Making Sense of Structure/Reaction 'Non-observables'

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What is a ‘non-observable’?

- I don’t mean *in-principle* non-observables
  - T.D. Lee: “The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”
  - E.g., you can’t measure absolute position or time
- True observables are directly measurable quantities
  - E.g., cross sections and energies
  - Association with a Hermitian operator is not enough!
- I mean scale- and scheme-dependent quantities
  - E.g., spectroscopic factors depend on scheme (do ANC’s?)
  - Questions to address:
    - Is there a consistent extraction from experiment such that they can be applied in other processes?
    - Can one convert between different prescriptions?
    - What is the ambiguity or convention dependence?
  - Note: Many quantities can be *in-practice* observables depending on the physics context (e.g., negligible ambiguity)

Partial list of ‘non-observables’ references

- *Equivalent Hamiltonians in scattering theory*, H. Ekstein, (1960)
- *Measurability of the deuteron D state probability*, J.L. Friar, (1979)
- *Problems in determining nuclear bound state wave functions*, R.D. Amado, (1979)
- *Nucleon nucleon bremsstrahlung: An example of the impossibility of measuring off-shell amplitudes*, H.W. Fearing, (1998)
- *Are occupation numbers observable?*, rjf and H.-W. Hammer, (2002)
- *Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors*, A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)
- *Non-observability of spectroscopic factors*, B.K. Jennings, (2011)
- *How should one formulate, extract, and interpret ‘non-observables’ for nuclei?*, rjf and A. Schwenk, (2010) [in J. Phys. G focus issue on Open Problems in Nuclear Structure Theory, edited by J. Dobaczewski]

From rjf and A. Schwenk essay [J. Phys. G 37 (2010)]

- The general structure is that a measured quantity such as a cross section is decomposed as a convolution of subsidiary pieces, usually based on a factorization principle.
- This decomposition is not unique, and so we refer here to the extracted quantities as ‘non-observables’.
- The quotes are intended to soften the implication that it is improper to talk about them; nevertheless, unless the conventions (e.g., scale and scheme dependence) are controlled and specified, there will be ambiguities that will be entangled with the structure and reaction approximations.
- The challenge is to formulate and carry out experimental extractions and theoretical calculations of non-observables systematically and consistently.

Parton distributions as paradigm [Marco Stratman]

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

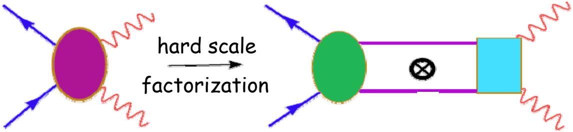
short-distance "Wilson coefficient"

choice of the factorization scheme

Parton distributions as paradigm [Marco Stratman]

Factorization schemes

pictorial representation of factorization:



e.g. $F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \hat{F}_2^a(x, \frac{Q}{\mu_f})$

the separation between long- and short-distance physics is not unique

long-distance parton density

short-distance Wilson coefficient

1. **choice of** μ_f: defines borderline between long-/short-distance
2. **choice of scheme**: re-shuffling finite pieces

Parton distributions as paradigm: Lessons

- The momentum distribution for a given hadron is not unique
  - With parton distributions one would not talk about the results at a particular Q^2 as being “the” quark or gluon momentum distribution as opposed to distributions for lower or higher Q^2.
  - Dependence on Q^2, which serves as the resolution scale and can be changed by renormalization group (RG) evolution, and the PDF analysis at NLO must be performed in a specific renormalization and factorization scheme (e.g., MS or DIS)
  - Controlled factorization allows PDF’s from one process to be used in other processes (and at other scales)!
  - For consistency, hard-scattering cross section calculations used for the input processes or that use the extracted PDFs have to be implemented with the same scheme
  - There is careful treatment of the uncertainties in the PDFs; not considered sufficient to just compare different extractions. Instead, Lagrange Multiplier and Hessian techniques have been developed to estimate PDF uncertainties.
- Can we formulate our stucture/reaction theory to have the same control as with PDFs using factorization?

What are the low-energy nuclear physics analogs?

- E.g., from D. Bazin ECT* talk, 5/2011

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_f - J_i| \leq j \leq J_f + J_i} S_j^{if} \sigma_{sp}$$

- Questions:
  - How general/robust is this factorization?
  - What does it mean to be *consistent* between structure and reaction models?
  - How does scheme dependence come in?
  - What are the trade-offs? (E.g., does simpler structure mean more complicated reaction?)

