

Application of Three-Body methods in Direct Nuclear Reactions: 6 He (p,p) 6 He

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Direct Reactions:

- \bullet Elastic & inelastic scattering
- \bullet Few-particle transfer (stripping, pick-up)
- \bullet Charge exchange
- •

Knockout (d,p) reaction

Three-Body Problem

Single Scattering

Three-body problem with particles:

 $o - i - (A-1)$ -core

o – i : NN interaction

 $i - (A-1)$ core : e.g. mean field force

Phenomenological Optical Potentials parameterize single scattering term

 $\overline{0}$

 $A-1$

Microscopic Optical Potentials "Folding Models" for closed shell nuclei

- \bullet Watson Multiple Scattering
	- Elster, Weppner, Chinn, Thaler, Tandy, Redish
		- Separation of p-A and n-A optical potential
		- Based on NN t-matrix as interaction input
		- Treating of interaction with (A-1)-core via mean field and as implicit three-body problem
- \bullet Kerman-McManus-Thaler (KMT)
	- Crespo, Johnson, Tostevin, Thompson
		- Based on NN t-matrix as input
		- Couple explicitly to (A-1) core
		- Introduce cluster ansatz for halo targets within coupled channels
- G-matrix folding
	- Arellano, Brieva, Love
		- Based on NN g-matrix
		- Improving local density approximation
	- Picked up by Amos, Karataglidis and extended to exotic nuclei

RIKEN: 6He(p,p) 6He @ 71 MeV

S. Sakaguchi et al. arXiv: 1106.3903 [nucl-exp]

Task: Optical Potential for Halo Nucleus

RIKEN: 6He(p,p) 6He

S. Sakaguchi et al. arXiv: 1106.3903

We adopted a standard Woods-Saxon optical potential with a spin-orbit term of the Thomas form:

$$
U_{\text{OM}}(R) = -V_0 f_r(R) - i W_0 f_i(R)
$$

+
$$
4i a_{id} W_d \frac{d}{dR} f_{id}(R)
$$

+
$$
V_s \frac{2}{R} \frac{d}{dR} f_s(R) L \cdot \sigma_p + V_{\text{C}}(R)
$$
 (1)

with

$$
f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x}A^{1/3}}{a_x}\right)\right]^{-1} \qquad (2)
$$

$$
(x = r, i, id, \text{ or } s).
$$

Scattering: Lippmann-Schwinger Equation

- \bullet LSE: T = V + V G $_{\rm 0}$ T
- $\bullet\,$ Hamiltonian: $\, {\mathsf H} = {\mathsf H}_0^{} + {\mathsf V}$
- $\bullet\;$ Free Hamiltonian: $\;$ H $_{\rm 0}$ = h $_{\rm 0}$ + H $_{\rm A}$
	- $-$ h $_{\rm 0}$: kinetic energy of projectile '0'
	- H_A: target hamiltonian with $\ H_{\text{A}}\ket{\Phi}$ = E_A $\ket{\Phi}$
- V: interactions between projectile '0' and target nucleons 'i' $\;\;\mathsf{V}=\Sigma$ $\mathsf{A}_{\mathsf{i}=0}\;\mathsf{V}_{\mathsf{0}\mathsf{i}}$
- Propagator is (A+1) body operator

$$
- G_0(E) = (E - h_0 - H_A + i \varepsilon)^{-1}
$$

Elastic Scattering

- $\bullet~$ In- and Out-States have the target in ground state $\Phi_{\rm o}$
- Projector on ground state $\mathsf{P} = |\Phi_0\rangle\langle\Phi_0|$
	- With $1 = P + Q$ and $[P,G_0] = 0$
- For elastic scattering one needs
- • $P T P = P U P + P U P G0(E) P T P$
- Or
	- **T = U + U G 0(E) P T**
	- **U = V + V G 0(E) Q U** ⇐ **optical potential**

Standard: $U^{(1)} \approx \Sigma^{A}{}_{i=0} \tau_{0i}$ (1st order) with

$$
\boldsymbol{\tau}_{0i} = \boldsymbol{v}_{0i} + \boldsymbol{v}_{0i}~\boldsymbol{G}_0(E)~\boldsymbol{Q}~\boldsymbol{\tau}_{0i}
$$

$\tau_{0i} = \rm v_{0i} + \rm v_{0i}~G_{0}(E)$ Q τ_{0i}

- $G_0(E) = (E h_0 H_A + i \varepsilon)^{-1} = (A + 1)$ body operator
	- Standard **"impulse approximation":**
	- Average over ${\sf H}_{\sf A} \Rightarrow$ c-number
	- $\;\rightarrow$ $\mathrm{G}_{0}(\mathrm{e})$ ==: two body operator
- Deal with **Q**

