{f_\tau(\vec{r},\vec{p}) f_{\tau'}(\vec{r},\vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}$$

Single nucleon potential within the HF approach using a modified Gogny force:

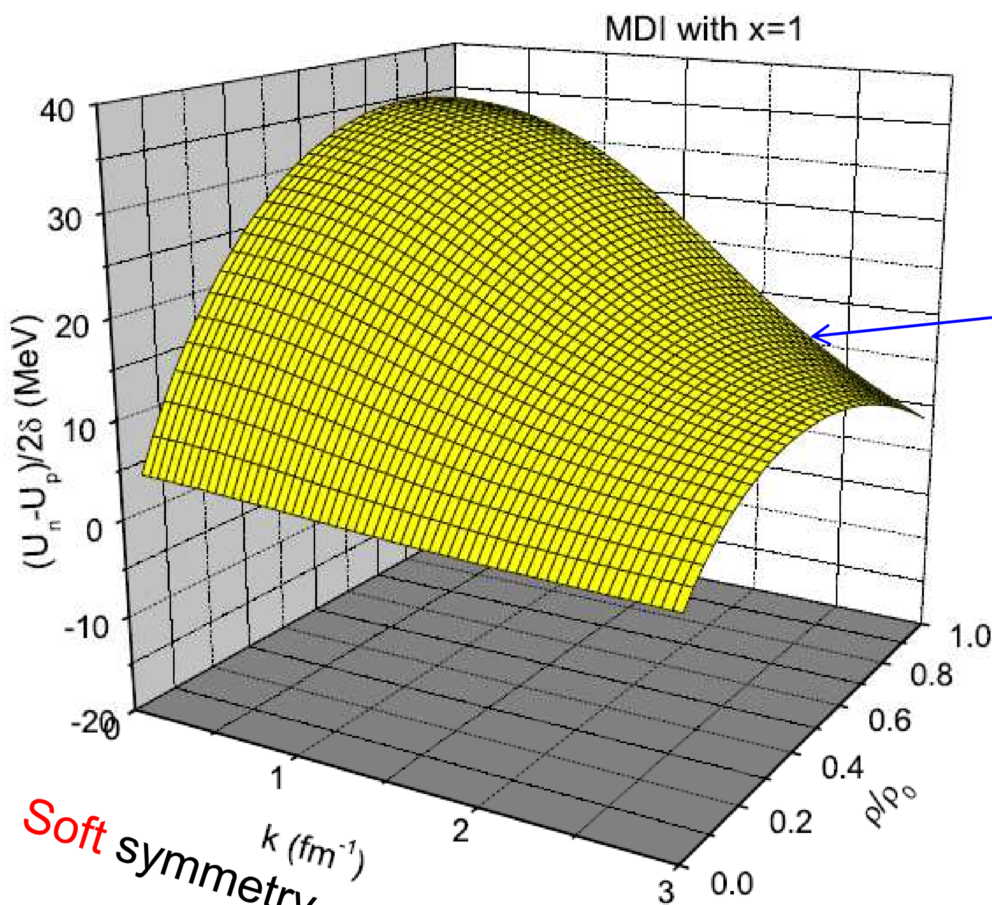
$$U(\rho,\delta,\vec{p},\tau,x) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma (1 - x\delta^2) - 8\tau x \frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}$$
$$+ \frac{2C_{\tau,\tau}}{\rho_0}\int d^3p' \frac{f_\tau(r,p')}{1 + (p - p')^2/\Lambda^2} + \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3p' \frac{f_{\tau'}(r,p')}{1 + (p - p')^2/\Lambda^2}$$

$$\tau,\tau' = \pm\frac{1}{2},\ A_l(x) = -121 + \frac{2Bx}{\sigma+1},\ A_u(x) = -96 - \frac{2Bx}{\sigma+1},\ K_0 = 211\,MeV$$

C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

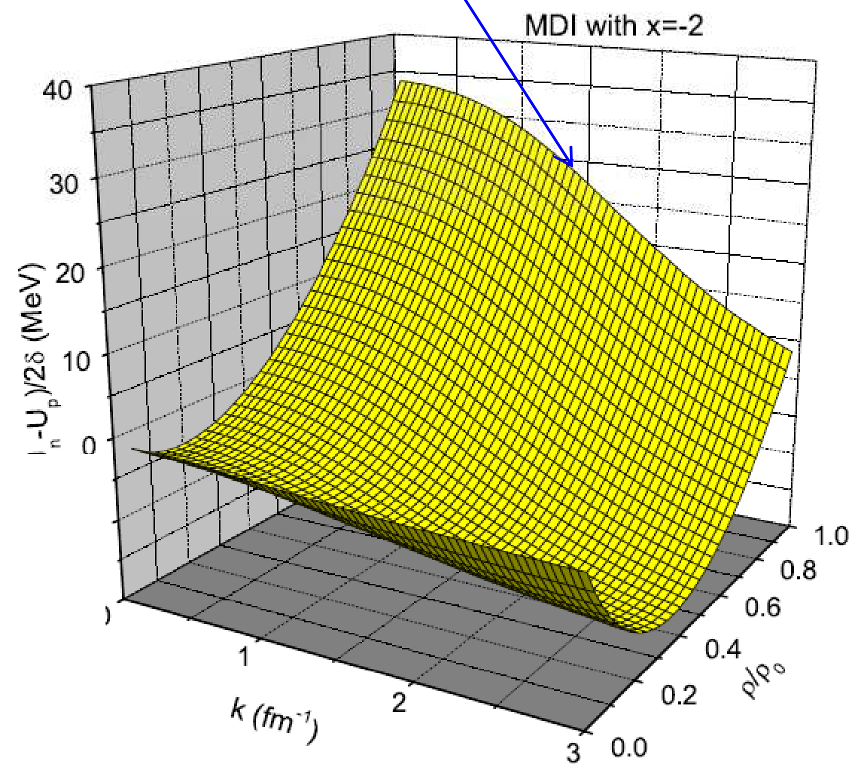
B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC **69**, 034614; NPA **735**, 563 (2004).