The source of convention or scheme dependence

- General form: cross section as convolution
  - but individual parts are not unique
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | O | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) UOU^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle$$

  But the matrix elements of O itself between the transformed states are in general modified:

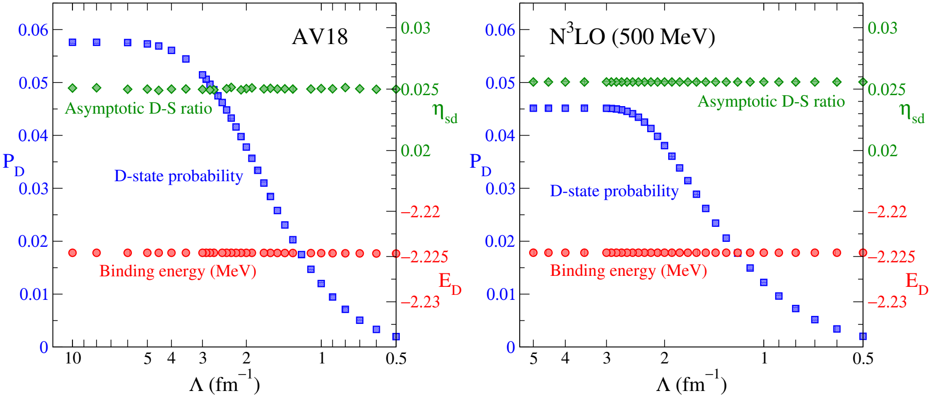
$$\widetilde{O}_{mn} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a(\mathbf{r}) | \Psi_0^A \rangle \text{ changes}$$

- Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only.
- Claim: In a low-energy effective theory, there is no preferred set of states (or preferred Hamiltonian) so transformations that modify *short-range* unresolved physics generate equally acceptable states. So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ ambiguous.

Quantities that vary with convention or scheme

- deuteron D-state probability [e.g., Friar, PRC **20** (1979)]
- off-shell effects (e.g., from NN bremsstrahlung) [Fearing/Scherer, PRC **62** (2000)]
- occupation numbers [Hammer/rjf, PLB **531** (2002)]
- spectroscopic factors [Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- proton radius (cf. charge radius) [Polyzou, PRC **58** (1998)]
- short-range part of wave functions (SRC's)
- wound integrals
- short-range potentials; e.g., contribution of short-range 3-body forces
- and so on . . .

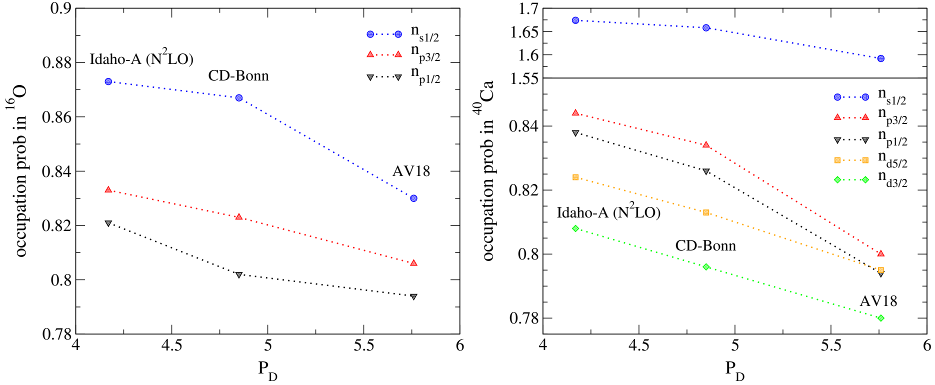
Deuteron true and scheme-dependent observables



- Unitary transformations labeled by Λ ($V_{\mathrm{low}\,k}$ here)
  $\Longrightarrow$ soften interactions by lowering resolution (how far?)
  $\Longrightarrow$ reduced short-range and tensor correlations
- D-state probability changes (cf. spectroscopic factors)
- Asymptotic D-S ratio is unchanged (cf. ANC's)

Correlation of P_D with spectroscopic factors

Calculations from Gad and Muether, Phys. Rev. C **66**, 044361 (2002)



- Increased occupation probability with increased non-locality and correlated reduction in short-range tensor strength
- Is the correlation quantitatively predictable?

Cutoff dependence in coupled cluster calculations

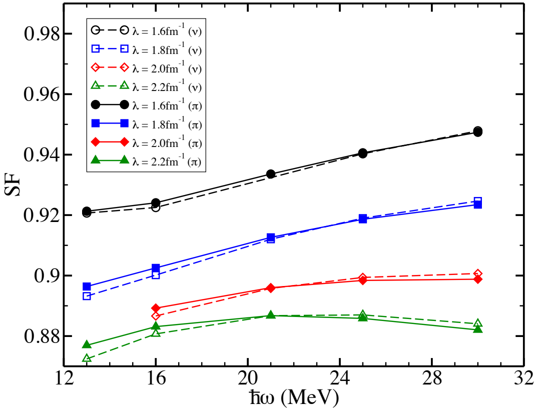


FIG. 4: (Color online) Spectroscopic factor $\mathrm{SF}(1/2^-)$ for neutron and proton removal as a function of the oscillator spacing $\hbar\omega$ for nucleon-nucleon interactions with different cutoffs in a model space with $N = 6$.

