

# Application of Three-Body methods in Direct Nuclear Reactions: <sup>6</sup>He (p,p) <sup>6</sup>He

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## **Direct Reactions:**

- Elastic & inelastic scattering
- Few-particle transfer (stripping, pick-up)
- Charge exchange
- Knockout

(d,p) reaction



Three-Body Problem



# Single Scattering

Three-body problem with particles:

o - i - (A-1)-core

o-i: NN interaction

i - (A-1) core : e.g. mean field force

**Phenomenological Optical Potentials parameterize single scattering term** 

0

A-1

## Microscopic Optical Potentials "Folding Models" for closed shell nuclei

- Watson Multiple Scattering
  - Elster, Weppner, Chinn, Thaler, Tandy, Redish
    - Separation of p-A and n-A optical potential
    - Based on NN t-matrix as interaction input
    - Treating of interaction with (A-1)-core via mean field and as implicit three-body problem
- Kerman-McManus-Thaler (KMT)
  - Crespo, Johnson, Tostevin, Thompson
    - Based on NN t-matrix as input
    - Couple explicitly to (A-1) core
    - Introduce cluster ansatz for halo targets within coupled channels
- G-matrix folding
  - Arellano, Brieva, Love
    - Based on NN g-matrix
    - Improving local density approximation
  - Picked up by Amos, Karataglidis and extended to exotic nuclei

## RIKEN: <sup>6</sup>He(p,p)<sup>6</sup>He @ 71 MeV

S. Sakaguchi et al. arXiv: 1106.3903 [nucl-exp]

Task: Optical Potential for Halo Nucleus



# RIKEN: <sup>6</sup>He(p,p)<sup>6</sup>He

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We adopted a standard Woods-Saxon optical potential with a spin-orbit term of the Thomas form:

$$U_{\rm OM}(R) = -V_0 f_r(R) - i W_0 f_i(R) + 4i a_{id} W_d \frac{d}{dR} f_{id}(R) + V_s \frac{2}{R} \frac{d}{dR} f_s(R) \mathbf{L} \cdot \boldsymbol{\sigma}_p + V_{\rm C}(R)$$
(1)

with

$$f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x}A^{1/3}}{a_x}\right)\right]^{-1}$$
(2)  
(x = r, i, id, or s).



## Scattering: Lippmann-Schwinger Equation

- LSE:  $T = V + V G_0 T$
- Hamiltonian:  $H = H_0 + V$
- Free Hamiltonian:  $H_0 = h_0 + H_A$ 
  - h<sub>0</sub>: kinetic energy of projectile '0'
  - H<sub>A</sub>: target hamiltonian with H<sub>A</sub>  $|\Phi\rangle$  = E<sub>A</sub>  $|\Phi\rangle$
- V: interactions between projectile '0' and target nucleons 'i'  $V = \Sigma^{A}_{i=0} v_{0i}$
- Propagator is (A+1) body operator

$$- G_0(E) = (E - h_0 - H_A + i\epsilon)^{-1}$$

# **Elastic Scattering**

- In- and Out-States have the target in ground state  $\Phi_0$
- Projector on ground state  $P = |\Phi_0\rangle\langle\Phi_0|$ 
  - With 1=P+Q and  $[P,G_0]=0$
- For elastic scattering one needs
- PTP = PUP + PUPGO(E)PTP
- Or
  - $T = U + U G_0(E) P T$
  - $U = V + V G_0(E) Q U \leftarrow optical potential$

U τ<sub>ni</sub>

Standard: 
$$\mathbf{U}^{(1)} \approx \Sigma^{\mathbf{A}}_{i=0} \tau_{0i}$$
 (1<sup>st</sup> order)  
with  $\tau_{0i} = \mathbf{v}_{0i} + \mathbf{v}_{0i} \mathbf{G}_{0}(\mathbf{E})$ 

# $\tau_{0i} = v_{0i} + v_{0i} G_0(E) Q \tau_{0i}$

- $G_0(E) = (E h_0 H_A + i\epsilon)^{-1} == (A+1)$  body operator
  - Standard "impulse approximation":
  - Average over  $H_A \Rightarrow c$ -number
  - $\rightarrow G_0(e) ==:$  two body operator
- Deal with Q
  - Define "two-body" operator t<sub>0i</sub> free by
  - $t_{0i}^{\text{free}} = v_{0i} + v_{0i} G_0(e) t_{0i}^{\text{free}}$
  - and relate via integral equation to  $au_{oi}$

-  $\tau_{oi} = t_{0i}^{\text{free}} - t_{0i}^{\text{free}} G_0(e) \tau_{oi}$  [integral equation]