Symmetry potential



Soft symmetry energy

At ρ_0
Constrained by the isovector part of the optical potential



Corresponding to the stiff symmetry energy

$$
\begin{aligned}
E_{\rm sym}(\rho) &= \frac{1}{2}\left(\frac{\partial^2 E}{\partial \delta^2}\right)_{\delta=0} \\
&= \frac{8\pi}{9mh^3\rho}p_f^5 + \frac{\rho}{4\rho_0}(A_l - A_u) - \frac{Bx}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^{\sigma} \\
&+ \frac{C_l}{9\rho_0\rho}\left(\frac{4\pi}{h^3}\right)^2 \Lambda^2\left[4p_f^4 - \Lambda^2 p_f^2 \ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}\right] \\
&+ \frac{C_u}{9\rho_0\rho}\left(\frac{4\pi}{h^3}\right)^2 \Lambda^2\left[4p_f^4 - p_f^2(4p_f^2+\Lambda^2)\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}\right]
\end{aligned}
$$

Isospin-dependence of nucleon-nucleon cross sections in neutron-rich matter

The effective mass scaling model:

$$\sigma_{medium} / \sigma_{free} \approx \left(\frac{\mu^*_{NN}}{\mu_{NN}} \right)^2$$

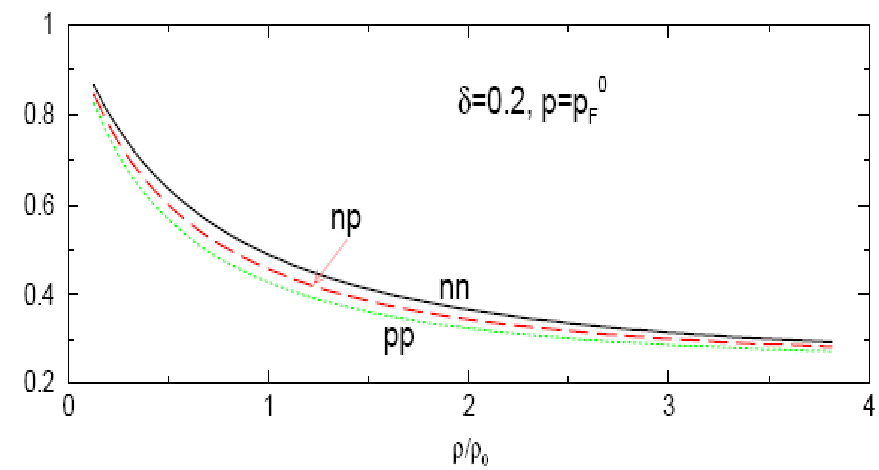
μ^*_{NN} is the reduced effective mass of the colliding nucleon pair NN

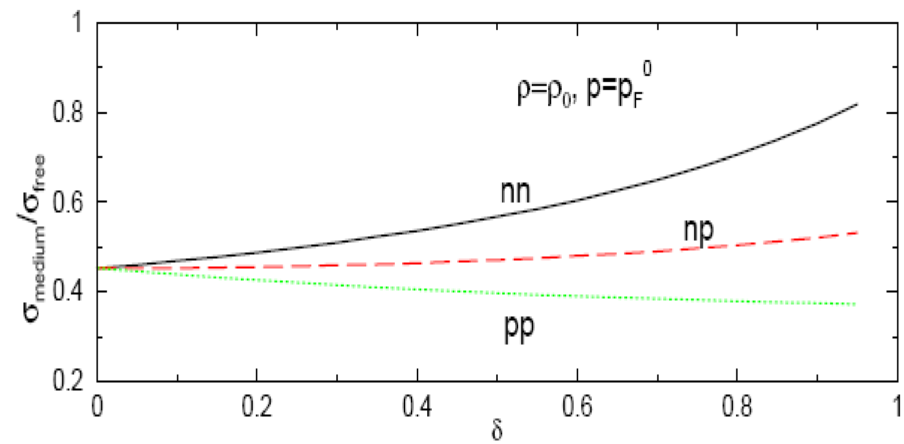
Applications in symmetric nuclear matter:

J.W. Negele and K. Yazaki, PRL 47, 71 (1981)
V.R. Pandharipande and S.C. Pieper, PRC 45, 791 (1992)
M. Kohno et al., PRC 57, 3495 (1998)
D. Persram and C. Gale, PRC65, 064611 (2002).

Application in neutron-rich matter:
nn and pp xsections are splitted due to the neutron-proton effective mass slitting

$\sigma_{medium} / \sigma_{free}$ in neutron-rich matter at zero temperature





Bao-An Li and Lie-Wen Chen, Phys. Rev. C72, 064611 (2005).