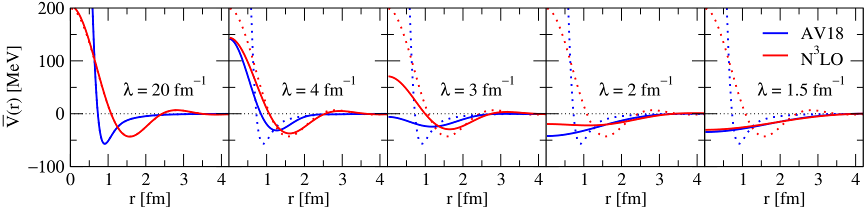
[From Ø. Jensen et al., PRC **82**, 014310 (2010)]

- SF increases as SRG resolution λ decreases from 2.2 to 1.6 fm^{-1}

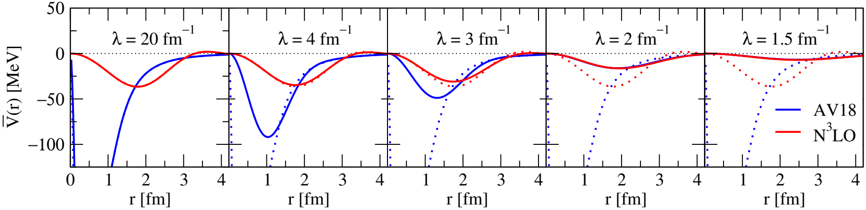
Wave functions are more single-particle-like as Λ/λ decreases, but do reaction operators become significantly less one-body?

Changing the scheme: (short-range) NN potential

- $V_{\text{low }k}$ or SRG unitary transformations to soften interactions
- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\ V_\lambda(r, r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]

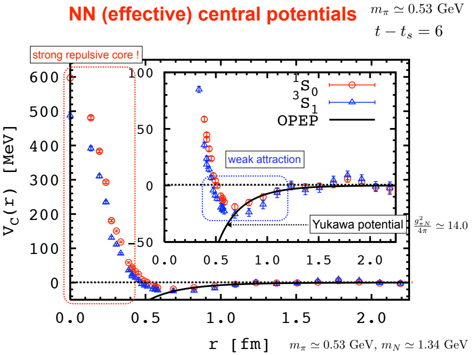


- Tensor part (S-D mixing) [graphs from K. Wendt]



Determining the nuclear potential from lattice QCD

[S. Aoki, *Hadron interactions in lattice QCD*, arXiv:1107.1284]



Bethe-Salpeter amplitude

$$\varphi_E(\vec{r}) = \langle 0 | N(\vec{x}, 0) N(\vec{y}, 0) | 2N, E \rangle$$

Nucleon fields

2N state with energy E

- define non-local $U(\mathbf{x}, \mathbf{y})$

$$[E - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x}, \mathbf{y})\varphi_E(\mathbf{y})$$

- Expand $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta(\mathbf{x} - \mathbf{y})$ to get AV18 form of local V

- Why not just calculate energy as function of separation $\Longrightarrow V(r)$?
  - Only works in heavy mass limit (e.g., works for B-mesons)
- But is this unique? No!
  - choice of nucleon interpolating field $\Longrightarrow$ different $V(\mathbf{x})$
  - choice of “wave function” smearing (changes overlap)

Determining the nuclear potential from lattice QCD

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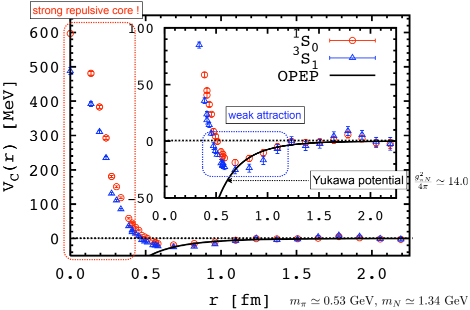
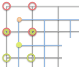
NN (effective) central potentials $m_\pi \simeq 0.53$ GeV $t - t_s = 6$

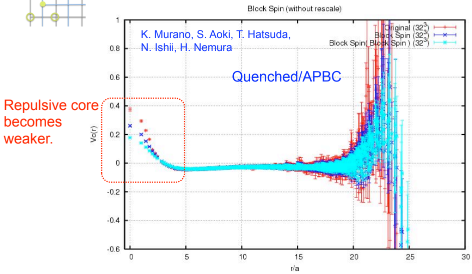


Smearing and potentials

Preliminary



Wave function is smeared. → "smeared" potential



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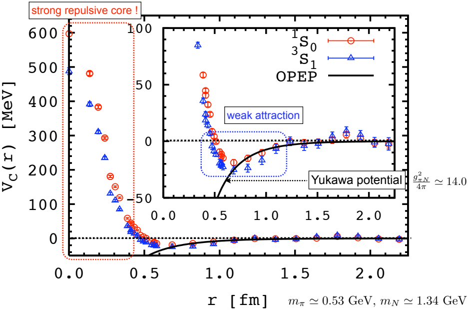

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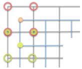

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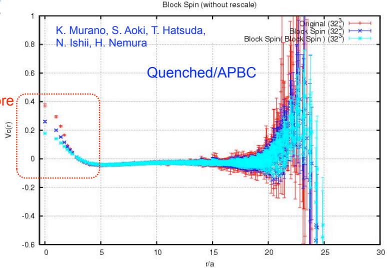
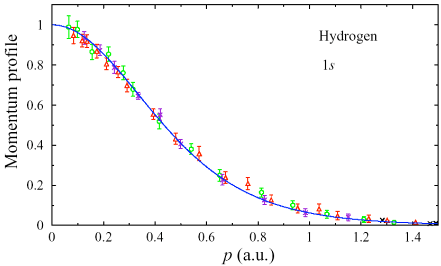
Repulsive core becomes weaker.