 $\mathcal{L}_{\mathcal{A}}$

- Define "two-body" operator t_{oi}^{free} by
- $-$ t_{0i}^{free} = v_{0i} + v_{0i} G₀(e) t_{0i}^{free}
- and relate via integral equation to $\bm{\tau}_{\text{oi}}$

– ^τ**oi = t0ifree - t0ifree G0(e)** ^τ**oi [integral equation]**

 Important for keeping correct iso-spin character of optical potential

$$
-\qquad \qquad \mathbf{U}^{(1)} = \Sigma_{i=1}^{A} \tau_{oi} =: \mathbf{N} \tau_{n} + \mathbf{Z} \tau_{p}
$$

First order Watson optical potential $\mathbf{U}^{(1)}=\Sigma$ $\mathbf{A}_{\mathbf{i}=1}$ $\mathbf{\tau}_\mathrm{oi}$ $=:\Sigma$ **N i=1** τ **n ⁺** Σ **P i=1** τ **p**

- •Important for treating N ≠Z nuclei
- •Be sensitive to proton vs. neutron scattering
- •In general

 $-$ **t**_{pp} \neq **t**_{np} and ρ _p \neq ρ _n

• These differences enter in a non-linear fashion into first order Watson optical potential

 $\tau_\alpha = \mathbf{t}_\alpha - \mathbf{t}_\alpha \ \mathbf{G_0}^\alpha(\mathbf{e}) \ \tau_\alpha$, α=n,p

• **The Watson ansatz allows introducing a cluster ansatz for a nucleus very naturally**

Isospin effects in elastic p+A scattering, Chinn, Elster, Thaler, PRC47, 2242 (1993)

More formal:

- $T_{el} = PUP + PUPG_0(E)PT_{el}.$ Elastic scattering : \bullet
- \bullet First order Watson O.P.:

$$
\langle \mathbf{k'} | \langle \phi_A | P U P | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k'}, \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k'} | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle
$$

 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

Proton scattering: $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$

Calculate: $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

 $\langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 \mathbf{k}_4 \dots \mathbf{k}_A | \phi_A \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4 \dots + \mathbf{k}_A - \mathbf{p}_0) \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \dots \zeta_{A-1} | \phi_A \rangle.$

$$
\langle \hat{\tau}_{01} \rangle = \int \prod_{j=1}^{A} d\mathbf{k}'_j \int \prod_{l=1}^{A} d\mathbf{k}_l \langle \phi_A | \zeta_1' \zeta_2' \zeta_3' \zeta_4' \dots \zeta_{A-1}' \rangle \delta(\mathbf{p}' - \mathbf{p}'_0) \langle \mathbf{k}' \mathbf{k}'_1 | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \mathbf{k}_1 \rangle
$$

$$
\prod_{j=2}^{A} \delta(\mathbf{k}'_j - \mathbf{k}_j) \delta(\mathbf{p} - \mathbf{p}_0) \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \dots \zeta_{A-1} | \phi_A \rangle,
$$
 (2.48)

With single particle density matrix :

$$
\rho(\zeta_1', \zeta_1) \equiv \int \prod_{l=2}^{A-1} d\zeta_1' \int \prod_{j=2}^{A-1} d\zeta_3 \langle \phi_A | \zeta_1' \zeta_2' \zeta_3' \zeta_4 \dots \zeta_{A-1} \rangle \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \dots \zeta_{A-1} | \phi_A \rangle.
$$

$$
\langle \hat{\tau}_{01} \rangle = \int d\zeta_1' \int d\zeta_1 \langle \mathbf{k}' | \zeta_1' + \frac{\mathbf{p}_0'}{A} | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} | \zeta_1 + \frac{\mathbf{p}_0}{A} \rangle \rho(\zeta_1', \zeta_1)
$$

$$
\delta(\frac{A-1}{A} \mathbf{p}_0' - \zeta_1' - \frac{A-1}{A} \mathbf{p}_0 + \zeta_1).
$$