Pion ratio probe of symmetry energy at supra-normal densities

GC Coefficients²	π^+	π^0	π^-
nn	0	1	5
pp	5	1	0
np(pn)	1	4	1

a) Δ(1232) resonance model
in first chance NN scatterings:
(negelect rescattering and reabsorption)

$$\frac{\pi^-}{\pi^+} = \frac{5N^2 + NZ}{5Z^2 + NZ} \approx \left(\frac{N}{Z}\right)^2$$

R. Stock, Phys. Rep. 135 (1986) 259.

b) Thermal model:

(*G.F. Bertsch, Nature 283 (1980) 281*; A. Bonasera and G.F. Bertsch, PLB**195** (1987) 521)

$$\frac{\pi^-}{\pi^+} \propto \exp[2(\mu_n - \mu_p)/kT]$$

$$\mu_n - \mu_p = (V^n_{asy} - V^p_{asy})\delta - V_{Coul} + kT\{\ln\frac{\rho_n}{\rho_p} + \sum_m \frac{m+1}{m} b_m (\frac{1}{2}\lambda_T^3)^m (\rho_n^m - \rho_p^m)\}$$

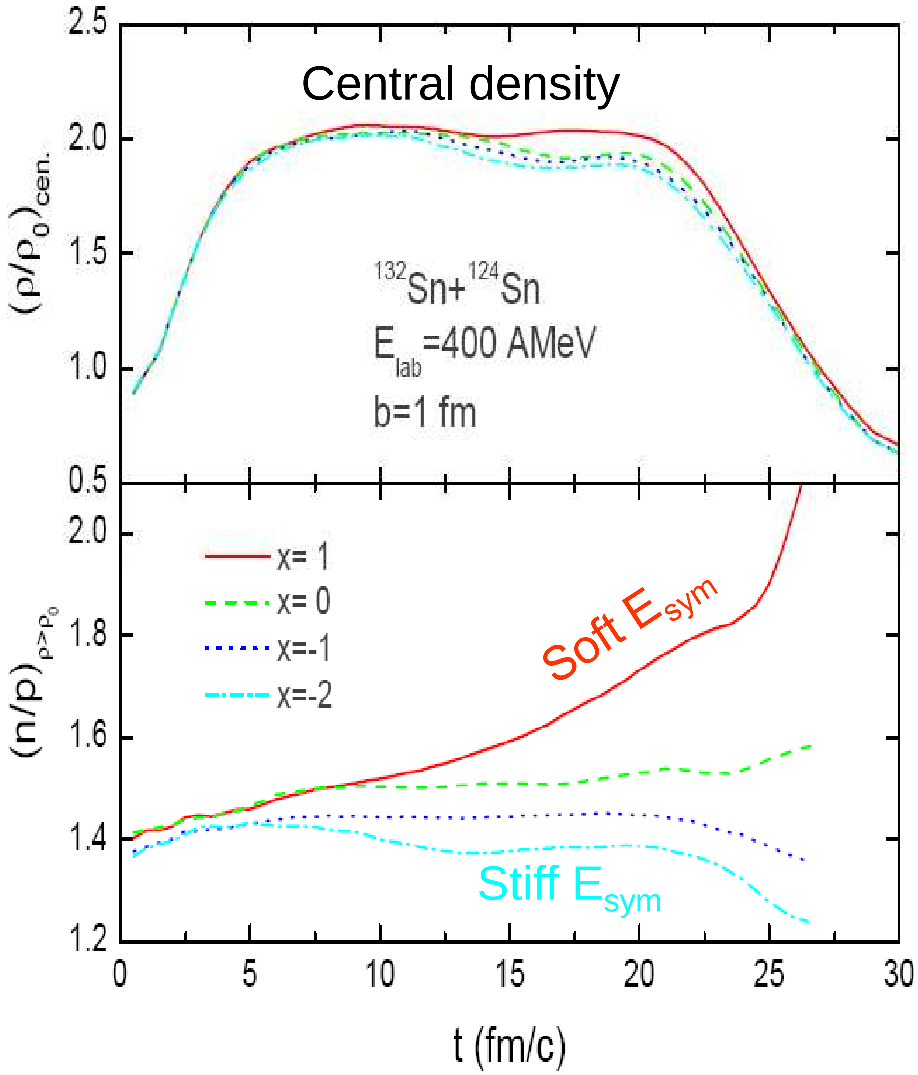
H.R. Jaqaman, A.Z. Mekjian and L. Zamick, PRC (1983) 2782.

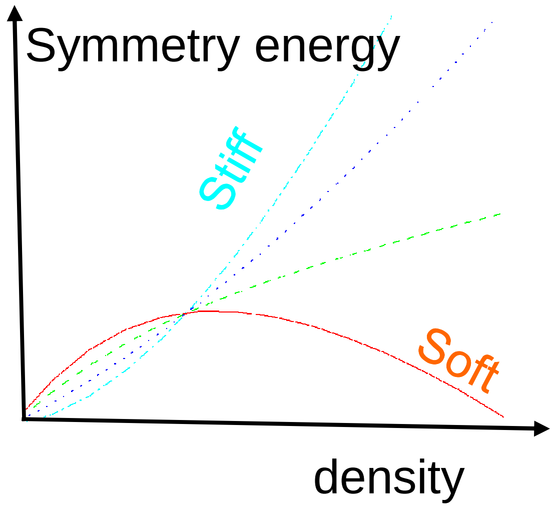
c) Transport models (more realistic approach):