- "...the potential depends on the choice of nucleon operator..." which "...is considered to be a 'scheme' to define the potential."
- "Is such a scheme-dependent quantity useful? The answer to this question is probably 'yes', since the potential is useful to understand or describe the phenomena."
- Claim: useful to choose a scheme that yields good convergence of the velocity expansion (close to local)

Are wave functions measurable? [from W. Dickhoff]

Atoms studied with the (e,2e) reaction

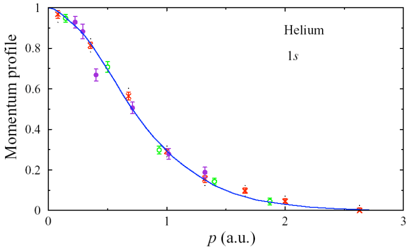


$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms …

Helium
in Phys. Rev. A8, 2494 (1973)



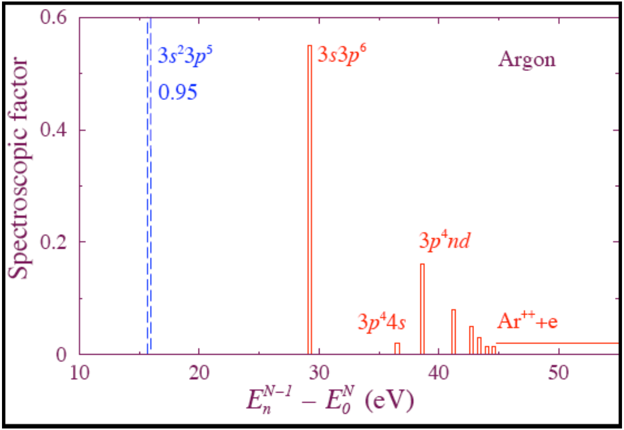
- But compare approximations for $(e, 2e)$ on atoms to those for $(e, e'p)$ on nuclei! (Impulse approx., FSI, vertex, . . .)

Spectroscopic factors in atoms

For a bound final N-1 state the spectroscopic factor is given by $S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}} \middle| \Psi_0^N \right\rangle \right|^2$

For H and He the 1s electron spectroscopic factor is 1

For Ne the valence 2p electron has S=0.92 with two additional fragments, each carrying 0.04, at higher energy.



Argon
3p and 3s
strength

Closed-shell
atoms
n(α) = 0 or 1

One-body scattering, small scheme dependence $\Longrightarrow$ robust SF

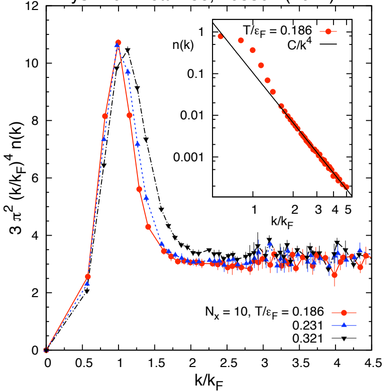
Unitary cold atoms: Is $n(k)$ observable?

- Tail of momentum distribution + contact [Tan; Braaten/Platter]

$$n(k) \xrightarrow{k\to\infty} \frac{C}{k^4}$$

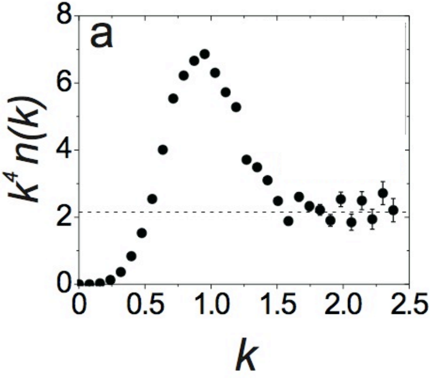
Theory (lattice)

J. E. Drut, T. A. Lähde, T. Ten
Phys. Rev. Lett. **106**, 205302 (2011)



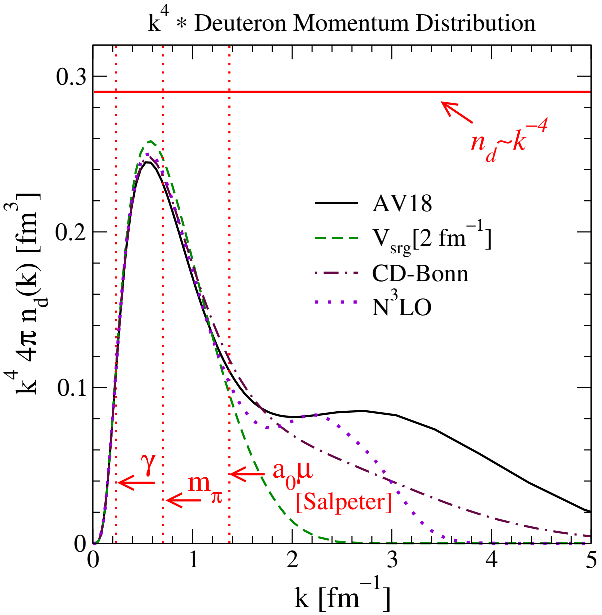
Experiment

J. T. Stewart et al
PRL **104**, 235301 (2010)



- When $R/a_s \ll kR \ll 1 \Longrightarrow$ tiny scheme dependence

Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



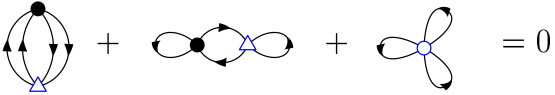
- E.g., extract from electron scattering?
- No region where $1/a_s \ll k \ll 1/R$
- Scheme dependent high-momentum tail!
- $n(k)$ from $V_{\rm SRG}$ has *no* high-momentum components!
- But $n(k)$ from $U a_k^\dagger a_k U^\dagger$ is unchanged $\Longrightarrow$ two-body operator!