 Important for keeping correct iso-spin character of optical potential

$$U^{(1)} = \Sigma^{A}_{i=1} \tau_{oi} =: N \tau_{n} + Z \tau_{p}$$

First order Watson optical potential  $U^{(1)} = \sum_{i=1}^{A} \tau_{oi} =: \sum_{i=1}^{N} \tau_{n} + \sum_{i=1}^{P} \tau_{p}$ 

- Important for treating N≠Z nuclei
- Be sensitive to proton vs. neutron scattering
- In general

 $- \quad \boldsymbol{t_{pp}} \neq \boldsymbol{t_{np}} \quad \text{and} \ \ \boldsymbol{\rho_p} \neq \boldsymbol{\rho_n}$ 

• These differences enter in a non-linear fashion into first order Watson optical potential

 $\tau_{\alpha} = t_{\alpha} - t_{\alpha} G_0^{\alpha}(e) \tau_{\alpha}, \quad \alpha = n, p$ 

• The Watson ansatz allows introducing a cluster ansatz for a nucleus very naturally

Isospin effects in elastic p+A scattering, Chinn, Elster, Thaler, PRC47, 2242 (1993)

## More formal:

- Elastic scattering :  $T_{el} = PUP + PUPG_0(E)PT_{el}$ .
- First order Watson O.P.:

$$\langle \mathbf{k}' | \langle \phi_A | PUP | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$



 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$ 

Proton scattering:  $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$ 

## **Calculate:** $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

 $\langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 \mathbf{k}_4 \dots \mathbf{k}_A | \phi_A \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4 \dots + \mathbf{k}_A - \mathbf{p}_0) \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \dots \zeta_{\mathbf{A}-1} | \phi_A \rangle.$ 

$$\langle \hat{\tau}_{01} \rangle = \int \prod_{j=1}^{A} d\mathbf{k}_{j}' \int \prod_{l=1}^{A} d\mathbf{k}_{l} \, \langle \phi_{A} | \zeta_{1}' \zeta_{2}' \zeta_{3}' \zeta_{4}' \dots \zeta_{A-1}' \rangle \delta(\mathbf{p}' - \mathbf{p}_{0}') \, \langle \mathbf{k}' \mathbf{k}_{1}' | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \mathbf{k}_{1} \rangle$$

$$\prod_{j=2}^{A} \delta(\mathbf{k}_{j}' - \mathbf{k}_{j}) \delta(\mathbf{p} - \mathbf{p}_{0}) \, \langle \zeta_{1} \zeta_{2} \zeta_{3} \zeta_{4} \dots \zeta_{A-1} | \phi_{A} \rangle,$$

$$(2.48)$$

With single particle density matrix :

$$\rho(\zeta_{1}',\zeta_{1}) \equiv \int \prod_{l=2}^{A-1} d\zeta_{1}' \int \prod_{j=2}^{A-1} d\zeta_{j} \langle \phi_{A} | \zeta_{1}' \zeta_{2}' \zeta_{3}' \zeta_{4}' \dots \zeta_{A-1}' \rangle \langle \zeta_{1} \zeta_{2} \zeta_{3} \zeta_{4} \dots \zeta_{A-1} | \phi_{A} \rangle.$$

$$\langle \hat{\tau}_{01} \rangle = \int d\zeta_{1}' \int d\zeta_{1} \langle \mathbf{k}' \zeta_{1}' + \frac{\mathbf{p}_{0}'}{A} | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \zeta_{1} + \frac{\mathbf{p}_{0}}{A} \rangle \rho(\zeta_{1}',\zeta_{1})$$

$$\delta(\frac{A-1}{A}\mathbf{p}_{0}' - \zeta_{1}' - \frac{A-1}{A}\mathbf{p}_{0} + \zeta_{1}).$$

## Cluster Folding Optical Potential $(n+n+\alpha)$

$$\mathbf{p}_{j_i} = \frac{1}{A} (A_{s_i} \mathbf{p}_i - A_i \mathbf{p}_{s_i})$$

**Correlation Density** 

Jacobi momenta

$$\rho_{corr}(\mathbf{p}_{j_1}, \mathbf{p}_{j_1}') \equiv \int \prod_{l=2}^{N_c} d\mathbf{p}_{j_l'} \int \prod_{m=2}^{N_c} d\mathbf{p}_{j_m} \langle \phi_A | \mathbf{p}_{j_1}' \mathbf{p}_{j_2}' \dots \mathbf{p}_{j_{N_c}}' \rangle \langle \mathbf{p}_{\mathbf{j}_1} \mathbf{p}_{\mathbf{j}_2} \dots \mathbf{p}_{\mathbf{j}_{N_c}} | \phi_A \rangle$$