Bao-An Li, Phys. Rev. Lett. 88 (2002) 192701, and several papers by others

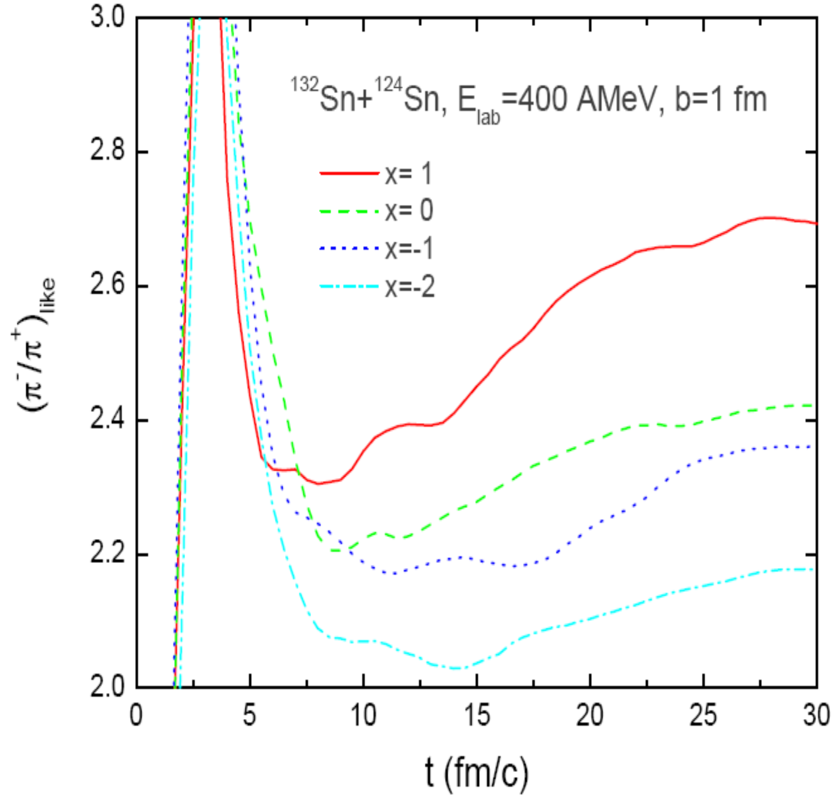
Formation of dense, asymmetric nuclear matter