Using EFT and field redefinitions as tool

- EFT: $\mathcal{L}_{\rm eft} = \psi^\dagger\big[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M}\big]\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$
  - general short-range interactions, but not unique!
- Try simple field redefinition to check scheme dependence:

$$\psi \longrightarrow \psi + \alpha\frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi \qquad \alpha \sim \mathcal{O}(1) \Longrightarrow \text{“natural”} \Longrightarrow \text{estimate!}$$

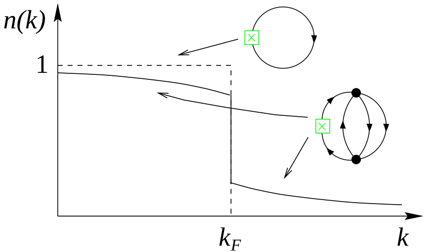
  - “new” vertices: 2–body off-shell $\triangle$, 3–body $\circ \propto \frac{8\pi\alpha}{\Lambda^3} C_0(\psi^\dagger\psi)^3$
  - asymptotic “on-shell” quantities (S-matrix elements) must be unchanged by redefinition
- Energy density is model (α) independent *if* all terms kept
  - sum of new terms is zero, so energy is unchanged



- What about momentum occupation number?

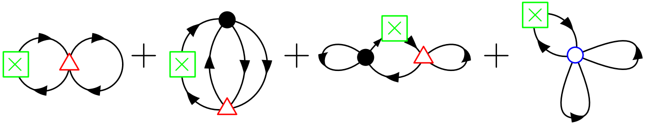
Occupation No. $\Longrightarrow$ Momentum Distribution

- Insert $a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \Longrightarrow \boxtimes$



- But nonzero contribution $\Delta n(k)$ from induced vertices:

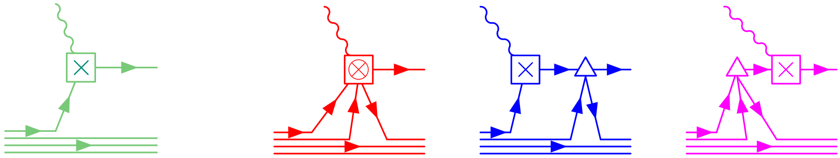
$\Delta n(k) =$



- There is no *preferred* definition for transformed operator
  - $\Longrightarrow$ only defined for specific convention
  - $\Longrightarrow$ momentum distributions for different schemes differ

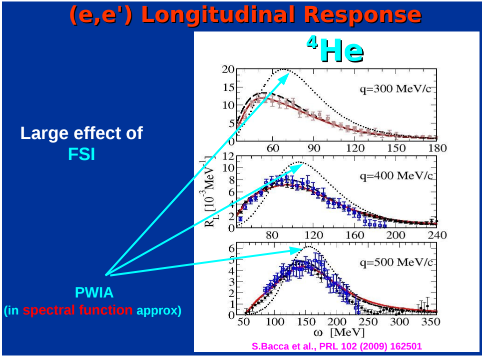
Analysis of (e,e'p) Experiments? [cf. (e,2e) on atoms]

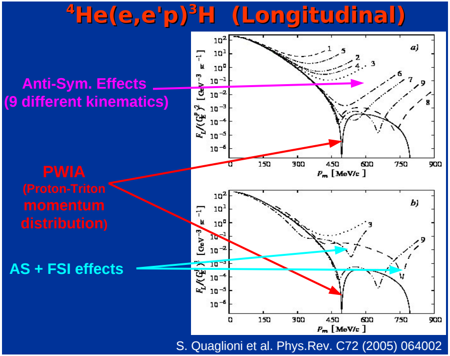
- Suppose external source $J(x)$ coupled to fermions
  - EFT: need most general current coupled to $J(x)$ for all α
- Consider lowest order with simplest ($\alpha = 0$) current
  - if $\alpha = 0$, just impulse approximation $J\psi^\dagger\psi$



  - if $\alpha \neq 0$ [recall $\psi \longrightarrow \psi + \alpha\frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi$], then same cross section *only* if vertex contribution from modified operator *and* modified final (and initial) state interactions are included
- There are *always* contributions from all three at each order
  - sub-leading pieces are mixed by field redefinitions
    $\Longrightarrow$ isolating $J\psi^\dagger\psi$ is model dependent
  - How large is ambiguity? Set by natural size $\alpha \sim \mathcal{O}(1)$