$$
\begin{array}{ll}\n\text{Better Variables:} & \mathbf{k} = \mathbf{K} - \frac{1}{2}\mathbf{q} & \zeta_1 = \mathbf{P} + \frac{A-1}{2A}\mathbf{q} \\
& \mathbf{k}' = \mathbf{K} + \frac{1}{2}\mathbf{q} & \zeta_1' = \mathbf{P} - \frac{A-1}{2A}\mathbf{q}.\n\end{array}
$$
\n
$$
\langle \hat{\tau}_{01} \rangle = \langle \frac{1}{2} (\mathbf{K} - \mathbf{P} + \frac{2A-1}{2A}\mathbf{q} - \frac{\mathbf{p}_0'}{A}) | \hat{\tau}_{01}(\hat{\mathcal{E}}) | \frac{1}{2} (\mathbf{K} - \mathbf{P} - \frac{2A-1}{2A}\mathbf{q} - \frac{\mathbf{p}_0}{A}) \rangle
$$
\n
$$
\rho(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}).\n\end{array} \tag{2.59}
$$
\n
$$
\begin{array}{ll}\nU_{el}(\mathbf{q}, \mathbf{K}) = \sum_{i=n, p} \int d\mathbf{P} \hat{\tau}_{0i}(\mathbf{q}, \frac{1}{2} (\frac{A+1}{A}\mathbf{K} - \mathbf{P}), \hat{\mathcal{E}}) \rho_i(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}) \\
& \mathbf{k}' - \frac{\mathbf{p}_0}{\mathbf{p}_0} \\
& \mathbf{k}' - \frac{\mathbf{p}_0}{\mathbf{p}_0} \\
& \mathbf{k} \end{array} \qquad \mathbf{k}' - \frac{\mathbf{p}_0}{\mathbf{p}_0} \\
\mathbf{k}' - \frac{\mathbf{p}_0}{\mathbf{p}_0} \\
\mathbf{p}_0\n\end{array}
$$

Cluster Folding Optical Potential (n+n+ α **)**

$$
\mathbf{p}_{j i} = \frac{1}{A} (A_{s i} \mathbf{p}_{i} - A_{i} \mathbf{p}_{s i})
$$

Correlation Density

Jacobi momenta

$$
\rho_{corr}(\mathbf{p}_{j_1},\mathbf{p}_{j_1}')\equiv \int \prod_{l=2}^{N_c} d\mathbf{p}_{j_l}' \int \prod_{m=2}^{N_c} d\mathbf{p}_{j_m} \langle \phi_A\vert \mathbf{p_j}_1' \mathbf{p_j}_2'... \mathbf{p_j}_{N_c} \rangle \; \langle \mathbf{p_{j_1}p_{j_2}}... \mathbf{p_{j_{N_c}}} \vert \phi_A \rangle
$$

Cluster optical potential

$$
U_{el}(\mathbf{q}, \mathbf{K}) = \sum_{c=1, N_c} \sum_{i=n_c, p_c} \int d\mathbf{P} \, d\mathcal{P}_{j_c} \, \rho_{corr}(\mathcal{P}_{j_c})
$$

$$
\hat{\tau}_{0i} \left(\mathbf{q}, \frac{1}{2} \left(\frac{A+1}{A} \mathbf{K} - \mathbf{P} \right), \mathcal{E} \right) \, \rho_{c_i} \left(\mathbf{P} - \frac{A-1}{2A} \mathbf{q}, \mathbf{P} + \frac{A-1}{2A} \mathbf{q} \right)
$$

Optical Potential for 6He as cluster α**+n+n**

Cluster folding potential for ⁶He+p

$$
\sum_{i=n,p}^{6\text{He}} U_{el}(\mathbf{q}, \mathbf{K}) = U_{\alpha} + 2U_{n} =
$$
\n
$$
\sum_{i=n,p} \int d\mathbf{P} \, d\mathcal{P}_{j_{\alpha}} \, \rho_{corr}(\mathcal{P}_{j_{\alpha}}) \, \hat{\tau}_{0i} \left(\mathbf{q}, \frac{1}{2} \left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \, \rho_{\alpha i} \left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right)
$$
\n
$$
+ 2 \int d\mathbf{P} \, d\mathcal{P}_{j_{n}} \, \rho_{corr}(\mathcal{P}_{j_{n}}) \, \hat{\tau}_{0n} \left(\mathbf{q}, \frac{1}{2} \left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \, \rho_{n} \left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right).
$$

For calculation:NN t-matrix: Nijmegen II potential Densities:COSMA density $== s & p$ - shell harmonic oscillator wave functionsFitted to give rms radius of ⁶He

and for 4He: Gogny density

6He (p,p) 6He @ 71 MeV

6He (p,p) 6He @ 71 MeV

4He (p,p) 4He

4He (p,p) 4He

Black: Free NN-tmatrix

Red: HFB mean field included

6He (p,p) 6He

6He (p,p) 6He

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4i a_{id} W_d \frac{d}{dR} f_{id}(R)
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+
$$
V_s \frac{2}{R} \frac{d}{dR} f_s(R) L \cdot \sigma_p + V_{\text{C}}(R)
$$
 (1)

with

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f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x}A^{1/3}}{a_x}\right)\right]^{-1} \qquad (2)
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(x = r, i, id, \text{ or } s).
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S.P. Weppner (preliminary): Phenomenological OP for 4He, 6Li, 6He