**Cluster optical potential** 

$$\begin{aligned} U_{el}(\mathbf{q}, \mathbf{K}) &= \sum_{c=1, N_c} \sum_{i=n_c, p_c} \int d\mathbf{P} \ d\mathcal{P}_{j_c} \ \rho_{corr}(\mathcal{P}_{j_c}) \\ &\hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \ \rho_{ci}\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right) \end{aligned}$$

Optical Potential for <sup>6</sup>He as cluster α+n+n



# Cluster folding potential for <sup>6</sup>He+p

$${}^{^{6}\text{He}}U_{el}(\mathbf{q},\mathbf{K}) = U_{\alpha} + 2U_{n} =$$

$$\sum_{i=n,p} \int d\mathbf{P} \, d\mathcal{P}_{j_{\alpha}} \, \rho_{corr}(\mathcal{P}_{j_{\alpha}}) \, \hat{\tau}_{0i}\left(\mathbf{q},\frac{1}{2}\left(\frac{A+1}{A}\mathbf{K}-\mathbf{P}\right),\mathcal{E}\right) \, \rho_{\alpha i}\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right)$$

$$+ 2\int d\mathbf{P} \, d\mathcal{P}_{j_{n}} \, \rho_{corr}(\mathcal{P}_{j_{n}}) \, \hat{\tau}_{0n}\left(\mathbf{q},\frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}),\mathcal{E}\right) \, \rho_{n}\left(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\mathbf{P}+\frac{A-1}{2A}\mathbf{q}\right).$$

For calculation: NN t-matrix: Nijmegen II potential Densities: COSMA density == s & p- shell harmonic oscillator wave functions Fitted to give rms radius of <sup>6</sup>He

and for <sup>4</sup>He: Gogny density

#### <sup>6</sup>He (p,p) <sup>6</sup>He @ 71 MeV



<sup>6</sup>He (p,p) <sup>6</sup>He @ 71 MeV

![](_page_18_Figure_1.jpeg)

### <sup>4</sup>He (p,p) <sup>4</sup>He

![](_page_19_Figure_1.jpeg)

## <sup>4</sup>He (p,p) <sup>4</sup>He

![](_page_20_Figure_1.jpeg)

#### **Black: Free NN-tmatrix**

#### **Red: HFB mean field included**

<sup>6</sup>He (p,p) <sup>6</sup>He

![](_page_21_Figure_1.jpeg)

## <sup>6</sup>He (p,p) <sup>6</sup>He

![](_page_22_Figure_1.jpeg)

# RIKEN: <sup>6</sup>He(p,p)<sup>6</sup>He

S. Sakaguchi et al. arXiv: 1106.3903

We adopted a standard Woods-Saxon optical potential with a spin-orbit term of the Thomas form:

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(1)

with

$$f_x(R) = \left[1 + \exp\left(\frac{R - r_{0x}A^{1/3}}{a_x}\right)\right]^{-1}$$
(2)  
(x = r, i, id, or s).

![](_page_23_Figure_6.jpeg)

S.P. Weppner (preliminary): Phenomenological OP for <sup>4</sup>He, <sup>6</sup>Li, <sup>6</sup>He

Ansatz for parameters:  $V_i \rightarrow V_i (A_0 + (A-4) B_0 + (N-Z)C_0)$ 

![](_page_24_Figure_2.jpeg)

S.P. Weppner (preliminary): Phenomenological OP for <sup>4</sup>He, <sup>6</sup>Li, <sup>6</sup>He

Ansatz for parameters:  $V_i \rightarrow V_i (A_0 + (A-4) B_0 + (N-Z)C_0)$ 

![](_page_25_Figure_2.jpeg)

Finding: surface term for <sup>6</sup>He is negative.

Non-standard

Back to derivation of optical potential:

# NN amplitude $f_{NN}(k'k;E) \approx \langle k' | t_{NN}(E) | k \rangle$

Invariant Ja	Amplitude Az
11	A (q, x, q, x)
$(\overline{\sigma}_A + \overline{\sigma}_2) \cdot \overline{W}$	$C(q, \mathcal{X}, \tilde{q}, \tilde{\mathcal{X}})$

How do those enter the optical potential?