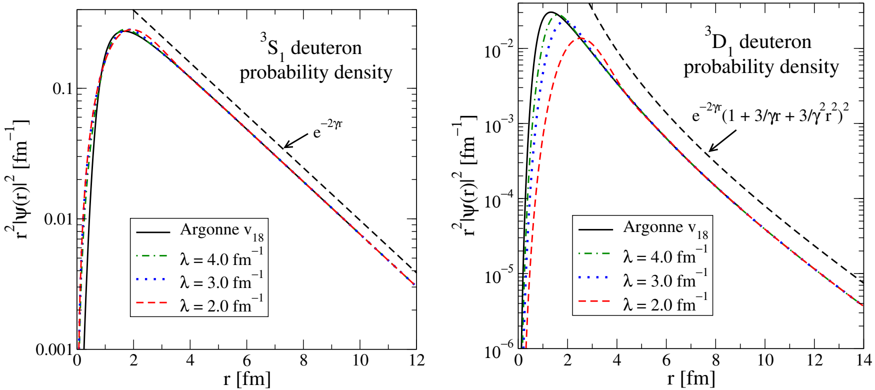
Ab initio electron scattering with LIT [from G. Orlandini]





- Ab initio calculations of longitudinal (e, e') response functions show importance of FSI for quasi-elastic regime
  - PWIA fails for quasi-elastic peak and at low ω
  - FSI effects decrease with q in peak but not at low ω
- Direct proton knockout and neglect of FSI tested for $(e, e'p)$
  - Both antisymmetrization effects and FSI play important roles
  - Approximate estimates of FSI effects can be poor

Why are ANC's different? Coordinate space

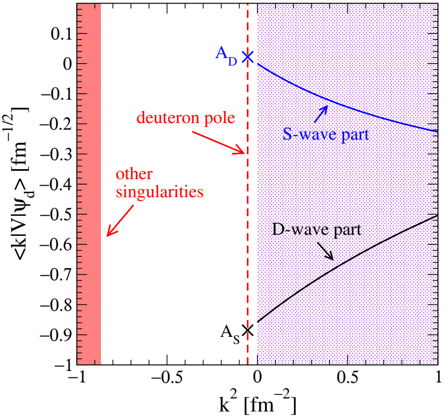


- ANC's, like phase shifts, are asymptotic properties
  $\Longrightarrow$ short-range unitary transformations do not alter them
  [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle\mathbf{k}|\psi_n\rangle + \langle\mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle\mathbf{k}|\psi_n\rangle$$
$$\Longrightarrow \langle\mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle\mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$
2. $$\langle\mathbf{r}|\psi_n\rangle = \int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}\langle\mathbf{k}|\psi_n\rangle \xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$
3. integral dominated by pole from 1.
4. extrapolate $\langle\mathbf{k}|V|\psi_n\rangle$ to $k^2 = -\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Inverse scattering puzzle: A_n uniquely determined because assumed longest-range part of V from one-pion exchange
- Next vertex singularity at $-(\gamma + m_\pi)^2 \Longrightarrow$ same for FSI

More questions and some possible answers

How should one choose a scheme/convention?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - (Near-) local potential: quantum Monte Carlo methods work
  - Low-k potential: many-body perturbation theory works, or to make microscopic connection to shell model
- Better interpretation or intuition $\Longrightarrow$ predictability
- Use range of schemes to test calculations and learn physics

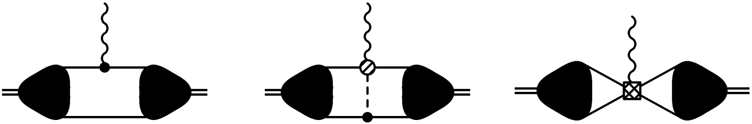
Can we (should we) use a reference Hamiltonian?

- That is, to allow us to make comparisons
- If so, which one? (cleanest extraction from experiment?)

More questions and some possible answers

How do we match Hamiltonians and operators?

- Use EFT perspective
  - E.g., electromagnetic currents [D.R. Phillips, nucl-th/0503044]



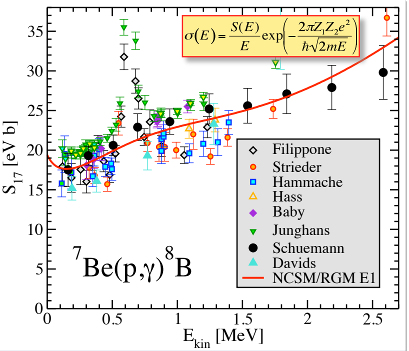
$O(e)$ $\quad$ $O(eP^3)$ $\quad$ $O(eP^5)$

  - Model independent because complete (up to some order)
  - Can identify consistent operator and interaction
  - Tells you when new info is required
- Use RG as tool to evolve consistent operators

Can EFT or RG help to construct optical potentials?

Is it ok to fine-tune SRG λ? [e.g. Navratil, Quaglioni, Roth]

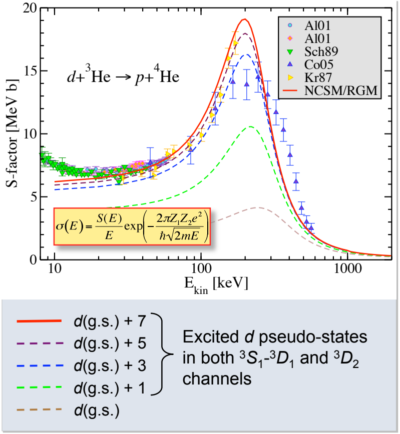
$^7Be(p,\gamma)^8B$ astrophysical S-factor



Ab initio theory predicts simultaneously both normalization and shape of S_{17}.