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2$$





π^-/π^+ probe of dense matter

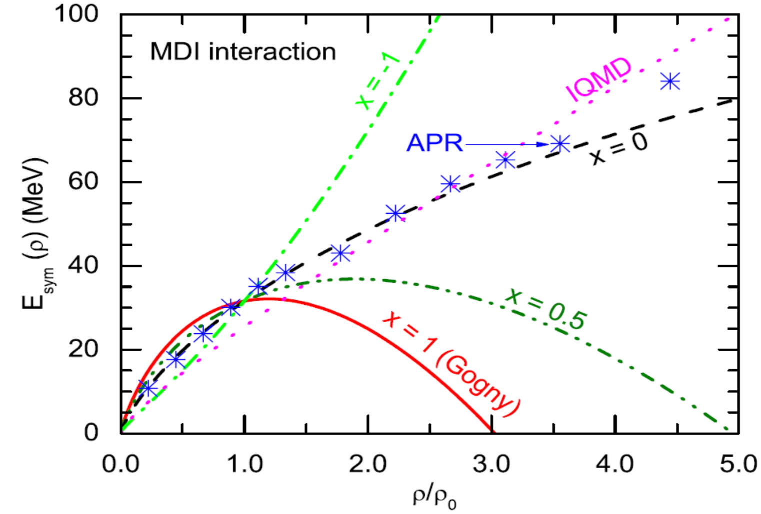


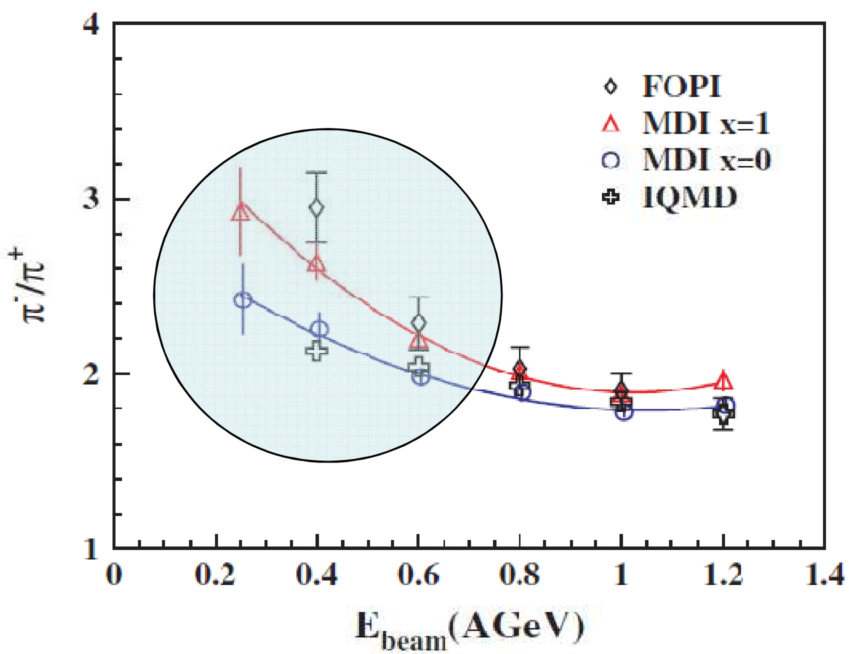
n/p ratio at supra-normal densities

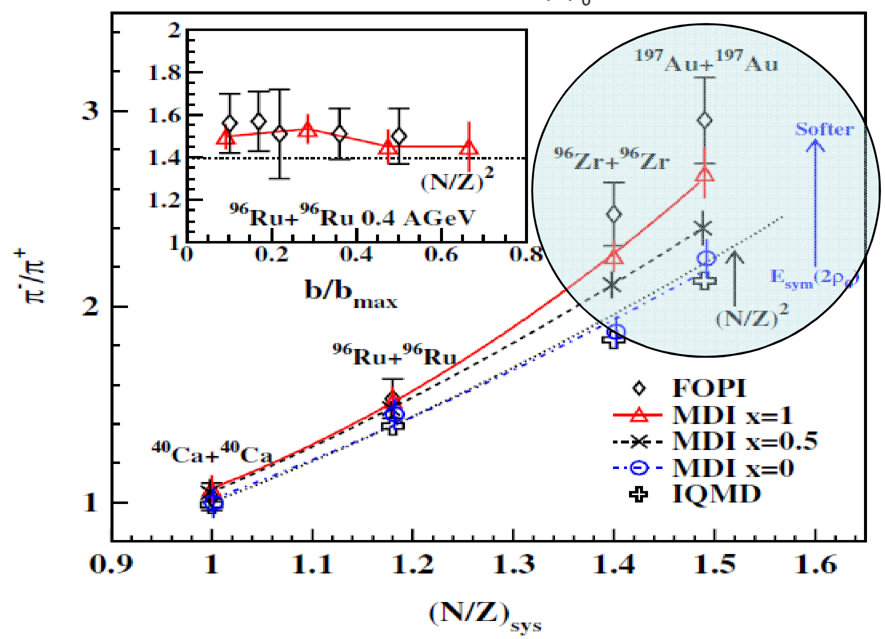
Circumstantial Evidence for a Super-soft Symmetry Energy at Supra-saturation Densities

Data: **W. Reisdorf et al. NPA781 (2007) 459**

Calculations: IQMD and IBUU04





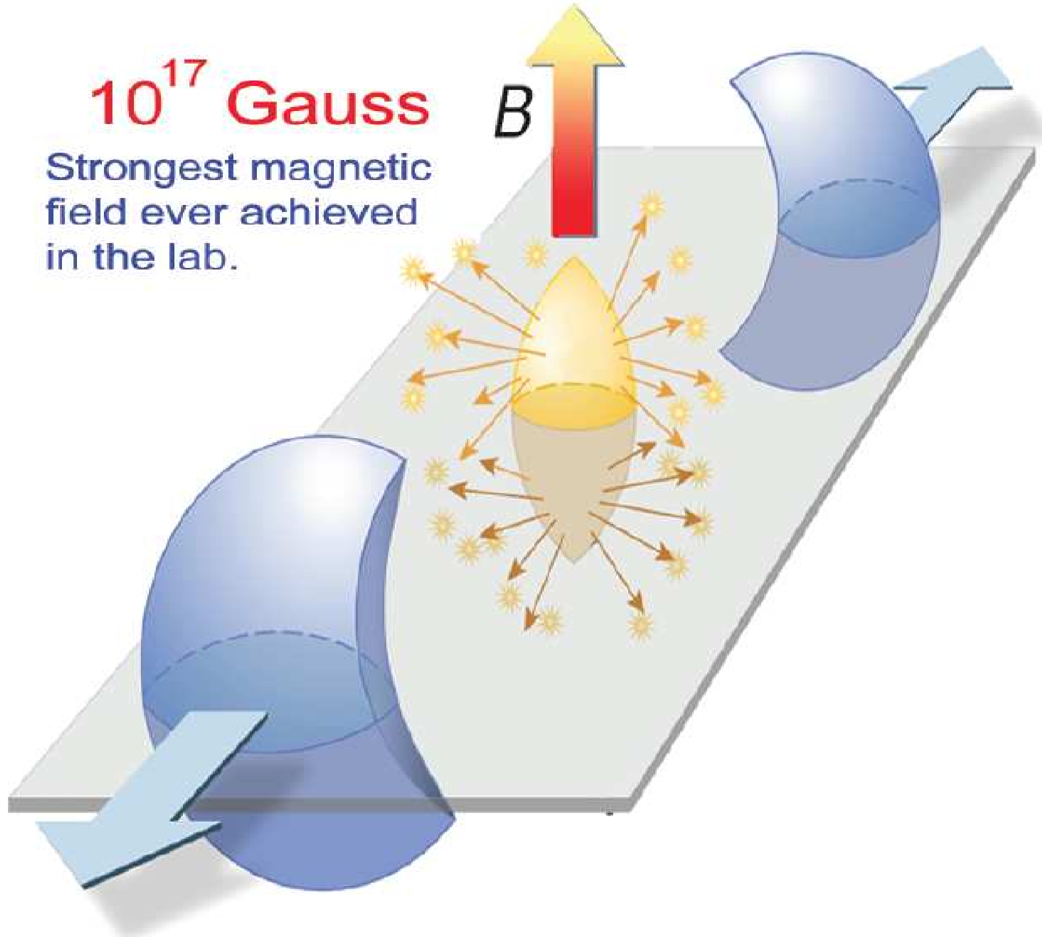


A super-soft nuclear symmetry energy is favored by the FOPI data!!!