## More formal:

- Elastic scattering :  $T_{el} = PUP + PUPG_0(E)PT_{el}$ .
- First order Watson O.P.:

$$\langle \mathbf{k}' | \langle \phi_A | PUP | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$

![](_page_27_Figure_4.jpeg)

 $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$ 

Proton scattering:  $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$ 

### More precisely:

(A+1) state: 
$$|\phi_n(A)\varphi_k\rangle = |\omega_n(A-1)\chi_n\varphi_k\rangle$$
  
 $\langle \varphi_{k'}\phi_n(A)|t|\phi_n(A)\varphi_k\rangle = \langle \varphi_{k'}\chi_n(p')\omega_n(A-1)|t|\omega_n(A-1)\chi_n(p)\varphi_k\rangle$   
 $+ \langle \varphi_{k'}\chi_n(p')|t|\chi_n(p)\varphi_k\rangle \langle \omega_n(A-1)|\omega_n(A-1)\rangle$ 

e.g. L·S term =Wolfenstein C :  $i [(\boldsymbol{\sigma}(1) + \boldsymbol{\sigma}(2)] \cdot \hat{n} C(\mathbf{q}, \mathbf{Q})] \qquad \hat{n} = \frac{\mathbf{q} \times \mathbf{Q}}{|\mathbf{q} \times \mathbf{Q}|}$ 

 $iC(\mathbf{q}, \mathbf{Q})\langle \varphi_{k'}\chi_n(p')| \left[ (\boldsymbol{\sigma}(1) + \boldsymbol{\sigma}(2)) \cdot \hat{n} \right] |\varphi_k\chi_n(p)\rangle =$ 

- $= iC(\mathbf{q}, \mathbf{Q})\langle \varphi_{k'}\chi_n(p') | \boldsymbol{\sigma}(1) \cdot \hat{n} | \varphi_k \chi_n(p) \rangle + iC(\mathbf{q}, \mathbf{Q}) \langle \varphi_{k'}\chi_n(p') | \boldsymbol{\sigma}(2) \cdot \hat{n} | \varphi_k \chi_n(p) \rangle$
- $= iC(\mathbf{q}, \mathbf{Q}) \langle \varphi_{k'} | \boldsymbol{\sigma}(1) \cdot \hat{n} | \varphi_k \rangle \langle \chi_n(p') | \chi_n(p) \rangle + iC(\mathbf{q}, \mathbf{Q}) \langle \varphi_{k'} | \varphi_k \rangle \langle \chi_n(p') | \boldsymbol{\sigma}(2) \cdot \hat{n} | \chi_n(p) \rangle$

$$= iC(\mathbf{q}, \mathbf{Q}) \boldsymbol{\sigma} \cdot \hat{n} \rho(\mathbf{q}, \mathbf{Q}) + iC(\mathbf{q}, \mathbf{Q}) \langle \chi_n(p') | \boldsymbol{\sigma}(2) \cdot \hat{n} | \chi_n(p) \rangle$$

$$(5)$$
Regular spin-orbit Only zero for closed shell nuclei

# NN amplitude $f_{NN}(k'k;E) \approx \langle k' | t_{NN}(E) | k \rangle$

6 invariant amplitudes off-shell (5 on-shell)

Juvariant Ju	Amplitude Az
41	$A(q, \mathbf{x}, \mathbf{\bar{q}}, \mathbf{\bar{x}})$
$(\overline{\sigma}_A + \overline{\sigma}_2) \cdot \overline{W}$	$C(q, \mathcal{X}, \overline{q}, \overline{\mathcal{X}})$
پندهنی ته کسامس می دو دو دو در د	લે છે. છે. છે. હે હે છે. છે. છે. છે. છે. છે.
GA.W GZ.W	$B(q, \mathcal{X}, \vec{q}, \vec{\mathcal{X}}) = M()$
Contribute $\vec{\sigma}_1 \cdot \hat{q}  \vec{\sigma}_2 \cdot \hat{q}$	$E(q, x, \overline{q}, \overline{x})$ or $g()$
$\nabla_{\mathbf{x}} \cdot \hat{\mathbf{x}} = \nabla_{\mathbf{x}} \cdot \hat{\mathbf{x}}$	F (q, X, q, X) H () (Wolfenstein) (Noshi zaki)
$(\overline{v}_1, \hat{q}, \overline{v}_2, \hat{\chi} + \overline{v}_1, \hat{\chi}, \overline{v}_2, \hat{q})$ $\overline{q}_1, \overline{\chi}$	D (q, x, \$ \$ ) = 0 m-shell

![](_page_30_Figure_0.jpeg)

## Status

- Cluster ansatz implemented into Watson optical potential for <sup>6</sup>He
  - Correlation visible in  $d\sigma/d\Omega$  at forward angles
  - Good description of <sup>4</sup>He important
  - Cluster ansatz can be implemented for <sup>8</sup>He in similar fashion
- Good global phenomenological optical potential fit to <sup>4</sup>He, <sup>6</sup>Li and <sup>6</sup>He with non-standard surface term
  - Guidance from microscopic calculations needed
- For non-closed shell nuclei all NN Wolfenstein amplitudes contribute to the optical potential

   Calculations with COSMA p-shell neutrons under way
- On CE wishlist:  $d\sigma/d\Omega$  and  $A_y$  at another energy (150 or 200 MeV/nucleon)