- arXiv:1105.5977
- SRG $\lambda = 1.86\,\mathrm{fm}^{-1}$
- match separation energy

$^3He(d,p)^4He$ astrophysical S-factor



- arXiv:1009.3965
- SRG $\lambda = 1.5\,\mathrm{fm}^{-1}$
- low $\lambda \Longrightarrow$ more convergent

What about long-range correlations?

- SF calculations with FRPA
- N^3LO Hamiltonian
  - Soft $\Longrightarrow$ small SRC
  - SRC contribution changes dramatically with lower resolution
- Compare short-range correlations (SRC) to long-range correlations from particle-vibration coupling
- LRC $\gg$ SRC!!
- Are long-range correlations scheme dependent?

C. Barbieri, PRL 103 (2009)

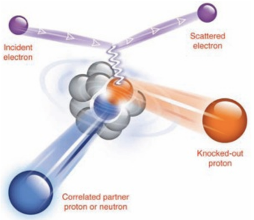
TABLE I. Spectroscopic factors (given as a fraction of the IPM) for valence orbits around ^{56}Ni. For the SC FRPA calculation in the large harmonic oscillator space, the values shown are obtained by including only SRC, SRC and LRC from particle-vibration couplings (full FRPA), and by SRC, particle-vibration couplings and extra correlations due to configuration mixing (FRPA + ΔZ_α). The last three columns give the results of SC FRPA and SM in the restricted $1p0f$ model space. The ΔZ_αs are the differences between the last two results and are taken as corrections for the SM correlations that are not already included in the FRPA formalism.

	10 osc. shells			Exp. [29]	$1p0f$ space		
	FRPA (SRC)	Full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	−0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	−0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	−0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	−0.03
^{57}Cu:							
$\pi 1p_{1/2}$	0.96	0.66	0.62		0.80	0.76	−0.04
$\pi 0f_{5/2}$	0.96	0.60	0.58		0.80	0.78	−0.02
$\pi 1p_{3/2}$	0.96	0.67	0.65		0.81	0.79	−0.02
^{55}Co:							
$\pi 0f_{7/2}$	0.95	0.73	0.71		0.89	0.87	−0.02

Parton distributions as paradigm: Factorization

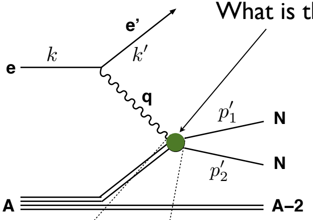
- PDF analysis: part of convolution for cross section can be calculated reliably for given experimental conditions so that the remaining part can be extracted as a universal quantity, to be related to other processes and kinematic conditions
- For hard-scattering processes with large momentum transfer scale Q, *factorization* allows separation of momentum and distance scales in reaction
  - The time scale for binding interactions in the rest frame is time dilated in the center-of-mass frame, so the interaction of an electron with a hadron in deep-inelastic scattering is with single non-interacting partons
  - Short-distance part calculated systematically in low-order perturbative QCD; long-distance part identified in PDF's (momentum distribution for partons in hadrons)
- PDF's relate deep inelastic scattering of leptons, Drell-Yan, jet production, and more
  - Measure in limited set of reactions and then perturbative calculations of hard scattering and PDF evolution enable first principles predictions of cross sections for other processes

Case study: Large Q^2 electron scattering



Subedi et al., Science 320, 1476 (2008)

What is this vertex?



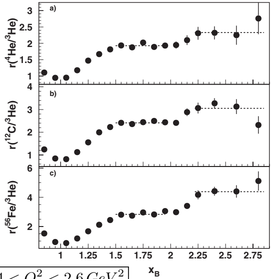
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

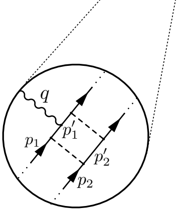
$$x_B = \frac{Q^2}{2m_N\nu}$$

Higinbotham, arXiv:1010.4433



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)



SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

---

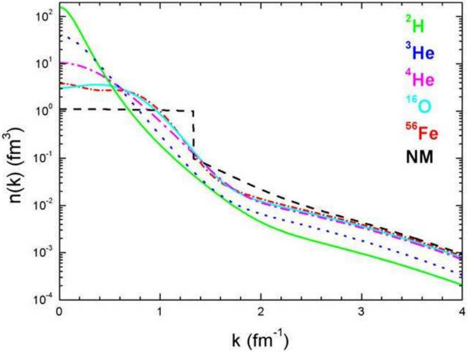
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

- K. Hebeler/E. Anderson $\Longrightarrow$ evolve *operators* to low resolution

Nuclear scaling: $n_A(p) \approx C_A n_D(p)$ at large p

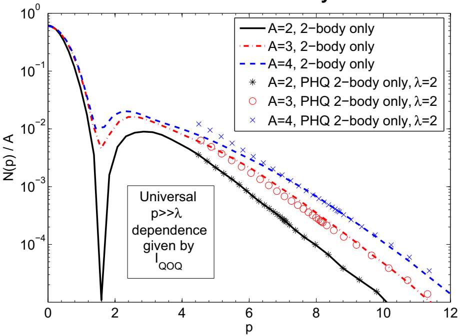
- Two-nucleon correlation model



From C. Ciofi degli Atti and S. Simula.