Z.G. Xiao, B.A. Li, L.W. Chen, G.C. Yong and M. Zhang, Phys. Rev. Lett. 102 (2009) 062502

A new issue: magnetic effects on the pion ratio

In off-central Au+Au collisions at RHIC,
D. Kharzeev, L. McLerran and H.Warringa, NPA803:227 (2008)



In sub-Coulomb barrier U+U collisions, the magnetic field B is on the order of 10^{14} G
J. Rafelski and B. Muller, PRL 36, 517 (1976)

Model

The Boltzmann-Uhling-Uhlenbeck transport equation including electomagnetic fields:

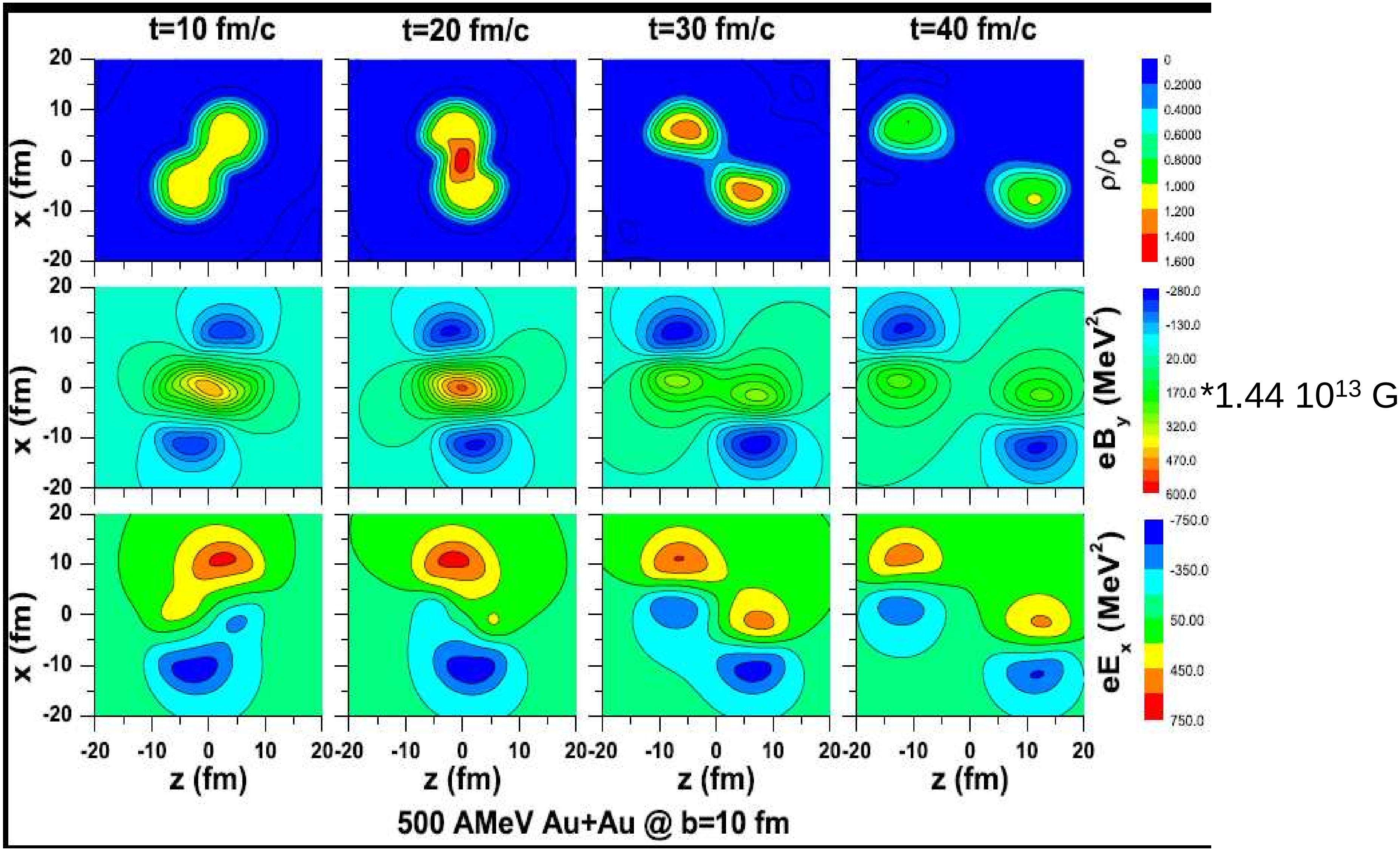
$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{P}}{E}\nabla_r - (\nabla_r \boldsymbol{U} - q\boldsymbol{v} \times \boldsymbol{B} - q\boldsymbol{E})\nabla_p\right] f(\boldsymbol{r}, \boldsymbol{p}, t) = I(\boldsymbol{r}, \boldsymbol{p}, t). \quad (1)$$

Electric and magnetic field strength with Liénard-Wiechert potentials and the retardation effect are calculated according to

$$e\boldsymbol{E}(\boldsymbol{r}, t) = \frac{e^2}{4\pi\epsilon_0} \sum_n Z_n \frac{c^2 - v_n^2}{(cR_n - \boldsymbol{R}_n \cdot \boldsymbol{v}_n)^3} (c\boldsymbol{R}_n - R_n \boldsymbol{v}_n); \quad (2)$$