Ansatz for parameters: $V_i \rightarrow V_i (A_0 + (A-4) B_0 + (N-Z)C_0)$

S.P. Weppner (preliminary): Phenomenological OP for 4He, 6Li, 6He

Ansatz for parameters: $V_i \rightarrow V_i (A_0 + (A-4) B_0 + (N-Z)C_0)$

Finding: surface term for ⁶He is negative.

Non-standard

Back to derivation of optical potential:

NN amplitude $f_{NN}(k'k;E) \approx \langle k' | t_{NN}(E) | k \rangle$

How do those enter the optical potential ?

More formal:

- $T_{el} = PUP + PUPG_0(E)PT_{el}.$ Elastic scattering : \bullet
- \bullet First order Watson O.P.:

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$$

 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

Proton scattering: $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$

More precisely:

(A+1) state:
$$
|\phi_n(A)\varphi_k\rangle = |\omega_n(A-1)\chi_n\varphi_k\rangle
$$

$$
\langle \varphi_{k'}\phi_n(A)|t|\phi_n(A)\varphi_k\rangle = \langle \varphi_{k'}\chi_n(p')\omega_n(A-1)|t|\omega_n(A-1)\chi_n(p)\varphi_k\rangle
$$

$$
+ \langle \varphi_{k'}\chi_n(p')|t|\chi_n(p)\varphi_k\rangle \langle \omega_n(A-1)|\omega_n(A-1)\rangle
$$

e.g. L⋅S term $\hat{n} = \frac{\mathbf{q} \times \mathbf{Q}}{|\mathbf{q} \times \mathbf{Q}|}$ $i\left[\left(\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)\right]\cdot\hat{n} \ C(\mathbf{q},\mathbf{Q})\right]$ =Wolfenstein C :

 $iC(\mathbf{q},\mathbf{Q})\langle \varphi_{k'}\chi_n(p')| [(\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2))\cdot\hat{n}]|\varphi_k\chi_n(p)\rangle =$

- = $iC(\mathbf{q},\mathbf{Q})\langle\varphi_{k'}\chi_n(p')|\boldsymbol{\sigma}(1)\cdot\hat{n}|\varphi_k\chi_n(p)\rangle+iC(\mathbf{q},\mathbf{Q})\langle\varphi_{k'}\chi_n(p')|\boldsymbol{\sigma}(2)\cdot\hat{n}|\varphi_k\chi_n(p)\rangle$
- = $iC(\mathbf{q},\mathbf{Q}) \langle \varphi_{k'}|\boldsymbol{\sigma}(1)\cdot\hat{n}|\varphi_k\rangle \langle \chi_n(p')|\chi_n(p)\rangle + iC(\mathbf{q},\mathbf{Q}) \langle \varphi_{k'}|\varphi_k\rangle \langle \chi_n(p')|\boldsymbol{\sigma}(2)\cdot\hat{n}|\chi_n(p)\rangle$

$$
= iC(\mathbf{q}, \mathbf{Q}) \sigma \cdot \hat{n} \rho(\mathbf{q}, \mathbf{Q}) + iC(\mathbf{q}, \mathbf{Q}) \langle \chi_n(p') | \sigma(2) \cdot \hat{n} | \chi_n(p) \rangle
$$
\n(5)

\nRegular spin-orbit only zero for closed shell nuclei

NN amplitude $f_{NN}(k'k;E) \approx \langle k'|t_{NN}(E)|k\rangle$

6 invariant amplitudes off-shell (5 on-shell)

Status

- Cluster ansatz implemented into Watson optical potential for 6He
	- Correlation visible in dσ/dΩ at forward angles
	- Good description of 4He important
	- Cluster ansatz can be implemented for 8He in similar fashion
- Good global phenomenological optical potential fit to ⁴He, ⁶Li and ⁶He with non-standard surface term
	- Guidance from microscopic calculations needed
- For non-closed shell nuclei all NN Wolfensteinamplitudes contribute to the optical potential
	- Calculations with COSMA p-shell neutrons under way
- On CE wishlist: dσ/dΩ and A_y at another energy (150 or 200 MeV/nucleon)