- Conventional explanation: Dominance of NN potential and short-range correlations (Frankfurt et al.)

- Test case: A bosons in toy 1D model



- Alternative: *factorization*

$$\int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \left[I_{QOQ} K_\lambda(k') K_\lambda(k) \right] \psi_\lambda(k)$$

- $I_{QOQ} \equiv$

$$\int_\lambda^\infty dq' \int_\lambda^\infty dq \left[Q_\lambda(q') \widehat{O}(q', q) Q_\lambda(q) \right]$$

- universal p dependence from I_{QOQ}
- C_A factor from low-momentum m.e.

Factorization with SRG [Anderson et al., arXiv:1008.1569]

- If $k < \lambda$ and $q \gg \lambda \Longrightarrow$ factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

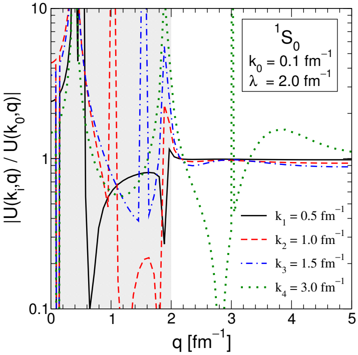
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2 dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\Big] \gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

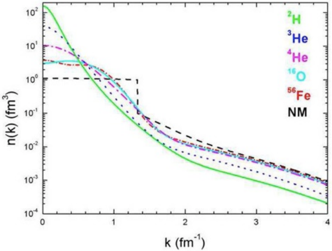
$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i) Q_\lambda(q)}{K_\lambda(k_0) Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

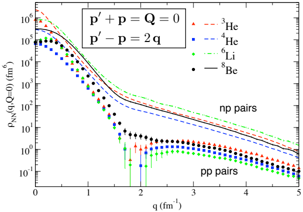
- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$



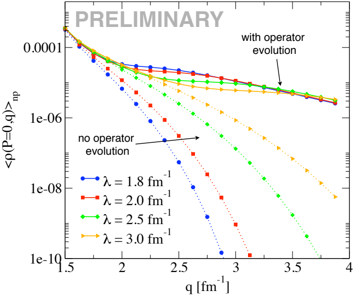
Simpler calculations of pair densities

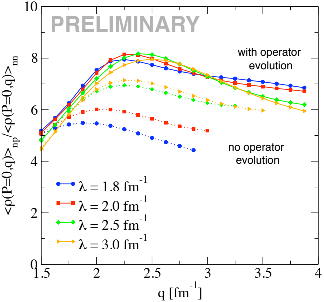


taken from Ciofi degli Atti, Simula PRC 53, 1689 (1996)



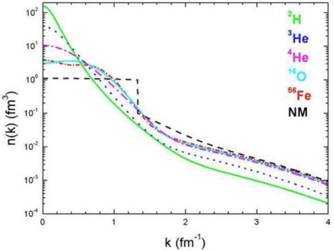
Schiavilla et al. PRL 98, 132501 (2007)





Many-body perturbation theory may be sufficient at low resolution!

Simpler calculations of pair densities



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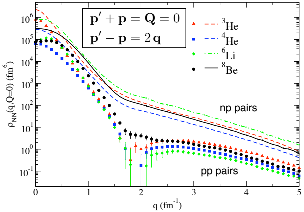
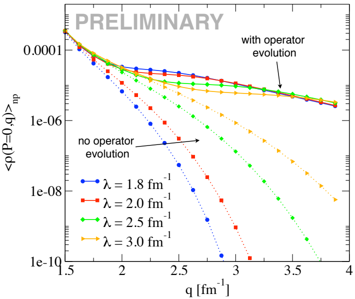
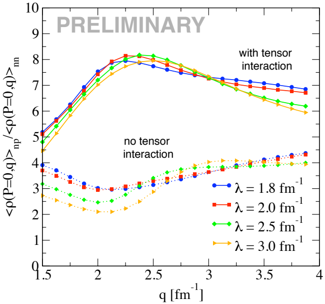


Schiavilla et al. PRL 98, 132501 (2007)





Many-body perturbation theory may be sufficient at low resolution!

Recap

- Summary points
  - Scheme-dependent observables are robust for some systems, but not generally for nuclei! Must specify it! (e.g., $V_{NN\cdots N}$, $\widehat{O}$'s)
  - SF's are scheme-dependent, ANC's are (generally) not
    - surface-integral formulation sounds promising!
  - Unitary transformations show *natural* scheme dependence
  - Parton distribution functions as a paradigm
    $\Longrightarrow$ Can we have controlled factorization?
- Questions for which EFT/RG may help
  - How should one choose a scheme/convention?
  - Can we (should we) use a reference Hamiltonian?
  - What *is* the scheme-dependence of SF's and other quantities?
  - Is the assumption of one-body operators viable? Required?
  - How do we match Hamiltonians and operators?
  - What can EFT or RG say about N-nucleus optical potentials?
  - What is the role of short-range/long-range correlations?
  - . . . and many more!