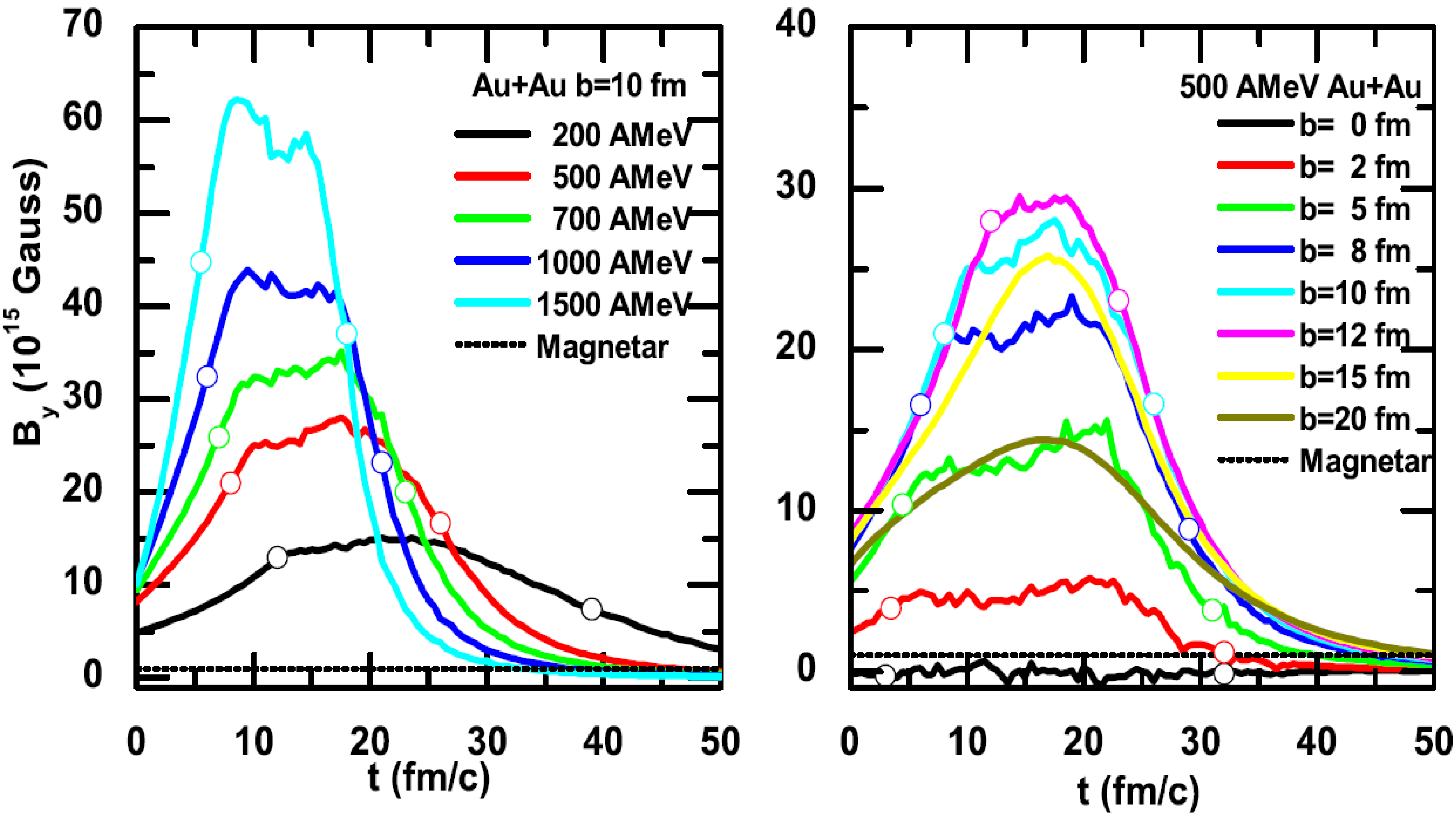
$$e\boldsymbol{B}(\boldsymbol{r}, t) = \frac{e^2}{4\pi\epsilon_0 c} \sum_n Z_n \frac{c^2 - v_n^2}{(cR_n - \boldsymbol{R}_n \cdot \boldsymbol{v}_n)^3} \boldsymbol{v}_n \times \boldsymbol{R}_n. \quad (3)$$

$\boldsymbol{R}_n = \boldsymbol{r} - \boldsymbol{r}'_n$, $\boldsymbol{r}'_n$ is the position vector of particle, $\boldsymbol{v}_n$ is particle velocity. $\boldsymbol{v}_n$ and $\boldsymbol{R}_n$ are taken at the retarded time $t_{\mathrm{r}n} = t - |\boldsymbol{r} - \boldsymbol{r}'_n(t_{\mathrm{r}n})|$.

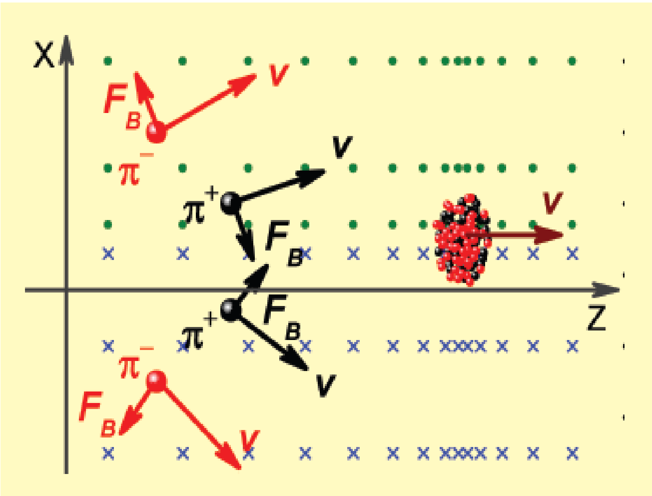


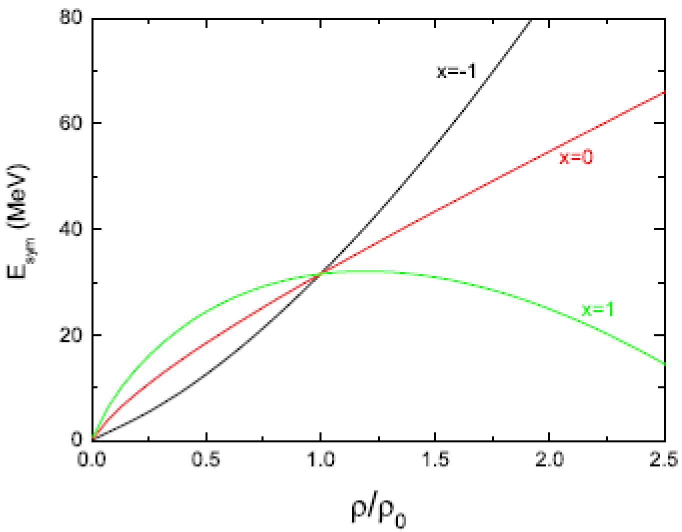
Ou Li and Bao-An Li arXiv:1107.3192v1 [nucl-th] 16 Jul 2011 arXiv:1107.3192

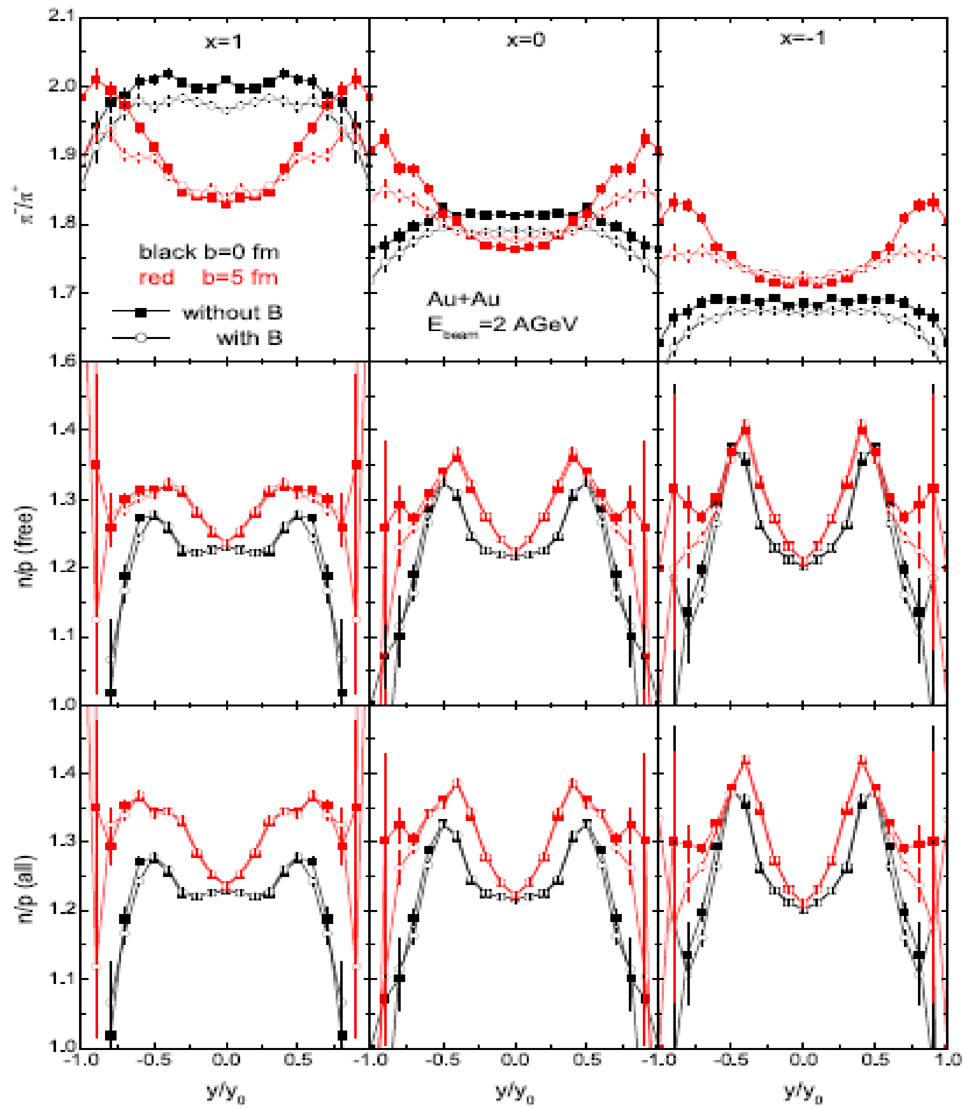
Beam energy and impact parameter dependence of magnetic field created in heavy-ion collisions



Significant Magnetic effects on the pion ratio







Conclusion and Perspectives

- Conclusion: No!
- Perspectives: a lot!

Thank you!